Homework5

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Exercise 1

Let

$$y = exp(-e^{-(x-c/\beta)})$$

then

$$x = -\beta log(log1/y) + c = F^{-1}(y)$$

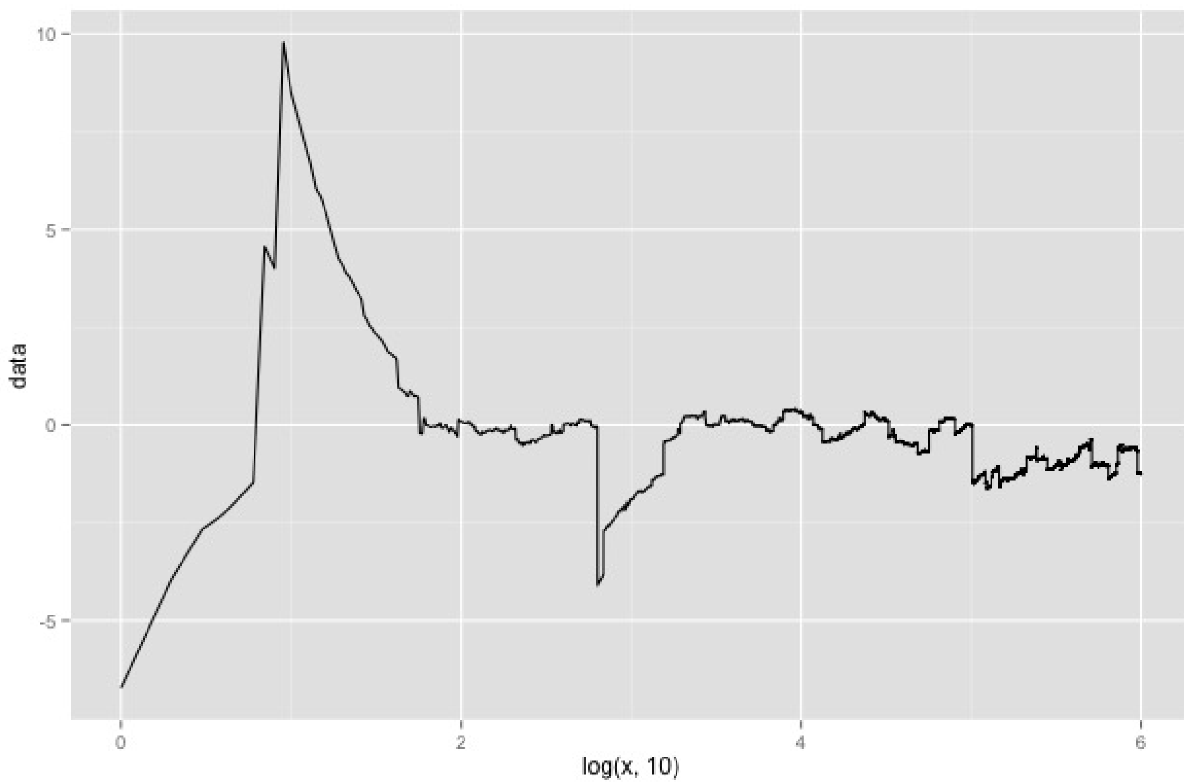
which is the inverse function of cdf. Therefore the random variable that ~ Gumbel(c, β) can be generate from $F^{-1}(y_i)$ where y_i ~ Uniform(0,1)

Exercise2

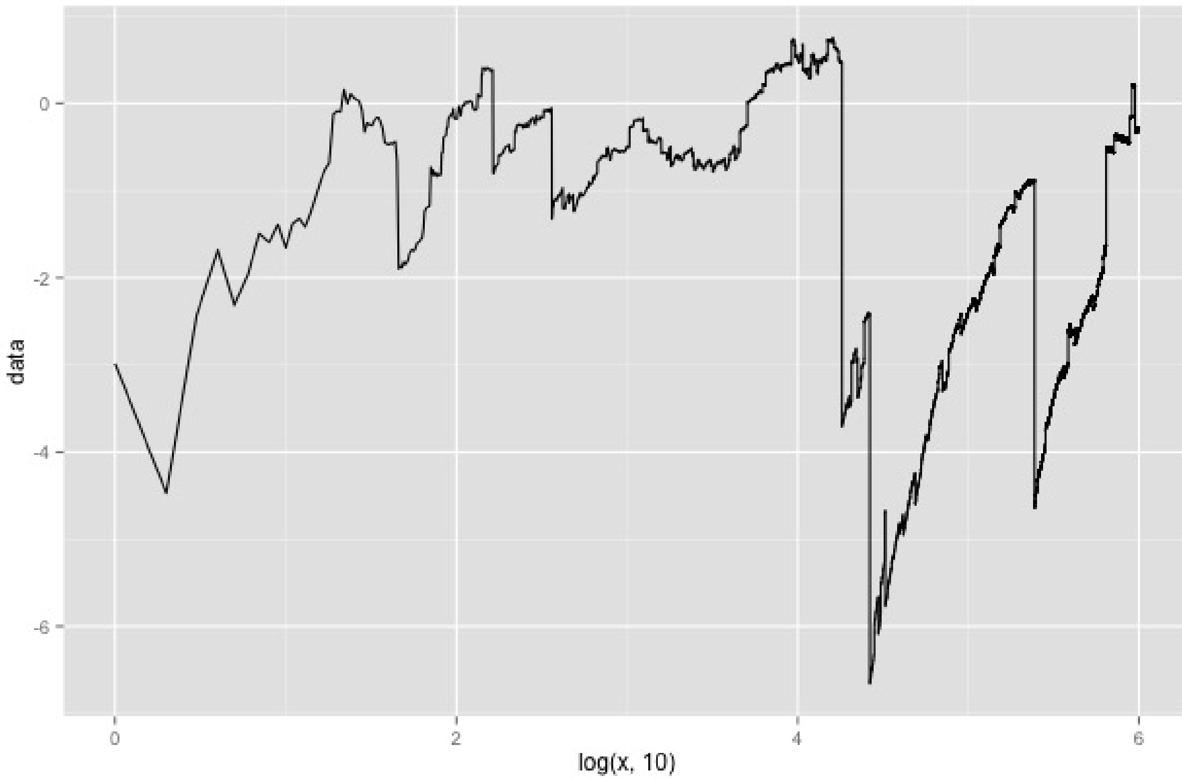
```
N <- 10^6
data <- rep(0,N)
data [1] <- rcauchy(1,0,1)
for (i in 2:N){
    data[i] <- (data[i-1]*(i-1) + rcauchy(1,0,1))/i
}

x <- seq(1:N)
df <- data.frame(x,data)

p <- ggplot(df, aes(log(x,10), data))
p + geom_line()
```

Therefore, from 4 examples we can see Cauchy(0,1) cannot converge at $N = 10^6$

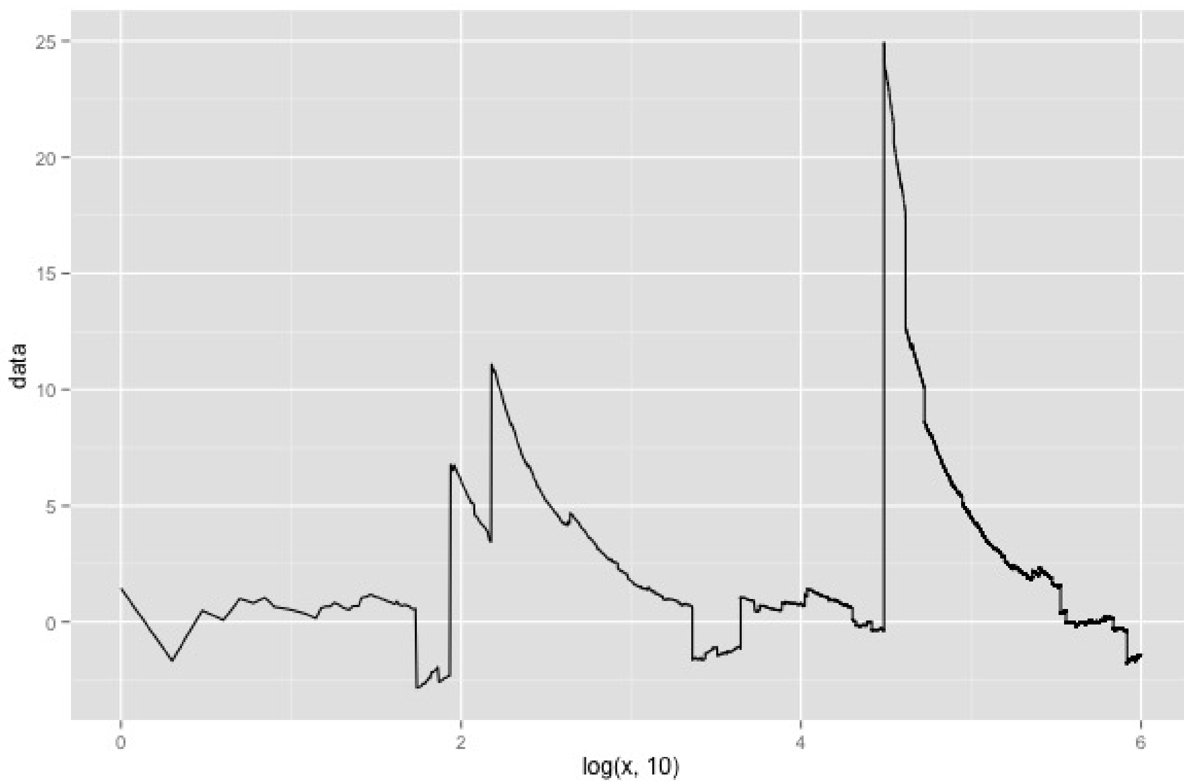
Sample 1



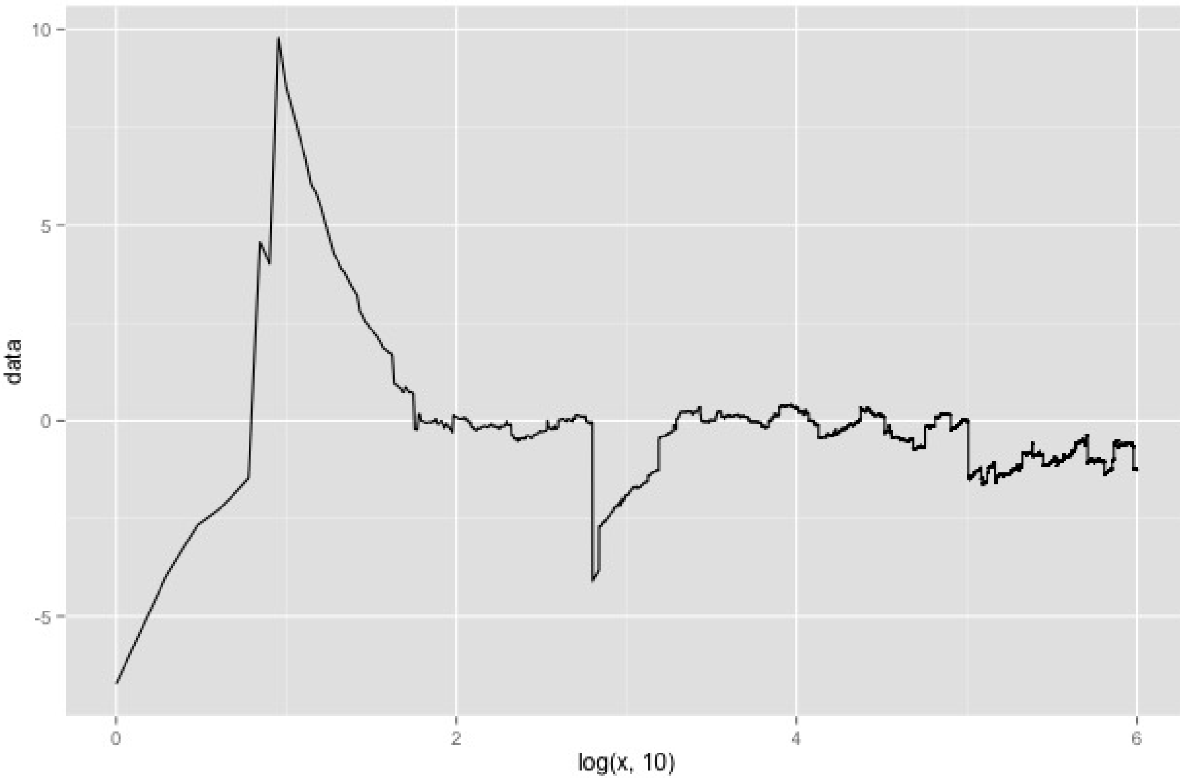
Sample 2



Sample 3



Sample 4



Exercise 3

(a)

$$p(\theta|x_{1:n}) = \frac{p(\theta)p(x_{1:n}|\theta)}{p(x_{1:n})}$$

$$\frac{p(\theta|x_{1:n})}{p(x_{1:n}|\theta)} = \frac{p(\theta)}{p(x_{1:n})}$$

Take derivative of both side Then we have

$$\int \frac{p(\theta|x_{1:n})}{p(x_{1:n}|\theta)} = frac1p(x_{1:n})$$

Therefore

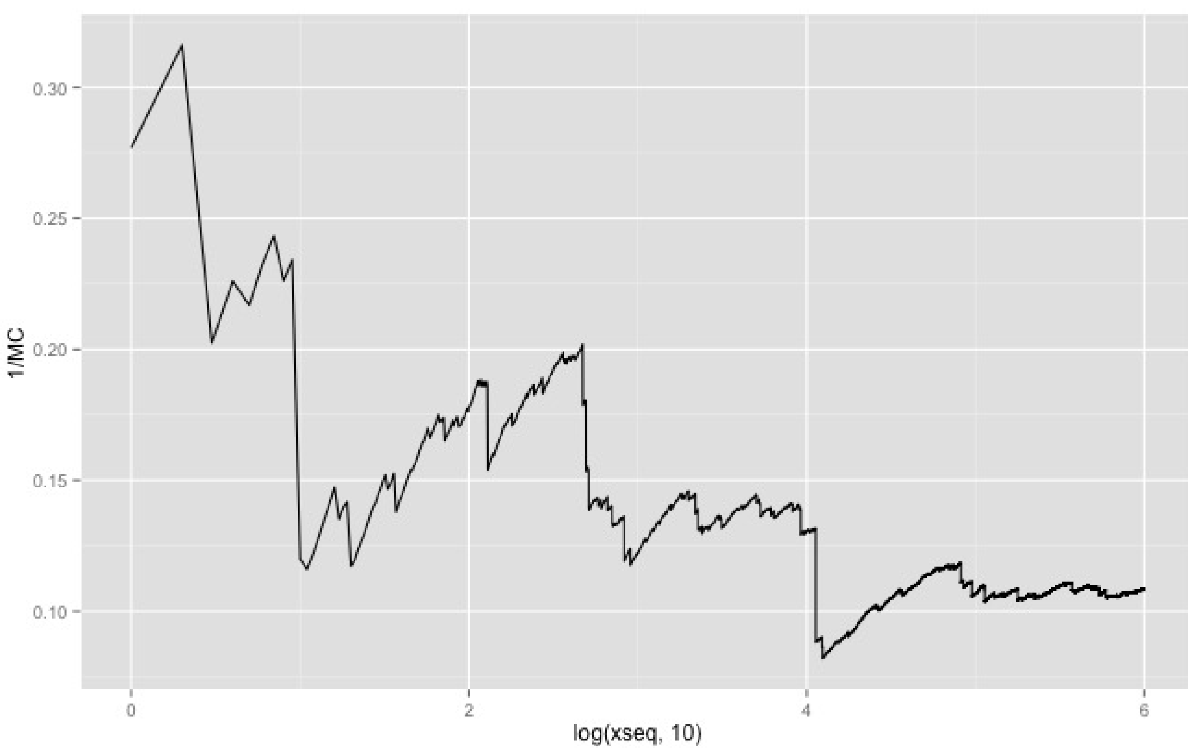
$$p(x_{1:n}) = \frac{1}{\frac{1}{N}\sum 1/p(x_{1:n}|\theta)}$$

(b)

```
x <- 2

N = 10^6
lambda <- 1
lambda0 <- 1/(10^2)
MC<- rep(0,N)


M <- (lambda0*0+lambda*2) / (lambda0+1)
L <- lambda0+1
theta <- rnorm(N,M ,sqrt(L^-1))

  MC<- rep(0,N)
  for (i in 1:N){
    MC[i] <- 1/dnorm(x,theta[i],sqrt(lambda^-1))
  }
```

Results:

```
> 1/mean(MC)
[1] 0.07345119
> dnorm(2,0,sqrt(lambda^-1 + lambda0^-1))
[1] 0.03891791
```



From the graph we can observe that HMA method is approaching the true value but still not converge when N = 10ˆ6. Therefore it is a slow process.

(c)

```
x <- 2

N = 10^6
lambda <- 1
lambda0 <- 1/(100^2)
MC<- rep(0,N)


M <- (lambda0*0+lambda*2) / (lambda0+1)
L <- lambda0+1
theta <- rnorm(N,M ,sqrt(L^-1))

  MC<- rep(0,N)
  for (i in 1:N){
    MC[i] <- 1/dnorm(x,theta[i],sqrt(lambda^-1))
  }
```

Result

```
> 1/mean(MC)
[1] 0.095079
> dnorm(2,0,sqrt(lambda^-1 + lambda0^-1))
[1] 0.003988426
```

Therefore still not converge to the true value

Exercise 5

$Z = X_k$ where k $= \min\{k, x_k \in A\}$

$$p(Z \in S) = \sum p(x_k \in S, x_1 \notin A ... x_{k-1} \notin A)$$

$$= \sum p(x_k \in S) p(x_1 \notin A) ... p(x_{k-1} \notin A)$$

$$= \sum p(x \in S)(1 - p(x \in A)^{k-1})$$

$$= \frac{p(x \in S)}{p(x \in A)}$$

$$= p(x \in S | x \in A)$$