lab9

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Question 1

let $\frac{1}{\tau} = \sigma^2$

$$x|\mu, \tau \sim LogNormal(\mu, \tau)$$

$$\mu \sim N(\mu_0, \tau_0)$$

$$\tau\ simGamma(\alpha, \beta)$$

$$p(x|\mu, \tau) = \frac{\sqrt{\tau}}{\sqrt{2\pi}x} \exp(-\frac{1}{2}\tau(\ln x - \mu)^2)$$

The likelihood will be

$$L(x_{1:n}|\mu, \tau) \propto \tau^{n/2} \exp(-1/2\tau \sum_{i=1}^{n}(\ln x_i - \mu)^2)$$

in the form of sigma it will be

$$L(x_{1:n}|\mu, \sigma) \propto \sigma^n \exp(-1/2\frac{1}{\sigma^2} \sum_{i=1}^{n}(\ln x_i - \mu)^2)$$

Question 2

Gibbs Sampler for mu

$$\mu|x, \tau \sim N(\mu|\mu_0, \tau_0)L(x_{1:n}|\mu, \tau) \propto exp(-0.5((n\tau + tau_0)\mu^2 - 2(\tau \sum_{i=1}^{n} \ln x_i + \tau_0\mu_0)\mu)) = N(\mu', \tau')$$

where

$$\mu' = \frac{\tau * \sum \ln x_i + \tau_0\mu_0}{n\tau + \tau_0}$$

$$\tau' = n\tau + \tau_0$$

Gibbs Sampler for sigma

let $\frac{1}{\tau} = \sigma^2$

$$\tau|\mu, x \sim Gamma(\tau|\alpha, \beta)L(x_{1:n}|\mu, \sigma) \propto \tau^{\alpha-1+n/2}exp(-\tau(\beta + \frac{\sum(\ln x_i - \mu)^2}{2})) = Gamma(\alpha', \beta')$$

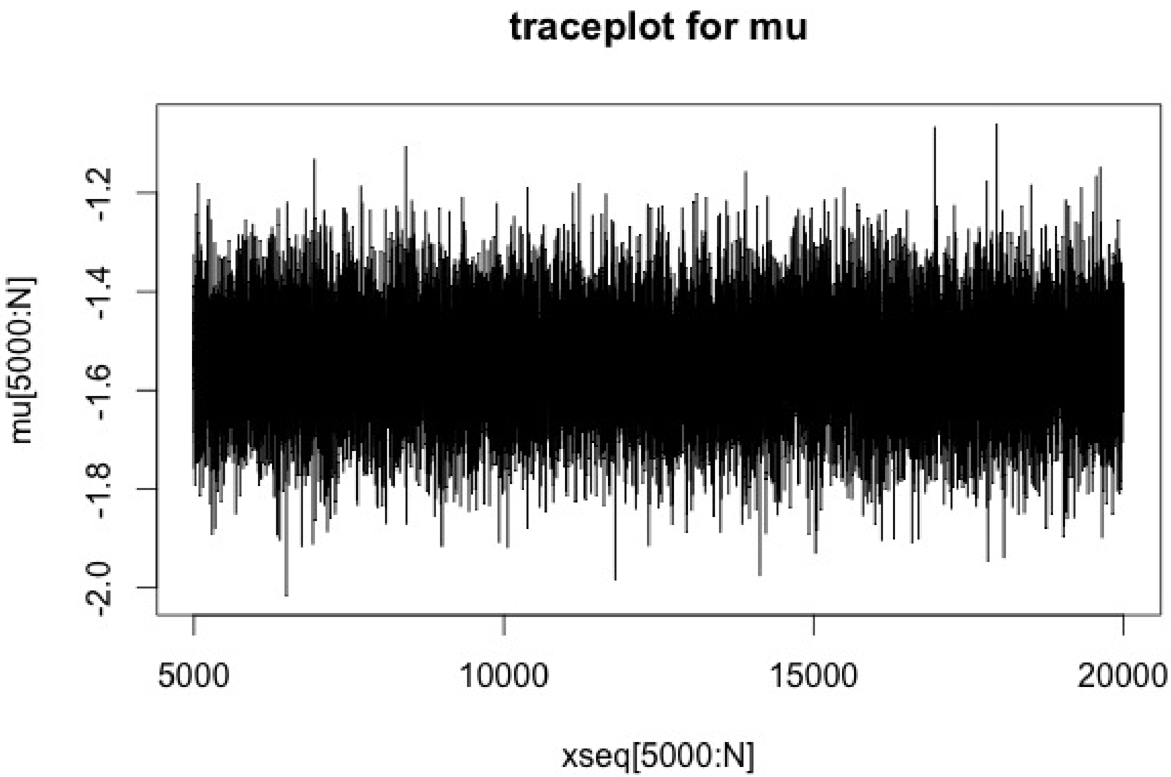
where

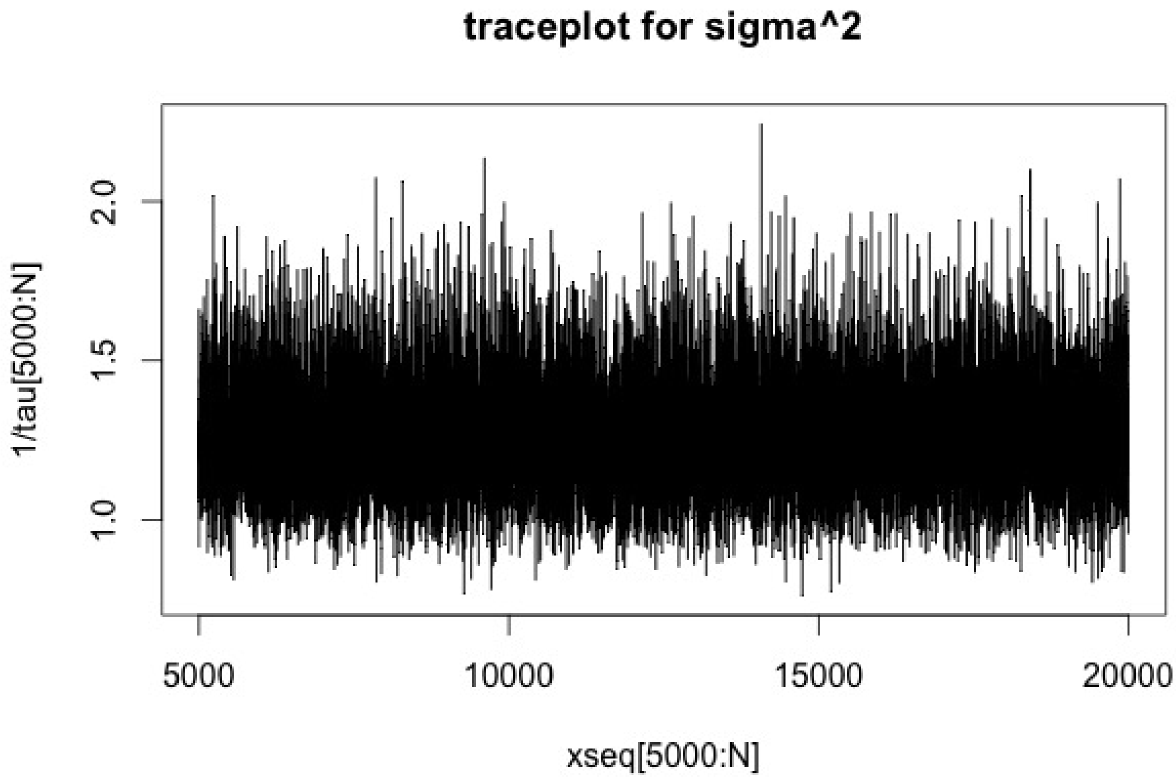
$$\alpha' = \alpha + n/2$$

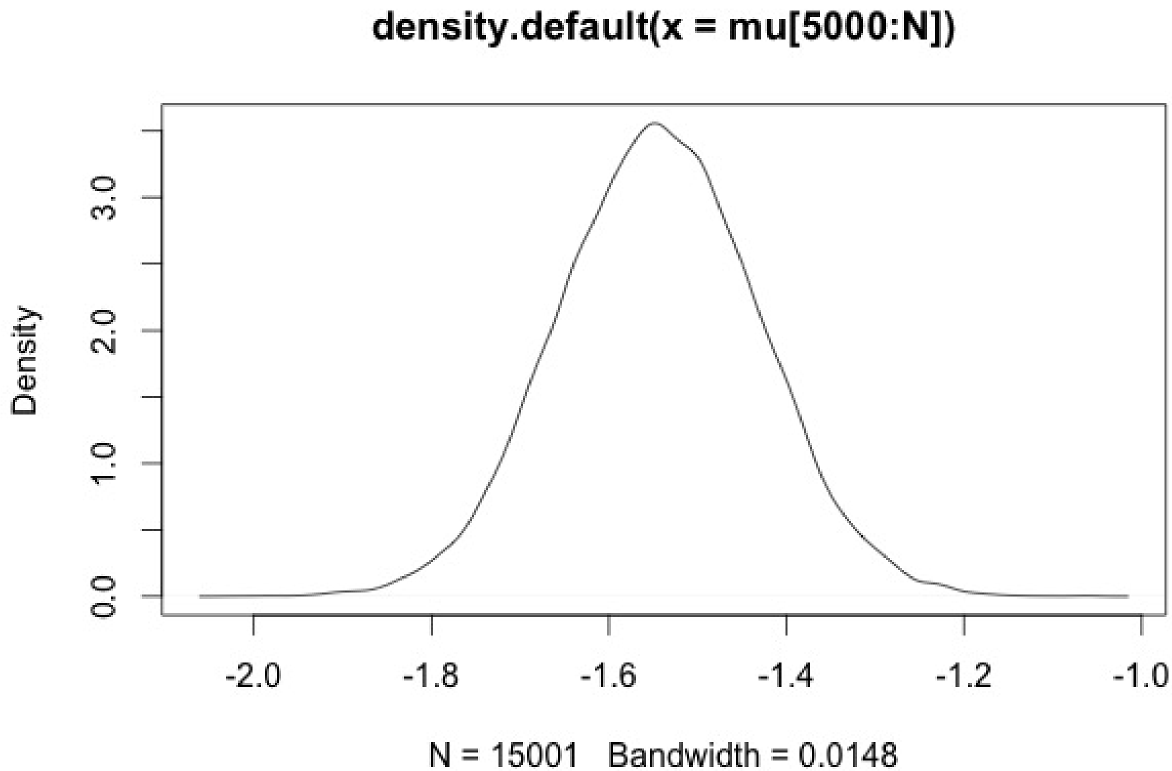
$$\beta' = \beta + \frac{\sum(\ln x_i - \mu)^2}{2}$$

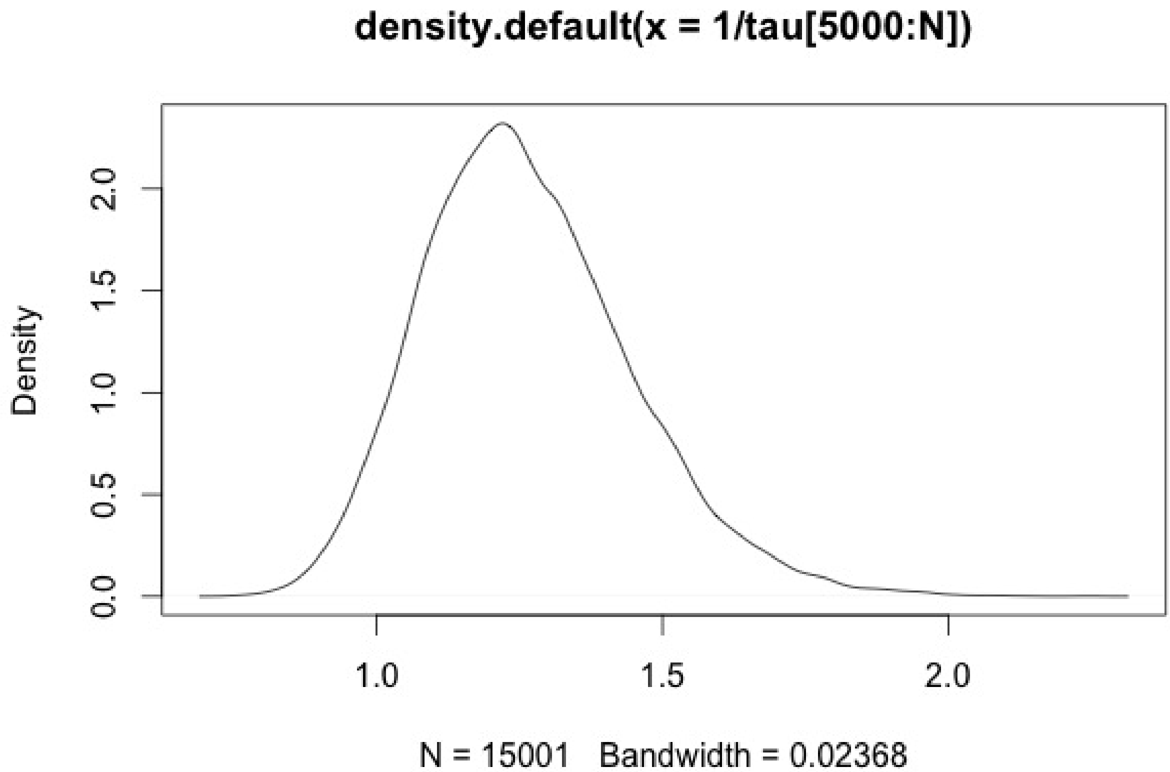
```
data <-read.csv("/Users/Dino/Downloads/data.txt")
N <- 20000
n <- nrow(data)

a<- b<- 0.1


mu<-rep(0,N)
tau<-rep(0,N)

mu0<- 0
t0<-1

for (i in 2:N){
  ts <- n * tau[i-1] + t0
  mus <- (tau[i-1] * sum(log(data)) + t0 * mu0) / ts

  mu[i] <- rnorm(1,mus,1/sqrt(ts))

  tau[i] <- rgamma(1,a + n/2,b + sum((log(data)-mu[i])^2)/2)


}

xseq <- seq(1,N)
```

```
plot(xseq[5000:N],mu[5000:N],type = 'l',main = "traceplot for mu")
plot(xseq[5000:N],1/tau[5000:N], type='l',main = "traceplot for sigma^2")
```


traceplot for mu
mu[5000:N]
-1.2
-1.4
-1.6
-1.8
-2.0
5000
10000
15000
20000
xseq[5000:N]


traceplot for sigma^2
1/tau[5000:N]
2.0
1.5
1.0
5000
10000
15000
20000
xseq[5000:N]


density.default(x = mu[5000:N])
Density
0.0
1.0
2.0
3.0
-2.0
-1.8
-1.6
-1.4
-1.2
-1.0
N = 15001 Bandwidth = 0.0148



Question 3

```
xmean <-exp(mu[5000:N]+1/(2*tau[5000:N]))
##mean
mean(xmean)
```

```
## [1] 0.4066496
```

```
##confidential interval
quantile(xmean,c(0.025,0.975))
```

```
##      2.5%     97.5%
## 0.3093079 0.5499059
```

```
xvar <- (exp(1/tau[5000:N])-1)*exp(2*mu[5000:N]+1/tau[5000:N])
##variance
mean(xvar)
```

```
## [1] 0.4644035
```

```r
##confidential interval
quantile(xvar,c(0.025,0.975))
```

```
##      2.5%     97.5% 
## 0.1686864 1.1825442
```