STAT601 LAB5

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1

$$p(N,\beta|y) \propto p(y|N,\beta)p(N)p(\beta) = \frac{25^N \beta^y (1-\beta)^{N-y} e^{-25}}{y!(N-y!)}$$

2

$$P(N|y,\beta) = \frac{P(N,y,\beta)}{p(y,\beta)} \propto \frac{p(y|N,\beta)p(N)p(\beta)}{p(y|\beta)} \propto p(y|N,\beta)p(N) = \binom{N}{y} p^y (1-\beta)^{N-y} \frac{25^N}{N!} e^{-25} \propto \frac{(25(1-\beta))^{N-y}}{(N-y)!}$$

Therefore, N-y ~ Poisson$(25(1-\beta))$

$$p(\beta|y,N) \propto p(y|N,\beta)p(\beta) \propto (1-\beta)^{N-y}\beta^y$$

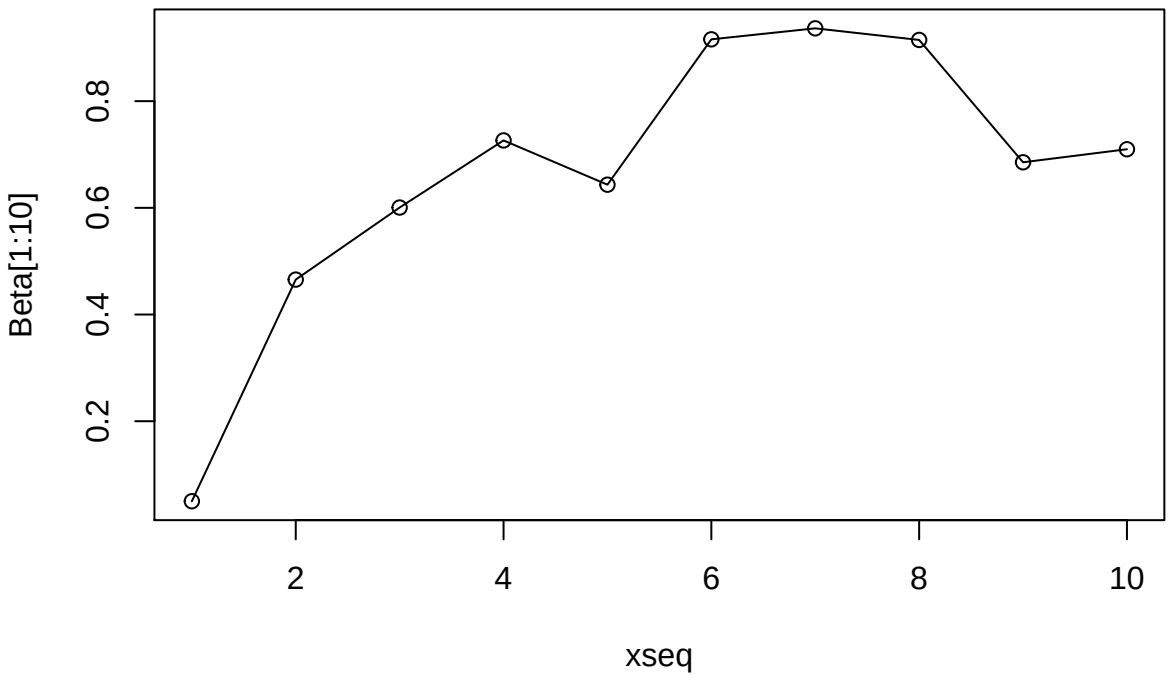
Therefore, β~Beta(y+1,N-y+1)

3

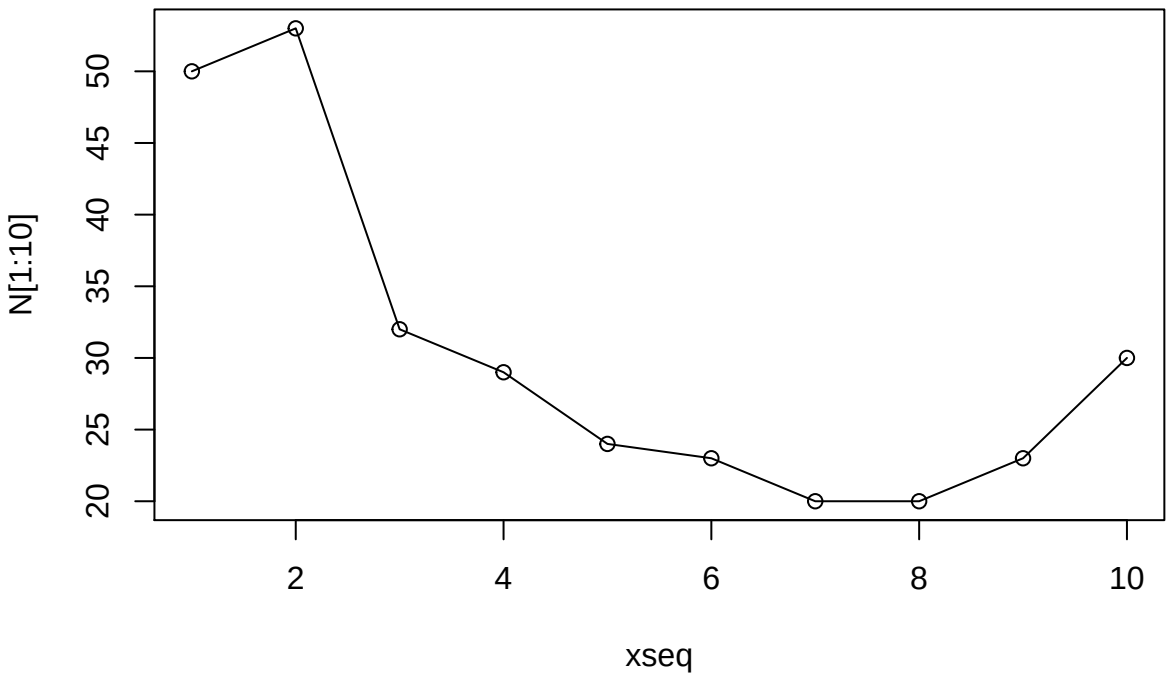
```
Sample <- 10000

N<-rep(0,Sample)
Beta<-rep(0,Sample)
N[1] <- 50
Beta[1] <- 0.05

for (i in 2:Sample){
  N[i] <- rpois(1,25*(1-Beta[i-1])) + 20
  Beta[i] <- rbeta(1,21,N[i]-20+1)
}
```

4

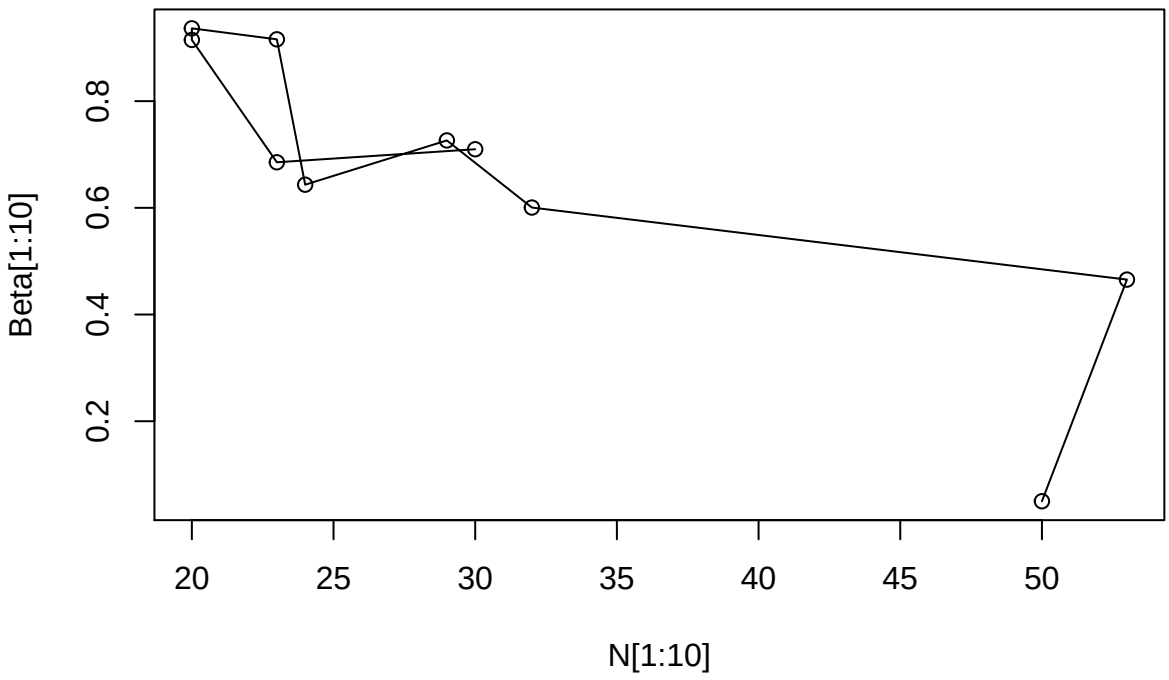
```
xseq <- seq(1,10)
plot(xseq,Beta[1:10])
lines(xseq,Beta[1:10])
```




```
plot(xseq,N[1:10])
lines(xseq,N[1:10])
```




```
#2D traceplot
plot(N[1:10],Beta[1:10])
lines(N[1:10],Beta[1:10])
```



5

```
#central 90% posterior credible interval for beta
quantile(Beta[-c(1:1000)],c(0.05,0.95))
```

```
##        5%       95%
## 0.5549397 0.9707958
```

6

```
#probability that exactly 20 people were polled
sum(N[-c(1:1000)]==20)/(Sample-1000)
```

```
## [1] 0.07733333
```