GEOPHYSICS, VOL. XXXI, NO. 2 (APRIL, 1966), PP. 362–371 4 FIGS., 1 TABLE

THE GRAVITATIONAL ATTRACTION OF A RIGHT RECTANGULAR PRISM†

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The derivation of a closed expression is presented to calculate the vertical component of the gravitational attraction of a right rectangular prism, with sides parallel to the coordinate axis. As any configuration can be expressed as the sum of prisms of various sizes and densities, the computation of the total gravitational effect of bodies of arbitrary shapes at any point outside of or on the boundary of the bodies is straightforward. To calculate the gravitational effect of the "unit" building element a subroutine called Prism has been developed, tested, and incorporated, in one program to calculate terrain corrections, and in another program for three-dimensional analysis of a gravity field.

I. INTRODUCTION

A number of papers have been published on methods of computing the gravitational attraction of simple forms such as the sphere, cylinder, ellipsoid, and prism. For most of these cases, only approximate expressions have been obtained, such that there are restrictions limiting the validity of the expressions near the computation point. In this paper a closed expression is developed for the gravitational attraction of a prism which is valid for any point outside of or on the boundary of the prism. It is possible to describe any arbitrary configuration in terms of building blocks composed of prisms of various dimensions and densities and, hence, to compute the vertical component of the gravitational attraction of any given mass distribution at arbitrarily selected points.

II. THE ATTRACTION OF A PRISM

The magnitude of the attraction of an elementary mass on a unit mass at distance r is given by:

$$\Delta F = G\rho \frac{\Delta v}{r^2}, \tag{1}$$

where G is the gravitational constant, ρ the density and Δv the volume element.

If the angle enclosed by r and the vertical axis is denoted by γ, then the vertical component of the attraction of a body can be obtained by integrating $\Delta F \cos\gamma$ over the volume, i.e.,

$$F_z = G\rho \int_V \frac{dv}{r^2} \cos\gamma = G\rho \int_V \frac{zdz}{r^3}. \tag{2}$$

The problem is simply to carry out this integration for a prism.

Using the cartesian coordinate system shown in Figure 1, (2) becomes:

$$F_z = G\rho \int_{x_1}^{x_2} dx \int_{y_1}^{y_2} dy \cdot \int_{z_1}^{z_2} \frac{zdz}{\sqrt{(x^2+y^2+z^2)^3}}. \tag{3}$$

Carrying out the integration with respect to z and without substituting the limits, one finds:

$$I_1 = \int \frac{zdz}{\sqrt{(x^2+y^2+z^2)^3}} = -\frac{1}{\sqrt{x^2+y^2+z^2}}. \tag{4}$$

Integrating (4) with respect to y gives:

$$I_2 = \int I_1 dy = \int \frac{dy}{\sqrt{x^2+y^2+z^2}} = \ln\left(y + \sqrt{x^2+y^2+z^2}\right). \tag{5}$$

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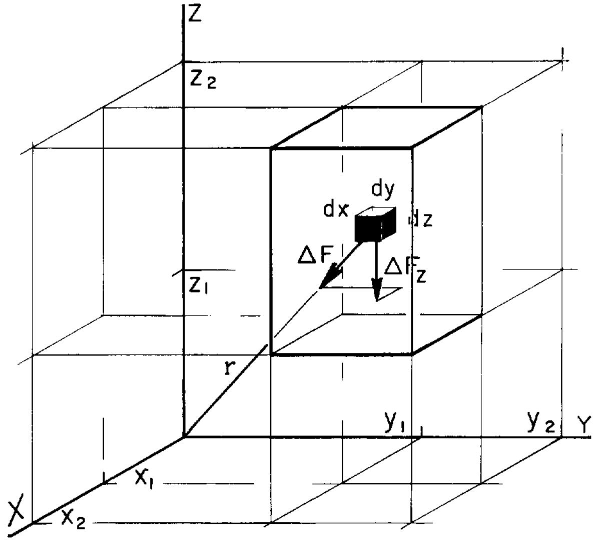

FIG. 1. A right rectangular prism with the volume element and its relation to the Cartesian coordinate system.

The integration of (5) with respect to x is slightly more complicated. One can proceed as follows:

$$I_3 = \int I_2 dx = \int \ln\left(y + \sqrt{x^2+y^2+z^2}\right) dx$$

$$= x \ln\left(y + \sqrt{x^2+y^2+z^2}\right) - \int \frac{x^2 dx}{\left(y + \sqrt{x^2+y^2+z^2}\right)\sqrt{x^2+y^2+z^2}}. \tag{6}$$

Letting

$$u = y + \sqrt{x^2+y^2+z^2},$$

then

$$x^2 = (u-y)^2 - y^2 - z^2,$$

and

$$dx = \frac{(u-y)du}{\sqrt{(u-y)^2 - y^2 - z^2}};$$

thus the integral in (6) becomes

$$I = \int \frac{x^2 dx}{\left(y + \sqrt{x^2+y^2+z^2}\right)\sqrt{x^2+y^2+z^2}} = \int \frac{\sqrt{u^2 - 2uy - z^2}}{u} du$$

$$= u^2 - 2uy - z^2 - y \ln\left(u - y + \sqrt{u^2 - 2uy - z^2}\right) - z \arcsin \frac{-uy - z^2}{u\sqrt{z^2+y^2}}.$$

Transforming back to the original variable and noting that

$$u^2 - 2uy - z^2 = \sqrt{y^2 + x^2 + z^2 + 2y\sqrt{x^2+y^2+z^2} - 2y^2 - 2y\sqrt{x^2+y^2+z^2} + z^2 - z^2} = x,$$

then

$$I = x - y \ln\left(x + \sqrt{x^2+y^2+z^2}\right) - z \arcsin - \frac{z^2 + y^2 + y\sqrt{x^2+y^2+z^2}}{\left(y + \sqrt{x^2+y^2+z^2}\right)\sqrt{y^2+z^2}}.$$

When the limits of integration are substituted, the first term in I drops out and the following general expression to calculate the vertical component of the attraction of a prism is obtained:

$$F_z = G\rho \left|\left|\left| x \ln (y + r) + y \ln (x + r) - z \arcsin \frac{z^2 + y^2 + yr}{(y + r)\sqrt{y^2 + z^2}} \right|_{z_1}^{z_2} \right|_{y_1}^{y_2} \right|_{x_1}^{x_2}, \tag{7}$$

where equation (7) is valid only when the limits z_1, z_2; y_1, y_2; and x_1, x_2 are substituted. When either the x, y, or both axes are crossed, the integration must be carried out from the lower limit to the axis, then from the axis to the upper limit, the sum of these integrations giving the required effect. To describe all possible situations in the procedure followed in this paper to evaluate equation (7), explicit expressions for four sets of limits are required. Two of these expressions are given in full below:

(a) The prism is completely contained within any one of the four xy quadrants; then using the absolute values of the limits, F_z takes the form:

$$\begin{aligned}
F_z/G\rho = {} & x_2 \ln (y_2 + \sqrt{x_2^2 + y_2^2 + z_1^2}) - x_1 \ln (y_2 + \sqrt{x_1^2 + y_2^2 + z_1^2}) \\
& + y_2 \ln (x_2 + \sqrt{x_2^2 + y_2^2 + z_1^2}) - y_2 \ln (x_1 + \sqrt{x_1^2 + y_2^2 + z_1^2}) \\
& + z_1 \arcsin \frac{z_1^2 + y_2^2 + y_2^2\sqrt{x_2^2 + y_2^2 + z_1^2}}{(y_2 + \sqrt{x_2^2 + y_2^2 + z_1^2})\sqrt{y_2^2 + z_1^2}} \\
& - z_1 \arcsin \frac{z_1^2 + y_2^2 + y_2\sqrt{x_1^2 + y_2^2 + z_1^2}}{(y_2 + \sqrt{x_1^2 + y_2^2 + z_1^2})\sqrt{y_2^2 + z_1^2}} \\
& - x_2 \ln (y_1 + \sqrt{x_2^2 + y_1^2 + z_1^2}) + x_1 \ln (y_1 + \sqrt{x_1^2 + y_1^2 + z_1^2}) \\
& - y_1 \ln (x_2 + \sqrt{x_2^2 + y_1^2 + z_1^2}) + y_1 \ln (x_1 + \sqrt{x_1^2 + y_1^2 + z_1^2}) \\
& - z_1 \arcsin \frac{z_1^2 + y_1^2 + y_1\sqrt{x_2^2 + y_1^2 + z_1^2}}{(y_1 + \sqrt{x_2^2 + y_1^2 + z_1^2})\sqrt{y_1^2 + z_1^2}} \\
& + z_1 \arcsin \frac{z_1^2 + y_1^2 + y_1\sqrt{x_1^2 + y_1^2 + z_1^2}}{(y_1 + \sqrt{x_1^2 + y_1^2 + z_1^2})\sqrt{y_1^2 + z_1^2}} \\
& - x_2 \ln (y_2 + \sqrt{x_2^2 + y_2^2 + z_2^2}) + x_1 \ln (y_2 + \sqrt{x_1^2 + y_2^2 + z_2^2}) \\
& - y_2 \ln (x_2 + \sqrt{x_2^2 + y_2^2 + z_2^2}) + y_2 \ln (x_1 + \sqrt{x_1^2 + y_2^2 + z_2^2}) \\
& - z_2 \arcsin \frac{z_2^2 + y_2^2 + y_2\sqrt{x_2^2 + y_2^2 + z_2^2}}{(y_2 + \sqrt{x_2^2 + y_2^2 + z_2^2})\sqrt{y_2^2 + z_2^2}} \\
& + z_2 \arcsin \frac{z_2^2 + y_2^2 + y_2\sqrt{x_1^2 + y_2^2 + z_2^2}}{(y_2 + \sqrt{x_1^2 + y_2^2 + z_2^2})\sqrt{y_2^2 + z_2^2}} \\
& + x_2 \ln (y_1 + \sqrt{x_2^2 + y_1^2 + z_2^2}) - x_1 \ln (y_1 + \sqrt{x_1^2 + y_2^2 + z_1^2}) \\
& + y_1 \ln (x_2 + \sqrt{x_2^2 + y_1^2 + z_2^2}) - y_1 \ln (x_1 + \sqrt{x_1^2 + y_1^2 + z_2^2}) \\
& + z_2 \arcsin \frac{z_2^2 + y_1^2 + y_1\sqrt{x_2^2 + y_1^2 + z_2^2}}{(y_1 + \sqrt{x_2^2 + y_1^2 + z_2^2})\sqrt{y_1^2 + z_2^2}} \\
& - z_2 \arcsin \frac{z_2^2 + y_1^2 + y_1\sqrt{x_1^2 + y_1^2 + z_2^2}}{(y_1 + \sqrt{x_1^2 + y_1^2 + z_2^2})\sqrt{y_1^2 + z_2^2}} .
\end{aligned} \tag{8}$$

As a special case of (8), letting $z_1 = 0$ and $z_2 = h$ and rearranging the terms, (8) becomes:

$$
\begin{aligned}
F_z/G\rho = {} & x_2\left\{\ln\frac{y_2+\sqrt{x_2^2+y_2^2}}{y_2+\sqrt{x_2^2+y_2^2+h^2}} - \ln\frac{y_1+\sqrt{x_2^2+y_1^2}}{y_1+\sqrt{x_2^2+y_1^2+h^2}}\right\} \\
& - x_1\left\{\ln\frac{y_2+\sqrt{x_1^2+y_2^2}}{y_2+\sqrt{x_1^2+y_2^2+h^2}} - \ln\frac{y_1+\sqrt{x_1^2+y_1^2}}{y_1+\sqrt{x_1^2+y_1^2+h^2}}\right\} \\
& + y_2\left\{\ln\frac{x_2+\sqrt{x_2^2+y_2^2}}{x_2+\sqrt{x_2^2+y_2^2+h^2}} - \ln\frac{x_1+\sqrt{x_1^2+y_2^2}}{x_1+\sqrt{x_1^2+y_2^2+h^2}}\right\} \\
& - y_1\left\{\ln\frac{x_2+\sqrt{x_2^2+y_1^2}}{x_2+\sqrt{x_2^2+y_1^2+h^2}} - \ln\frac{x_1+\sqrt{x_1^2+y_1^2}}{x_1+\sqrt{x_1^2+y_1^2+h^2}}\right\} \\
& + h\left\{\arcsin\frac{y_2^2+h^2+y_2\sqrt{x_2^2+y_2^2+h^2}}{(y_2+\sqrt{x_2^2+y_2^2+h^2})\sqrt{y^2+h^2}}\right. \\
& - \arcsin\frac{y_2^2+h^2+y_2\sqrt{x_1^2+y_2^2+h^2}}{(y_2+\sqrt{x_1^2+y_2^2+h^2})\sqrt{y_2^2+h^2}} \\
& - \arcsin\frac{y_1^2+h^2+y_1\sqrt{x_2^2+y_1^2+h^2}}{(y_1+\sqrt{x_2^2+y_1^2+h^2})\sqrt{y_1^2+h^2}} \\
& \left. + \arcsin\frac{y_1^2+h^2+y_1\sqrt{x_1^2+y_1^2+h^2}}{(y_1+\sqrt{x_1^2+y_1^2+h^2})\sqrt{y_1^2+h^2}}\right\}. \qquad (9)
\end{aligned}
$$

Designating the terms within the brackets by $T_1, T_2, \cdots T_{12}$ one obtains the simple form:

$$F_z/G\rho = x_2\{T_1-T_2\} - x_1\{T_3-T_4\} + y_2\{T_5-T_6\} - y_1\{T_7-T_8\} + h\{T_9-T_{10}-T_{11}+T_{12}\}. \qquad (10)$$

(*b*) The y axis is crossed, i.e. the signs of x_1 and x_2 are different. Since the vertical component of the attraction of the same mass below and above the y axis is the same, the integral is evaluated from 0 to x_2 and from 0 to x_1 (absolute values):

$$
\begin{aligned}
P_1 = {} & x_2\left\{ln\frac{y_2+\sqrt{x_2^2+y_2^2}}{y_2+\sqrt{x_2^2+y_2^2+h^2}} - \ln\frac{y_1+\sqrt{x_2^2+y_1^2}}{y_1+\sqrt{x_2^2+y_1^2+h^2}}\right\} - 0 \\
& + y_2\left\{\ln\frac{x_2+\sqrt{x_2^2+y_2^2}}{x_2+\sqrt{x_2^2+y_2^2+h^2}} - \ln\frac{y_2}{\sqrt{y_2^2+h^2}}\right\} \\
& - y_1\left\{\ln\frac{x_2+\sqrt{x_2^2+y_1^2}}{x_2+\sqrt{x_2^2+y_1^2+h^2}} - \ln\frac{y_1}{\sqrt{y_1^2+h^2}}\right\} \\
& + h\left\{\arcsin\frac{y_2^2+h^2+y_2\sqrt{x_1^2+y_2^2+h^2}}{(y_2+\sqrt{x_2^2+y_2^2+h^2})\sqrt{y_2^2+h^2}}\right. \\
& - \arcsin\frac{y_2^2+h^2+y_2\sqrt{y_2^2+h^2}}{(y_2+\sqrt{y_2^2+h^2})\sqrt{y_2^2+h^2}} \\
& - \arcsin\frac{y_1^2+h^2+y_1\sqrt{x_2^2+y_1^2+h^2}}{(y_1+\sqrt{x_2^2+y_1^2+h^2})\sqrt{y_1^2+h^2}} \\
& \left. + \arcsin\frac{y_1^2+h^2+y_1\sqrt{y_1^2+h^2}}{(y_1+\sqrt{y_1^2+h^2})\sqrt{y_1^2+h^2}}\right\},
\end{aligned}
$$

and

$$P_2 = x_1 \left\{ \ln \frac{y_2 + \sqrt{x_1^2 + y_2^2}}{y_2 + \sqrt{x_1^2 + y_2^2 + h^2}} - \ln \frac{y_1 + \sqrt{x_1^2 + y_1^2}}{y_1 + \sqrt{x_1^2 + y_1^2 + h^2}} \right\} - 0$$

$$+ y_2 \left\{ \ln \frac{x_1 + \sqrt{x_1^2 + y_2^2}}{x_1 + \sqrt{x_1^2 + y_2^2 + h^2}} - \ln \frac{y_2}{\sqrt{y_2^2 + h^2}} \right\}$$

$$- y_1 \left\{ \ln \frac{x_1 + \sqrt{x_1^2 + y_1^2}}{x_1 + \sqrt{x_1^2 + y_1^2 + h^2}} - \ln \frac{y_1}{\sqrt{y_1^2 + h^2}} \right\}$$

$$+ h \left\{ \arcsin \frac{y_2^2 + h^2 + y_2\sqrt{x_1^2 + y_2^2 + h^2}}{(y_2 + \sqrt{x_1^2 + y_2^2 + h^2})\sqrt{y_2^2 + h^2}} \right.$$

$$- \arcsin \frac{y_2^2 + h^2 + y_2\sqrt{y_2^2 + h^2}}{(y_2 + \sqrt{y_2^2 + h^2})\sqrt{y_2^2 + h^2}}$$

$$- \arcsin \frac{y_1^2 + h^2 + y_1\sqrt{x_1^2 + y_1^2 + h^2}}{(y_1 + \sqrt{x_1^2 + y_1^2 + h^2})\sqrt{y_1^2 + h^2}}$$

$$\left. + \arcsin \frac{y_1^2 + h^2 + y_1\sqrt{y_1^2 + h^2}}{(y_1 + \sqrt{y_1^2 + h^2})\sqrt{y_1^2 + h^2}} \right\}.$$

Using the terms $T_1, T_2 \cdots T_{12}$, P_1 and P_2 take the following form:

$$P_1 = x_2\{T_1 - T_2\} + 0 + y_2 \left\{ T_5 - \ln \frac{y_2}{\sqrt{y_2^2 + h^2}} \right\} - y_1 \left\{ T_7 - \ln \frac{y_1}{\sqrt{y_1^2 + h^2}} \right\}$$
$$+ h\{T_9 - 1 - T_{11} + 1\},$$

$$P_2 = x_1\{T_3 - T_4\} + 0 + y_2 \left\{ T_6 - \ln \frac{y_2}{\sqrt{y_2^2 + h^2}} \right\} - y_1 \left\{ T_8 - \ln \frac{y_1}{\sqrt{y_1^2 + h^2}} \right\}$$
$$+ h\{T_{10} - 1 - T_{12} + 1\}.$$

Then adding P_1 and P_2 one finds:

$$F_z/G\rho \equiv P_1 + P_2 = x_2\{T_1 - T_2\} + x_1\{T_3 - T_4\} + y_2\{T_5 + T_6\} - y_1\{T_7 + T_8\}$$
$$+ h\{T_9 + T_{10} - T_{11} - T_{12}\} - 2 \left\{ y_2 \ln \frac{y_2}{\sqrt{y_2^2 + h^2}} - y_1 \ln \frac{y_1}{\sqrt{y_1^2 + h^2}} \right\}. \quad (11)$$

Other special cases, when crossing the x axis (y_1 and y_2 having different signs) or crossing the origin (both y_1, y_2 and x_1, x_2 with different signs), can be obtained similarly.

Subsequent to this development of equation (7), it was found that Sorokin (1951) and Haáz (1953) also published solutions to this problem. The solutions they obtained are given below using their notation. Sorokin carried out the integration differently and obtained (page 370, equation (426)):

$$\Delta g = - f\sigma \Bigg|_{\zeta_1}^{\zeta_2} \Bigg|_{\eta_1}^{\eta_2} \Bigg|_{\xi_1}^{\xi_2} \xi \ln (\eta + R)$$
$$+ \eta \ln (\xi + R) + \zeta \operatorname{arctg} \frac{\zeta R}{\xi \eta} \Bigg|\Bigg|\Bigg|.$$

Table 1. Calculated terrain corrections for twelve gravity stations shown in Figure 2

Station number	UTM Coordinates X_m	UTM Coordinates Y_m	h_{ft}	Δg	Δg_A	ϵ
	2	3	4	5	6	7
8174	6 793 889.7	569 909.1	1795	1.09	1.11	±.11
9810	6 794 529.7	573 172.9	1622	3.31	3.36	±.14
8193	6 792 197.2	571 837.6	1831	1.44		
9815	6 795 525.5	571 852.6	1936	2.04	2.08	±.13
9813	6 795 878.6	571 687.2	2063	3.68	3.69	±.16
8135	6 796 148.7	571 687.4	1908	1.62		
1	6 793 921.4	570 604.2	−380	8.29	8.26	±.10
3	6 794 073.7	571 291.7	−685	13.43	13.43	±.06
5	6 794 229.6	571 229.0	−745	14.45	14.46	±.06
7	6 794 350.0	572 610.0	−780	13.07		
14	6 794 861.6	571 697.5	−805	14.39		
16	6 793 164.5	571 631.4	−440	9.03		

Haáz applied Euler's theorem of homogeneous functions to the second derivative of the potential of a prism, thus obtaining the first derivative and the potential itself without integration. His result for the vertical component of the attraction (page 62) is quoted below:

$$\phi_z = - a \log (b + r) - b \log (a + r) + c \operatorname{arctg} \frac{ab}{cr} \cdot$$

Although these equations, including (7), do not seem to agree, it has been verified that they are identical.

Having obtained the expressions for all cases, a subroutine called Prism has been written in Fortran II. Although the arithmetic is fairly simple, some care must be exercised to obtain the proper special case to be used for a given set of input values. Extensive testing shows that the subroutine provides the correct value for the vertical component of the gravitational attraction of a prism on a unit particle at P, if P is outside or on the boundary of the prism. In the following section two applications of the Prism subroutine are given. See also Nagy (1966).

III. APPLICATIONS

a) *Terrain corrections*

The Prism subroutine has been used in a program to calculate terrain corrections. The principle of the method is described as follows: the local area, whose terrain effect is to be taken into account, is subdivided into prisms by a grid system with intervals dx and dy. The bases of these prisms are at sea level, and the tops are defined by the estimated elevations, $H_{i,j}$. The elevation difference between a compartment and the station, together with the horizontal coordinates of the compartment, are calculated and fed into the Prism subroutine, which computes the exact gravitational effect of that compartment on the gravity station. The sum of the effects of all compartments gives the terrain correction. More detail will be given in a forthcoming paper.

This program has been applied for an area 10×10 km surrounding the New Quebec Crater (Figure 2). A grid interval of 100 m for both x and y has been used producing 10,000 compartments. The elevation of the water level in the crater is 1,620 feet above sea level, with a maximum depth of 810 ft. The surrounding topography varies from 1,530 to 2,156 ft. For the calculations it was assumed that the top of each prism was a plane surface parallel to the xy plane, and that all prisms had the same density. The standard deviations of $H_{i,j}$ for compartments on land was estimated at ±5 ft and for compartments on water at ±25 ft. Terrain corrections were calculated for 130 gravity stations of which twelve are shown in Figure 2. The coordinates, elevation and computed terrain corrections of these stations are listed in Table 1.

To assess the effect of the errors in the elevations, $H_{i,j}$, on the computed terrain corrections, provision was made for error analysis by using the Monte Carlo technique. Pseudo-random numbers of magnitude proportional to the standard deviation of $H_{i,j}$, were superimposed on each

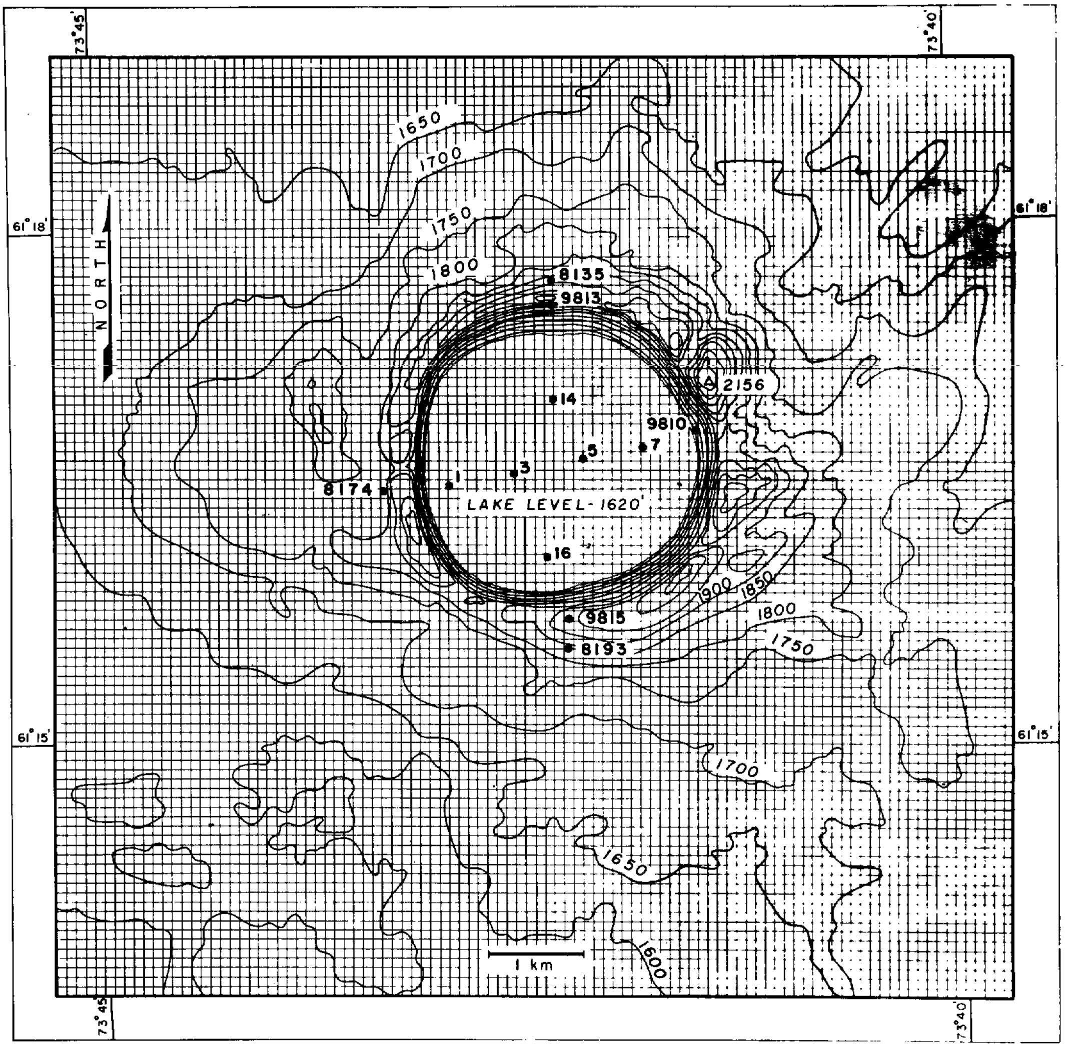

FIG. 2. The New Quebec Crater with surrounding topography. Contour interval 50 ft.

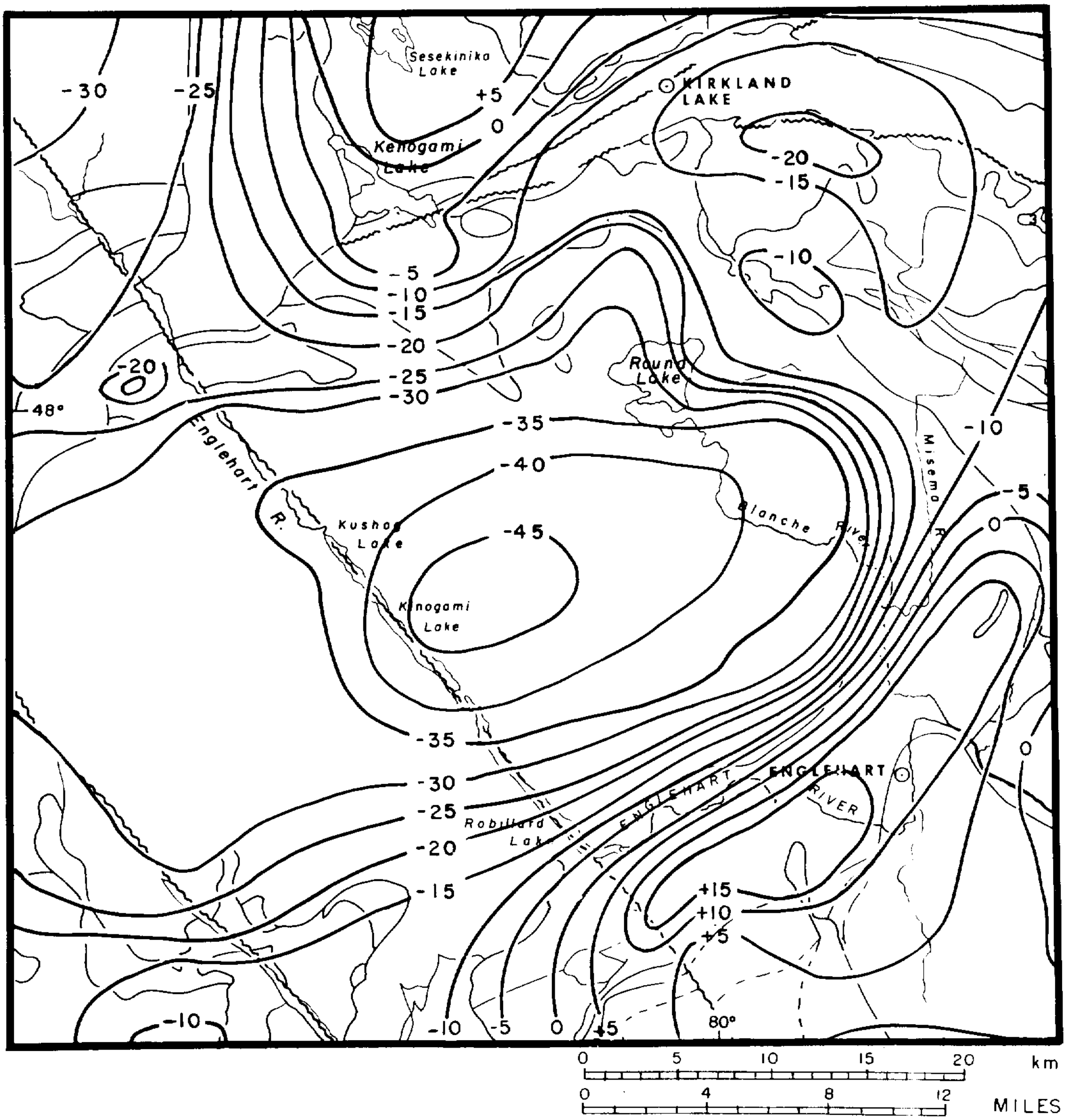

FIG. 3. Residual Bougouer anomaly map for the vicinity of Kirkland Lake, Ontario. Contour interval 5 mgal.

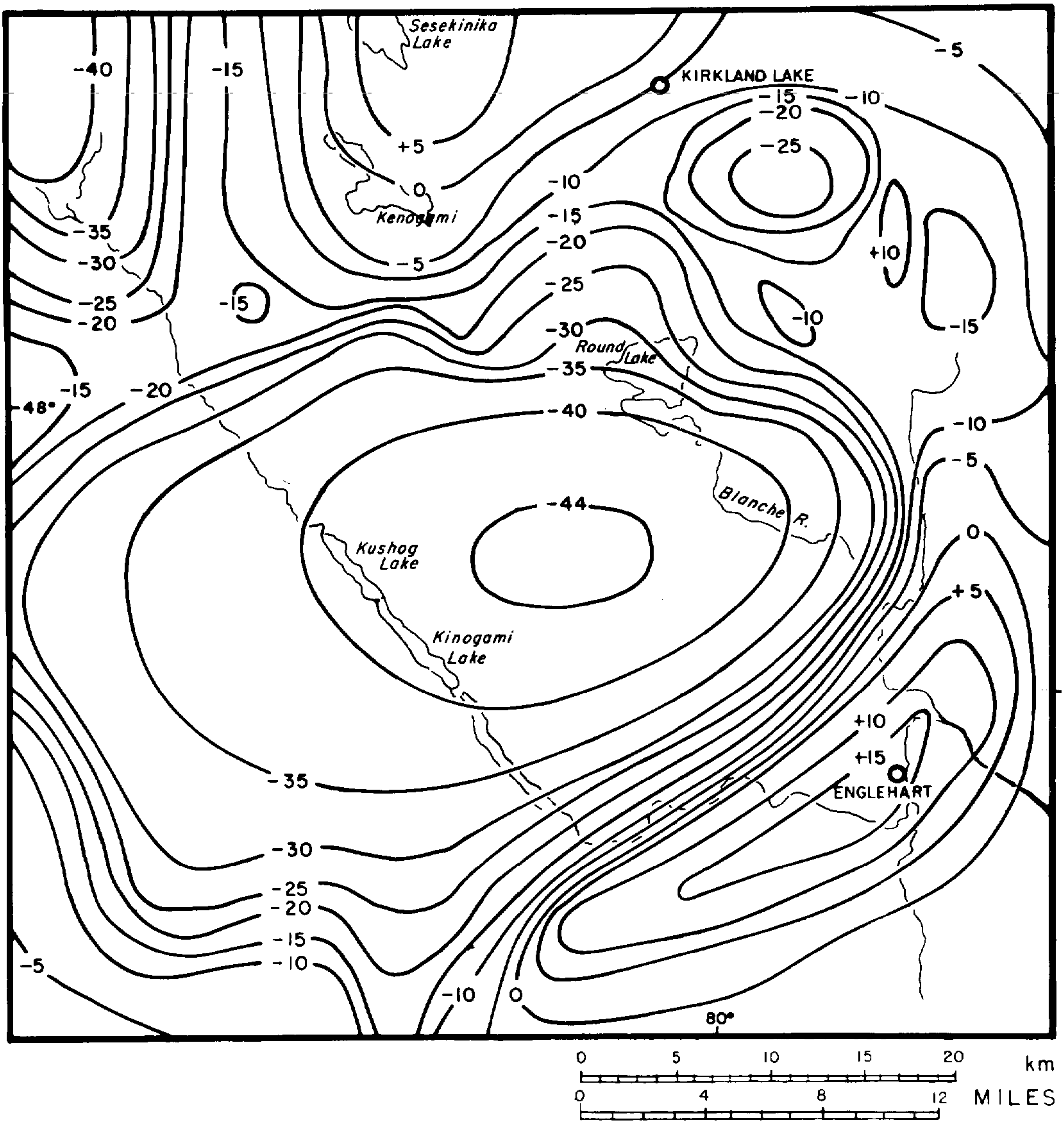

FIG. 4. Synthetic gravity contour map resulting from three-dimensional analysis.

estimated elevation. Then the terrain correction was calculated with this new set of input data. This computation has been repeated 36 times for each station using new sets of random numbers each time. Table 1 gives the average, Δg_A, of the 36 calculated terrain corrections with their respective standard deviations, ϵ, for seven stations. As can be seen from Table 1 the terrain correction Δg calculated from the original input (column 5) differs from Δg_A by less than $\pm\epsilon$.

b) *Three-dimensional analysis*

On the suggestion of J. van Boeckel of the Dominion Observatory, another program, for three-dimensional analysis, has been developed around the Prism subroutine. The principle involved is simple: the sum of the gravitational effects of prisms of given dimensions and densities is calculated at specific points. These initial dimensions and densities are determined from existing information, usually from geological maps of the area. It is noted here that there is no restriction on the dimensions of prisms or their distance from the computation point (other than that the point may not be inside a prism). The calculated values then are compared to the given anomalies and the differences are successively eliminated by modifying the block arrangement, number of blocks and/or densities. This program has been used by van Boeckel to explain the residual gravity anomaly field shown in Figure 3. After four modifications the gravitational attraction of 80 blocks was evaluated at 625 grid points. The values were plotted and the resulting contoured map is shown in Figure 4.

The two examples above illustrate the power and flexibility of the application of the Prism method to problems associated with gravity fields. The application to other problems is limited only by the availability of input data and by memory space in the computers. For example, in terrain corrections one could easily include variations in density.

It is now possible by using the Prism subroutine to test some of the assumptions about the geology of an area; to analyze different sources of errors and calculate their effect on the output; to derive different degrees of approximations for the gravitational effect of the prism for practical computations and estimate their accuracies; to obtain "regional" gravity anomalies; to carry out geological corrections to obtain "residuals"; and to compare different isostatic hypotheses.

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DISCUSSIONS

DISCUSSION ON "THE GRAVITATIONAL ATTRACTION OF A RIGHT RECTANGULAR PRISM," BY DEZSO NAGY (GEOPHYSICS, APRIL, 1966, PP. 362–371)

The recent paper by Dezsö Nagy, "The gravitational attraction of a right rectangular prism," is certainly of great interest. It might be pointed out, however, that in his textbook published in 1930, MacMillan already gave a formula for computing the potential of this body, as well as an extremely simple method to compute the derivatives along a coordinate axis. The Dover reprint is widely available.

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DISCUSSION ON "THE GRAVITATIONAL ATTRACTION OF A RIGHT RECTANGULAR PRISM," BY DEZSO NAGY (GEOPHYSICS, APRIL, 1966, PP. 362–371)

You might be interested to note that the results obtained by Nagy (1966) and published in Geophysics concerning the vertical component of the gravitational attraction of a right rectangular prism not only had been published previously by Sorokin and Haáz but also were derived and published (in English) 136 years ago by Everest (1830, p. 94–97). Everest calculated closed expressions which are equivalent to that of Nagy for the horizontal and vertical gravitational effects of a rectangular parallelopiped and used these equations to estimate the topographic deflection of the plumb bob due to the Satpura Range in India.

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REPLY BY AUTHOR TO DISCUSSIONS BY J. CL. DE BREMAECKER AND CHARLES E. CORBATO

I thank both Drs. Corbató and De Bremaecker for their references to earlier works in connection with the gravitational attraction of the prism. It may be of some interest, at least from the historical point of view, to give some further references, which were found after my paper went to press. First of all, reference is made to Mader's closed expression (Tarczy-Hornoch, A., Personal communication, May, 1966) derived in connection with geoid computations. Another solution was presented by Ansel both in the form of an infinite series, which he preferred for numerical computations, and in a closed form. A more general solution was given recently by Kolbenheyer who expressed the vertical component of the attraction of a right prismatic body as the potential difference between the top and bottom boundary surfaces. Applying his solution to the prism, the same closed expression results as was obtained in different forms in the above references (as well as Everest, Sorokin, Haáz and Nagy).

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DISCUSSION ON "THE EFFECT OF A CONDUCTING OVERBURDEN ON ELECTROMAGNETIC PROSPECTING MEASUREMENTS," BY W. LOWRIE AND G. F. WEST (GEOPHYSICS, AUGUST, 1965, PP. 624–632)

The recent paper by Lowrie and West (1965) may prove to be a very valuable contribution in the study of the effect of overburden in geo-electromagnetics. The model experiments and their results look very impressive, but the interpretation of curves appears incomplete and incorrect in some respects. I have following comments for the paper:

(A) In my opinion the authors have failed to present the importance of the results obtained for the moderate values of α_o. According to them, "The presence of an overburden with a moderate value of α_o does not cause any profound changes in the qualitative nature of the 'conductor' anomaly, although it does alter the anomaly amplitudes substantially."

A careful and detailed study of Figures 2 and 3 reveals that:

(i) The in-phase anomalies for moderate values of α_o (B-curves) always show a definite increase in anomaly as compared to the case of no overburden (A-curves).

(ii) While for large values of α_o (C-curves) the in-phase anomaly always shows a general decrease in the magnitude of the anomaly.

These two sets of curves (B- and C-curves) show a definite difference even in the qualitative nature of the anomaly. The B-curves (in-phase curves) indicate in most unambiguous terms that for suitable moderate values of α_0 for the overburden the vertical conductor shows up better than in the case of absence of overburden. This is a very curious and significant result establishing that a decrease of conductivity contrast from maximum value may result in better detectibility. I would like to add here that my theoretical analysis of Electromagnetic Screening by a coating also supports the possibility of such an anomalous response.

(B) I disagree with their conclusion that overburden makes the vertical conductor seem more conducting and more deeply buried. It seems quite impossible to make such an unambiguous interpretation from the curves presented in the paper.

The general decrease of the in-phase anomaly (in C-curves) can be interpreted by various combinations of depth and conductivity variations. Similar infinite combinations would be possible for the out-of phase anomalies also. Actually for the case of increase of out-of phase anomaly index for $\alpha_c = 25$ and $\alpha_o = 3.5$ it could be stated that the ore body appears shallower and less conducting.

(C) The titles of the Figures 2 and 3 should be interchanged.

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REPLY BY AUTHORS TO DISCUSSION BY J. G. NEGI

We should like to express our appreciation to Dr. Negi for his interest in our paper. Our comments on his points are the following:

A. The phenomenon noted by Dr. Negi certainly occurs. We had hoped that our summary graphs (Figures 5, 6 and 7) and our conclusion 3 expressed this sufficiently clearly. The statement of ours that Dr. Negi quoted does not seem really to be in question. Some confusion may have resulted from our use of the phrase "qualitative nature of the anomaly." We meant this to mean the shape or form of the anomaly profile, as distinct from the (peak-to-peak) anomaly amplitude.

B. It might help to explain that the common practice in North America is to interpret horizontal-loop EM anomalies by considering both phases together, rather than individually. A standard method is to plot the in-phase and the quadrature anomaly indices (or amplitudes) on a diagram such as Figure 4, in order to find the conductivity-thickness product and the depth to top of the conductor (a sheetlike conductor of large dip and strike extent is assumed). The measured data define a point on the phasor diagram; thus α_c $(=\sigma_c\mu_o\omega s_c l)$ and h_c/l are immediately obtained. Since μ_o, ω, and l are known $\sigma_c s_c$ and h_c can be found directly.

If this procedure is followed when a conductive overburden is present, the interpretation will be in error as we described. For example, if the observed anomaly indices are 20 percent in-phase and 10 percent quadrature, Figure 4 gives $h_c/l = 0.28$ and $\alpha_c = 16$. However, Figure 7 should have been used in the interpretation if, for instance, an overburden of $\alpha = 1.0$ was actually present. The correct result (assuming the other assumptions of the interpretation method to be true) is $h_c/l = 0.18$ and $\alpha_c = 5$. Thus, as we stated in conclusion 4, if the overburden is neglected the values obtained for h_c/l and α_c are too large.

C. Correct. Another misprint occurs in the last line of the abstract – in-phase should read quadrature.

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