Homework 5 for "Convex Optimization"

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November 11, 2019

1 Algorithms for ℓ_1 minimization

Consider the ℓ_1-regularized problem

$$\min_x \quad \frac{1}{2}\|Ax-b\|_2^2+\mu\|x\|_1, \tag{1.1}$$

where $A\in\mathbb{R}^{m\times n}$, $b\in\mathbb{R}^m$ and $\mu>0$ are given. Test matrices:

```
seed = 97006855;
ss = RandStream('mt19937ar','Seed',seed);
RandStream.setGlobalStream(ss);
n = 1024;
m = 512;
A = randn(m,n);
u = sprandn(n,1,0.1);
b = A*u;
mu = 1e-3;
```

See `http://bicmr.pku.edu.cn/~wenzw/courses/Test_l1_regularized_problems.m`

1. Solve (1.1) using CVX by calling different solvers mosek and gurobi.
2. First write down an equivalent model of (1.1) which can be solved by calling mosek and gurobi directly, then implement the codes.
3. First write down, then implement the following algorithms in Matlab (or Python):
   (a) Projection gradient method by reformulating the primal problem as a quadratic program with box constraints
   (b) Subgradient method for the primal problem
   (c) Gradient method for the smoothed primal problem
   (d) Fast gradient method for the smoothed primal problem
   (e) Proximal gradient method for the primal problem
   (f) Fast proximal gradient method for the primal problem

(g) Augmented Lagrangian method for the dual problem

(h) Alternating direction method of multipliers for the dual problem

(i) Alternating direction method of multipliers with linearization for the primal problem

(j) Anderson acceleration for the primal or dual problem

(k) Proximal inertial method for the primal problem

(l) Block coordinate method for the primal problem

4. Extra Credit: Write down and implement the deterministic version of AdaGrad, Adam, RMSProp, Momentum for solving (1.1). A description of these algorithms can be found at

   `http://www.deeplearningbook.org/contents/optimization.html`

5. Requirement:

   (a) The interface of each method should be written in the following format

   ```
   [x, iter, out] = method_name(x0, A, b, mu, opts);
   ```

   Here, x0 is a given input initial solution, A, b and mu are given data, opts is a struct which stores the options of the algorithm, iter is the number of iterations when the termination condition of the algorithm is satisfied, out is a struct which saves all other output information.

   (b) Compare the efficiency (cpu time) and accuracy (checking optimality condition) in the format as `http://bicmr.pku.edu.cn/~wenzw/courses/Test_l1_regularized_problems.m`

   (c) Prepare a report including

   - detailed answers to each question
   - numerical results and their iterpretation

   (d) Pack the report and all of your codes in one file named as "l1-hw-StudentID.zip" and send it to TA: pkuopt@163.com

   (e) If you get significant help from others on one routine, write down the source of references at the beginning of this routine.

   (f) Due date

   - Nov. 18: 1, 2, 3 (a), (b)
   - Nov. 25: 3 (c), (d), (e), (f)
   - Dec. 9: 3 (g), (h), (i)
   - Dec. 17: 4 (optional)