Vaccine Implementation with Social Networks

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1 Introduction

For problem descriptions, see:
https://www.research.ibm.com/haifa/ponderthis/challenges/February2021.html.

2 Model Formulations

2.1 Set

K: indexed by k, set of direction in the order of {N, E, S, W}

2.2 Parameters

m: number of rows
n: number of columns
D_{ijk}: $= 1$, if the vaccination of cell $[i, j], i \in \{1..m\}, j \in \{1..n\}$ is made before the vaccination in the neighbour in the drection of $k \in K$; $= 0$, otherwise
T: maximum time stages

2.3 Decision variables

X_{ijt}: $= 1$, if cell $[i, j], i \in \{0..m + 1\}, j \in \{0..n + 1\}$ has positive vaccination in time $t \in \{0..T\} = 0$, otherwise

2.4 Model

Objective:

$$Minimize \sum_{i=1..m} \sum_{j=1..n} X_{ij,0}$$

Constraints:
Dummy boundary at rows 0 and $m+1$ and cols 0 and $n+1$:

$$X_{ijt} = 1, \quad (i = 0 \; or \; i = m+1 \; or \; j = 0 \; or \; j = n+1) \; and \; \forall t \in \{0..T\} \quad (1)$$

$X_{ij,t+1}$ takes vaccine only if trusted neighbours take vaccine in time t:

$$D_{ij,N} X_{ij,t+1} <= X_{i+1,j,t}, \quad \forall i \in \{1..m\}, j \in \{1..n\}, t \in \{0..T-1\} \quad (2)$$

$$D_{ij,E} X_{ij,t+1} <= X_{i,j+1,t}, \quad \forall i \in \{1..m\}, j \in \{1..n\}, t \in \{0..T-1\} \quad (3)$$

$$D_{ij,S} X_{ij,t+1} <= X_{i-1,j,t}, \quad \forall i \in \{1..m\}, j \in \{1..n\}, t \in \{0..T-1\} \quad (4)$$

$$D_{ij,W} X_{ij,t+1} <= X_{i,j-1,t}, \quad \forall i \in \{1..m\}, j \in \{1..n\}, t \in \{0..T-1\} \quad (5)$$

$$X_{i,j,t+1} >= D_{ij,N} X_{i+1,j,t} + D_{ij,E} X_{i,j+1,t} + D_{ij,S} X_{i-1,j,t} + D_{ij,W} X_{i,j-1,t} - \sum_k D_{ijk} + 1,$$

$$\forall i \in \{1..m\}, j \in \{1..n\}, t \in \{0..T-1\} \quad (6)$$

Requirement for vaccination for all cells:

$$X_{ij,T} = 1, \quad \forall i \in \{1..m\}, j \in \{1..n\}, \quad (7)$$

Maintaining vaccine condition once vaccined:

$$X_{i,j,t+1} >= X_{ijt}, \quad \forall i \in \{1..m\}, j \in \{1..n\}, t \in \{0..T-1\} \quad (8)$$