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Probabilities of Allen interval relations

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My Webpage

Probabilities of interval relations

The interval algebra of Allen 1983 is one of the most commonly used approaches to time and events in natural language processing (Jurafsky and Martin 2023, chapter 22). The aim of this project is to introduce probabilities on relations between intervals, based on transitions between states, conceived at different granularities and recording varying extents of history.

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Presentation

Relational Graph

Python Notebook

Dissertation

https://xiejiax.github.io/My-Blog/

Overview

- Background
- Research Content
- Result
- Demo of Project
- Conclusions & Future Work

Background

The study of **temporal relationships** is essential across various fields, including *computer science, artificial intelligence, and natural language processing*.

To better model and understand the temporal relationships between events, **Allen** introduced an **interval algebra theory** to describe **13** possible relations between two time intervals.

These relations provide a systematic framework for representing and reasoning about temporal information.

Background

1.Equal (Eq):

```
                |-----A-----|
                |-----B-----|
```

2.Before (B):

```
     |-----A-----|
                     |-----B-----|
```

3.After (Bi) (Inverse of Before):

```
          |-----B-----|
                          |-----A-----|
```

4.Meets (M):

```
     |-----A-----|
                |-----B-----|
```

5.Met by (Mi) (Inverse of Meets):

```
          |-----B-----|
                     |-----A-----|
```

6.Overlaps (O):

```
          |-----A-----|
     |-----B-----|
```

7.Overlapped by (Oi) (Inverse of Overlaps):

```
               |-----B-----|
          |-----A-----|
```

Background

8.Starts (S):

```
|-----A-----|
|-----------B-----------|
```

9.Started by (Si) (Inverse of Starts):

```
|-----------A-----------|
|-----B-----|
```

10.Finishes (F):

```
              |-----A-----|
|------------B------------|
```

11.Finished by (Fi) (Inverse of Finishes):

```
|------------A------------|
              |-----B-----|
```

12.During (D):

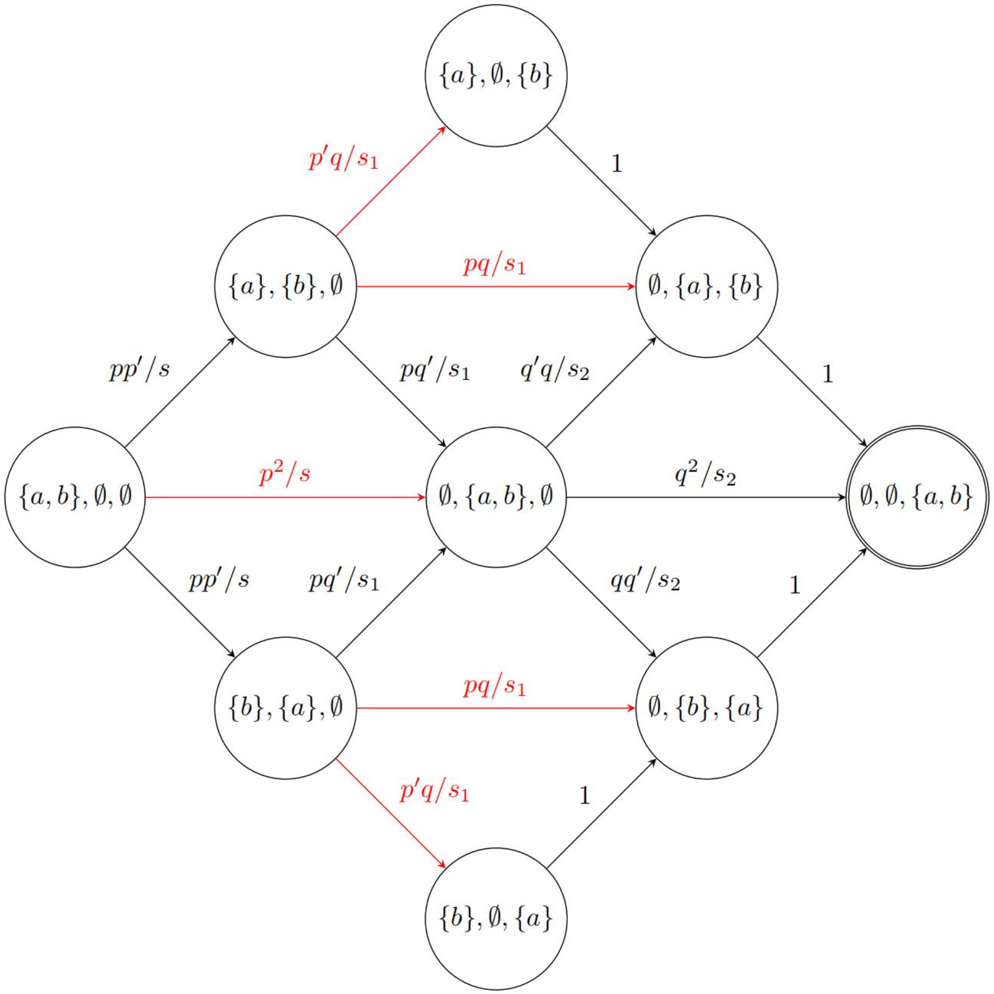
```
       |-----A-----|
|-----------B-----------|
```

13.Contains (Di) (Inverse of During):

```
|-----------A-----------|
       |-----B-----|
```

Background

Motivation



The probability of birth is ***p***, and the probability of death is ***q***

Where p':=1-p, q':=1-p and

$s := 2pp' + p^2$
$s_1 := pq' + p'q + pq$
$s_2 := 2qq' + q^2$

Background

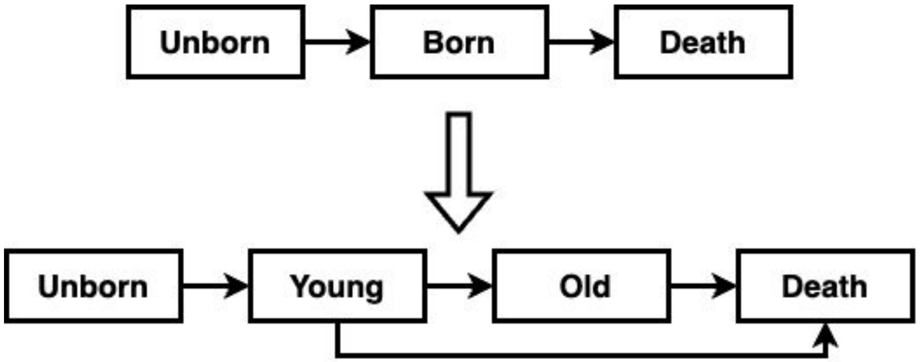
Motivation

Allen relation	probabilities	p = q	p, q = 0.5	p = q → 0
eq	p^2q^2/ss_2	p^4/s^2	1/9	0
b, bi	pp'^2q/ss_1	$p^2p'2/s^2$	1/9	1/4
m, mi	$p^2p'q/ss_1$	p^3p'/s^2	1/9	0
s, si	p^2qq'/ss_2	p^3p'/s^2	1/9	0
o, oi	$p^2p'qq'^2/ss_1s_2$	$p^3p'3/s^3$	1/27	1/8
d, di	$p^2p'qq'^2/ss_1s_2$	$p^3p'3/s^3$	1/27	1/8
f, fi	$p^2p'q^2q'/ss_1s_2$	$p^4p'^2/s^3$	1/27	0

Background

Motivation

Allen relation	probabilities	p = q	p, q = 0.5	p = q → 0
o, oi	$p^2p'qq'^2/ss_1s_2$	$p^3p'^3/s^3$	1/27	1/8
d, di	$p^2p'qq'^2/ss_1s_2$	$p^3p'^3/s^3$	1/27	1/8



P_old > P_young

P_overlap > P_during

Research Content

1.Definition of Survival States

This study aims to use **Finite Automaton** to simulate the **13** types of **Allen interval probabilities** in the context of life and death processes, including **Age** and **Gender** impacts on different survival states.

We assume that the probability of **Overlap** should be greater than **During** and that **females** will have lower death rates than **males**.

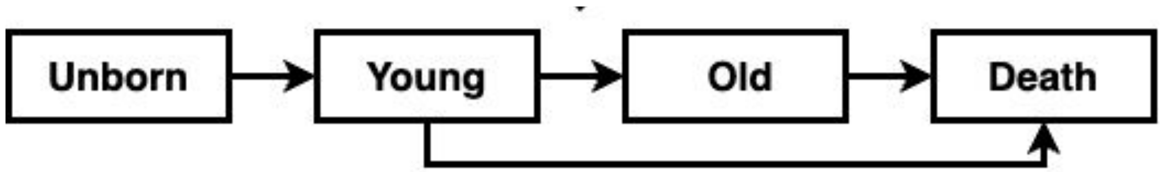
Research Content

1.Transition Logic

To model the life and death process, I defined four survival states for both **males** and **females**:

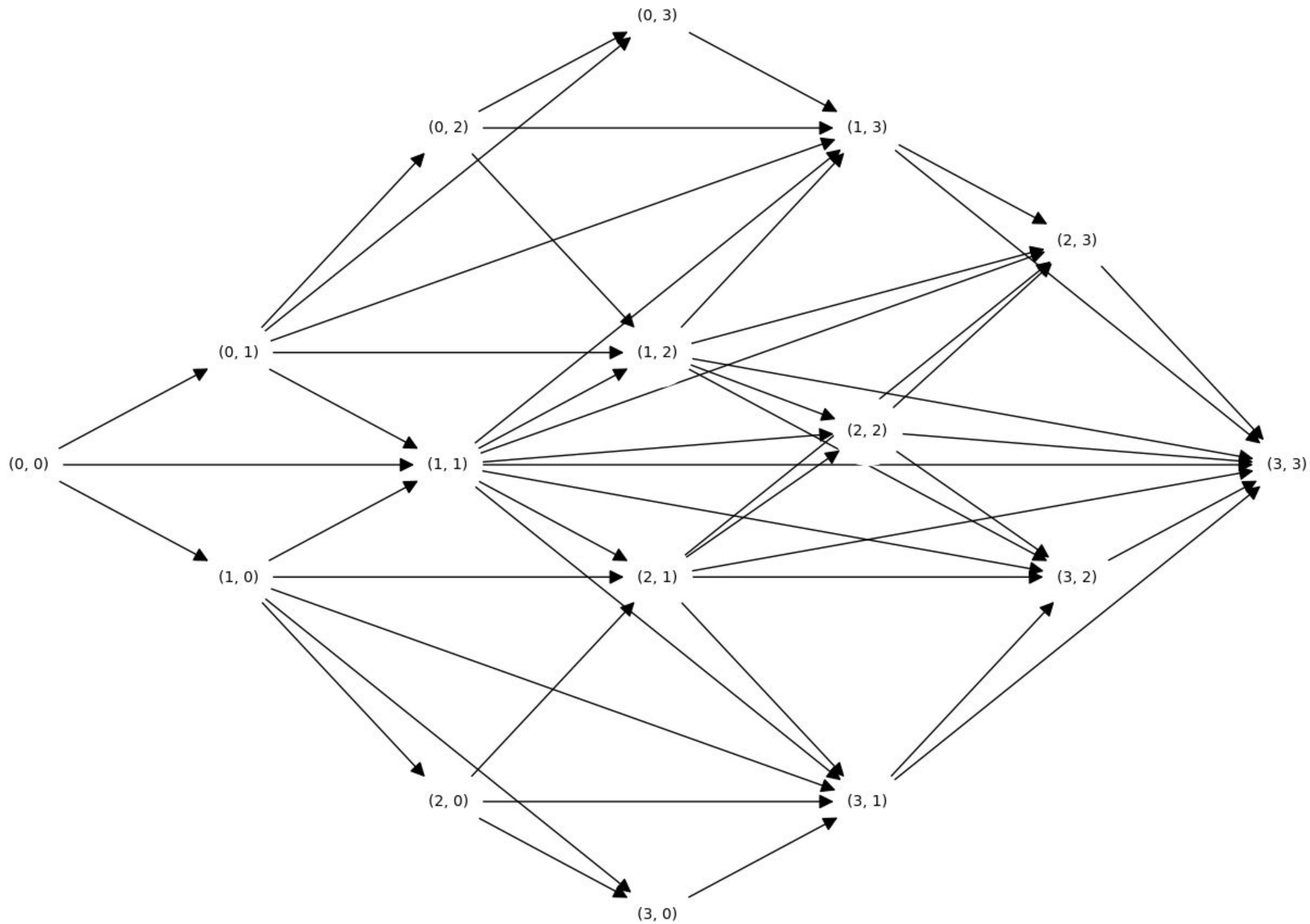
```
["unborn=0","young=1","old=2","dead=3"]
```

Each state is assigned a number and rules are set such that unborn can only transition to young.



Research Content

2. Directed Graph



There are 126 paths

Research Content

3. Interval relationship path classification

Next, the 126 paths are classified according to the rules of the 13 Allen interval relationships.

```
1. Equal (Eq):
Number of eq_paths: 6
[[0, 0], [1, 1], [2, 1], [2, 2], [3, 3]]
[[0, 0], [1, 1], [2, 1], [3, 3]]
[[0, 0], [1, 1], [1, 2], [2, 2], [3, 3]]
[[0, 0], [1, 1], [1, 2], [3, 3]]
[[0, 0], [1, 1], [2, 2], [3, 3]]
[[0, 0], [1, 1], [3, 3]]

2. Before (B):
Number of before_paths: 4
[[0, 0], [1, 0], [2, 0], [3, 0], [3, 1], [3, 2], [3, 3]]
[[0, 0], [1, 0], [2, 0], [3, 0], [3, 1], [3, 3]]
[[0, 0], [1, 0], [3, 0], [3, 1], [3, 2], [3, 3]]
[[0, 0], [1, 0], [3, 0], [3, 1], [3, 3]]

3. After (Bi) (Inverse of Before):
Number of after_paths: 4
[[0, 0], [0, 1], [0, 2], [0, 3], [1, 3], [2, 3], [3, 3]]
[[0, 0], [0, 1], [0, 2], [0, 3], [1, 3], [3, 3]]
[[0, 0], [0, 1], [0, 3], [1, 3], [2, 3], [3, 3]]
[[0, 0], [0, 1], [0, 3], [1, 3], [3, 3]]

4. Meets (M):
Number of meets_paths: 4
[[0, 0], [1, 0], [2, 0], [3, 1], [3, 2], [3, 3]]
[[0, 0], [1, 0], [2, 0], [3, 1], [3, 3]]
[[0, 0], [1, 0], [3, 1], [3, 2], [3, 3]]
[[0, 0], [1, 0], [3, 1], [3, 3]]

5. Met by (Mi) (Inverse of Meets):
Number of met_by_paths: 4
[[0, 0], [0, 1], [0, 2], [1, 3], [2, 3], [3, 3]]
[[0, 0], [0, 1], [0, 2], [1, 3], [3, 3]]
[[0, 0], [0, 1], [1, 3], [2, 3], [3, 3]]
[[0, 0], [0, 1], [1, 3], [3, 3]]

6. Overlap (O):
Number of overlaps_paths: 18
[[0, 0], [0, 1], [1, 1], [2, 1], [2, 2], [2, 3], [3, 3]]
[[0, 0], [0, 1], [1, 1], [2, 1], [2, 3], [3, 3]]
[[0, 0], [0, 1], [1, 1], [1, 2], [2, 2], [2, 3], [3, 3]]
[[0, 0], [0, 1], [1, 1], [1, 2], [1, 3], [2, 3], [3, 3]]
[[0, 0], [0, 1], [1, 1], [1, 2], [1, 3], [3, 3]]
[[0, 0], [0, 1], [1, 1], [1, 2], [2, 3], [3, 3]]
[[0, 0], [0, 1], [1, 1], [1, 3], [2, 3], [3, 3]]
[[0, 0], [0, 1], [1, 1], [1, 3], [3, 3]]
[[0, 0], [0, 1], [1, 1], [2, 2], [2, 3], [3, 3]]
[[0, 0], [0, 1], [1, 1], [2, 3], [3, 3]]
[[0, 0], [0, 1], [0, 2], [1, 2], [2, 2], [2, 3], [3, 3]]
[[0, 0], [0, 1], [0, 2], [1, 2], [1, 3], [2, 3], [3, 3]]
[[0, 0], [0, 1], [0, 2], [1, 2], [1, 3], [3, 3]]
[[0, 0], [0, 1], [0, 2], [1, 2], [2, 3], [3, 3]]
[[0, 0], [0, 1], [1, 2], [2, 2], [2, 3], [3, 3]]
[[0, 0], [0, 1], [1, 2], [1, 3], [2, 3], [3, 3]]
[[0, 0], [0, 1], [1, 2], [1, 3], [3, 3]]
[[0, 0], [0, 1], [1, 2], [2, 3], [3, 3]]

7. Overlapped by (Oi) (Inverse of Overlap):
Number of overlapped_by_paths: 18
[[0, 0], [1, 0], [2, 0], [2, 1], [3, 1], [3, 2], [3, 3]]
[[0, 0], [1, 0], [2, 0], [2, 1], [3, 1], [3, 3]]
[[0, 0], [1, 0], [2, 0], [2, 1], [2, 2], [3, 2], [3, 3]]
[[0, 0], [1, 0], [2, 0], [2, 1], [3, 2], [3, 3]]
[[0, 0], [1, 0], [1, 1], [2, 1], [3, 1], [3, 2], [3, 3]]
[[0, 0], [1, 0], [1, 1], [2, 1], [3, 1], [3, 3]]
[[0, 0], [1, 0], [1, 1], [2, 1], [2, 2], [3, 2], [3, 3]]
[[0, 0], [1, 0], [1, 1], [2, 1], [3, 2], [3, 3]]
[[0, 0], [1, 0], [1, 1], [3, 1], [3, 2], [3, 3]]
[[0, 0], [1, 0], [1, 1], [3, 1], [3, 3]]
[[0, 0], [1, 0], [1, 1], [1, 2], [2, 2], [3, 2], [3, 3]]
[[0, 0], [1, 0], [1, 1], [1, 2], [3, 2], [3, 3]]
[[0, 0], [1, 0], [1, 1], [2, 2], [3, 2], [3, 3]]
[[0, 0], [1, 0], [1, 1], [3, 2], [3, 3]]
[[0, 0], [1, 0], [2, 1], [3, 1], [3, 2], [3, 3]]
[[0, 0], [1, 0], [2, 1], [3, 1], [3, 3]]
[[0, 0], [1, 0], [2, 1], [2, 2], [3, 2], [3, 3]]
[[0, 0], [1, 0], [2, 1], [3, 2], [3, 3]]
```

Research Content

3. Pathways from Start to End

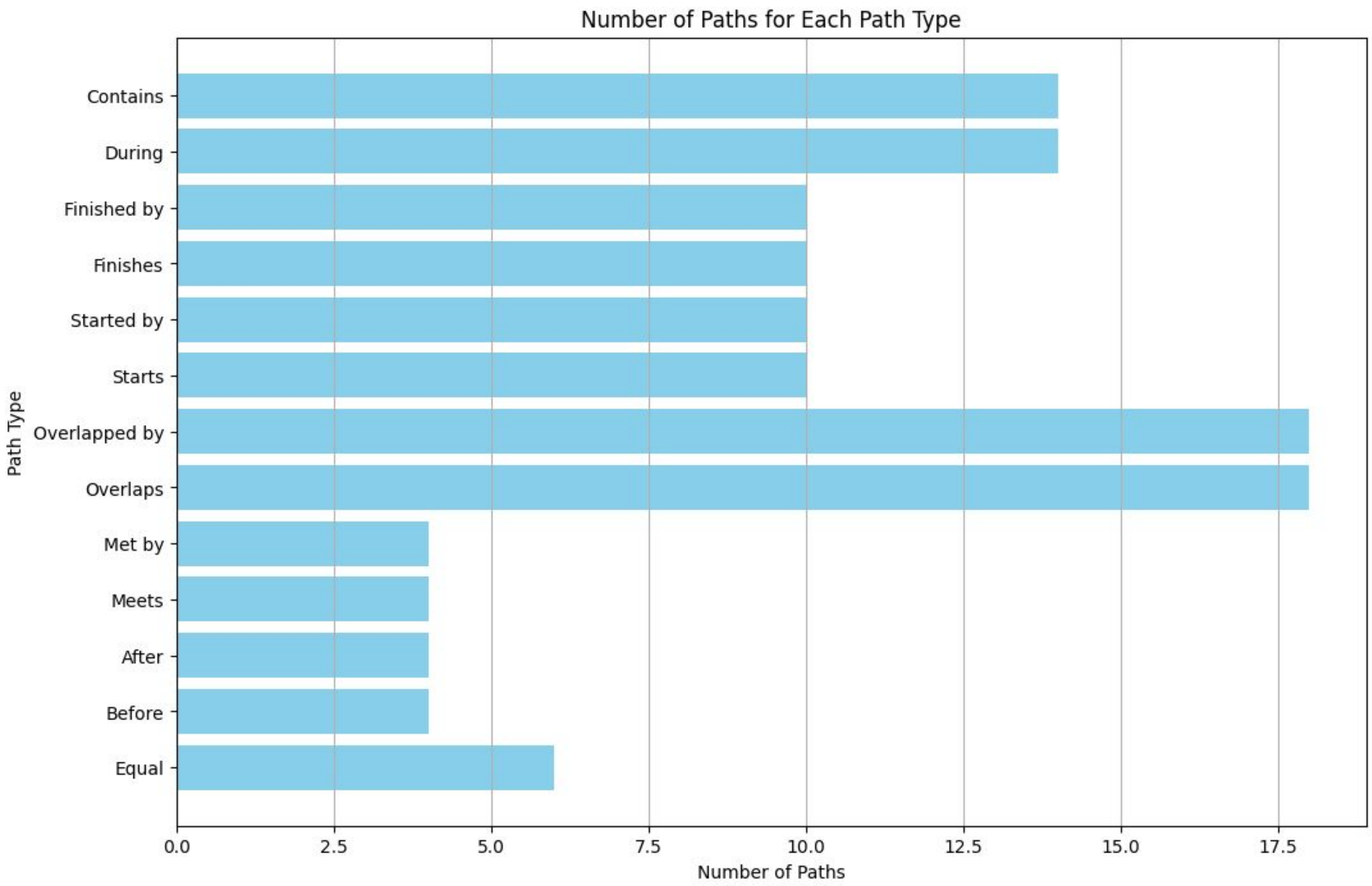
```
8. Starts (S):
Number of starts_paths: 10
[[0, 0], [1, 1], [2, 1], [3, 1], [3, 2], [3, 3]]
[[0, 0], [1, 1], [2, 1], [3, 1], [3, 3]]
[[0, 0], [1, 1], [2, 1], [2, 2], [3, 2], [3, 3]]
[[0, 0], [1, 1], [2, 1], [3, 2], [3, 3]]
[[0, 0], [1, 1], [3, 1], [3, 2], [3, 3]]
[[0, 0], [1, 1], [3, 1], [3, 3]]
[[0, 0], [1, 1], [1, 2], [2, 2], [3, 2], [3, 3]]
[[0, 0], [1, 1], [1, 2], [3, 2], [3, 3]]
[[0, 0], [1, 1], [2, 2], [3, 2], [3, 3]]
[[0, 0], [1, 1], [3, 2], [3, 3]]

9. Started by (Si) (Inverse of Starts):
Number of started_by_paths: 10
[[0, 0], [1, 1], [2, 1], [2, 2], [2, 3], [3, 3]]
[[0, 0], [1, 1], [2, 1], [2, 3], [3, 3]]
[[0, 0], [1, 1], [1, 2], [2, 2], [2, 3], [3, 3]]
[[0, 0], [1, 1], [1, 2], [1, 3], [2, 3], [3, 3]]
[[0, 0], [1, 1], [1, 2], [1, 3], [3, 3]]
[[0, 0], [1, 1], [1, 2], [2, 3], [3, 3]]
[[0, 0], [1, 1], [1, 3], [2, 3], [3, 3]]
[[0, 0], [1, 1], [1, 3], [3, 3]]
[[0, 0], [1, 1], [2, 2], [2, 3], [3, 3]]
[[0, 0], [1, 1], [2, 3], [3, 3]]
```

```
10. Finishes (F):
Number of finishes_paths: 10
[[0, 0], [0, 1], [1, 1], [2, 1], [2, 2], [3, 3]]
[[0, 0], [0, 1], [1, 1], [2, 1], [3, 3]]
[[0, 0], [0, 1], [1, 1], [1, 2], [2, 2], [3, 3]]
[[0, 0], [0, 1], [1, 1], [1, 2], [3, 3]]
[[0, 0], [0, 1], [1, 1], [2, 2], [3, 3]]
[[0, 0], [0, 1], [1, 1], [3, 3]]
[[0, 0], [0, 1], [0, 2], [1, 2], [2, 2], [3, 3]]
[[0, 0], [0, 1], [0, 2], [1, 2], [3, 3]]
[[0, 0], [0, 1], [1, 2], [2, 2], [3, 3]]
[[0, 0], [0, 1], [1, 2], [3, 3]]

11. Finished by (Fi) (Inverse of Finishes):
Number of finished_by_paths: 10
[[0, 0], [1, 0], [2, 0], [2, 1], [2, 2], [3, 3]]
[[0, 0], [1, 0], [2, 0], [2, 1], [3, 3]]
[[0, 0], [1, 0], [1, 1], [2, 1], [2, 2], [3, 3]]
[[0, 0], [1, 0], [1, 1], [2, 1], [3, 3]]
[[0, 0], [1, 0], [1, 1], [1, 2], [2, 2], [3, 3]]
[[0, 0], [1, 0], [1, 1], [1, 2], [3, 3]]
[[0, 0], [1, 0], [1, 1], [2, 2], [3, 3]]
[[0, 0], [1, 0], [1, 1], [3, 3]]
[[0, 0], [1, 0], [2, 1], [2, 2], [3, 3]]
[[0, 0], [1, 0], [2, 1], [3, 3]]
```

```
12. During (D):
Number of during_paths: 14
[[0, 0], [0, 1], [1, 1], [2, 1], [3, 1], [3, 2], [3, 3]]
[[0, 0], [0, 1], [1, 1], [2, 1], [3, 1], [3, 3]]
[[0, 0], [0, 1], [1, 1], [2, 1], [2, 2], [3, 2], [3, 3]]
[[0, 0], [0, 1], [1, 1], [2, 1], [3, 2], [3, 3]]
[[0, 0], [0, 1], [1, 1], [3, 1], [3, 2], [3, 3]]
[[0, 0], [0, 1], [1, 1], [3, 1], [3, 3]]
[[0, 0], [0, 1], [1, 1], [1, 2], [2, 2], [3, 2], [3, 3]]
[[0, 0], [0, 1], [1, 1], [1, 2], [3, 2], [3, 3]]
[[0, 0], [0, 1], [1, 1], [2, 2], [3, 2], [3, 3]]
[[0, 0], [0, 1], [1, 1], [3, 2], [3, 3]]
[[0, 0], [0, 1], [0, 2], [1, 2], [2, 2], [3, 2], [3, 3]]
[[0, 0], [0, 1], [0, 2], [1, 2], [3, 2], [3, 3]]
[[0, 0], [0, 1], [1, 2], [2, 2], [3, 2], [3, 3]]
[[0, 0], [0, 1], [1, 2], [3, 2], [3, 3]]

13. Contains (Di) (Inverse of During):
Number of contains_paths: 14
[[0, 0], [1, 0], [2, 0], [2, 1], [2, 2], [2, 3], [3, 3]]
[[0, 0], [1, 0], [2, 0], [2, 1], [2, 3], [3, 3]]
[[0, 0], [1, 0], [1, 1], [2, 1], [2, 2], [2, 3], [3, 3]]
[[0, 0], [1, 0], [1, 1], [2, 1], [2, 3], [3, 3]]
[[0, 0], [1, 0], [1, 1], [1, 2], [2, 2], [2, 3], [3, 3]]
[[0, 0], [1, 0], [1, 1], [1, 2], [1, 3], [2, 3], [3, 3]]
[[0, 0], [1, 0], [1, 1], [1, 2], [1, 3], [3, 3]]
[[0, 0], [1, 0], [1, 1], [1, 2], [2, 3], [3, 3]]
[[0, 0], [1, 0], [1, 1], [1, 3], [2, 3], [3, 3]]
[[0, 0], [1, 0], [1, 1], [1, 3], [3, 3]]
[[0, 0], [1, 0], [1, 1], [2, 2], [2, 3], [3, 3]]
[[0, 0], [1, 0], [1, 1], [2, 3], [3, 3]]
[[0, 0], [1, 0], [2, 1], [2, 2], [2, 3], [3, 3]]
[[0, 0], [1, 0], [2, 1], [2, 3], [3, 3]]
```

Research Content

3. Pathways from Start to End



Research Content

4.State Transition Probabilities

1. The birth rate from unborn to youth.
2. The Aging rate from youth to old age.
3. A lower death rate from youth to death.
4. A higher death rate from old age to death.
5. Lower death rates for females during youth and old age compared to males.

```
def arSimulate(probBorn,proGrowth,prob_M_Young_Die,prob_F_Young_Die,prob_M_Old_Die, prob_F_Old_Die,trials):
    redRuns = simulateRed(probBorn,proGrowth,prob_M_Young_Die,prob_F_Young_Die,prob_M_Old_Die, prob_F_Old_Die,trials)
    dic = scoreRed(redRuns)
    die = die_pro(redRuns)
    p = probDic(dic,trials)
    d = probDic(die,trials)
    return p,d
```

Result

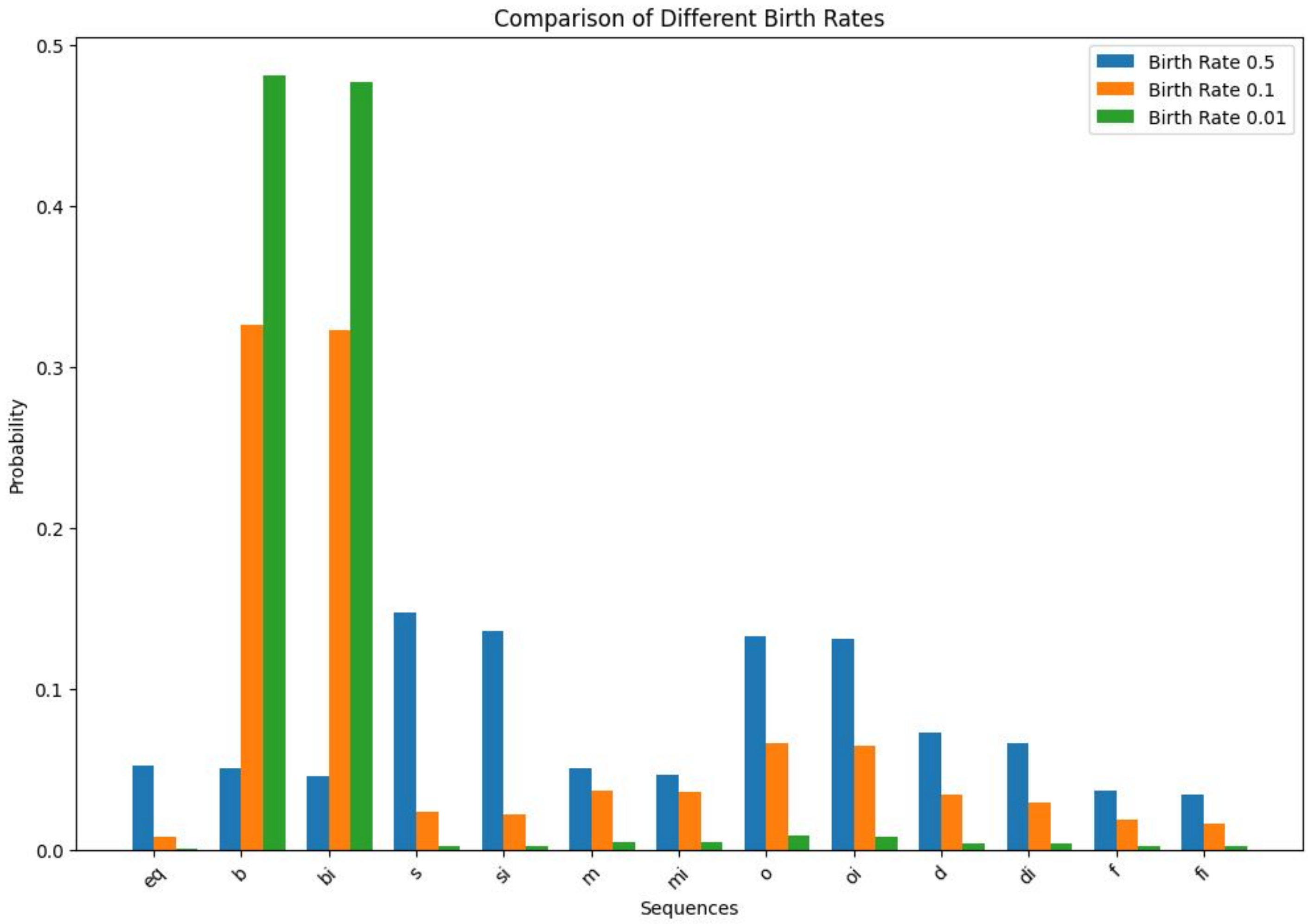
State Transition Probabilities

Scenario	Birth Rate	Aging Rate	Male Young Death Rate	Female Young Death Rate	Male Old Death Rate	Female Old Death Rate	Trials	Eq	B	Bi
1	0.50	0.5	0.1	0.08	0.5	0.48	10k	0.049	0.049	0.038
2	0.10	0.5	0.1	0.08	0.5	0.48	10k	0.005	0.344	0.314
3	0.01	0.5	0.1	0.08	0.5	0.48	10k	0.001	0.475	0.488

Scenario	S	Si	M	Mi	O	Oi	D	Di	F	Fi
1	0.138	0.147	0.048	0.041	0.132	0.125	0.088	0.068	0.04	0.037
2	0.029	0.023	0.043	0.039	0.071	0.067	0.017	0.019	0.011	0.018
3	0.001	0.001	0.003	0.006	0.010	0.010	0.002	0.001	0.001	0.001

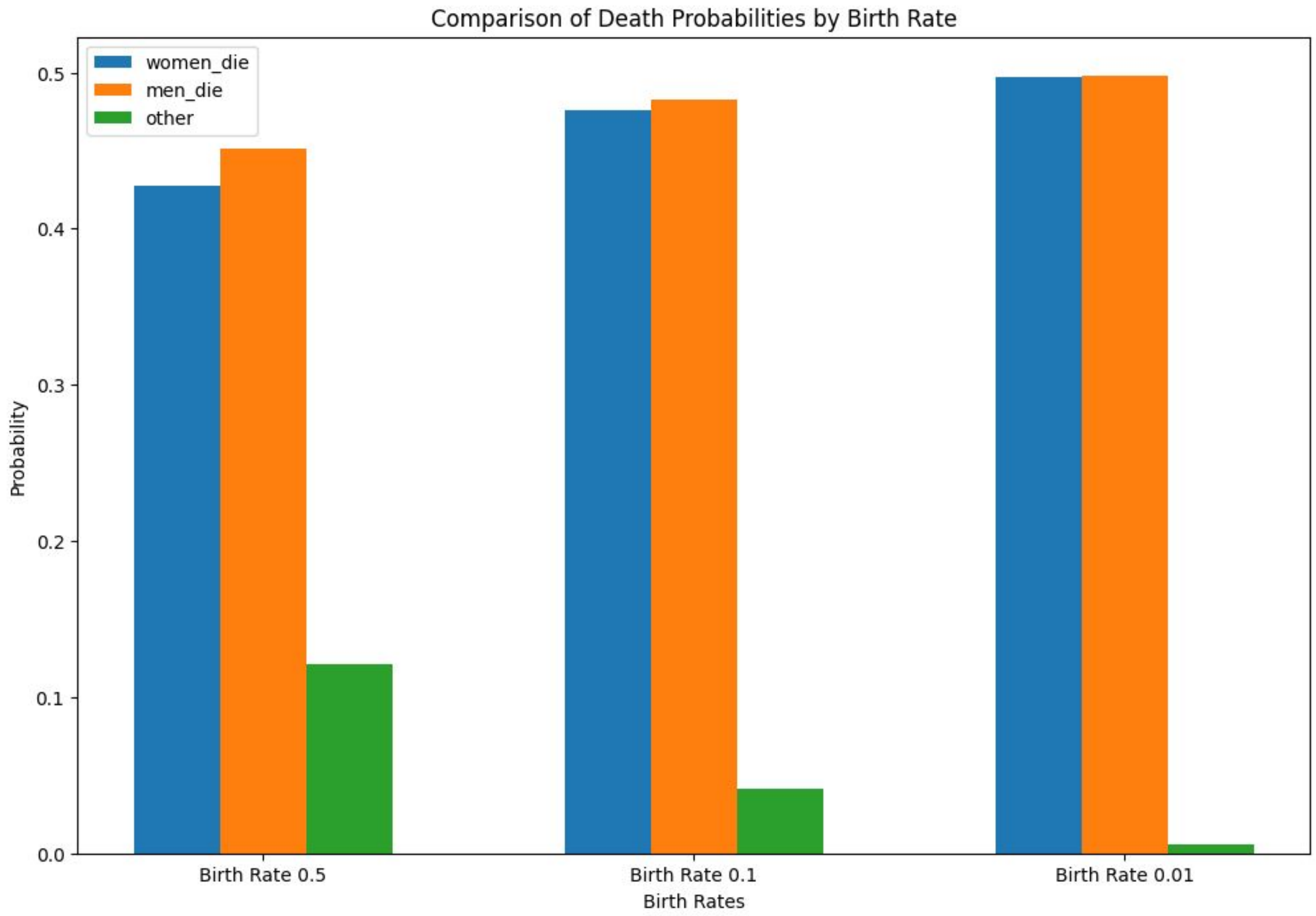
Result

State Transition Probabilities



Result

State Transition Probabilities



Demo of Project

jiaxuan_code.ipynb

File Edit View Insert Runtime Tools Help Last saved at 8:05 PM

+ Code + Text

[3]

```python
import networkx as nx
from itertools import product

# Define states and corresponding numerical representations
states = ["unborn", "young", "old", "deceased"]
state_numbers = {state: i for i, state in enumerate(states)}

# Define transition rules using numbers
transitions = {
    0: [1],      # unborn -> young
    1: [2, 3],   # young -> old, dead
    2: [3],      # old -> dead
    3: []        # dead -> (no transitions)
}

# Create a dictionary to store the adjacency list
adjacency_list = {}

# Generate all possible combinations of states for two species
combinations = list(product(range(4), repeat=2))

# Generate nodes and their adjacent nodes based on combinations
for combo in combinations:
    state_a, state_b = combo
    adjacency_list[combo] = []
    # Transition a
    for next_state_a in transitions[state_a]:
        adjacency_list[combo].append((next_state_a, state_b))
    # Transition b
    for next_state_b in transitions[state_b]:
        adjacency_list[combo].append((state_a, next_state_b))
    # Simultaneous transition of a and b
```

✓ Connected to Python 3 Google Compute Engine backend

Conclusions & Future Work

1. Simulation Results Summary

1. Impact of Age on Mortality Rates

- *The model accurately reflected higher mortality rates for older individuals.*
- *Higher frequencies of "**Overlap**" and "**During**" relations in scenarios with higher death rates for older individuals.*

2. Gender-Specific Death Rates

- *Gender differences in death rates were accurately reflected.*
- *Lower death rates for females led to higher probabilities of interval relations involving longer female lifespans.*

Conclusions & Future Work

Key Contributions of the Study

1. Advancement in Allen's Interval Algebra Research

- *Introduced innovative approaches for probability research in Allen's interval algebra.*
- *Verified theoretical assumptions through simulations of life and death processes.*

2. Enhanced Understanding of Temporal Relationships

- *Explored probabilities of different survival states.*
- *Examined gender-specific differences to deepen reasoning in temporal relationships.*

Conclusions & Future Work

Reflection

- **Future development direction**:

  *At present, my research has completed the automated exploration of **four survival states**.However, current research has not yet achieved automated exploration of **more survival states (five or more)**.*

- **Real-world applicability:**
  Epidemiology, Social sciences, Artificial Intelligence.

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The University of Dublin

Questions ?

Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

Thank You

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[2] Tim Fernando. and Carl Vogel. Prior probabilities of allen interval relations over finite orders. In Proceedings of the 11th International Conference on Agents and Artificial Intelligence - Volume 2: NLPinAI,, pages 952–961. INSTICC, SciTePress, 2019.

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