

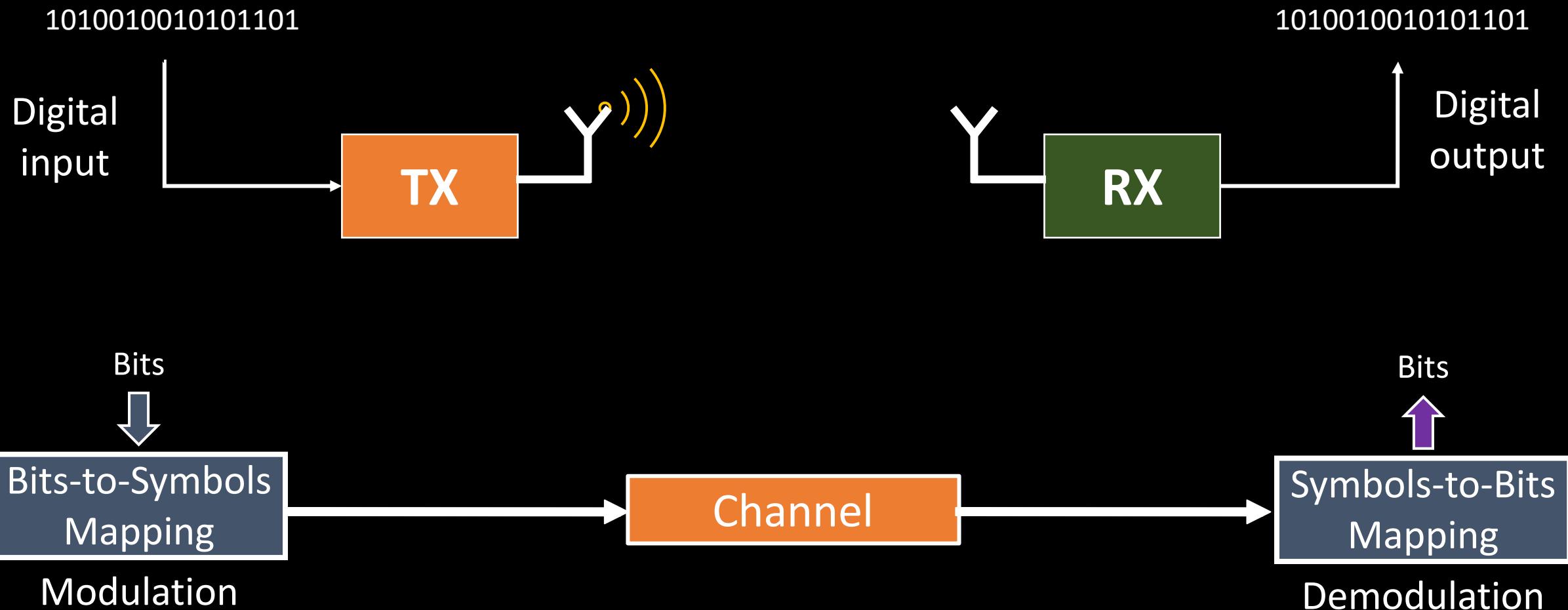
CSE610 Special Topics on Mobile Network & Mobile Sensing

Yaxiong Xie

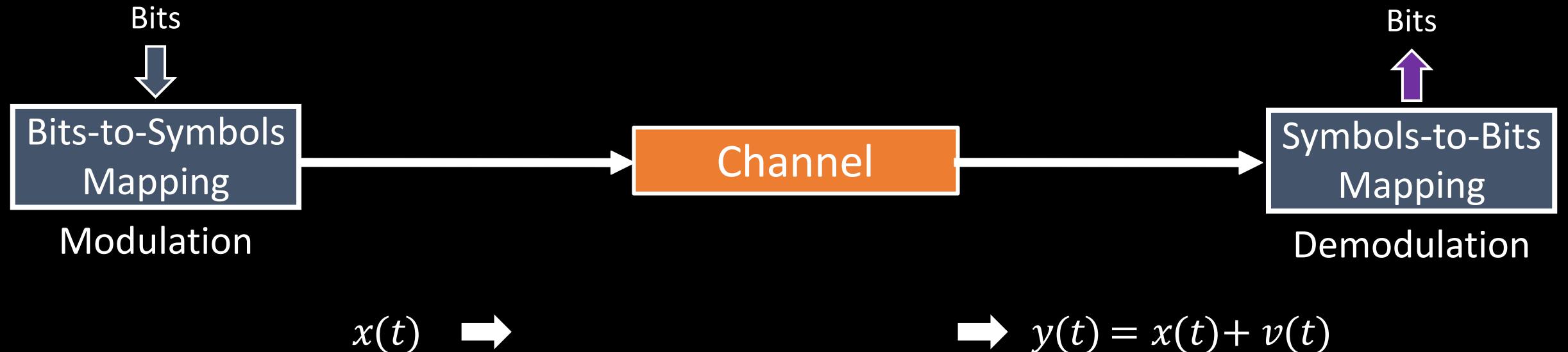
Lecture 2: Channel + OFDM + MIMO



Wireless Channel



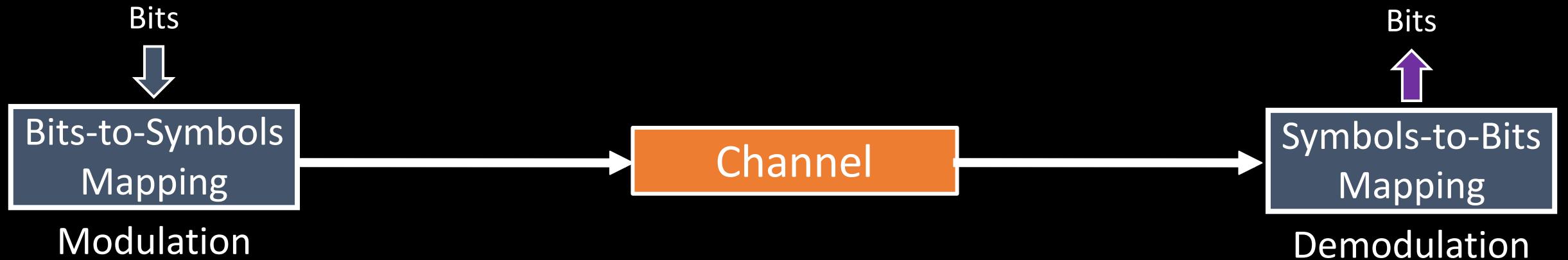
Wireless Channel



Channel adds noise!

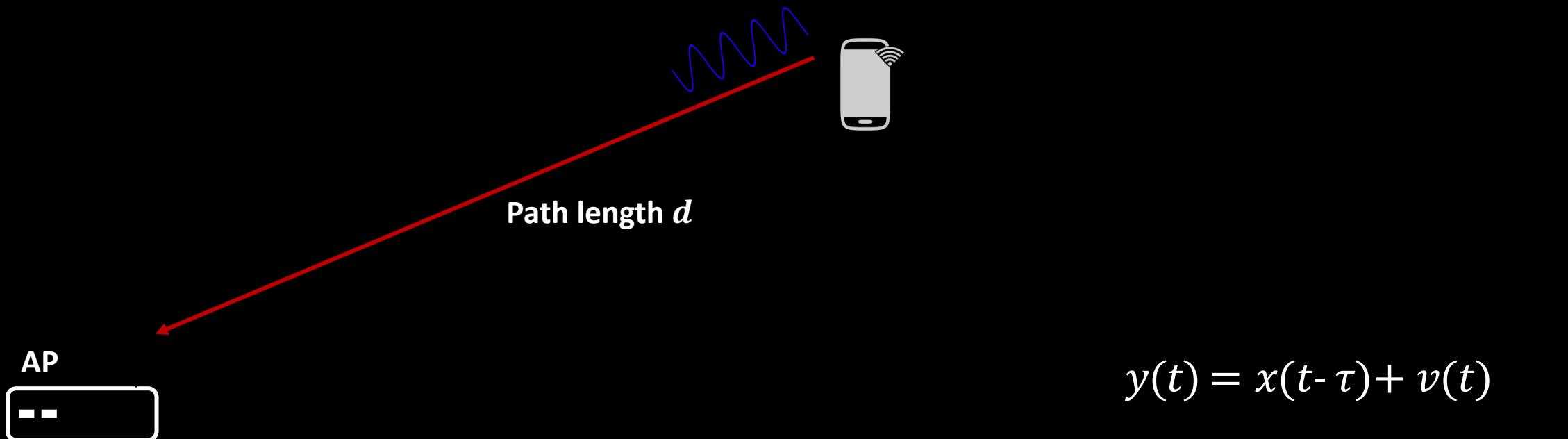
$$v(t) \sim N(0, \sigma)$$

Wireless Channel



Channel delays the signal!

Wireless Channel



Channel delays the signal!

$$\tau = \frac{d}{c}$$

The distance the signal travels

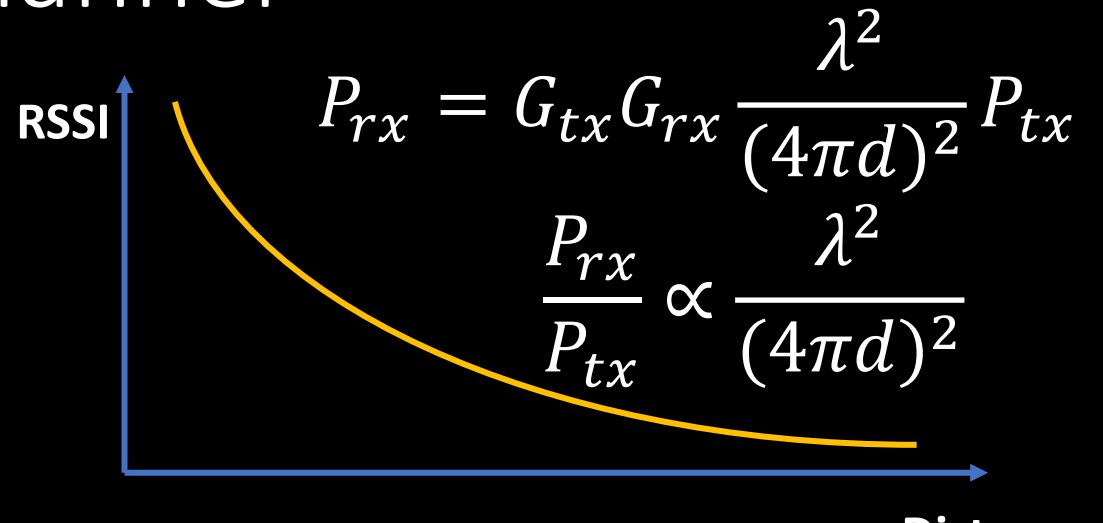
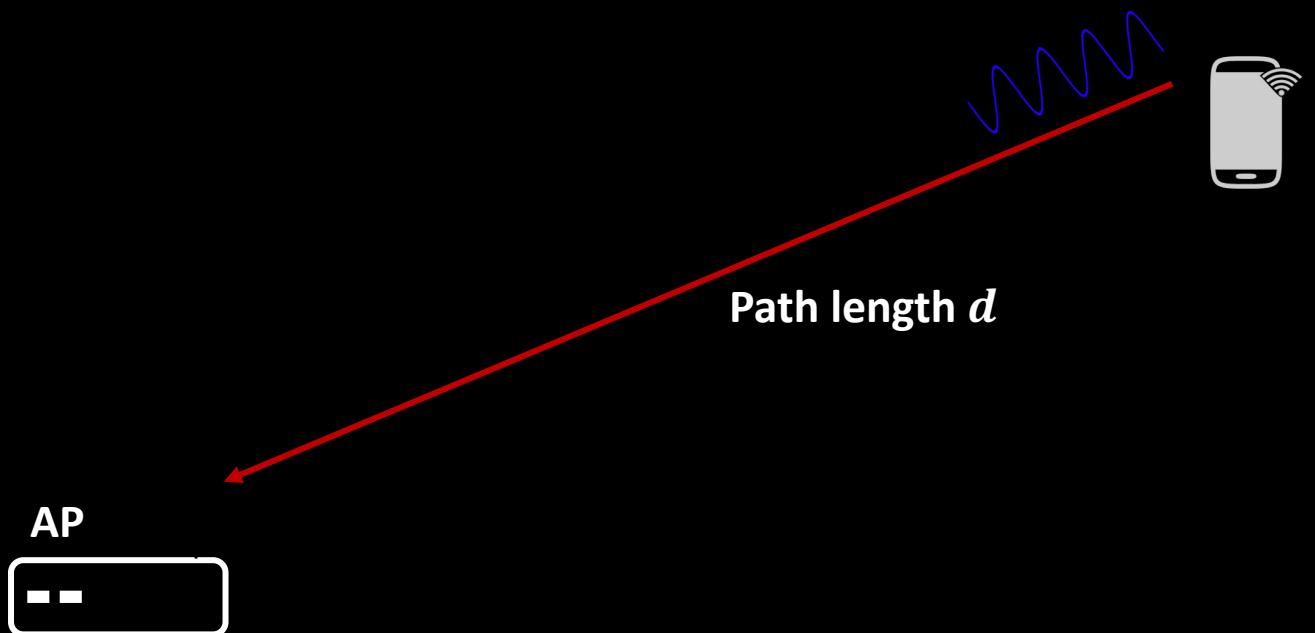
Speed of light

Wireless Channel



Channel attenuates the signal (Pathloss)

Wireless Channel



$$y(t) = \mathbf{h}x(t-\tau) + v(t)$$

Channel attenuates the signal (Pathloss)

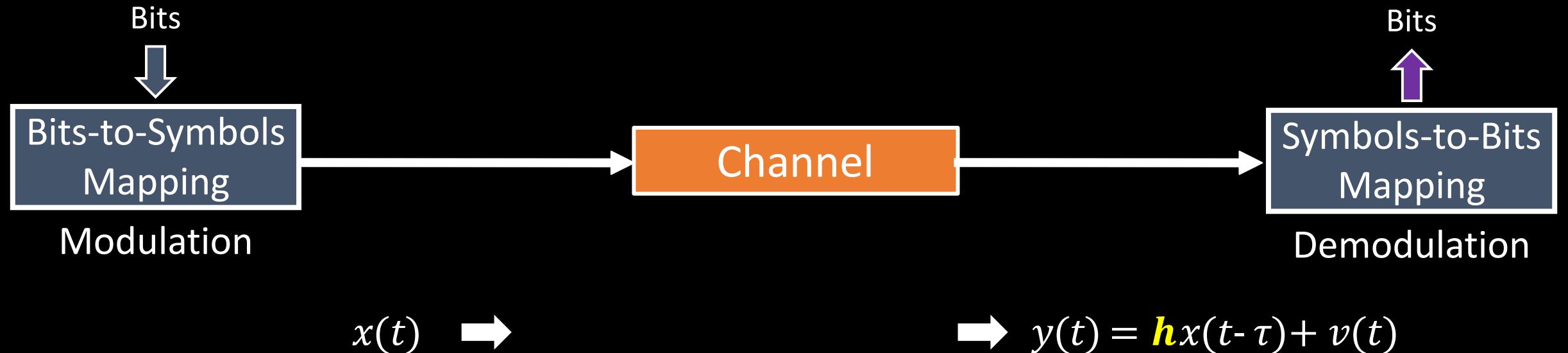
$$P_{rx} = G_{tx}G_{rx} \frac{\lambda^2}{(4\pi d)^2} P_{tx}$$



$$|\mathbf{h}| \propto \frac{\lambda}{d}$$

Wavelength of the carrier frequency

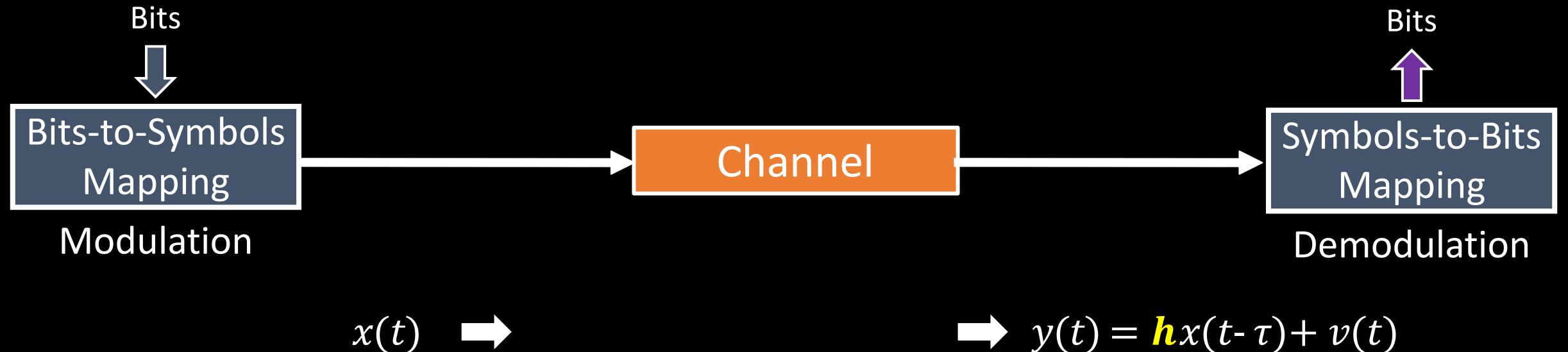
Wireless Channel



Channel rotates the signal (Adds Phase)

$$h \propto \frac{\lambda}{d} e^{j\phi}$$

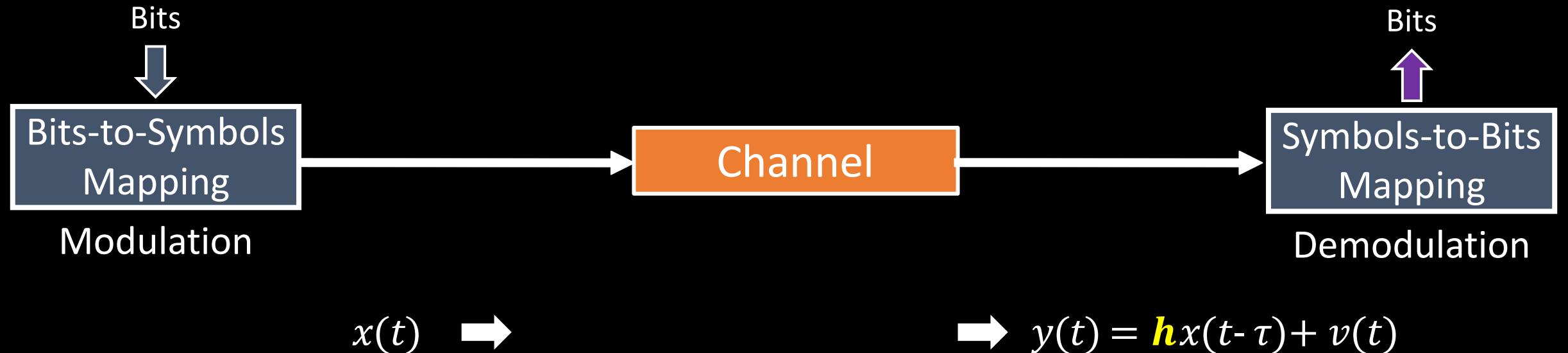
Wireless Channel



Channel rotates the signal (Adds Phase)

$$h \propto \frac{\lambda}{d} e^{j\phi} = \frac{\lambda}{d} e^{j2\pi d/\lambda}$$

Wireless Channel



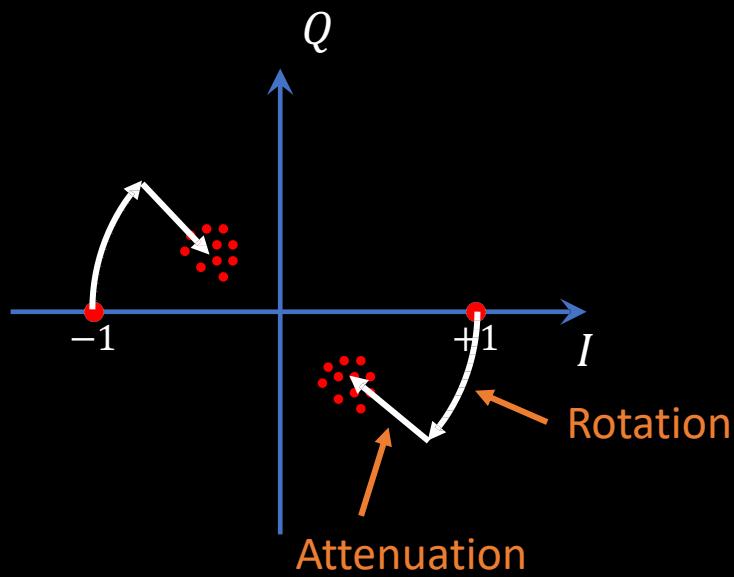
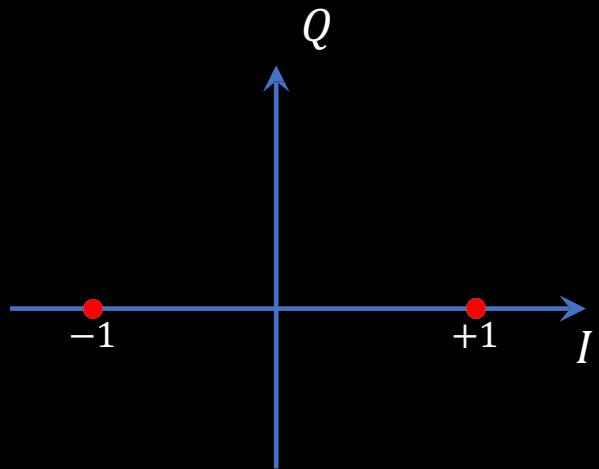
Wireless channel:

- Add the noise
- Delay the signal
- Attenuate the signal
- Rotate the signal

$$h \propto \frac{\lambda}{d} e^{j\phi} = \frac{\lambda}{d} e^{j2\pi d/\lambda}$$

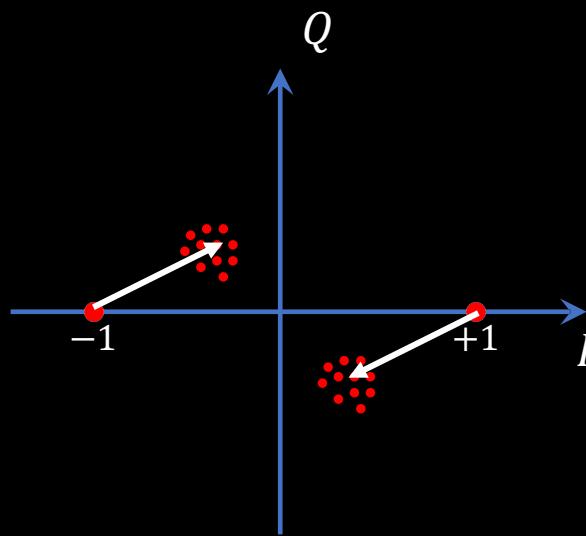
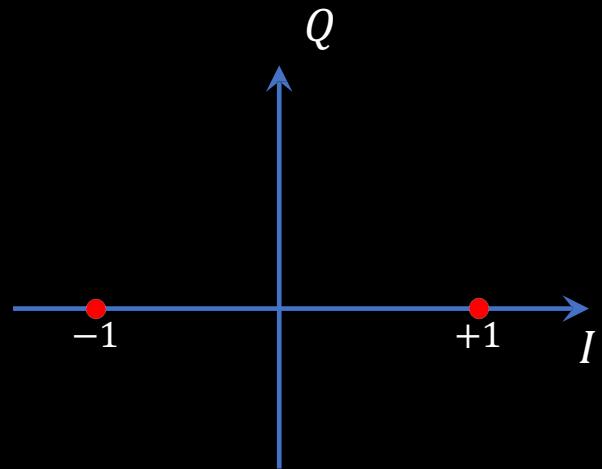
The Impact of Wireless Channel

Considering BPSK



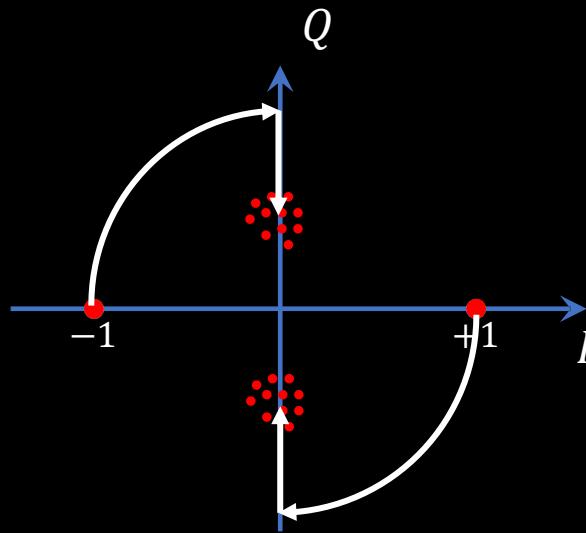
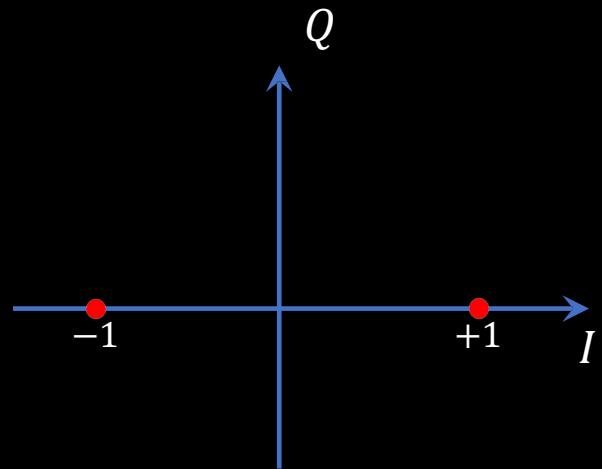
The Impact of Wireless Channel

Considering BPSK



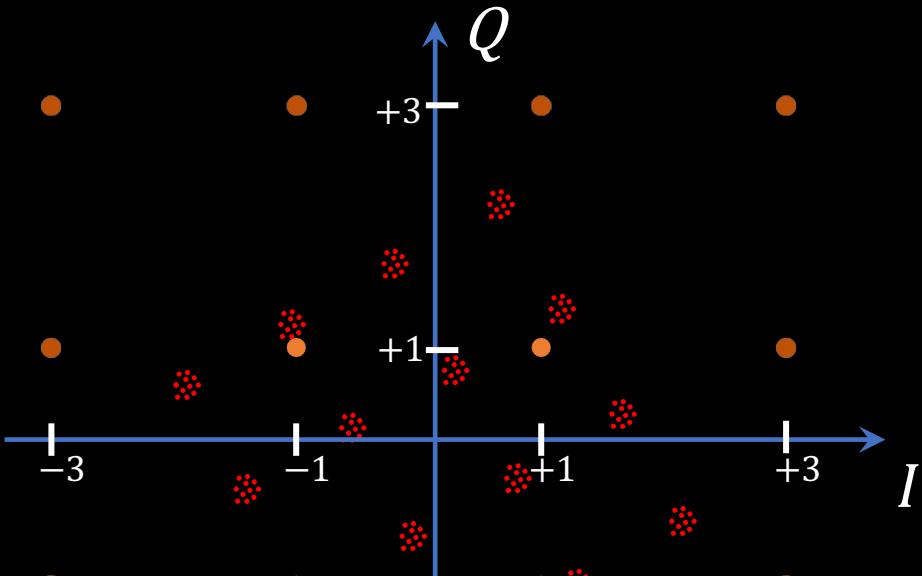
The Impact of Wireless Channel

Considering BPSK



The Impact of Wireless Channel

Considering QAM



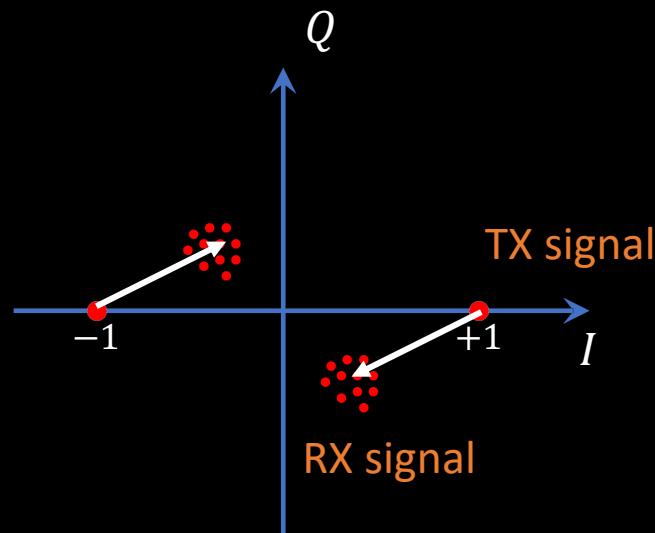
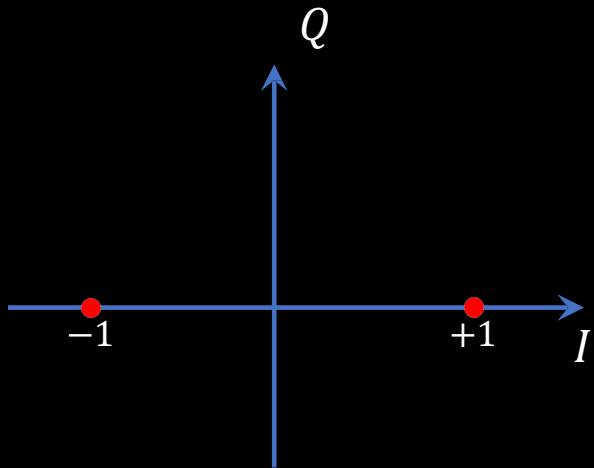
We need to estimate and correct for the channel \mathbf{h} !



Channel Estimation and & Correction

Channel Estimation

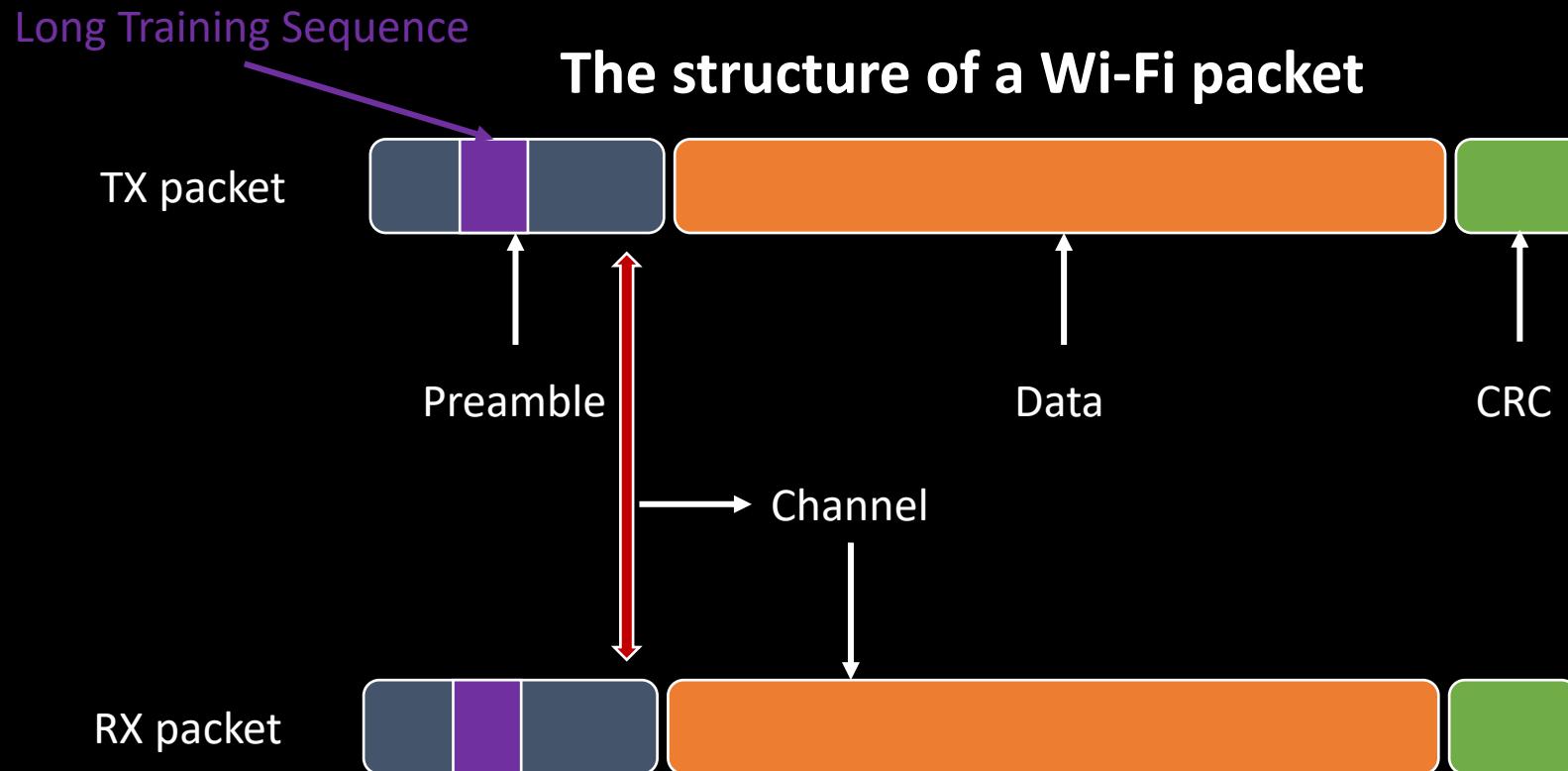
Considering BPSK



Generally, the receiver only gets the RX signals.

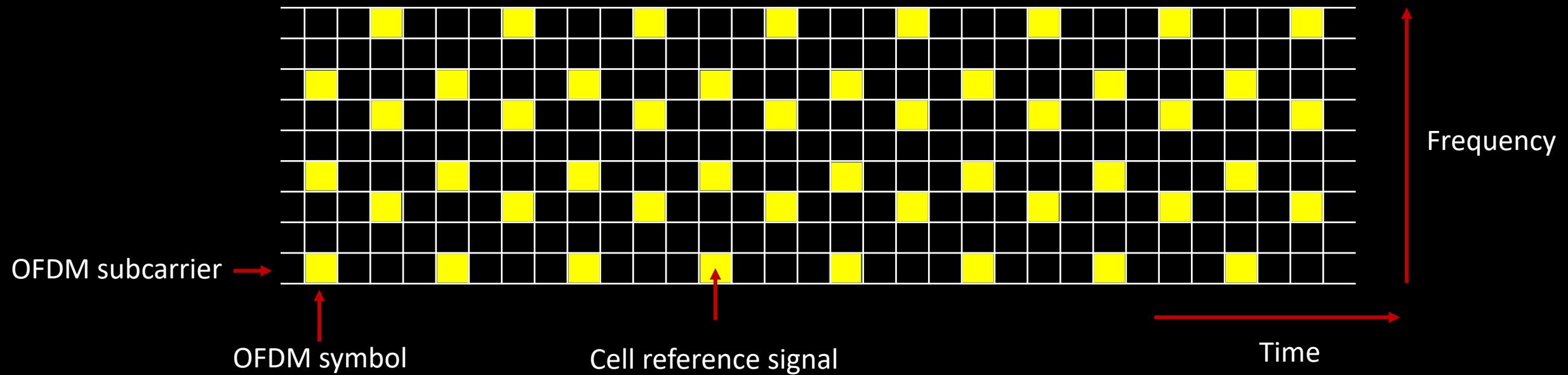
Channel Estimation

Wi-Fi: Send Training Sequence (Preamble Bits): Known Bits

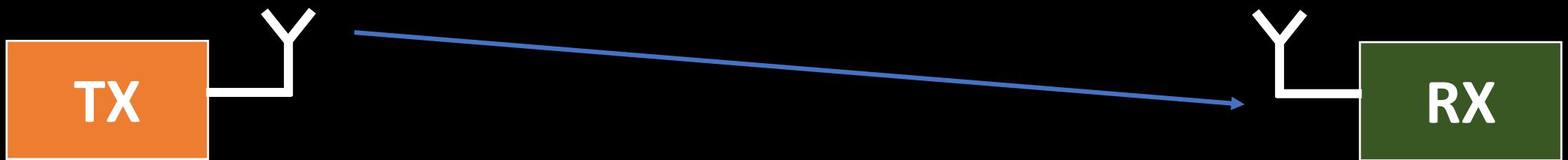


Channel Estimation

Cellular: Send Training Sequence: Reference Signal



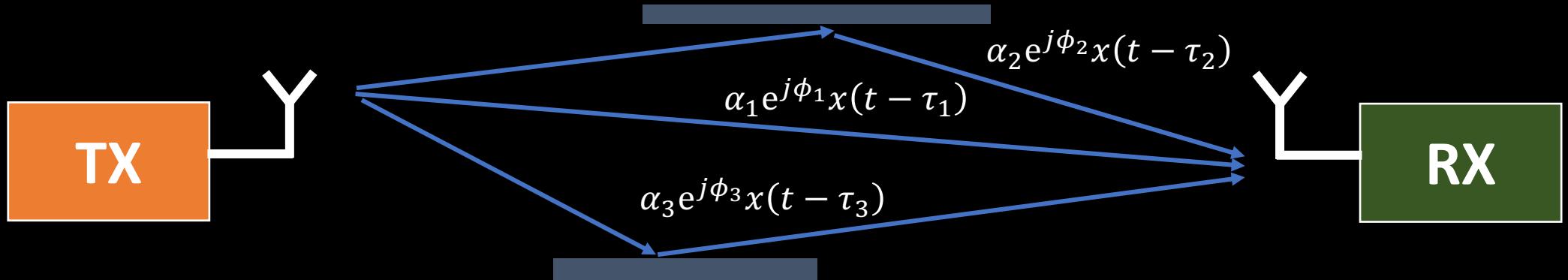
Multipath Channel



$$h \propto \frac{\lambda}{d} e^{j\phi} = \frac{\lambda}{d} e^{j2\pi d/\lambda}$$

Assumes single path!

Multipath Channel

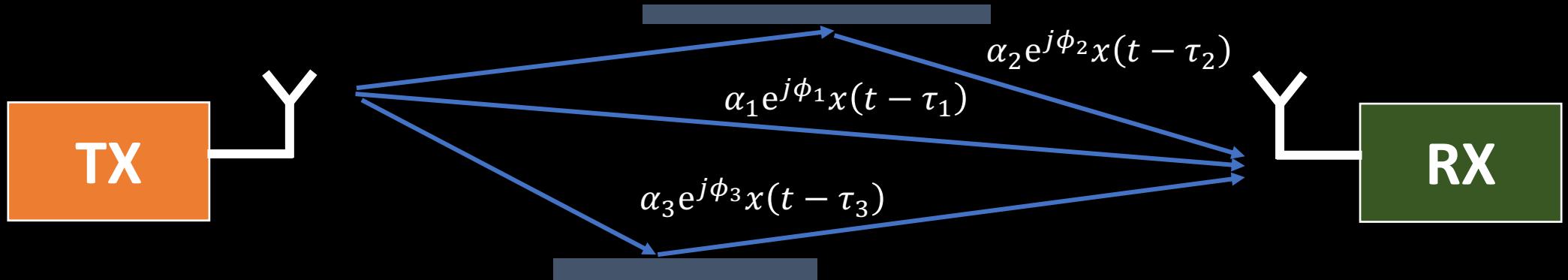


Multipath Propagation: radio signal reflects off objects ground, arriving at destination at slightly different times

Signal of a single path:

$$y(t) = \underbrace{\alpha e^{j\phi}}_{\text{Signal attenuation}} x(t - \tau_1) \underbrace{x}_{\text{Phase rotation}}$$

Multipath Channel



Multipath Propagation: radio signal reflects off objects ground, arriving at destination at slightly different times

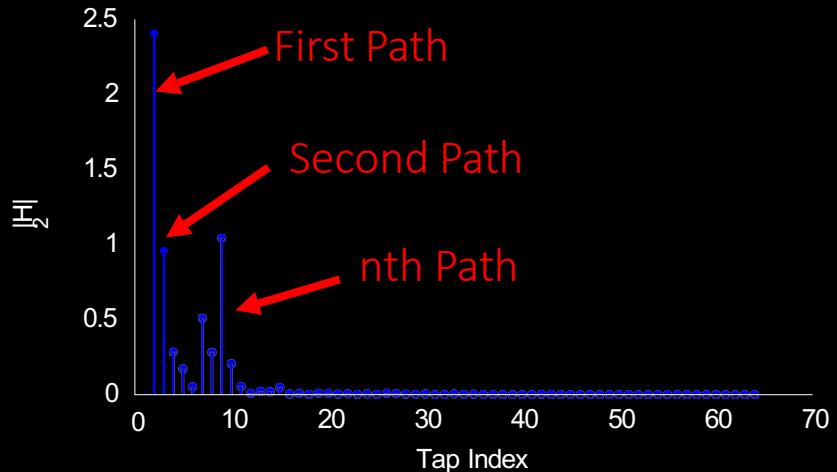
$$y(t) = \alpha_1 e^{j\phi_1} x(t - \tau_1) + \alpha_2 e^{j\phi_2} x(t - \tau_2) + \alpha_3 e^{j\phi_3} x(t - \tau_3) + \dots$$

$$y(t) = \sum_k \alpha_k e^{j\phi_k} x(t - \tau_k) = \sum_k h(\tau_k) x(t - \tau_k) = h(t) * x(t)$$

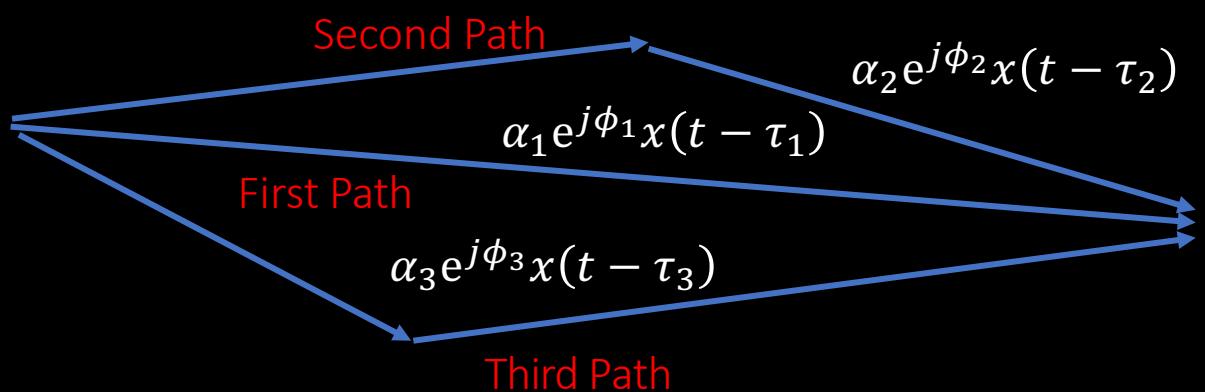
$h(t)$ is channel impulse response.

Multipath Channel

$$y(t) = \sum_k \alpha_k e^{j\phi_k} x(t - \tau_k) = \sum_k h(\tau_k) x(t - \tau_k) = h(t) * x(t)$$

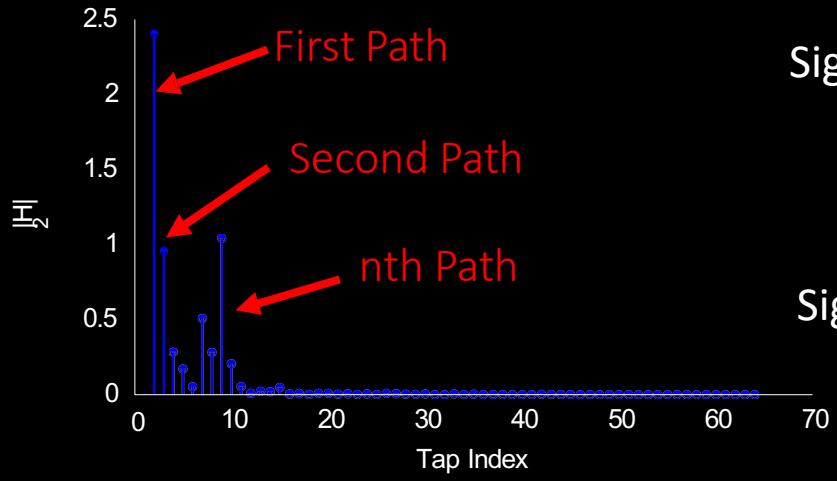


Multi-tap Channel

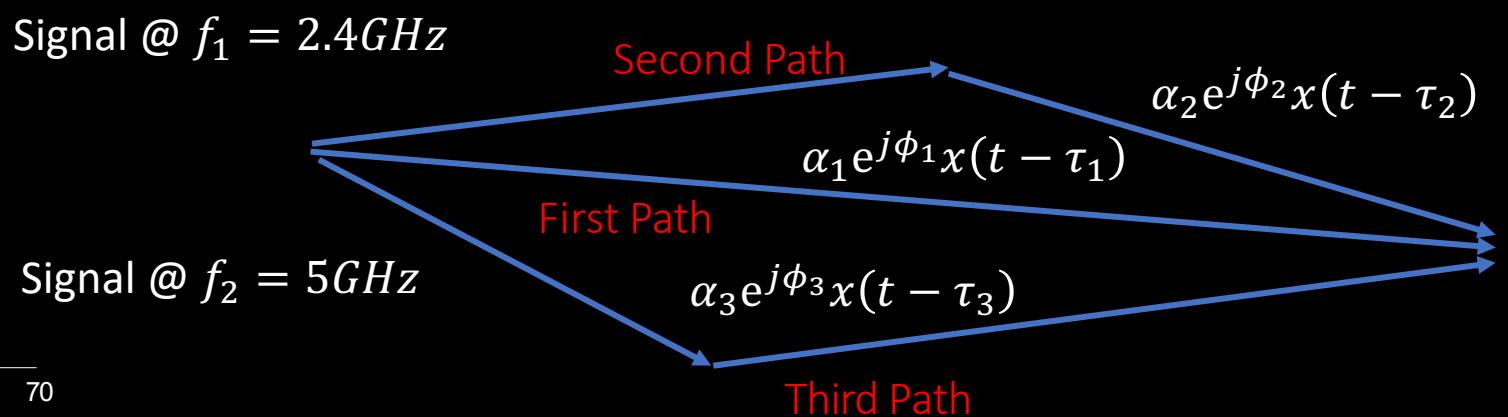


Multipath Channel

$$y(t) = \sum_k \alpha_k e^{j\phi_k} x(t - \tau_k) = \sum_k h(\tau_k) x(t - \tau_k) = h(t) * x(t)$$



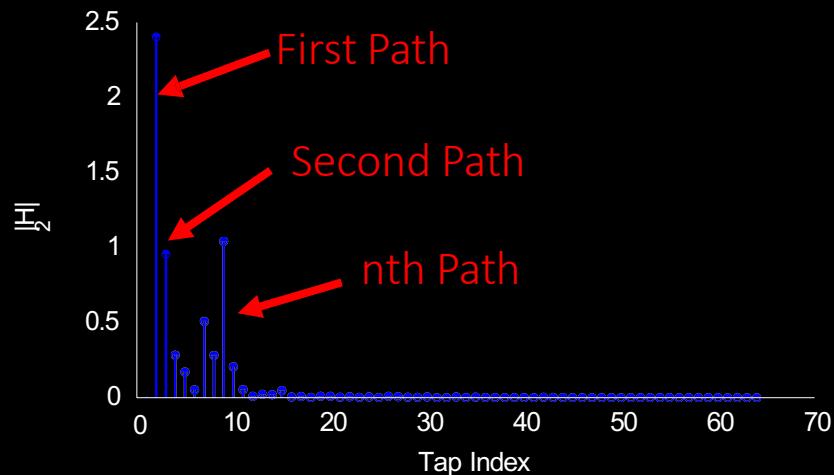
Multi-tap Channel



Multipath Channel

$$y(t) = \sum_k \alpha_k e^{j\phi_k} x(t - \tau_k) = \sum_k h(\tau_k) x(t - \tau_k) = h(t) * x(t)$$

Example of 2 paths with distance $d_1 = 1m, d_2 = 1.6m$



$$h = h_1 + h_2 = \frac{\lambda}{d_1} e^{j2\pi d_1/\lambda} + \frac{\lambda}{d_2} e^{j2\pi d_2/\lambda}$$

Signal @ $f_1 = 2.4GHz, \lambda_1 = 12cm$

$$h = 0.12 e^{j\frac{2\pi}{3}} + 0.113 e^{j\frac{5\pi}{3}} \approx 0.006$$

Signal @ $f_2 = 5GHz, \lambda_1 = 12cm$

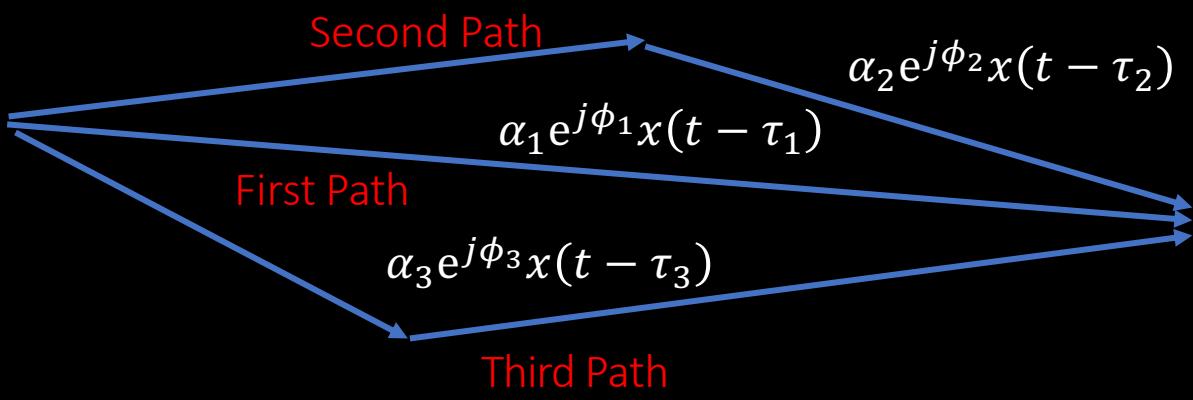
$$h = 0.06 e^{j\frac{5\pi}{3}} + 0.05 e^{j\frac{5\pi}{3}} \approx 0.116$$



17×
(24dB)

Multipath Channel

Signal @ $f_1 = 2.4\text{GHz}$



$$h \approx 0.006$$

Signal @ $f_2 = 5\text{GHz}$

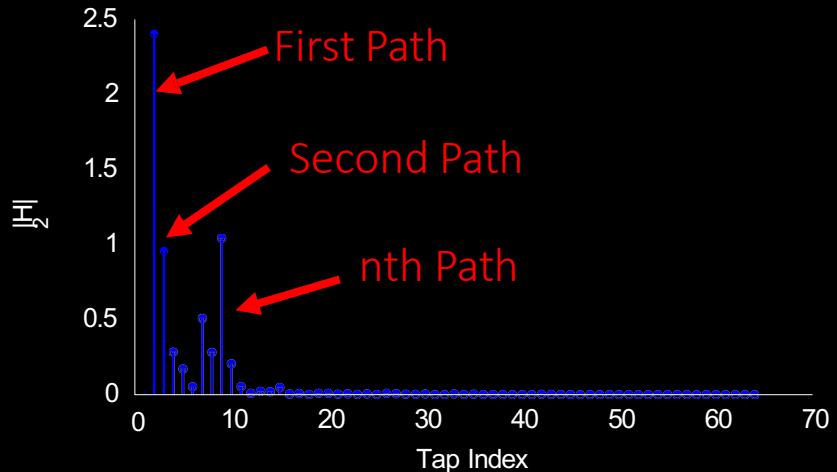
The same multipath channel

$17\times$
(24dB)

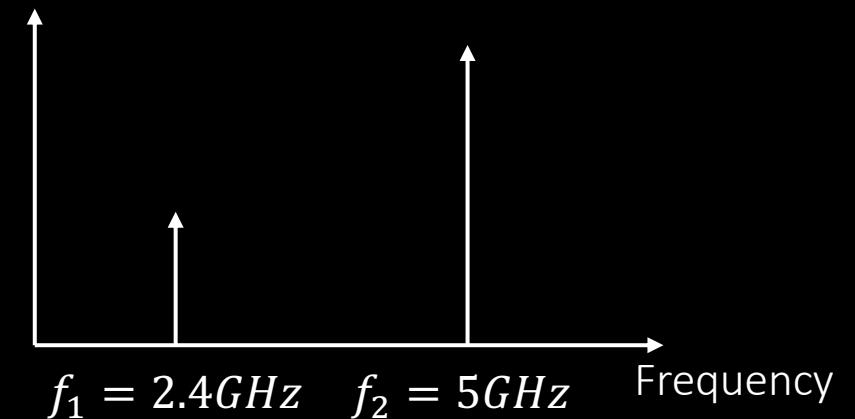
$$h \approx 0.116$$



Multipath Channel



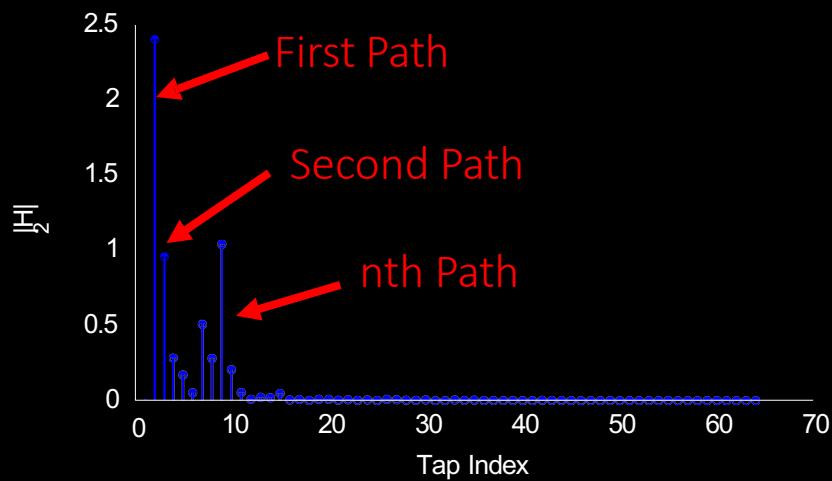
Multi-tap Channel



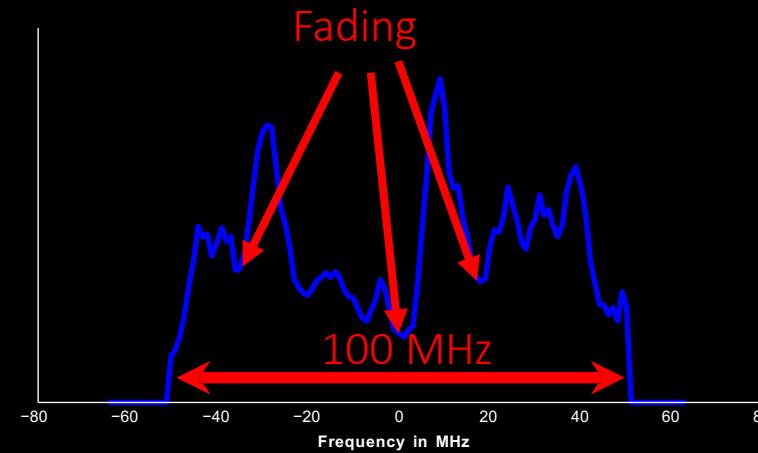
Channel Fading

Symbols arriving along different paths sum up destructively

Multipath Channel

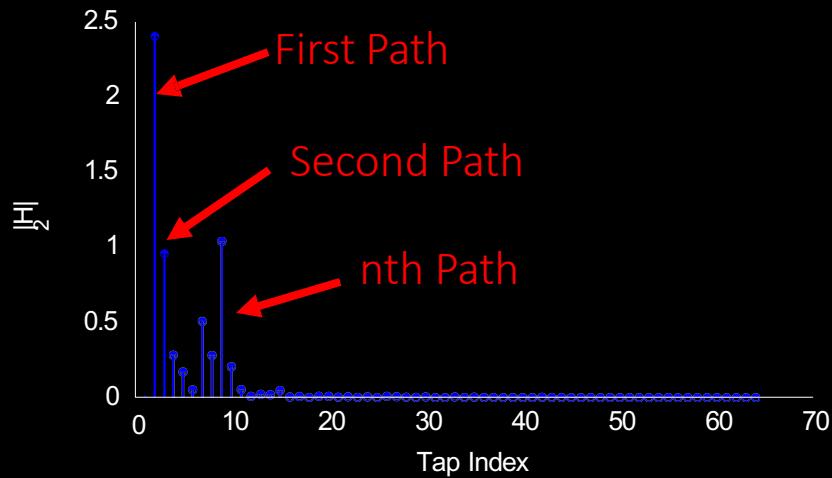


Multi-tap Channel

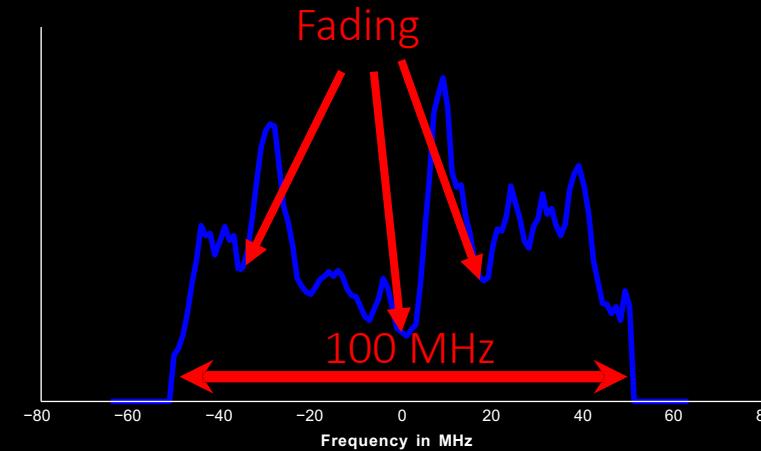


Frequency Selective Fading

Multipath Channel



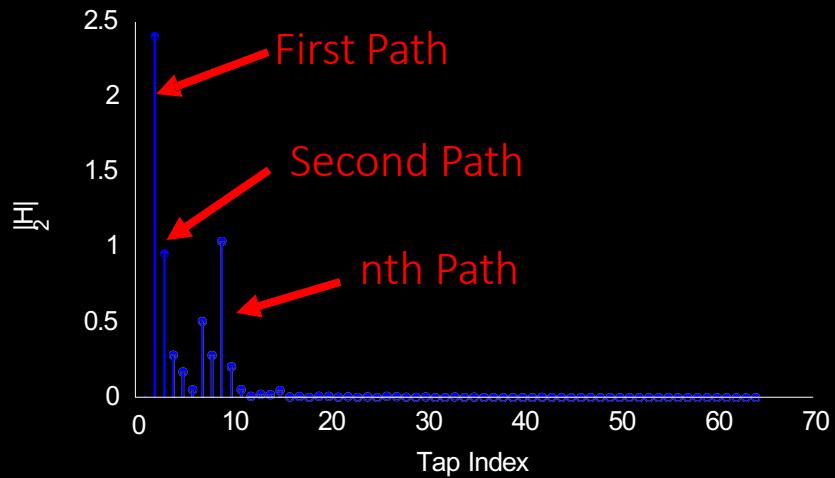
Multi-tap Channel



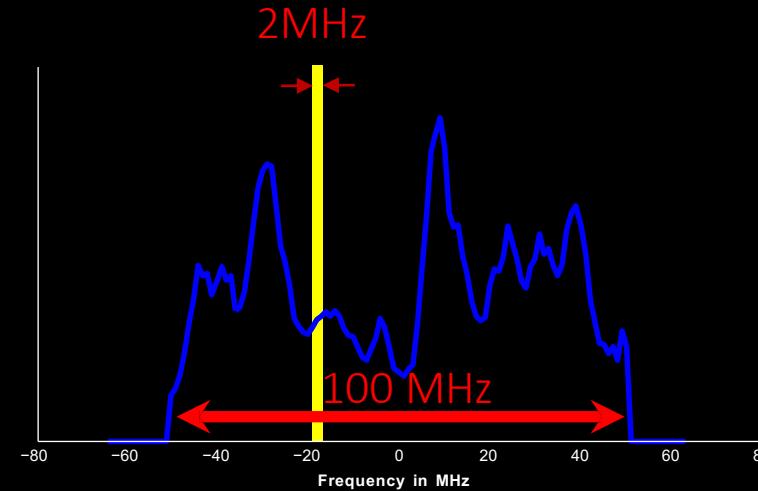
Frequency Selective Fading

How can we estimate such a complicated wireless channel?

Channel Estimation for Wideband Channel



Multi-tap Channel



Frequency Selective Fading

Channel of a narrowband can be treated as flat

Single Carrier Modulation

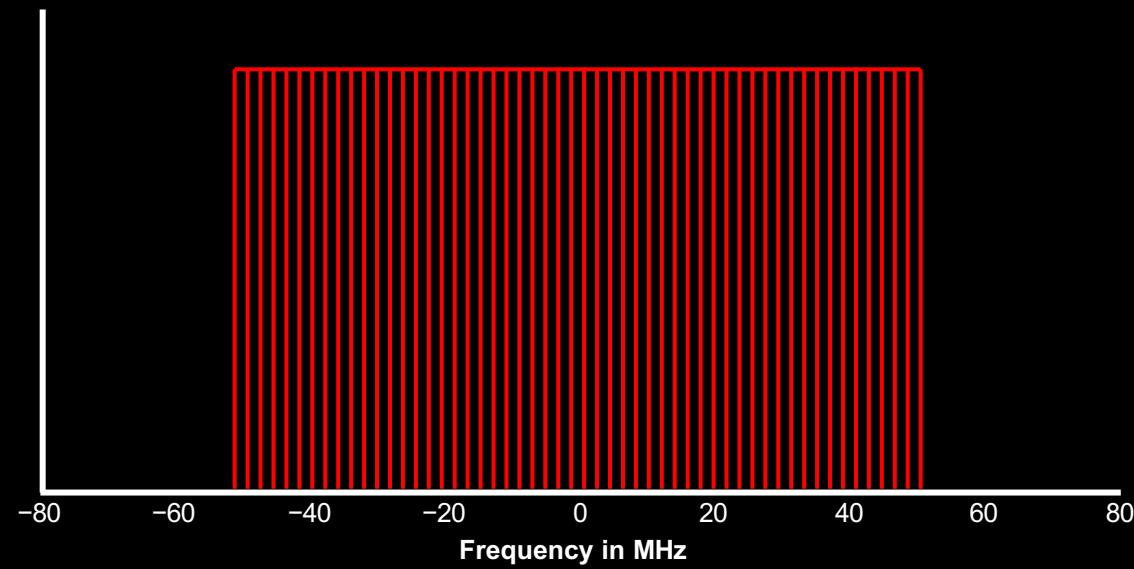


Single Carrier Modulation



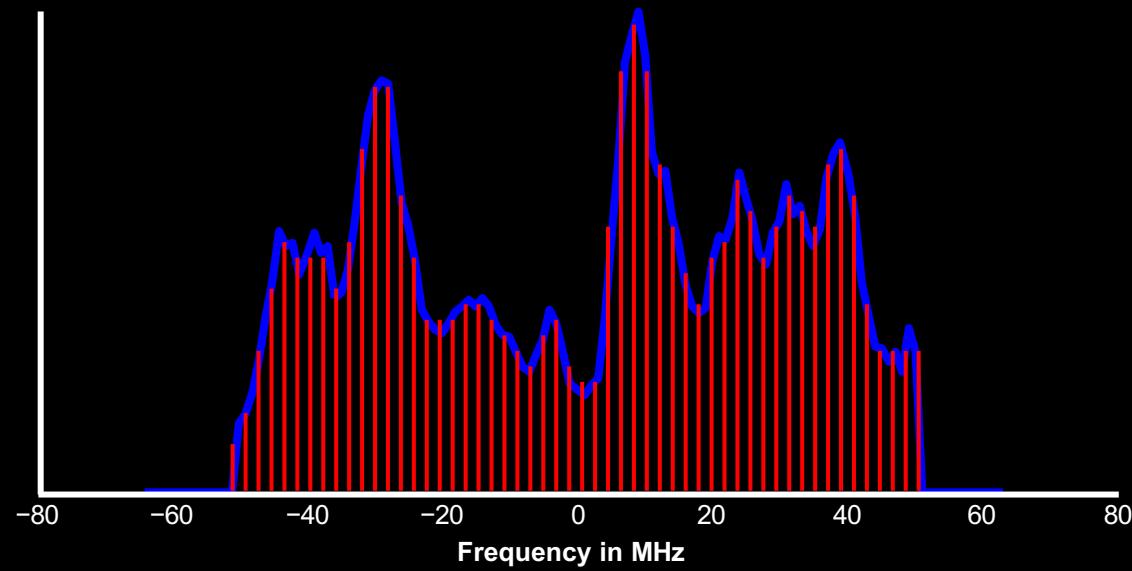
Multi-Carrier Modulation

- Divide the spectrum into many narrow bands
- Transmits symbols on different carriers in the narrow bands
- Channel is Flat → estimate channel of each narrow band



Multi-Carrier Modulation

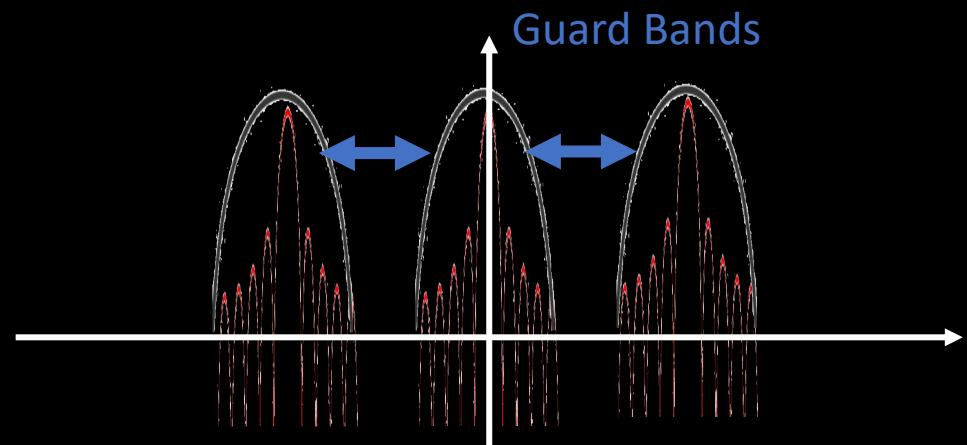
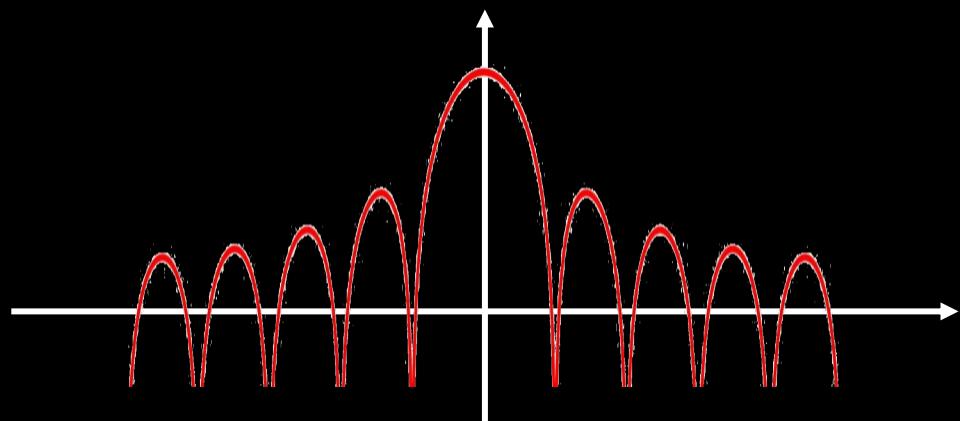
- Divide the spectrum into many narrow bands
- Transmits symbols on different carriers in the narrow bands
- Channel is Flat → estimate channel of each narrow band



Not That Simple!

Multi-Carrier Modulation

- Divide the spectrum into many narrow bands



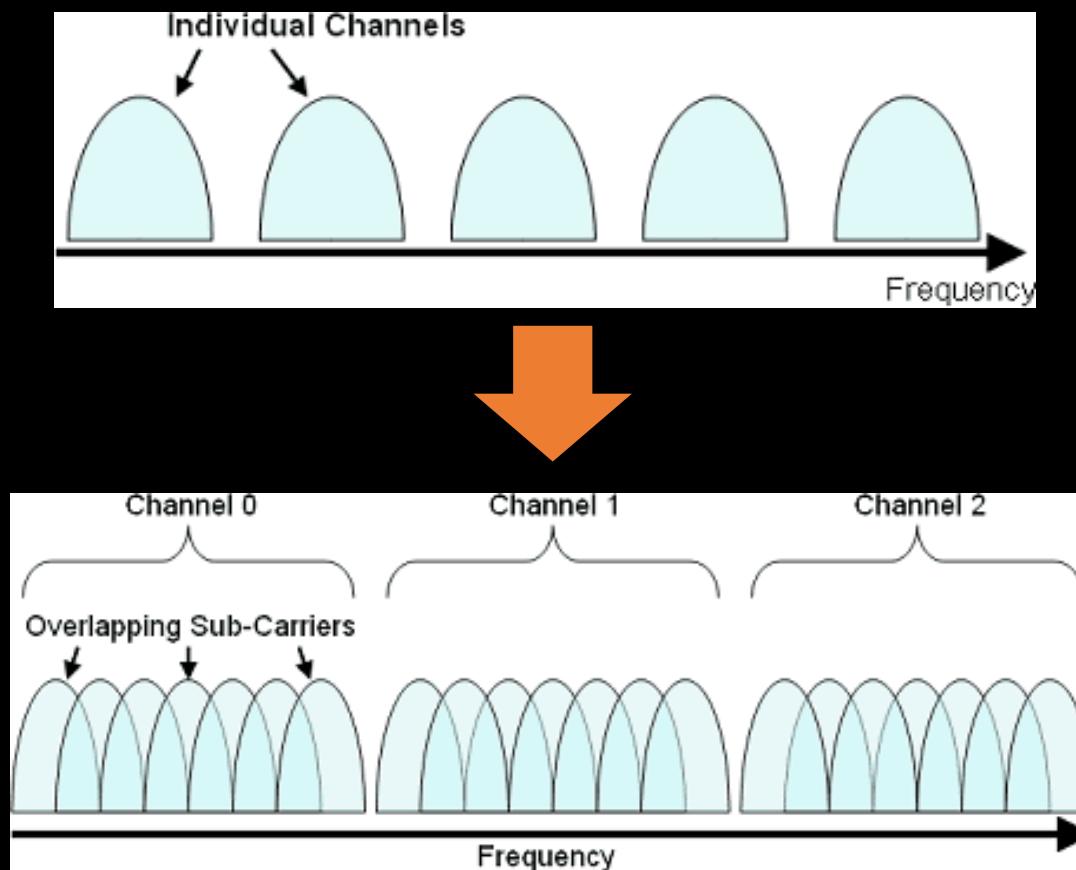
- Significant leakage between adjacent subcarriers
- Need guard bands → very very inefficient

Solution: Make the Sub-Carriers Orthogonal

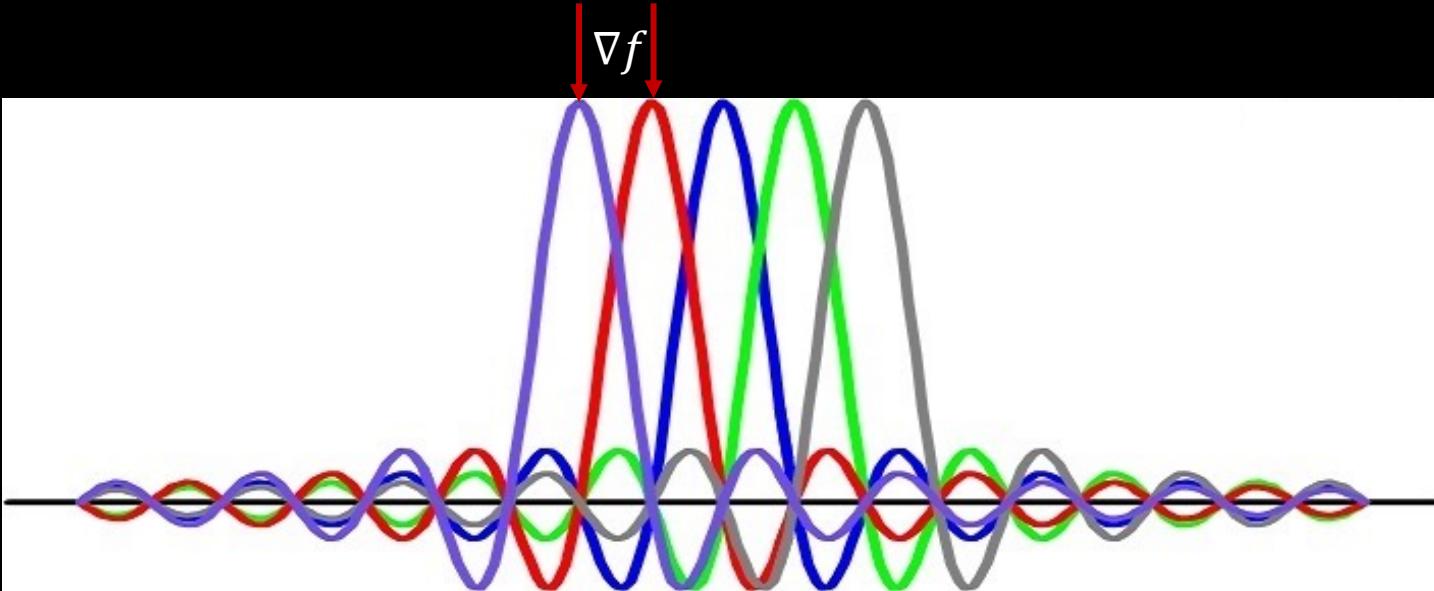
Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

Make the Sub-Carriers Orthogonal



OFDM: Orthogonal Frequency Division Multiplexing



- Subcarriers are orthogonal: At the sub-carrier frequency, the sampled value has zero leakage from other subcarriers.

How to achieve this?

OFDM: Orthogonal Frequency Division Multiplexing

Use DFT: Discrete Fourier Transform

$$\text{N-Point DFT: } X(f_i) = \frac{1}{N} \sum_{t=0}^{N-1} x(t) e^{-j \frac{2\pi f_i t}{N}}$$

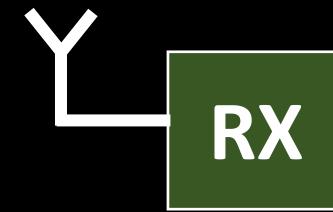
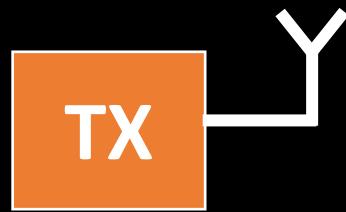
$$\text{N-Point IDFT: } x(t) = \sum_{f_i=0}^{N-1} X(f_i) e^{j \frac{2\pi f_i t}{N}}$$

Send symbols in Frequency Domain

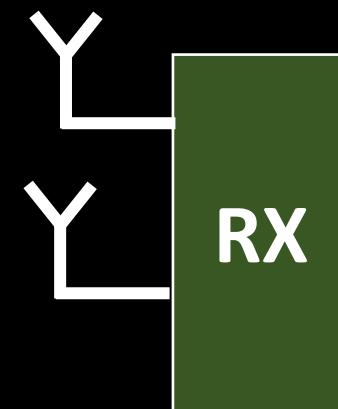
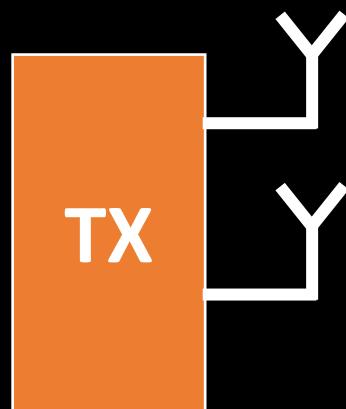
$X(f) = s(n) \rightarrow$ Compute and transmit $x(t)$ using IDFT

MIMO: Multiple Input Multiple Output

So far: single input single output

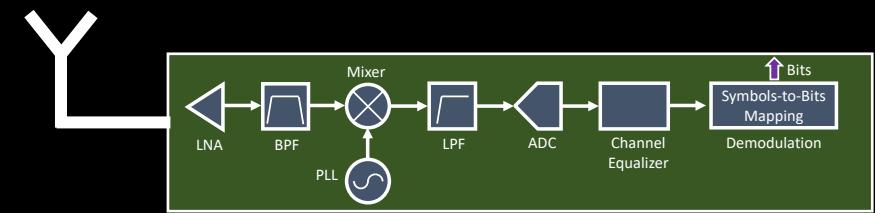
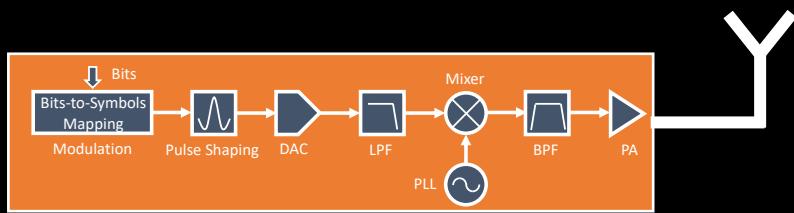


MIMO: multiple input multiple output

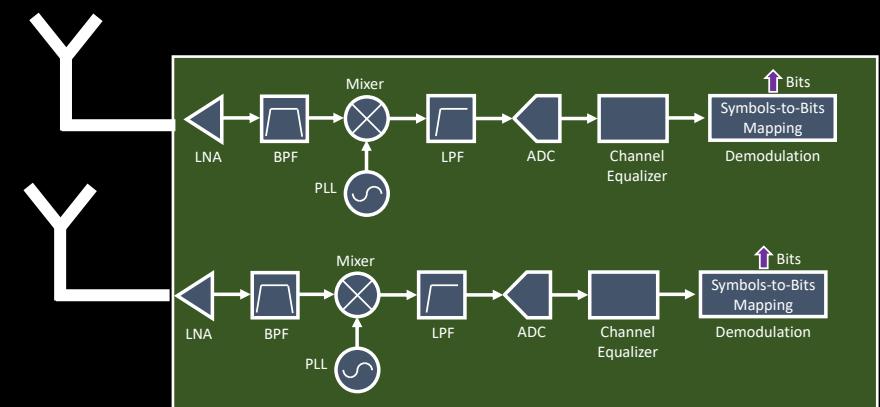
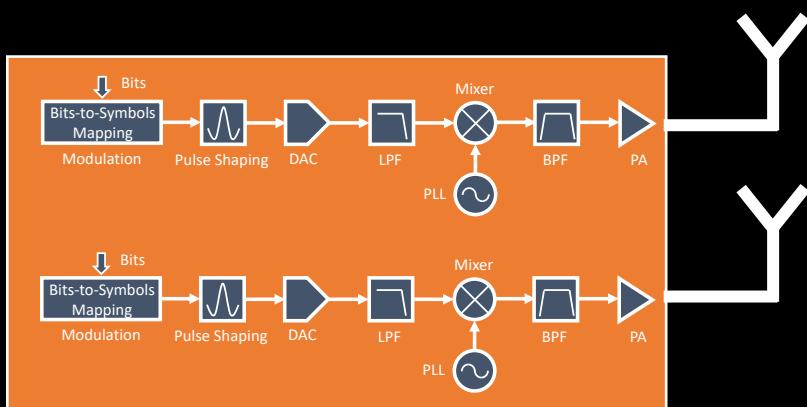


MIMO: Multiple Input Multiple Output

So far: single input single output



MIMO: multiple input multiple output



MIMO Gains

Diversity Gain:

- Send/Receive the same packet on multiple antennas
- Increase SNR of the received packets
→ transmit at higher data rates

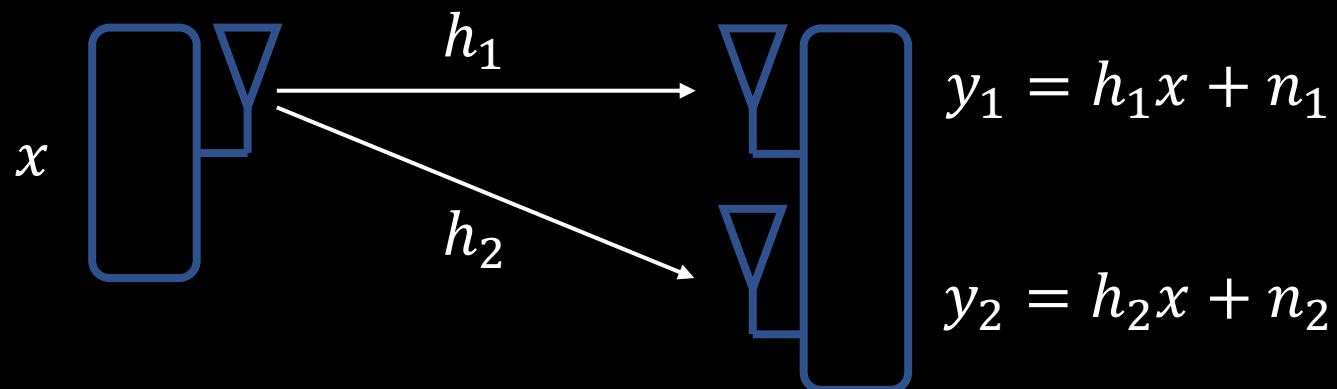
Multiplexing Gain:

- Send multiple packets at the same time
- $N \times N$ MIMO → $N \times$ more packets

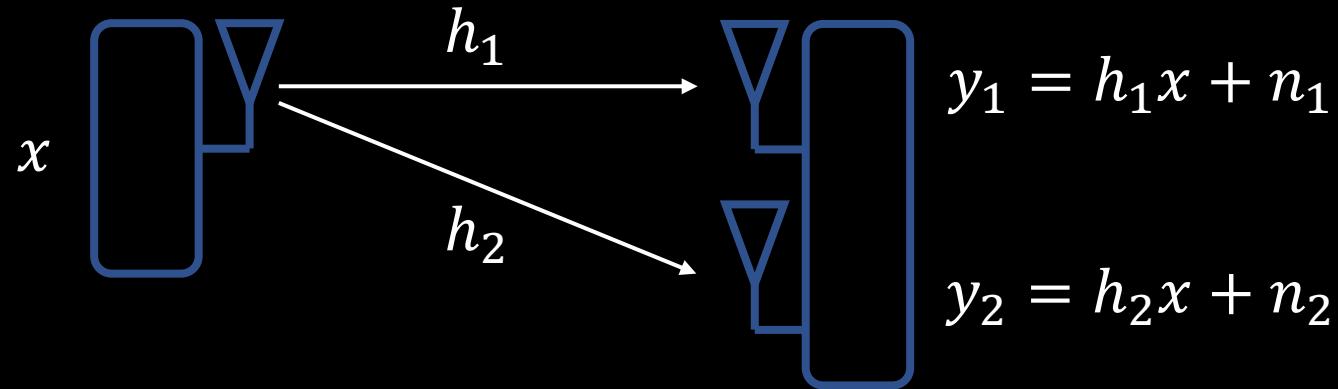
MIMO Gains

Diversity Gain:

- Send/Receive the same packet on multiple antennas
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→ transmit at higher data rates



Diversity Gains



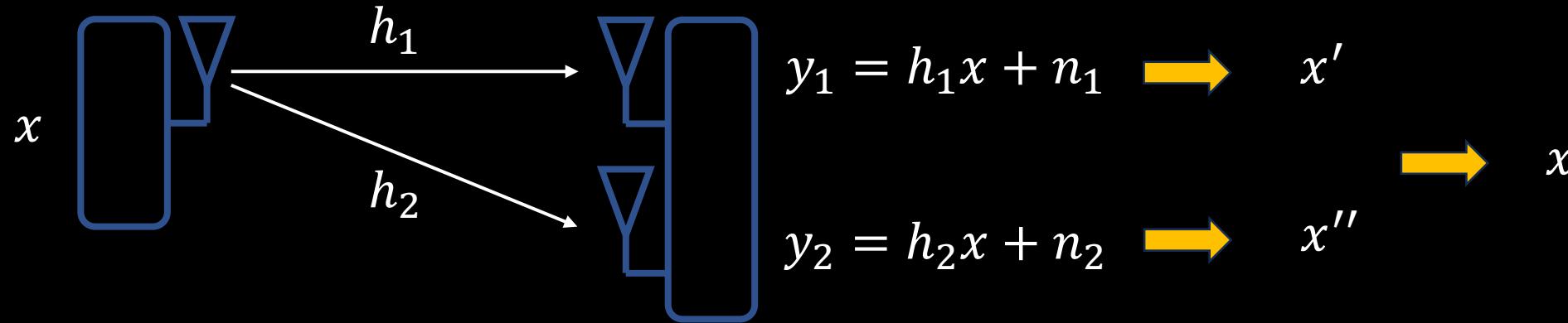
How to best decode x ?

Option 1: Add the received signals

$$\begin{aligned}y_1 + y_2 &= h_1x + n_1 + h_2x + n_2 \\&= (h_1 + h_2)x + n_1 + n_2\end{aligned}$$

Channels can sum up destructively! $h_1 + h_2 \approx 0$

Diversity Gains



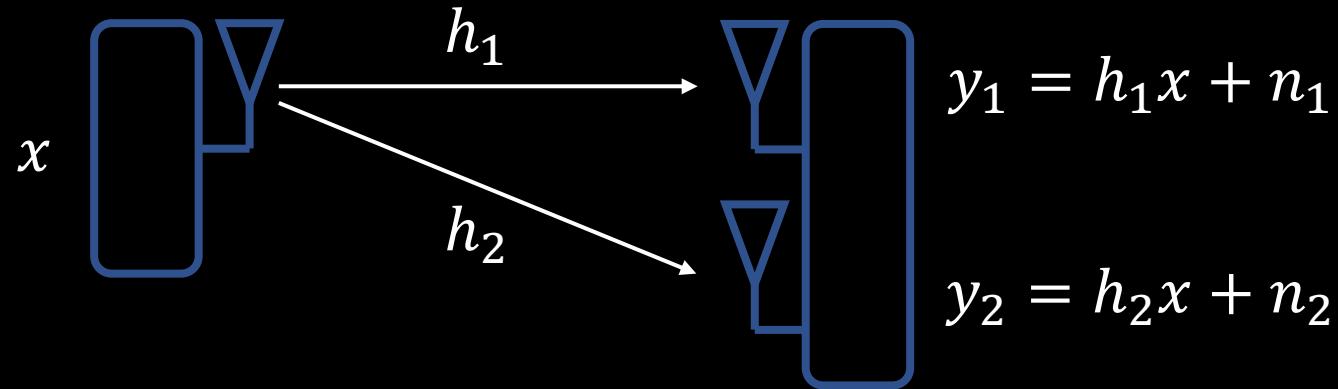
How to best decode x ?

Option 1: Add the received signals

Option 2: Decode independently

Sub-optimal!

Diversity Gains



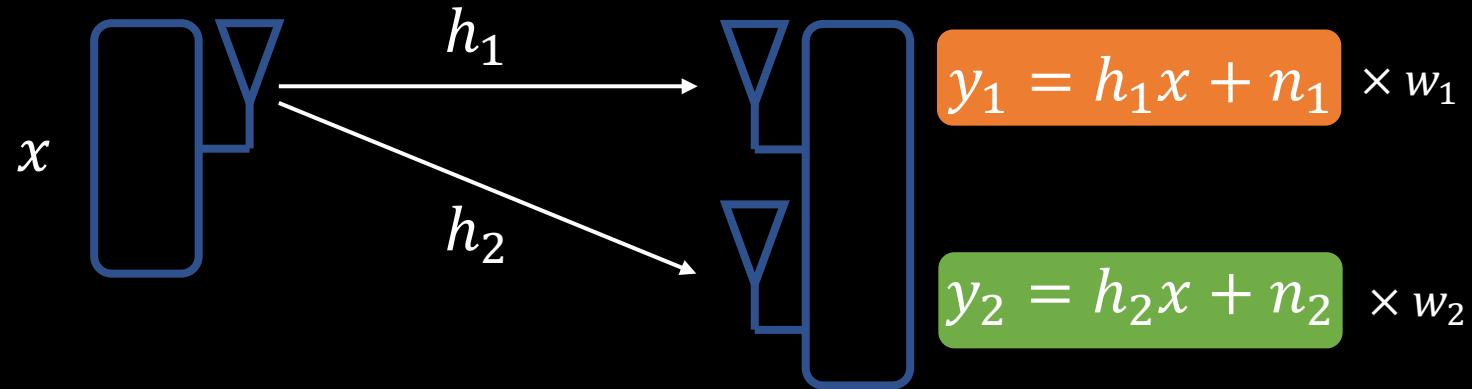
How to best decode x ?

Option 1: Add the received signals

Option 2: Decode independently

Optimal Solution: Maximum Ratio Combining (MRC)

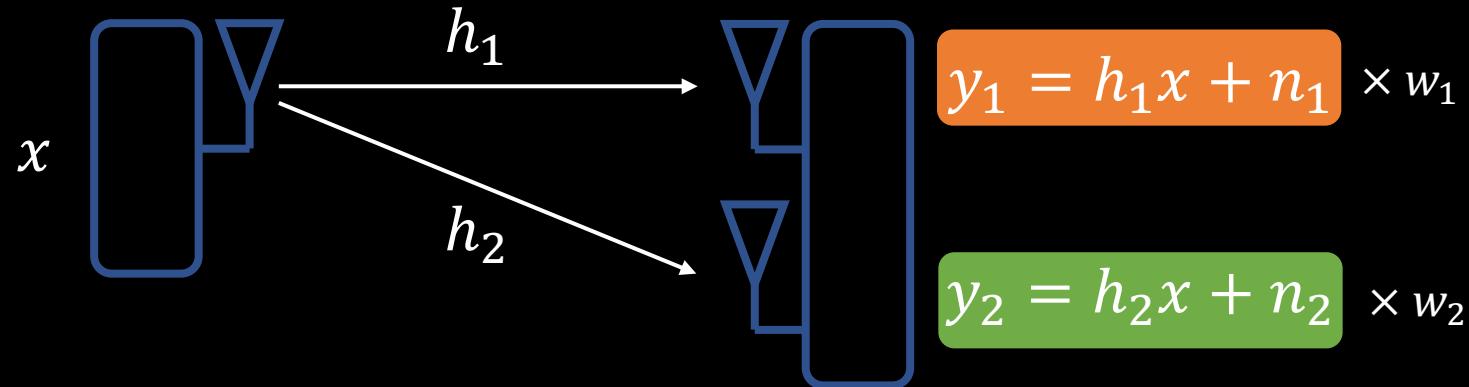
Diversity Gains: Maximum Ratio Combining



$$y_1 + y_2 = (h_1 + h_2)x + n_1 + n_2$$

Channels can sum up destructively! $h_1 + h_2 \approx 0$

Diversity Gains: Maximum Ratio Combining

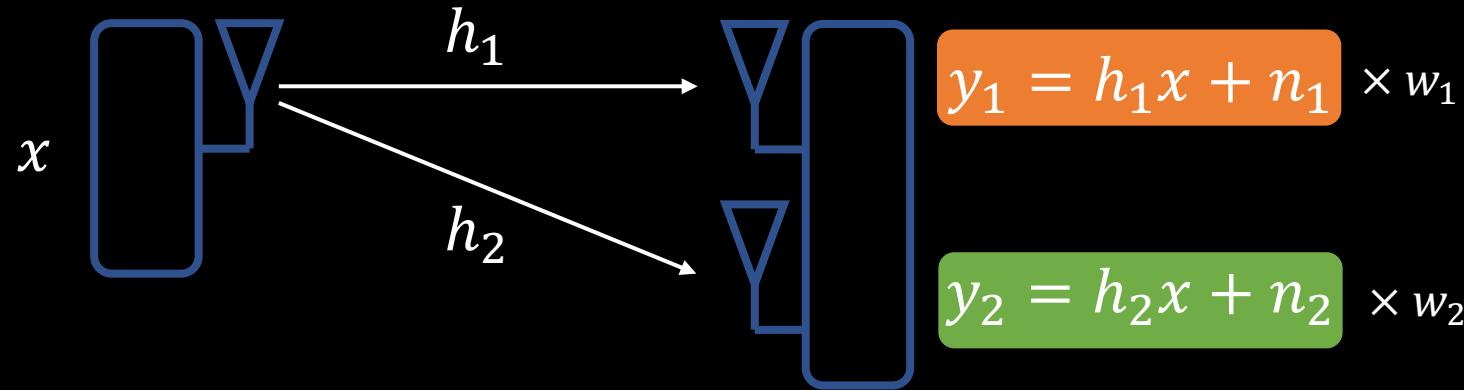


$$w_1 y_1 + w_2 y_2 = (w_1 h_1 + w_2 h_2)x + w_1 n_1 + w_2 n_2$$

The optimal weight is:

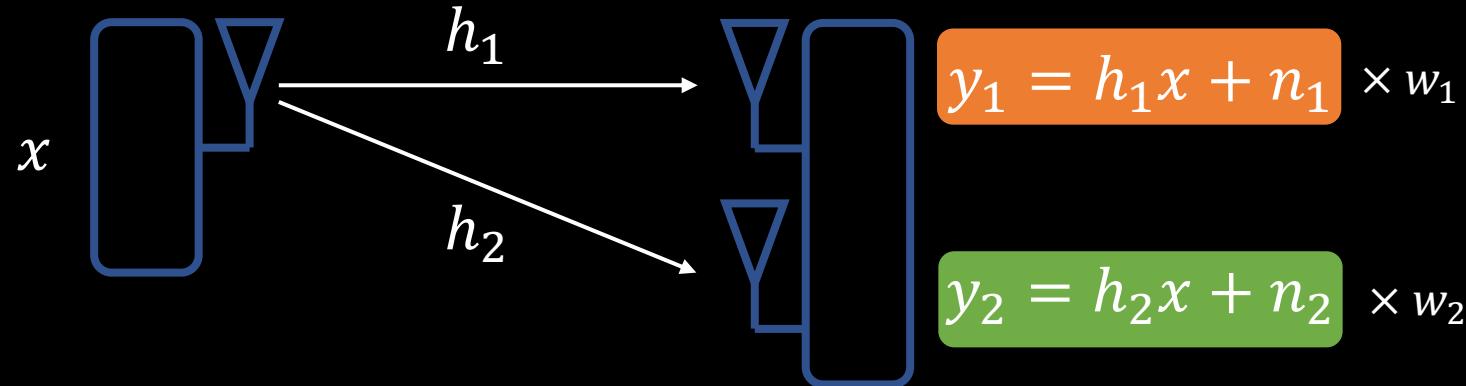
$$\begin{cases} w_1 = h_1^* \\ w_2 = h_1^* \end{cases}$$

Diversity Gains: Maximum Ratio Combining



$$\begin{aligned}w_1y_1 + w_2y_2 &= (h_1^*h_1 + h_1^*h_2)x + h_1^*n_1 + h_1^*n_2 \\&= (|h_1|^2 + |h_2|^2)x + h_1^*n_1 + h_1^*n_2\end{aligned}$$

Diversity Gains: Maximum Ratio Combining



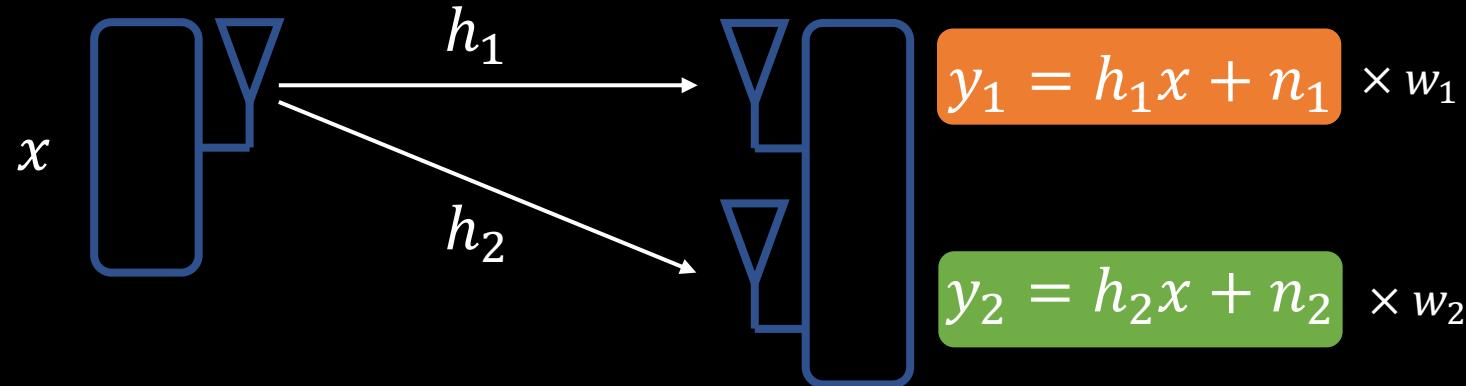
$$\begin{aligned}w_1 y_1 + w_2 y_2 &= (h_1^* h_1 + h_1^* h_2)x + h_1^* n_1 + h_1^* n_2 \\&= (|h_1|^2 + |h_2|^2)x + h_1^* n_1 + h_1^* n_2\end{aligned}$$

Let $P = D[|x|^2]$ and $N_0 = D[|n_1|^2] = D[|n_2|^2]$

$$\text{Signal Power} = (|h_1|^2 + |h_2|^2)^2 P$$

$$\begin{aligned}\text{Noise Power} &= D[|h_1^* n_1 + h_1^* n_2|^2] = D[|h_1^* n_1|^2] + D[|h_1^* n_2|^2] \\&= |h_1|^2 D[|n_1|^2] + |h_2|^2 D[|n_2|^2] = (|h_1|^2 + |h_2|^2) N_0\end{aligned}$$

Diversity Gains: Maximum Ratio Combining



$$\begin{aligned}w_1 y_1 + w_2 y_2 &= (h_1^* h_1 + h_1^* h_2)x + h_1^* n_1 + h_1^* n_2 \\&= (|h_1|^2 + |h_2|^2)x + h_1^* n_1 + h_1^* n_2\end{aligned}$$

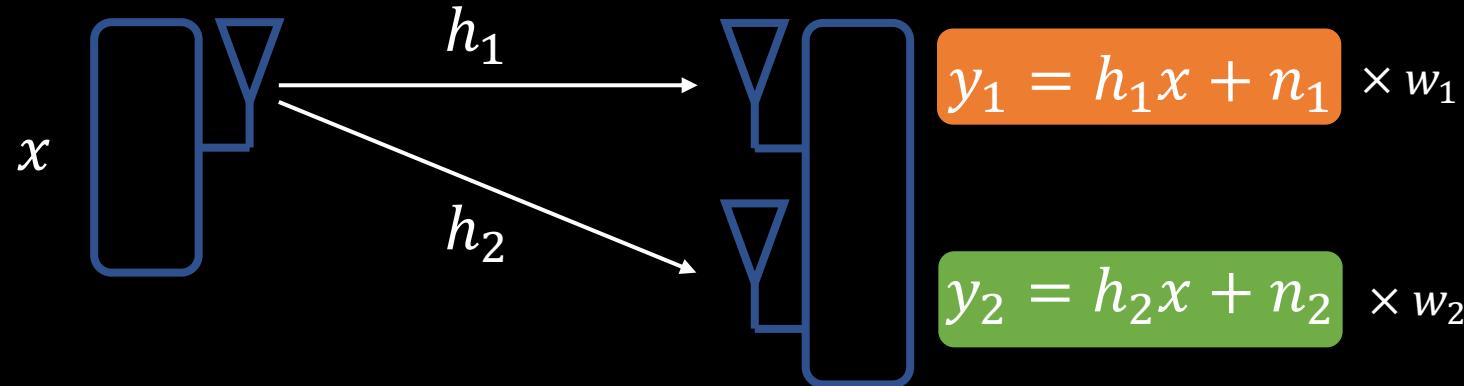
Let $P = D[|x|^2]$ and $N_0 = D[|n_1|^2] = D[|n_2|^2]$

$$\text{Signal Power} = (|h_1|^2 + |h_2|^2)^2 P$$

$$\text{Noise Power} = (|h_1|^2 + |h_2|^2) N_0$$

$$\text{SNR} = \frac{(|h_1|^2 + |h_2|^2)^2 P}{(|h_1|^2 + |h_2|^2) N_0} = (|h_1|^2 + |h_2|^2) \frac{P}{N_0}$$

Diversity Gains: Maximum Ratio Combining



$$\begin{aligned}w_1y_1 + w_2y_2 &= (h_1^*h_1 + h_1^*h_2)x + h_1^*n_1 + h_1^*n_2 \\&= (|h_1|^2 + |h_2|^2)x + h_1^*n_1 + h_1^*n_2\end{aligned}$$

With Maximum Ratio Combining and Diversity:

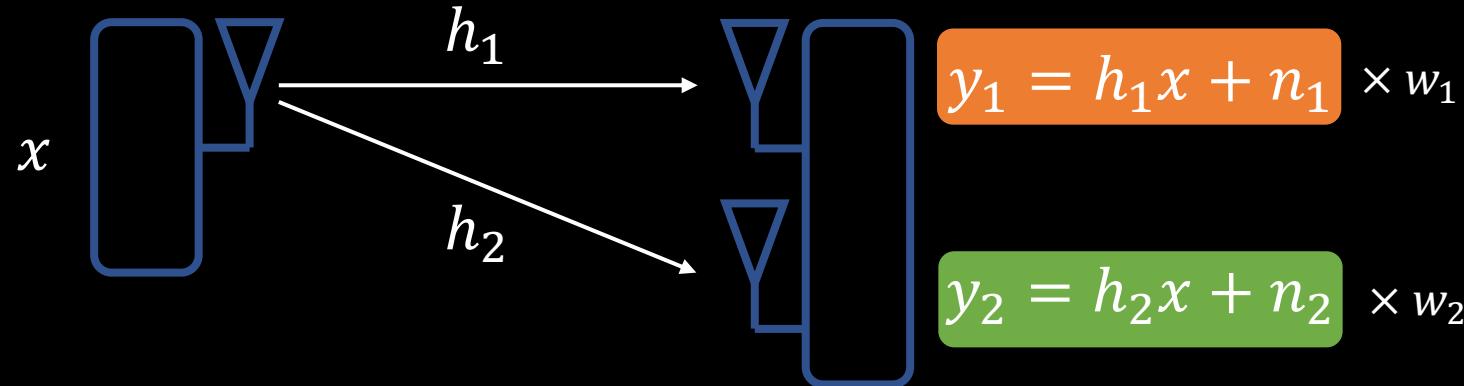
$$\text{SNR} = (|h_1|^2 + |h_2|^2) \frac{P}{N_0}$$

Single receiver and no Diversity:

$$\text{SNR} = |h_1|^2 \frac{P}{N_0} \quad \text{Or}$$

$$\text{SNR} = |h_2|^2 \frac{P}{N_0}$$

Diversity Gains: Maximum Ratio Combining



With Maximum Ratio Combining and Diversity:

$$\text{SNR} = (|h_1|^2 + |h_2|^2) \frac{P}{N_0}$$

Single receiver and no Diversity:

$$\text{SNR} = |h_1|^2 \frac{P}{N_0} \quad \text{Or}$$

$$\text{SNR} = |h_2|^2 \frac{P}{N_0}$$

- If $|h_1|^2 \approx |h_2|^2$. \rightarrow We almost double the SNR !

MIMO Gains

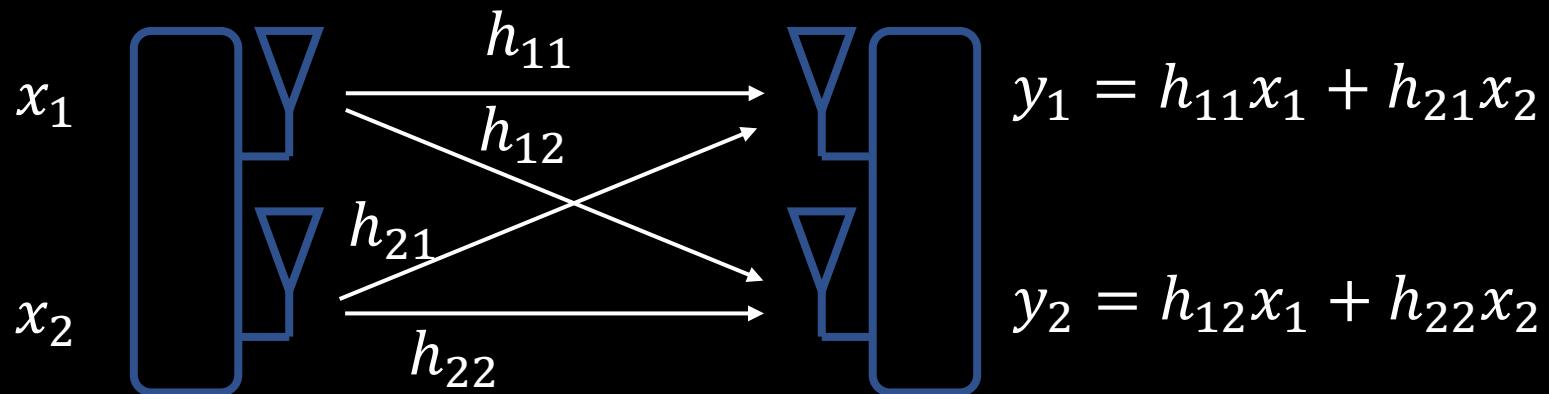
Diversity Gain:

- Send/Receive the same packet on multiple antennas
- Increase SNR of the received packets
→ transmit at higher data rates

Multiplexing Gain:

- Send multiple packets at the same time
- $N \times N$ MIMO → $N \times$ more packets

Multiplexing Gains



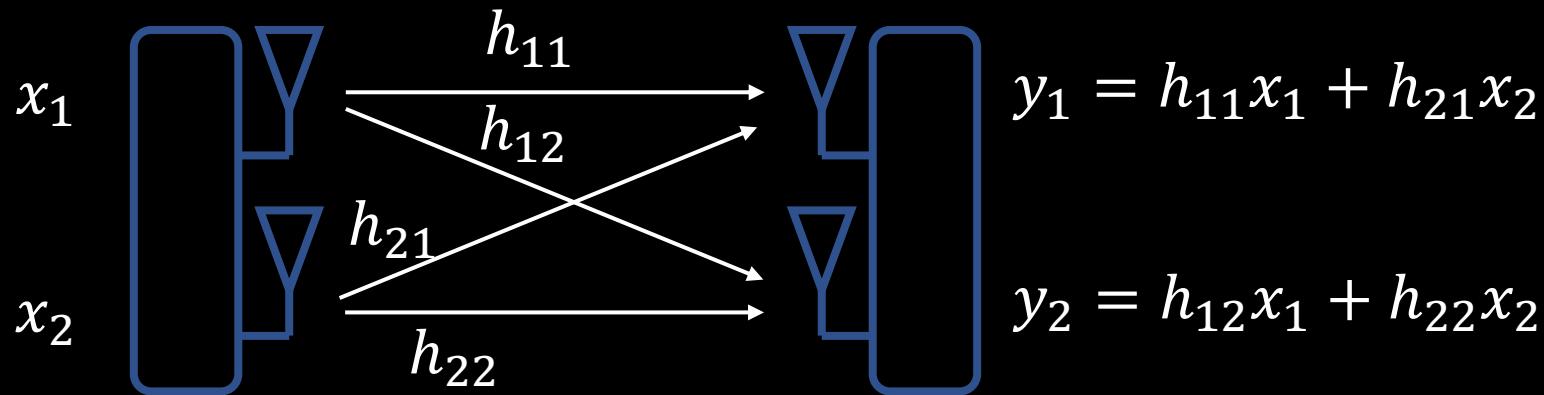
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad \longrightarrow \quad \mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

How to recover x_1 and x_2 ?

Estimate \mathbf{H} , compute \mathbf{H}^{-1} , and invert the channel

$$\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{y} = \mathbf{H}^{-1}\mathbf{H}\mathbf{x} + \mathbf{H}^{-1}\mathbf{n} = \mathbf{x} + \mathbf{H}^{-1}\mathbf{n}$$

Multiplexing Gains

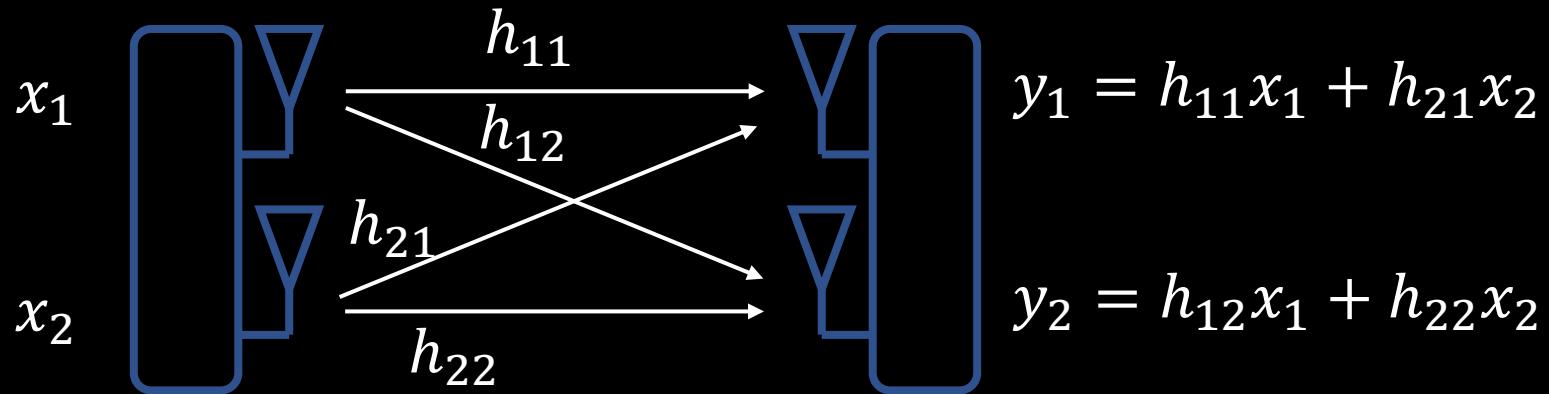


$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad \longrightarrow \quad \mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

How to recover x_1 and x_2 ?

We transmit two packet at one time! → Double the throughput!

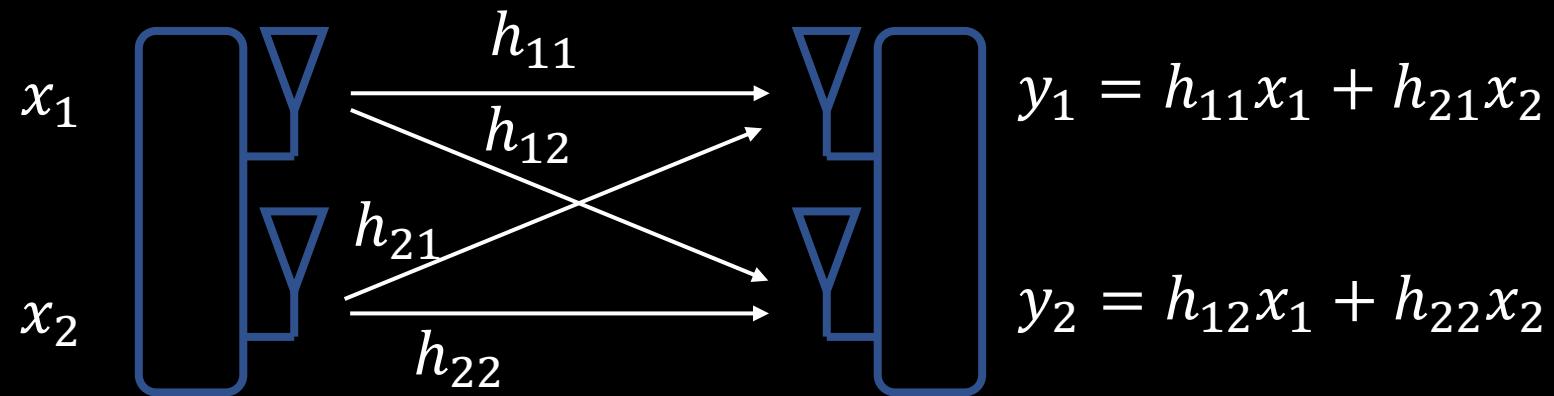
MIMO Channel Estimation



How to estimate the channels: $h_{11} \ h_{12} \ h_{21} \ h_{22}$?



MIMO Channel Estimation



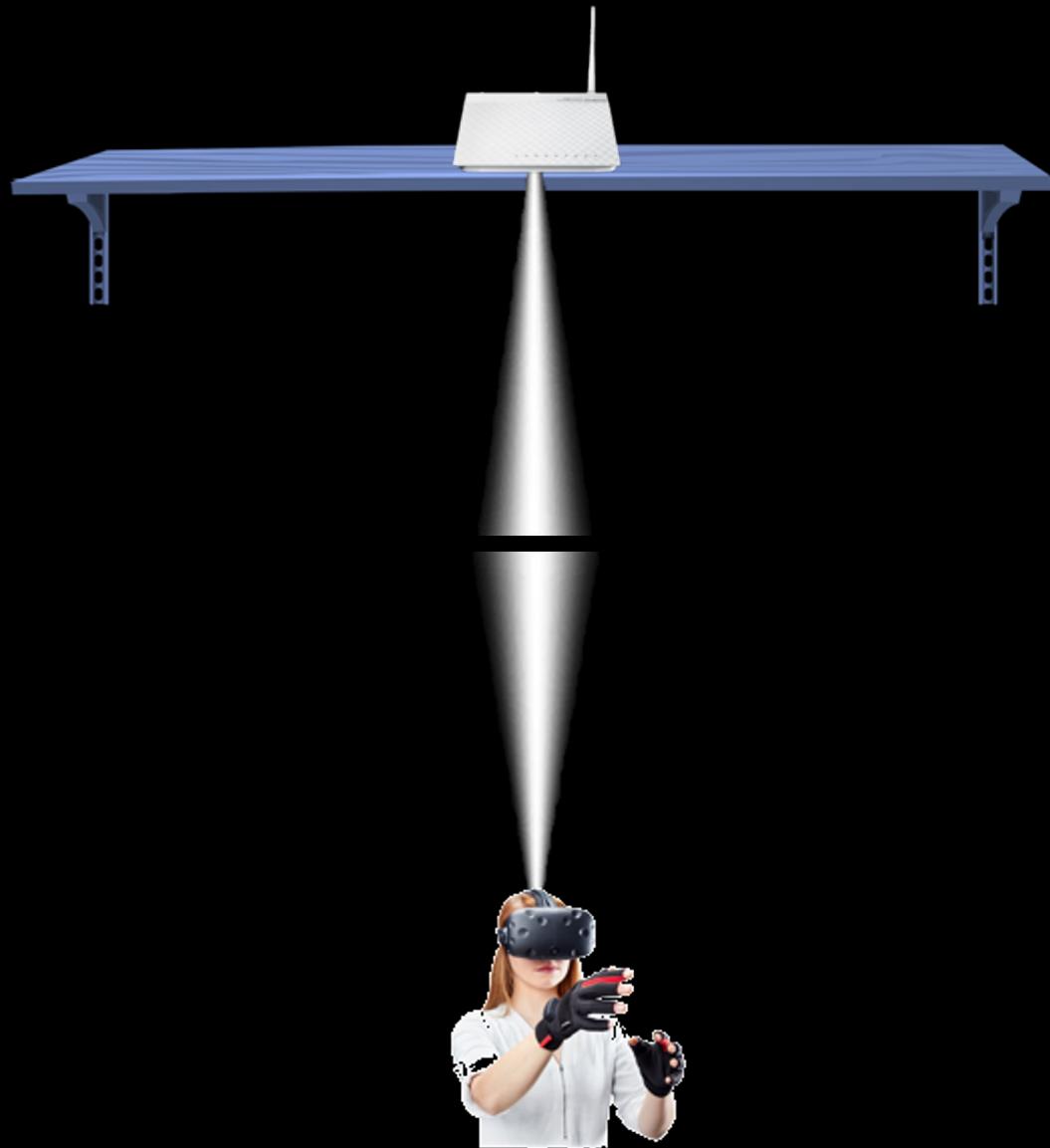
How to estimate the channels: $h_{11} \ h_{12} \ h_{21} \ h_{22}$?



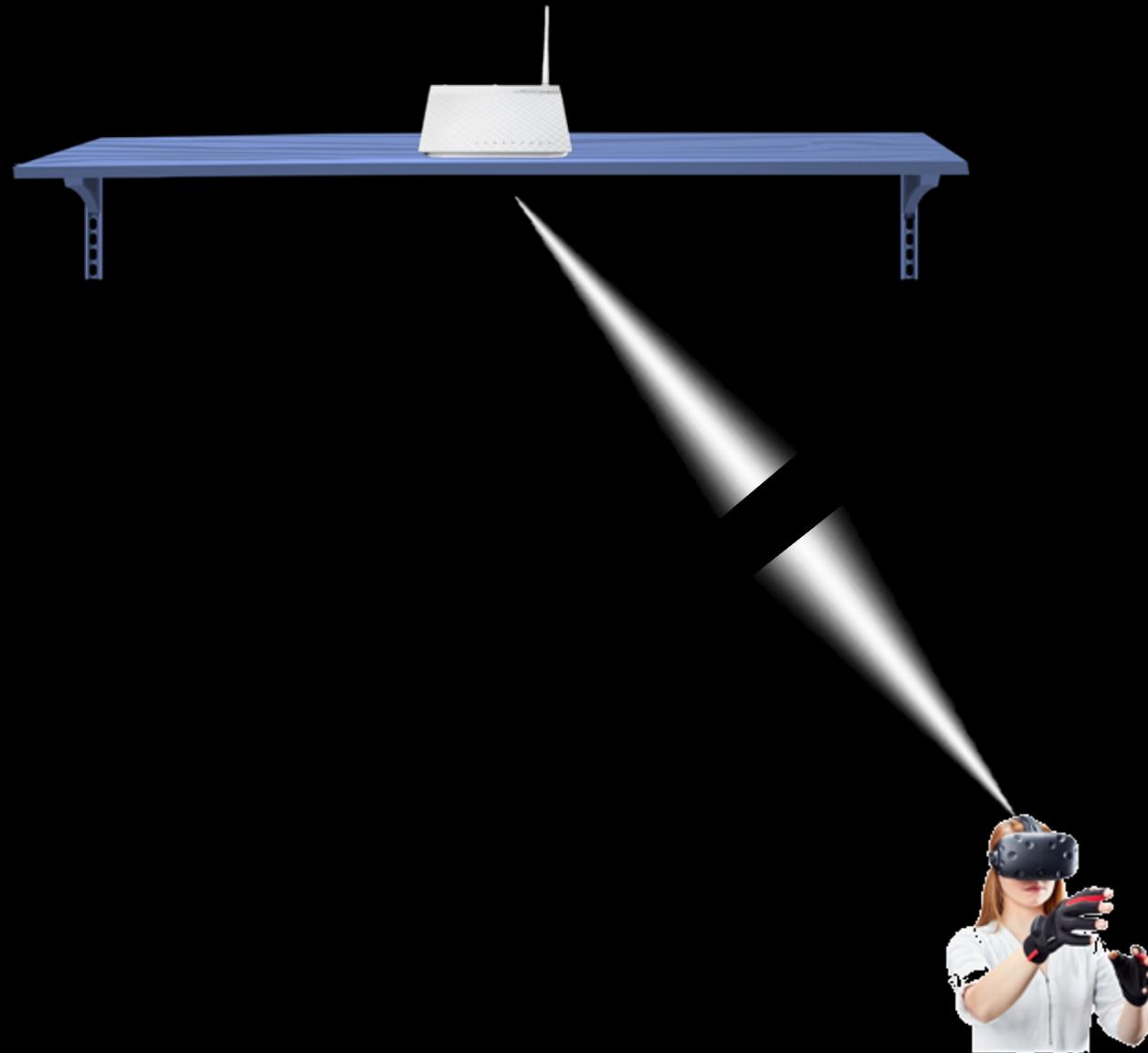
MIMO Application: Beamforming



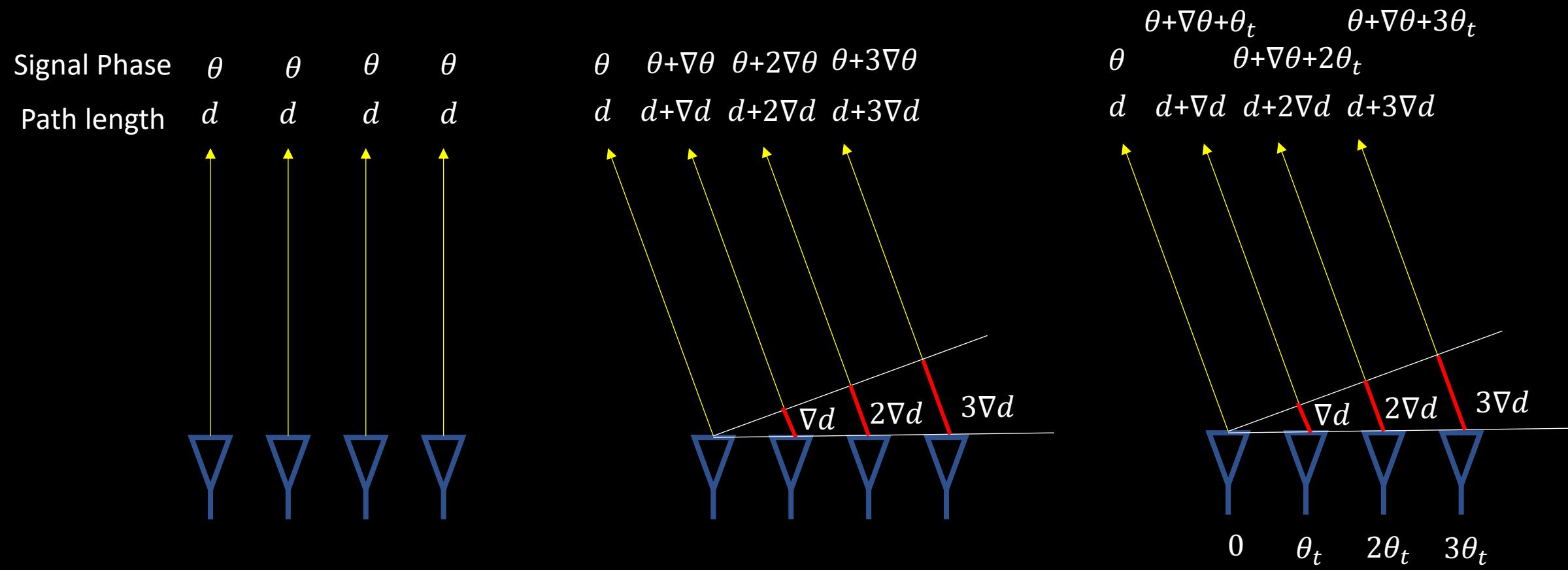
MIMO Application: Beamforming



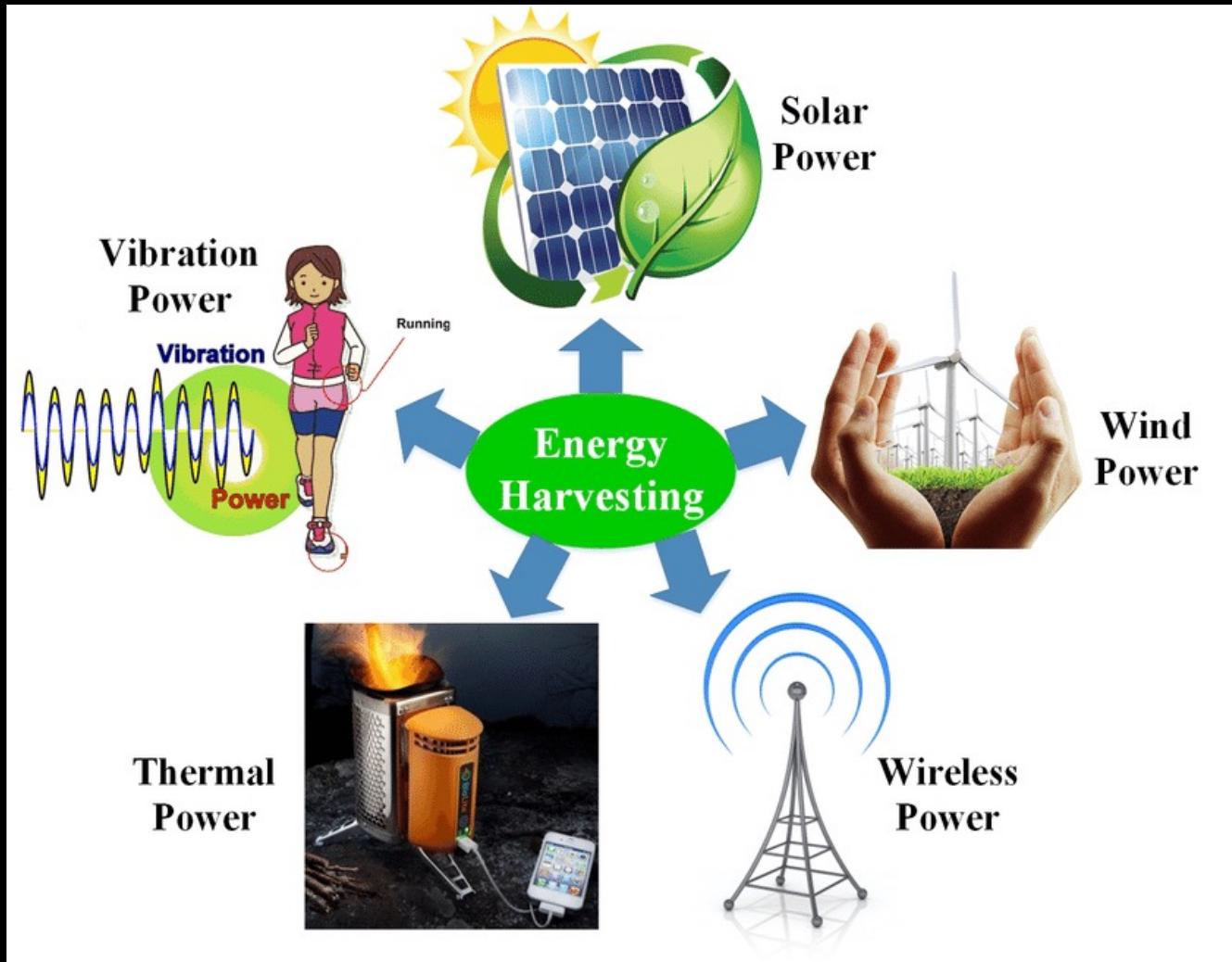
MIMO Application: Beamforming



MIMO Application: Beamforming



MIMO Application: Wireless Charging



Wireless Charging: Current Status

- Near-field charging
- Device still needs to be close to the charger



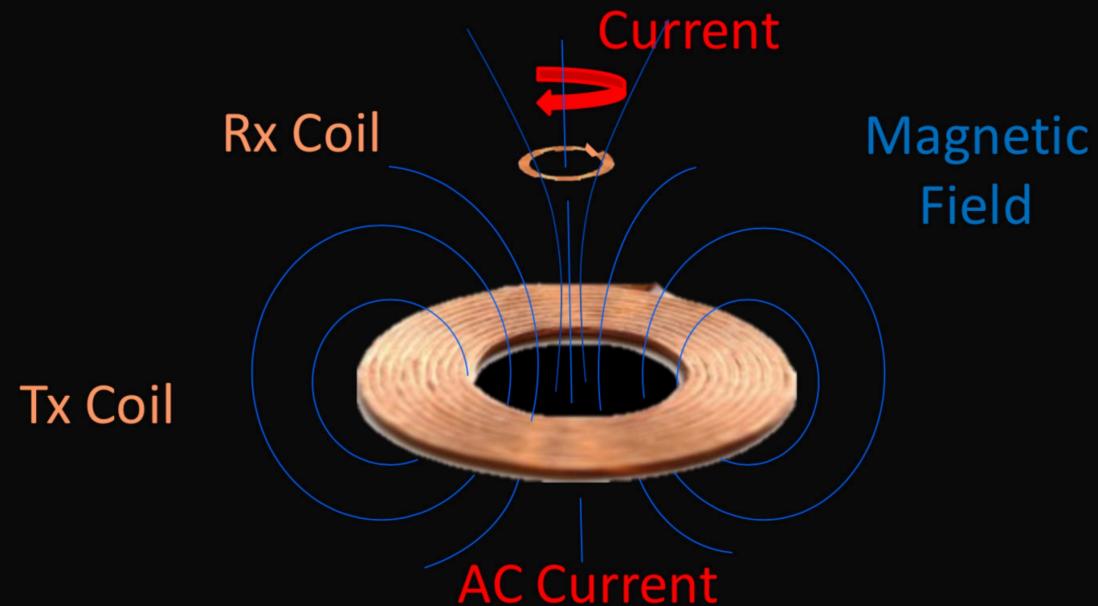
Wireless Charging: Current Status

- Not as efficient as wired charging
- 20% or higher power is wasted
- It's a lot harder to use a phone during the wireless charging process (at least for now)
- Usually slower than wired charging
- Plastic and glass cases needed (no metal)

Wireless Charging: Current Status

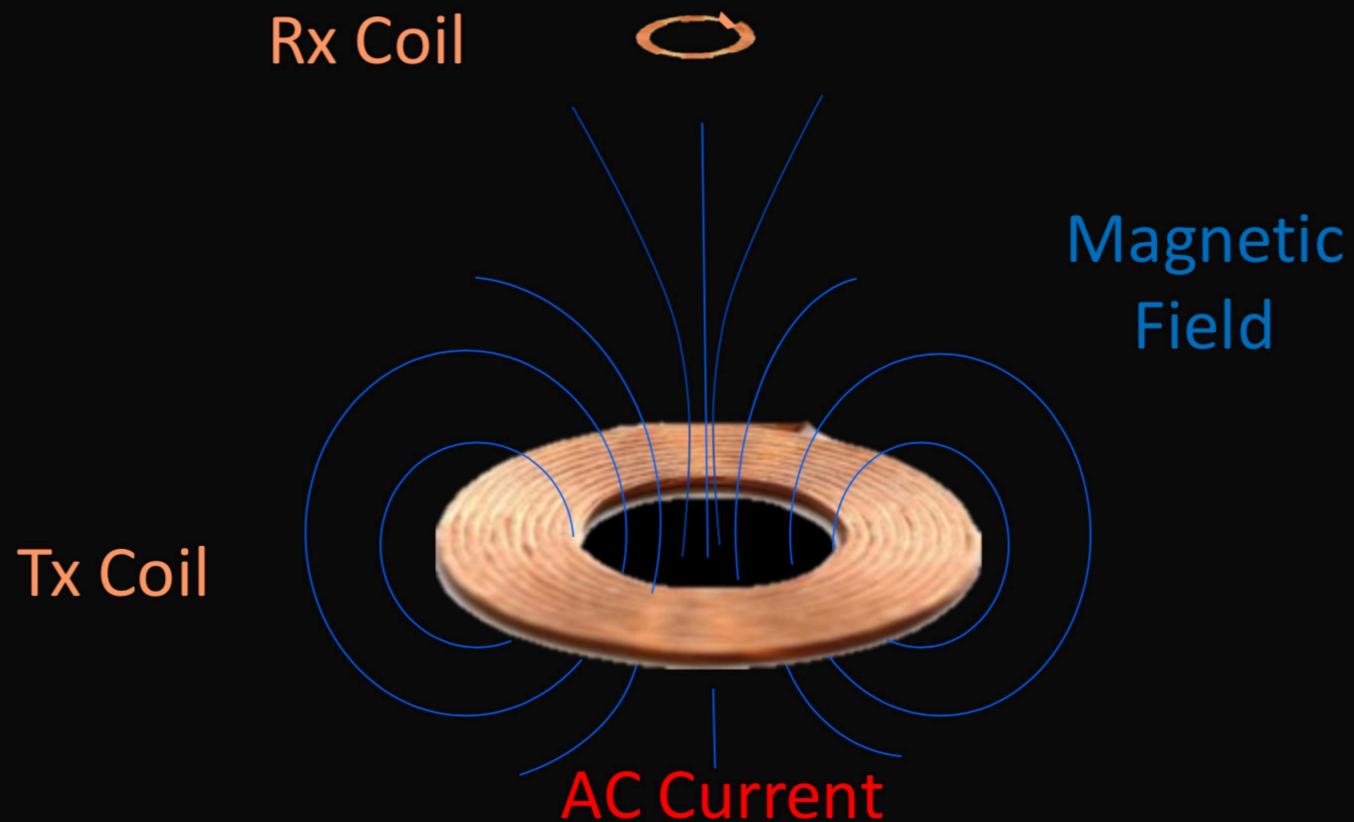
Employ magnetic field to generate current for charging

Magnetic field generates a current in Rx → Power in Rx



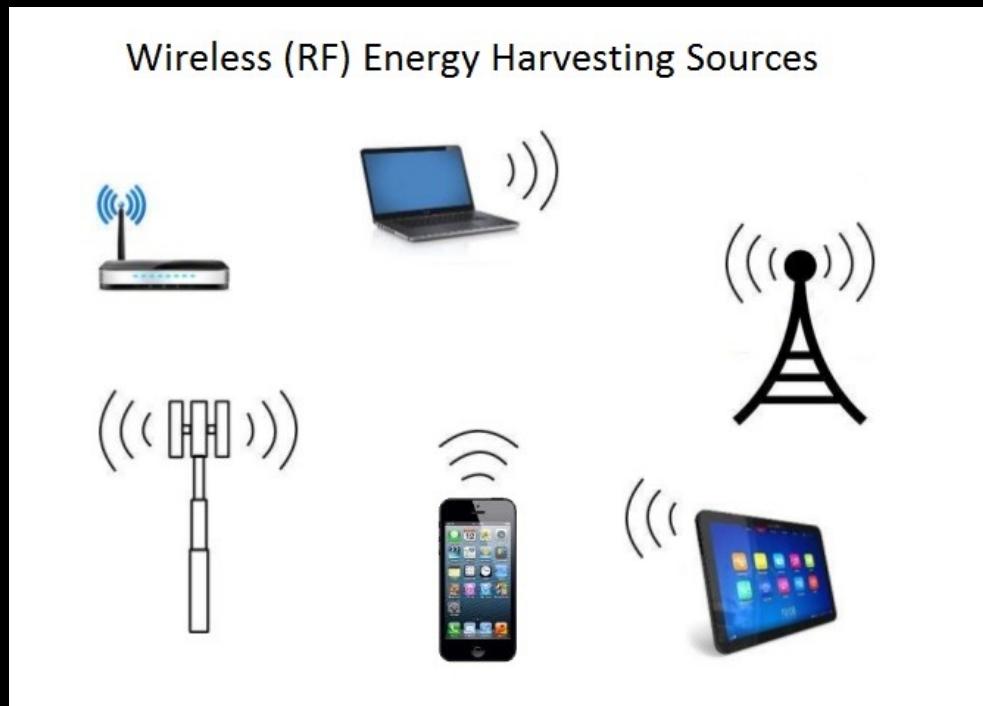
Wireless Charging: Current Status

Magnetic field doesn't cross Rx → No Rx current



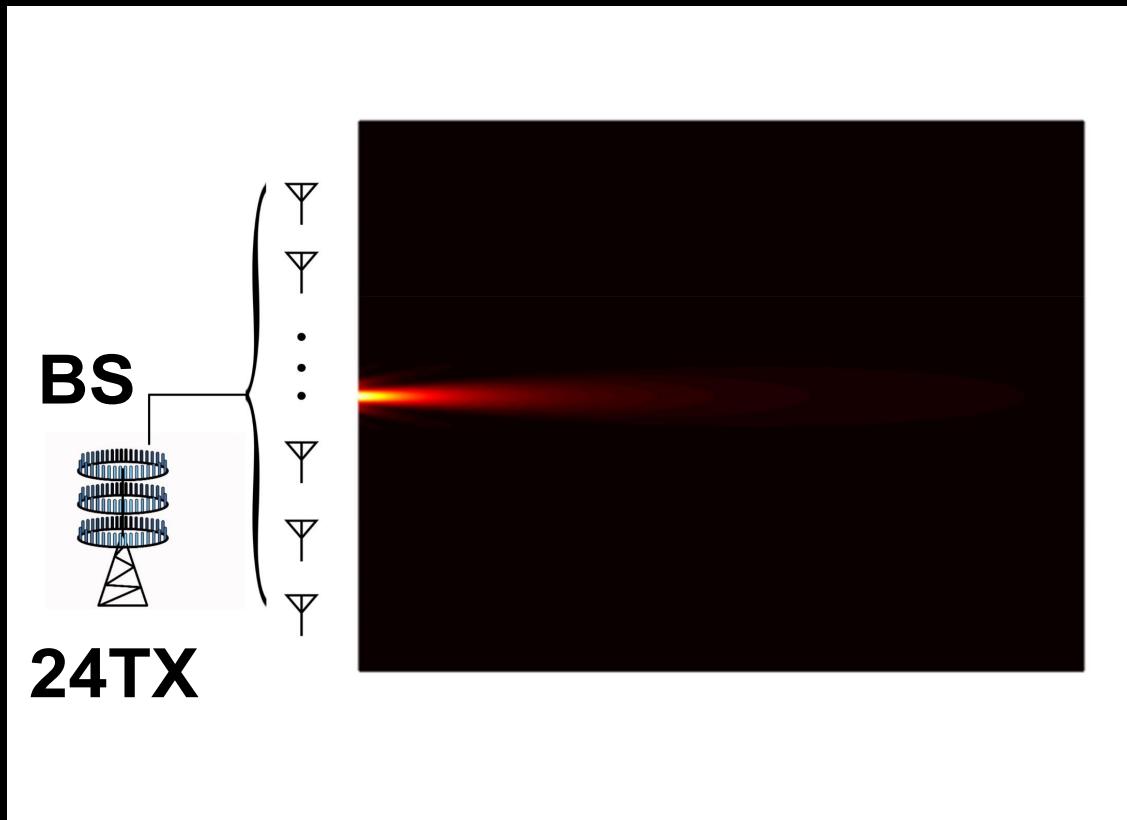
Wireless charging in the future??

- Far-field charging
- Meters away



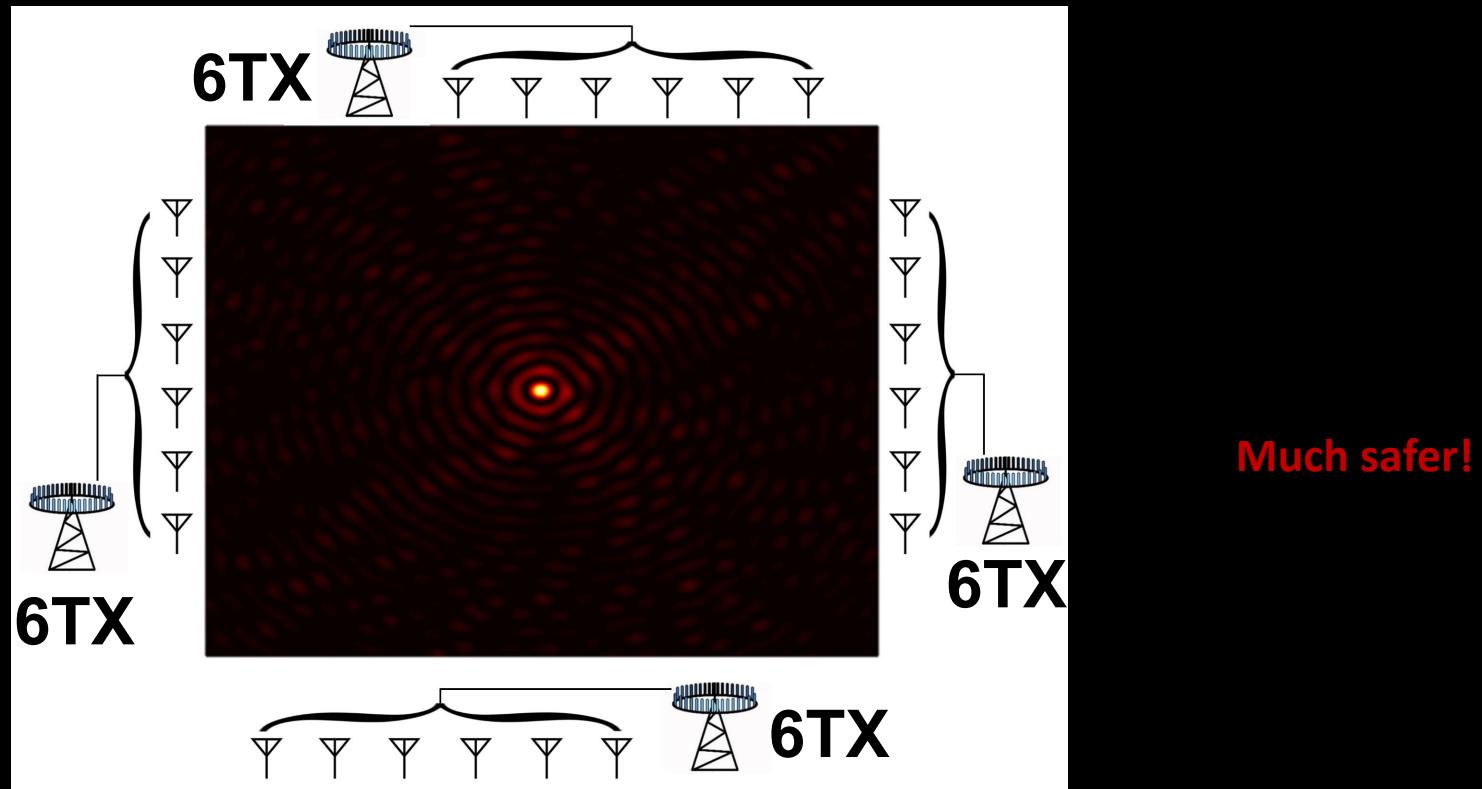
Wireless Charging: Beamforming

- Can concentrate power at one direction



Wireless Charging: Beamforming

- Can concentrate power at one direction

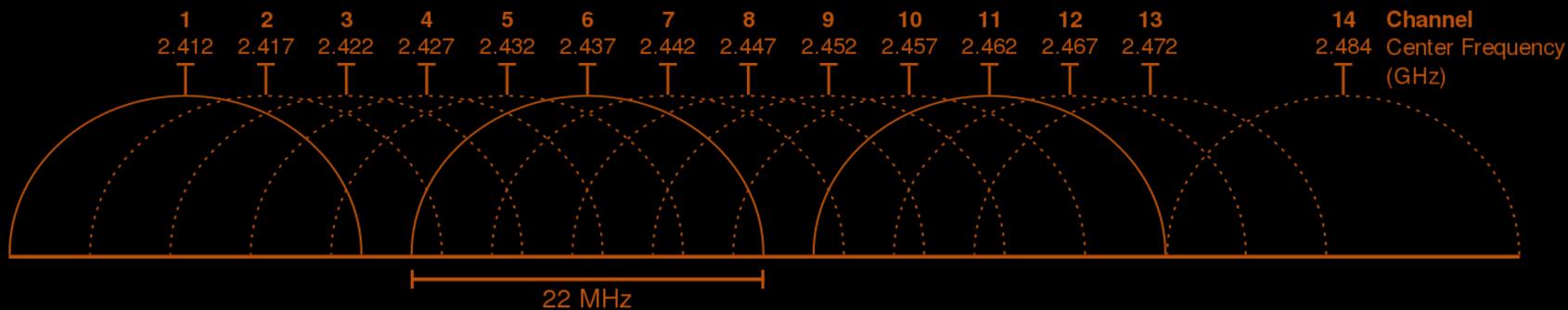


Beamforming

- Safety
- Efficiency
- Cold start
- Mobility
- Implant charging

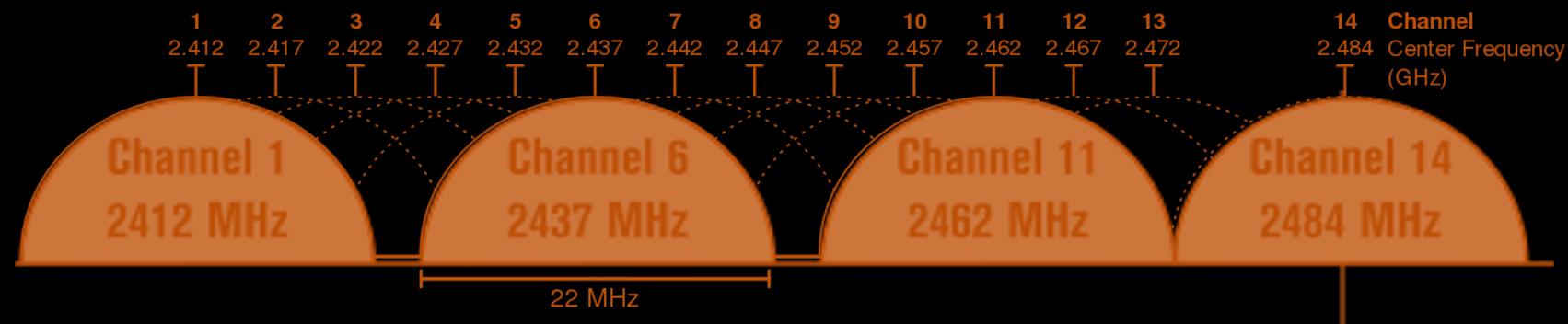
MIMO + OFDM has been widely used in
many communication system,
such as Wi-Fi and Cellular

WiFi Standards from WiFi 1 to WiFi 6



Version	Year	Technology	Modulation	Freq.	Bandwidth	Maximum Data Rate
WiFi 1 (802.11b)	1999	DSSS	DBPSK, DQPSK,	2.4 GHz	22 MHz	11 Mb/s

WiFi Standards from WiFi 1 to WiFi 6



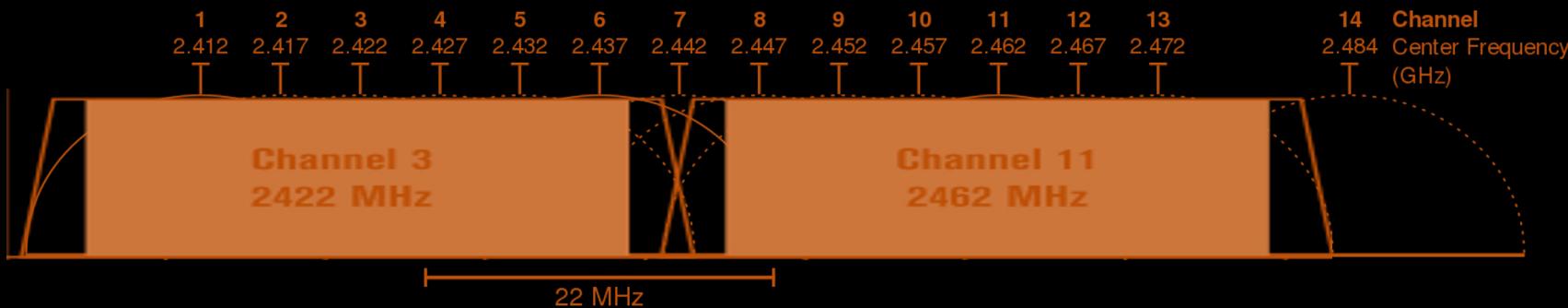
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WiFi Standards from WiFi 1 to WiFi 6



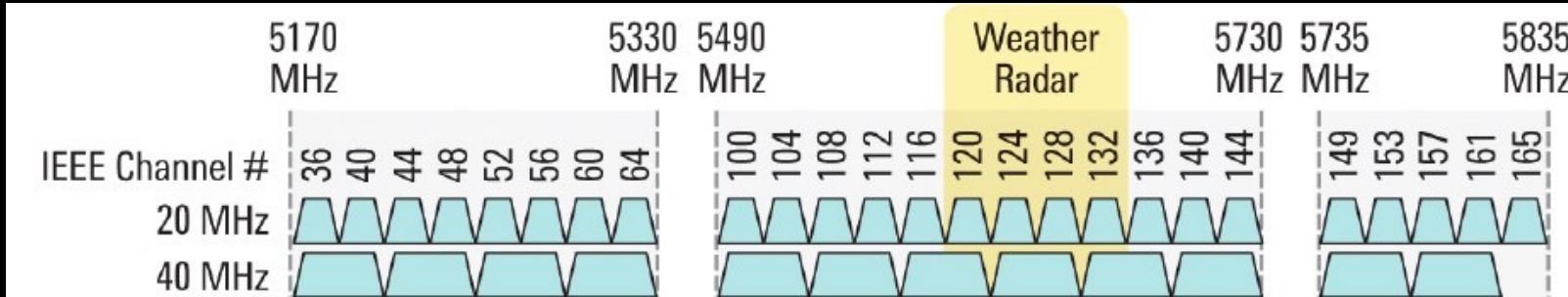
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WiFi 4 (802.11n)	2009	OFDM (N=64) MIMO (4x4)	BPSK, QPSK, 16-QAM, 64-QAM	2.4 GHz 5 GHz	20 MHz 40 MHz	600 Mb/s

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WiFi Standards from WiFi 1 to WiFi 6



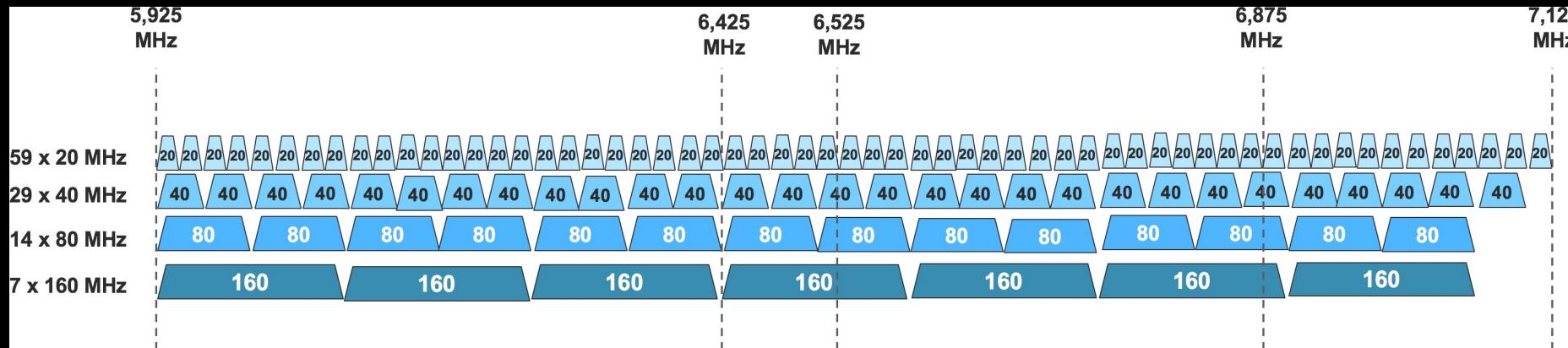
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WiFi 5 (802.11ac)	2014	OFDM (N=64, 128, 256, 512) MIMO (8x8) MU-MIMO (Downlink)	BPSK, QPSK, 16-QAM, 64-QAM 256-QAM	5 GHz	20 MHz 40 MHz 80 MHz 160 MHz	6.933 Gb/s

WiFi Standards from WiFi 1 to WiFi 6



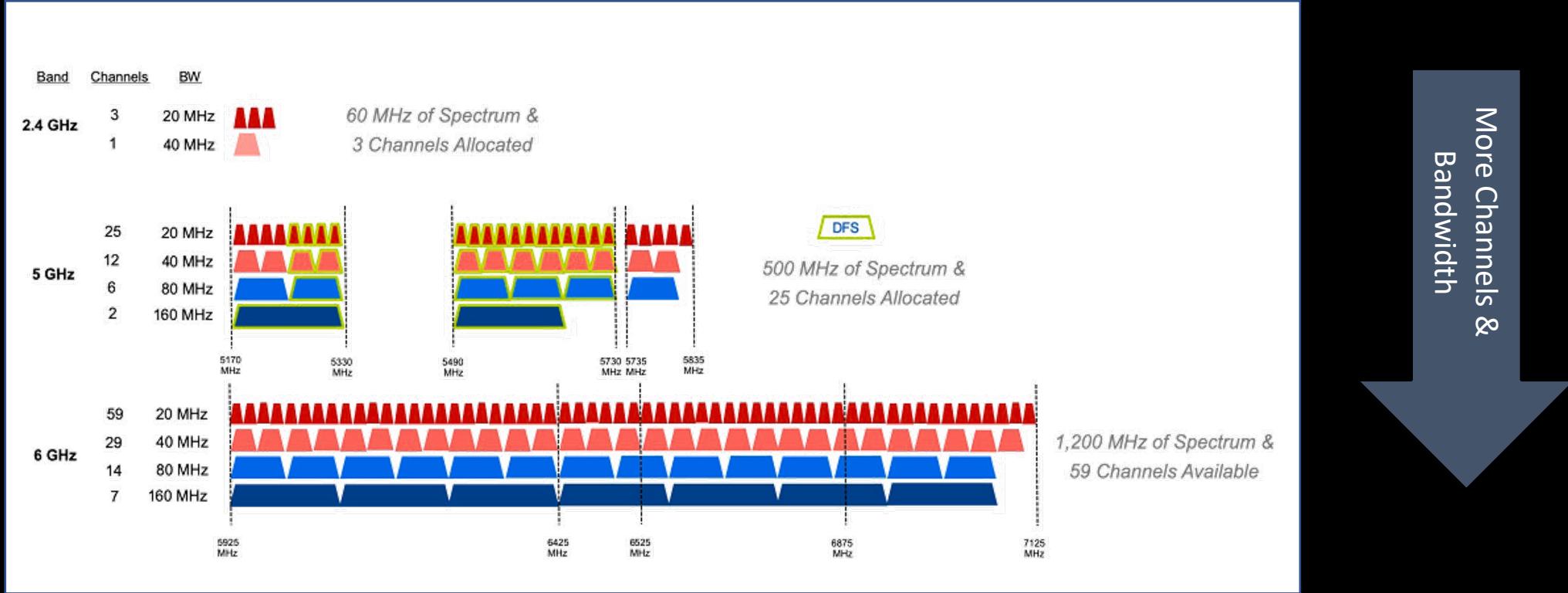
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WiFi 6 (802.11ax)	2019	OFDM (N=256, 512, 1024, 2048) MIMO (8x8) MU-MIMO (Up & Down) OFDMA	BPSK, QPSK, 16-QAM, 64-QAM 256-QAM, 1024-QAM	2.4 GHz 5 GHz	20 MHz 40 MHz 80 MHz 160 MHz	9.608 Gb/s

WiFi Standards from WiFi 1 to WiFi 6



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WiFi 6 (802.11ax) WiFi 6E	2020	OFDM (N=256, 512, 1024, 2048) MIMO (8x8) MU-MIMO (Up & Down) OFDMA	BPSK, QPSK, 16-QAM, 64-QAM 256-QAM, 1024-QAM	2.4 GHz 5 GHz 6 GHz	20 MHz 40 MHz 80 MHz 160 MHz	9.608 Gb/s

WiFi Standards from WiFi 1 to WiFi 6



WiFi 1



More MIMO Antennas



WiFi 5/6

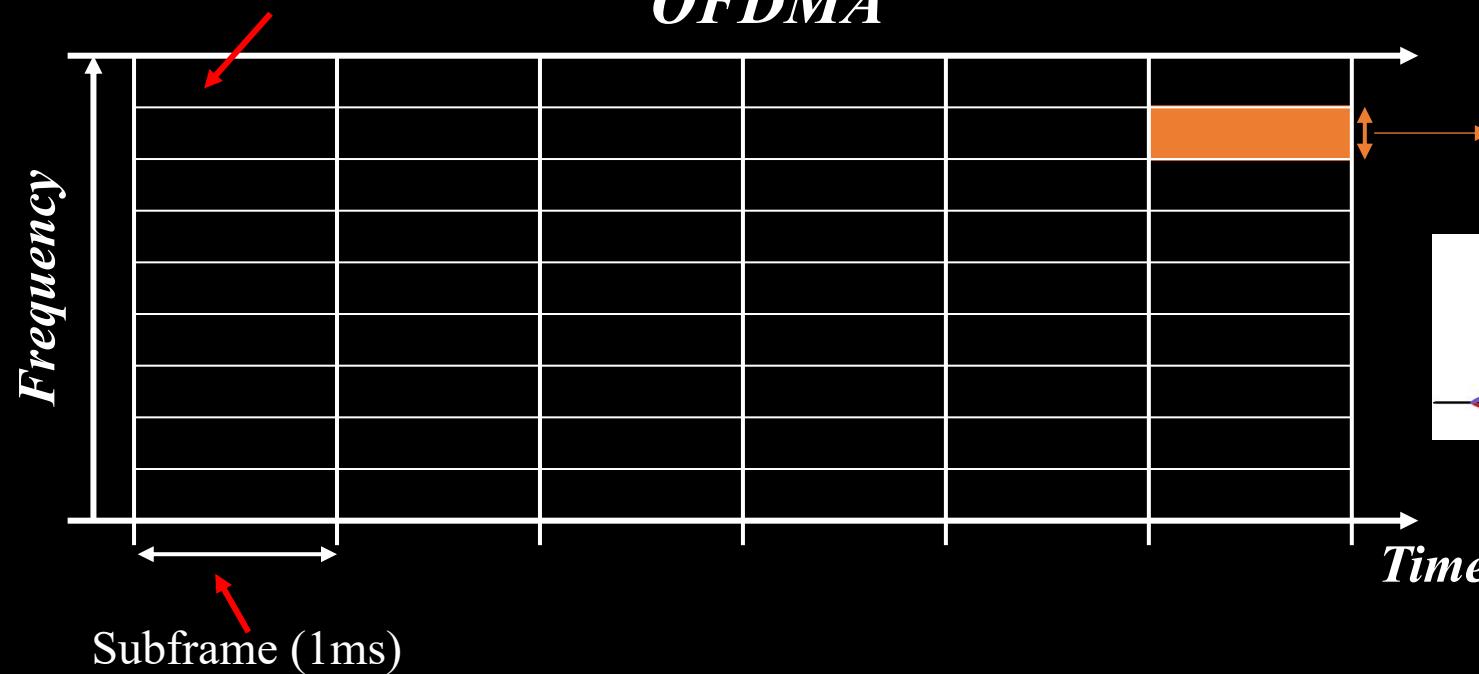


Cellular Physical Layer: OFDMA

Orthogonal Frequency-Division Multiple Access

Physical resource block (PRB)

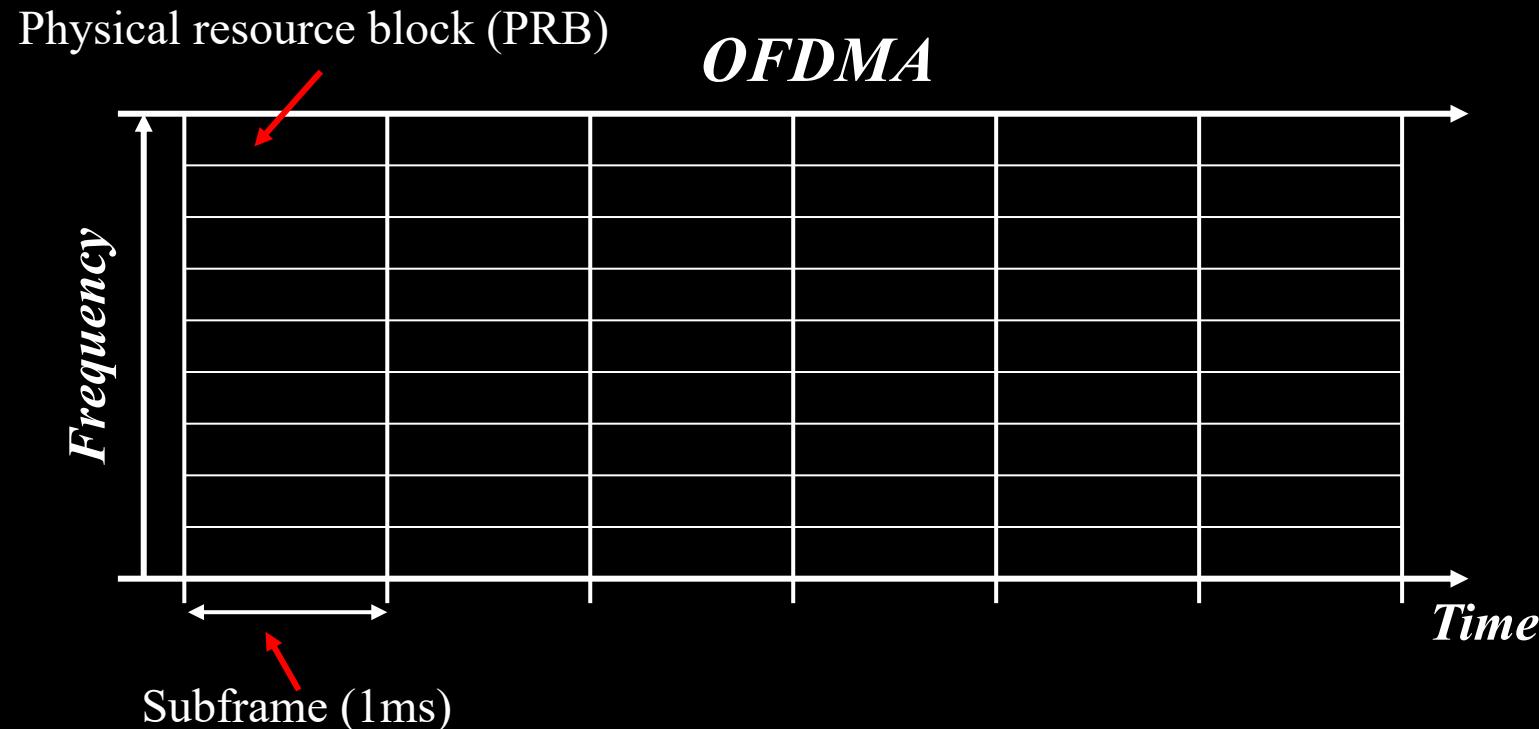
OFDMA



Each PRB has 12 subcarriers

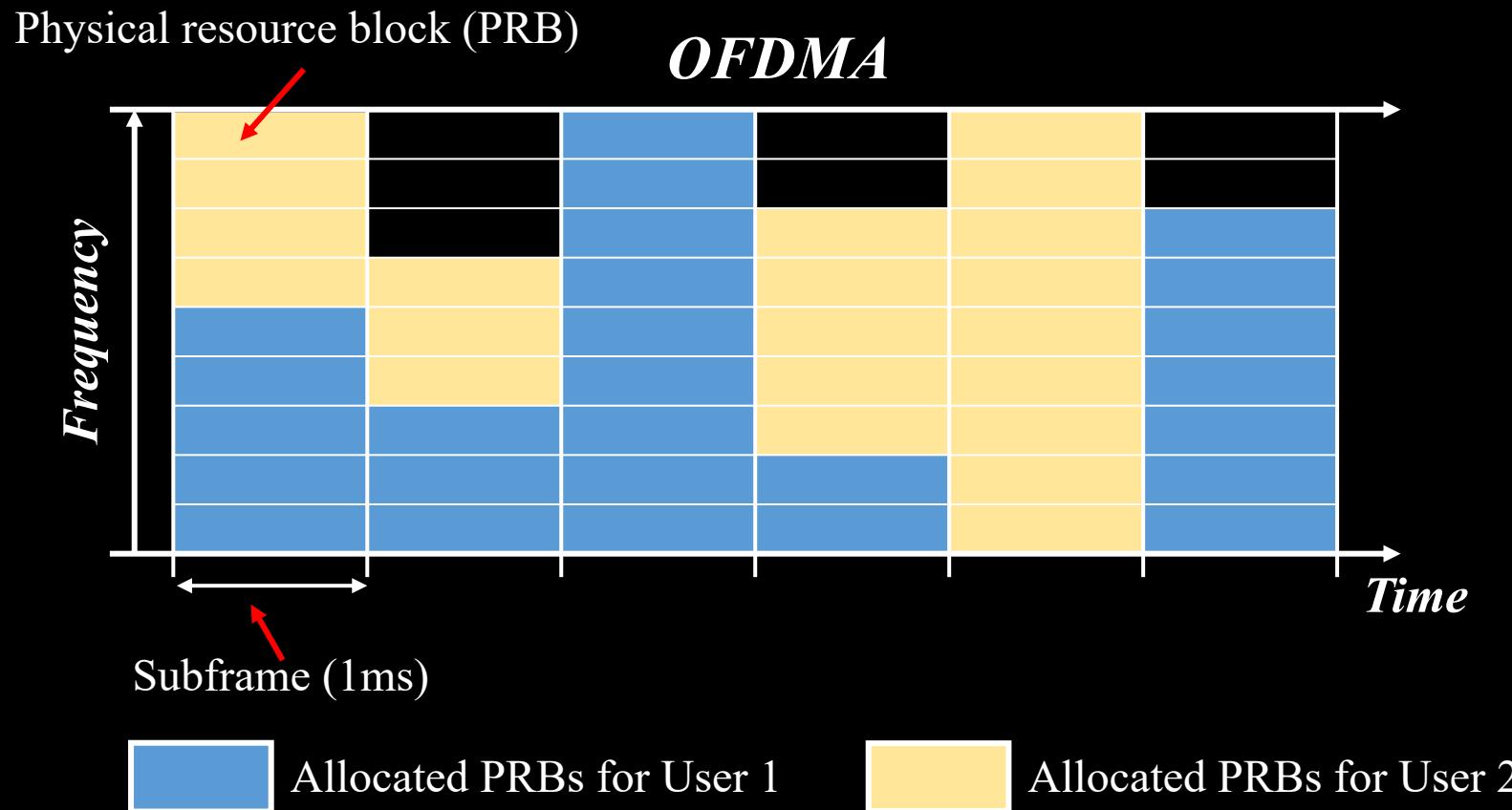
Cellular Physical Layer: OFDMA

Orthogonal Frequency-Division Multiple Access

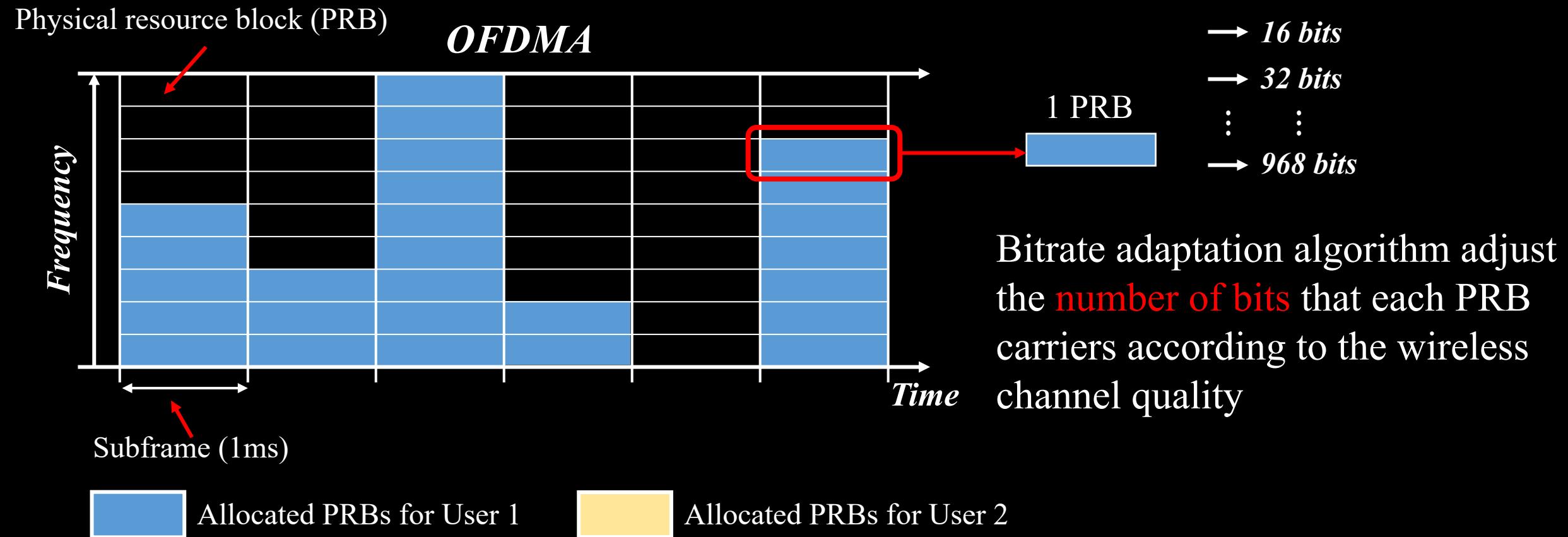


Cellular Physical Layer: OFDMA

Orthogonal Frequency-Division Multiple Access



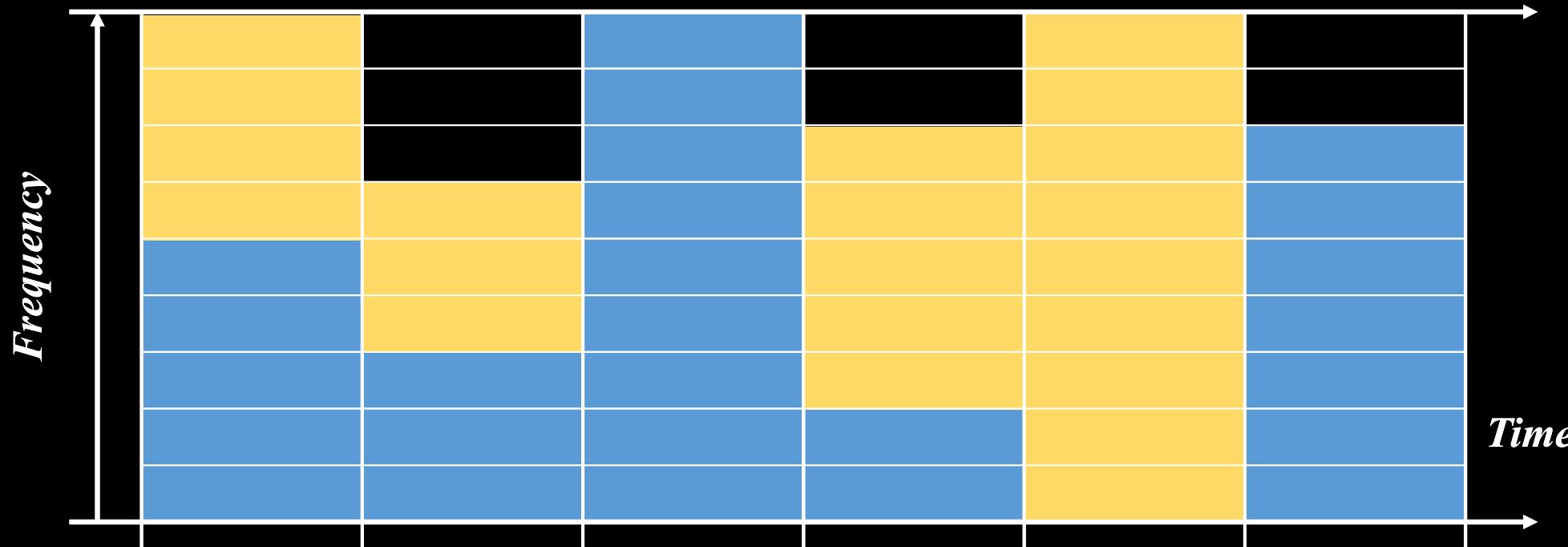
Cellular Physical Layer: Bit rate adaptation



Bandwidth Allocation + Bitrate Adaptation



Bandwidth Allocation + Bitrate Adaptation



Bandwidth Allocation + Bitrate Adaptation

