Frugal Calibration of Mutual Coupling in Large Scale Antenna Array

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***Abstract*—In a time-division duplexing (TDD) system, the uplink (UL) and downlink (DL) channel reciprocity gets impaired due to the mismatches of the antenna radio frequency (RF) electronics. Meanwhile, in massive multiple-input multiple-output (MIMO), as the antenna elements are placed close to each other, the mutual coupling effects among the adjacent antennas also become pronounced. Different coupling matrices for transmission and reception also hurt the end-to-end UL/DL channel reciprocity. Aiming to restore the end-to-end channel reciprocity with a small amount of over-the-air (OTA) resources, we propose to estimate the gain mismatches of the RF electronics and the mutual coupling mismatches separately. Specifically, we rely on the pilot exchanges among the BS antennas and the symmetry of the antenna array to recover the transmit and receive gains of all the RF components. Unlike prior calibration works, our schemes can recover the off-diagonal elements in the calibration matrix as well. Our designs are also corroborated with numerical simulations.**

***Index Terms*—Massive MIMO, TDD, reciprocity, calibration, mutual coupling**

I. Introduction

In a time-division duplexing (TDD) multiple-input multiple-output (MIMO) communication system, the physical uplink (UL) and downlink (DL) channels are reciprocal [1]. However, due to the mismatches of the radio frequency (RF) gains at different antennas, the end-to-end UL and DL channels become non-reciprocal. Many approaches have been proposed to estimate and calibrate this non-reciprocity. In particular, the existing schemes can be classified into two type, i.e. the "Self-Calibration" method [2]–[4] and the "Over-the-Air" (OTA) calibration method [5]–[7], depending on whether the served users participate in the calibration process or not.

By installing a massive antenna array at the base station (BS), massive MIMO can bring huge improvements in both spectrum efficiency and energy efficiency [8]. It has been regarded as one key enabling technology for the next-generation communication system. As the antennas are placed compactly, the distance between adjacent antennas becomes small. Accordingly, the mutual coupling between adjacent antennas would have a significant impact on the overall channel gain [9]. Furthermore, the effects of mutual coupling on the UL and DL channels are usually not reciprocal due to the design variations among the antennas [10].

Recently, realizing the importance of TDD reciprocity calibration in massive MIMO systems, there have been many works addressing various calibration issues in massive MIMO. In particular, in [11], [12], the authors proposed some methods to reduce the OTA calibration complexity. The mutual coupling effects in massive MIMO were analyzed in [10], [13]. Furthermore, the strong coupling between adjacent antennas were exploited for self-calibration in [14], [15]. As far as the authors' knowledge, there have been no viable schemes addressing the calibration task when considering the non-reciprocal mutual coupling during transmission and reception. In this paper, we first show the RF gain mismatches and the mutual coupling mismatches can be estimated separately. This will be able to save a significant amount of OTA resources to restore the end-to-end channel reciprocity. We then put forth efficient methods to estimate both diagonal and off-diagonal elements of the calibration matrix containing all the RF gain mismatches. Taking advantage of the strong coupling, our proposed estimation schemes rely on the pilot exchanges among the adjacent BS antennas over the air and exhibit significant performance improvement over the conventional calibration schemes.

The rest of the paper is organized as follows. In Section II, we describe the mutual coupling effect and highlight the reciprocity calibration problem. Section III gives novel calibration methods by exploiting the mutual coupling effect. Numerical simulation results are discussed in Section IV and Section V concludes the paper.

Notations: Lowercase boldface $\mathbf{h}$ and uppercase boldface $\mathbf{A}$ denote vectors and matrices respectively. Notations $(\cdot)^T$, $(\cdot)^H$, and $\mathtt{Tr}(\cdot)$ denote transpose, Hermitian transpose, and trace respectively. $\mathbf{I}_K$ stands for the identity matrix of size $K \times K$. h_i denotes the i-th entry of the vector $\mathbf{h}$. $A_{i,j}$ denotes the (i,j)-th entry of the matrix $\mathbf{A}$. $\mathsf{Diag}\{\cdots\}$ denotes the diagonal matrix with diagonal elements inside the curly brackets. $\mathsf{diag}\{\mathbf{A}\}$ denotes the vector formed by the diagonal elements of $\mathbf{A}$. The element-wise division between $\mathbf{a}$ and $\mathbf{b}$ is denoted by $\mathbf{a}./\mathbf{b}$.

II. Preliminary and Problem Statement

In this section, we provide the necessary background for the antenna coupling effect [9] and demonstrate that the mutual coupling has to be calibrated to effect the end-to-end DL and UL channel reciprocity in TDD systems.

A. Mutual Coupling Effect

1) Coupling in Transmission Mode: Assume two antennas, e.g. antenna m and antenna n of an array, are positioned

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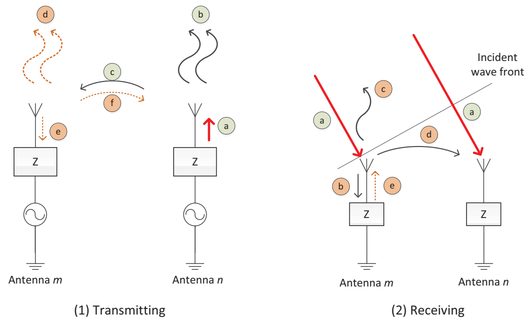

Fig. 1. Mutual coupling in transmission and reception modes.

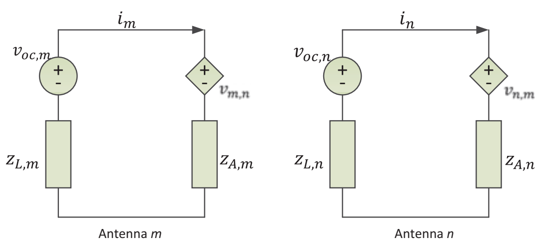

Fig. 2. Thevenin equivalent circuits of an antenna array [10]: $v_{oc,m}$ denotes the open-circuit voltage source; i_m represents the current in the equivalent circuit; $v_{m,n}$ stands for the coupled voltage due to the coupling between antenna-m and antenna-n; $z_{m,n} = v_{m,n}/i_n$ is the mutual impedance; $z_{A,m}$ is the self impedance; $z_{L,m}$ is the load impedance.

relative to each other as shown in Fig. 1. As a source is attached to antenna n, the generated energy (a) will be radiated into space (b) and toward antenna m (c). The energy incident on antenna m sets up currents which have a tendency to re-scatter some of the energy (d)+(f) and allow the remaining to travel toward the generator of m (e). Some of the re-scattered energy may be redirected back toward antenna n (f). This process will continue indefinitely.

2) Coupling in Reception Mode: As also shown in Fig. 1, a plane wave (a) impinges on the antenna array and strikes antenna m first, where it causes a current flow. Part of this incident wave will be re-scattered into space (c) and the other will be directed toward antenna n (d) where it will add vectorially with the incident wave (a). The other part will travel into its feed (b).

3) Coupling Matrices: According to the open-circuit voltage method [16], each antenna element in the array can be modelled as a Thevenin equivalent circuit as illustrated in Fig. 2. The transmitting coupling matrix $\mathbf{\Psi}_t$ is defined as $\mathbf{v}_{A,t} = \mathbf{\Psi}_t \mathbf{v}_{oc,t}$, where $\mathbf{v}_{A,t} := [v_{A,1}, \cdots, v_{A,M}]^T$ is the self-impedance voltage vector and $\mathbf{v}_{oc,t} := [v_{oc,1}, \cdots, v_{oc,M}]^T$ denotes the open-circuit voltage vector [10]. Similarly, the receiving coupling matrix $\mathbf{\Psi}_r$ is defined as $\mathbf{v}_{L,r} = \mathbf{\Psi}_r \mathbf{v}_{oc,r}$, where $\mathbf{v}_{L,r} := [v_{L,1}, \cdots, v_{L,M}]^T$ is the load-impedance voltage vector and $\mathbf{v}_{oc,r} := [v_{oc,1}, \cdots, v_{oc,M}]^T$ denotes the open-circuit voltage vector in the case of signal reception. Due to the symmetry of the mutual impedances, i.e. $z_{m,n} = z_{n,m}$, with applications of the Kirchhoff law, the following relationship between $\mathbf{\Psi}_t$ and $\mathbf{\Psi}_r$ can be obtained (details can be found in [10]):

$$\mathbf{\Psi}_t = \mathbf{Z}_A \mathbf{\Psi}_r^T \mathbf{Z}_L^{-1}, \tag{1}$$

where $\mathbf{Z}_A := \mathsf{Diag}\{z_{A,1}, \cdots, z_{A,M}\}$ is a diagonal matrix containing all the self-impedances along its diagonal and $\mathbf{Z}_L := \mathsf{Diag}\{z_{L,1}, \cdots, z_{L,M}\}$ contains all the load-impedances along the diagonal.

B. Problem Description

We consider a massive MIMO system where a BS with M antennas serving K $(M \gg K)$ single-antenna users. The end-to-end UL and DL channels, i.e. $\mathbf{G}_{UL}$ and $\mathbf{G}_{DL}$, can be expressed as follows [10]:

$$\mathbf{G}_{UL} = \mathbf{C}_{BS,r} \mathbf{\Psi}_r \mathbf{H}^T \mathbf{C}_{UE,t}, \tag{2}$$

$$\mathbf{G}_{DL} = \mathbf{C}_{UE,r} \mathbf{H} \mathbf{\Psi}_t C_{BS,t}, \tag{3}$$

where $\mathbf{H}$ represents the reciprocal physical propagation channel between the BS and the users, $\mathbf{\Psi}_t(\mathbf{\Psi}_r)$ is the coupling matrix during transmission(reception) at the BS, $\mathbf{C}_{BS,t}(\mathbf{C}_{BS,r})$ denotes the transmitter(receiver) RF gain matrix at the BS, and $\mathbf{C}_{UE,t}(\mathbf{C}_{UE,r})$ denotes the transmitter(receiver) RF gain matrix at the users (a.k.a. user equipment (UE)). In particular, these matrices are diagonal and of the following forms:

$$\mathbf{C}_{BS,t} = \mathsf{Diag}\{\alpha_1, \alpha_2, \cdots, \alpha_M\}, \tag{4}$$

$$\mathbf{C}_{BS,r} = \mathsf{Diag}\{\beta_1, \beta_2, \cdots, \beta_M\}, \tag{5}$$

$$\mathbf{C}_{UE,t} = \mathsf{Diag}\{a_1, a_2, \cdots, a_K\}, \tag{6}$$

$$\mathbf{C}_{UE,r} = \mathsf{Diag}\{b_1, b_2, \cdots, b_K\}. \tag{7}$$

From (2) and (3), the DL and UL channels can be related by

$$\mathbf{G}_{DL} = \mathbf{C}_{UE,r} \mathbf{C}_{UE,t}^{-1} \mathbf{G}_{UL}^T \mathbf{C}_{BS,r}^{-1} (\mathbf{\Psi}_r^T)^{-1} \mathbf{\Psi}_t \mathbf{C}_{BS,t}. \tag{8}$$

In practice, there exist minor differences between the antennas, which lead to the discrepancies among the self-impedances and the load-impedances of the antennas. As a result of (1), we see $(\mathbf{\Psi}_r^T)^{-1} \mathbf{\Psi}_t \neq \rho \mathbf{I}$ where ρ is a scalar. Even when the RF gains have been perfectly calibrated, i.e. $\mathbf{C}_{UE,r} = \mathbf{C}_{UE,t}$ and $\mathbf{C}_{BS,r} = \mathbf{C}_{BS,t}^{-1}$, from (8), we see the overall DL and UL channels are still not reciprocal due to the mutual coupling effects. Specifically, assuming all the RF gain matrices are equal to the identity matrix, we obtain

$$\mathbf{G}_{DL} = \mathbf{G}_{UL}^T (\mathbf{\Psi}_r^T)^{-1} \mathbf{\Psi}_t. \tag{9}$$

From (9), we see the UL and DL channels are not reciprocal unless $(\mathbf{\Psi}_r^T)^{-1} \mathbf{\Psi}_t = \rho \mathbf{I}$. During conventional calibrations, mutual coupling effects are typically neglected and the focus is on the estimation of the mismatches in RF gains, i.e. $\mathbf{C}_{BS,t}^{-1} \mathbf{C}_{BS,r}$. In massive MIMO systems, as the distance between adjacent antenna elements becomes very small, the mutual coupling effects can become pronounced and should not be ignored. Therefore, we also need efficient ways to estimate the differences between the transmitting and receiving mutual coupling matrices. Accordingly, we can compensate the differences to re-establish the end-to-end DL and UL channel reciprocity in the case of massive MIMO.

III. Mutual Coupling Calibration Method

In this paper, considering each user has only one antenna and there is no joint reception across the users, we assume the antennas at the users have been calibrated, i.e.

$$\mathbf{C}_{UE,r}\mathbf{C}_{UE,t}^{-1} = \mathbf{I}_K. \tag{10}$$

According to (8), we have $\mathbf{G}_{DL} = \mathbf{G}_{UL}^T\mathbf{M}$, where

$$\mathbf{M} := \mathbf{C}_{BS,r}^{-1}(\mathbf{\Psi}_r^T)^{-1}\mathbf{\Psi}_t C_{BS,t}. \tag{11}$$

The ultimate goal of calibration is thus to estimate the the *calibration matrix* $\mathbf{M}$. Define $\mathbf{W} := (\mathbf{\Psi}_r^T)^{-1}\mathbf{\Psi}_t$. From (4) and (5), the overall calibration matrix $\mathbf{M}$ in (11) can be expressed as

$$\mathbf{M} = \begin{bmatrix} \frac{\alpha_1}{\beta_1}W_{1,1} & \frac{\alpha_2}{\beta_1}W_{1,2} & \cdots & \frac{\alpha_M}{\beta_1}W_{1,M} \\ \frac{\alpha_1}{\beta_2}W_{2,1} & \frac{\alpha_2}{\beta_2}W_{2,2} & \cdots & \frac{\alpha_M}{\beta_2}W_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\alpha_1}{\beta_M}W_{M,1} & \frac{\alpha_2}{\beta_M}W_{M,2} & \cdots & \frac{\alpha_M}{\beta_M}W_{M,M} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{\alpha_1}{\beta_1} & \frac{\alpha_2}{\beta_1} & \cdots & \frac{\alpha_M}{\beta_1} \\ \frac{\alpha_1}{\beta_2} & \frac{\alpha_2}{\beta_2} & \cdots & \frac{\alpha_M}{\beta_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\alpha_1}{\beta_M} & \frac{\alpha_2}{\beta_M} & \cdots & \frac{\alpha_M}{\beta_M} \end{bmatrix} \odot \tag{12}$$
$$\begin{bmatrix} W_{1,1} & W_{1,2} & \cdots & W_{1,M} \\ W_{2,1} & W_{2,2} & \cdots & W_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ W_{M,1} & W_{M,2} & \cdots & W_{M,M} \end{bmatrix} := \mathbf{P} \odot \mathbf{W},$$

where $\odot$ represents element-wise matrix product (a.k.a. Hadamard product). By separating the calibration matrix $\mathbf{M}$ into two independent matrices, i.e. $\mathbf{P}$ and $\mathbf{W}$, we can estimate them separately via different mechanisms. Note the matrix $\mathbf{W}$ is determined by the mutual coupling matrices: $\mathbf{\Psi}_t$ and $\mathbf{\Psi}_r$. Meanwhile, the mutual coupling effects depend on the positions of the antenna elements. Once the antenna array is fixed, the matrix $\mathbf{W}$ can be regarded fixed as well. We can estimate $\mathbf{W}$ at the beginning and treat it as constant between two calibration efforts. From (8), (10), and (12), we have

$$\mathbf{G}_{DL} = \mathbf{G}_{UL}^T(\mathbf{P} \odot \mathbf{W}). \tag{13}$$

From (13), in order to obtain an estimate of $\mathbf{W}$, we need to have knowledge about $\mathbf{G}_{DL}$, $\mathbf{G}_{UL}$, and $\mathbf{P}$. The UL channel $\mathbf{G}_{UL}$ can be estimated by the BS with UL pilots. The DL channel $\mathbf{G}_{DL}$ can be estimated by the UE first and then fed back to the BS. The remaining problem is how to acquire the *RF gain calibration matrix* $\mathbf{P}$. Either with the "Self-Calibration" or relying on the "Over-The-Air (OTA)" exchanges of messages, conventional calibration methods only aimed to estimate the diagonal entries in $\mathbf{P}$. In OTA approaches, some UEs are employed to assist the calibration task at the BS. Note that, to acquire $\mathbf{P}$ separately, the OTA calibration cannot be employed since the non-reciprocal mutual coupling effects will affect the system as in (12). To the best of the authors' knowledge, there have been no prior works explicitly addressing the estimation of the off-diagonal elements in the matrix $\mathbf{P}$ in the context of massive MIMO. Next we will describe some novel "Self-Calibration" algorithms to estimate $\mathbf{P}$, especially the off-diagonal entries.

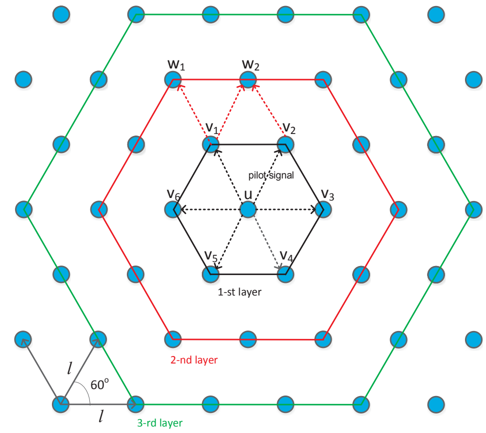

Fig. 3. Hexagonal grid antenna array and layers of different distances to the origin node u.

A. Pilot Exchanges Among Neighboring Antennas

During the "Self-Calibration", pilots are exchanged among the BS antennas. In [14], the authors proposed a similar "Self-Calibration" method by taking advantage of the strong coupling between adjacent antennas. However, only the diagonal entries of $\mathbf{P}$ were acquired with the approach in [14].

Similar to [14], we consider an antenna array on a hexagonal grid[1] as shown in Fig. 3. All the antennas are thin dipoles with distance between neighbors being $l = 0.5\lambda$, where λ denotes the wavelength corresponding to the nominal carrier frequency. First, let's examine the effects of mutual coupling among three adjacent antennas in Fig. 3, i.e. antenna-u, antenna-v_1 and antenna-v_2. In particular, we use h_{u,v_1}, $h_{v_1,u}$ h_{u,v_2}, $h_{v_2,u}$, h_{v_1,v_2}, and h_{v_2,v_1} to represent the corresponding mutual coupling gains between each antenna pair. Due to the symmetric distances between these 3 antennas, we have $h_{u,v_1} = h_{v_1,u} = h_{u,v_2} = h_{v_2,u} = h_{v_1,v_2} = h_{v_2,v_1} = h$ [14], [17][2]. With this symmetry, each antenna can transmit a pilot signal to its 6 (or less for those boundary antennas) adjacent antennas one-by-one. In each time, only the 6 closest neighbors are turned on to pick up the pilots. After this process, we can obtain a signal matrix $\mathbf{Y}$ whose (u, v)-th element, i.e. $Y_{u,v}$ ($1 \le u, v \le M$), denotes the signal received by antenna-v due to the transmission from antenna-u. When antenna-u and antenna-v are adjacent, we have

$$Y_{u,v} = \beta_v \cdot h \cdot \alpha_u + n_{u,v}, \tag{14}$$

[1]Note our approach also works for other antenna array layout, e.g. the rectangular antenna array. The only difference is the set of adjacent antennas of the same distance to one reference antenna

[2]When there exist small variations in the mutual coupling gains: $\{h_{u,v_1}, h_{v_1,u}, h_{u,v_2}, h_{v_2,u}, h_{v_1,v_2}, h_{v_2,v_1}\}$, we can still make this equality assumption and treat the variations as observation noise in (14). Simulated performance is given in Section IV.

where β_v is the receive RF gain for antenna-v, α_u is the transmitter RF gain of antenna-u, and $n_{u,v} \sim CN(0, \sigma^2)$ denotes the receiver noise. Note $Y_{u,v} = 0$ if antenna-u and antenna-v are not adjacent to each other.

B. Diagonal First Scheme

In this section, we provide one method to recover $\mathbf{P}$ by estimating the diagonal entries first. The off-diagonal elements are then estimated in a layer-by-layer iterative fashion.

1) Estimate of Diagonal Elements: In order to estimate the diagonal entries of $\mathbf{P}$, we can employ the Neighbor Least-Squares (N-LS) method as proposed in [15]. Define $\mathbf{c} := \gamma \left(\frac{\beta_1}{\alpha_1}, \frac{\beta_2}{\alpha_2}, \cdots, \frac{\beta_M}{\alpha_M}\right)^T$, where γ is a tunable scalar. Then $\mathbf{c}$ can be recovered by minimizing the following cost function:

$$J(\mathbf{c}) = \sum_{u=1}^{M} \sum_{v \in \mathcal{A}_u} \left| c_u Y_{u,v} - c_v Y_{v,u} \right|^2, \tag{15}$$

where $\mathcal{A}_u$ stands for the set of neighbor antennas to antenna-u. To minimize (15), one can simply set the gradient $\nabla J(\mathbf{c})$ to zero. To avoid the trivial solution $\mathbf{c} = \mathbf{0}$, we fix the first entry of $\mathbf{c}$ to $c_1 = 1$. Defining vector $\tilde{\mathbf{c}}$ as $\mathbf{c}^T = [c_1, \tilde{\mathbf{c}}^T]$, the optimal $\tilde{\mathbf{c}}$ is as follows:

$$\tilde{\mathbf{c}} = -(\mathbf{A}_1^H \mathbf{A}_1)^{-1} \mathbf{A}_1^H \mathbf{a}_1 c_1, \tag{16}$$

where $\mathbf{A} := [\mathbf{a}_1, \mathbf{A_1}]$ and the (u, v)-th element is defined as

$$A_{u,v} = \begin{cases} \sum_{i \in \mathcal{A}_u} |Y_{u,i}|^2, & u = v \\ -Y_{u,v}^* Y_{v,u}, & u \neq v, v \in \mathcal{A}_u \\ 0, & \text{otherwise} \end{cases}. \tag{17}$$

2) Estimate of Off-Diagonal Elements: Now we show how to estimate the off-diagonal elements in $\mathbf{P}$, i.e. α_u/β_v, $u \neq v \in [1, M]$. The antenna array can be viewed as a graph where each antenna corresponds to one node. Each node in the graph just connects with the closest neighboring nodes with edges. For clear elaboration, we define the "distance" between antenna-u and antenna-v as the length of the shortest path. The distance between adjacent antennas is assumed to be 1. With respect to the node at the origin, the set of nodes whose distances to the origin are equal to d form the d-th layer (see also Fig. 3).

An iterative method can be utilized to estimate the off-diagonal elements. In particular, during the k-th iteration, we will recover the ratios between the RF gains of the node-u and the nodes among the k-th layer of node-u.

- *Layer-1 Procedure:* Note all the nodes in the first layer are adjacent to node-u. In Fig. 3, node-$v_1 \sim$ node-v_6 are among the first layer of node-u. Define $\mathbf{P}_1^{(u)} := [\alpha_u/\beta_{v_1}, \cdots, \alpha_u/\beta_{v_6}]^T$. Since node-$u$ and node-v_i are adjacent, there must be another node, e.g. node-v_j, which is adjacent to both of them. With the definition of $c_{v_j} = \gamma\beta_{v_j}/\alpha_{v_j}$, by ignoring the measurement noise in (14), we can have

$$\frac{Y_{u,v_j}}{Y_{v_j,v_i}} \cdot \frac{1}{c_{v_j}} = \frac{1}{\gamma} \frac{\alpha_u}{\beta_{v_i}}. \tag{18}$$

In the presence of noise, by selecting different neighboring nodes, the estimate of α_u/β_{v_i} can be further improved. In particular, when we have node-v_{j_1} and node-v_{j_2} adjacent to both node-u and node-v_i, we can obtain a better estimate of α_u/β_{v_i} via averaging:

$$\left[\frac{\alpha_u}{\beta_{v_i}}\right]_{\texttt{avg}} = \frac{1}{2}\left(\frac{Y_{u,v_{j_1}}}{Y_{v_{j_1},v_i}} \cdot \frac{1}{c_{v_{j_1}}} + \frac{Y_{u,v_{j_2}}}{Y_{v_{j_2},v_i}} \cdot \frac{1}{c_{v_{j_2}}}\right). \tag{19}$$

- *Layer-k Procedure ($k \geq 2$):* Assuming node u' is among the k-th layer of node-u, we can find a shortest path from u to u', e.g. $u \to t_1 \to t_2 \to \cdots \to t_{k-1} \to u'$. Then $\alpha_u/\beta_{u'}$ can be calculated in a recursive manner as follows:

$$\frac{\alpha_u}{\beta_{u'}} = \frac{1}{2}\left(\frac{\alpha_u}{\beta_{t_1}} \cdot \frac{\alpha_{t_1}}{\beta_{u'}} \cdot c_{t_1} + \frac{\alpha_u}{\beta_{t_{k-1}}} \cdot \frac{\alpha_{t_{k-1}}}{\beta_{u'}} \cdot c_{t_{k-1}}\right), \tag{20}$$

where $\alpha_{t_1}/\beta_{u'}$ and $\alpha_u/\beta_{t_{k-1}}$ have been obtained during the iteration for layer-$(k-1)$. When there are L shortest paths from u to u', we can average over the L paths to improve the estimation quality as:

$$\left[\frac{\alpha_u}{\beta_{u'}}\right]_{\texttt{avg}} = \frac{1}{L}\sum_{l=1}^{L}\left[\frac{\alpha_u}{\beta_{u'}}\right]_{\texttt{path}-l}. \tag{21}$$

C. Batch Estimation Scheme

The "Diagonal First Scheme" re-uses the widely adopted LS formulation in (15) to recover the diagonal entries of the RF gain calibration matrix $\mathbf{P}$ first. The off-diagonal entries of $\mathbf{P}$ are then obtained iteratively in a layer-by-layer fashion. In this section, unlike the prior works, we propose to estimate $\{\alpha_m, \beta_m\}_{m=1}^M$ jointly in a batch way. To this end, with all the available observations in $\mathbf{Y}$, the maximum-likelihood (ML) estimates can be obtained by solving the following optimization problem:

$$\begin{aligned} \underset{\mathbf{X}_t, \mathbf{X}_r}{\text{minimize}} \quad & f(\mathbf{X}_t, \mathbf{X}_r) = \|\mathbf{Y} - \mathbf{X}_t \mathcal{A} \mathbf{X}_r\|_F^2 \\ \text{subject to} \quad & \mathbf{X}_t = \mathsf{Diag}\{x_{t,1}, \cdots, x_{t,M}\} \\ & \mathbf{X}_r = \mathsf{Diag}\{x_{r,1}, \cdots, x_{t,M}\}, \end{aligned} \tag{22}$$

where $\|\cdot\|_F$ denotes the matrix Frobenius norm and the matrix $\mathcal{A}$ characterizes the adjacency relationship of antennas as follows:

$$\mathcal{A}_{i,j} := \begin{cases} 1, & i\text{-th antenna and } j\text{-th antenna are adjacent;} \\ 0, & \text{otherwise.} \end{cases}$$

Neglecting the noise term in (14), we can have $\mathbf{Y} = h\mathbf{C}_{BS,t}\mathcal{A}\mathbf{C}_{BS,r}$, where h is the common mutual coupling gain between adjacent antennas as in (14). It can be shown that, due to the connectivity of the antenna array graph, the solutions minimizing the objective function in (22) to zero are as follows:

$$\mathbf{X}_t = \mu h \mathbf{C}_{BS,t}, \quad \mathbf{X}_r = \nu \mathbf{C}_{BS,r}, \tag{23}$$

where μ and ν are arbitrary scalars such that $\mu \cdot \nu = 1$.

In the presence of measurement noise, in order to recover $\mathbf{X}_t$ and $\mathbf{X}_r$ in (22) which minimize the biconvex objective function $f(\mathbf{X}_t, \mathbf{X}_r)$, we can exploit the block-coordinate descent (BCD) principle as follows [19]:

1) **Initialization:** Initial guesses of $\mathbf{X}_t^{(0)}$, $\mathbf{X}_r^{(0)}$;
2) **Iterations:**
Set $j \leftarrow 0$;
repeat
Given $\mathbf{X}_t^{(j)}$, obtain $\mathbf{X}_r^{(j+1)}$ as:

$$\mathbf{X}_r^{(j+1)} = \arg\min_{\mathbf{X}_r} f(\mathbf{X}_t^{(j)}, \mathbf{X}_r) = \mathsf{Diag}\left\{\mathsf{diag}\{\mathcal{A}\mathbf{X}_t^{(j)H}\mathbf{Y}\}./\mathsf{diag}\{\mathcal{A}\mathbf{X}_t^{(j)H}\mathbf{X}_t^{(j)}\mathcal{A}\}\right\};$$

Given $\mathbf{X}_r^{(j+1)}$, obtain $\mathbf{X}_t^{(j+1)}$ as:

$$\mathbf{X}_t^{(j+1)} = \arg\min_{\mathbf{X}_t} f(\mathbf{X}_t, \mathbf{X}_r^{(j+1)}) = \mathsf{Diag} \left\{\mathsf{diag}\{\mathbf{Y}\mathbf{X}_r^{(j+1)H}\mathcal{A}\}./\mathsf{diag}\{\mathcal{A}\mathbf{X}_r^{(j+1)}\mathbf{X}_r^{(j+1)H}\mathcal{A}\}\right\};$$

$j \leftarrow j+1$;
until termination test satisfied.

In the Appendix, the proof of the above results are outlined and the convergence behavior is analyzed.

D. Frugal Calibration via Separation

Through exchanging pilot signals among BS antennas, we can obtain the RF gain calibration matrix $\mathbf{P}$ subject to a single unknown scaling parameter. By exchanging pilot signals between the BS antennas and the users, we can estimate the channel matrix $\mathbf{G_{DL}}$ and $\mathbf{G_{UL}}$. Then the matrix $\mathbf{W}$ can be estimated according to (13). Note that in a typical massive MIMO system, the number of served UEs is less than the number of antennas at the BS. Thus one time of measurement of $\mathbf{G}_{DL}$ and $\mathbf{G}_{UL}$ is not enough to determine $\mathbf{W}$. One solution is to collect the channel measurements multiple times in different channel coherence time slots.

Once we have an updated $\mathbf{W}$, we can treat it as a constant matrix during the following calibration efforts. Given the UL channel $\mathbf{G}_{UL}$, as long as we acquire the matrix $\mathbf{P}$ according to the proposed procedures by exchanging pilots among the BS antennas, the BS will be able to infer the DL channel matrix and design optimal DL beamforming vectors accordingly.

It is important to note that the estimate of $\mathbf{W}$ necessitates the OTA exchanges of pilots between the BS and the users. On the other hand, the estimation of $\mathbf{P}$ can be completed solely by the BS without requiring any assistance from the users. Thus, by separating the calibration task in the case mutual coupling into two parts as in (13), a significant amount of OTA overheads can be saved.

IV. Simulated Performance

To compare the conventional diagonal-only calibration method [15] and the two calibration methods proposed in Section III for recovery of the whole calibration matrix $\mathbf{P}$, we assume the matrix $\mathbf{W} := (\mathbf{\Psi}_r^T)^{-1}\mathbf{\Psi}_t$ in (12) has been acquired. We consider two scenarios, i.e. the "uniform coupling gain" case and the "non-uniform coupling gain" case. For convenience, we refer to the conventional diagonal-only calibration method in [15] as "Conventional Scheme". Fig. 4 and Fig. 5 depict the DL sum spectral efficiency of a massive MIMO system configured as follows:

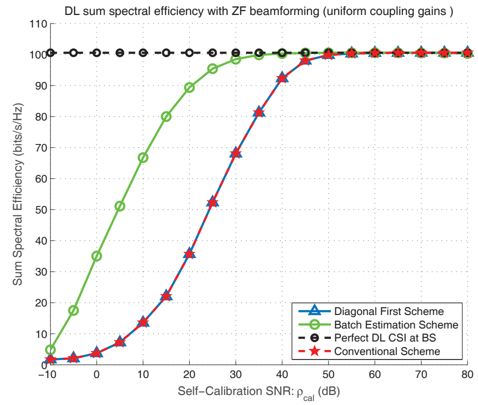

Fig. 4. DL sum spectral efficiency with ZF beamforming in the case of uniform coupling gains. ρ_{cal} refers to the observation SNR in (14).

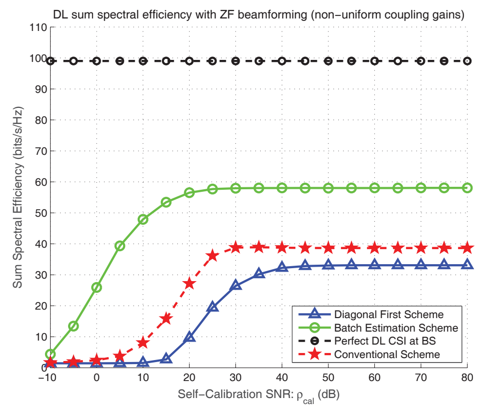

Fig. 5. DL sum spectral efficiency with ZF beamforming and non-uniform coupling gains.

- One 16×16 antenna array is installed at the serving BS as illustrated in Fig. 3. The BS serves 12 single-antenna users. The entries of the reciprocal physical channel $\mathbf{H}$ are modelled as IID zero-mean complex Gaussian. The total DL transmission power is normalized with the thermal noise at the user and is set at 10 dB (path loss in the DL is included);
- The logarithms of the magnitudes of the RF gains: $\{\ln|\alpha_m|, \ln|\beta_m|\}_{m=1}^M$ are zero-mean Gaussian with variance $\sigma_\alpha^2 = \sigma_\beta^2 = 1$. The phases of the RF gains are uniformly distributed over $[-\pi, \pi]$;
- The BS acquires perfect UL channels and relies on the calibrated UL channels to perform the DL zero-forcing (ZF) beamforming [13].

Fig. 4 considers the "uniform coupling gain" case, where all the BS antennas are designed identically. Clearly, this is an ideal scenario. For any two adjacent antennas u and v, the coupling gain $h_{u,v}$ is equal to a constant. Meanwhile, the mutual coupling matrices $\mathbf{\Psi}_t$ and $\mathbf{\Psi}_r$ in (1) become reciprocal [10], i.e. $(\mathbf{\Psi}_r^T)^{-1}\mathbf{\Psi}_t \propto \rho\mathbf{I}$. From Fig. 4, we see our "Batch Estimation Scheme" always outperforms the "Conventional Scheme", especially in the low SNR regime. The "Diagonal First Scheme" gives the same performance as the "Conventional Scheme". This is due to the fact that these two methods estimate the diagonal elements of the calibration matrix $\mathbf{P}$ in the same way. Note that in the "uniform coupling gain" case, we only need to recover the diagonal entries of $\mathbf{P}$.

In the "non-uniform coupling gain" case, the perturbation in the individual antenna design is taken into account and the transmission and reception mutual coupling matrices are not reciprocal anymore. Variations of the coupling gains and the self-impedances are modelled as $h_{u,v} = h^r \xi^r_{u,v} + jh^i \xi^i_{u,v}$ and $Z_u = Z^r \xi^r_u + jZ^i \xi^i_u$, where $\xi^r_{u,v}, \xi^i_{u,v}, \xi^r_u, \xi^i_u$ denote the relative perturbations. To configure the above parameters appropriately, we employ the professional electromagnetic (EM) simulation software "CST Studio Suite" [18] to compute the actual mutual coupling gains and the impedance values with a realistic antenna array. In particular, we arrange 256 dipoles as in Fig. 3 in parallel [14]. The length of the dipoles are uniformly distributed in $[70, 80]$mm. From the CST simulation results, we find $\ln \xi^r_{u,v}, \ln \xi^i_{u,v}, \ln \xi^r_u, \ln \xi^i_u$ are normally distributed with the following variances: $\sigma^2_{h^r} = 0.0080, \sigma^2_{h^i} = 0.2200, \sigma^2_{Z^r} = 0.0448, \sigma^2_{Z^i} = 0.1361$. Fig. 5 depicts the performance of different calibration methods with the above realistic antenna array containing 256 dipoles. Our "Batch Estimation Scheme" has worse performance in Fig. 5 than in Fig. 4 due to the non-uniform coupling gains. The performance degradation of the "Conventional Scheme" can be attributed to the neglect of the off-diagonal elements in the calibration matrix $\mathbf{P}$. On the other hand, our proposed "Batch Estimation Scheme" still outperforms the "Conventional Scheme" significantly. Also note the "Diagonal First Scheme" suffers the most degradation because it relies on the uniform coupling gain assumption heavily.

V. Conclusion

In a TDD massive MIMO system, besides the RF gain mismatches, the mutual coupling mismatches between the UL and the DL channels also need to be calibrated. In this paper, we have proposed one systematic scheme to estimate these different mismatches separately to save the OTA overheads. In addition to retrieving the diagonal entries in the calibration matrix, our proposed algorithms also recover the off-diagonal elements as well. In particular, the proposed "Batch Scheme" enjoys ML optimality and exploits all the neighboring observations efficiently to recover all the entries in the calibration matrix subject to one common scaling parameter. Simulation results demonstrate our proposed calibration method enables better channel reciprocity and higher system throughput than those conventional LS-based calibration methods.

Appendix

The partial derivatives of the objective function $f(\mathbf{X}_t, \mathbf{X}_r)$ in (22) can be expressed as

$$\begin{aligned} \partial f(\mathbf{X}_t, \mathbf{X}_r)/\partial \mathbf{X}_r^H &= \mathcal{A}\mathbf{X}_t^H \mathbf{X}_t \mathcal{A} \mathbf{X}_r - \mathcal{A}\mathbf{X}_t^H \mathbf{Y}, \\ \partial f(\mathbf{X}_t, \mathbf{X}_r)/\partial \mathbf{X}_t^H &= \mathcal{A}\mathbf{X}_r^H \mathbf{X}_r \mathcal{A} \mathbf{X}_t - \mathbf{Y}\mathbf{X}_r^H \mathcal{A}. \end{aligned}$$

Notice $f(\mathbf{X}_t, \mathbf{X}_r)$ is biconvex. The BCD updates in Section III-C can be obtained by setting the above derivatives to 0.

According to Theorem 4.5 and Theorem 4.9 in [19], we can see the proposed BCD iterations ensure the following convergence behavior:

$$\begin{aligned} &\lim_{j\to\infty} f(\mathbf{X}_t^{(j)}, \mathbf{X}_r^{(j)}) = f_0, \\ &\lim_{j\to\infty} \|\mathbf{X}_t^{(j+1)} - \mathbf{X}_t^{(j)}\|_F = 0, \\ &\lim_{j\to\infty} \|\mathbf{X}_r^{(j+1)} - \mathbf{X}_r^{(j)}\|_F = 0, \end{aligned}$$

where f_0 denotes the limiting value of the objective function evaluated after each BCD iteration.

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