# Hierarchical Visible Light Communication System

*Xin Xie and Xiliang Luo*
School of Information Science and Technology
ShanghaiTech University
Shanghai, China
Email:{xiexin, luoxl}@shanghaitech.edu.cn

***Abstract*—Visible light communication (VLC) has attached a lot of interests due to its safety, practicability, and efficiency. In a VLC system with multiple light-emitting-diodes (LEDs), two different types of channels named imaging channel and non-imaging channel can be used to transmit data. In this paper, basing on the channel characteristics, we propose a novel hierarchical VLC system which supports two different types of receivers simultaneously. Meanwhile, we also demonstrate how to achieve maximum throughput with optimal power allocation. Numerical results are also provided to corroborate our designs.**

***Index Terms*—VLC, MIMO, Hierarchical Communication**

## I. Introduction

Visible light communication (VLC), originally proposed back in 2000 [1], [2], is a promising technology which can provide both high-rate communication and everyday lighting utilizing low-cost LEDs [3]. In a VLC communication system, orthogonal frequency division multiplexing (OFDM) modulation is typically used to increase the transmission rate [4]–[7]. Due to the non-negativity constraint of the optical intensity, the input symbols to the inverse Fast Fourier Transform (IFFT) engine must satisfy the Hermitian symmetry. Various modified OFDM schemes have been proposed, e.g. asymmetrically-clipped optical OFDM (ACO-OFDM) [4], DC-biased optical OFDM (DCO-OFDM) [5] and so on. Another method to boost the rate is to employ multiple LEDs at the transmitter [8]. When these LEDs are placed close enough, they have almost identical channel gains toward the receiver. With multiple photodetectors (PD) placed at the receiver, we have a VLC-MIMO system. Close placement of the transmit LEDs makes this MIMO channel highly correlated. One way to mitigate the channel correlation is to put an imaging lens in front of the receiver, which helps identify independent data streams originated from different LEDs. For example, the usage of hemispherical lens and fish-eye lens have be proposed in [9], [10]. Meanwhile, the capacity of a VLC channel with an imaging lens has been analyzed in [11].

In this paper, through exploiting the channel characteristics of both the imaging channel and the non-imaging channel, we propose a hierarchical visible light communication (HVLC) system. The proposed HVLC system is capable of serving receivers of different receiving capabilities. Independent data streams can be transmitted towards the receiver with an imaging lens and the one without an imaging lens. We also show the whole HVLC system can achieve optimum rate region with judicious power allocation.

978-1-5090-1317-3/16/$31.00 ©2016 IEEE

## II. Channel Model

### A. Non-Imaging Channel Model

The emission of an LED is modelled as a Lambertian pattern with line-of-sight (LOS) path dominating the whole propagation [2]. The channel from an LED to a PD can be modelled as a flat fading channel whose gain is related to the angle of incidence, angle of irradiance, and distance from the transmitter to the receiver. In a VLC multiple-input single-output (MISO) system where the transmitter consists of $N$ LEDs, the received signal can be expressed as:

$$r = \mathbf{h}^{\mathbf{T}}(\mathbf{x} + \mathbf{i}^{\mathbf{bias}}) + w, \tag{1}$$

where $\mathbf{x} = (x_1, x_2, ..., x_N)^{\mathbf{T}}$ is the $N \times 1$ bipolar transmitted signal vector, $\mathbf{i}^{\mathbf{bias}} = (i_1^{bias}, i_2^{bias}, ..., i_N^{bias})^{\mathbf{T}}$ denotes bias current added onto $\mathbf{x}$, and $w$ denotes the additive white Gaussian noise (AWGN) at the receiver with variance $\sigma_w^2$. The channel vector $\mathbf{h}$ describes the channel gain from each LED to the receiver. In particular, the channel gain from the $k$-th LED to the receiver can be expressed as [2]:

$$h_k = \begin{cases} \frac{(m+1)A}{2\pi d_k^2}\cos^m(\psi_k)\cos(\varphi_k)T_s(\varphi_k)g(\varphi_k) & \varphi_k \le \varphi_c \\ 0 & \varphi_k > \varphi_c \end{cases}, \tag{2}$$

where $A$ denotes the area of the PD, $m$ stands for the Lambertian order which is related to the LED semiangle at half-power $\psi_{\frac{1}{2}}$, i.e. $m = \frac{-\ln 2}{\ln(\cos\psi_{\frac{1}{2}})}$, $d_k$ refers to the distance from the $k$-th LED to the receiver, $\varphi_k$ is the angle of incidence, $\psi_k$ is the angle of irradiance, $T_s(\varphi_k)$ is the gain of the optical filter, $g(\varphi_k)$ is the gain of optical concentrator, and $\varphi_c$ is the filed-of-view (FOV) of the receiver. The gain of optical concentrator is as follows:

$$g(\varphi_k) = \begin{cases} \frac{n^2}{\sin^2(\varphi_c)} & 0 \le \varphi_k \le \varphi_c \\ 0 & \varphi_k > \varphi_c \end{cases}, \tag{3}$$

where $n$ is the refractive index of the optical concentrator.

In a VLC MISO system, there are strong correlations among the channels from different LEDs to the receiver. Meanwhile, under some conditions, the signal received at the receiver is just the summation of the transmitting intensities from different LEDs with a constant scaling, which is referred to as *spatial summing*. In our proposed hierarchical system, the spatial summing is employed in the non-imaging receiver, so it is necessary to obtain the sufficient condition under which the spatial summing property can be employed.

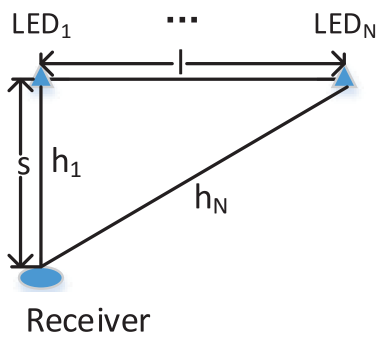


Fig. 1. Non-imaging channel in VLC.

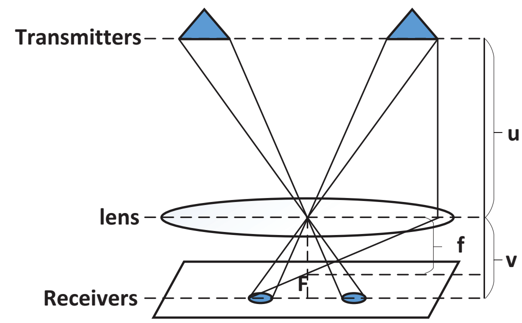


Fig. 2. Imaging channel in VLC.

In the typical indoor environment shown in Fig. 1, $N$ LEDs are fixed at the ceiling, a single PD is placed perpendicular to the floor of the room and looking at the ceiling as the non-imaging receiver. Assume the first LED is exactly located above the receiver with a height: $s$ and a channel gain: $h_1$, and the distance from the first LED to the last one is $l$. The channel gain from the last LED to the receiver is donated as $h_N$. According to (2), we have

$$h_1 = \frac{(m+1)A}{2\pi s^2} T_s(0)g(0), \tag{4}$$

and

$$h_N = \frac{(m+1)A}{2\pi(s^2+l^2)} \cos^{m+1}(\varphi_N)T_s(\varphi_N)g(\varphi_N). \tag{5}$$

Let $x_k$ be transmitted from the $k$-th LED, $\forall 1 \le k \le N-1$. Let $x_N = y - \sum_{k=1}^{N-1} x_k$ be transmitted from the $N$-th LED. The received signal at the receiver is

$$r = h_N y + \sum_{k=1}^{N-1}(h_k - h_N)x_k + \sum_{k=1}^{N} h_k i_k^{bias} + w. \tag{6}$$

In general, $i_k^{bias}$ represents LED brightness and depends on user's requirement. After eliminating the bias current, the bipolar received signal is

$$\tilde{r} := r - \sum_{k=1}^{N} h_k i_k^{bias} = h_N y + \sum_{k=1}^{N-1}(h_k - h_N)x_k + w. \tag{7}$$

Considering $y$ is the desired signal, the signal to interference plus noise ratio (SINR) of this received signal is

$$\eta = \frac{\mathbf{E}\{(h_N y)^2\}}{\mathbf{E}\{(\sum_{k=1}^{N-1}(h_k - h_N)x_k)^2\} + \sigma_w^2}. \tag{8}$$

Let $\mathbf{E}\{x_k\} = \mathbf{E}\{y\} = 0$, $\mathbf{E}\{x_k^2\} = \mathbf{E}\{y^2\} = 1$, $1 \le k \le N-1$. Assuming $\{x_k\}_{k=1}^{N-1}$ and $y$ are mutually independent, we can have

$$\mathbf{E}\left\{\left[\sum_{k=1}^{N-1}(h_k - h_N)x_k\right]^2\right\} \le (N-1)(h_1 - h_N)^2. \tag{9}$$

The interference power can be ignored only when it is small enough compared with the noise power. Thus we can have the following result:

**Proposition 1:** *Assuming $T_s(0) = g(0) = T_s(\varphi_N) = g(\varphi_N) = m = 1$, in order to experience negligible interference in (8), we need to design the system such that the following condition is met:*

$$\frac{1}{s^2} - \frac{s^2}{(s^2+l^2)^2} \le \sqrt{10^{-2}\frac{\pi^2\sigma_w^2}{(N-1)A^2}}. \tag{10}$$

*In particular, when the above condition is met, the total amount of interference power will be no more than 1% of the thermal noise power.*

In a scenario with 5 transmit LEDs being fixed at a hight of 2m above the receiver, for $A = 0.1\text{cm}^2$ and $\sigma_w^2 = -146.6\text{dBm}$, Proposition 1 dictates $l \le 0.08\text{m}$. This means the distance between two LEDs should be less than $8cm$ to ensure negligible interference due to the channel difference.

### B. Imaging Channel Model

For an imaging channel, a lens is placed between the transmitter and the receiver as illustrated in Fig. 2. To be able to obtain a clear image at the receiving plane, we must have $1/u + 1/v = 1/f$, where $u$ is the object distance and $v$ which is the image distance, and $f$ is the focal length of the lens. The magnification of the imaging lens is given by $M = v/u = f/(u-f)$. An imaging MIMO channel can be divided into two parts: 1) the free space channel from the transmitter to the lens; 2) the imaging channel from the lens to receivers. The imaging channel gain between the $j$-th LED to the $i$-th PD is given by the the fraction of the area of the image of the $j$-th LED that is incident on the $i$-th PD [11], i.e.

$$h'_{ij} = \frac{\text{Area}(\text{image}_j \bigcap \text{photodiode}_i)}{\text{Area}(\text{image}_j)}, \tag{11}$$

Obviously, $0 \le h'_{ij} \le 1$. Note $h'_{ij}$ is determined by several parameters such as the position of receiver and the lens magnification factor. Under some ideal situations, the imaging channel is an $N \times N$ identity matrix, which means that the image of each LED falls on one PD exactly. Accordingly, the inter-channel interference (ICI) vanishes. Otherwise, one LED image projects onto multiple PDs, when the system performance degrades severely due to the existence of ICI.

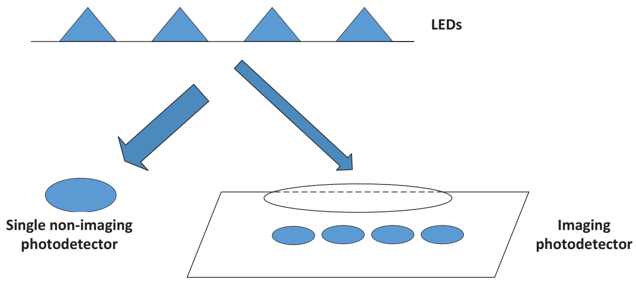


Fig. 3. A hierarchical VLC system.

Regarding the free space channel gain from the transmitter to the lens, it is similar to the channel gain in the non-imaging MISO channel as follows:

$$h''_{ij} = \frac{(m+1)A_i}{2\pi d_{ij}^2}\cos^m(\psi_{ij})\cos(\varphi_{ij})T_s(\varphi_{ij})g(\varphi_{ij}). \quad (12)$$

The overall channel gain matrix $\mathbf{H}$ can be obtained from the free space channel gain and the imaging channel gain as

$$\mathbf{H}_{ij} = h'_{ij}h''_{ij}. \quad (13)$$

## III. Hierarchical VLC System

A hierarchical VLC system is illustrated in Fig. 3, where $N$ LEDs are deployed on the ceiling as the transmitter, a single PD is placed on the floor as a non-imaging receiver, and $N$ PDs together with a lens above are functioning as an imaging receiver. The signal: $x_k(1 \le k \le N-1)$ is transmitted from the $k$-th LED with signal power $p_k$. Meanwhile, $x_N$ satisfies $x_N = y - \sum_{k=1}^{N-1} x_k$, and the power of $y$ is $p_y$. According to (7), the received bipolar signal at the non-imaging receiver is

$$r^{nim} = h_N y + \sum_{k=1}^{N-1}(h_k - h_N)x_k + w. \quad (14)$$

As we have shown that the second term can be ignored when it is small enough compared with noise power. The signal to noise ratio (SNR) is then: $\eta = \frac{h_N^2 p_y}{\sigma_w^2}$.

For the imaging receiver, the free space channel gain can be considered as a constant $\kappa$, i.e, $h''_{ij} = \kappa(1 \le i,j \le N)$, when the distances to the lens satisfy the condition in Proposition 1. Then the overall channel gain matrix can be obtained as

$$\mathbf{H} = \kappa\begin{pmatrix} h'_{11} & \cdots & h'_{1N} \\ \vdots & \ddots & \vdots \\ h'_{N1} & \cdots & h'_{NN}\end{pmatrix}. \quad (15)$$

Thus the received signal vector at the imaging receiver can be expressed as

$$\mathbf{r}^{im} = \mathbf{H}(\mathbf{x} + \mathbf{i}^{\mathbf{bias}}) + \mathbf{w}. \quad (16)$$

After subtracting the bias current, the received bipolar signal at the $k$-th PD is

$$\tilde{r}_k^{im} = \kappa\left[(h'_{kk} - h'_{kN})x_k + \sum_{j=1,j\neq k}^{N-1}(h'_{kj} - h'_{kN})x_j + h'_{kN}y\right] + w. \quad (17)$$

Denote $g_{kj} := \kappa(h'_{kj} - h'_{kN})$, $\mu_k := \kappa h'_{kN}$. Eq. (17) can be rewritten as

$$\tilde{r}_k^{im} = g_{kk}x_k + \sum_{j=1,j\neq k}^{N-1} g_{kj}x_j + \mu_k y + w. \quad (18)$$

The average SINR can be obtained as

$$\gamma_k = \frac{g_{kk}^2 p_k}{\sum_{j=1,j\neq k}^{N-1} g_{kj}^2 p_j + \mu_k^2 p_y + \sigma_w^2}. \quad (19)$$

In order to transmit data streams without distortion, the input current $I_{input}$ to an LED should be within a specified dynamic range, i.e, $I_{input} \in [I_L + I_{bias}, I_U + I_{bias}]$. In this paper, we set $I_U = -I_L$, so the input bipolar signal should satisfy $|x| < I_U$. However, if $x$ corresponds to an OFDM signal, the signal exhibits Gaussian distribution and would exceed the dynamic range of the LED with a certain probability. In the proposed system, DCO-OFDM is employed for the bipolar signal $x_k(1 \le k \le N-1)$ and $y$, so $x_k \sim \mathcal{N}(0, p_k)$, $y \sim \mathcal{N}(0, p_y)$. According to the system performance requirement, we have the following requirements:

$$P_r(|x_k| > I_U^k) \le \beta, \quad (20)$$

where $I_U^k$ corresponds to the dynamic range of the $k$-th LED and $\beta$ is related to system requirement. Furthermore, we have $p_k \le t_k$, $1 \le k \le N$, where $t_k$ is the corresponding bound to ensure $P_r(|x_k| > I_U^k) \le \beta$.

Given the HVLC system as in Fig. 3, we would like to maximize the imaging receiver throughput while the performance of the non-imaging communication is not less than a threshold. We also assume that the digital pre-distortion (DPD) has been employed so that the LED nonlinearity can be ignored. The problem is then formulated as an optimization problem as follows:

$$\begin{aligned} \underset{p_y,p_1,p_2,\cdots,p_{N-1}}{\text{maximize}} \quad & \textstyle\sum_{k=1}^{N-1}\log(1+\gamma_k) \\ \text{subject to} \quad & 0 \le p_k \le t_k, 1 \le k \le N-1 \\ & p_y + \textstyle\sum_{k=1}^{N-1} p_k \le t_N \\ & \eta \ge \eta_t, \end{aligned} \quad (21)$$

where $\eta_t$ represents the performance threshold for the non-imaging receiver.

It can be seen the optimal value of the objective function is achieved when the third constraint in (21) is active, i.e, $\eta = \eta_t$. The problem can be rewritten as follows

$$\begin{aligned} \underset{p_1,p_2,\cdots,p_{N-1}}{\text{maximize}} \quad & \textstyle\sum_{k=1}^{N-1}\log(1+\gamma_k) \\ \text{subject to} \quad & 0 \le p_k \le t_k, 1 \le k \le N-1 \\ & \textstyle\sum_{k=1}^{N-1} p_k \le t_N - \frac{\eta_t\sigma_w^2}{h_N^2}, \end{aligned} \quad (22)$$

where $\gamma_k$ can be rewritten as

$$\gamma_k = \frac{g_{kk}^2 p_k}{\sum_{j=1,j\neq k}^{N-1} g_{kj}^2 p_j + \hat{\sigma}_k^2} \quad (23)$$

and

$$\hat{\sigma}_k^2 = \frac{\eta_t\sigma_w^2}{h_N^2}\mu_k^2 + \sigma_w^2. \quad (24)$$

This problem is a typical power allocation problem with ICI and is a non-convex optimization problem due to the expression of $\gamma_k$. However, the MAPEL algorithm proposed in [12] can be used to achieve the global optimal solution.

On the other hand, when the position of the PDs can be adjusted adaptively, the ICI can be eliminated, i.e, $\mathbf{H} = \kappa \mathbf{I}$ in (15), where $\mathbf{I}$ represents an identity matrix. The problem in (22) can be rewritten as

$$\begin{array}{ll} \underset{p_1,p_2,\cdots,p_{N-1}}{\text{maximize}} & \sum_{k=1}^{N-1} \log(1+\frac{\kappa^2 p_k}{\sigma_w^2}) \\ \text{subject to} & 0 \leq p_k \leq t_k, 1 \leq k \leq N-1 \\ & \sum_{k=1}^{N-1} p_k \leq t_N - \frac{\eta_t \sigma_w^2}{h_N^2}. \end{array} \tag{25}$$

The above problem is convex. We can apply the KKT optimality conditions to obtain the following result:

**Proposition 2:** *In the absence of ICI in the imaging receiver, the optimal power allocation maximizing the throughput towards the imaging receiver obeys the following water-filling rule:* $\forall k \in [1, N-1]$,

$$p_k^* = \begin{cases} 0 & \xi^* < \frac{\sigma_w^2}{\kappa^2} \\ \xi^* - \frac{\sigma_w^2}{\kappa^2} & \frac{\sigma_w^2}{\kappa^2} < \xi^* < \frac{\sigma_w^2}{\kappa^2} + t_k \\ t_k & \xi^* > \frac{\sigma_w^2}{\kappa^2} + t_k \end{cases}, \tag{26}$$

*where* $\xi^* > 0$ *is chosen such that* $\sum_{i=1}^{N-1} p_k^* = t_N - \frac{\eta_t \sigma_w^2}{h_N^2}$.

## IV. Numeric Results

We simulate one hierarchical VLC system with the parameters listed in Table 1.

**Table 1. Simulation parameters**

| $\sigma_w^2$ | $N$ | $h_N$ | $\kappa$ in (15) | $\beta$ in (20) |
|---|---|---|---|---|
| $-146.6$dBm | 5 | $3 \times 10^{-6}$ | $1 \times 10^{-6}$ | 0.001 |

The imaging channel matrix between the lens to the PDs is

$$\mathbf{H}' = \begin{pmatrix} 0.80 & 0.12 & 0.02 & 0.01 & 0 \\ 0.10 & 0.73 & 0.05 & 0.03 & 0.02 \\ 0.05 & 0.10 & 0.89 & 0.06 & 0.03 \\ 0.04 & 0.03 & 0.03 & 0.85 & 0.05 \\ 0.01 & 0.02 & 0.01 & 0.05 & 0.9 \end{pmatrix}. \tag{27}$$

Assume the dynamic ranges of the LEDs are different and the values are $I_U = (0.45, 0.47, 0.51, 0.53, 0.55)$. Fig. 4 shows the feasible rate regions when the two types of receivers work simultaneously. The MAPEL algorithm is adopted to derive the rate region in the case of ICI. The simple water-filling rule in Proposition 2 is utilized when there is no ICI (i.e. "Ideal scenario" in Fig. 4). Clearly, ICI in the imaging receiver causes the rate region to shrink. Meanwhile, as the non-imaging receiver demands more transmission power, the imaging receiver will have to lower down the transmission rate.

## V. Conclusion

In this paper we analyze the characteristics of the non-imaging channel and the imaging channel in a VLC system with multiple transmit LEDs. Basing on these characteristics, we propose a novel hierarchical VLC (HVLC) system, where the non-imaging receiver and the imaging receiver can communicate with transmitter simultaneously. We also show this HVLC system can achieve optimal rate region by allocating the signal power appropriately.

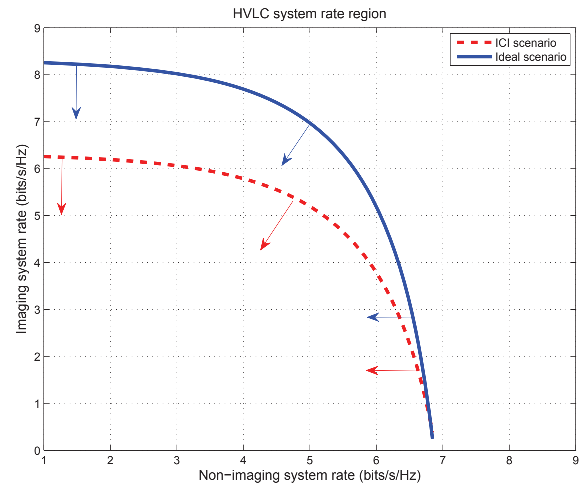


Fig. 4. Rate region of a hierarchical VLC system. Ideal scenario: identity imaging channel matrix; ICI scenario: imaging channel matrix as in (27).

## References


[1] Y. Tanakat, S. Haruyamat, and M. Nakagawat, "Wireless optical transmissions with white colored LED for wireless home links", in *Proc. PIMRC*, London, UK, pp. 1325-1329, Sep. 2000.

[2] T. Komine and M. Nakagawa, "Fundamental analysis for visible-light communication system using LED lights", *IEEE Trans. Consum. Electron.* vol. 50, no. 1, pp. 100-107, Feb. 2004.

[3] H. Elgala, R. Mesleh, and H. Haas, "Indoor optical wireless communication: Potential and state-of-the-art," *IEEE Commun. Mag.*, vol. 49, no. 9, pp. 56-62, Sep. 2011.

[4] J. Armstrong, "OFDM for optical communications", *J. Lightw. Technol.* vol. 27, no. 3, pp. 189-204, Feb. 2009.

[5] M. Z. Afgani, H. Haas, H. Elgala, and D. Knipp, "Visible light communication using OFDM", in *Proc. TRIDENTCOM*, Barcelona, Spain, 2006.

[6] A. H. Azhar, T. A. Tran, and D. O'Brien, "A gigabit/s indoor wireless wransmission using mimo-ofdm visible-light communications", *IEEE Photon. Technol. Lett.*, vol. 25, no. 2, pp. 171-174, Jan. 2013.

[7] J. Armstrong and B. J. C. Schmidt, "Comparison of asymmetrically clipped optical ofdm and dc-biased optical ofdm in awgn", *IEEE Commun. Lett.*, vol. 12, no. 5, pp. 343-345, May 2008.

[8] M. S. A. Mossaad, S. Hranilovic, S. Member, and L. Lampe, "Visible light communications using ofdm and multiple leds", *IEEE Trans. Commun.*, vol. 63, no. 11, pp. 4304-4313, Nov. 2015.

[9] T. Q. Wang, Y. A. Sekercioglu, and J. Armstrong, "Analysis of an optical wireless receiver using a hemispherical lens with application in mimo visible light communications", *J. Lightw. Technol.*, vol. 31, no. 11, pp. 1744-1754, Jun. 2013.

[10] T. Chen, L. Liu, B. Tu, Z. Zheng, and W. Hu, "High-spatial-diversity imaging receiver using fisheye lens for indoor mimo vlcs", *IEEE Photon. Technol. Lett.*, vol. 26, no. 22, pp. 2260-2263, Nov. 2014.

[11] P. M. Butala, H. Elgala, and T. D. C. Little, "SVD-VLC: A novel capacity maximizing vlc mimo system architecture under illumination constraints", in *Proc. Globecom Workshop*, pp.1087-1092, Atlanta, GA, Dec. 2013.

[12] L. Qian, Y. Zhang, and J. Huang, "MAPEL: Achieving global optimality for a non-convex wireless power control problem", *IEEE Trans. Wireless Commun.*, vol. 8, no. 3, pp. 1553-1563, Mar. 2009.