This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3062024, IEEE Transactions on Smart Grid

Impact of Communication Packet Delivery Ratio on Reliability of Optimal Load Tracking and Allocation in DC Microgrids

Masoud H. Nazari, *Senior Member, IEEE,* Siyu Xie, Le Yi Wang, *Fellow, IEEE,* George Yin, *Fellow, IEEE,* and Wen Chen, *Senior Member, IEEE*

***Abstract*—Communication systems introduce uncertainties that directly impact power system management. Quantitative analysis of such impact is essential for power system reliability and resilience. This paper establishes rigorous relationships between communication packet delivery ratio and errors on optimal load tracking and allocation (OLTA) in DC microgrids (MGs). By modeling channel uncertainties using packet delivery ratio and communication network topologies, impact of communication packet loss on the distributed OLTA algorithm for DC MGs is studied. It is shown that communication packet loss directly affects the convergence rate of the distributed OLTA algorithm under intermittent renewable generations. The results of this paper quantitatively characterize a practical criterion for securing reliability of OLTA solutions under communication uncertainty. Simulation studies using a real-world DC MG are conducted to validate the theoretical findings.**

Keywords. DC microgrids, optimal load tracking and allocation, reliability, packet delivery ratio, randomly switching network.

I. Introduction

CYBER-physical power systems naturally integrate communication and control for system coordination and resilience enhancement. For instance, microgrids (MGs) rely ubiquitously on communication systems for data exchange [1]. New power system architectures employ communication systems to exchange data, including Wi-Fi and Wi-MAX (IEEE 802.16), cellular (GPRS/UMTS/LTE), WLAN [2]. Customer premise communications include ZigBee, WiFi (802.11), IEEE 1901/ G.hn/ Home-plug/PRIME PLC, BPL, and wireless.

Reliability of communication systems is essential for power systems and has drawn increased attention [3]–[5]. In particular, wireless communications inherently encounter random uncertainties such as channel interruption, packet loss, and delays [6], creating random link interruption and consequently randomly switching communication networks. Such communication uncertainties directly impact many time-sensitive power system management tasks. This paper focuses on analysis of such impact on optimal load tracking and allocation (OLTA) problems under intermittent renewable generations.[1]

To ensure power system reliability, sensing, management, and control must be sufficiently fast so that optimal solutions can be quickly attained after perturbations. Fast convergence of the power system algorithms, particularly time sensitive algorithms, is essential for the reliability of the power system. This paper proves that communication packet loss significantly slows down the convergence of the distributed OLTA algorithm. In particular, the paper establishes the quantitative relationship between communication uncertainties and errors in distributed OLTA for DC MGs.

DC MGs are common and efficient in distribution-level smart grids. They support renewable generation and electric vehicle integration and can simplify control design by avoiding unnecessary ac/dc conversions. Many advanced control methods have been designed for high-performance DC MGs. Various optimization methods have been employed for DC MG power management, including primal-dual distributed optimal control algorithms [7], optimal power sharing strategy with line resistance [8], distributed optimal control architecture for multiple dc-dc converters [9], cooperative and distributed secondary/primary control framework [10]. Reference [11] proposed a decentralized current-sharing control strategy for fast transient response. Reference [12] proposed a trust-based cooperative controller to mitigate adverse effects of attacks on communication links and controller hijacking, [13] studied the impact of communication delay on the small-signal stability of MGs with distributed secondary frequency and voltage control, and [14] considered the asynchronous distributed power control problem of hybrid MGs. The previous results did not investigate stochastic communication uncertainties. In our early work, distributed optimization methods were introduced with rigorous convergence analysis [15]–[17].

Impact of communications on networked control systems has been investigated from different perspectives such as noisy communication channels [18], added constant delays [19], [20], and event-trigger-based strategies [21]–[23]. Reference

Masoud H. Nazari is with the Department of Electrical and Computer Engineering, Wayne State University, Detroit, Michigan 48202 <masoud.nazari@wayne.edu>

Siyu Xie is with the Department of Electrical and Computer Engineering, Wayne State University, Detroit, Michigan 48202 <syxie@wayne.edu>

Le Yi Wang is with the Department of Electrical and Computer Engineering, Wayne State University, Detroit, Michigan 48202 <lywang@wayne.edu>

George Yin is with the Department of Mathematics, University of Connecticut, Storrs, CT 06269-1009, <gyin@uconn.edu>

Wen Chen is with the Division of Engineering Technology, Wayne State University, Detroit, MI 48202, USA <wchenc@wayne.edu>

The research was supported in part by the Army Research Office under grant W911NF-19-0176.

[1] Although stochastic communication uncertainties will significantly affect frequency and voltage regulation, time for contingency clearance, among many other power efficiency and reliability issues, for concreteness this paper will treat only OLTA issues.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3062024, IEEE Transactions on Smart Grid

[24] modeled communication systems as a time-varying network topology in terms of mobility and established its impact on the network control quality. In [25], an in-depth study of coordinated control and communication design was conducted in which TCP-based communication protocols were employed. Reference [26] is concentrated on block erasure channels and [27] established safety distances. Impact of communication packet delivery ratio on highway platoon performance was studied in [28].

This paper models the power system communication network by a graph whose topology switches randomly. The random switching is represented by a sequence of independent and identically distributed random matrices. This modeling approach for communication uncertainties has been applied to diversified application domains such as binary channels [29], radio communication systems [30], fading channels [31], and mobile systems [32]. But, its application in power systems is new.

In our previous work [15]–[17], distributed OLTA algorithms are investigated assuming fixed and connected network topologies. This paper analyzes the impact of communication erasure channels on the convergence of distributed OLTA algorithms. Channel erasure is represented by a binary-valued random variable such that an erasure becomes a channel interruption in a given time interval, leading to a randomly switching network topology. Rigorous stochastic analysis of the impact of the communication uncertainties on the distributed power management algorithms is relatively an open field.

The main contributions of this paper are as follows: 1) Implementing stochastic network models to represent block erasure channels and communication network dynamics in DC MGs. 2) Embedding packet delivery ratio and network topology structure into the distributed OLTA algorithms for DC MGs, laying a foundation for rigorous analysis of integrated communication and optimization schemes. 3) Quantitatively characterizing the fundamental relationship between packet delivery ratios and convergence rates of the distributed OLTA to develop a practical criterion for securing reliability of load tracking under communication uncertainties. 4) Showing that the convergence rates of our algorithms can achieve asymptotically the Cramér-Rao lower bound. This fundamental result from statistics links the "information contents" of data to the "best possible" mean-square estimation errors. Consequently, the impact relationship established in this paper is fundamental and information-oriented, and can be used as a foundation for studying optimal resource allocation, reliability analysis, and other related topics.

The rest of the paper is arranged as follows. Section II formulates the problem, presents global optimality conditions, and develops distributed control algorithms for DC MG power management. Section III introduces stochastic models for erasure channels in communication systems. Impact of erasure channels on OLTA is analyzed in Section IV, in which error bounds, strong convergence, and asymptotic optimality errors are derived. By establishing convergence rates as functions of the erasure probabilities, impact of communication erasure channels is analyzed. Section V presents simulation case studies to show the impact of erasure channels on power system reliability. Finally, Section VI concludes the paper with a summary of the main findings and potential extensions.

II. Distributed Optimization for DC Power Management

A. Communication Packet Loss and Power System Reliability

The main question that this paper aims to study is: How reliable must the communication system be so that distributed OLTA algorithms can be performed with acceptable accuracy and reliability? The accuracy of OLTA for DC MGs is defined according to the error magnitude and duration from the optimal value.[2] Communication systems' reliability is modeled as reduced packet delivery ratio leading to a stochastic description of network topologies. Under high communication uncertainties, distributed OLTA algorithms will have fundamentally slower convergence rates, implying that it will take more steps to reach a desired accuracy on the OLTA solutions and hence increase the time for system recovery, which negatively affects the power system reliability.

It is observed that power system communications have critical timing specifications. IEEE 1646 standard mandates maximum time for data transfer. For example, power system protection messages need to be delivered within 4 milliseconds (ms), and monitoring and control data within 16 ms. The results of this paper further quantify how probability of packet delivery should be controlled to meet the system requirements.[3]

B. OLTA in DC MGs

We first specify a class of OLTA problems for DC MGs. For a DC MG with n subsystems that have their own generation capability and loads, the OLTA aims to find an "optimal" way to determine the amount of electricity each subsystem should produce.

The control variables, loads, and network variables can be current, power, or other meaningful physical variables. However, for clarity and consistency of presentation, we use "current" as the typical interpretation. During the kth time interval, the subsystem i has a controllable current u_k^i that represents all controllable assets, such as traditional generators, and battery energy storage systems. Likewise, the stochastic uncertain load current l_k^i represents all assets with stochastic behavior, such as renewable generators, commercial and residential loads, charge stations for electric vehicles (EVs), and/or trolleybuses on feeder lines. The current from node i to node j is denoted by the network state x_k^{ij} with

[2]This type of requirements is a commonly used quality metric: For example, IEEE Standard $1250-2018$ specifies reliability requirements on voltage quality. A voltage swell (or sag) occurs when the RMS voltage exceeds (or falls below) the nominal voltage by $10-80\%$ for 0.5 cycle to 1 minute. Undesirable voltage flicker is the scenario when random variations in the RMS voltage are within $90-110\%$ of the nominal value. More sever voltage quality deterioration involves "under-voltage" (or "over-voltage") when the nominal voltage drops below 90% (or rises above 100%) for more than 1 minute.

[3]Communication systems' bandwidth allocation and transmission power are commonly used to ensure a required packet delivery ratio.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3062024, IEEE Transactions on Smart Grid

$x_k^{ij} = -x_k^{ji}$. The vectors, containing the control, loads, and network states, will be denoted by $u \in \mathbb{R}^{n\times 1}$, $\ell \in \mathbb{R}^{n\times 1}$, and $x \in \mathbb{R}^{m\times 1}$, respectively. This paper does not directly deal with voltage management. However, bus-to-bus voltage variations are directly related to the link currents that are part of performance indices. As a result, voltage variations can be regulated by using higher weights on the link currents [4]. The network topology will be represented by an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V}$ is the set of buses and $\mathcal{E}$ is the set of transmission lines. [5] Here we will use the following separable and quadratic performance measure

$$\min J = f(u, x) = \frac{1}{2}(x^\top Q x + u^\top R u) \tag{1}$$

with $Q \in \mathbb{R}^{m\times m}$ and $R \in \mathbb{R}^{n\times n}$ are diagonal positive definite matrices. The variables are related by the equality constraints $u = Ax + l$ where $A = [a^{ij}] \in \mathbb{R}^{n\times m}$, see [15] for details. Due to the physical characteristics of the power system (the power network is connected), A is full column rank, meaning that for a given u, the dependent variable x is uniquely generated by the physical power system.

As a result, $x = (A^\top A)^{-1} A^\top (u - l)$. This is a global relationship since each u^i influences all x. Note that if x is given, the computation of u is local and can be easily performed, which is the local Kirchhoff's current law on the subsystems. However, given u, computation of x requires global information since a change on an input current will affect all network currents. When local input currents are changed, *the physical power system* will generate x for any given u. In other words, if one can *measure* x, there is no need to *calculate* x. This cyber (calculating u) and physical (measuring x) interaction allows us to develop distributed algorithms. This important aspect will be illustrated in Section V.

The mathematics formulation in this paper considers equality constraints. However, typical inequality bounds can be accommodated. Most common inequality constraints in power system optimization problems are bounds on control inputs, loads, and currents on transmission lines. These bounds can be readily included in our algorithms. We only assume that the actual optimal solutions are in the interior of the bounding sets, which simply mean that the normal optimal operation is within the rated bounds. During algorithm implementation, when a variable moves outside the bounds, a projection operator can be applied to force it back into the feasible region. Our results on convergence, convergence rate, and fundamental impact of packet delivery ratio remain valid and intact, which will be illustrated by Example 5.2 in Section V. More general inequality constraints that involve interconnected coupling of multiple variables and network dependence are far more complicated and beyond the scope of this paper.

[4]The scope of this paper is on load tracking and allocation in DC MGs. The reader is referred to [17] for an integrated load distribution and voltage regulation.

[5]Traditional power grids are tree types. Some common DC MGs employ connected radial systems that do not contain loops. The results of this paper cover such systems but are not limited to them.

C. Optimality Conditions

Theoretically, the global optimal solution can be obtained by the Lagrange Multiplier method: For $\tilde{\beta} \in \mathbb{R}^{n\times 1}$,

$$L = \frac{1}{2}(x^\top Q x + u^\top R u) + \tilde{\beta}^\top (u - Ax - l) \tag{2}$$

with the optimality condition $\begin{cases} Qx - A^\top \tilde{\beta} &= 0; \\ u - Ax - l &= 0; \\ Ru + \tilde{\beta} &= 0. \end{cases}$ The optimal solution is then

$$x^* = -(Q + A^\top R A)^{-1} A^\top R l, \quad u^* = Ax^* + l. \tag{3}$$

However, this solution requires global information and is not feasible in a distributed framework.

D. Distributed Algorithm with Embedded Communication Uncertainty

We first seek distributed optimization on x, and then update u accordingly. From (1) and $u^i = \sum_{j\in\mathcal{N}_i} x^{ij} + l^i$,

$$J = \frac{1}{2}\Big(\sum_{(i,j)\in\mathcal{E}} q^{ij}(x^{ij})^2 + \sum_{i\in\mathcal{V}} r^i \big(\sum_{j\in\mathcal{N}_i} x^{ij} + l^i\big)^2\Big).$$

Since $x^{ij} = -x^{ji}$,

$$J^{ij} = \frac{1}{2}\left((q^{ij} + q^{ji})(x^{ij})^2 + r^i(x^{ij} + l^i)^2 + r^j(-x^{ij} + l^j)^2\right),$$

which implies that $\delta^{ij} = \frac{\partial J}{\partial x^{ij}} = b^{ij} x^{ij} + c^{ij}$, with $b^{ij} = q^{ij} + q^{ji} + r^i + r^j$ and $c^{ij} = r^i l^i - r^j l^j$. Note that if the load l is a constant vector, then c^{ij} is constant, which is known only to nodes i and j and can be used by these nodes in a strictly distributed algorithm.

Following the same order as x, define the vectors $\delta = [\delta^{ij}] \in \mathbb{R}^{m\times 1}$ and $C = [c^{ij}] \in \mathbb{R}^{m\times 1}$, and matrix $B = \mathrm{diag}[b^{ij}] \in \mathbb{R}^{m\times m}$. Then, $\delta = Bx + C$. Since $Q > 0$ and $R > 0$ are diagonal, each element of the matrix B is positive. Thus, B is positive definite. The communication network is assumed to be identical to $\mathcal{G}$. However, during implementation, packet loss and channel interruption may cause the cyber link to be randomly disconnected. This is represented by an indicator function $\lambda_k^{ij} = \begin{cases} 1, & \text{if the link } (i,j) \text{ is connected at step } k, \\ 0, & \text{if the link } (i,j) \text{ is broken at step } k, \end{cases}$ which is a random variable. Denote $\lambda_k = \mathrm{diag}[\lambda_k^{ij}] \in \mathbb{R}^{m\times m}$ in the same order as x.

Next, we distinguish the cyber update (calculated value) from the *physical variable* x. We use y^{ij} to denote the *computed value* in the updating algorithm, Algorithm 1. Variable $y = [y^{ij}] \in \mathbb{R}^{m\times 1}$ has the same order as x. The main updating step in Algorithm 1 uses measured $\widehat{x}_k = x_k + d_k$ in the form $y_{k+1} = y_k - \mu_k \lambda_k (Bx_k + C + Bd_k)$, where the step size μ_k is designed.

The control u is updated from the computed y_k by the distributed relationship $u_k = Ay_k + l$. As a result, the updating algorithm can be represented by the following cyber-physical interactive steps.

Algorithm 1:

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3062024, IEEE Transactions on Smart Grid

1) Initial Condition: u_0 is selected, and y_0 is equal to the initial measured $\widehat{x}_0$.
2) From u_k at step k, the control and state are updated by

$$\begin{aligned} x_k &= (A^\top A)^{-1}A^\top(u_k - l),\ \widehat{x}_k = x_k + d_k \\ y_{k+1} &= y_k - \mu_k\lambda_k(B\widehat{x}_k + C),\ u_{k+1} = Ay_{k+1} + l, \end{aligned}$$

where x_k, $\widehat{x}_k$, y_{k+1}, and u_{k+1} are the physical implementation, distributed measurement, distributed cyber update, and distributed control update, respectively.

According to Algorithm 1, $y_{k+1} = (A^\top A)^{-1}A^\top(u_{k+1} - l)$. Without noise or error in communication, $x_{k+1} = y_{k+1}$. While the updating algorithm is strictly distributed, for convergence analysis, we use a more generic expression

$$x_{k+1} =(I - \mu_k\lambda_k B)x_k - \mu_k\lambda_k C - \mu_k\lambda_k B d_k.$$

During real-time implementation of Algorithm 1, if the link (i, j) is broken at step k, the corresponding variables will not be updated. This is equivalent to using the modified (random) gradient $\lambda^{ij}\delta^{ij}$, or in the compact form $\lambda\delta$ for the entire system.

Define $M(\lambda_k) = \lambda_k B \in \mathbb{R}^{m\times m}, P(\lambda_k) = \lambda_k C \in \mathbb{R}^{m\times 1}$. Then,

$$x_{k+1} = (I - \mu_k M(\lambda_k))x_k - \mu_k P(\lambda_k) - \mu_k M(\lambda_k)d_k. \quad (4)$$

Convergence analysis is carried out on (4). Note that (4) contains two stochastic processes, i.e., the noise d_k and network topology switching process λ_k.

The optimal solution x^* satisfies

$$M(\lambda_k)x^* + P(\lambda_k) = \lambda_k B x^* + \lambda_k C = 0 \quad (5)$$

for all k. Define the optimality error $\widetilde{x}_k = x_k - x^*$. Then by (4) and (5), we can show that

$$\widetilde{x}_{k+1} = (I - \mu_k M(\lambda_k))\widetilde{x}_k - \mu_k M(\lambda_k)d_k. \quad (6)$$

From (4) and (6), one can see that

$$\widetilde{u}_k = u_k - u^* = A\widetilde{x}_k, \quad (7)$$

which means that the convergence of x_k implies the convergence of u_k. Thus, the convergence analysis of x_k is our main focus.

III. Communication Block Erasure Channels

Consider a DC MG supported by a wireless network. In Algorithm 1, data exchange among subsystems relies on communication channels. Within a decision time interval, an erasure channel can either deliver the data packet or lose it [28]. The loss of a packet may be caused by erasure of one or multiple bits within the packet during transmission [6], [33]. During data transmission in the kth time interval, the source generates a data block, which will be coded and transmitted to the receiver. Due to channel uncertainties, the received codeword is subject to possible erasure of bits. After decoding and error correction, the receiver either acknowledges receipt of the data, or indicates a packet erasure. Data re-sending is permitted only within the decision time interval. Probability of successful packet delivery is defined as the *packet delivery ratio*.

We assume that packet loss from all channels is mutually independent, and each channel's packet loss is an independent and identically distributed (i.i.d) sequence of random variables with probability p_k to be linked and $1-p_k$ to be disconnected.[6] Applying this scenario to all channels, we have a randomly switching network topology such that the probability for each topology is generated from individual link connection probabilities. As a result, all related matrices in the distributed power management algorithms are random.

When block erasure channels are mutually independent, λ_k^{ij} is an independent random variable. Suppose λ_k^{ij} is stationary and its packet delivery probability is $P\{\lambda_k^{ij} = 1\} = p^{ij}$. Let I_S be the indicator function of the event S. For a given cyber network configuration $\lambda_k = \mathrm{diag}[\lambda_k^{ij}]$, its probability of occurrence can be derived from channel packet delivery probabilities:

$$p_{\lambda_k} = \prod_{(i,j)} \Big(I_{\lambda_k^{ij}} p^{ij} + (1 - I_{\lambda_k^{ij}})(1 - p^{ij}) \Big). \quad (8)$$

IV. Main Properties

In this section, we establish convergence and rate of convergence of Algorithm 1. Let the set of possible network configurations be Λ. Since (4) is a stochastic approximation procedure, we can use the general framework in Kushner and Yin [35] to analyze its asymptotic properties. Recall that $A \in \mathbb{R}^{n\times m}$, $B \in \mathbb{R}^{m\times m}$, $\lambda_k \in \mathbb{R}^{m\times m}$, and for each $\lambda \in \mathbb{R}^{m\times m}$, $M(\lambda) \in \mathbb{R}^{m\times m}$, and $P(\lambda) \in \mathbb{R}^{m\times 1}$.

Assumption 4.1: (a) $\mathcal{G}$ is connected.[7] (b) The noise $\{d_k \in \mathbb{R}^{m\times 1}\}$ is a sequence of i.i.d. random variables such that $E[d_1] = \mathbf{0}_{m\times 1}$ and $E[d_1 d_1^\top] = \Sigma_d \in \mathbb{R}^{m\times m}$, where Σ_d is symmetric and positive definite. (c) $\{\lambda_k \in \mathbb{R}^{m\times 1}\}$ is i.i.d. with mean $\overline{\lambda} \in \mathbb{R}^{m\times m}$ and covariance $E[\lambda_1\lambda_1^\top] = \Sigma_\lambda \in \mathbb{R}^{m\times m}$. Both the mean and covariance are positive definite.

Note that the observation noises (from measurements, or computation, or communications) are stochastic, but their statistical coefficients are known, including their means and variances. When an observation error has non-zero mean, it is a measurement bias that will naturally introduce a bias on optimal solutions. However, this bias can be easily corrected: Since we know the mean μ, a simple subtraction of μ from the observation data will lead to a modified observation error sequence whose mean is then zero. Such transformation techniques are widely used in control systems and related fields. In practice, even when the mean is unknown, estimation algorithms can be added, leading to adaptive optimization algorithms. The topic of adaptation is beyond the scope of this paper and will not be further discussed.

A. Optimality Error Bounds

Define $\overline{M} = E[M(\lambda_1)] = E[\lambda_1]B = \overline{\lambda}B \in \mathbb{R}^{m\times m}, \overline{P} = E[P(\lambda_1)] = E[\lambda_1]C = \overline{\lambda}C \in \mathbb{R}^{m\times 1}$. Since both B and $\overline{\lambda}$

[6] Probabilistic models for erasure channels can be derived from channels' signal-to-noise ratio (SNR). In a VANET framework under low density parity-check (LDPC) convolutional coding [34], such models were derived in [25].

[7] Recall that an undirected graph is called connected if there is a path from each node in the graph to every other node.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3062024, IEEE Transactions on Smart Grid

are diagonal and positive definite, the following results can be obtained.

Proposition 4.1: Under *Assumption 4.1*, the matrix $\overline{M}$ is positive definite.

Since λ_k and d_k are independent, and $E[d_k] = \mathbf{0}_{m\times 1}$, the mean $\eta_k = E[\widetilde{x}_k]$ satisfies $\eta_{k+1} = (I - \mu_k \overline{M})\eta_k$. Also, $\overline{M}$ is positive definite, if the step size is a constant $\mu_k = \mu$ and sufficiently small, $\gamma = \|I - \mu\overline{M}\| < 1$. This implies that $|\eta_k| \le \gamma^k |\eta_0|$ which converges to 0 exponentially.

For any deterministic matrix $X \in \mathbb{R}^{s\times t}$, the Euclidean norm is defined as $\|X\| = (\lambda_{\max}(XX^\top))^{\frac{1}{2}}$, and for any random matrix A, its norm is defined as $\|A\| = \{E[\|A\|^2]\}^{\frac{1}{2}}$.

Theorem 4.1: Suppose that $\mu_k = \mu$. Under *Assumption 4.1*, for all $k > 0$, there exists a constant $\mu^* = 1/\lambda_{\max}\{B\} > 0$, for any $\mu \in (0, \mu^*)$,

$$\|\widetilde{x}_k\| \le (1-\mu c)^k \|\widetilde{x}_0\| + N\sqrt{\mu}, \tag{9}$$

where $c \in (0,1)$ and $N > 0$ are two constants which are defined in the proof.

Proof: Recall that $M(\lambda_k) = \lambda_k B$. For any $\mu \in (0,\mu^*)$, $0 \le \mu M(\lambda_k) < I$ holds. Since $\overline{M}$ is positive definite, we know by *Theorem 2.1* in [36] that for any $k \ge i \ge 0$,

$$\left\| \prod_{j=i+1}^{k} (I - \mu M(\lambda_j)) \right\| \le (1-\mu c)^{k-i}, \tag{10}$$

where $c = 1 - (1 - \mu^* \lambda_{\min}\{\overline{M}\})^{1/64\mu^*} \in (0,1)$. Using (6), (10), and *Assumption 4.1*, by [37], we can obtain that $\|\widetilde{x}_{k+1}\| \le (1-\mu c)^{k+1}\|\widetilde{x}_0\| + N\sqrt{\mu}$, where $N = \sup_k \|d_k\|(\max_{k\ge 0}\{e^{-ck}\sqrt{k}\} + \frac{1}{c^2 e^{-2c}(1-e^{-c})}) > 0$. This completes the proof. ■

Theorem 4.1 establishes a range of step size choices that ensures the convergence of the OTLA algorithm. This is an important foundation for the theme of this paper since only within this range, the step size becomes a feasible design parameter.

Denote the mean-square error of $\widetilde{x}_k$ as $\Sigma_{\widetilde{x},k} = E[\widetilde{x}_k \widetilde{x}_k^\top]$, and $\Sigma_{\widetilde{x}} = \lim_{k\to\infty} \Sigma_{\widetilde{x},k}$, when the limits exist.

Theorem 4.2: Under *Assumption 4.1*, for all $k \ge 1$, there exists a constant $\mu^* > 0$ such that for any $\mu \in (0,\mu^*)$,

1) The error variance

$$\begin{aligned}\Sigma_{\widetilde{x},k+1} =& (I-\mu\overline{M})^{k+1}\Sigma_{\widetilde{x},0}(I-\mu\overline{M})^{k+1} \\ &+ \mu^2 \sum_{\ell=0}^{k} (I-\mu\overline{M})^{\ell}\overline{M}\Sigma_d\overline{M}(I-\mu\overline{M})^{\ell}.\end{aligned} \tag{11}$$

2) $\Sigma_{\widetilde{x}}$ is the solution to the Lyapunov equation $(I - \mu\overline{M})\Sigma_{\widetilde{x}}(I-\mu\overline{M}) - \Sigma_{\widetilde{x}} = -\mu^2\overline{M}\Sigma_d\overline{M}$, or explicitly

$$\Sigma_{\widetilde{x}} = \mu^2 \sum_{\ell=0}^{\infty} (I-\mu\overline{M})^{\ell}\overline{M}\Sigma_d\overline{M}(I-\mu\overline{M})^{\ell}. \tag{12}$$

Proof: 1) By (6) and *Assumption 4.1*, because $\Sigma_{\widetilde{x},k+1} = E[\widetilde{x}_{k+1}\widetilde{x}_{k+1}^\top]$,

$$\begin{aligned}\Sigma_{\widetilde{x},k+1} &= (I-\mu\overline{M})\Sigma_{\widetilde{x},k}(I-\mu\overline{M}) + \mu^2\overline{M}\Sigma_d\overline{M} \\ &= (I-\mu\overline{M})^{k+1}\Sigma_{\widetilde{x},0}(I-\mu\overline{M})^{k+1} \\ &\quad + \mu^2 \sum_{\ell=0}^{k} (I-\mu\overline{M})^{\ell}\overline{M}\Sigma_d\overline{M}(I-\mu\overline{M})^{\ell}.\end{aligned}$$

2) By (11) and *Assumption 4.1*, $I - \mu\overline{M}$ is stable, and

$$\Sigma_{\widetilde{x}} = \lim_{k\to\infty} \Sigma_{\widetilde{x},k} = \mu^2 \sum_{\ell=0}^{\infty} (I-\mu\overline{M})^{\ell}\overline{M}\Sigma_d\overline{M}(I-\mu\overline{M})^{\ell}.$$

Then $(I-\mu\overline{M})\Sigma_{\widetilde{x}}(I-\mu\overline{M}) = \mu^2\sum_{\ell=1}^{\infty}(I-\mu\overline{M})^{\ell}\overline{M}\Sigma_d\overline{M}(I-\mu\overline{M})^{\ell} = \Sigma_{\widetilde{x}} - \mu^2\overline{M}\Sigma_d\overline{M}$. The desired result thus follows. ■

Theorem 4.2 establishes the concrete, explicit, and accurate expression of the mean-square error on the optimal solution of the OTLA algorithm, that results from uncertainties in the communication system. Consequently, it represents the fundamental impact of packet delivery ratio and step size selection on obtaining the optimal solution. Since $\|I - \mu\bar{M}\| < 1$ and $0 \le \mu M(\lambda_k) < I$, the summation in (12) with ℓ from 0 to infinity is convergent to a finite limit. This value is actually the solution to the corresponding continuous-time Lyapunov equation[8].

B. Convergence and Convergence Rate

For practical applications, strong convergence (also called convergence with probability one (w.p.1)) is important since it ensures that almost all sample paths will be convergent. To achieve strong convergence, the step size must be chosen carefully.

Assumption 4.2: The step size satisfies the following conditions: $\mu_k \ge 0$, $\mu_k \to 0$ as $k \to \infty$, and $\sum_k \mu_k = \infty$.

The limit ODE (ordinary differential equation) is $\dot{x} = -\overline{M}x + \overline{P}$, whose equilibrium point is precisely the optimal solution $x^* = \overline{M}^{-1}\overline{P}$. Using the ODE method in stochastic approximation [35], define $t_k = \sum_{j=0}^{k-1}\mu_j$, $\varpi(t) = \max\{k : t_k \le t\}$, the piecewise constant interpolation $x^0(t) = x_k$ for $t \in [t_k, t_{k+1})$, and the shift sequence $x^k(t) = x^0(t + t_k)$. By Gronwall's inequality[9] and a standard argument, we can establish the following assertion: Under (A1), for any $0 < T < \infty$, $\sup_k E[|x_k|^2] \le K$ and $\sup_{0\le t\le T} E[|x^k(t)|^2] \le K$, for some $K > 0$. Thus, the following results can be obtained.

Theorem 4.3: Under *Assumptions 4.1* and *4.2*, Algorithm 1 converges to the optimal solution w.p.1.

While the actual proof of Theorem 4.3 will be skipped, the main ideas can be summarized as follows. If the step sizes are selected according to Assumption 4.2, the interpolated sequence $\{x^k(\cdot)\}$ is uniformly bounded and equicontinuous.[10] By Ascoli-Arzéla's theorem,[11] we can extract a subsequence $\{x^{k_\ell}(\cdot)\}$, which converges to $x(\cdot)$ on any compact intervals w.p.1 such that $x(\cdot)$ is a solution. The ODE has a unique

[8] If $P = \sum_{k=0}^{\infty} A^k Q A^k$, where A is symmetric and a contraction, then $APA - P = -Q$ is the Lyapunov equation. This is a set of linear equations and can be solved exactly, for example, by the Matlab function $P = dlyap(A,Q)$.

[9] A simplified version of Gronwall's inequality states that if for some $\beta > 0$, $\dot{x}(t) \le \beta x(t)$, then $u(t) \le u(0)e^{\beta t}$. This is used to derive a bound on the growth rate of the function from its derivative bound.

[10] Equicontinuous functions are continuous and have equal variation over a given neighborhood. This property is important for Algorithm 1 to converge to the optimal solution and is guaranteed if the step size is selected properly as in Assumption 4.2. In practice, a relatively small and constant step size may be selected to achieve a near-optimal solution in power system applications.

[11] This theorem provides conditions under which a sequence of a given family of real-valued continuous functions defined on a closed and bounded interval will contain a uniformly convergent subsequence, see [38].

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3062024, IEEE Transactions on Smart Grid

equilibrium point, which is the optimal solution. Now, by using the Lyapunov method, the equilibrium point x^* is an asymptotically stable point, since $-\overline{M}$ is stable by *Proposition 4.1*. This theoretical result leads to the desired property for Algorithm 1.

Corollary 4.1: Under the conditions of *Theorem* 4.3, the iterates $\{x_k\}$ generated by the OTLA algorithm (4) converge to the optimal solution $x_k \to x^*$ w.p.1 as $k \to \infty$.

For simplicity, we omit the verbatim proof and refer the reader to [35, Chapters 5 and 6].

Next, we demonstrate the convergence rate of the OTLA algorithm. Define $v_k = (x_k - x^*)/\sqrt{\mu_k}$, which takes a continuous-time interpolation as $v^0(t) = v_k$ for $t \in [t_k, t_{k+1})$, and define $v^k(t) = v^0(t + t_k)$. As in [35, Chapter 10], we can show that $v^k(\cdot)$ converges weakly to $v(\cdot)$ such that $v(\cdot)$ is a solution of an appropriate stochastic differential equation. The scaling factor $\sqrt{\mu_k}$ together with the asymptotic covariance gives the desired rate of convergence. The standard central limit theorem argument yields that $\frac{1}{\sqrt{k}} \sum_{j=\kappa}^{\kappa+k-1} M(\lambda_j) d_j$ converges weakly to $N(0, \Sigma)$, where $N(0, \Sigma)$ is a normal random variable whose variance is given by $\Sigma = E[M(\lambda_1) d_1 d_1^\top M^\top(\lambda_1)] = E[M(\lambda_1) \Sigma_d M^\top(\lambda_1)] \in \mathbb{R}^{m \times m}$.

C. Impact of Packet Delivery Ratio on Convergence Rate

Before we establish the convergence rate of Algorithm 1, we recall that the word "convergence rate" is the property of how fast the sequence x_k and u_k moves toward the optimal solution. A typical form of the convergence rate is $\|x_k - x^*\| \leq \frac{c}{k^p}$. Therefore, smaller c and larger p are desirable. If convergence becomes faster, it takes less time to return to the desired range of optimal point. Thus, the power system can recover faster after a disturbance.

Now, we show how the packet loss ratio will affect convergence rate of Algorithm 1. We first emphasize that under the given observation noise and packet delivery ratio, there is a fundamental lower bound on the achievable convergence rate of Algorithm 1, called the Cramér-Rao (CR) bound [39], which is the lower bound on the variance of unbiased estimators of a fixed and unknown parameter given the observation data points. It is the reciprocal of the Fisher information. The Fisher information is calculated from the variance of the natural logarithm of the likelihood function. Any algorithm that can achieve the CR bound is the "fastest" algorithm. To achieve the CR bound, we derive an improved algorithm that provides the "optimal" convergence rate in the sense of the CR bound. This is done by using iterate averaging to build a revised algorithm. For simplicity, we take $\mu_k = 1/k^\gamma$, where $(1/2) < \gamma < 1$. Then,

$$x_{k+1} = x_k - \frac{1}{k^\gamma}(M(\lambda_k)x_k + P(\lambda_k) + M(\lambda_k)d_k), \qquad (13)$$

and $\overline{x}_k = \sum_{j=0}^{k-1} x_j/k$. Thus, the following results can be obtained:

Theorem 4.4: $\sqrt{k}(\overline{x}_k - x^*)$ converges weakly to a normal random variable with mean 0 and asymptotic covariance $\Sigma^* = \overline{M}^{-1} \Sigma \overline{M}^{-1}$.

While we omit the proof for simplicity [12], we point out that the interpretation is that $\overline{x}_k - x^*$ is asymptotically normal (Gaussian distributed) with covariance Σ^*/k (or in short, $\overline{x}_k - x^* \sim N(0, \Sigma^*/k)$). For (13), references [40]–[42] showed that $\bar{x}_k$ will converge to its limit at a convergence rate that approaches asymptotically the corresponding CR lower bound. This implies that the improved algorithm can achieve the CR bound asymptotically. In this sense, Σ^* is the "smallest" covariance possible.

The asymptotic covariance is a common error measure and a main performance indicator for the convergence rate (mean-square convergence). Hence, it is a measure of reliability for DC MGs against communication uncertainties. Note that this is a general performance indicator, not limited to distributed algorithms. As a result, we use the error covariance matrix Σ^* to evaluate how fast convergence to the optimal solution can be achieved and how the convergence rate depends on the packet delivery ratio.

For simplification, here we use a single packet delivery ratio ρ for all channels. This simplification allows us to graphically illustrate the relationship between ρ and the corresponding Cramér-Rao lower bound. As a result, Theorem 4.4 can be better understood. However, the results of this paper can be easily extended to the case when each channel has a distinct packet delivery ratio.

Theorem 4.5: Under *Assumption 4.1*, if all links have a uniform packet delivery probability ρ (probability $1 - \rho$ for erasure), then $\Sigma^* = \frac{1}{\rho}\Sigma_d$.

Proof: Under the hypothesis, the covariance matrix $\Sigma^* = \overline{M}^{-1} \Sigma \overline{M}^{-1}$ is specified to $\overline{M} = E[M(\lambda_1)] = E[\lambda_1 B] = \rho B$, and $\Sigma = E[M(\lambda_1)\Sigma_d M^\top(\lambda_1)] = E[\lambda_1 B \Sigma_d B \lambda_1] = \rho B \Sigma_d B$, leading to $\Sigma^* = \frac{1}{\rho}\Sigma_d$. ■

From the explicit formula Σ^*, it can be observed that the higher the packet delivery ratio ρ is, the smaller Σ^* is and in turn the faster the convergence becomes. Consequently, this relationship explicitly links the communication reliability to the accuracy of OLTA in DC MGs. Examples will be given in the next section to illustrate the theoretical findings.

Note that the main goal of this paper is to establish the quantitative and fundamental relationship between communication packet loss and optimization, rather than an intuitive or qualitative trend. This relationship will become a foundation for resource allocation (on communication systems) and reliability assessment (on power systems). To obtain this fundamental relationship, we must first establish the "best possible" convergence rate over "all possible algorithms". This is achieved by introducing an algorithm that can achieve asymptotically the Cramér-Rao lower bound. This fundamental result from statistics links the "information contents" of data to the "best possible" mean-square estimation error. Since our algorithm can achieve this bound asymptotically, the impact relationship established in this paper is fundamental and information-oriented: Packet loss reduces information contents in data, and its impact on optimization is accurately characterized by the convergence rate (mean-square error sequence) toward the optimal solution.

[12] We refer the reader to Chapter 11 of [35] for mathematical details.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3062024, IEEE Transactions on Smart Grid

V. Case Studies and Discussions

In this section, we illustrate how the packet delivery ratio impacts the convergence rate of the OLTA algorithm in DC MGs. The convergence rate directly affects the reliability and resilience of the DC MGs in load tracking and power balancing, as discussed in Section II.

Example 5.1: Consider a DC MG with four nodes, shown in Fig. 1. The nodes have the generation capacities $\gamma = [\gamma^1, \gamma^2, \gamma^3, \gamma^4]^\top$. Suppose that the total load is L. The OLTA can be formulated as a weighted consensus problem, in which the optimal solution is obtained when the incremental costs of the generators are equal. Theoretically, this can be formulated as: For a given total load L, node i should contribute u^i, $i = 1, \ldots, 4$, satisfying $\frac{u^1}{\gamma^1} = \frac{u^2}{\gamma^2} = \frac{u^3}{\gamma^3} = \frac{u^4}{\gamma^4}$ and $\sum_{i=1}^4 u^i = L$. For this example, $\gamma = [12, 15, 20, 28]^\top$ kA and the total load is $L = 53.9$ kA. The initial power dispatch is $x_0 = [12, 14, 10.9, 17]^\top$ kA, which is not optimal. The grid is interconnected by a topology with 3 links $\mathcal{G} = \{(1,2), (2,3), (3,4)\}$.

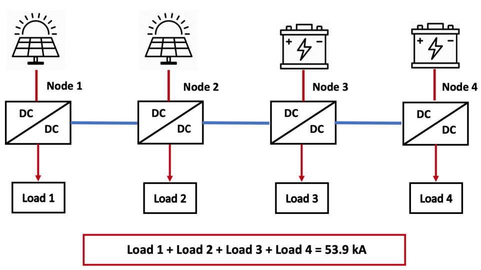


Fig. 1 The topology of the DC MG with four nodes.

During algorithm implementation, all observed variables are subject to random additive noises. Noises are independent and identically distributed Gaussian random variables with mean 0 and variance 0.2. Fig. 2 shows trajectories of nodal power productions, weighted powers, and power dispatching errors, when all links have the packet delivery ratio 0.9.

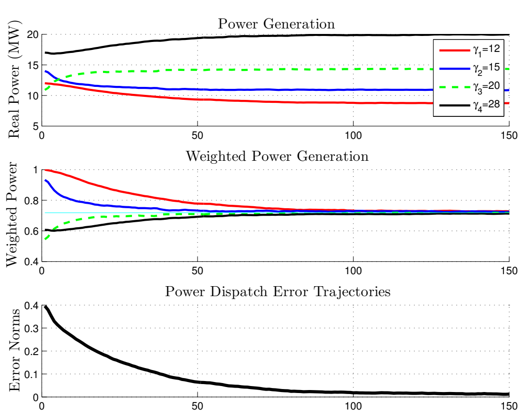


Fig. 2 Generation Power Trajectories

To further understand the impact of the packet delivery ratio on convergence rates, Fig. 3 compares error trajectories under three different scenarios. As shown in Fig. 3, by increasing the packet delivery ratio, the convergence rate of the OTLA algorithm for the four node system increases. To relate this to power system reliability, suppose that each iteration takes τ ms (milliseconds), which includes channel connection overhead, transmission, measurement time, and data processing delays. The slower the convergence rate, the more steps it takes for the OTLA algorithm to reach to the optimal solution. This implies that it takes longer time to recover from a disturbance or contingency, leading to a detrimental impact on the efficiency and reliability of power systems.

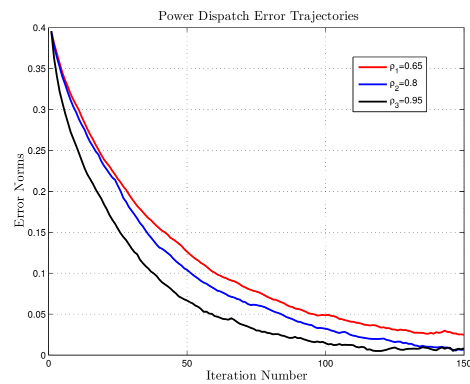


Fig. 3 Impact of different packet delivery ratio: 0.65, 0.8, and 0.95

To achieve 0.025 per unit (p.u.) error threshold, Table I lists convergence time for different packet delivery ratios. Apparently, improving packet delivery ratio can directly increase the power system reliability. This implies a desirable approach of control-communication co-design in which distributed optimization algorithms and communication resource allocation (such as transmission power) must be coordinated.

TABLE I Convergence under Different Packet Delivery Ratios.

Packet Delivery Ratio	Iteration Number	Time
0.65	140	1.4 s
0.8	125	1.25 s
0.95	78	0.78 s

Example 5.2: The "Junbaose" district in the Beijing Dual-Source Trolleybus System is selected for this case study. The DC MG in the Beijing Dual-Source Trolleybus System is shown in Fig. 4. Certain details of the DC MG were presented in [15] and are summarized here for self-containment and clarification. The dual-source trolleybuses can be driven by electricity from either the power lines or on-board batteries. When the duration of over-current in one feeder is more than the set value, the protection circuit would trip, and the dual-source trolleybuses would have to drive by battery until the over-load contingency is cleared. There are 130 km supply lines, which are divided into 75 km segments and powered by the feeders. The trolleybuses pass between segments by using on-board battery power. $4-8$ power segments are connected to a power station. The feeder is about 2 km. Note that a segment can be represented abstractly as a node, although the internal structure, circuits, protection system, and load condition can be highly complex. Segments can exchange electricity and form a connected network, as shown in Fig. 4. Each segment controls its AC/DC converter by using

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3062024, IEEE Transactions on Smart Grid

the PWM-based power electronic control system. Over-load situations can be mitigated by using the OLTA algorithm, so that the power supply network can serve as many trolleybuses as possible.

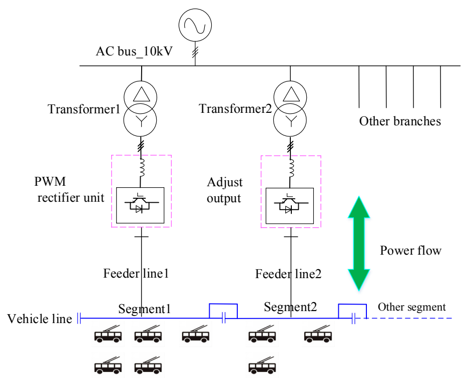


Fig. 4 Diagram of a new trolleybus power supply network [15]

The Junbaose power supply contains 6 feeders with the topology shown in Fig. 5. Its communication network is identical to the physical network. We assume that all lines are operated within the physical limit of 1100 A. Also, the load conditions and system parameters are obtained from the physical system measurements, see [15], [16] for system details.

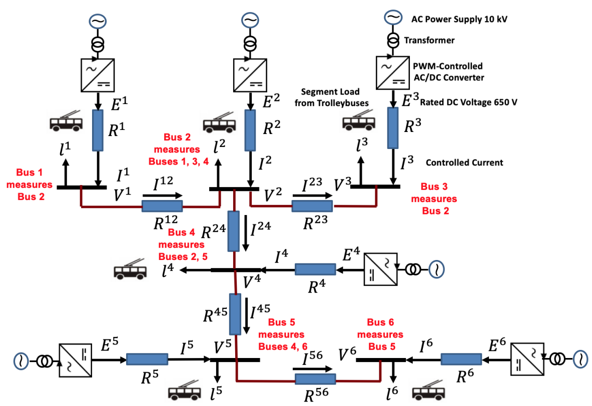


Fig. 5 The DC power network of the Junbaose trolley bus system.

The resistance of the power lines is 0.2 (Ω) per kilometer. Thus, the resistance of the power network is calculated and shown in Table II [15]. This leads to the power loss weighting matrix $Q = \mathrm{diag}\{0.4, 0.38, 0.34, 0.31, 0.36\}$.

TABLE II
Link Resistance of Junbaose Trolleybus Network.

Link Number	(1,2)	(2,3)	(2,4)	(4,5)	(5,6)
Link Length (km)	2	1.9	1.7	1.55	1.8
Line Resistance (Ω)	0.4	0.38	0.34	0.31	0.36

The network has a communication network, which follows the power grid topology. Thus, $u = [I^1, I^2, I^3, I^4, I^5, I^6]^\top$, $x = [I^{12}, I^{23}, I^{24}, I^{45}, I^{56}]^\top$, and $\ell = [\ell^1, \ell^2, \ell^3, \ell^4, \ell^5, \ell^6]$ with $A = [1\,0\,0\,0\,0; -1\,1\,1\,0\,0; 0\,-1\,0\,0\,0; 0\,0\,-1\,1\,0; 0\,0\,0\,-1\,1; 0\,0\,0\,0\,\,-1]$. Then, $u = Ax + \ell$ holds. Here, we assume that the initial currents of feeder lines are $I_0^1 = 713A, I_0^2 = 811A, I_0^3 = 960A, I_0^4 = 844A, I_0^5 = 887A, I_0^6 = 823A$ and the loads on the segments are $\ell^1 = 681A, \ell^2 = 783A, \ell^3 = 1009A, \ell^4 = 842A, \ell^5 = 921A, \ell^6 = 803A$.

The OLTA algorithm has two objectives: 1) Balancing the feeder currents, namely $I_k^i \to \bar{I}, i = 1, \ldots, 6$, where $\bar{I} = \sum_{i=1}^6 I_0^i/6$; 2) Reducing the power losses (or transmission costs) $\sum_{ij}^5 (I^{ij})^2 Q^{ij} = x^\top Q x$. Note that the first criterion is to avoid over-load by optimally allocating loads to different segments.

Thus, the performance index is given as follows:

$$\min_{x,u} J = \frac{1}{2}\Big(x^\top Q x + (u - \bar{I}z)^\top (u - \bar{I}z)\Big),$$
$$s.t. \quad u = Ax + \ell, \tag{14}$$

where z is a column vector of all 1's. The optimal solution is $x^* = -(Q + A^\top A)^{-1} A^\top \ell$ and $u^* = Ax^* + \ell$.

We assume that the link observation noise is a sequence of i.i.d. Gaussian random variables with mean zero and variance 6. [13] We also assume that at each time instant k, the currents of feeder lines should be in the range of $[0A, 900A]$, which are bounding constraints for the optimization problem. Here, we add projection operators to u_k^i: If u_k^i is out of the required range, we will project it slightly inside the bound. The trajectories for u_k^3 with and without projection operators are given and compared in Fig. 6, where there is no packet loss. It illustrates that the projection operators have been activated during the beginning of the iteration steps. Apparently our algorithms still converge to the optimal values under bounding constraints. Fig. 6 also shows that the inequality constraints may slow down the convergence rate. In other words, at each iteration if u_k^3 is out of the range $[0A, 900A]$, it will be projected inside the bound to ensure that the inequality constraint is satisfied. Thus, it takes more iterations to converge. It is shown in Fig. 6 that the trajectory without the inequality constraint converges within 30 iterations, while the trajectory with the inequality constraint needs at least 50 iterations to converge.

Moreover, the error trajectories of $\|u_k - u^*\|$ are shown in Fig. 7 under four different packet delivery ratios: 0.4, 0.6, 0.8, 1. Therefore, by increasing the packet delivery ratio, the OTLA algorithm converges faster.

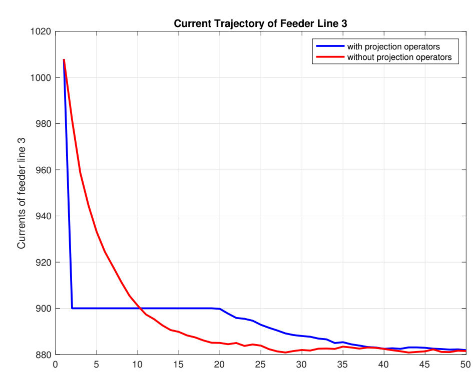


Fig. 6 Current trajectory of feeder line 3 with and without projection operators

[13] This is an estimate on the variations on combined measurement and communication errors on signals.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3062024, IEEE Transactions on Smart Grid

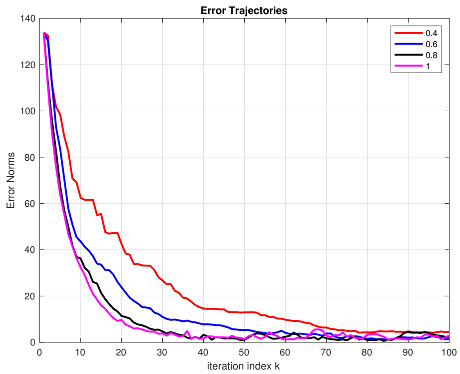


Fig. 7 Error trajectories with different packet delivery ratios

VI. Conclusions

Modern DC MGs increasingly rely on wireless communication systems for data exchange. Rigorous analysis of the impact of communication systems on DC MGs is essential for their efficient and reliable operation. To facilitate such analysis, the common erasure channels in communication systems are modeled by using probabilistic descriptions of packet delivery ratio. The proposed channel model is congregated to derive a stochastic description of the information network topology in DC MGs. Stochastic approximation methods are used for the convergence analysis of distributed OLTA algorithms in DC MGs. By embedding the information network switching model into the OLTA algorithm and performing stochastic analysis, one can establish a rigorous relationship between the packet delivery ratio of communication erasure channels, algorithm convergence rate, and accuracy of OLTA solutions. In other words, increased packet delivery ratio can reduce communication latency, which in turn can reduce the solution error and accelerate the convergence rate of the OLTA algorithm. Consequently, this increases the reliability of DC MGs.

There are several potential future directions for extending the results. For instance, more complicated communication channel models can be considered, such as Markov chains. Also, the methods can be extended to AC power systems and other power system algorithms such as frequency regulation and automatic demand management. Furthermore, integration with vehicle control systems will lead to more comprehensive integrated grid-vehicle management systems.

References

[1] M. Kuzlu, M. Pipattanasomporn, and S. Rahman, "Communication network requirements for major smart grid applications in han, nan and wan," *Computer Networks*, vol. 67, pp. 74 – 88, 2014.

[2] V. C. Gungor, D. Sahin, T. Kocak, S. Ergut, C. Buccella, C. Cecati, and G. P. Hancke, "Smart grid technologies: Communication technologies and standards," *IEEE Transactions on Industrial Informatics*, vol. 7, no. 4, pp. 529–539, Nov 2011.

[3] Y. Wang, W. Li, and J. Lu, "Reliability analysis of wide-area measurement system," *IEEE Transactions on Power Delivery*, vol. 25, no. 3, pp. 1483–1491, July 2010.

[4] G. Celli, E. Ghiani, F. Pilo, and G. G. Soma, "Reliability assessment in smart distribution networks," *Electric Power Systems Research*, vol. 104, pp. 164–175, 2013.

[5] B. Falahati, Y. Fu, and L. Wu, "Reliability assessment of smart grid considering direct cyber-power interdependencies," *IEEE Transactions on Smart Grid*, vol. 3, no. 3, pp. 1515–1524, Sept 2012.

[6] A. Lapidoth, "The performance of convolutional codes on the block erasure channel using various finite interleaving techniques," *IEEE Transactions on Information Theory*, vol. 40, no. 5, pp. 1459–1473, 1994.

[7] Z. Wang, F. Liu, Y. Chen, S. H. Low, and S. Mei, "Unified distributed control of stand-alone dc microgrids," *IEEE Transactions on Smart Grid*, vol. 10, no. 1, pp. 1013–1024, 2019.

[8] A. Khorsandi, M. Ashourloo, and H. Mokhtari, "A decentralized control method for a low-voltage dc microgrid," *IEEE Transactions on Energy Conversion*, vol. 29, no. 4, pp. 793–801, 2014.

[9] M. Baranwal, A. Askarian, S. Salapaka, and M. Salapaka, "A distributed architecture for robust and optimal control of dc microgrids," *IEEE Transactions on Industrial Electronics*, vol. 66, no. 4, pp. 3082—3092, 2019.

[10] V. Nasirian, S. Moayedi, A. Davoudi, and F. L. Lewis, "Distributed cooperative control of dc microgrids," *IEEE Transactions on Power Electronics*, vol. 30, no. 4, pp. 2288–2303, 2015.

[11] H. Wang, M. Han, R. Han, J. M. Guerrero, and J. C. Vasquez, "A decentralized current-sharing controller endows fast transient response to parallel dcdc converters," *IEEE Transactions on Power Electronics*, vol. 33, no. 5, pp. 4362–4372, 2018.

[12] S. Abhinav, H. Modares, F. L. Lewis, and A. Davoudi, "Resilient cooperative control of dc microgrids," *IEEE Transactions on Smart Grid*, vol. 10, no. 1, pp. 1083–1085, 2019.

[13] X. Wu, Y. Xu, J. He, C. Shen, G. Chen, J. C. Vasquez, and J. M. Guerrero, "Delay-dependent small-signal stability analysis and compensation method for distributed secondary control of microgrids," *IEEE Access*, vol. 7, pp. 170 919–170 935, 2019.

[14] Z. Wang, L. Chen, F. Liu, P. Yi, M. Cao, S. Deng, and S. Mei, "Asynchronous distributed power control of multi-microgrid systems," *IEEE Transactions on Control of Network Systems*, 2020.

[15] D. Zhang, J. Jiang, L. Y. Wang, and W. Zhang, "Robust and scalable management of power networks in dual-source trolleybus systems: a consensus control framework," *IEEE Transactions on Intelligent Transportation Systems*, vol. 17, no. 4, pp. 1029–1038, Nov. 2015.

[16] D. Zhang, L. Y. Wang, J. Jiang, and W. Zhang, "Optimal power management in dc microgrids with applications to dual-source trolleybus systems," *IEEE Transactions on Intelligent Transportation Systems*, vol. 19, no. 4, pp. 1188–1197, April 2018.

[17] E. Sindi, L. Y. Wang, M. Polis, G. Yin, and L. Ding, "Distributed optimal power and voltage management in dc microgrids: applications to dual-source trolleybus systems," *IEEE Transactions on Transportation Electrification*, vol. 4, no. 3, pp. 778–788, June 2018.

[18] J. S. Freudenberg and R. H. Middleton, "Feedback control performance over a noisy communication channel," in *2008 IEEE Information Theory Workshop*. IEEE, 2008, pp. 232–236.

[19] R. Luck and A. Ray, "Experimental verification of a delay compensation algorithm for integrated communication and control systems," *International Journal of Control*, vol. 59, no. 6, pp. 1357–1372, 1994.

[20] E. A. Coelho, D. Wu, J. M. Guerrero, J. C. Vasquez, T. Dragičević, u. Stefanović, and P. Popovski, "Small-signal analysis of the microgrid secondary control considering a communication time delay," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 10, pp. 6257–6269, 2016.

[21] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, "Distributed event-triggered control for multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 5, pp. 1291–1297, 2012.

[22] S. Wen, X. Yu, Z. Zeng, and J. Wang, "Event-triggering load frequency control for multiarea power systems with communication delays," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 2, pp. 1308–1317, 2016.

[23] R. Han, L. Meng, J. M. Guerrero, and J. C. Vasquez, "Distributed nonlinear control with event-triggered communication to achieve current-sharing and voltage regulation in dc microgrids," *IEEE Transactions on Power Electronics*, vol. 33, no. 7, pp. 6416–6433, 2017.

[24] L. Moreau, "Stability of multiagent systems with time-dependent communication links," *IEEE Transactions on automatic control*, vol. 50, no. 2, pp. 169–182, 2005.

[25] L. Xu, L. Y. Wang, G. Yin, and H. Zhang, "Communication information structures and contents for enhanced safety of highway vehicle platoons," *IEEE Transactions on vehicular Technology*, vol. 63, no. 9, pp. 4206–4220, 2014.

[26] L. Xu, L. Y. Wang, G. Yin, and H. Zhang, "Impact of communication erasure channels on the safety of highway vehicle platoons," *IEEE*

Transactions on Intelligent Transportation Systems, vol. 16, no. 3, pp. 1456–1468, June 2015.

[27] T. Richardson and R. Urbanke, *Modern coding theory*. Cambridge university press, 2008.

[28] T. Nguyen, L. Y. Wang, G. Yin, H. Zhang, S. E. Li, and K. Li, “Impact of communication erasure channels on control performance of connected and automated vehicles,” *IEEE Transactions on Vehicular Technology*, vol. 67, no. 1, pp. 29–43, 2018.

[29] B. Fritchman, “A binary channel characterization using partitioned markov chains,” *IEEE transactions on Information Theory*, vol. 13, no. 2, pp. 221–227, 1967.

[30] H. S. Wang and N. Moayeri, “Finite-state markov channel-a useful model for radio communication channels,” *IEEE transactions on vehicular technology*, vol. 44, no. 1, pp. 163–171, 1995.

[31] Q. Zhang and S. A. Kassam, “Finite-state markov model for rayleigh fading channels,” *IEEE Transactions on communications*, vol. 47, no. 11, pp. 1688–1692, 1999.

[32] F. Babich and G. Lombardi, “A markov model for the mobile propagation channel,” *IEEE Transactions on Vehicular Technology*, vol. 49, no. 1, pp. 63–73, 2000.

[33] A. G. I Fabregas and G. Caire, “Coded modulation in the block-fading channel: Coding theorems and code construction,” *IEEE Transactions on Information Theory*, vol. 52, no. 1, pp. 91–114, 2006.

[34] D. J. Costello Jr, A. E. Pusane, S. Bates, and K. S. Zigangirov, “A comparison between ldpc block and convolutional codes,” in *Proc. Information Theory and Applications Workshop*, 2006, pp. 6–10.

[35] H. Kushner and G. Yin, *Stochastic approximation and recursive algorithms and applications*. 2nd ed. New York, NY, USA: Springer-Verlag, 2003.

[36] L. Guo, “Stability of recursive stochastic tracking algorithms,” *SIAM Journal on Control and Optimization*, vol. 32, no. 5, pp. 1195–1225, 1994.

[37] L. Guo and L. Ljung, “Performance analysis of general tracking algorithms,” *IEEE Transactions on Automatic Control*, vol. 40, no. 8, pp. 1388–1402, 1995.

[38] W. Rudin, *Principles of Mathematical Analysis*. McGraw-hill New York, 1976, vol. 3.

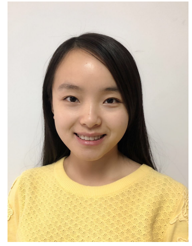
[39] T. Anderson, *An introduction to multivariate statistical analysis*. John Wiley & sons, New York, USA, 1984.

[40] G. Yin, “On extensions of polyak’s averaging approach to stochastic approximation,” *Stochastics: An International Journal of Probability and Stochastic Processes*, vol. 36, no. 3-4, pp. 245–264, 1991.

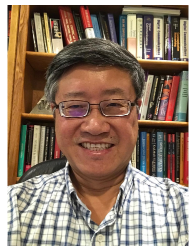
[41] B. T. Polyak and A. B. Juditsky, “Acceleration of stochastic approximation by averaging,” *SIAM journal on control and optimization*, vol. 30, no. 4, pp. 838–855, 1992.

[42] H. F. Chen, “Asymptotically efficient stochastic approximation,” *Stochastics: An International Journal of Probability and Stochastic Processes*, vol. 45, no. 1-2, pp. 1–16, 1993.

Masoud H. Nazari received his dual Ph.D. degree in electrical & computer engineering, and engineering & public policy from Carnegie Mellon University, Pittsburgh, PA, USA, in 2012. He was a Postdoctoral Fellow from 2013 to 2015 in the Electrical & Computer Engineering Department at the Georgia Institute of Technology. He was an Assistant Professor of Electrical Engineering (EE) with California State University, Long Beach, USA. He is currently an Assistant Professor of ECE with Wayne State University, Detroit, MI, USA. He is the Primary Investigator of a $2.5 million California Energy Commission Project to develop an innovative building energy management system. He has over 40 publications in the field. Dr. Nazari is the IEEE Senior Member in the Power and Energy Society (PES) and was the recipient of the Best Paper Award in the 2017 North American Power Symposium.



Siyu Xie received the B.S. degree in information and computing science (systems control) from Beijing University of Aeronautics and Astronautics in 2013, and Ph.D. degree in control theory from Academy of Mathematics and Systems Science, Chinese Academy of Sciences in 2018. She is currently a postdoctoral fellow at the Department of Electrical and Computer Engineering, Wayne State University, USA. Her research interests include networked systems, distributed optimization problems for power systems, distributed adaptive filters, machine learning, compressive sensing and distributed control.



Le Yi Wang (S’85-M’89-SM’01-F’12) received the Ph.D. degree in electrical engineering from McGill University, Montreal, Canada, in 1990. Since 1990, he has been with Wayne State University, Detroit, Michigan, where he is currently a Professor in the Department of Electrical and Computer Engineering. His research interests are in the areas of complexity and information, system identification, robust control, information processing and learning, as well as medical, automotive, communications, power systems, and computer applications of control methodologies. He was a plenary speaker in many international conferences. He serves on the IFAC Technical Committee on Modeling, Identification and Signal Processing. He was an Associate Editor of the IEEE Transactions on Automatic Control and several other journals.

George Yin (S’87-M’87-SM’96-F’02) received the B.S. degree in mathematics from the University of Delaware in 1983, and the M.S. degree in electrical engineering and the Ph.D. degree in applied mathematics from Brown University in 1987. He joined Wayne State University in 1987, became Professor in 1996, and University Distinguished Professor in 2017. He moved to the University of Connecticut in 2020. His research interests include stochastic processes, stochastic systems theory, and applications. He served as Co-chair for a number of conferences, was on the Board of Directors of the American Automatic Control Council, and was Chair of the SIAM Activity Group on Control and Systems Theory. He is Editor-in-Chief of SIAM Journal on Control and Optimization. He was an Associate Editor of Automatica 1995-2011, IEEE Transactions on Automatic Control 1994-1998, and Senior Editor of IEEE Control Systems Letters 2017-2019. He is a Fellow of IFAC and a Fellow of SIAM.

Wen Chen (SM’09) received the Ph.D. degree from Simon Fraser University, British Columbia, Canada in 2004. He was a postdoctoral researcher, from 2005 to 2007, at University of Louisiana at Lafayette, Louisiana, USA. In 2007, he became a Control Systems Engineer at Paton Controls and was hired by Triconex, Houston, Texas, USA in 2008. He joined Division of Engineering Technology, Wayne State University, USA, in 2009, as an Assistant Professor, where he is currently an Associate Professor. His research interests have been in the area of control and diagnosis of industrial systems.