Compressive Massive MIMO Calibration

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***Abstract*—Massive multiple-input multiple-output (MIMO) offers ample spectral efficiency gains by installing hundreds of antenna elements at one single base station (BS). To learn the downlink (DL) channel state information (CSI) from the uplink (UL) channel measurements at the BS, it is critical to be able to take full advantage of time-division duplexing (TDD) channel reciprocity. However, due to different radio frequency branches for transmitting and receiving, antenna calibration at the BS is necessary to effect this reciprocity between the end-to-end DL and UL channels. In massive MIMO, this task becomes extremely challenging since all the existing antenna calibration approaches do not scale well with the size of the antenna array. In this paper, we rely on compressive sensing (CS) theory to develop an efficient way to calibrate the massive antenna array at the BS, which only requires CSI feedback in the order of $\mathbb{O}(\log N)$ instead of $\mathbb{O}(N)$ as in those conventional calibration schemes. Simulations demonstrate our approach can maintain the whole large scale antenna system in a "calibrated" state with only a small amount of feedback overhead.**

Keywords: Massive MIMO, Compressive Sensing, Reciprocity Calibration, Time-Division Duplexing, TDD

I. Introduction

Massive multiple-input multiple output (MIMO) is regarded as one of the enabling technologies for next generation wireless communications [1]–[3]. Considering each base station (BS) is equipped with hundreds of antenna elements, to avoid the huge overhead for CSI feedback as required in frequency-division duplexing (FDD) systems, it is critical to rely on the time-division duplexing (TDD) channel reciprocity to learn the downlink (DL) channel state information (CSI) from the uplink (UL) channel measurements at the BS. However, in reality, the transmit and receive chains consist of different analog circuits. As a result, the effective channel gain in the DL is different from that in the UL in both magnitude and phase. Antenna calibration (a.k.a. reciprocity calibration) is thus required to restore the channel reciprocity.

Existing antenna calibration schemes, e.g. the "Self-Calibration Method" [4]–[7] and the "Over-the-Air (OTA) Method" [8]–[11], work well in current MIMO systems with a small number of antennas. In the case of massive MIMO, it turns not practical to design DL pilots for all the antennas at the BS for a mobile station (MS) to measure all the DL channels. Meanwhile, CSI feedback would constitute a significant amount of overhead. Furthermore, the classical "Self-Calibration" approach requires extra costly analog switches and attenuators wiring all the antenna ports together, which makes this scheme costly to apply.

In this paper, we develop an efficient way to calibrate the massive antenna array at the BS. The key idea is to exploit the sparsity structure in the relative changes between consecutive calibration efforts. Resorting to the compressive sensing (CS) theory [12], [13], we show that only an $\mathbb{O}(\log N)$ amount of CSI feedback is required to keep the massive MIMO system calibrated.

II. TDD Reciprocity Calibration

Due to different transmit and receive circuitry, we see different effective gains even when the same antennas are used for DL and UL communications in TDD systems. Let $\{\alpha_k, \beta_k\}_{k=1}^N$ denote the transmit and receive gains for the antennas at the BS, and $\{a_m, b_m\}_{m=1}^M$ stand for the corresponding gains at the MS. The end-to-end DL and UL channel matrices can be expressed as

$$\begin{aligned} H_{DL} &= \mathtt{Diag}\left\{\frac{b_1}{a_1}, \frac{b_2}{a_2}, ..., \frac{b_M}{a_M}\right\} \cdot H_{UL}^T \cdot \\ &\quad \mathtt{Diag}\left\{\frac{\alpha_1}{\beta_1}, \frac{\alpha_2}{\beta_2}, ..., \frac{\alpha_N}{\beta_N}\right\}, \qquad (1) \\ H_{UL}^T &= \mathtt{Diag}\{a_1, a_2, ..., a_M\} \cdot H_{PHY} \cdot \\ &\quad \mathtt{Diag}\{\beta_1, \beta_2, ..., \beta_N\}, \qquad (2) \end{aligned}$$

where H_{PHY} is an $M \times N$ matrix standing for the physical propagating channels between the antennas, and $\mathtt{Diag}\{\cdots\}$ denotes the diagonal matrix with diagonal elements being those defined inside the curly brackets. Without taking additional steps, the DL CSI cannot be inferred from the measured UL CSI at the BS. To restore the TDD channel reciprocity, antenna calibration at the BS is necessary. The goal of antenna calibration is to pre(post)-adjust the gains of the transmit(receive) chains such that we can establish the following relationship:

$$\frac{\alpha_1}{\beta_1} \cdot \frac{1}{w_1} = \frac{\alpha_2}{\beta_2} \cdot \frac{1}{w_2} = \cdots = \frac{\alpha_N}{\beta_N} \cdot \frac{1}{w_N}, \qquad (3)$$

where w_k is called calibration coefficient for antenna k. After picking one particular antenna as the reference antenna and setting its calibration coefficient to 1, the calibration coefficients in (3) become relative coefficients. Given (3), we can pre-adjust the transmit gains as: $\tilde{\alpha}_k := \alpha_k / w_k$ and obtain:

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$$\frac{\tilde{\alpha}_1}{\beta_1} = \frac{\tilde{\alpha}_2}{\beta_2} = \cdots = \frac{\tilde{\alpha}_N}{\beta_N}. \tag{4}$$

After accomplishing the calibration in (3), we can simply rely on the restored channel reciprocity to design the DL beamforming vector at the BS with the UL CSI.

Either the classical "Self-Calibration" scheme or the "OTA" scheme can be used to achieve the calibration goal in (3). However, all these conventional methods do not scale well with N, i.e. the number of antennas in the array. As N goes large, the required system overhead for the antenna calibration task is in the order of $\mathbb{O}(N)$. This will significantly limit the system performance. In the next section, we will describe one novel scheme to calibrate antenna systems of large size while only requiring the assisting MS to feed back CSI for $\mathbb{O}(\log N)$ channels.

III. Compressive Massive MIMO Calibration

Consider a typical massive MIMO system depicted in Fig. 1, where the BS is equipped with N antennas in addition to one reference antenna (e.g. antenna 0 in Fig. 1), and each MS has a single antenna. Next, we propose a novel approach which modifies the classic "OTA" method so that the calibration of the massive antenna array can be achieved with only a limited amount of feedback from the MS. In order to proceed with the details about our calibration scheme at time t, we make the following three assumptions:

- *AS1*: The antenna system has been calibrated at previous time $t-1$ with the relative calibration coefficient for the k-th antenna at time $t-1$ with respect to antenna 0 defined as: $k \in [1, N]$,
$$w_k(t-1) := \frac{\alpha_k(t-1)}{\beta_k(t-1)} \cdot \frac{\beta_0(t-1)}{\alpha_0(t-1)}; \tag{5}$$
- *AS2*: Antenna 0 is taken as the reference antenna for the calibration at time t such that $\beta_0(t)/\alpha_0(t) = \beta_0(t-1)/\alpha_0(t-1)$;
- *AS3*: Only $S \ll N$ antenna elements experience hardware glitches in their radio frequency (RF) front ends (FE) and observe changes in their relative calibration coefficients from time $t-1$ to time t.

"AS1" presumes a previously calibrated antenna system[1]. In order for "AS2" to hold well, we need to select the reference antenna carefully enough. Basically, we want to pick a particular antenna that is not expected to change its transmit and receive gains from time $t-1$ to time t. "AS3" holds valid for a realistic system where a majority of antennas will remain calibrated after the previous calibration effort [7], [14]. In the case of massive MIMO, due to low-cost hardware implementation [1], [2], the RF FEs are allowed to experience various hardware glitches and even failures sporadically. "AS3" allows for glitches in the RF circuitry of a massive antenna array with a small probability. On one hand, it establishes minimum requirements on the RF electronics

[1]The conventional full-scale calibration with either the "Self-Calibration Method" or the "OTA Method", albeit costly, is assumed to be performed regularly, e.g. once a day [7].

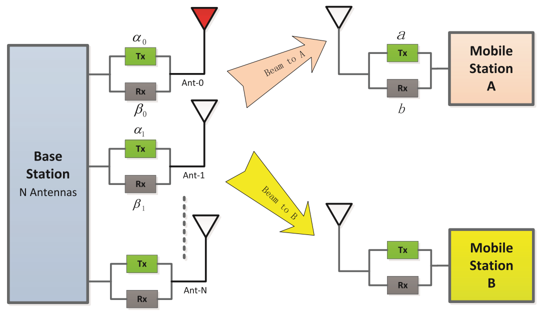

Fig. 1. Massive MIMO system.

in massive MIMO. On the other hand, it also serves as the guideline for choosing the time interval between consecutive calibration times.

To calibrate the antenna array at time t, we let the BS form a "virtual" antenna using a vector $\mathbf{x} = [x_1, x_2, ..., x_N]^T$. The received signal at the assisting MS (e.g. MS A in Fig. 1) is then

$$y = s \cdot b(t) \cdot \sum_{k=1}^{N} h_{PHY,k}(t) \cdot \alpha_k(t) \cdot x_k, \tag{6}$$

where s denotes the transmitted pilot symbol, $b(t)$ is the antenna receive gain at the MS, $h_{PHY,k}(t)$ stands for the physical propagating channel between BS antenna k and the MS, and $\alpha_k(t)$ represents the transmit gain of BS antenna k at time t. From (6), the MS can estimate the aggregate channel for this "virtual antenna" as

$$h_{virtual}^{DL}(t) = b(t) \cdot \sum_{k=1}^{N} h_{PHY,k}(t) \cdot \alpha_k(t) \cdot x_k. \tag{7}$$

Then, the assisting MS sends UL pilots for the BS to acquire the UL CSI from this MS towards each antenna at the BS:

$$h_k^{UL}(t) = a(t) \cdot \beta_k(t) \cdot h_{PHY,k}(t). \tag{8}$$

After the CSI of the "virtual antenna" in (7) is fed back to the BS, the BS can compute the following quantity η with (8):

$$\eta := \rho_0 \sum_{k=1}^{N} x_k \cdot h_k^{UL}(t) \cdot w_k(t-1) - h_{virtual}^{DL}(t), \tag{9}$$

where $\rho_0 := h_0^{DL}(t)/h_0^{UL}(t) = b(t)\alpha_0(t)/a(t)\beta_0(t)$ denotes the ratio of DL to UL channel measurement for the reference antenna at the BS (antenna 0). This ratio is made available at the BS through the feedback from the assisting MS.

Per "AS1", the calibration coefficients at time $t-1$ for each antenna $\{w_k(t-1)\}_{k=1}^{N}$ are available. At time t, the calibration coefficients for some antennas experience changes in either the amplitude or the phase. These changes $\{\delta_k(t)\}_{k=1}^{N}$ can be modelled as: $w_k(t) = w_k(t-1) - \delta_k(t)$. From the definition of $h_k^{UL}(t)$ in (8) and the definition of $h_{virtual}^{DL}(t)$ in (7), eq. (9) becomes

$$\eta \quad = \quad \rho_0 \sum_{k=1}^{N} x_k \cdot h_k^{UL}(t) \cdot \delta_k(t). \tag{10}$$

Note we obtain one linear equation about the unknown calibration coefficients changes $\{\delta_k(t)\}_{k=1}^N$. Conventionally, we need N independent equations to solve all the unknowns $\{\delta_k(t)\}_{k=1}^N$. Yet, with the sparsity assumption made in "AS3", we can resort to the compressive sampling theory to circumvent this dilemma by letting the BS form M "virtual" antennas in the order of $\mathbb{O}(\log N)$. The MS estimates the aggregate DL channels for these M "virtual" antenna and then feeds back the measured CSI to the BS. Let the i-th "virtual" antenna correspond to the vector of $[x_{i,1}, x_{i,2}, ..., x_{i,N}]^T$, $i = 1, 2, ..., M$. Stacking all these vectors together, we can form an M-by-N matrix X whose (i, j)th element is defined as: $X_{i,j} = x_{i,j}$, $i = 1, 2, ..., M$, $j = 1, 2, ..., N$. Then the BS can construct the following M equations following (10):

$$\mathbf{q} := \begin{pmatrix} \eta_1/\rho_0 \\ \eta_2/\rho_0 \\ \vdots \\ \eta_M/\rho_0 \end{pmatrix} = X \cdot \begin{pmatrix} h_1^{UL}(t)\delta_1(t) \\ h_2^{UL}(t)\delta_2(t) \\ \vdots \\ h_N^{UL}(t)\delta_N(t) \end{pmatrix} = X \cdot \mathbf{d}. \quad (11)$$

Since $\mathbf{d}$ is simply the product of the relative changes and the UL CSI, under "AS3", we know $\mathbf{d}$ is also sparse.

When there is no error in estimating the UL CSI $\{h_k^{UL}(t)\}_{k=1}^N$ and the "virtual" DL CSI $\{h_{virtual,k}^{DL}(t)\}_{k=1}^M$, the equality constraint in (11) is exact. Thus, we can formulate the following l_1 optimization problem to recover the unknown sparse vector $\mathbf{d}$:

$$\mathbf{d}_{opt} = \underset{\mathbf{d} \in C^N}{\texttt{argmin}} \|\mathbf{d}\|_{l_1}, \quad \texttt{s.t.} \quad \mathbf{q} = X \cdot \mathbf{d}. \quad (12)$$

Compressive sensing theory [12], [13] tells us these "virtual" antennas should be randomly created from an orthonormal basis with a small coherence value with respect to the representing basis where the signal of interest enjoys sparsity. Given that the representing basis is simply the identity matrix in our case, one can easily verify that the $N \times N$ FFT matrix $\mathcal{F}_N$ with element defined as: $\mathcal{F}_N^{k,n} = 1/\sqrt{N} e^{-j2\pi kn/N}$ enjoys the lowest possible coherence value of 1. Thus, our "virtual" antennas could be generated as follows:

$$X = \mathcal{A} \cdot \mathcal{F}_N, \quad (13)$$

where $\mathcal{A}$ is an $M \times N$ matrix selecting M rows from $\mathcal{F}_N$ uniformly at random. Note the BS has the knowledge about all the UL channels: $\{h_k^{UL}(t)\}$. After recovering $\mathbf{d}$, the BS can then derive the values of $\{\delta_k(t)\}$ and the corresponding calibration coefficients at time t, i.e. $w_k(t) = w_k(t-1) - \delta_k(t)$. Summarizing we can establish the following main result on massive MIMO calibration:

Proposition 1: *Under Assumptions $AS1 \sim 3$, without measurement errors, a massive array of size N can be calibrated by solving the l_1 optimization in (12) when M ($\sim \mathbb{O}(\log N)$) "virtual" antennas are generated according to (13) and fed back by the assisting MS.*

In reality, there are errors in estimating those DL and UL channels. Taking into account the channel estimation errors, similar to the definition of η in (9), we can compute $\tilde{\eta}$ as

$$\begin{aligned} \tilde{\eta} &:= \rho_0 \sum_{k=1}^N x_k \cdot \hat{h}_k^{UL}(t) \cdot w_k(t-1) - \hat{h}_{virtual}^{DL}(t) \\ &= \rho_0 \sum_{k=1}^N x_k \hat{h}_k^{UL}(t)\delta_k(t) + \rho_0 \sum_{k=1}^N x_k e_k w_k(t) - f. \quad (14) \end{aligned}$$

where e_k are the UL channel estimation errors and f stands for the error in estimating the "virtual" DL channel. Accordingly, eq. (11) becomes

$$\begin{aligned} \tilde{\mathbf{q}} &:= \begin{pmatrix} \tilde{\eta}_1/\rho_0 \\ \tilde{\eta}_2/\rho_0 \\ \vdots \\ \tilde{\eta}_M/\rho_0 \end{pmatrix} = X \cdot \begin{pmatrix} \hat{h}_1^{UL}(t)\delta_1(t) \\ \hat{h}_2^{UL}(t)\delta_2(t) \\ \vdots \\ \hat{h}_N^{UL}(t)\delta_N(t) \end{pmatrix} + \mathbf{e} \\ &:= X \cdot \tilde{\mathbf{d}} + \mathbf{e}, \quad (15) \end{aligned}$$

where $\mathbf{e}$ is the vector of estimation errors. Let f_m stand for the error in estimating the DL channel for the m-th "virtual" antenna. The m-th entry of the error vector $\mathbf{e}$ can be expressed as follows: $e_m = \sum_{k=1}^N x_{m,k} e_k w_k(t) - f_m/\rho_0$. Given (15), we can see the equality constraint in (12) does not hold anymore. To recover the calibration vector in a robust way, we can solve the following optimization problem instead:

$$\tilde{\mathbf{d}}_{opt} = \underset{\tilde{\mathbf{d}} \in \mathbb{C}^N}{\texttt{argmin}} \|\tilde{\mathbf{d}}\|_{l_1}, \quad \texttt{s.t.} \quad \|X \cdot \tilde{\mathbf{d}} - \tilde{\mathbf{q}}\|_{l_2} \le \epsilon. \quad (16)$$

Here ϵ can be determined by the power of the estimation error as $\epsilon^2 = \sigma_e^2 \sum_{m=1}^M \sum_{k=1}^N |x_{m,k}|^2 |w_k(t)|^2 + \sigma_f^2 \frac{M}{|\rho_0|^2}$, where we have assumed the same variance in $\{e_k\}_{k=1}^N$ and the same variance for $\{f_m\}_{m=1}^M$. With the compressive sampling theory, we can establish another main result on massive MIMO calibration:

Proposition 2: *Under Assumptions $AS1 \sim 3$, when M ($\sim \mathbb{O}(\log N)$) "virtual" antennas are generated according to (13), a massive antenna array of size N can be calibrated by solving the l_1 optimization in (16) when the "virtual" antenna matrix X exhibits a small isometry constant.*

Given the above result, "AS3" can be relaxed to allow a small number of antennas having significant changes due to hardware glitches while the remaining majority of antennas experiencing small drifting in their relative calibration coefficients, e.g. due to temperature variations.

IV. Simulated Performance

Simulation assumptions are listed as follows:

1) The physical propagation channels between the BS and the MS are modeled as i.i.d. complex circular symmetric Gaussian with zero mean and unit variance;
2) All the antennas at the BS are balanced in the sense that the magnitudes of their receive/transmit gains are the same;
3) Between consecutive calibration efforts, only S antennas at the BS see new receive/transmit gains with random phases uniformly distributed in the interval of $[0, 2\pi]$.

Fig. 2 depicts the calibration error probabilities for different array sizes in the case of clean channel measurements. As N

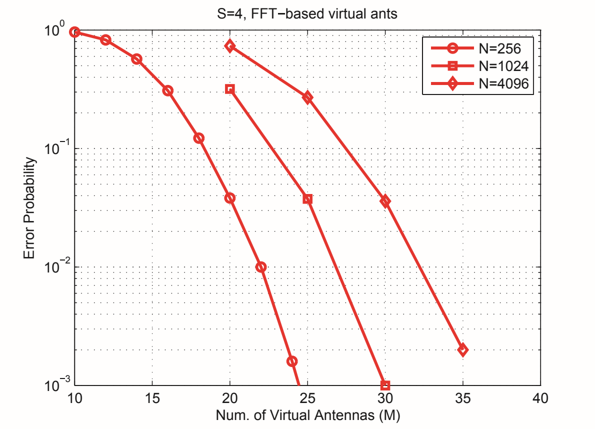

Fig. 2. Calibration performance with clean measurements.

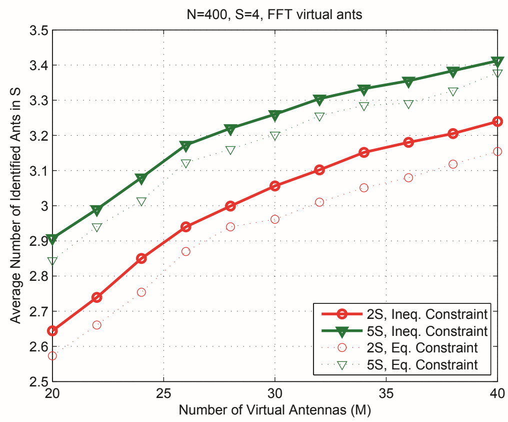

Fig. 3. Calibration performance with noisy channel measurements: -30dB channel estimation mean square error (MSE) relative to channel power ($2S(5S)$ means $\kappa = 2(\kappa = 5)$).

goes from 256 to 4096, when targeting 0.1% error probability, the number of needed "virtual" antennas only increases by $35/25 = 1.4 \approx \log(4096)/\log(256)$ times, which aligns very well with Proposition 1. In the case of noisy channel measurements, we cannot hope for exact recovery. Denote the set of $S = 4$ antennas with phase changes as $\mathcal{S}$. After solving the problem (12) with equality constraint or the problem (16) with inequality constraint, we claim κS antennas corresponding to the largest entries in $\mathbf{d}_{opt}$ or $\tilde{\mathbf{d}}_{opt}$ as the antennas out of calibration. Fig. 3 plots the average number of antennas in set $\mathcal{S}$ identified by the above approach. After identifying the antennas necessitating re-calibration, conventional OTA method can be applied with reasonable feedback overhead considering $\kappa S \ll N$. From the results in Fig. 3, we also see the inequality constraint in (16) leads to better performance than the equality constraint in (12).

V. Conclusion

In this paper, we have developed a novel compressive sensing based algorithm to calibrate the massive MIMO system with a limited amount of feedback from the assisting MS. In contrast to conventional antenna calibration approaches which require an $\mathbb{O}(N)$ amount of time or frequency resources to effect end-to-end DL-UL channel reciprocity, the proposed algorithm can calibrate the system of size N with only an $\mathbb{O}(\log N)$ amount of CSI feedback by exploiting the inherent sparsity structure in the relative changes between consecutive calibrations of the massive antenna array.

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