Writings of Charles S. Peirce

A CHRONOLOGICAL EDITION

Volume 5
1884–1886

Writings of Charles S. Peirce

Volume 5

Peirce ca. 1884

Writings of

CHARLES S. PEIRCE

A CHRONOLOGICAL EDITION

Volume 5
1884–1886

CHRISTIAN J. W. KLOESEL, *Editor*
NATHAN HOUSER, *Associate Editor* URSULA NIKLAS, *Assistant Editor*
MARC SIMON, *Textual Editor* ALETA HOUSER, *Copy Editor*
ANDRÉ DE TIENNE, *Assistant Editor* CATHY L. CLARK, *Editorial Associate*
MAX H. FISCH, *Editor Emeritus*

Indiana University Press
Bloomington and Indianapolis

Preparation of this volume has been supported in part by grants from the Program for Editions of the National Endowment for the Humanities, an independent federal agency.

CENTER FOR
SCHOLARLY EDITIONS
AN APPROVED EDITION
MODERN LANGUAGE
ASSOCIATION OF AMERICA

Harvard University Press holds the copyright to those parts of this volume that first appeared in *Collected Papers of Charles Sanders Peirce* (Vols. 1–6 edited by Charles Hartshorne and Paul Weiss, 1931–1935; 7–8 by Arthur W. Burks, 1958).

© 1993 by Peirce Edition Project
All rights reserved

No part of this book may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying and recording, or by any information storage and retrieval system, without permission in writing from the publisher. The Association of American University Presses' Resolution on Permissions constitutes the only exception to this prohibition.

The paper used in this publication meets the minimum requirements of American National Standard for Information Sciences—Permanence of Paper for Printed Library Materials, ANSI Z39.48-1984.

∞™

Manufactured in the United States of America

Library of Congress Congress Cataloging-in-Publication Data
(Revised for volume 5)

Peirce, Charles S. (Charles Sanders), 1839–1914.
Writings of Charles S. Peirce.

Vol. 5– : Christian J.W. Kloesel, editor.
Includes bibliographies and indexes.
Contents: v. 1. 1857–1866.—v. 2. 1867–1871.—[etc.]—v. 5. 1884–1886.
1. Philosophy. I. Fisch, Max Harold, 1900– II. Kloesel, Christian J. W. III. Title.
B945.P4 1982 191 79–1993
ISBN 0-253-37201-1 (v. 1)
ISBN 0-253-37205-4 (v. 5)

1 2 3 4 5 97 96 95 94 93

Indiana University-Purdue University at Indianapolis

Peirce Edition Project

Christian J. W. Kloesel, Director
Nathan Houser, Associate Editor
André De Tienne, Assistant Editor
Ursula Niklas, Assistant Editor*
Marc Simon, Textual Editor**
Aleta Houser, Copy Editor and Production Manager
Cathy L. Clark, Editorial Associate

Max H. Fisch, Editor Emeritus
Edward C. Moore, Founding Editor

Contributing Editors
(Vol. 5)

Douglas Anderson
Irving I. Anellis
Maryann Ayim
Robert Burch
Glenn Clark
Jonathan R. Eller
Ivor Grattan-Guinness
Carl R. Hausman
Richard A. Tursman
Shea Zellweger
J. Jay Zeman

Advisory Board

Don L. Cook
Jo Ann Boydston
Arthur W. Burks
Gérard Deledalle
Umberto Eco
Carolyn Eisele
John Gallman
Karen Hanson
Robert H. Hirst
Christopher Hookway
Kenneth L. Ketner
Edward C. Moore
Paul Nagy
Klaus Oehler
Vincent Potter
Hilary Putnam
Don D. Roberts
Israel Scheffler
Thomas A. Sebeok
William A. Stanley
Richard A. Tursman
President, Peirce Society

*served until July 1990
**served until July 1991

This page intentionally left blank

Contents

This page intentionally left blank

Preface

Editions differ in what they select and how they arrange and edit their texts. Our selecting, arranging, and editing in the *Writings of Charles S. Peirce: A Chronological Edition* are guided by the belief that Peirce's writings are, as he said of Plato's, "worthy of being viewed as the record of the entire development of thought of a great thinker" and that the development of his thought is eminently worth studying; for Peirce contributed to an exceptionally wide range of disciplines—in mathematics, the natural and social sciences, and the humanities—while aiming always at eventual synthesis, with a primary focus in logic, more and more broadly conceived.

The need for a comprehensive, chronologically arranged edition of Peirce's writings began to be acutely felt after Murray Murphey's *The Development of Peirce's Philosophy* appeared in 1961. At the "Arisbe Conference" in Milford, Pennsylvania, in October 1973, some twenty-five Peirce scholars discussed the relative merits of several alternative plans for such an edition, and settled on a selected but strictly chronological one. Indiana University assumed responsibility for the preparation of the new edition in 1975, and the Peirce Edition Project was established at the Indianapolis campus, Indiana University-Purdue University at Indianapolis. Supporting grants from the National Endowment for the Humanities and the National Science Foundation began in July 1976, and the Project got underway with a full-time staff of three. An Advisory Board and a group of Contributing Editors were appointed and, after a meeting with the former in November 1977, general policies and procedures were adopted. In the meantime, copies of most of Peirce's lifetime publications and of his manuscripts deposited in the Houghton Library of Harvard University had been acquired—and materials from other depositories were added later. Since 1991, the year of Professor Max Fisch's retirement, the Project has had a full-time staff of five.

When work toward the new edition began in 1975, the only edition of Peirce's writings in more than one volume was the eight-volume *Collected Papers* (1931–35, 1958). But in 1976 there appeared the four volumes (in five) of *The New Elements of Mathematics.* By that time the first part of Peirce's *Contributions to THE NATION* had been published; parts 2, 3, and 4 followed in 1978, 1979, and 1988. In 1977 there appeared the *Complete Published Works,* a 149-microfiche edition accompanied by a printed *Comprehensive Bibliography* (revised and enlarged by 12 fiches in 1986). And in 1985, Carolyn Eisele published two volumes of *Historical Perspectives on Peirce's Logic of Science.* These are all valuable editions, but none is critical and none conveys a comprehensive sense of Peirce's entire work. His known writings, published and unpublished, would fill over a hundred volumes if all manuscript drafts and versions and the several thousand manuscript pages of discarded computations and scratch-sheet calculations were included. But any edition in fewer than sixty volumes might fairly be called "Selected Writings."

The present *critical* edition will consist of thirty volumes. It will include every philosophical and logical article that Peirce published during his lifetime; those scientific and mathematical articles that shed particular light on the development of his thought and remind us of the immediate scientific and mathematical background of the work he was doing in philosophy and logic; and those of his papers—no matter what their field or subject-matter—that are most important to an understanding of the development of his thought and of his work as a whole. The most distinctive feature of our edition is that Peirce's writings are arranged in a single chronological order: those he published as of their dates of publication (or oral presentation), those he did not publish as of their dates of composition. But to allow the reader to discern the degree of coherence and unity of Peirce's thought during a given period, every series of papers is presented complete and uninterrupted, as of the date of the first paper in the series. About one-half of the writings included in our edition will be from hitherto unpublished manuscripts. Even what is not new will often seem new by virtue of the fresh context provided for it by the chronological sequence. In all cases, even when we repeat what has appeared before, we have returned to the original manuscripts and publications and have edited them anew. We also include in each volume a few of Peirce's letters, published or unpublished, that are relevant to his work during the period. Except in the case of some long technical-scientific papers, we publish only complete selections in the text; related excerpts are given as notes in the editorial apparatus.

Recently a growing number of readers of Peirce have come to him from semiotics, the theory and study of signs, and they regard him as one of the founders of that discipline. From the beginning, Peirce conceived of logic as coming entirely within the scope of the general theory of signs, and all of his work in philosophical logic was done within that framework. At first he considered logic a branch of semeiotic (his preferred spelling), but he later distinguished between a narrow and a broad sense of logic; in the broad sense it was coextensive with semeiotic. Eventually he abandoned the narrow sense, and the comprehensive treatise on which he was working during the last decade of his life was entitled "A System of Logic, considered as Semeiotic."

Our edition will facilitate the tracing of this and of other developments of Peirce's thought, and it may yield answers to questions that have so far been difficult to pursue. Who were the thinkers whose writings Peirce studied most intensively, in what order, and at what stages of development of his own thought? What were the questions with which he began, and what others did he take up and when? To what questions did his answers change, and what was the sequence of changes? When and to what extent were his philosophic views modified by his own original researches in mathematics and the sciences, and by the major scientific discoveries of his time? In each distinguishable period, to what degree did he bring his thought to systematic completeness? Did he have a single system from beginning to end, with only occasional internal adjustments? To encourage the pursuit of questions like these and to enable the reader to trace the whole development of Peirce's thought—and to trace that thought as articulated in critically edited and reliable texts: these are the primary goals of our edition.

Each volume contains several distinct sections. The largest and most important, the text of Peirce's writings, is preceded by a Chronology, which lists the most significant dates and events in his life and work, and by an Introduction, which provides the biographical and historical background for the writings. The editorial apparatus is continued, following the text, with Notes, a Bibliography of Peirce's References, and a Chronological List of every paper he is known to have written, whether published or not, during the period covered by the volume. The Introduction and Chronological List thus frame the writings that appear between them, and they provide a comprehensive sense of Peirce's work in mathematics, the sciences, philosophy and logic, and the other areas to which he contributed. Then follows the Essay on Editorial Method, which explains our editing policies and principles, and a section called Symbols, which defines all symbols and abbreviations used both in the text and in the Textual Apparatus. The latter provides, for each item in the text (when applicable), two kinds of records: first, a record of the textual decisions that have been made, which consists of (untitled) headnotes, Textual Notes, Emendations, Line-End Hyphenation, and Rejected Substantive Variants; second, and this is a new feature in the present volume, a selected record of the changes Peirce made in his text, titled Alterations. (Rejected Substantive Variants are derived from the historical collation list, which also contains variants in accidentals, and Alterations from a complete list of Peirce's alterations in the manuscripts. Both lists are prepared early in the editing process and, although neither is included in our volumes, they are available to interested persons for the cost of photocopying. The principles that guide the selection of alterations published in the Textual Apparatus are described in detail in the Essay on Editorial Method.) The Textual Apparatus is followed by Line-End Hyphenation in the Edition Text, which lists hyphenated compounds that must retain their hyphens when transcribed or quoted from the critical text of the given volume, and by an Index. (A comprehensive index and bibliography is planned for a separate later volume.)

The writings included in the edition have been prepared according to the standards of the Modern Language Association's Committee on Scholarly Editions, and they appear in "clear" text: that is, excepting a few editorial symbols that represent physical problems in the manuscripts, everything in the main section is Peirce's own, including the footnotes. In some instances, we have supplied titles. Titles of published items are printed in italic type, those of unpublished items in roman. Each title is preceded by the item number in the present volume and followed by a source note or identifying number—published items are identified by P number and the bibliographic information listed in the *Comprehensive Bibliography;* unpublished items by MS number and the date of composition. (Further information concerning manuscript or publication appears in the headnote for each item in the Textual Apparatus.) MS numbers refer not to the Harvard arrangement (as given in Richard S. Robin's *Annotated Catalogue of the Charles S. Peirce Papers*) but to the new arrangement of Peirce's writings established in Indianapolis, which also includes those known to exist in depositories other than the Houghton Library. Reassembling the thousands of scattered pages and sequences of pages that were formerly in "fragment folders," and arranging all manuscripts chronologically (Peirce himself having dated only about a fourth of them), has involved a great deal of preliminary work and will continue until shortly before publication of the final volume. If further papers turn up too late to appear in their chronological places, they may be included in later supplements.

Finally, it must be said that restraint and accuracy have been the guiding principles in our editing and that our critical text represents what Peirce wrote, not what we think he should have written. This is true of published as well as unpublished writings; but because of editorial and compositorial interference in the printing process, the former are more likely to be emended than the latter, and at times even regularized. It should also be mentioned that, among unpublished writings, we distinguish between public and private documents (the latter including diaries, notebooks, and letters or drafts of letters) and that such private documents are reproduced almost without change. In any case, in our editing we correct typographical errors, but retain Peirce's inconsistencies in spelling and punctuation when they reflect acceptable nineteenth-century standards and practices. We make other changes only when some evidence suggests that Peirce's intention warrants them—and all such changes are listed, by page and line numbers, in the Textual Apparatus. As a further aid to the reader of Peirce's text, we have introduced four sets of symbols into the critical text. Titles and other text supplied by the editors appear in italic brackets; italic brackets enclosing three ellipsis points indicate one or more lost manuscript pages; italic brackets enclosing a blank indicate that an incomplete discussion occurs before the end of the manuscript page; and sets of double slashes mark the beginning and end of Peirce's undecided alternate readings, with the single slash dividing the original from the alternative inscription.

Acknowledgments

We are indebted to Indiana University and the National Endowment for the Humanities for their support of the Peirce Edition Project; to the Harvard University Department of Philosophy for permission to use the original manuscripts, and to the officers of the Houghton Library, especially Melanie Wisner, for their cooperation regarding the Charles S. Peirce Papers; to Webb Dordick for his research assistance in the Harvard libraries; to Marjorie Ciarlante of the National Archives; to Thomas C. Cadwallader, David Frisby, H. S. Harris, J. J. Kockelmans, Robert D. Madison, José Vericat, Douglas E. Wilson, and all other scholars who have given us expert help at various points; to Christine Wertheim for her valuable service during her time as a visiting research associate in the Project; to the eleven contributing editors of this volume; to our editorial and administrative support staff members Beth Sakaguchi and Beth Van Vorst Greene, and to former members Janine Beckley and John F. Hirschman; and to Max H. Fisch, our Editor Emeritus who, after more than fifty years of Peirce research and a well-deserved retirement in 1991, donated his many books and papers to Indiana University.

We are also indebted to the Interlibrary Loan department of Indiana University-Purdue University at Indianapolis for continued good service; to the Harvard University Archives for permission to publish two letters from the Abbot Papers; to the Princeton University Library for permission to publish a letter from the Allan Marquand Papers; and to the Texas Tech University Institute for Studies in Pragmaticism for permission to use duplicates of its annotated electroprint copy of the Harvard Peirce Papers. A final note of thanks goes to three administrative officers of Indiana University: Chancellor and Vice President Gerald L. Bepko, Executive Vice Chancellor and Dean of the Faculties William M. Plater, and Dean John D. Barlow of the School of Liberal Arts.

This page intentionally left blank

Chronology

1839 Born on 10 Sept. in Cambridge, MA, to Benjamin and Sarah Hunt (Mills) Peirce
1855 Entered Harvard College
1859 Graduated (A.B.) from Harvard
Temporary aide in U.S. Coast Survey, fall to spring '60
1860 Studied classification with Agassiz, summer-fall
1861 Entered Lawrence Scientific School at Harvard
Appointed regular aide in Coast Survey, 1 July
1862 Married to Harriet Melusina Fay, 16 Oct.
1863 Graduated *summa cum laude* (Sc.B.) in chemistry from Lawrence Scientific School
1865 Harvard lectures on "The Logic of Science," spring
Began Logic Notebook, 12 Nov.; last entry in Nov. '09
1866 Lowell Institute lectures on "The Logic of Science; or Induction and Hypothesis," 24 Oct.–1 Dec.
1867 Elected to American Academy of Arts and Sciences, 30 Jan.
1869 First of ca. 300 *Nation* reviews, in Mar.; last in Dec. '08
Assistant at Harvard Observatory, Oct. '69–Dec. '72
Harvard lectures on "British Logicians," Dec.–Jan.
1870 First Survey assignment in Europe: 18 June–7 Mar. '71
1872 Founding member of Cambridge Metaphysical Club, Jan.
In charge of Survey office, spring-summer
Put in charge of pendulum experiments, beginning in Nov.
Promoted to rank of Assistant in the Survey, 1 Dec.
1875 Second Survey assignment in Europe: Apr. '75–Aug. '76
Served as first official American delegate to International Geodetic Association in Paris, 20–29 Sept.
1876 Separated from Melusina in Oct.
1877 Elected to National Academy of Sciences, 20 Apr.
Third Survey assignment in Europe: 13 Sept.–18 Nov.
Represented U.S. at International Geodetic Association conference in Stuttgart, 27 Sept.–2 Oct.
1878 *Photometric Researches* published in Aug.
1879 Lecturer in Logic (till '84) at Johns Hopkins University
First meeting of JHU Metaphysical Club, 28 Oct.

1880 Elected to London Mathematical Society, 11 Mar.
Fourth Survey assignment in Europe: Apr.–Aug.
French Academy address on value of gravity, 14 June
1881 Elected to American Association for the Advancement of Science in Aug.
1883 *Studies in Logic* published in spring
Divorced from Melusina, 24 Apr.
Married to Juliette Froissy (Pourtalès), 30 Apr.
Fifth and final Survey assignment in Europe: May–Sept.
1884 In charge of Office of Weights and Measures, Oct.–22 Feb. '85
1888 Purchased "Arisbe," outside Milford, PA
1889 Contributor to *Century Dictionary*
1891 Resigned from Coast and Geodetic Survey, 31 Dec.
1892 Lowell lectures on "The History of Science," 28 Nov.–5 Jan.
1893 *Petrus Peregrinus* announced; prospectus only published
"Search for a Method" announced by Open Court; not completed
1894 "The Principles of Philosophy" (in 12 vols.) announced by Henry Holt Co.; not completed
"How to Reason" rejected by both Macmillan and Ginn Co.
1895 "New Elements of Mathematics" rejected by Open Court
1896 Consulting chemical engineer (till '02), St. Lawrence Power Co.
1898 Cambridge lectures on "Reasoning and the Logic of Things," 10 Feb.–7 Mar.
"The History of Science" announced by G. P. Putnam's; not completed
1901 Contributor to *Dictionary of Philosophy and Psychology*
1902 Grant application for "Proposed Memoirs on Minute Logic" rejected by Carnegie Institution
1903 Harvard lectures on "Pragmatism," 26 Mar.–17 May
Lowell lectures on "Some Topics of Logic," 23 Nov.–17 Dec.
1907 Harvard Philosophy Club lectures on "Logical Methodeutic," 8–13 Apr.
1909 Last published article, "Some Amazing Mazes"
1914 Died on 19 April

Introduction

The years 1884–1886 were a time of transition for Peirce. When the period began, he intended to make Baltimore his permanent home, confident that his connection with Johns Hopkins was secure. His main work in life would be logic. But he soon learned of the fateful resolution of the Johns Hopkins Executive Committee that his contract would not be renewed, and he knew that his days there were numbered. After his shock had subsided and he had reluctantly yielded to the inevitable, his focus shifted back to his scientific work for the Coast and Geodetic Survey. During these years he probably spent more time on science, either with pendulum observations or with the reduction of scientific data and the preparation of reports, than on all other activities combined. He was on location for the Survey much of the time; almost continuously between July 1884 and February 1886 he directed pendulum operations at a succession of sites extending from Washingon, DC to Key West, Florida and Madison, Wisconsin. It might have been a time of passage to a long and influential career in science, but a scandal led to Peirce's estrangement from the Survey and considerably dampened his enthusiasm for government service. By 1886 his scientific interest shifted from gravity research and metrology to such subjects as the study of color and the history of science, which were outside his sphere of responsibility for the Survey, and after the brief resurgence of his enthusiasm for science, philosophy came again to dominate his thought.[1]

[1]In writing this introduction, I have depended a great deal on the results of Max H. Fisch's many years of research, contained in his files at the Peirce Edition Project. The best accounts of Peirce's intellectual development are in Fisch, *Peirce, Semeiotic, and Pragmatism,* eds. Kenneth L. Ketner and Christian J. W. Kloesel (Bloomington: Indiana University Press, 1986) and Murray G. Murphey, *The Development of Peirce's Philosophy* (Cambridge: Harvard University Press, 1961). For a general account of Peirce's life, see Joseph Brent, *Charles Sanders Peirce: A Life* (Bloomington: Indiana University Press, 1993).

To reduce the number of footnotes, I do not give references for items that can be easily located by keeping the following in mind: all manuscript references (according to either the Peirce Edition Project or Harvard arrangement) are to the Peirce Papers at Harvard University which also contain the correspondence between Peirce and the members of his family; correspondence with employees of the Coast Survey is in Record Group 23 in the National Archives. NEM refers to *The New Elements of Mathematics,* ed. Carolyn Eisele (The Hague: Mouton, 1976), and EP to *The Essential Peirce,* eds. Nathan Houser and Christian Kloesel (Bloomington: Indiana University Press, 1992).

These years mark the end of what Max Fisch calls Peirce's "cosmopolitan period," a time devoted mainly to science and frequent travels in Europe and throughout the United States and Canada.[2] That period began in June 1870, when Peirce sailed for Europe to arrange for scientific observations of the 22 December solar eclipse,[3] and ended in April 1887, when he and his second wife, Juliette, moved to Milford, Pennsylvania, a Pocono Mountain resort town. The following year the Peirces settled on a farm on the Delaware River just outside Milford in the home they would soon name "Arisbe," and in the years that followed, except for frequent trips to New York City (some for extended intervals) and occasional trips to Cambridge, Peirce stayed in Pennsylvania.

From a different point of view, that of Peirce's intellectual growth, the years 1884–1886 mark a new beginning. According to Murray Murphey, this is the start of the fourth and final phase of Peirce's intellectual development, stemming from his discovery of quantification and set theory.[4] In 1883 Peirce's Johns Hopkins student Oscar Howard Mitchell had introduced indices into algebraic logic in a way Peirce recognized as the key to quantification.[5] Over the following months (see items 20–22), Peirce developed a theory of quantification that by 1885 (items 30–32) took a very modern form. At about the same time, he first came into contact with the work of Georg Cantor (see MS 530), which must have been a stimulus for the investigations he had already begun on number theory. He had probably also begun to reflect deeply on the nature of continuity, for by 1 April 1884, while working on the definition for the *Century Dictionary,* he lamented that "continuity" had never been adequately defined: "Kant's definition, to which I am ashamed to say I have hitherto given my adhesion, is ridiculous when you come to think of it" (MS 528).[6] By that time Peirce had begun to formulate his unique theory of the continuum that would ground his anti-Cantorian set theory (and theory of number). His discovery of the quantifier and set theory marks the beginning of a major phase of his intellectual development because, as Murphey has shown, Peirce was forced to make major revisions to his theory of reality and to his categories.[7] The groundwork for this final period in Peirce's thought was laid in works included in the present volume.

Peirce wrote or published over one hundred papers and reports between 1884 and 1886, ranging in subject matter from the measurement of sensations and the price of sugar to the algebra of logic and philosophical categories. Most of the fifty-eight items included in the present volume belong to science, logic, or philosophy; the Study of Great Men, which fills eighty pages (items 2–19), was described by Peirce as comparative biography, and thirty-two pages (item 57) are devoted to definitions of words beginning with the

[2]See Fisch, p. 227.

[3]Discussed in W2 Introduction, pp. xxxi–xxxiv.

[4]See Murphey, p. 3.

[5]See Oscar Howard Mitchell, "On a New Algebra of Logic," in *Studies in Logic,* ed. C. S. Peirce (Boston: Little, Brown, & Co., 1883), pp. 72–106.

[6]Quoted in Fisch, p. 233.

[7]See Murphey, ch. 15.

letter "e" that Peirce drafted for the *Century Dictionary.* At least one paper (item 24) belongs most appropriately with psychology, and there are four private letters and several reviews.

For much of this three-year period the Peirces were itinerant. They stayed in Baltimore for the first half of 1884, where Peirce's contract with Johns Hopkins ran until September, but spent the summer in Virginia, where Peirce was conducting gravity experiments and looking for sites for new pendulum stations. They were in Washington for much of the latter part of 1884 and the first part of 1885, where from October to February Peirce was in charge of the Office of Weights and Measures and directed pendulum operations at the Smithsonian. From March 1885 through February 1886 the Peirces traveled for extended visits to Key West, Ann Arbor, Madison, and Ithaca, all on Coast Survey business. Finally in 1886, engaged only in local operations and soon to be relieved of fieldwork so that he could devote his time to preparing reports, he settled down in New York City, where he and Juliette stayed until their move to Pennsylvania the following year.

The year 1884 may have been the worst of Peirce's life (although Joseph Brent's biography reveals that there were bitter years still ahead). Peirce and Juliette had been married for only a few months and had hardly settled into their newly leased house when they learned of the decision by the Johns Hopkins trustees to let Peirce go.[8] It is clear now—though it was not then—that Peirce's dismissal was a sign that society could not tolerate his disregard for its conventions. His unique and somewhat arrogant individuality was too much at odds with the tenor of the times, and especially the conservatism of Baltimore. (Ironically, in less than a decade Peirce would express grave reservations about the American propensity to regard one's character as an individual as more important than one's social character.) By mid-year Peirce and Juliette knew that they would have to give up their Baltimore house and dispose of the elegant home furnishings they had so painstakingly chosen. The shock and disappointment of this turn of events led to ill health and despair.[9]

It must have come as a relief when in July 1884 Peirce was assigned to Fort Monroe in Virginia to make gravity determinations, and then to reconnoiter for one or two more stations in the mountains of the Virginias and North Carolina. Traveling with Juliette, Peirce arrived at Fort Monroe at the bottom of Chesapeake Bay on 23 July and proceeded to set up a station. The Superintendent's Report for the fiscal year ending June 1885 (P 331) indicates that only Peirce Pendulum No. 3 was swung and that Peirce was pleased with the results. In addition to gravity measurements, Peirce experimented with the use of an instrument, called a noddy, for measuring the swaying of the pendulum support and worked up some theoretical results for the 1884 Report (item 42). In September Peirce looked for new mountain

[8]For more on Peirce's life during the early months of 1884, see W4: xxxv–xxxvi and lxii–lxx.

[9]For a fuller account of the impact of Peirce's dismissal on his life, see Brent, ch. 3.

stations, but not finding any that met the Superintendent's specifications, he returned to Washington toward the end of the month and on 1 October was put in charge of the Office of Weights and Measures, an agency of the Coast Survey.

Peirce began his tenure with a great deal of energy and enthusiasm and even announced to Superintendent Julius Hilgard that he would write a book on the history of standards. Before year's end he had traveled to Boston, Providence, Hartford, New York, and Philadelphia and had met with electricians and manufacturers of gauges and other machinery in order to determine how best to meet the need for metrical standards as recommended by the U.S. Electrical Conference. In mid-October he attended the scientific session of the National Academy of Sciences in Newport, where he presented three papers: "On Gravitation Survey" (P 281), "On Minimum Differences of Sensibility," co-authored with Joseph Jastrow (P 282), and "On the Algebra of Logic" (P 283). His paper on logic must have been a preview, along the lines of items 20–22, of the paper he would soon finish for the *American Journal of Mathematics* (item 30). The paper on gravity surveys elaborated on a program for future gravity determinations that Peirce had proposed to Hilgard in his 1 October letter (item 23) and that was summarized in the 24 October issue of *Science* (pp. 396–97):

> Mr. C. S. Peirce explained some of the errors still needing correction in Pendulum observations, particularly such as were due to the flexure of the pendulum. He presented the outline of a scheme for a gravitation survey of the entire country, indicating the position of points in the eastern portion of the country which he thought most desirable to occupy, in which the stations would be about two hundred miles apart, regions of geological disturbance avoided, but their sides occupied, together with the summits of the higher mountains. Seven or eight stations could be occupied in a year, and thus a series of curves secured which would give us the form of the geoid; i.e., of the surface beneath the continent where the force of gravity was uniform.

While in Newport he took time to investigate an old stone mill to try to determine what standard of length had been used for its construction, probably thinking that he might help settle a dispute about the mill's origin. He argued that the construction of such a building would have required a "drawing to scale" and therefore a unit of length, which he assumed was either the English or the Norse foot. In December he published his findings in *Science* (item 26) and might have established himself in the popular mind as an authority in the field of measurement—but alas, his conclusions favored a Norse origin for the mill and it was soon revealed that it was English (which fact had already appeared in print).[10] In fairness to Peirce it must be said that he stated his purpose as purely metrological and that he had deliberately declined to offer an archeological opinion.

On 30 December 1884 Peirce attended the meeting of the American Metrological Society at Columbia College in New York City. He read a paper on the determination of gravity (P 270) and gave an account of his measures of the Old Stone Mill. He also participated in a discussion of the adequacy

[10] See also the note for 139–43.

of the standards of weight and measure in the United States and pointed out some of the deficiencies in the current system. As a result of his revelations, the Society passed a resolution recommending the appointment of a committee to advise Congress on the need for establishing an efficient bureau of standards.

Peirce had managed to finish this difficult year with a burst of energy. Perhaps he had resigned himself to a non-academic life and had readjusted to the idea of a life of science. Toward the end of the year, he began a series of five lengthy occupations that would continue to the end of January 1886, at stations in Washington, Key West, Ann Arbor, Madison, and Ithaca. The occupations began at the Smithsonian, where all four Peirce Pendulums were measured (compared to standards) in preparation for the elaborate fieldwork ahead, and continued there under his direction through February 1885.

Probably the most important of Peirce's scientific writings to appear in print in 1884 was his "Determinations of Gravity at Allegheny, Ebensburgh, and York, Pa., in 1879 and 1880" (item 1). It had been scheduled to appear in two previous *Coast Survey Reports,* but he had not been able to finish it because he was overextended with his dual appointment at Johns Hopkins. It is an important work, in part because it connects American geodetic methods and results with European geodesy. Another 1884 publication, *Observations with the Meridian Photometer During the Years 1879–1882* (P 271), makes heavy use of Peirce's scientific findings, in particular his *Photometric Researches* (P 118).

When his Johns Hopkins classes ended in the spring of 1884, Peirce might have stopped his university related research, but a momentum had built up that carried him along for several months. In the fall of 1883 he had begun teaching a course on the psychology of great men, a subject he had found interesting since boyhood. As early as 1860, in his "Private Thoughts," he had reflected on greatness and had concluded that a great man should be revered "notwithstanding his mistakes" (W1:5). The subject of human character was a topic of general interest and was addressed on more than one occasion at the Johns Hopkins Metaphysical Club: the work of Francis Galton was discussed on at least two occasions, and William James's "Great Men, Great Thoughts, and the Environment" was the topic for one meeting. A special interest of Peirce while at Johns Hopkins was the application of statistics to different subjects, and comparative biography lent itself to the illustration of statistical investigations that depended largely on impressionistic data. Some years later Peirce wrote that he had "cast about for a subject that might afford valuable training in such inductive investigation [as] the members of my class might need in future life and which they would not be likely to acquire in their other classes" (CP 7.256). The course, although poorly enrolled (like many of Peirce's other courses), was an apparent success. A carefully thought out program was followed that involved reading standard biographies and extracting relevant information (the question of relevance having been settled beforehand), compiling impressionistic lists of great men (and a few women), and submitting the resulting data to statistical examination. The study was carried on informally by Peirce and his students after the course had ended, apparently up to the time of his final departure from Baltimore near the end of 1884. In the absence of a complete record of the research it is unclear how far the study had progressed by then, and it seems likely that results were spread among the papers of the participants. The parts that are published here (items 2–19) give only a sampling of the methods and results as preserved in pages that remain with the Peirce Papers, and it may appear that the study never achieved any considerable success. But later discussions in Peirce's writings[11] and references to the study made by Jastrow suggest otherwise. When in 1894 the American educator Albert Yoder asked G. Stanley Hall, then of Clark University, about the study of greatness, Hall referred him to Peirce. Some years after he had left Baltimore, possibly stimulated by the 1891 publication of *The Man of Genius* by Cesare Lombroso and the 1892 *New Calendar of Great Men* (based on Comte's positivist calendar),[12] Peirce's interest in the old study was rekindled. An invitation to give the 1892 Lowell Lectures led Peirce to write to Augustus Lowell suggesting his "Comparative Biography of Great Men" as a topic for the series:

> It refers, not to the *eminent* men whom Galton has studied, but to a higher order, the *phenomena* of the history of mankind. A list of about 300 of such men would be formed and discussed and a method for the comparative study of them developed. Comparative lives of a few of them would be given,—a sort of scientific Plutarch,—scientific I mean in the treatment, not so exclusively as to the subjects. Finally, a large number of general questions relating to the nature, kinds, causes, and characters of greatness would be inductively considered.[13]

Peirce took up the study again at the turn of the century and in 1901 published a paper on "The Century's Great Men of Science." In a manuscript related to that paper (Harvard MS 1125), Peirce explained more fully his distinction between eminent men and truly great men:

> the native capacity of the lesser great men, like that of the merely eminent men, is due to the accidental coöperation of a thousand minute independent causes such as operate one way or another upon all of us, while the greater ones do somewhat partake of the nature of monstrous births in that their exceptional natures are largely due to causes that very rarely operate at all.

When Peirce recalled the original course of study, it was always with the greatest fondness for his students: "It was one of those matchless classes—the very salt of the earth,—which it was my privilege to enjoy in Baltimore."

Peirce taught two courses in the spring of 1884: one was the second half of his advanced logic course and the other a course on probabilities. His last

[11] See, for example, CP 7.256–66, where the study is described in some detail.

[12] *The New Calendar of Great Men: Biographies of the 558 Worthies of all ages and countries in the Positivist Calendar of August Comte,* ed. Frederick Harrison (London: Macmillan, 1892).

[13] Peirce to Lowell, 6 December 1891. Quoted in Carolyn Eisele, *Studies in the Scientific and Mathematical Philosophy of Charles S. Peirce,* ed. R. M. Martin (The Hague: Mouton, 1979), pp. 141–42.

two advanced logic students were Henry Taber and Joseph Jastrow. Taber had planned to write his dissertation on logic but after Peirce's dismissal had to give it up because, as he explained in a letter to Paul Weiss dated 3 September 1931, Peirce's successor "was quite ignorant of formal logic except the very rud[i]ments." Taber held Peirce in high esteem: "I have been told that James, or perhaps it was Royce, I have forgotten which, had said that Peirce impressed him as potentially the most powerful intellect he had ever known. I would certainly subscribe to this estimate of Peirce's powers."

Peirce's other advanced student, Jastrow, went on to become a respected psychologist and a well-known debunker of the paranormal.[14] In addition to logic (and probability theory—both Jastrow and Taber took the course on probabilities along with five other students, including William E. Story), Jastrow studied experimental psychology independently with Peirce. In his 1930 autobiography Jastrow said that "it was Charles S. Peirce, one of the most exceptional minds that America has produced, who stimulated me most directly."[15] Peirce suggested to Jastrow that they undertake an experiment to test Fechner's claim that human sensations are subject to a limitation he called a *Differenzschwelle* (the minimum perceptible difference of sensation). Below this threshold it was said to be impossible to discern differences of intensity. Peirce and Jastrow conducted elaborate experiments between 10 December 1883 and 7 April 1884 that constituted the first psychological investigation undertaken at Johns Hopkins and one of the earliest studies in experimental psychology in North America.[16] Peirce described the experiment in a letter to Simon Newcomb dated 7 January 1908:

> I note that you ac[c]ept as *established* the dictum of Gustav Theodor Fechner that the least sensible ratio of light is 101/100. If you will look in volume III Mem. of the U. S. Nat. Acad. of Sci. you will find a paper by me and my then student in logic Joseph Jastrow devoted to the question whether there is or is not such a thing as a "Differenz-Schwelle" or least perceptible difference of sensation. . . . [We] began with sensations of pressure and for a reason I will shortly mention we ended there. At once, using such precautions as any astronomer would use in observing faint nebulas, without any practice we found that if there were any least perceptible ratio of pressure, it was twenty or thirty times nearer unity than the psychologists had made it to be. We afterward tried to do the same thing for light; but were stopped by the utter impossibility of getting a piece of Bristol board containing a square inch of uniform luminosity. No doubt this might have been overcome. But Jastrow and I were severally pressed with other work and we dropped the investigation—contenting ourselves with what we had done.[17]

They had good reason to be content. Their report (item 24), presented to the National Academy of Sciences on 17 October 1884 and published in the

[14]See Thomas A. Sebeok, *Semiotics in the United States* (Bloomington: Indiana University Press, 1991), pp. 114–15.

[15]"Joseph Jastrow," in *A History of Psychology in Autobiography,* ed. Carl Murchison (Worcester, MA: Clark University Press, 1930), vol. 1, pp. 135–62.

[16]See Thomas C. Cadwallader, "Charles S. Peirce (1839–1914): The First American Experimental Psychologist," *Journal of the History of the Behavioral Sciences* 10 (1974): 291–98.

[17]Quoted in Eisele (1979), p. 87.

Academy's *Memoirs* in 1885, is described by Stephen M. Stigler as unexcelled in the nineteenth century and "a good example of a well-planned and well-documented experiment today."[18] Stigler points out that the study was the first to employ a "precise, mathematically sound randomization scheme," and also the first to require subjects to state their confidence in their choice (weight A is lighter or heavier than weight B) and to choose even when the level of confidence was zero. Ian Hacking, who also discusses the experiment, points out that Peirce's understanding of the importance of randomization was at least three decades ahead of his time.[19] Yet Peirce's idea was forcefully rejected by E. B. Titchener for being out of touch with psychological reality, and it was not reintroduced until R. A. Fisher's *Design of Experiments* appeared in 1935.[20] Hacking also remarks on the interesting last paragraph of item 24 where Peirce and Jastrow indicate that their conclusion has important bearings on such questions as women's insight and telepathic phenomena. The word "telepathy" was less than two years old, according to Hacking. It is noteworthy that at about this time the American Society for Psychical Research was being formed (an organizational meeting was held in Boston on 23 September) to "ascertain the truth in regard to the alleged psychical phenomena" and to expose "charlatan spiritualism."[21]

In addition to his gravimetric work, his Johns Hopkins classes and the study of great men, and his work with Jastrow, as well as other activities not yet considered, Peirce somehow managed to devote very productive time in 1884 to algebraic logic. In his 1880 paper in the *American Journal of Mathematics* (W4: item 19) he had given the first definitions of logical addition and multiplication suitable for modern Boolean algebra and, as Arthur Prior has shown, the system developed in that paper, with only slight enhancements, gives a complete basis for the classical propositional calculus.[22] The paper was intended as the first part of a much longer work on formal logic, but, though Peirce started several continuations (see items 20–22), certain difficulties and discoveries held him back. The greatest difficulty concerned the problem of distribution, which had arisen as a result of his claim that he could easily prove the law of distribution but had omitted a proof because it was too tedious. Ernst Schröder rejoined that Peirce must be mistaken because the independence of one of the distribution principles could be demonstrated—thus showing that the full law could not be proved. Peirce was convinced at first that Schröder was right but later reasserted that distribution could indeed be proved for his system. His position is often said

[18]Stephen M. Stigler, "Mathematical Statistics in the Early States," *Annals of Statistics* 6 (1978): 248.

[19]See Ian Hacking, "Telepathy and Randomization," *Isis* 79 (1988): 427–51.

[20]R. A. Fisher, *The Design of Experiments* (Edinburgh: Oliver & Boyd, 1935).

[21]*Science* (17 October 1894): 370. References to the Society had appeared even earlier in the popular press.

[22]See Arthur N. Prior, "The Algebra of the Copula," in *Studies in the Philosophy of Charles Sanders Peirce,* eds. Edward C. Moore and Richard S. Robin (Amherst: University of Massachusetts Press, 1961), pp. 79–84.

to amount to the claim that every lattice is distributive, but that is almost certainly a misconstrual of Peirce's views.[23]

Between 1880 and 1885 Peirce developed a conception of truth values (a sentence has the value v if it is true or f if it is false) and created a semantics for his algebraic logic. Items 20–22 show him in the process of discovery. Stimulated no doubt by Schröder's 1883 paper to the British Association for the Advancement of Science, which argued against Peirce's distribution claim, but also by his 1882 edition of his father's *Linear Associative Algebra* and the recent publication of *Studies in Logic,* Peirce filled these short manuscripts with brilliant flashes of insight. In addition to the systematic introduction of truth values, we find an early statement of truth-function analysis ("it is clear that the truth of a general formula may be tested by trying whether it will always hold when either v or f is substituted throughout for each letter" [p. 112]), the development of quantifiers (following their anticipation in W4: item 66) and remarks about their significance for distinguishing logic from mathematics, the groundwork for "Peirce's law," matrix representations of universes of discourse, the idea that the elementary logical operations are insertion and deletion (item 20), and a great deal more. The idea that the copula of inclusion might be abandoned in favor of disjunction and conjunction with rules only for insertion (amplification) and deletion (simplification), an idea Peirce got from Mitchell, may be seen as an anticipation of the idea on which Gentzen based his system of natural deduction.[24] As late as the summer of 1884 Peirce was still working on a continuation of his 1880 paper, but within a few months he would be ready to relegate reference to the earlier paper to a footnote in what would become his most influential work on logic.

Peirce had delivered his (and Jastrow's) paper on minimum sensibility to the National Academy in October. The success of that study may have encouraged him in his use of statistical methods, for it was soon followed by a paper on the "Success of Predictions" (item 25) in which, according to Stigler, he derived "a latent structure measure of association for 2 $\times$ 2 tables."[25] In this work Peirce addresses the question whether meteorologists could successfully predict tornadoes. He finished the year with a discussion in the pages of the *New York Evening Post* and the *Nation* on the economics of the sugar trade with Cuba (items 27–28).

The year 1884 had been difficult, but by its end Peirce had reoriented himself to a life of science. He was ready to start the Smithsonian occupation, the beginning of more than a year mostly on location—away from Baltimore! And he had his definitions to write for the *Century Dictionary,* something he had been working on for over a year. Perhaps the worst was over.

[23]See W4: xlvi–xlvii and note 184.3 for further discussion of the 1880 "Algebra of Logic."

[24]See Irving Anellis, "Forty Years of 'Unnatural' Natural Deduction and Quantification," *Modern Logic* 2 (1991): 113–52, especially 115. For another discussion of Peirce's anticipation of Gentzen, see Don D. Roberts, "The Existential Graphs and Natural Deduction," in Moore and Robin, pp. 109–21.

[25]Stigler (1978), 249.

By a cruel turn of events, Peirce had hardly settled himself to the harsh reality of his dismissal from Johns Hopkins when he had to face a whole new episode of bitter and painful events. It began without much fanfare as a broad investigation of four federal scientific agencies which had outgrown their original charters: the Geological Survey, the Coast and Geodetic Survey, the Signal Service of the U.S. Army, and the Hydrographic Office of the U.S. Navy. The investigation was conducted by a joint commission of the U.S. Senate under the chairmanship of Senator William B. Allison of Iowa and was undertaken to examine the structure and operation of the four agencies for economy, efficiency, legality, and utility.[26] The Commission heard testimony from more than fifty federal employees between 4 December 1884 and 28 February 1885 and in a subsequent session about a year later. Peirce was called to testify on 24 January, one of only ten Survey employees questioned by the Commission. Even though he had been in charge of the Gravimetric Survey for many years, he was questioned almost exclusively about the work of the Office of Weights and Measures, which he had directed only since October 1884. This may have had something to do with the 30 December 1884 resolution of the American Metrological Society, for the Commission's questioning and Peirce's testimony (item 29) bear a marked resemblance to the discussion at Columbia one month earlier. Peirce made it plain that many, if not most, of the standards in the United States were in great need of improvement. One example that caught the interest of the commissioners was that the hollow brass weights used to weigh out gold for coins minted at Philadelphia and Denver actually measured out different amounts of gold because of the buoyancy of air. The result was that coins minted at Denver contained too much gold. Peirce's testimony before the Allison Commission and the resolution of the American Metrological Society were the first two steps toward the creation of the National Bureau of Standards.[27]

In the course of about four months as head of the Office of Weights and Measures Peirce had established a noticeable momentum toward an improved agency. He had written to Superintendent Hilgard on 27 September with an impressive plan for his first six months of service, including the preparation of at least five reports on metrological research (including his long-awaited report on the spectrum meter), some new computations and comparisons, an inventory of instruments and records, the commencement of an index of results, the preparation of a history of instruments and standards, and the systematic collection of foreign publications on metrology for the library. Given Peirce's obvious enthusiasm, it is surprising that on 22 February 1885, less than five months after his appointment, Peirce declined further service as head of Weights and Measures. The reason for his action was revealed to Simon Newcomb in a letter dated 10 June 1899: "[I] only left because [Hilgard's] physical condition was such as to cause me embar-

[26]See Thomas G. Manning, *Government in Science* (Lexington: University of Kentucky Press, 1967), pp. 122–23.

[27]See Fisch, p. 409.

rassment which I thought required me to quit Washington."[28] Hilgard's condition would soon be revealed to the world.

It is not clear whether the findings of the Allison Commission directly damaged Peirce or the Coast Survey, but the mere fact that the investigation was conducted revealed that federal science in America was entering a new age, a time when the value of work would be judged by its immediate practical (economic) benefits. When Grover Cleveland took office in March as the 22nd president of the United States, and his Democratic Party took control of the government, anti-scientific sentiment had won the day. Dedicated to reducing the federal bureaucracy, Cleveland found a ready target in the Coast Survey.[29]

On 25 July 1885 the *Washington Post* broke a story with the headline, "Exhorbitant [*sic*] Expenditures. Coast Survey Officials Suffer Penalty for Extravagance. Superintendent Hilgard Suspended, Several Subordinates Dismissed and an Investigating Committee Appointed." The *Post* announced that the Treasury Department had for some time been dissatisfied with the Coast Survey accounts and, after an audit, had found them to be "entirely unsatisfactory." Cleveland had dismissed Hilgard and had appointed Frank M. Thorn, chief clerk of the Internal Revenue Bureau, to head a commission to take charge of the Coast Survey office and to conduct a full investigation. On 7 August the *Post* reported that "the actual condition of the office of [the] survey was one of demoralization, and its workings [were] inefficient, unjust, and to some extent disreputable." Hilgard was accused of misappropriating federal funds and was reported to be an alcoholic, which was widely known and probably contributed to the decision of the Treasury Department to investigate the Survey.[30] In a paragraph dealing with Peirce, it was reported that for several years he had been performing pendulum experiments "without restriction or limitation" and that the "meager value" of his work was substantially destroyed by its cost. Peirce was shocked and indignant. He wrote a letter of protest and rebuttal from Ann Arbor on 10 August that was published four days later in the *New York Evening Post* (P 300) and later in *Science* (P 317). His chief concern, the main reason for his indignation, was that unscientific men had been permitted to judge the importance of his work and had judged it to be of "meager value."[31] He announced that he would resign if that estimate was accepted by the Survey. It is to Peirce's credit that the scientific community, even within the Survey, immediately came to his defense. Charles A. Schott wrote to Benjamin A. Colonna: "I trust you will be able to induce Mr P. to reconsider his action and for the sake of the scientific reputation of the Survey, *continue the work*, now that we are on the eve of reaping the practical benefit of his researches."

[28]Quoted in Eisele (1979), p. 82.

[29]See Thomas G. Manning, "Peirce, the Coast Survey, and the Politics of Cleveland Democracy," *Transactions of the Charles S. Peirce Society* 11 (1975): 187–94. See also Brent, ch. 3.

[30]See Manning (1975), 188.

[31]The source of the "meager value" remark appears to be a disposition of B. A. Colonna on the condition of the Survey (see L91a: 127).

And at the 28 August 1885 executive meeting of the American Association for the Advancement of Science a resolution was passed on Peirce's behalf which chastised the Treasury Department for referring the question of the value of scientific work to non-scientists. The Association recognized that an ominous note had been sounded by the Cleveland administration and perhaps foresaw that the shift of power from scientists to bureaucrats would bring about a period of decline for pure science in America. Its resolution urged that the head of the Coast Survey "should be appointed by the President, with the advice and consent of the Senate," and "should have the highest possible standing among scientific men and should command their entire confidence." Peirce's scientific reputation had been vindicated. Still the scandal was a great strain on him. In October he wrote to William James that

> This horrid & sickening business of the Survey makes me long intensely for University life. The villainous things which I hear whispered, the Vandal methods of trying to set things right, the accusations of which I have myself been the subject, combine to make me loathe the Survey so, that I would rather keep a pea-nut stand than stay in it one minute longer than my duty requires me to do.

About six months later, the Allison Commission found that the Geological Survey, headed by John W. Powell, was extravagant in its operations and sought passage of a congressional bill to restrict its work and limit its publications. Again the scientific community was outraged and none more than Peirce. He wrote to Powell assuring him of strong support within the Coast Survey and proposed a plan of attack: "Let the congressmen hear of science, no longer as merely giving reasons, but as an *interest,* saying *We want so and so.* . . . There are a hundred votes in the house to be commanded in this way."[32]

Tumultuous though it was, 1885 was one of Peirce's most productive years in the field. On 1 March, after concluding the Smithsonian occupation and within a few days of his resignation from Weights and Measures, Peirce left for Key West. He set up a station in an Army barracks which he occupied until 1 May. Using only Peirce Pendulum No. 2, he discovered that the residual difference between gravity in Washington and Key West was smaller than he had expected. From the Superintendent's Report on Peirce's work for the year ending June 1885, it appears that the Key West results helped determine his next major assignment.

> Referring to the fact that the residual difference of gravity between Washington and Key West is somewhat smaller than he had anticipated, Mr. Peirce expresses the opinion that the question to which gravity research should be directed more particularly for the present is, whether lines of equal residual gravity can be traced upon the map, or whether the merely local variations will mask those that are progressive, and that for this purpose lines of stations a thousand miles or so in length should be run with stations three degrees apart. . . . The first endeavor should therefore be to run an east and west line.

[32]Peirce to Powell, 2 May 1886; quoted in Manning (1967), pp. 138–39.

By this time Peirce should have been able to replace the somewhat defective American-made set of (Peirce) pendulums that he had designed a few years earlier. He had ordered new pendulums from Gautier in Paris during his final European assignment in 1883 but had been directed by Hilgard to return to Washington before they were finished. As a result of a series of inadvertencies, including possibly some disingenuousness on the part of Gautier, communications broke down between the Coast Survey and the Paris manufacturer—and it became a matter of some dispute who if anyone was at fault. As it turned out, the Gautier pendulums were never acquired, and this became the cause of both regret and embitterment for Peirce.[33]

In July Peirce was directed to make a reconnaissance for an east-west line of three or four stations approximately along the forty-third parallel and as far west as the Mississippi River. Peirce chose Ann Arbor, Madison, and Ithaca and, in August, began operations in Ann Arbor. The American Association for the Advancement of Science held meetings in Ann Arbor while Peirce was stationed there, and it was then that the executive committee passed its resolution of support for him. Operations at the University of Wisconsin began early in October and continued throughout the month. The station was set up in Library Hall (today called Music Hall), which had a clock connected to the University's Washburn Observatory. Peirce's main contact at the University was Edward S. Holden, Director of the Observatory, with whom he became very friendly (and with whom he would correspond for many years). From Madison Peirce proceeded to Ithaca, where work began by early December. En route the Peirces passed through Niagara Falls, where because of a miscalculation on Peirce's part, compounded by the delay of a payment from Washington, they were "stranded" for fifteen days. Being stranded in Niagara was not altogether a bad thing, as is revealed in a letter to Holden dated 8 January 1885: "Mrs. Peirce doesn't progress very fast. We stayed a fortnight in Niagara & that did her ever so much good." Peirce spent some of that time working on mathematical problems related to "the effect of the air on the period of the pendulum" (Peirce to Thorn, 7 November 1885), a problem in hydrodynamics that had never been satisfactorily treated. He was impressed with the Falls and frequently referred to it in illustrations in subsequent writings (see especially item 54). Peirce arrived in Ithaca on 19 November 1885, where he found his foreman demoralized because of the delay. By the end of the month Peirce had discharged the man and had hired a Cornell graduate student to replace him. Peirce's host at Cornell was E. A. Fuertes, Dean of the Faculty of Engineering, who so greatly impressed Peirce that he worked behind the scenes to get him appointed as superintendent of the Survey. Operations in Ithaca continued to the end of January.

At each of these stations, and then again at the Smithsonian, Peirce swung Peirce Pendulums Nos. 2 and 3 (a meter and yard respectively). A

[33]For a fuller account of the Gautier pendulum episode, see W4: xxxiv and Brent, pp. 143–44, 167, and 180.

description of the procedures employed for these operations appeared in the 1886 *Report,* which illustrates the laboriousness of gravity determinations (for which the data were hand-recorded for subsequent manual reduction and computation).

Two new pendulum stands had been constructed of improved design, so that two pendulums could be oscillated simultaneously on two supports. Each swinging consisted of five thousand oscillations with heavy end up and fifteen thousand with heavy end down, except that one-fourth of the swingings in the latter position were of double length. There were thus about six hundred thousand oscillations with heavy end down and one hundred and sixty thousand with heavy end up at each station.

The paper that resulted from this series of occupations was Peirce's second major memoir on gravity—the first was his "Measurements of Gravity at Initial Stations in America and Europe" (P 161; W4: item 13)—and, according to Victor Lenzen, would have been an influential work in geodesy had it appeared in 1890 when Peirce finally had it ready for publication.[34] But on the advice of Newcomb, then Superintendent of the Coast Survey Thomas C. Mendenhall declined to print Peirce's memoir (P 385) and, having decided that Peirce's usefulness to the Survey had come to an end, asked for his resignation.

Two papers appeared in 1885 as appendices to the 1884 *Coast Survey Report* and both deal with Peirce's investigations of the flexure of pendulum stands. "On the Use of the Noddy" (item 42) describes a method he devised for measuring flexure, and "Note on the Effect of the Flexure of a Pendulum upon its Period of Oscillation" (item 43) discusses the degree of disturbance caused by flexure, an effect which he concluded must be considerable "for all the reversible pendulums which have ever been constructed." In the second paper Peirce introduced the expression "kinetic potency" to avoid using the more standard expression "potential energy," which he said "grates upon the ear of a student of Aristotelian philosophy."

Except for his scientific work and the Coast Survey scandal, logic dominated Peirce's life, at least until August. As a result of the stimulating insights of the summer of 1884 and the realization that he had moved too far from his 1880 paper (W4: item 19) to write a second part, he refocused his attention on a new formalization of logic, self-consciously motivated by his notational discoveries. In the new paper, "Algebra of Logic: A Contribution to the Philosophy of Notation" (item 30), Peirce considered the different kinds of signs necessary for a fully adequate logic system, and he concluded that it is necessary to have tokens (conventional or general signs, usually called symbols), indexes (demonstrative signs), and icons (signs of resemblance). This is the first published application of his revised theory of signs to algebraic logic, which he had begun to formulate in item 22.[35]

It is in this paper, appearing in February 1885, that Peirce introduces truth-values in giving his decision procedure for theoremhood, and the first

[34]See Victor F. Lenzen, "An Unpublished Scientific Monograph by C. S. Peirce," *Transactions of the Charles S. Peirce Society* 5 (1969): 5–24.

[35]See also item 62 in W4.

theorem proved by that method is his fourth icon, $(((p \prec q) \prec p) \prec p)$, which marks the difference between classical and positive logic.[36] The axioms for first-order logic are given in the first five icons, although the fourth (the negation principle) can be used to reduce Peirce's basis to the Tarski-Bernays axiomatic system for implicational logic.[37] Here quantifiers are first introduced in their standard form and Peirce anticipates the modern distinction between first- and second-order logic.[38] Furthermore, he provides the basis for a complete quantification theory with identity[39] and in his discussion of procedures for working with his calculus shows remarkable insight into modern methods, even introducing something very much resembling what today is called Skolem normal form. The paper was widely read and had considerable influence on the development of symbolic logic. It is cited as a key work by many notable logicians, including Peano, Whitehead, Lewis, and Tarski; and through Schröder, Peirce's most influential follower, its results reached many others, including Löwenheim and Skolem. Łukasiewicz often quoted the first paragraph of Part II to show that Peirce was a precursor in conceiving of many-valued logics.[40] Even Bertrand Russell read the paper (along with the 1880 paper) at the turn of the century,[41] but how much he was influenced by Peirce, directly or indirectly, is far from clear. In addition to its place in the history of exact logic, for which it is justly acclaimed, item 30 represents an advance in semiotic theory and an important stage in Peirce's systematic thought.[42]

Throughout the early months of 1885 and into the summer, Peirce worked on a continuation of item 30, which he justly believed set the stage for a whole new era in logic. On 25 June he wrote to William James: "I have not sent out any copies of my new memoir . . . because the paper is not yet completed & the most important part of it is to come. But I consider it as the beginning of a new life for Formal Logic." When he had finished the second part, which extended his theory of quantification and what he called his general algebra of logic, he submitted it to Newcomb, then editor of the *American Journal of Mathematics*. (J. J. Sylvester, who had agreed in principle to publish it, had returned to England to take up a chair at Oxford.) Newcomb read Peirce's paper and agreed to publish it only if Peirce said that it was mathematics, not logic. Peirce refused and

[36]This principle has been known since Łukasiewicz as Peirce's Law.

[37]See Atwell R. Turquette, "Peirce's Icons for Deductive Logic," in Moore and Robin, pp. 95–108, especially p. 101.

[38]See Richard Martin, *Peirce's Logic of Relations and Other Studies* (Dordrecht, Holland: Foris Publications, 1980), p. 63.

[39]See J. Jay Zeman, "Peirce's Philosophy of Logic," *Transactions of the Charles S. Peirce Society* 22 (1986): 1–22, especially 7.

[40]See Henry Hiż , "Peirce's Influence on Logic in Poland," forthcoming in *Studies in the Logic of Charles Sanders Peirce,* eds. Nathan Houser, Don D. Roberts, and James Van Evra (Bloomington: Indiana University Press, 1994).

[41]See Irving Anellis, Review of Volumes 1–4, *Writings of Charles S. Peirce, Modern Logic* 3 (1992): 77–92, especially 87.

[42]See item 56 for a related but somewhat fuller presentation of Peirce's revised theory of signs.

the paper was rejected.[43] He retold this incident to James in August 1905 and, as he often did, gave him a lesson in logic at the same time. In explaining how to draw certain inferences in his general algebra of logic, he made use of the principle $\Pi_x l_{xx} \prec \Pi_x \Sigma_y l_{xy}$, which led him to reflect:

I do not know whether I ever stated this in print or not. It is a part of a principle thoroughly developed by me in a memoir which Newcomb practically refused to print in 1885 or 1886 which is the reason why I have never since printed anything on logic which could not be put in popular form. I there called the principle (of which this is a very small part) the principle of identification and diversification. It holds good strictly even if there is no x. From "every phenix would burn itself" it follows that "every phenix would burn something." It is somewhat remarkable since $\Pi_x l_x$ does *not* warrant $\Sigma_x l_x$.[44]

Item 31 appears to be part of the paper Newcomb rejected. Together with the related item 32—logical investigations carried out over a six-day stretch in May—it represents logic in its most advanced state until after the turn of the century.

Except for logic and the revision of his theory of signs, Peirce had not wrestled with philosophy for over a year—since his lecture on design and chance. But sometime during the summer of 1885 he turned again to speculative philosophy. Several events and circumstances stand out as instrumental in Peirce's return to philosophy at that time. There was the Coast Survey scandal that thoroughly demoralized him and destroyed his commitment to federal service and a life of experimental science, and there was Newcomb's rejection of his pioneering logic paper, which dampened his enthusiasm for that most formal and technical branch of philosophy. These two events led Peirce to close off avenues he might otherwise have followed, which helped clear the way for his resumption of speculative philosophy. His travels to Michigan, Wisconsin, and Cornell may also have played a part; perhaps someone at one of the universities re-ignited his enthusiasm for philosophy, or perhaps Peirce deliberately turned to philosophy as the field most likely to secure him a teaching position. Brent speculates that Peirce may have had something like that in mind,[45] and certainly it is clear that he wanted a university post. He had written to James in June about giving a fall course at Harvard, and in October he wrote to his brother James Mills (usually called Jem) that teaching was the life he desired. He knew from his experience at Johns Hopkins (and with Newcomb) that logic was not very marketable, and may therefore have decided to recast his academic profile in the direction of traditional philosophy.

Possibly all these factors played a part in Peirce's shift of focus back to philosophy in the summer of 1885, but probably the most influential event

[43]In one recollection of this incident (cited in NEM 3: 1069), Peirce said that Newcomb rejected *a proposal* on the grounds that the planned work was not mathematics and that, as a result, the paper remained unwritten. If that was the case, items 31 and 32 may constitute Peirce's most finished results. But Peirce's earlier accounts tend to confirm that he submitted a finished paper for publication.

[44]NEM 3: 816–17.

[45]Brent, p. 177.

was the publication of Josiah Royce's *Religious Aspect of Philosophy,* which appeared mid-year. In his book Royce argued for the existence of God from the possibility of error and, almost as a challenge to Peirce, defended his position against the "modern Thrasymachus" who held that all we can conclude from the *possibility* of error is the *possibility* of God. Peirce recognized himself as the modern Thrasymachus and took up the challenge—and in doing so, according to Fisch, "turns some of the *will-bes* of his *Popular Science Monthly* series into *would-bes,* and thereby takes a short step from his earlier nominalistic pragmatism toward his later realistic pragmaticism."[46] Peirce wrote a long review (item 33) for Edward L. Youmans's *Popular Science Monthly,* but, as he explained to William James in a letter of 28 October, it was never printed: "I wrote to Youmans,—at his particular request,—a notice of Royce's book. I was a long time over the book & wrote I thought something really very good, for me; but Youmans wouldn't print it, i.e. he made such a wry mouth that I relieved him of it." In the review, Peirce criticized Royce's idealism as being too much like that of Hegel, whose "capital error . . . which permeates his whole system in every part of it is that he almost altogether ignores the Outward Clash." Peirce repeated the thesis of item 30, that three kinds of signs are indispensable in all reasoning, and emphasized the necessity for indexes to refer to individuals: "one such index must enter into every proposition, its function being to designate the subject of discourse." There is no doubt that Royce's book, in conjunction with his own recent discoveries in logic and his revised theory of signs, had a profound effect on Peirce. It was then that Peirce returned to his categories and to a reassessment of Kant.

In an unfinished draft of the review (MS 540), Peirce reflected that Kant's entire system of thought stood on his logic, in particular on his analysis of propositions. He then mused:

> If we assume then that the logical distinctions of propositions are necessarily involved in reasoning and take their origin in the nature of the human mind, then so also do these conceptions, *cause, reality,* etc., which are essentially presupposed in those distinctions. . . . Thus, the whole system of Kant depends upon the truth and necessity of the system of formal logic which furnishes these distinctions of propositions. If the latter system is artificial, the Kantian philosophy must fall to the ground; yet even then it would seem that there must be in place of that a true system which would be based in a similar way upon the correct analysis of formal logic.

But, he concluded, Kant's system *is* artificial: "the traditional distinctions of propositions rest nearly all of them upon mere accidents of language." In Kant's wake, Peirce was ready to put forward "the correct analysis of formal logic" upon which "a true system" of thought might be founded: "there are three conceptions which enter necessarily into formal logic at every turn and under a thousand shapes,—namely, the ideas of First, Second, and Third; or, more accurately expressed, An, Other, and Medium." Here we have a preview of what was to come, a new architectonic system of thought based on Peirce's categories to replace Kant's. In his finished review, but more in-

[46]Fisch, pp. 190–91.

tensely in items 34–37, Peirce worked out the revisions to his theories of categories and signs in his quest for "the correct analysis of formal logic." These papers constitute the spadework for his new system of thought—his architectonic evolutionary philosophy—and are the precursors of his book "One, Two, Three."

Three reviews from the latter part of 1885, which are included in the present volume, may have contributed to—or resulted from—the resumption of Peirce's interest in speculative philosophy. In November Peirce reviewed Raymond Perrin's *Religion of Philosophy* for the *Nation* (item 39). Even though he was thoroughly unimpressed with the book, it is interesting that Perrin's purpose, revealed in the paragraph-long full title, was to reduce "the categories of thought, or the most general terms of existence, to a single principle, thereby establishing a true conception of God." Around the same time Peirce reviewed Thomas K. Abbott's translation of Kant's *Introduction to Logic* (item 40) and John Fiske's *The Idea of God* (item 41). The review of Kant's *Logic* is unfinished, but there is enough to see that Peirce's reassessment of Kant occasioned by his response to Royce is uppermost in his mind. In his one-paragraph review of Fiske, unpublished and probably unfinished, he refocuses directly on the old "Design and Chance" conceptions of evolution and chance, but perhaps more importantly on what elements are necessary for explaining the "whole development of the world." Peirce mentions in particular Fiske's claim that the events of the universe are the result neither of chance nor of blind necessity, and he counters that "minds . . . formed under the influence of physical science" hold that events *are* brought about by force and chance and that the place of freedom, if granted at all, is very limited as compared with that of necessity. Of course Peirce was one of the physical scientists who granted a small, though profoundly important, role for freedom. A fourth review in late 1885—of Clifford's *Common Sense of the Exact Sciences* published in the *Nation* on 3 September (item 38)—does not address architectonic or cosmological questions of the sort touched on above, but it does contain an early indication of Peirce's relativism—absolute position in space and absolute velocity have no meaning. Also in the review is an interesting reference to the ideas of Peirce's old school friend Francis Ellingwood Abbot, who was within a few days of finishing his *Scientific Theism.*

In the closing months of 1885, Peirce formulated the general outlines of the project that would grow into his 1887–88 "Guess at the Riddle." Although there is no full articulation of his guess to be found in the writings of 1885, it is clear from the final incomplete paragraph of item 35 that he had already made it. In his 25 October letter to Jem (quoted below), he spoke of the "momentous thing" he had to say and on its importance for molecular science and psychology. Three days later he wrote to William James (in the same letter in which he mentioned his review of Royce): "I have something very vast now. . . . It is . . . an attempt to explain the laws of nature, to show their general characteristics and to trace them to their origin & predict new laws by the laws of the laws of nature."

Peirce spent the first month of 1886 swinging pendulums in Ithaca, but was back in New York by 1 February. On the 3rd his brother Jem held a reception for him in Cambridge. Abbot was there and wrote of the proceedings in his diary:

Attended a meeting of "philosophers," including John Fiske, James, Royce, and Perry, at Prof. J. M. Peirce's, 4 Kirkland Place, to welcome Prof. Chas. S. Peirce, of Johns Hopkins, (my classmate), and hear from him a new "logical theory of Evolution." Peirce begins with absolute or pure potentiality, with absolute chance or negation of all law, even logical, to evolve at last Absolute Being and Absolute Law—in fact, to evolve Infinity out of Zero, God out of Nothing. Brilliant, ingenious, and—impossible. Had a wine supper, during which Charley continued to spin his glistening cobweb.

Peirce had written to Abbot from Ithaca (items 44 and 45) about his new book, *Scientific Theism,* and had probably already written the review that appeared in the 11 February issue of the *Nation* (item 46). Abbot, more the iconoclast than Peirce, had spun his own cobweb which, if not glistening, was at least alluring. Abbot's would be the second book of the period to exert a considerable influence on the course of Peirce's thought.

Having just returned from directing pendulum operations at three major universities and, as a matter of prudence, still planning for a life of science, Peirce might have held forth on a topic related to experimental science. But Abbot's diary reveals what purpose had taken hold of Peirce's mind. Three months earlier, on 25 October, Peirce had written to Jem:

All this [the difficulties in the Survey] has awakened me to the duty of making some effort to do that thing for which I am in the world, namely, to set forth the true nature of logic, and of scientific methods of thought and discovery. I have a great and momentous thing to say on this subject. Without it, molecular science must remain at a stand-still. It must continue what it is, idle guess-work. The true theory of the constitution of matter, which can only be based on sound scientific logic, must have the most important consequences in every direction. On psychology too, which is to be the great science of the coming hundred years, logic must exert weighty influence. About logic I have something to say which other men have not thought of, and probably may not soon think of. Perhaps I cannot get an opportunity to develope this. To do it I must sit down quietly to it & to teaching, and not live in boxes. . . . But it is *certain* that so long as I stay in the Survey my destiny will not be fulfilled.

Peirce was "straining at the bit" to get back to work on his neo-Kantian architectonic, but Survey work would keep him from it for a few months longer.

After his return from Ithaca Peirce took charge of pendulum operations at the Stevens Institute in Hoboken, where the British Kater pendulums had to be measured before their return to Herschel, and again at the Smithsonian to remeasure the pendulums he had used for the operations of the past year. But as Peirce had anticipated, the Survey was changing with Thorn in control, and it soon became apparent that leadership in geodetic science was being transferred to Schott.[47] Thorn did make an effort to bring Peirce around to his own way of doing things, perhaps out of a genuine appreciation of his powers, but more likely because Peirce represented a great investment on the part of the Survey; however, he never grasped the full serious-

[47]See Brent, p. 177.

ness of Peirce's ordeal nor its disruptive effect on his capacities. Peirce, of course, wanted to proceed with his plan for gravity determinations as set out in his letters to Hilgard of 1 October 1884 (item 23) and 30 June 1885 (and reiterated in a letter of 27 October 1885 to Thorn), but he was swamped with volumes of unreduced data from years of work that had to be turned into publishable reports—which was all Thorn seemed to want. He did manage to get three short papers into print in 1886 (items 51–53) which took some account of the 1885 work at Key West, but there was still an abundance of data on gravity and on flexure to be worked up for publication and, more importantly, a major unfinished paper on the spectrum meter which represented a great outlay of time and money. There was also the report on the 1882–83 gravity work in the Arctic, carried out for Peirce by the ill-fated Greely party. Lieutenant Greely returned from Lady Franklin Bay late in 1884 to a hero's welcome, and he made it known that the work he had carried out for Peirce was for him a matter of much satisfaction. Two years had passed since he had handed over the Arctic gravity records to Peirce, and he was anxious for results.[48] There was also a lot of unreduced data from various less extensive operations, some gravitational and others metrological, and to make matters worse, Peirce now had a mass of data from the just completed gravity operations in Washington, Ann Arbor, Madison, and Ithaca.[49]

In the Survey scandal of the previous year, one of the main criticisms of Peirce had been that his work was of "meager value." It is true, of course, that however great its potential, his results were not of much use until the raw data were reduced and reports were written for publication. Thorn saw the risks and in August officially relieved Peirce of his field duties so that he might devote all his time to preparing his reports.[50] He wrote to Peirce on 26 October: "It seems to us here that the feeling in Congress indicates that the whole future of the pendulum work of the Survey will depend upon your success in giving us some systematic work and adequate returns for the thousands of dollars already spent in pendulum research." Thus came the end of Peirce's long period of leadership in geodetic science for the Survey.

The year 1886 was a watershed in Peirce's intellectual life. It was the year when his guess at the riddle of the universe became fully articulated and then grew into the hypothesis that would guide the course of his thought for years to come. By the summer, when he began writing the book that was to set out his new system of thought (items 47–50), the guess was featured in his opening chapter:

We must . . . suppose an element of absolute chance, sporting, spontaneity, originality, freedom, in nature. We must further suppose that this element in the ages of the past

[48]Peirce's report on the Arctic gravity work will be included in W6, and a more detailed account of the Peirce-Greely interaction will be given in the introduction to that volume.

[49]See Brent, pp. 196–201.

[50]Victor F. Lenzen, "Charles S. Peirce as Mathematical Geodesist," *Transactions of the Charles S. Peirce Society* 8 (1972): 90–105, especially 98.

was indefinitely more prominent than now, and that the present almost exact conformity of nature to law is something that has been gradually brought about. . . . If the universe is thus progressing from a state of all but pure chance to a state of all but complete determination by law, we must suppose that there is an original, elemental, tendency of things to acquire determinate properties, to take habits. This is the Third or mediating element between chance, which brings forth First and original events, and law which produces sequences or Seconds. . . . [T]his tendency must itself have been gradually evolved; and it would evidently tend to strengthen itself. . . . Here then is a rational physical hypothesis, which is calculated to account, or all but account for everything in the universe except pure originality itself. (p. 293)

On 20 August Peirce wrote to Holden at the University of Wisconsin that his "evolutionist speculation" had grown into "a great working hypothesis of science, destined to play a great part in the future." He said that the skeleton of his ideas had "filled itself out on the philosophical side, so that my book will be a real manual of philosophy, leaving no question untouched."

It is remarkable how many lines of thought came together at this point—how many influences led Peirce to his guess. The story is too complex to be given here in full—the best account so far is by Fisch[51]—but a sketch of some of the main factors will further illustrate the significance of the writings of the present volume.

With his 17 January 1884 lecture on design and chance Peirce had taken a stand on the question of determinism, declaring that absolute chance was a real agency in the evolution of the universe *and even in the evolution of law itself*. This thesis became a fundamental doctrine of his evolutionary cosmology and was a major factor in preparing him for his guess. It is important to remember, however, that the path to "Design and Chance" was itself very complex and that the roots of Peirce's tychist logic extend deep into his early thought.[52]

Although the general thesis of an evolving universe, a universe subject to the originating influence of absolute chance, was crucial to his guess, it was not in itself sufficient. It might be more accurate to say that the key was his theory of categories, which had virtually lain dormant since first expounded in 1867 (W2: item 4). Only after he had revived his theory of categories, stimulated by his study of Royce, was Peirce ready to make his guess. But what was also revived by the study of Royce was Peirce's Kant-inspired attraction for architectonic philosophy, for system building, and the belief that with his categories he could improve on Kant. By the end of 1885 Peirce knew that by combining his evolutionary speculations with his revived and revised categories he was on to something vast.

It is not clear exactly when he started thinking of his project as the elaboration of a guess or when he first conceived of his guess as solving the

[51]See Fisch, ch. 12.

[52]See W4: lxvii–lxix; Fisch, ch. 12; and Brent, pp. 174–76. An excellent account of Peirce's early indeterminism, especially as developed in his 1866 Lowell Lectures, was given by Paul D. Forster in Toronto on 9 October 1992: "The Logical Foundations of Peirce's Indeterminism" will be published in the proceedings of the conference, entitled "New Topics in the Philosophy of C. S. Peirce" and held at Trinity College, University of Toronto.

riddle of the universe in the fullest sense—so that *everything* would be included within the scope of his solution. A number of factors have already been mentioned that probably led Peirce to think along those lines, including the books he reviewed in 1885. Another important factor was his work for the *Century Dictionary,* which led him to reconsider Greek philosophy. Fisch emphasizes that nearly all the Greek philosophers were evolutionary cosmologists and he flatly states that "it was by way of the Greeks, and especially by way of Empedocles, Aristotle, and Epicurus, . . . that Peirce arrived at his own evolutionary cosmology."[53] In his earliest sketch of the book that was to present his new system of thought—in thirteen chapters, beginning with a chapter on fundamental conceptions and ending with one on theism—Peirce related his purpose to that of pre-Socratic philosophy: "I am going . . . to propound a hypothesis about the constitution of the universe," he began, and then pointed out that the "very first philosophical conception that appeared in early Greece was that of primal matter" (pp. 294, 295). He then defended the pre-Socratic approach to the problem of accounting for the world, which was to "first determine where their account was to begin." That called for a *guess!* "Now every intellectual undertaking must in its inception strike out with an original ejaculation of thought. A guess has always to be made" (p. 296). It may have been these reflections that led Peirce to cast his cosmological speculations as a *guess* at the riddle of the universe. Thus his great project had both a distinct Kantian character—its rigorous architectonic structure with its foundation in the categories—and an equally distinct Greek character—its evolutionary and cosmological approach self-consciously grounded on a guess.

In considering how Peirce reached the mature conception of his cosmological project, a factor not to be left out is the influence of Abbot. When Peirce had written to Abbot in December 1885 and again in early 1886 about *Scientific Theism,* he had said little to indicate that he was impressed with the scope of the book, which is in some respects suggestive of Peirce's own cosmological project. It was a sort of manifesto for a "scientific theism" and introduced the Scientific Method as the new deity: "the head has been too long sacrificed to the heart in religion."[54]

> Science maintains that the universe it knows is actual existence, perish who or what may,—affirms the uttermost reality of its own conquests,—claims to have solved by victorious wit not a few of the Sphinx-riddles propounded to mankind by the *Weltgeist,*—and testifies that it finds the universe intelligible wherever it can bring to bear its unfailing method of research and discovery. It indignantly spurns the sophistry which would explain away its hard-won cosmical truths as the phenomenist's merely subjective "representations"—real while he wakes, potential only while he sleeps.[55]

Abbot asserted that "the great principle of the *Infinite Intelligibility of the Universe* is the corner-stone of Scientific Theism" and that the key to philoso-

[53]Fisch, p. 233.
[54]Francis Ellingwood Abbot, *Scientific Theism* (London: Macmillan, 1885), p. 217.
[55]Ibid., pp. 121–22.

phy and to the explanation of the universe lies in the conception of organic teleological evolution, not in the mechanistic ideas of Spencer and Haeckel.

In his letters and review Peirce appears to have been mainly interested in Abbot's theory of reality, especially as it concerned relations. Initially he opposed Abbot: "I am not only phenomenalist, but also idealist" (p. 280)—two positions Abbot abhorred. But before long he had been converted to Abbot's view, and in his definition of "realism" for the *Century Dictionary* Peirce included a lengthy quotation from *Scientific Theism* as the primary illustration. In his first letter Peirce had indicated that he agreed with Abbot's "universal endocosmic teleology," although he showed little interest in his cosmology. But it is interesting that Abbot mentions "Sphinx-riddles" very near the time when Peirce began to conceive of his projected book as his "Guess at the Riddle" and not long before he decided that it should appear with a vignette of the Sphinx printed below the title. By 1887, or at the latest 1888, Peirce would succinctly express his guess by speculating that "three elements are active in the world: first, chance; second, law; and third, habit-taking," and then add that "Such is our guess of the secret of the sphynx" (EP 1: 277; CP 1.409–10).

But it was not only Abbot who reminded Peirce of cosmological riddles and the Sphinx. During his Johns Hopkins years he suffered two losses—one great and the other at least sobering—which somehow may have primed him for his cosmological turn and which may also shed light on his understanding of the riddle that he supposed he had solved.

Peirce's father, Benjamin, the single greatest influence in his life, had died on 6 October 1880. Just eight months earlier, at the Peabody Institute of Johns Hopkins, he had delivered a series of six lectures on "Ideality in the Physical Sciences,"[56] where he spoke of some ancient tablets, recently discovered in Nineveh, on which was recorded an account of Babylonian cosmogony.

> In the first tablet are placed, side by side, the two primitive sources of creation,—Chaos and Ideality. They stand silent and immovable,—imperturbable meditation and inactive mass,—like the sphinx by the pyramid. There they might have remained eternally unproductive. But the tablet's next record is the birth of Motion. . . . the divine energy of creation.[57]

Later in the lectures Benjamin remarked that "Nature's riddles are man's intellectual nourishment. . . . To shrink from them is cowardice and want of faith." Peirce had been deeply shaken by the loss of his father and had almost immediately become the vehicle for the continuation of Benjamin's mathematical thought. He may well have imagined that to some extent his father's mind could live in him—and perhaps it was true that Benjamin's more metaphysical, more speculative thought found a place in his mind to rest and await a revival. In the coming years, as Peirce delved deeply into the prime-

[56] These lectures were first given at the Lowell Institute in Boston in 1879. They were edited by Jem after Benjamin's death and published in 1881.

[57] This and the immediately following quotation are from Benjamin Peirce's *Ideality in the Physical Sciences* (Boston: Little, Brown, and Co., 1881), pp. 43–44 and 183.

val origins of the universe, he must have known that, were it possible, his father would have smiled on him.

There was another seer in Peirce's life, the old family friend Ralph Waldo Emerson. After Emerson's death in April 1882, Peirce often raised his ghost when expressing profound thoughts, especially the elusive connection between thinking and what is thought, between seeing and what is seen. "Of thine eye I am eyebeam," says Emerson's Sphinx. In *Nature,* which Peirce must have known from his youth, Emerson stated the riddle more precisely—and it might have stayed in the back of Peirce's mind as a motivation for later cosmological speculations:

> The laws of moral nature answer to those of matter as face to face in a glass. "The visible world and the relation of its parts, is the dial plate of the invisible." The axioms of physics translate the laws of ethics. . . . This relation between the mind and matter is not fancied by some poet, but stands in the will of God, and so is free to be known by all men. It appears to men, or it does not appear. When in fortunate hours we ponder this miracle, the wise man doubts if at all other times he is not blind and deaf . . . for the universe becomes transparent, and the light of higher laws than its own shines through it. It is the standing problem which has exercised the wonder and the study of every fine genius since the world began; from the era of the Egyptians and the Brahmins to that of Pythagoras, of Plato, of Bacon, of Leibniz, of Swedenborg. There sits the Sphinx at the roadside, and from age to age, as each prophet comes by, he tries his fortune at reading her riddle.[58]

That is the riddle. For Peirce, at the end of 1886, the universe was becoming transparent and the light of higher laws was shining through—the laws of the laws of nature. An exciting path of thought lay open to him, and he had a clear conception of where it would lead. But the book that would found a new era in philosophy was barely started. It remained for Peirce to work up the details and consequences of his grand hypothesis into a systematic philosophy that would leave no question untouched.

It would be a mistake to suppose that, while Peirce delved into cosmology and system building, he discontinued his work in logic and the foundations of mathematics. He had a remarkable capacity for carrying out concurrent investigations, and his interest in those areas continued almost unabated for the rest of his life. His short paper on the properties of number (item 45), which he typed at Ithaca on 5 January 1886, seems rather more anomalous than most other work of the period in that it does not fit easily into the context of his other studies, and it is not mentioned anywhere in his correspondence. He did participate in the university's intellectual life while he was at Cornell, and he gave at least two lectures to engineers and mathematicians. An announcement in the 3 December 1885 *Cornell Daily Sun,* which introduced Peirce as "the son of one of the most eminent mathematicians of [the] century," encouraged anyone interested in the mathematical intricacies of pendulum operations to attend his lecture the following afternoon: "Professor Peirce's ready command of language, thorough acquaintance with the subject and pleasing delivery will make the occasion profitable

[58]This quotation, and the idea that it may be connected with Peirce's guess, appears in an unpublished manuscript by John Sheriff.

and enjoyable." It is not unlikely that a mathematician at Cornell stimulated Peirce's interest in number theory and that item 45 was prepared for discussion or presentation. On the other hand, after his 1880–81 work on the axioms of arithmetic, Peirce always remained interested in further developing his theory of number and related conceptions of mathematical continua, and item 45 might have been part of this ongoing study.

After Newcomb had declined to publish the continuation of item 30, Peirce did not pursue other routes to publication—and seven years passed before he would again publish on symbolic logic. But his 1886 work in logic (items 54–56) shows that he continued to write with publication in mind. These items, all incomplete, appear to be continuations of his earlier work on a general logic book (see W4: items 30, 31, and 61) that would eventually turn into "How to Reason" (more commonly known as the "Grand Logic"). Items 55 and 56 may be more directly related to the continuation of item 30, although the set of papers that Peirce had planned for the *American Journal of Mathematics* was probably only a reconception of the plan for his book. On the other hand, by mid-1886 Peirce was beginning to wonder how he might supplement his income, and his 1886 work may have been part of a plan he was concocting to make logic pay.

Together, these items consolidate many important logical ideas from the years immediately past, and they anticipate some key ideas that Peirce would soon develop. This is especially true of item 54 which, although to some extent elementary, contains valuable discussions of modality and possible worlds, the importance of observation and the limitations of syllogistic reasoning for mathematics, the importance of temporality for logic, and some interesting spatial conceptions suggestive of his later Existential Graphs. Peirce points out explicitly (p. 331) that part of the business of logic is to teach useful ways of constructing diagrams, and he claims that "the ordinary business of life is best conducted without too much self-criticism" (p. 327) and that ordinary day-to-day reasonings are "better performed unconsciously than they would be if we were to try to interfere with them by a captious and hypochondriac logic" (p. 328). In some respects item 54 seems to be an expansion and development of the 1880 paper on the algebra of logic, while he might have intended to further develop the 1885 paper in a second book on quantitative logic. Item 56 expands somewhat on Peirce's revised theory of signs as set out in the 1885 paper.

Another endeavor that extended throughout the period of this volume was Peirce's lexicographical work for the *Century Dictionary.* He had been recruited in 1882 by Benjamin Eli Smith, a Johns Hopkins graduate assistant who soon became managing editor for the *Century,* and by 1883 Peirce had begun to write definitions. He was responsible for several subject areas and contributed over five thousand definitions (see W4: lvi). Edited by the great American linguist William Dwight Whitney, the *Century Dictionary* still stands as America's greatest single contribution to lexicography. Peirce worked diligently on the dictionary project until he left Johns Hopkins, but it is difficult to determine exactly how much he produced during the following two years. There seems to have been a hiatus in his dictionary work after his departure from Baltimore, although by 1886 he was again hard at work on his definitions.

The method employed in the preparation of the *Century Dictionary* was to distribute to its contributors relevant pages from the *Imperial Dictionary,* for which the Century Company held rights, to serve as a basis for the new work, and then to supply them with selections of quotations using new or difficult words. Item 57 illustrates how (at a fairly early stage) Peirce carried out his work for some words in "e"; he would continue in this vein for the rest of the alphabet, until the first edition appeared in 1889–91. But even then Peirce's dictionary work was not finished, for he set to work at once rewriting definitions for a supplement that appeared in 1909. (A fuller account of Peirce's lexicographical work will be given in W7, which will include a substantial selection of his published definitions.)

It is perhaps a fitting sign of Peirce's mind that at the close of 1886 he paused from his logical and cosmological speculations, and from his lexicographical work, to point out to his former student, Allan Marquand, the key to moving from one age of computing to the next. With his simple circuit diagrams (item 58) Peirce provided the clue that might have opened the way to modern electrical computing. But even though Marquand followed Peirce's advice and had elaborate wiring diagrams drawn up,[59] and although reference to these diagrams was made in the article on logical machines in Baldwin's *Dictionary,* it was to no avail. Peirce probably should have pursued his idea, but he was not really very interested in computing, for he did not conceive of computers capable of effective inductive reasoning—especially weak inductive reasoning—which he believed to be the foundation of human intelligence.

Looking back, we see that Peirce's stint at Johns Hopkins had been a time of great originality and remarkable accomplishments (recounted in the introduction to W4), accomplishments due in part to stimulation from brilliant colleagues and students. Yet in the short period of the present volume we see an even greater concentration of brilliance. There may not be a richer three-year stretch in Peirce's life, nor one that gave rise to so many critical turning points. It is true that many of his most remarkable advances were continuations of work begun earlier; for example, his 1885 contributions to logic were direct outgrowths of a creative surge that had begun as early as 1879 and should be regarded as fruits of his Johns Hopkins experience. But we cannot say the same for his philosophy, where his creative surge began with the "Design and Chance" lecture of January 1884. Although there were many influences that led to that lecture, the ideas expressed there stand out as a starting point for a new line of thought.

It is hard to tell what really set Peirce going in a new direction—what actually *moved* him. Perhaps the shock of his dismissal from Johns Hopkins (and thus from university life) threw him into the state of disequilibrium—a state soon intensified by his troubles at the Survey—that triggered his cre-

[59]See Kenneth Laine Ketner, "The Early History of Computer Design," *Princeton University Library Chronicle* 45 (1984): 187–224.

ative energy. The finality of his dismissal and the shocking discontinuity it forced on the course of his life must have brought a kind of freedom, a time when he could look forward without looking back. At such times, fresh ideas, either new or drawn from some reservoir of the past, are likely to have unusual impact—especially when they are in conflict with a present course of thought. So in early 1884, at the most pronounced moment of disruption and uncertainty in Peirce's life, he surrendered to his long-held attraction for the idea of the efficacy of chance. He opened his lecture on design and chance by remarking on a new element in intellectual history: the tendency to question the exact truth of axioms. By the summer of 1886 he would begin his book on the categories with the abrupt assertion: "This is the day for doubting axioms" (p. 292). In Peirce's own words we have a good summation of where he stood at the time.

To assess how the work in this volume contributes to the overall development of Peirce's thought—a task far too complex to be fully addressed here—it will be helpful to follow the guidance of Fisch and Murphey. The measure Fisch uses to gauge Peirce's general intellectual development is how far he had progressed from his early nominalism (some say his early weak realism) toward his ever more encompassing realism.[60] In Fisch's broadest characterization Peirce can be classified as a one-, two-, or three-category realist, depending on whether he acknowledged the reality of Thirdness, of Thirdness and Secondness, or of all three categories. Peirce did not accept the reality of actuality, or Secondness, until about 1890, and it was seven years later, in 1897, when he finally accepted the reality of possibility, or Firstness—and only then became a three-category realist. Thus, during the period of the present volume, Peirce was still a one-category realist, accepting only the reality of Thirdness. However, his intellectual progress did not occur in two or three great leaps but in a series of many steps. Some of the most significant developments leading to his acceptance of the reality of Secondness are directly related to the work of this period. According to Fisch, these include

> his work on the logic of relations and on truth-tables, indices, and quantification; the resulting reformulation of his categories; his work and that of Cantor and Dedekind on transfinite numbers; the appearance in 1885 of provocative books by Royce and Abbot; and . . . a fresh review of the history of philosophy for purposes of defining philosophical terms for the *Century Dictionary*.[61]

Murphey divides Peirce's intellectual development into four periods or systems, each characterized by a distinctive approach to the categories. Peirce's growth from his earliest to his latest system of thought was the result of successive discoveries in logic, each requiring revisions to the categories because of Peirce's architectonic approach to philosophy. Murphey and Fisch agree that it is how the categories stand in relation to Peirce's theory of reality that best measures his development.

[60]See Fisch, ch. 10.
[61]Fisch, p. 189.

When the period covered by the present volume began, Peirce was just entering the final and longest phase of his intellectual life, according to Murphey's account. The logical discoveries that led Peirce to this final phase were his discoveries of quantification and set theory. In particular it was Peirce's discovery of the index, following Mitchell, that led him to understand the importance of individuality and of reference to the individual.[62] Fisch also emphasizes the importance of this discovery and points out that it forced Peirce to revise both his theory of signs and his theory of categories. It was at this time that Peirce began to stress that "the actual world cannot be distinguished from a world of imagination by any description. Hence the need of pronouns and indices" (p. 164). Toward the end of 1885, in his review of T. K. Abbott's translation of Kant's *Logic* (item 40), Peirce remarked that Kant's statement that no general description of existence is possible "is perhaps the most valuable proposition that the *Critic* contains."

According to Murphey, Peirce's new understanding of the fundamental importance of reference to the individual led him to abandon his definition of reality as "that which is thought in the final opinion to which inquiry will lead," which equates the real with the end of a series of cognitions.[63] That theory of reality was a *constitutive* doctrine. Murphey says that sometime between 1880 and 1890 Peirce abandoned the constitutive principle for a weaker *regulative* principle, which held only that "in order to make certain that agreement will be pursued it is necessary to hope that ultimate agreement will come." Peirce's 1885 study of Royce may have played the essential role in leading him to this revision. There Peirce claimed that a skeptic (like himself) can fruitfully embrace God's omniscience "as a regulative but not a speculative conception" (p. 229). Christopher Hookway also points to that review as giving an early account of Peirce's "moderate fallibilism."[64]

Other important doctrines and themes in Peirce's later thought seem also to be prefigured, if not directly grounded, in the writings of this volume. For example, in the Royce review just cited, Peirce briefly discusses his theory of the existence of God, which he says he hoped soon to get into print: "I think that the existence of God, as well as we can conceive of it, consists in this, that a tendency toward ends is so necessary a constituent of the universe that the mere action of chance upon innumerable atoms has an inevitable teleological result." We can see here the interplay of Peirce's theology with his methodology and cosmology and that his conception of chance had begun to spread throughout his thought. Earlier in this period, in April 1884, he had asked to teach a fall course at Johns Hopkins on the logic of religion, and in May he had delivered a paper on that subject to the Metaphysical Club, where he discussed proofs of the existence of God (see W4: lxvi). It was noted above that the final chapter of "One, Two, Three," Peirce's projected treatise on evolutionary cosmology, was entitled "Theism."

[62]Indices do appear in Peirce's earlier work, especially in his "New List" (W2: item 4), but not as signs that refer directly to individuals. See Murphey, pp. 299–300.

[63]Murphey, p. 301.

[64]Christopher Hookway, *Peirce* (London: Routledge & Kegan Paul, 1985), p. 73.

There are other "turning points," not mentioned above, that belong to this time. For example, Peirce's reading of Kempe's 1886 "Memoir on the Theory of Mathematical Form" (cf. MS 583) was an important stimulus in turning his thoughts to logical diagrams and the development of his Existential Graphs.[65] And there are indications that his study of Abbot's *Scientific Theism* may have led him to investigate how relations are represented in thought and to reflect deeply on the importance of diagrams for understanding thinking as a process (see pp. 287–88). Also in the Abbot review there is an early statement of Peirce's vortex solution to the mind-body problem.

But while these years represent a new beginning in the development of Peirce's philosophy, they effectively mark the end of his life of science. It is true that he spent the next few years working up scientific results for publication and he sometimes attempted to revive the goodwill he once had in the Survey (until his forced resignation on 31 December 1891), but he was never again given the chance to work in the field as a professional geodesist. In 1899 he tried to reenter the world of professional science as Inspector of Standards for the Office of Weights and Measures, but he was foiled again by Newcomb, his old nemesis.[66] Occasionally Peirce would resume old investigations such as his study of color—he began some color experiments in June of 1886 which continued until June of 1887—but most later work in experimental science was sporadic and connected with some scheme or other in his (never successful) quest for prosperity. It is ironic that Peirce's geodetic work continued to contribute and even to bring great credit to the Survey, though without acknowledgment or benefit to him.[67]

For his life in general, these years were a time of dislocation and uncertainty—a mid-life crisis of massive proportions and in the most literal sense. This is reflected early in the period in a 1 May 1884 letter from his mother: "I am longing to hear of your cologne water, your lectures, your Actuaryship & whatever other schemes you may have thought of & trust they will not all die out 'like the baseless fabric of a vision.' " (Peirce had concocted a cologne water that he hoped to market.) Was his mother paraphrasing her late husband, Charles's father: "how can we be sure that our intellectual picture of the external world is not a human creation, and the fabric of a vision?"[68] She wrote again the following month, on 6 June:

> I have received to-day the little bottle of Cologne you promised me for my pocket! Now I must enlarge my pockets or your intention cannot be carried out to the letter! . . . soon I suppose you will be leaving Baltimore for the summer. Oh! my dear Charley—how much I feel for your discouragements, & troubles—& how I wish I could in any way help you! At such times how much we all miss your dear Father—always so ready with advice of the best kind, & any possible help for you all! I hope you will not resign from the CS. until you are *sure* of something better.

[65]See Don D. Roberts, *The Existential Graphs of Charles S. Peirce* (The Hague: Mouton, 1973), pp. 20ff.

[66]See Brent, pp. 152 and 266–67.

[67]Ibid., p. 198.

[68]B. Peirce (1881), p. 23.

As the years progressed, Peirce's attention shifted, often abruptly and erratically, between science and philosophy. When 1886 came to an end, his mind must have been a swirl of ideas about logic and categories and evolutionary cosmology—and about the various scientific reports he was working hard to finish. But prominent in his consciousness was the realization that somehow he had to make a living. For the life he wanted—the life he had promised Juliette—he needed more money than he could make at the Survey, and even that source of income was tenuous. Perhaps Juliette could go on stage—it was said that she had great talent. In 1886 she began to study acting with Steele MacKaye, New York playwright and theater manager, and Peirce toyed with the idea that she might become a great success: "then a difficult question will arise for me between my duty to Humanity in the abstract, and my duty to this Lady in the concrete."[69] But Peirce knew that this was not the solution to his financial problems. Was there not a way to make a living from what he knew best: logic? He finished the period making plans. On 4 January 1887 he wrote to his cousin, Henry Cabot Lodge, asking for a short-term loan to fund a new venture:

> I have quite a reputation for my knowledge of the logic and methods of science. I have worked out a long series of practical exercises to teach the whole art of reasoning from beginning to end. There are throughout the country thousands of young men and women to whom these lessons would be of more real service than almost anything they could study. The question is, first, how many of them I could teach. Now I have planned a system which I won't trouble you with, with passages written out answering every conceivable difficulty in the whole course, type-writers, and assistants (upon whom I can lay my hands when I need them) by which I can write say 500 letters a day, or take charge of 1500 students. I propose to charge $30 in advance for 30 lessons, the entire course being about 200. . . . I want to begin by sending out a hundred thousand [circulars] in order to ascertain what number of circulars has to be sent to gain one scholar in the long run. I guess about a thousand. . . . This scheme, or some modification of it which I will find, must pay.

Peirce did not get the loan, but his circular was already written and would soon appear in *The Century Magazine* and other popular publications. If he could not live his life teaching logic at a university, he would make his living teaching logic in some other way.[70] So he thought.

NATHAN HOUSER

[69] Peirce to E. S. Holden, 20 August 1886.

[70] Selections from Peirce's correspondence course in logic will be included in the next volume of the present edition.

Determinations of Gravity at Allegheny, Ebensburgh, and York, Pa., in 1879 and 1880

Item 1 ***P 290:*** **Coast Survey Report 1883, *473–87***

I.—GRAVITY AT THE ALLEGHENY OBSERVATORY.

The Allegheny Observatory is situated in—

Latitude 40° 27′ 41″.6 north,

Longitude $5^{h}\ 20^{m}\ 2^{s}.93$ west of Greenwich.

It stands 1,140 feet (=348 metres) above the mean sea-level.[1] From a few yards in front of the observatory the descent is very sharp into the valley of the Ohio, and as this has been formed by erosion, it must be supposed to diminish the acceleration of gravity, perhaps by the one hundred thousandth part. Unfortunately the necessary calculation, which a topographical sketch would enable us to perform at once, remains impossible for the present.

The operations were conducted nearly as described in my "Measurements of Gravity at Initial Stations." The Repsold reversible pendulum was oscillated *in vacuo* on the Geneva support, in the cellar of the observatory, the feet of the support resting on iron bars laid upon other bars let into the great pier of the equatorial at one end and into a stone wall at the other.

Measures of the length of the pendulum were commenced 1879, January 2; but owing to the difficulty of maintaining a tolerably constant temperature in any part of the observatory that was otherwise suited for a comparing-room, no valuable results were obtained before January 18; and even after that date, it was found necessary to reject the work of several days, owing to bad conditions. The first

1. The latitude and longitude here given have been extracted from the *American Ephemeris.* The elevation is from data furnished to Professor Langley by the Allegheny City surveyor and by the engineer of the Pennsylvania Railway.

series of measures of length was completed February 1. Four swingings of the pendulum were made on February 6 and 7 with heavy end up, and two swingings on February 8 and 9 with heavy end down. On February 10, the position of the centre of mass was determined and the knives were interchanged. Two days were then lost in trying to make the vacuum chamber stanch; after which two swingings were made with heavy end down, February 13 and 14, and four with heavy end up February 15, 16, and 17. On February 18 and 20, the flexure of the apparatus was measured, and these measures were supplemented by others on March 4. From February 22 to March 2, the pendulum was measured. The thermometers were compared from 1878, December 19 to 31, and again 1879, March 3.

The following table gives a synopsis of the results of the swingings, the period being corrected for the rate of the clock and for arc of oscillation, and being reduced to 15° C and to a pressure of one million absolute C. G. S. units. The approximate pressure in millimetres of mercury and the approximate temperature centigrade are also shown. It is unnecessary to say that the air-pump was never brought into action during any swinging.

The agreement of the resulting periods is, as far as it goes, favorable to the plan of swinging *in vacuo*. It will be noticed that the oscillations were continued down to a small amplitude, but there seems to have been no increased error upon this account. Following the synopsis will be found a table of the errors of the partial swingings formed by intermediate transits, as shown on pages 502–503.

HEAVY END UP. KNIFE No. 2.

Date.	Temperature.		Pressure.		Half arc in terms of radius.		Number of oscillations.	Corrected period.
	Maximum.	Minimum.	Beginning.	End.	Beginning.	End.		
1879.	°	°	*mm.*	*mm.*				*s.*
February 6	0.3	0.3	23	25	.023	.003	20,891	1.0066466
6	0.8	0.4	29	36	.030	.003	21,406	1.0066428
7	0.5	0.3	43	46	.030	.002	21,420	1.0066399
7	0.7	0.4	20	20	.034	.005	19,742	1.0066430
							83,459	1.0066431

HEAVY END DOWN. KNIFE No. 1.

Date.	Maximum.	Minimum.	Beginning.	End.	Beginning.	End.	Number of oscillations.	Corrected period.
February 8	0.7	0.1	13	14	.033	.002	74,805	1.0064533
9	0.3	−0.1	14	15	.035	.002	74,680	1.0064515
							150,485	1.0064524

HEAVY END DOWN. KNIFE No. 2.

February 13 . . .	1.5	−0.5	17	40	.033	.002	61,844	1.0064471
14 . . .	−0.3	−1.3	18	40	.035	.002	67,626	1.0064470
							129,470	1.0064470

HEAVY END UP. KNIFE No. 1.

February 15 . . .	−0.6	−1.1	17	29	.034	.004	19,822	1.0066370
16 . . .	−1.0	−1.2	17	35	.034	.004	20,766	1.0066337
16 . . .	−0.9	−1.1	15	35	.034	.003	22,588	1.0066380
17 . . .	−0.7	−0.9	21	37	.036	.003	20,848	1.0066411
							84,024	1.0066375

Errors of partial and total swingings.

Heavy end up.						Heavy end down.					
Knife No. 2.			Knife No. 1.			Knife No. 2.			Knife No. 1.		
Partial swingings.			Partial swingings.			Partial swingings.			Partial swingings.		
Error in 7th place.	Sq. root. No. oscill.	Product in 5th place.	Error in 7th place.	Sq. root. No. oscill.	Product in 5th place.	Error in 7th place.	Sq. root. No. oscill.	Product in 5th place.	Error in 7th place.	Sq. root. No. oscill.	Product in 5th place.
+43	57	25	+ 6	70	4	− 4	77	3	+80	78	63
+ 2	77	2	− 1	87	1	+ 48	83	40	+19	178	34
+24	83	20	− 3	86	3	+ 29	87	25	−27	94	25
+74	69	51				−180	31	56	− 9	88	8
			−66	85	56	+ 43	73	31	−35	87	30
+ 3	94	3	−13	79	10	+ 10	198	20			
−22	79	17	−18	85	15	− 61	93	57	−39	82	32
− 3	80	2							+38	88	33
			−15	82	12	+ 1	78	1	+53	83	44
−66	63	42	+28	92	26	+ 35	84	29	− 6	192	11
+30	59	18	+13	83	11	− 12	88	11	−23	95	22
−76	82	62				− 9	81	7	Mean of products.		29
−16	85	15	−28	81	23	− 11	178	20			
			+49	85	41	− 43	99	43			
+36	87	31	+99	83	82	− 19	82	16			
− 7	87	6	Mean of products.		26	Mean of products.		26			
−58	73	42									
Mean of products.		24									
Whole swingings.			Whole swingings.			Whole swingings.			Whole swingings.		
+32	145	46	00	141	00	+ 6	273	16	+ 8	249	20
− 6	146	8	−33	144	48	− 12	275	33	+ 7	260	18
−35	146	51	+10	150	15	Mean of products.		24	Mean of products.		19
− 4	140	6	+41	144	59						
Mean of products.		28	Mean of products.		30						

The errors given are differences from the following periods, deduced from the final results:

$$T_d \text{ (knife 1)} = 1^{s}.0064527 \qquad T_u \text{ (knife 2)} = 1^{s}.0066434$$
$$T_d \text{ (knife 2)} = 1.0064463 \qquad T_u \text{ (knife 1)} = 1.0066370$$

The errors are multiplied by the square roots of the number of oscillations, and the products are shown to be constant in the mean. It is also noticeable that this constant has the same value whichever end is up. Several obvious inferences might be made. In particular, it will be seen that the error of the result depends only on the total number of oscillations, no matter how they may be separated by intervals of rest.

Time was observed by Mr. F. W. Very, Professor Langley's assistant, with the instruments of the observatory, a fine 8-inch transit and the sidereal clock (Frodsham 1358). The chronometer, Negus 1589, was used for the pendulum observations; and this chronometer as well as two others (Hutton 202 and Bond 380) were compared upon the chronograph with the clock three times a day, between 3 and 4 o'clock in the afternoon and between 9 and 10 morning and evening.

The corrections to the chronometer used were obtained by assuming that between certain dates certain timepieces moved with absolute uniformity, the changes of rate being supposed to be sudden. This is the same method of reduction used in my previous work, and appears to me most consonant with observed facts in regard to the running of timepieces. The standards used were as follows:

Date.	Sidereal time.	Timepiece assumed uniform from each time to next.
	h. m.	
February 4..........	6 18	Frodsham, 1358.
6..........	5 25	Do.
9..........	6 47	Do.
13..........	7 14	Hutton, 202.
15..........	8 02	Frodsham, 1358.
21..........	7 12	Do.

The results of the comparisons of the length of the pendulum with the pendulum metre were as follows:

MEASURES OF LENGTH.

FIRST SERIES.

Date. 1879.	Pend. − standard. μ
January 18	+26.1
January 21	+24.6
January 22	+26.4
January 23	+20.3
Mean	+24.3

SECOND SERIES.

January 25	+22.8
January 29	+25.5
January 31	+23.2
February 1	+18.6
Mean	+22.5

THIRD SERIES.

February 22	+11.3
February 23	+10.2
February 24	+ 9.9
February 25	+ 9.1
February 26	+12.1
March 1	+15.0
March 2	+11.6
Mean	+11.3

These results have to be diminished by $200^{\mu}.4$, because they are referred to the mean of the three lines $999^{\text{mm}}.7$, $999^{\text{mm}}.8$, $999^{\text{mm}}.9$ of the standard instead of to the metre. They have then to be increased by $261^{\mu}.1$ in order to be referred to the metre adopted in my "Measurements of Gravity at Initial Stations." It follows that the length of the pendulum in terms of the metre adopted in my previous work (which is now known to be erroneous, but which is for the present adhered to, in order to avoid confusion) was

	m.
Before the interchange of knives	1.0000853
After the interchange of knives	1.0000732

The difference of the distances of the centre of mass from the two knife-edges was found to be $0^{\mathrm{m}}.39303$, to which the correction, +.00014, has to be applied.[2]

The experiments to determine the flexure of the support have already been published in the *Coast Survey Report* for 1881, pp. 375–377. The mean of the measurements of two observers shows that the flexure at the middle of the knife-edge, under a horizontal force equal to the weight of the pendulum, was $38^{\mu}.8$.

We now proceed to calculate [T^2 Rev.] and [T^2 Inv.], as in the paper above referred to. Only, it is to be remarked that, in consequence of what is said on page 72 of that paper (page 271 of the *Coast Survey Report* for 1876), one-seventh of the viscosity effect has to be subtracted in order to eliminate the effect of the bells; that is to say, T_d has to be diminished by 66×10^{-7} and T_u by 151×10^{-7}. The values have to be separately calculated for the experiments made before and after the interchange of the knives.

Before the interchange of knives.

	s.		*s.*
T_d	1.0064524	T_u	1.0066431
Bells and cylinder . .	−145	Bells and cylinder . .	−321
	1.0064379		1.0066110
T_d^2	1.0129172	T_u^2	1.0132657
Flexure	−270	Flexure	−118
Stretching	. . . .	. . . .	+ 10
Corrected T_d^2 . . .	1.0128902	Corrected T_u^2 . . .	1.0132549

After the interchange of knives.

T_d	1.0064470	T_u	1.0066375
Bells and cylinder . .	−145	Bells and cylinder . .	−321
	1.0064325		1.0066054
T_d^2	1.0129064	T_u^2	1.0032545
Flexure	−270	Flexure	−118
Stretching	. . . .	. . . .	+10
Corrected T_d^2 . . .	1.0128794	Corrected T_u^2 . . .	1.0132437

2. See "Measurements of Gravity at Initial Stations," p. 114 (*Coast Survey Report* for 1876, p. 313), where the correction is, however, applied with the wrong sign.

	Before interchange.	After interchange.
	s.	*s.*
Corrected T_d^2	1.0128902	1.0128794
Corrected T_u^2	1.0132549	1.0132437
$\frac{1}{2}(T_d^2 + T_u^2)$	1.0130725	1.0130615
$\frac{1}{2}(T_d^2 - T_u^2)$	−1824	−1822
$\frac{h_d - h_u}{h_d + h_u} \frac{1}{2}(T_d^2 - T_u^2)$	−717	−716
$\frac{h_d + h_u}{h_d - h_u} \frac{1}{2}(T_d^2 - T_u^2)$	−4638	−4633
[T^2 Inv.]	1.0130009	1.0129899
[T^2 Rev.]	1.0126087	1.0125982
[T^2 Inv.] − [T^2 Rev.]	3922	3917

The two values of [T^2 Rev.] combined with the two values of the length, give for the seconds pendulum at Allegheny:

	m.
Before the interchange of knives	0.9930479
After the interchange of knives	0.9930461
Mean	0.9930470

This is the final result from this station alone. But the correction for the erroneous length of the metre, as provisionally stated in the *Coast Survey Report* for 1881, page 463, is -162×10^{-7}, giving

m.
0.9930308;

and this may further be modified by the effect of measurements at other stations, and comparisons of [T^2 Inv.]. There is, however, reason to believe that such modification would be, in this case, insignificant.

Applying the correction for elevation, without continental attraction, diminished by one-tenth part, and the correction for latitude, as in my paper (*C. S. Report,* 1881, p. 445), we have

	m.
Seconds pendulum at Allegheny	0.9930308
Elevation	+979
Latitude	−21903
Reduced to equator and sea-level	0.9909384

This would be increased if the effect of the valley were taken into account. A topographical sketch of this vicinity is the most pressing need of the work at this time.

The details of the work at the Allegheny Observatory are given in the tables appended to the edition of this Appendix, which has been published separately.

II.—DETERMINATION OF GRAVITY AT EBENSBURGH.

Ebensburgh is the chief (though not the principal) town of Cambria County, Pennsylvania, in the Allegheny Mountains. The observations were made in the house and grounds of Mrs. Frances S. McDonald, on Centre Street. The place is shown on the county map by Beers (1867), where the house has marked under it "J. M. McDonald." It is at the southeast corner of the street next south from Highland Street. The transit pier is $23\frac{1}{2}$ metres south of the northern boundary and $28\frac{1}{3}$ metres east of the western boundary of the lot. The pendulum was observed in the cellar of the house.

The latitude of the station, $+40° 27'$, was determined by Mr. Marcus Baker by sextant observations upon the Sun, Jupiter, and Polaris. The longitude was determined by telegraphic exchanges with the Allegheny Observatory, the observers being Mr. F. W. Very and Mr. H. Farquhar, with the result:

	h.	*m.*	*s.*
Ebensburgh east of Allegheny,	0	5	9.2
Ebensburgh west of Greenwich,	5	14	53.7

The elevation of the station has been ascertained from that of the railway at the station, as communicated by the engineer of the Pennsylvania Railway. The pendulum station was connected with the railway by a line of levels. The elevation so found is 2,137 feet (=651 metres).

It was intended to conduct the operations as at Allegheny; but various difficulties compelled me to support the pendulum on the Repsold tripod, as at my European stations. The brass foot-rests were placed directly upon the hard clay floor of the cellar. The old knives which had been used in Europe and in the stations at Hoboken and at Allegheny were replaced by new ones, made by Messrs. Darling, Brown, and Sharpe, of Providence. The amplitude of oscillation was measured on a fine arc by Messrs. Stackpole & Brothers, which is divided into thousandths of the radius. The arc and transits were observed with a reading telescope carrying an objective corrected for use at a short distance by Byrne, of New York. The same eye-piece was constantly used. The telescope was placed at a distance of two metres from the pendulum; and no screen was interposed between them.

The general order of the pendulum experiments was as follows:

1879.		
August	14–21.—	Measurements of length.
September	5.—	Swinging, heavy end down; knife, 3-4.
		Swinging, heavy end up; knife, 7-8.
September	6.—	Swinging, heavy end up; knife, 7-8.
		Swinging, heavy end down; knife, 3-4.
		Centre of mass determined.
		Interchange of knives.
		Centre of mass determined.
September	7.—	Swinging, heavy end down; knife, 7-8.
		Swinging, heavy end up; knife, 3-4.
September	8.—	Swinging, heavy end up; knife, 3-4.
		Swinging, heavy end down; knife, 7-8.
September	10–13.—	Measurements of length.
September	14.—	Swinging, heavy end down; knife, 7-8.
		Swinging, heavy end up; knife, 3-4.
September	15.—	Swinging, heavy end up; knife, 3-4.
		Swinging, heavy end down; knife, 7-8.
September	16.—	Determination of centre of mass.
		Interchange of knives.
		Determination of centre of mass.
		Swinging, heavy end down; knife, 3-4.
		Swinging, heavy end up; knife, 7-8.
September	17.—	Swinging, heavy end up; knife, 7-8.
		Swinging, heavy end down; knife, 3-4.
September	18–25.—	Measurements of length.

A synopsis of the periods of oscillation at Ebensburgh is given below. These periods have received not only the reductions for arc, rate, temperature, and pressure, but also peculiar *à priori* corrections for flexure of the support, difference of knives, and injury to the pendulum. These I proceed to explain:

After half the swingings had been made, the pendulum was measured. In adjusting the microscopes a plumb-line was used; and to attach this it was necessary to remove the two forward nuts which bind the head of the support to the legs of the tripod. These were afterward replaced for the rest of the swingings, but instead of being tightened by a wrench they were only tightened by hand. This negligence was only discovered after all the swingings were completed, and it was then too late to repeat them. Elaborate experiments (see *Coast Survey Report* for 1881, Appendix 14) were accordingly instituted to determine the flexure of the support when the nuts in question were hand-tightened and when they were wrenched. The values given on page 388 of the *Report* have been used in the reductions, and the periods have accordingly received the following corrections:

	Heavy end down.	Heavy end up.
First four days	−.0000832	−.0000362
Last four days	−.0000895	−.0000390

The knives used at Ebensburgh and York, which are marked 3-4 and 7-8, have, at my request, been micrometrically examined by Assistant Edwin Smith, to determine the distance of the edges from the plane of the bearings. He obtained the following results:

Knife 3-4. At end marked 3, 122^{μ}. At end marked 4, 125^{μ}.
Knife 7-8. At end marked 7, 168^{μ}. At end marked 8, 170^{μ}.

On September 11 the record notes that a small spring belonging to the attachment of the knife at the *light* end of the pendulum was found to be broken. In consequence of this the pendulum must have lost mass, and the centre of mass should have been removed toward the heavy end. In examining the measures of the position of the centre of mass, we find that at York, the station occupied after Ebensburgh, the centre of mass was distant $0^{m}.30333$ from the knife-edge at the heavy end. In fact, using an empirical correction for the relative position of the knives, the individual results (16 in number) show a probable error of $\pm.000013$. At Ebensburgh, measures were made on September 6 and September 16. The four individual measures on September 16, with the correction for position of knives, give for h_u

m.
0.30330
0.30332
0.30330
0.30339

Rejecting the last observation, in which there seems to have been an erroneous reading, the others give $0^{m}.30331$, not differing sensibly from the value at York. The measures of the 6^{th} give

m.
0.30324
0.30330
0.30327
0.30328

These show a value sensibly smaller than that of the 16$^{\text{th}}$. The difference is such as would be produced by the loss of something less than a gramme at the *heavy* end. The distance between the knife-edges not having changed, no other changes can affect the result from the pendulum—considered as reversible—although the accident, whatever it was, must spoil the agreement of the different days. Although it does not affect the final result, I have, in the calculation, supposed that a gramme was lost at the *heavy* end, 2 centimetres beyond the knife-edge. The result of placing a small mass, m, on the pendulum at a distance of x metres and $l+x$ metres from the two knife-edges is easily found to be to increase the periods of oscillation by

$$\Delta T_d = T_d \frac{m}{M} \frac{x\,(l+x)}{2\,h_d\,l}$$

$$\Delta T_u = T_u \frac{m}{M} \frac{x\,(l+x)}{2\,h_u\,l}$$

where M is the mass of the reversible pendulum, l the distance between the edges, h_d and h_u the distances of the centre of mass from the two edges, and T_d and T_u the periods. In the present case we have $m = -1$, $M = 6308$, $x = +.02$, $l = 1$, $h_d = 0.7$, $h_u = 0.3$, $T_d = T_u = 1$. We have, therefore,

$$\Delta T_d = -.0000023$$

$$\Delta T_u = -.0000054$$

and *these corrections have been applied to the first four days,* so as to reduce the pendulum to its state at the end of the work at this station.

Synopsis of periods of oscillation.

	HEAVY END DOWN.	HEAVY END UP.
	Knife 7-8.	Knife 3-4.
1879.	*s.*	*s.*
September 5	1.0064424	1.0065264
September 6	1.0064377	1.0065054
	Knife 3-4.	Knife 7-8.
September 7	1.0064482	1.0065122
September 8	1.0064400	1.0064296

	Knife 3-4.	Knife 7-8.
September 14	1.0064377	1.0065024
September 15	1.0064389	1.0064789
	Knife 7-8.	Knife 3-4.
September 16	1.0064401	1.0065157
September 17	1.0064385	1.0064895

The period for September 8, with heavy end up, is obviously affected by an abnormal error. The Paris, Berlin, Kew, Hoboken observations show that the probable error of a period from a single swinging with heavy end up is $\pm 0^{s}.000006$. The period for September 8 differs from the mean of the others by $0^{s}.000077$, having thus an error about thirteen times the probable error, an event which would occur by chance only once in a million × million × million times. We may, therefore, safely say that on that day there was some extraordinary force tending to restore the pendulum to the vertical. The records of observations of arc show the following times of decrement on different days:

	From .0400 to .0180.	From .0180 to .0080.
	m.	*m.*
September 5	20.9	28.6
September 6	20.7	28.8
September 7	21.1	28.4
September 8	17.1	21.3
September 14	21.3	28.6
September 15	17.2	26.8
September 16	21.1	28.8
September 17	19.7	27.0
Mean 5, 6, 7, 14, 16	21.0	28.3

It thus appears that on the 8th there was some extraordinary force tending to bring the pendulum to rest. These facts suggest that a spider's line might on that day have connected the pendulum with the stand, and this supposition is somewhat strengthened by finding that on that day the operations commenced with oscillating the pendulum with heavy end up in the position in which it had been left the night before. On the 15th and 17th, also, the arc descended rapidly, the periods are very short, and the pendulum had been left over night with the heavy end up ready for the oscillations which were begun in this position in the morning. If there were spider-lines on these mornings, we should expect the disturbing influence to decrease as the arc descended. Whether this is so in regard to the effect on the decrement on the 8th it is difficult to say, but it certainly is so on the 15th and 17th. Transits were observed shortly after the arcs reached .0400, .0180, and .0080, so that there are two intervals from which periods can be deduced. These periods, corrected as in the synopsis, are

	HEAVY END UP.	
	First interval.	Second interval.
	s.	*s.*
September 8	1.0064130	1.0064385
September 15	1.0064423	1.0064931
September 17	1.0064683	1.0065020

These numbers certainly confirm the hypothesis of spider-lines; and I shall consequently entirely reject the work with heavy end up on September 8 and the first intervals on September 15 and 17. With these rejections the mean periods for pairs of days in which the circumstances were the same, except the time of beginning (for on alternate days the position of the pendulum at the first swinging alternated), are as follows:

	Heavy end down.	Heavy end up.
	s.	*s.*
	1.0064400	1.0065159
	1.0064441	1.0065122
	1.0064383	1.0064978
	1.0064393	1.0065088
Means,	1.0064404	1.0065087

The time observations at Ebensburgh were made with transit No. 5 carrying a reticule divided on glass by Prof. W. A. Rogers. The equatorial intervals of the five middle wires are sensibly equal to $2^{s}.583$. The pivot inequality was determined by Mr. Marcus Baker to be $+0^{s}.030$ with illumination west. Both lamps were in place during the whole of the observations, which were made by Mr. Henry Farquhar. The reductions were made by least squares, using Mr. Schott's weights of 1872. Separate azimuths were assumed for the two positions. The chronograph was a fillet-reed instrument, by Breguet. The battery consisted of two sulphate of copper gravity cells.

Chronometer Negus 1589 was always used for the star and pendulum observations, as this was undoubtedly our best chronometer. Chronometers Frodsham 2490, Hutton 202, and Bond 380, were compared with Negus twice daily. The two former break every second omitting the 0; the two latter break every even second, and also at 59^{s}. Frodsham and Bond were wound at 8.30 a.m.; Negus and

Hutton at 8.30 p.m. at first, afterward at 9 p.m. until September 23, and after that at 6 p.m. Chronometers Negus, Frodsham, and Bond were in their external cases. All four rested firmly on sand heaped on the cellar floor about 15 cm from an inner foundation wall and 30 cm from one another. They were placed in this order: Negus, Hutton, Frodsham, Bond. The boxes of Hutton, Frodsham, and Bond were never opened except to wind them. The daily range of temperature in the cellar averaged less than 5° C. The chronometers were compared with the clock of the Allegheny Observatory twice daily.

The measurements of length before the first interchange of knives were as follows:

	Pend.—standard.
	μ.
August 18	+16.4
19	+16.3
19	+16.9
20	+16.9
20	+21.5
21	+17.5
Mean	+17.6

But these measures are uncorrected for the difference of temperature between the pendulum and the standard; and in point of fact the former carried no thermometer. We may assume that the result should have a correction of $+2^{\mu}4$ on this account, because this is the mean value of the correction in the following series. With this correction the mean result is that the pendulum was longer than the standard by $20^{\mu}0$.

After the first interchange the results were these:

	Pend.—standard.
	μ.
September 10	+19.4
11	+18.6
12	+18.4
13	+19.5
Mean	+19.0

After the second interchange the results were as follows:

	Pend.—standard.
	μ.
September 23	+19.5
23	+20.3

	μ.
September 24	+21.5
24	+21.3
25	+17.0
25	+17.7
Mean	+19.5

We conclude that the pendulum preserved the same length at all times, and was $19^{\mu}.5$ longer than the standard. The latter at 15° C is $261^{\mu}.1$ longer than the metre assumed in the "Measurements of Gravity at Initial Stations"; so that in terms of that metre the length of the pendulum at 15° C was

$$1^{m}.0002806.$$

The difference in the distances of the centre of mass from the two knife-edges was found to be in one position

$$0^{m}.39351$$

and in the other

$$0^{m}.39352.$$

To these values must be applied a small correction, $+.14^{mm}$, which in the "Measurements of Gravity at Initial Stations" is correctly given, but is applied with the wrong sign.

The following is the calculation of the length of the seconds pendulum from the first four and last four days' oscillations at Ebensburgh:

	First days.	Last days.
	s.	*s.*
T_d	1.0064420	1.0064388
T_u	1.0065140	1.0065033
T_d^2	1.0129255	1.0129191
T_u^2	1.0130704	1.0130489
Corr. stretching	1.0130714	1.0130499
$\frac{1}{2}(T_d^2 + T_u^2)$	1.0129985	1.0129845
$\frac{1}{2}(T_d^2 - T_u^2)$	−730	−654
$(h_d + h_u) : (h_d - h_u)$	2.54045	2.54097
$[T^2$ Rev.]	1.0128131	1.0128187
Same in mean time	1.0072880	1.0072936
Length pend.	1.0002806	1.0002806
Sec. pend.	0.9930432	0.9930379

Seconds pendulum at Ebensburgh = $0^{m}.9930406$.

This is expressed in terms of the erroneous metre having the provisional correction -162×10^{-7}. Applying as for Allegheny the corrections for elevation and latitude, we have

Seconds pendulum at Ebensburgh	0.9930244
Elevation	+1827
Latitude	−21399
Corrected to equator and sea-level	0.9910672

In the tables appended to the edition of this Appendix which has been published separately are given the details of the work at Ebensburgh.

III.—DETERMINATION OF GRAVITY AT YORK.

York, Pa., is situated east of the Alleghenies in a comparatively plain country. The pendulum was oscillated in the cellar of the factory of Mr. A. B. Farquhar, near the railway station, on Duke Street. The transit was about a hundred yards to the east of the factory, on land belonging to Messrs. Billmeyer and Small, in Gay Alley. The co-ordinates of the station are:

Latitude, 39° 58′ north.
Longitude, $5^{h}\ 05^{m}\ 54^{s}$ west of Greenwich.
Elevation, 122 metres (373 feet).

The work at this station was conducted by Mr. Henry Farquhar, under my supervision. The pendulum observations were partly made according to a method of eye-and-ear coincidences invented by Mr. Farquhar. For the purpose of studying the effects of flexure, the Repsold reversible pendulum was oscillated on various supports, viz.: 1^{st}, on the Repsold tripod; 2^{nd}, on a solid support formed by bolting the head of the Repsold tripod to an oaken plank 2 inches thick; 3^{rd}, on the Geneva support and tripod, with the bells off and with the bells on (this to ascertain the effect of the bells); 4^{th}, on the Repsold tripod mounted on a wooden support; 5^{th}, on the Repsold tripod resting on pieces of India rubber.

Experiments were also made at this station upon the effect of substituting rollers for the knives as the bearings of the pendulum. The rollers were steel cylinders of 5^{mm} diameter, backed by steel planes. They were well constructed by Messrs. Darling, Brown, and Sharpe. The utmost pains were taken (here as well as in later experiments in Baltimore) to avoid the inclusion of dust between the roller and its support. Nevertheless the decrement of the amplitude was very rapid for arcs above .035 of the radius on each side of the vertical; and the periods show enormous variations.

The experiments on the effect of the bells of the Geneva support are also of interest, though they fail to give a very accurate evaluation of this constant.

The summary of the periods of oscillation at this station (except upon the Geneva support) has already been published in the *Coast Survey Report* for 1881, pages 423–424. This summary is here repeated, with the difference that the flexure corrections are now applied, that some errors of computation are corrected,[3] and that the experiments relating to the effect of the bells are added.

In drawing up the summary, besides the corrections for arc, pressure, temperature, and rate, the following have been applied:

3. The following table shows these corrections:

Support.	Method of observation.	Position heavy end.	Date.	Correction to last figure.	Cause of former error.
Repsold	Transits	Up	May 2.	−9	Error in subtraction had occasioned rejection of a transit.
Do. . . .	Coincidence	Down . . .	Mar. 19.	−9	Error of computation.
Do. . . .	. . Do.	. . Do.	Mar. 21.	−1	Do.
Stiffest . .	Transits	. . Do.	Apr. 4, *bis*.	−3	Mr. Farquhar thinks he recorded the wrong minute, a fault to which he was liable. Changing the minute, a rejected transit is brought into concordance with the others.

Cause.	Authority for amount.	Amount.	
		Heavy end down.	Heavy end up.
Knife, 7-8 (for 3-4, with reversed sign)	See Ebensburgh report*	−.000006	+.000015
Flexure Repsold support	*C. S. R.*, 1881, p. 424	−.000084	−.000036
Flexure stiffest support	*C. S. R.*, 1881, p. 423	−.000082	−.000009
Flexure Geneva support	*C. S. R.*, 1881, p. 399	−.000020	−.000009
Flexure wooden support	*C. S. R.*, 1881, p. 423	−.000123	−.000054
Flexure rubber support	Do.	−.000300	−.000131
Geneva cylinder	*C. S. R.*, 1876, p. 270	−.000004	−.000008
Geneva bells	*C. S. R.*, 1876, pp. 270, 271	−.000012	−.000028

*At the time the paper on the flexure of pendulum supports was drawn up Mr. Smith had not measured the knives. It was consequently necessary to determine this correction *a posteriori* and slightly different corrections were thus used in the synopsis given in that report, viz., −.000004 and +.000012.

PERIODS OF OSCILLATION AT YORK.

REPSOLD SUPPORT.

Method of transits.

HEAVY END DOWN.		HEAVY END UP.	
	Knife 7-8.		Knife 3-4.
1880.	*s.*	1880.	*s.*
April 7	1.006413	April 7	1.006467
April 30	1.006405	April 30	1.006446
	Knife 3-4.		Knife 7-8.
May 2	1.006418	May 2	1.006486
May 3	1.006418	May 3	1.006483

Method of coincidences.

	Knife 3-4.		Knife 7-8.
March 19	1.006432	March 19	1.006490
March 21	1.006407	March 21	1.006440
June 4	1.006413	June 4	1.006472
June 5	1.006407	June 4	1.006450
	Knife 7-8.		Knife 3-4.
March 22	1.006422	March 22	1.006488
March 23	1.006406	March 23	1.006494
June 6	1.006421	June 6	1.006472
June 6	1.006429	June 6	1.006466

PERIODS OF OSCILLATION AT YORK. *(Continued)*

STIFFEST SUPPORT.

Method of transits.

	HEAVY END DOWN.		HEAVY END UP.
	Knife 3-4.		Knife 7-8.
	s.		*s.*
March 31	1.006415	March 31	1.006467
April 2	1.006419	April 2	1.006472
	Knife 7-8.		Knife 3-4.
April 4	1.006410	April 4	1.006471
April 4	1.006417	April 4	1.006463

Method of coincidences.

	Knife 7-8.		Knife 3-4.
March 26	1.006419	March 26	1.006456
March 27	1.006423	March 27	1.006463
	Knife 3-4.		Knife 7-8.
March 28	1.006417	March 28	1.006461
March 29	1.006415	March 29	1.006463

WOODEN SUPPORT.

Method of coincidences.

	Knife 7-8.		Knife 3-4.
April 24	1.006420	April 24	1.006473
April 25	1.006417	April 25	1.006469
	Knife 3-4.		Knife 7-8.
April 27	1.006415	April 27	1.006470
April 28	1.006417	April 28	1.006488

RUBBER SUPPORT.

Method of coincidences.

	Knife 7-8.		Knife 3-4.
	s.		*s.*
April 18	1.006404	April 18	1.006484
April 20	1.006401	April 20	1.006482

GENEVA SUPPORT; BELLS OFF.

Method of transits.

	Knife 3-4.		Knife 7-8.
May 19	1.006425	May 19	1.006499
	Knife 7-8.		Knife 3-4.
May 22	1.006420	May 22	1.006488

Method of coincidences.

	Knife 3-4.		Knife 7-8.
May 18	1.006433	May 18	1.006509
	Knife 7-8.		Knife 3-4.
May 23	1.006431	May 23	1.006463

PERIODS OF OSCILLATION AT YORK. *(Continued)*

GENEVA SUPPORT; BELLS ON.

Method of coincidences.

	HEAVY END DOWN.		HEAVY END UP.
	Knife 7-8.		Knife 3-4.
	s.		
May 26	1.006432	May 26	Rejected.
May 27	1.006439	May 27	1.006485
May 29	1.006430	May 29	1.006459
	Knife 3-4.		Knife 7-8.
May 30	1.006432	May 30	1.006507
May 31	1.006437	May 31	1.006488

The means of the observed periods for the Repsold and stiffest supports are—

Method of transits.

	Heavy end down.	Heavy end up.
	s.	s.
Repsold support	1.006413 ± 1	1.006470 ± 5
Stiffest support	1.006415 ± 1	1.006468 ± 1
Weighted mean	1.006414 ± 1	1.006468 ± 1
Method of coincidences.		
Repsold support	1.006417 ± 3	1.006471 ± 5
Stiffest support	1.006419 ± 2	1.006461 ± 1
Weighted mean	1.006418 ± 2	1.006462 ± 1
General mean	1.006416 ± 1	1.006465 ± 1

It will be seen that the method of eye-and-ear coincidences is greatly inferior in accuracy, the eight observations taken in this way on the Repsold support being less valuable than the four by transits; and there can be little doubt that the means would be brought nearer to the truth by rejecting all the observations by these coincidences. We shall accordingly allow observations with this method only one-fourth weight. With these weights, the above periods become—

Corrected periods	1.006415	1.006468

The observations on the Geneva support, with the bells off, give

Heavy end down.	Heavy end up.
s.	s.
1.006424	1.006492

The differences from the corrected periods just ascertained are—

+.000009	+.000024

These numbers are in such a proportion as to indicate some force acting equally on the pendulum in its two positions. Experiments subsequently made in Baltimore, to be described in another memoir, leave no doubt that the effect is connected with the supporting planes of the Geneva receiver.

The observations with the bells on, all made by the method of coincidences, give—

Heavy end down.	Heavy end up.
s.	*s.*
1.006435	1.006485

From these numbers it would seem that the effect of the bells may be a little larger than was calculated; but the error, if any, can hardly be sensible when the receiver is pumped out.

The time observations were made with the same transit instrument used at Hoboken and at Ebensburgh. The eye-piece not being quite steady, the variations of collimation were considerable, and the instrument could not be kept free from dust. Time was kept by the four chronometers:

Negus 1589
Frodsham 2490
Hutton 202
Bond 380

They seem to have required cleaning, and show large diurnal variations. An attempt was made in the computations to take account of these, but not successfully.

The measurement of the pendulum on March 3 showed—

$$\text{Pendulum} - \text{standard} = +26\overset{\mu}{.}9$$

On May 7 and 8 three sets were taken with heavy end up, on which account $1\overset{\mu}{.}0$ has to be added to the results. (See "Measurements of Gravity at Initial Stations.") With this correction the results are as follows:

	$\mu.$
Pendulum − standard =	+26.9
	+23.4
	+25.8
Mean	+25.3

On June 9, the knives having been interchanged, four sets gave

$$
\begin{array}{rr}
 & \mu. \\
\text{Pendulum} - \text{standard} = & +27.8 \\
 & +25.5 \\
 & +31.3 \\
 & \underline{+30.0} \\
\text{Mean} & +28.6
\end{array}
$$

These figures are uncorrected for the difference of thermometers on the pendulum and standard, because such correction would make the accordance of the measures much less good. We must assume the excess of length of the pendulum in the first position to have been $+26^{\mu}.1$, and for the mean of the two positions $+27^{\mu}.3$. Since the standard is $+261^{\mu}.1$ longer at 15° C than the assumed metre, it follows that the length of the pendulum in terms of that metre (now known to be false) was

$$1^{m}.0002884.$$

I prefer to retain the erroneous metre for the present, in order to avoid further confusion.

The difference of the distances of the centre of mass from the two edges was found to be

Date.	Knife, 3-4 at heavy end.	Knife, 7-8 at heavy end.	First roller at heavy end.	Second roller at heavy end.
	m.	*m.*	*m.*	*m.*
March 22....	0.39343	0.39353	....	....
March 28....	0.39340	0.39349	....	....
April 26	0.39353	0.39351	....	....
May 10	....	....	0.39388	0.39387
May 30	0.39344	0.39353	....	....
Means	0.39345	0.39351	....	....

In the mean of the two positions of the knives we have 0.39348, to which .00014 has to be added on account of the error of the standard. (See "Measurements of Gravity at Initial Stations.")

The following is the calculation of the length of the seconds pendulum at York:

$T_d =$	1.006415	$T_u =$	1.006468
T_d^2	1.012871	T_u^2	1.012978
$\frac{1}{2}(T_d^2 + T_u^2)$	1.012925	Corr. stretching	1.012979
$\frac{1}{2}(T_d^2 - T_u^2)$	-54		
$\frac{h_d + h_u}{h_d - h_u} \frac{1}{2}(T_d^2 - T_u^2)$	-137		
$[T^2 \text{ Rev.}]$	1.012788		

Whence the length of the seconds pendulum in York referred to the metre heretofore used is:

	$0^{\text{m}}.993073$
Provisional correction to metre	-16
Elevation	$+104$
Latitude	-2146
Reduced to sea-level and equator	0.991015

These reductions have been made, like those of Allegheny, in accordance with the principles of my memoir on the ellipticity of the earth (*Coast Survey Report* for 1881, Appendix No. 15).

Details of the work at York are printed in tables appended to the edition of this Appendix which has been published separately.

This page intentionally left blank

[STUDY OF GREAT MEN]

Materials for an Impressionist List of 300 Great Men

Item 2 ***MS 470: Fall 1883–Fall 1884***

First Rank	*Provisionally Admitted*	*Doubtful*	*Provisionally Excluded*
1 Alexander	4 .Thomas à Becket	265 .Abel (math.)	
2 ?Archimedes	5 .Abelard	Aesop	
3 Aristotle	6 Aeschylus	?d'Alembert X	
	292 ?Agassiz (Exclude?)	16 !Apelles X	
	7 Alcibiades	17 Sir R. Arkwright X	
	?-Alfieri	284 Augustus	
	8 .Alfred the Great		
	9 .St. Thomas Aquinas		
	10 .Jeanne d'Arc		
	?-J. van Artevelde		
	11 Ph. v. Artevelde		
	12 .Athanasius		
	13 Attila		
	14 .St. Augustine		
	15 Marcus Aurelius		
18 Beethoven	19 .Sebastian Bach	279 von Baer	Baxter
	20 Francis Bacon	Chevalier Bayard	278 Bentley
	21 .Roger Bacon	Berzelius	St. Bernard
	22 Barneveldt	Boccaccio	Bp. Berkeley
	23 Jeremy Bentham	Botticelli	St. Ch. Borromeo
	24 Jacob Bernoulli	Giordano Bruno	
	25 John Bernoulli	von Buch	Buffon
	26 Bhas-cara *(algebraist)*	282 Burke	35 Bossuet X
		34 !Burns X	36 Balzac X
	27 .Bismarck		37 Bruce X
	28 .Bolívar		
	291 ?Boole		
	29 Boyle		
	30 Tycho Brahe		
	31 Robert Browning		
	32 !Buddha		
	33 .Byron		

First Rank	*Provisionally Admitted*	*Doubtful*	*Provisionally Excluded*
38 Caesar	43 Calderon	Canova	Cleopatra
39 Columbus	44 .Calvin	Carlyle	280 Clausius
40 ?Confucius	45 Camoens	62 !Cavour X	
41 Copernicus	46 .Cauchy	Cecil	
42 Cromwell	47 Cellini	63 Charles V X	
	48 Cervantes	Chopin	
	49 !Champollion	64 =Cicero X	
	50 !Charlemagne	Coleridge	
	51 Charles Martel	65 Constantine X	
	52 Charles XII	66 =Captain Cooke X	
	53 ?Chasles	67 Cortes X	
	54 ?Chaucer	Coulomb	
	55 Chrysostom	Admirable Crichton	
	56 .Cid		
	57 .Clive		
	58 Colbert		
	59 ??Comte		
	???Charlotte Corday		
	60 Cuvier		
	61 Cyrus		
68 Dante	69 !Darwin	Dalton	Dickens
	71 King David	=Danton	Diderot
	72 Democritus	=Darius	Dryden
	70 !Descartes	266 Sir Humphrey Davy X	75 Drake X
	73 De Witt	Deschapelles X	
	74 Dürer	267 Diez X	
		Dirichlet	
76 Euler	77 ?Geo. Eliot	Ehrenberg	
	78 X Queen Elizabeth	82 !Erasmus X	
	?-R. W. Emerson	Erigena	
	Epaminondas	van Eyck	
	79 Epicurus		
	80 !Euclid		
	81 Euripides		
	??Fabricius	88 =Farragut X	Fénelon
	83 Faraday	=Froissart	288 !!Charles James Fox
	84 .Fichte		
	85 B. Franklin		
	86 Frederick the Great		
	87 Fresnel		
89 Galileo	91 Galen	Vasco da Gama	!!Gambetta X
90 Gauss	92 Garibaldi	Garcilaso de la Vega	287 Grotius
	??-D. Garrick	100 !Gladstone X	
	93 Germanicus	Gluck	
	94 ?Gilbert	Godefroi	
	95 ?Giotto		

First Rank	*Provisionally Admitted*	*Doubtful*	*Provisionally Excluded*
	96 .!Goethe	Earl Godwin	
	??-Jean Goujon	U. S. Grant	
	??-The Gracchi	W. Grimm	
	97 Gregory the Great		
	98 Jacob Grimm		
	99 Gustavus Adolphus		
101 Homer	??Hafiz	117 Haller X	124 !Huber X Hamilton
	102 Sir Wm. Rowan Hamilton	118 =Hampden X	
	103 Hannibal	119 Händel X	
	104 Haroun al-Rashid	120 Warren Hastings X	
	105 Harvey	269 Haydn	
	106 .Hegel	Heine	
	107 .Helmholtz	Hermann	
	108 Henri IV	=Sir Wm. Herschel	
	273 ?-Henry V of England	.Hiawatha	
	109 Herodotus	285 .Horace	
	110 .Hipparchus	121 =Howard (philanthropist) X	
	111 Hippocrates	122 !Victor Hugo X	
	112 Hobbema	123 .Hunter X	
	113 Holbein		
	114 Huss		
	115 .Huyghens		
	?Hyder Ali		
	?Hypatia		
	116 A. v. Humboldt		
	270 Isaiah		
125 Jesus	126 Jacobi (math.)	271 Sir Wm. Jones	Jefferson X
	127 Jenghiz Khan	Jussieu	
	128 ??-?Jenner		
	129 .Julian		
130 Kant	293 ?Charles Kean	272 =John Knox	
131 Kepler	132 .Thomas à Kempis	=Kossuth	
133 Luther	??-?LaFayette	286 Lucian	Lafontaine
	134 Lagrange	Laurent (chemist)	W. S. Landor
	135 !Laplace	Legendre	John Law
	136 Lavater	=Leo X	Gen^{l}. R. E. Lee
	137 Lavoisier	Leonardo Fibonacci	Louis XIV
	138 Leibniz	146 !Leonidas X	Lobatchewsky
	139 Leonardo da Vinci	Lesseps	Lais
	140 -Lessing	Leverrier	
	?Liebig	Fra Lippo Lippi	
	141 Ab. Lincoln	Liszt X	
	142 Linnæus	Lotze	
	143 Dr. Livingstone		

First Rank	*Provisionally Admitted*	*Doubtful*	*Provisionally Excluded*
	144 .Locke	147 Lucretius X	
	145 Loyola	Raymond Lully	
	?-?Lyell		
148 ?Milton	151 Macchiavelli	170 !Massasoit (K. Philip) X	Hugh Miller
149 ?Moses	152 !Mahomet		Mitscherlich
150 Michaelangelo	153 Marlborough	Masséna	Murat
	154 Marius	Max Müller	Meissonier
	155 Marie Stuart	277 Mendelssohn	Lord Mansfield
	156 Mazarin	Michelet	281 Maxwell
	??Cosimo de' Medici	Millet	283 Moltke
		Johannes Müller	
	157 Lorenzo de' Medici		
	158 Mencius		
	159 James Mill		
	160 John Stuart Mill		
	?Miltiades		
	161 .Molière		
	162 Monge		
	163 Montaigne		
	164 Montesquieu		
	165 Sir Th. More		
	166 Paul Morphy		
	167 Mozart		
	168 Munkácsy		
	169 Mirabeau		
171 Napoleon Bonaparte	173 Nelson	Napier of Merchiston	Ninon de l'Enclos
172 Sir Isaac Newton	174 Niebuhr	Necker	
		Niccola	
	175 .Ockham	295 Oersted	Osman
	176 Omar I	Ohm	Omer Pacha
	177 Origen		
	178 Oxenstiern		
179 St. Paul	181 .B. Palissy, the Potter	194 Palestrina X	Palladio
180 Plato		Paracelsus	.Sir R. Peel
	182 Pascal	195 Pasteur X	Wm. Penn
	183 St. Patrick	196 Paul Veronese X	Polycarp
	184 Pericles	Ambrose Père	A. Pope
	185 Perugino	197 =Peter the Hermit X	Pym
	186 .Peter the Great		
	187 Phidias	Petrarch	
	188 Philip of Macedon	=Pizarro	
		Poniatowski	
	189 Pindar	Porson	
	190 .Wm. Pitt	Baptista Porta	
	??-?Marco Polo		
	191 Praxiteles		
	192 Priestley		
	?Pyrrhus		
	193 Pythagoras		

First Rank	*Provisionally Admitted*	*Doubtful*	*Provisionally Excluded*
198 Rafael	199 Rabelais	208 Ralegh X	Sir Joshua Reynolds
	200 ?Rachel	!Ramus	Rochefoucauld
	201 Racine	Ray	Rollo
	202 Rembrandt	Ricardo	
	203 Richelieu	Ristori	
	204 Jean Paul Richter	209 =Rumford X	
	205 Rienzi		
	206 J. J. Rousseau		
	207 Rubens		
210 Scipio Africanus	213 ?Sadi	Andrea del Sarto	Gen^{l}. Maurice Saxe
211 Shakespeare	214 Saladin	Scheele	Joe Smith
212 Socrates	215 !Savonarola	230 !Shelley	Agnes Sorel
	216 Schiller	269 !W. T. Sherman	275 !!Jonathan Swift
	217 Sir W. Scott	.Solon	.Sydenham
	218 Servetus	=Thad. Stevens	231 !Duns Scotus X
	219 Mrs. Siddons	274 Sir Ph. Sydney X	
	220 Sixtus V		
	221 ?-Adam Smith		
	222 Sobieski		
	223 Solomon		
	224 Sophocles		
	225 Ed. Spenser		
	226 Spinoza		
	227 Geo. Stephenson		
	228 Sully		
	229 Swedenborg		
	232 Talleyrand	Tacitus	Tancred
	233 Talma	Tasso	Tintoretto
	234 Thales	Tecumseh	Troyon
	235 Themistocles	276 Tennyson	245 .Turgot X
	236 Theodosius	Theodoric	
	237 Timour	241 =Thiers X	
	238 Titian	242 Thorwaldsen X	
	239 Turenne	243 Thucydides X	
	240 Turgeneff	244 =Toussaint l'Ouverture X	
		Trajan	
		Turner	
	246 .Velasquez	Gustavus Vasa	268 Lope de Vega
	247 ?Vesalius	Vauban	
	248 .Voltaire	Vidocq	
		290 !Vieta X	
		249 Virgil X	
250 Washington	252 Wagner	Wallenstein	Whitfield
251 William the Conqueror	253 Watt	Walther v. d. Vogelweide	259 !!Wilberforce X
	254 Wellington	257 !Wesley X	Sir Chr. Wren
	255 William of Orange	258 !Wiclif X	297 W. Weber

First Rank	*Provisionally Admitted*	*Doubtful*	*Provisionally Excluded*
	256 Wordsworth	William of Sens	
		William of Wyckham	
		Roger Williams	
		Wilson	
		Winckelmann	
		Wolsey	
	260 Xavier	=Ximenes	Xerxes
		=Brigham Young	
		261 Dr. Thomas Young X	
	262 Zenobia	Zeno the Stoic	Zeisberger
	263 Zoroaster	264 Zwingli X	Zinzendorf

My list of great Men

Item 3 *MS 471: Fall 1883–Fall 1884*

Alexander 1.7
Archimedes
Aristotle
à Becket
Abelard
Aeschylus
Alcibiades 6.0
Alfred
Aquinas
Jeanne d'Arc
Ph. van Artevelde
Athanasius

Attila 4.4
St. Augustine
M. Aurelius
Aristophanes
Apelles
Arkwright
Augustus 2.1
Beethoven
Bouddha
Bach
Fr. Bacon
Roger Bacon

Barneveldt 3.9
Bentham
Jacob Bernoulli
Berzelius
Boccaccio
Bismarck
Bolivar 4.0
Boyle
Tycho Brahe
Robert Browning
Burns
Byron

Bossuet 3.5
Balzac
Berkeley
Caesar
Columbus
Confucius
Copernicus 3.7
Cromwell
Calderon
Calvin
Camoens
Cauchy

Cellini 5.0
Cervantes
Champollion
Charlemagne
Charles Martel
Charles XII
Chasles 4.8
Chaucer
Chrysostom
The Cid
Clive
Apollonius of Perga

Comte 3.4
Cuvier
Cyrus
Cavour
Charles V
Cicero
Constantine 2.8
Bopp
Cortez
Dante
Darwin
Descartes

(Continued)

King David 3 ±	Faraday 3.5	Gilbert 2.7
Democritus	Fichte	Gluck
De Witt	B. Franklin	Gregory the Great
Dürer	Frederic the Great	Jacob Grimm
Dickens	Fresnel	Gustavus Adolphus
Euclid	Fermat	Gladstone
Euler 2.0	Galileo 1.5	Homer 1.3
Geo. Eliot	Gauss	Sir W. R. Hamilton
Q. Elizabeth	Goethe	Hannibal
Epicurus	Galen	Haroun al-Rashid
Euripides	Garibaldi	Harvey
Erasmus	Wm. Herschel	Hegel
Helmholtz	Hampden 4.6	Julian 4.0
Henri IV	Händel	Sir W. Jones
Henry V	Warren Hastings	Kant
Herodotus	Haydn	Kepler
Hipparchus	Howard	T. à Kempis
Hippocrates	Victor Hugo	Luther
Alex. v. Humboldt 3.1	John Hunter 3.0	Lagrange 3.5
Hawthorne	Isaiah	Laplace
Holbein	Jesus	Lavater
Huss	Jacobi	Lavoisier
Huygens	Jenghiz Khan	Leibniz
Haller	Jenner	Leonardo da Vinci
Lessing 3.6	Machiavelli 4.8	Montaigne 4.8
Lincoln	Marlborough	Montesquieu
Linnaeus	Lanfranc	Maxwell
Hume	Johannes Müller	Millet
Locke	Mazarin	Mozart
Loyola	Lorenzo de' Medici	Moltke
Leonidas 5.4	Mencius 2.8	Morphy
Ptolemy	James Mill	Mendelssohn
Milton	J. S. Mill	Napoleon
Moses	Mirabeau	Newton
Michelangelo	Molière	Nelson
Mahomet	Monge	Niebuhr

(Continued)

Ockham 2.5	Phidias 1.4	Rabelais 4.5
Omar I	Philip of Macedon	Rachel
Origen	Pindar	Racine
Oxenstiern	Pitt	Rembrandt
St. Paul	Praxiteles	Richelieu
Plato	Priestley	Jean Paul Richter
Palissy 3.0	Pythagoras 1.1	Rienzi 5.5
Pascal	Palestrina	J. J. Rousseau
Peter the Hermit	Sydenham	Rubens
Petrarch	Jacquard	Ralegh
Peter the Great	Raphael	Rumford
Pericles	Ricardo	Scipio Africanus
Shakespeare 1.0	Swedenborg 4.1	Talma 5.2
Socrates	Sophocles	Thales
Saadi	Spenser	Themistocles
Saladin	Spinoza	Theodosius
Savonarola	Stephenson	Timour
Schiller	Sully	Titian
Sir W. Scott 5.2	Emerson	Turenne 3.2
Schopenhauer	Shelley	Steiner
Siddons	Gen^{l} Sherman	Tennyson
Sixtus V	Duns Scotus	Sylvester
Adam Smith	Tintoretto	Grassmann
Sobieski	Talleyrand	Thucydides
Toussaint 4.3	Wellington	Grotius 4.1
Louis XIV	William of Orange	Garrick
Velasquez	Wordsworth	Belisarius
Vesalius	Wesley	Horace
Voltaire	Wiclif	Huber
Lope de Vega	Xavier	Knox
Virgil	Young 3.1	von Baer
Wallace	Zeno Skeptic	Bellini
Washington	Riemann	Fielding
William the Conqueror	Zoroaster	Keats
Wagner	Bentley	Bichat
Watt	Diez	

[Men of Feeling, Action, Thought]

Item 4 ***MS 475: Fall 1883–Fall 1884***

Men of Feeling

Musician

Bach
Beethoven
Chopin
Handel
Haydn
Mendelssohn
Mozart
Palestrina
Wagner

Novelists

Balzac
Cervantes
Geo. Eliot
Hawthorne
Scott
Turgeneff

Artist

Apelles
Canova
Cellini
Dürer
Giotto
Jean Goujon
Holbein
Leonardo da Vinci
Michelangelo
Millet
Muncaczy
Perugino
Phidias
Praxiteles
Rafael
Rembrandt
Rubens
Titian
Velasquez

Poet

Aeschylus
Rob. Browning
Burns
Byron
Calderon
Camoens
Chaucer
Dante
Euripides
Goethe
Homer
Horace
V. Hugo
Isaiah
Milton
Pindar
Edgar Poe
Saadi
Shakespeare
Schiller
Shelley
Sophocles
Spenser
Tennyson
Lope de Vega
Virgil
Wordsworth

Writer

Cicero
R. W. Emerson
Erasmus
Herodotus
Lavater
Montaigne
Montesquieu
Pascal
Rabelais
Jean Paul Richter
J. J. Rousseau
Voltaire

Dramatist

Lessing
Molière
Racine

Actor

Rachel
Mrs. Siddons
Talma

Other

Savonarola

Men of Action

Rulers

Alexander
Alfred
Augustus
Bolivar
Caesar
Charlemagne
Charles V
Constantine
Cromwell
Cyrus
K. David
Q. Elizabeth
Haroun al-Rashid
Henri IV
Jenghiz Khan
Julian
Abe Lincoln
Lorenzo de' Medici
Louis XIV
Napoleon
Pericles
Peter the Great
Philip of Macedon
Pitt
Sixtus V
Washington
William Conqueror
William of Orange

Generals

Alcibiades
Ph. van Artevelde
Attila
Belisarius
Charles Martel
Charles XII
Cid
Clive
Sir F. Drake
Frederick the Great
Farragut
Garibaldi
Gustavus Adolphus
Hannibal
Henry V
Moltke
Nelson
Omar I
Saladin
Scipio Africanus
Sobieski
Gen. Sherman
Themistocles
Theodosius
Timour
Toussaint
Turenne
Wallace
Wellington

Statesmen

Barneveldt
à Becket
Bismarck
Cavour
Colbert
De Witt
Gambetta
Gladstone
Grotius
Hampden
Warren Hastings
Mazarin
Mirabeau
Oxenstiern
Ralegh
Sully
Talleyrand
Thiers
Turgot

Personalities

Jeanne d'Arc
B. Franklin
Leonidas
Palissy
Rienzi

Philanthropist

Howard

Explorers

Lawyers

Men of Thought

Philos.

Aristotle
Abelard
Aquinas
Fr. Bacon
Bentham
Bp. Berkeley
Comte
Democritus
Descartes
Epicurus
Fichte
Ed. v. Hartmann
Hegel
Kant
Leibniz
Locke
Lucretius
J. Mill
J. S. Mill
Ockham
Plato
Schopenhauer
Duns Scotus
Spinoza
Zeno Stoic
Zeno Sceptic

Math.

Archimède
Abel
Jacob Bernoulli
Cauchy
Chasles
Euclid
Euler
Fresnel
Gauss
Rowan Hamilton
Huyghens
Jacobi
Lagrange
Laplace
Monge
Paul Morphy
Newton

Physicist

Roger Bacon
Berzelius
Boyle
Tycho Brahe
Copernicus
Faraday
Galileo
Gilbert
Helmholtz
Wm. Herschel
Hipparchus
A. von Humboldt
Kepler
Lavoisier
Maxwell
Melloni
Priestley
Rumford
Young

Moralist & Reformer

M. Aurelius
Mencius
Pythagoras
Socrates

Linguist

Bentley
Champollion
Diez
Jacob Grimm
Sir Wm. Jones

Other

Adam Smith

Men of Thought *(Continued)*

Religionists

Athanasius
St. Augustine
Bossuet
Buddha
Calvin
Chrysostom
Gregory the Great
Huss
Jesus
Th. à Kempis
Knox
Lanfranc
Loyola
Luther
Mahomet
Moses
Origen
St. Paul
Peter the Hermit
Swedenborg
Wesley
Wiclif
Xavier
Zoroaster
Zwingli

Inventors

Arkwright
Stephenson
Watt

Biologists

von Baer
Bichat
Cuvier
Darwin
Galen
Haller
Harvey
Hippocrates
Huber
John Hunter
Jenner
Linnaeus
Sydenham
Vesalius
Jo. Müller

Explorers

Columbus
Dr. Livingstone

Historian

Niebuhr
Thucydides

[Notes on Archimedes, Abel, Lagrange, and Gauss]

Item 5 ***MS 478: Fall 1883–Fall 1884***

Archimedes b. 287 B.C.
Friend & relation of Hiero. K of Syracuse
Made in his youth voyage to Egypt.
Euclid then taught at Alexandria

Abel born 1802 Norway (Father protestant minister)
1815 Entered cathedral school of Christiania
1818 Talent for Math. first observed
Soon necessary to reserve problems expressly for him
1821 quitted school, having read Poisson, Gauss, Lagrange
went to University of Christiania
Obtained first subscription from professors, afterward government pension to aid his studies at school & university
1824 Proved imposs. of solving eq. of 5^{th} degree.
In 1825 New grant from gov. Went to Berlin (met Crelle)
1828 Engaged to be married
1829 Died of phthisis.
Purity & Nobility of character & a rare modesty
Dissipated

Lagrange. born 1736. Did not at first have any predilection for math but at 16 was Professor of Math.
1759 Calculus of Variations

GAUSS. b. 1777 d 1855 Son of parents *peu* fortunés.
Could calculate before he could speak.

At 10 commenced calculus.
Distinguished (Lat & Gr) at gymnasium.
Protected by Duke of Brunswick
1795 to University of Göttingen—Kästner he didnt like
1795 Method of Least Squares
1796 Division of circle
1788–1801 worked on *Disquisitiones Arithmeticae*
He called Arith a divine science
1799 Proof that every equation has a root
Proof of Lagrange's theorem
Method of Calculating Easter
1800 Calc. elements of Ceres
1802 " " Pallas
1810 Medal for perturbations of Pallas
He had a strong aversion for a chair of mathematics wanted an obsy
1807 Appointed to Göttingen obsy
1809 *Theoria Motus*

Who is the greatest Math. of Germany? Pfaff, answered Laplace. I thought Gauss was his superior. Ah! But he is not the greatest math. of Germany but the greatest math. of Europe.

1811 Discovered Comet
1821–24 Geodetic connection Göttingen & Altona
Heliotrope
1831 Crystallography
1831 Magnetism
1837 Memoirs of bifilar magnetometer
1840 Theory of terrestrial magnetism
About this time powers began to fail
1849 Last memoir
1853 Health declined

His researches made to please himself; publication quite secondary. In his youth his ideas presented themselves quicker than he could set them down. Math queen of sciences. Arith queen of Math. Math principal means of developing human mind

Great facility for languages. At 63 learned Russian in 2 years

[Notes on Leonidas]

Item 6 ***MS 479: Fall 1883–Fall 1884***

Leonidas King of Sparta 17th of the Agid line.
B.C. 480

Had been king one year when Xerxes invaded Greece. Sent by resisting states with 8000 men to hold pass at Thermopylae. Persians by treacherous aid turned the pass & then Leonidas dismissed all the army except his 300 Spartan citizens, 700 Thespians and also the Theban contingent whom he suspected of treachery. These last laid down their arms. Every man of the rest died on the field.

It was a splendid act. Mark him 5.5

[Notes on Mencius]

Item 7 *MS 480: Fall 1883–Fall 1884*

Mencius sage of the Second Degree.
Mencius = Măng-tze i.e. Master Mang

The Mangs were one of the three great clans of Lû, all descended from the Marquis Hwan 700 B.C. Their power had been broken & the branch to which our philosopher belonged had settled in Tsâu.

Born about 380 B.C. Father died when child was 3 years old. His mother is celebrated in China as the model mother.

As young man he had been in communication with disciples of Confucius who was for him the chief of mortal men.

At the age of 40 he was accompanied by several eminent disciples.

He had a plan of reforming the kingdom. He went about trying to find a king to put it in practice. (There were practically 7 kingdoms in China though nominally unit and with a national feeling.) In China men of ability are much reverenced. Went about for 20 years from court to court everywhere much respected. Finally he recognized the impracticability of bringing about his plan, and retired to private life. Spent twenty years in conversing with his disciples and writing very elaborately & elegantly the record of his conversations.

His views of government were these. Royal government is an institution of God. This was the traditional view. But who should be the ruler? The people must judge from the character, public & private, of the man. He held to the truth of the old saying, Heaven sees as the people see, heaven hears as the people hear.

He said: "The people are the most important element in a nation; the altars to the spirits of the land & grain are the second; the sovereign is the lightest." A bad king should be dethroned.

Who should remove the bad king? First, the members of the royal house.—Second, if they failed to do their duty, any high minister of state should assume the office.—Failing this, a minister of heaven.—Not a mere assassin but someone should so conduct himself as to draw all eyes and all hearts to himself while remaining in his subordinate position. Finally, he should raise the standard of righteousness. This "minister of heaven" was what Mencius was 20 years trying to set up, holding that the house of *Châu* had forfeited its title to the throne.

The aims of government should be two. First, the prosperity of the people. Second, their education.

No one should occupy the throne who could be happy while his people were miserable or who delighted in war, or could bear to spend vast sums for palaces & parks from which the body of the people were altogether excluded.

Taxes should be light. Agriculture & commerce should be regulated, but in such a way as to encourage & stimulate them. Thus, systems of irrigation should be instituted. The first duty of a sovereign is to see that his people can get their livelihood. Otherwise they are forced to break the laws & the king becomes a "trapper."

He had a programme of 4 kinds of educational institutions to be established. But he chiefly insisted on the first not on this second duty of government.

Opinions were much divided at his time as to the goodness or badness or indifference of human nature. Some held that some men are naturally good, some bad. Mencius held that all men are naturally good. "The tendency of man's nature to goodness," he said, "is like the tendency of water to flow downwards. By striking water you may make it leap over your forehead; and by damming and leading it you may make it go up hill. But such movements are not according to the nature of water; it is the force applied which causes them. When men do what is not good, their nature has been dealt with in this way." How Chinese the simile!

Benevolence, Righteousness, Propriety, Wisdom. These are not infused into us from without. Men have these 4 principles just as they have their 4 limbs.

Some constituents of human nature are ignoble. They ought to be ruled. But they are not in their proper sphere Bad.

I find two definitions by Mencius of the great man.

The great man is he who does not lose his child-heart.

To dwell in love, the wide house of the world, to stand in propriety, the correct seat of the world, and to walk in righteousness, the great path of the world; when he obtains his desire for office, to practise his principles for the good of the people, and when that desire is disappointed, to practise them alone; to be above the power of riches and honours to make dissipated, of poverty and mean condition to make swerve from the right, and of power and force to make bend, these characteristics constitute the great man.

Mark him 2.6

[Notes on Michelangelo]

Item 8 *MS 481: Fall 1883–Fall 1884*

Name Michael Àgnolo Buonarroti-Simoni
Born. March 6. 1475. 2 AM
Poor noble
2nd child 5 in all. All boys.
Mother 19. Father 31 Full period
Father b 11 June 1444
Family among the leading people of Florence
One of the most disting. Florentine fam
Mother very strong physically
Never married

Childhood Idled at school & spent time lounging in the painters ateliers

Nat destiny To be a merchant a higher pursuit than painting; that was opposed by his family but he insisted.

Apr 1 1488 Articled to Ghirlandaio for 3 years. No premium. Masters to pay 6 gold florins 1st year, 8 second, 10 third

Soon after Ghirlandaio exclaimed "He knows more than I do myself" & soon became jealous.

Before 3 years turned him off

1489 Neglecting Ghirlandaio to practice sculpture. Attracts notice of Lorenzo the Magnificent. Michael Angelo taken into palace—clothed—boarded—& given 5 ducats monthly

Much given to sarcasm? Nose broken by fist of Torrigiano. Remained with Lorenzo till death of latter in 1492. So great his grief at death of his friend that he remained for several days unable

Childhood in Settignano 3 miles from Florence. Nurse wife of stone mason. Began to draw as soon as he could use his hand. Drawing firm & vigorous, an evidence of the boy's precocity.

"Dear Giorgio, if my mind is worth anything, I owe it to the clear air of your Arezzo country."

In the air of Florence there lies an immense stimulus to aspire after fame and honour. Impossible for anyone with a spark of literary enthusiasm to be associated with the society constantly assembled in Lorenzo's house without being stirred up to mental activity

He who allowed himself to rest in Florence stepped into the background.

After Lorenzo's death Michelangelo pursued his art for 2 years at his own expense.

22 January 1494 made snow statue for Piero de' Medici. Took him back to the palace

Cardiere sees vision of Lorenzo. Michelangelo takes it seriously. Much agitated. Sudden fear seized him. Thought the vengeance of heaven predicted by Savonarola was coming on the city. This fear extended to all minds. At last fled to Venice (1494 or 1495)

Great facility in learning. Impossible to guess how he could have learned all he knew

Decided mathematical facility

Musical. Composed music still extant

From Bologna to Venice

Michel alone had money. Afterward gave it to them & went to Aldovrandi's house

Violent character. Irritable. Sarcastic. Love of solitude amounting to disease. Intimately associated with most disting & celebrated men.

Zealously protected pupils. Generosity. Preferred unimportant people. Comforting unfortunate. Lived like a poor man. Frugal. Extraordinary activity. Very irregular. Months in meditation without touching chisel or brush. Worked with fury. Would impatiently attack marble without making precise calculations. Very little sleep.

Middle height. Broad shoulders. Slender & well proportioned

Dry nervous temperament

Complexion full of health & vigor

Round head. High temples. Broad square forehead thick

Rather small eyes.

Impetuous to the highest degree

Irascible (hasty) Kind to the poor

Sympathy remarkable

Long intimacy never heard from his mouth a single loose word.

[Notes on Ockham and Machiavelli]

Item 9 *MS 482: Fall 1883–Fall 1884*

Ockham d. Munich 1347 b Ockham in Surrey
Studied at Merton in Oxford
1300 by merit made Archdeacon of Stowe, but refused

Accepted other benefices; then gave them up to enter austere order St. Francis. Had Duns Scotus for master (No doubt). *Doctor invincibilis Venerabilis inceptor.* Began with logic.—Took violent part against popes in favor of civil power.

1321 Franciscan. Berengarius Talo. pronounced every private property of ecclesiastic abuse. Sentenced by Pope. Defended by all Franciscans. William wrote manifesto which was sent by Pope to French bishops. Ockham & Co were imprisoned Avignon. Took flight. Protected by Louis King of Bavaria. For 30 years continued to assail abuses of church.

Macchiavelli

Born 1469 Florence died 1527. One of most ancient families of Florence. Guelph. 66 of them held public offices. Did not become rich. Father jurisconsult & treasurer of the march of Ancona. Mother *Nelli* family, very good & distinguished family. Herself poetess. Father died when he was 16. Was brought up by mother. Never knew Latin well. The quotations necessary to adorn his works were furnished by Marcello Virgilio

1498 appointed chancellor of 2nd chancellery
also secretary of the office of the 10 judges of the peace
(who formed the government of the republic)

Florentine Secretary.

1512 Revolution. Secretary conducted general correspondence of republic. Registered deliberations of council, & drew up treaties.

These were the least part of duties. Represented Florence in 23 legations.

Intimate relations with Cesare Borgia

Got up an organized Florentine militia in 1506. Great trouble opposition inertia.

Narrations clear & agreeably written. Remarks on men & things //acute/fine// & judicious. Conversations reported in a lively & characteristic way. Macaulay

Got himself into diplomatic difficulties between Pope Julius II and Louis XII of France. Florence offered Pisa for seat of council against the pope—who delivered city to Medici by aid of Maximilian who had guaranteed integrity of Florence for 40000 ducats.

Macchiavelli deprived of office. Exiled recalled, arrested tortured

Too poor to live in Florence, retired to his estate La Strada

Then wrote *The Prince.* Intended simply to restore him to office.

Comedies

Discours on Titus Livius

1527 Medicis driven out. Macchiavelli's attachment to them kept him out of office. Died of chagrin. Left widow & *five* children in great poverty.

[Notes on Pythagoras]

Item 10 *MS 483: Fall 1883–Fall 1884*

Pythagoras born 569 died 470 B.C. 99 years old. Native Samos. Father Mnesarchus received freedom of Samos for having provisioned the island during a famine. He was without doubt a superior as well as wealthy man, and traced ancestry to Apollo. Had a very lucrative commerce with Egypt, Sicily, & Italy. The tyrant of Samos was Polycrates, allied with Pharaohs sent mercenaries to Egypt, was a cultivated King, supported Anacreon, Ibycus (poet), Theodorus (artist). Pythagoras as a child accompanied his father on a voyage or voyages. He had great passion for music from childhood. 551 (18 years old) with aid of Hermodamas his master fled from Samos (the tyrant having forbidden young men to leave) in order to Study. At Lesbos, he met Pherecydes. In 549 (20 years) at Miletus, studied under Anaximander and Thales. The latter advised him to go to Egypt

He set sail. Touched at Sidon, put himself in communication with the priests there—

On reaching Egypt repaired to royal city of Memphis. He wanted to be initiated into the priesthood. Being a caste there was great difficulty. Finally obtained order of King Amasis for this. Still had great difficulty—circumlocution office—finally received into noviciate by priests of Thebes. Terrible ordeal (Herod Book II). Passed with astonishing determination & courage. Circumcised.

Remained from 22 years to 44 years. Country conquered by Cambyses & Pythagoras with other priests carried captive to Babylon. There allied himself with Chaldean priests and magi.

In Egypt he had learned, divinity, geometry, arithmetic, transmigration of souls. In Babylon, astronomy & magical medicine. He met Zoroaster

Remained 12 years in Babylon and in 512 (aged 57 years) with the aid of Democedes, doctor of Darius, escaped to Greece.

Found his parents still living in Samos. At Delos placed a garland on the altar of ἀναιμάκτου τοῦ Γενήτορος Ἀπόλλωνος. Tended the sick-bed of Pherecydes till he died of *lice.* In Crete, was initiated by Epimenides in the mysteries of Idean Zeus.

Tried to found a school at Samos. Failed. In 510 (aged 59) landed at Sybaris. Going from there to Tarentum bought a whole seine of fish in order to set them free. At Cretona, he produced a profound impression. His public lectures were fashionable & thronged by persons who expected to hear curious theories. But he confined himself to practical morals. The inhabitants conferred on him the freedom of the city & appointed him censor of morals.

Opened a school. Married one of his pupils. She succeeded him to head of school. He had by her 7 children, 3 male 4 female

Two classes of scholars, *acousmatics & mathematics*—or hearers & students.

At Cretona Pythagoras was centre of aristocratic party. In a revolution at Sybaris certain aristocrats were driven out. Deputation sent to negotiate their return, among them friends of Pythagoras were murdered. Cretona at instigation of Pythagoras declared war 509 BC (aged 60) lasted 70 days ended in destruction of Sybaris. Pythagoras received a farm as booty. A rich man left him a fortune.

Organized a college σύστημα. In centre auditorium ὁμακόειον. Around gymnasium dormitories refectories συσσίτια. Motto of the college was κοινὰ τὰ τῶν φίλων. Very remarkable & deep motto this is as the motto of a school. Its *raison d'être.* These κοινὰ what are they? Material or spiritual goods?

Examination began by thorough physical scrutiny, scholar stript and every muscle, every organ examined. Physiognomy carefully noted. They next considered their biography, their physical instincts, then their habits, their tastes, their sentiments, their talents. If they passed this examination satisfactorily they were entered on probation. Afterward some were occasionally dismissed, their money & outfit being given back.

Such persons were treated as dead—& tombs were erected to them. The course of study began early & lasted 5 years. The first years learned to obey, to be silent, to be humble. Before speaking the pupil was taught to consider

The *acoustic* or *exoteric* pupils did not see Pythagoras at all, only heard him through a curtain lecture to the others. The day of admission as *mathematic* or *esoteric* was celebrated as a feast. Not all were admitted as *esoterics.* These latter had a right to take notes & write out the lectures.

The principle was We know as much as we remember. The exoterics committed to memory golden sayings. Honor first the Gods, then the heroes. Honor thy father & mother & next of kin. Learn to rule thyself in thy stomach & in sleep, in mildness & in rage. Reverence thyself. Do justice by word & act; act not inconsiderately and remember that we all have to die;—Keep the golden mean in drinking, eating, & exercising—Before sleeping review the deeds of the day & ask yourself. How have I been at fault, what have I done that I ought not to have done & what omitted

—Courage, the race of men are sprung from the Gods.

Nature showeth thee her secrets, it is thine to look at them—After life we leave our bodies and become Gods immortal.

There was also a catechism. What are the isles of the blessed? The Sun & Moon. What is it that the oracles at Delphi give? τετρακτύς. The ten $1 + 2 + 3 + 4$. What is the song of the Sirens? The world.

What is wisest? Numbers & the meanings of words applied to things. What is most beautiful? Music. What most powerful? γνώμη

Do not sit on a quart

Dont shake hands with too many

Dont eat your heart

Avoid beans

One, two

Hail, holy number, generator of gods & men. Zeus, is τέτρας, tetrad. He contains

1st Ether, pure space or monad, the male principle

2nd Matter, the dyad or female principle.

3rd Time, the triad, past present & future. ἀγήραος—unaging

4th Necessity, the περιέχον or encircling, the eternal night.

[Notes on Rabelais]

Item 11 *MS 484: Fall 1883–Fall 1884*

Rabelais

Son of innkeeper, apothecary, both or neither. Youngest of several brothers b (1495) *Sign of Lamprey.* This father had a house at Chinon in Touraine which was a cabaret later & in neighborhood he had the *clos of la Devinière* renowned for excellent wine. At the neighboring village of Seully, young Rabelais was put to school about 10 years old. He learned nothing & was sent to convent of la Baumette near Angers or to University of Angers. He there formed acquaintance of several boys afterward eminent in church.

Rabelais at his parents desire became *cordelier,* taking vow of ignorance & was made priest 1511. (But this date is wrong, 1519 is nearer right.) Remained 15 years in this convent at Fontenay-le-Comte in Poitou. Learned to love letters & hate monks. In 1523 Greek books were discovered in his cell. Confiscated & he imprisoned. Took flight with friend in same box.

Budaeus and others protected him & books were restored

Clement VII gave indulgence to change to Benedictines but he didn't stay long & left without leave, & took habit of secular priest, & began life of vagabond doctor & priest combined. Nothing very shocking about this.

Worked in bed. Was excellent theologian, mathematician, physician, jurisconsult, musician, astronomer, painter & poet.

1524–1530 supposed to be at Universities of Bourges & Paris

1534 Dismissed from post as doctor to great Hotel-Dieu of Lyon because twice absent without leave.

Employed by book seller & even made almanacs with predictions accompanied however with pious demonstrations that such predictions could be good for nothing.

Grateful. Did not forget old friends

His celebrated book written during meals to amuse his patients 1532

Became doctor to Cardinal Jean du Bellay.

Learned arabic. Studied archeology.

Demanded absolution of the pope. Granted in a bull 17 Jan 1536. Made M.D. 22 May 1537

Got a new bull to give him further liberty to do just what he was doing anyway

Adresse & esprit. Obtained privilege of printing 3rd Book of heroic deeds & sayings of Pantagruel

Rabelais always persecuted by monks & theologians, protected by prelates & princes

Death of Francis I put him to flight. He went to Rome for protection of Card. Bellay

Same time courted Bellays great enemy Chastillon

Soon after Curé of MEUDON from Bellays. Made a good *curé*. Resigned this curacy before publishing the 4th book of Pantagruel

Died 1553 in Paris

Questions on Great Men
[First Questionnaire]

Item 12 *MS 485: Fall 1883–Fall 1884*

Parents.

(5) Noble, bourgeois, or peasant?
(8) Parents' age?
10) Family ability?
52) Period of gestation?
53) Hereditary peculiarities in both directions?

Brothers and sisters.

(9) How many older?
11) Total number?

Childhood.

26) Childish precocity?
50) Peculiar education?
51) A solitary child?

Youth.

14) Favorableness of early circumstances?
27) Strong bent?
57) Wild oats?

Physique.

61) Size?
43) Health?

60) Strength?
60bis) Beauty?
55) Left-handed?
62) Other peculiarities?
62bis) Complexion?
65) How much sleep did he take?

Effect of age.

25) At what age did originality first strongly appear?
47) First great work at what age?
46) Best work at what age?
19) Was power retained to old age?
39) Was power of learning a new thing retained to advanced age?
18) Age at death?

VII Progeny.

59) Married? at what age?
12) Number of children?
13) Ability of children?

VIII Milieu.

(4) Century?
(3) Nation?
22) Did he appear at a particularly favorable moment?
17) Did he have any extraordinary good fortune?
16) Did he live in an age particularly given to the sort of thing that distinguished him?
15) Did his immediate predecessors or contemporaries come near to doing the thing that mainly distinguished him?
21) What did he find ready-made to his hand?
45) How appreciated by his contemporaries?
40) What sentiments did he excite in those who surrounded him?
41) Was he greatly beloved by anyone much above the average?
23) Did he begin by imitating a model?
24) Was that exemplar a personal acquaintance?
28) Did he develope greatness in other men?
(7) Followed by imitators?
66) Natural surroundings in childhood & youth?
67) Were his times especially in youth stormy?

IX Work.

(1) Kind of greatness?
(2) Degree of greatness?
20) What did he create?
30) What was his greatest conception?
29) Did he make much use of other men?
63) Did he work in bed or with his feet up?
64) Did he overwork so as to require cessation?
68) At what time of the day did he preferably work?
69) Did he work under stimulants?

X Genius.

31) Hallucinations
34) Excited by work
48) Work intermittent or continual
36) Strength of will
49) Energy
37) Delicacy of perception
38) Capacity of learning
33) Memory
32) Patience
35) Urged by a great longing to his work

XI Other peculiarities.

(6) Weaknesses moral
(6bis) " mental
56) Moral character
58) Religious character
42) Self-estimate
44) Opinions about nature of greatness

Questions on Great Men [Revised Questionnaire]

Item 13 ***MS 516: 8 November 1884***

1884, Nov. 8.

FAMILY.

Noble, commoner, or peasant?
Parents' age at birth of subject?
Family ability?
Mother's character?
Period of gestation?
Hereditary peculiarities in both directions?
How many older brothers and sisters?
Total number brothers and sisters?
At what age married?
Number of children?
Ability of children?

CHILDHOOD.

General facts
Childish precocity?
Peculiar education?

YOUTH.

General facts.
Wealthy?

Whether naturally thrown into the pursuit in which he was distinguished?
Circumstances otherwise favorable to development of his geni/us?/
Strong bent for pursuit in which distinguished?

PHYSIQUE.

Height?
Length of body
Length of legs.
Weight
Build of trunk
Build of limbs
Build of neck and head.
Whether any deformity.
Digestion good?
General health?
Cause of death?
Strength?
Beauty?
Left-handed?
Other peculiarities.

EFFECTS OF AGE.

General facts.
At what age originality first strongly appeared?
First great work at what age?
Best work at what age?
Was power retained to old age?
Age at death compared with expectation of life at age of greatest work?

ENVIRONMENT.

Century
Nation
Was his age much given to the sort of thing that he did?
How appreciated by his contemporaries?
What sentiments did he excite in those immediately about him?
Surrounding aspects of nature in childhood and youth and his taste therefor?

CHARACTER.

How much muscular exercise and what?
Sexuality, how shown?
Habits of eating
Taste, smell, touch, hearing, sight.
Whether musical?
Power of expression?
Perception of others' feelings
Insight into character
Fond of society?
Tenderness
Tendency to dwell on his feelings
Power of guessing? Shrewdness?
Memory, what kind?
Imagination, what kind?
Love of the marvellous
Gambling, novel-reading, etc.
Facility in learning
Tendency to change his opinions
Generalizing power
Prudence
Patience
Scrupulousness of honour and sense of justice.
Firmness and steadiness of purpose.
Animal courage, cheerfulness, coolness, resolution.
Irascibility.
Conceit, vanity, pride.
Love of power and power of command.
Deductive and mathematical power
Whether devout?
What extraordinary weaknesses
Opinion about the nature of greatness
Other interesting facts.

WORK AND GENIUS.

Kind of greatness
Degree of greatness
How much did he work?
How intensely did he work?
At what time of day did he preferably work?

Did he need to be alone to work?
Did he work under stimulants?
Other physical peculiarities and habits about working?
Did he overwork so as to require cessation?
Hallucinations
Excited by work?
Work intermittent or continual?
Inwardly urged to his work?

NAME.

FAMILY. Rank: Parents' age: Gestation:

Family ability: Longevity:

Mother:

Hereditary traits:

Parent's children: Married when: Children:

Children's ability:

CHILDHOOD:

Education:

Precocity:

YOUTH:

Wealth: Natural destiny:

Circumstances:

Bent:

PHYSIQUE: Sleep:

Height:	body to legs:	Frame:	Weight:
Build trunk:	limbs:	Set head:	Deformity:
Hair:	Head:	Forehead:	Ears:
Eyes:	Nose:	Lips:	Chin:
Fingers:	Hand-writing:	Skin:	Temperament:

Digestion: Health: Cause of death:

Strength: Beauty: Left-handed?

AGEING:

1st originality: 1st great work: Best work:

Retention of power: Fulfil. Expect. Life:

ENVIRONMENT. Century: Nation: Aspect of nature:

Spirit of times:

Contemporary appreciation:

Impression on intimates:

WORK. Long? Hard? When?

Alone? Stimulants?

How?

Excitedly? Continuously?

Desire for work: Overwork:

First recto, of a folded sheet, of Peirce's Questionnaire for the Study of Great Men

GREATNESS, Kind:	Degree:
CHARACTER:	Muscular exercise:
Eating:	
Sexuality:	
Senses:	
Music:	Eloquence:
Sympathy:	Generosity:
Insight:	
Society:	Fashion:
Tenderness:	
Subjectivity:	Impressionability:
Shrewdness:	Memory:
Sense of beauty:	Imagination:
Love of marvel:	Gambling, etc.
Facility in learning:	Change of opinions:
Generalizing:	
Prudence:	Acquisitiveness:
Patience:	
Honor and justice:	Sense of duty:
Steadiness and firmness:	
Courage:	
Decision:	
Irascibility:	Malice:
Conceit:	
Command:	
Mathematics:	Concentration:
Religion:	Public spirit:
Relaxations:	Humor:
Weaknesses:	
Opinion about greatness:	
Hallucinations:	
Absence of mind:	
Hypnotism:	Miracles:
Other facts:	
Biographies:	

Second recto of Peirce's Questionnaire, printed November-December 1884 (Max Fisch Papers)

[Remarks on Questions from the Final Questionnaire]

Item 14 ***MS 520: November–December 1884***

I have inserted remark[s] to explain some of the questio[ns]

NAME. We will call the person M

FAMILY. Rank and status **Parents' age:** at M's birth
Gestation: Whether M was a 7 months child. **Longevity:** in the family.
Mother: Great men are said to derive their qualities chiefly from their mothers. Would M's case support this or not? How favorable or unfavorable.
Parent's children: How many and whether of marked ability or vitality. **Married when:** That is when did M marry
Children: What children did M have.

CHILDHOOD: Its general circumstances.
Education: When began? How conducted? What was taught *in childhood?*
Precocity: as *a child.*

YOUTH: Its general circumstances.
Circumstances: That is, what particular surroundings influenced the future.
Bent: Whether as a youth a decided passion or bent for special pursuits.

PHYSIQUE:
Ratio body to legs: **Frame:** Large or small bones

AGEING:
1st originality: At what age? **1st great work:** Date. **Best work:** Date

WORK. **When?** That is, at what hours and seasons? **How?** Method and ways and tricks in working.

CHARACTER:
Subjectivity: Whether given to thinking about self and sensations or to losing oneself. Can he without difficulty at once tell how he feels at any moment, whether suffering any pain, etc.
Change of opinions: Does he readily change or the reverse.
Miracles: Has anything ever occurred of a strange and apparently supernatural kind.

[Questionnaire Responses for Michelangelo, Hobbes, and Locke]

Item 15 ***MS 522: November 1884–1890***

NAME. Michel Angelo (or Àgnolo) Buonarroti (or Buonarotti) Simoni

FAMILY. Rank: Poor & small nobility **Parents' age:** F. 31 M. 19
Gestation: Full
Mother fell from horse without injury.
Family ability: Fairly good. Held some offices in Florence **Longevity:** F 90 M 42. Paternal uncle 74. Gr. father 60 Gr. mother Alessandra
Mother: Francesca di Neri di Miniato del Sera
Brothers and S., He, 2nd **total:** 5 with him all male. **Married when:** Never
Children: None.

CHILDHOOD: Born at Caprese. Brought up at Settignano.
Education: Left with nurse (stonemason's wife) at Settignano. Then to grammar school in Florence. Idled at school. Would only draw.
Precocity: Began to draw as soon as he could use his hands. These drawings on wall of house exhibited in 18th Century. Quite powerful.

YOUTH: 13 years old, insisted against opposition of his family on being apprenticed to Ghirlandaio. Very soon outstripped master. Taken up by Lorenzo.
Wealth: V. moderate circumstances till Lorenzo took him. Then well off. **Natural Destiny:** To be merchant; artist's life despised by his family
Circumstances: At Lorenzo's house, enough to fire the genius of anybody with a spark of it.
Bent: Very strong

PHYSIQUE: Active **Sleep:** Little. Often rose in the night to work
Height: Middle **body to legs:** Well proportioned **Frame:** High cheek bones Bony & muscular
Weight: Light **Build trunk:** Broad shoulder **limbs:** Slender. **Set head:** Italian
Deformity: Nose broken by Torregiano, an artist. **Hair:** Raven black **Head:** High narrow
Forehead: High square 7 lines across it **Ears:** Not large. Set low and far back. Lobe not long **Eyes:** Rather small. Color of horn. Specks of yellow & blue.
Nose: Broken **Lips:** Thin. Serious looking mouth **Chin:** Projecting chin
Hand-writing: Extremely legible & composed **Skin:** Complexion lively **Temperament:** Nervous.
Health: Had stone 1550 Got well. In 1500 had swelling in side. **Cause of death:** Old age. 89. Lingering fever.
Strength: Strong **Beauty:** Ugly **Left-handed?** No.

AGEING:
1st great work: Pietà. About 1500 **Best work:** Moses about 1510 Last Judgment 1533–41
Retention of power: Energy & power to 86. Many poems later.

ENVIRONMENT. Aspect of nature: If my mind is worth anything, I owe it to the clear air of your Arezzo country.
Taste of times: Just in his line
Contemporary appreciation: Worship.
Impression on intimates: "He is a terrible man." No withstanding his influence. Influenced Raphael

WORK. Long? Very. **Hard?** Furiously
Alone? Never would have anybody with him. No assistant **Stimulants?** No.
Continuously? Irregularly. Months in reverie. Then to work. After working furiously at a thing for months would drop it; afterwards take it up again
Desire for work: Intense

CHARACTER:
Eating: Abstemious.
Sexuality: When 60, fell in love with Vittoria Colonna. A Platonic passion. Otherwise, sexuality deficient. Wedded to his art.
Music: Composer. **Eloquence:** Wonderful compression & energy. His poetry untranslatable

Sympathy: V. great. Loved ordinary & commonplace people best. **Generosity:** V. generous.
Society: Passion for solitude, yet intimate with every eminent man in Italy.
Impressionability: Greatly impressed with Savonarola
Thrown into sort of panic more than once.
Memory: Great memory.
Imagination: Awful
Facility in learning: One cannot imagine how he learned all he knew.

NAME. Thomas Hobbes

FAMILY. Rank: Obscure **Gestation:** Premature birth owing to fright about Armada.
Family ability: Ordinary common sense **Longevity:** He and brother very long lived
Hereditary traits: Father a choleric man, and ignorant person.
Parent's children: He was second son **Married when:** Not

CHILDHOOD: In period following Armada.
Education: Good, especially in Greek. First got sight of Euclid after 40.
Precocity: Somewhat so for language. Trans. *Medea* into Latin verse before 14.

YOUTH:
Wealth: Supported by Uncle **Natural destiny:** Tutor to Wm. Cavendish
Circumstances: Disorder in University of Oxford. Scholasticism disappearing.
Bent: For Geography—After 20 for classics.

PHYSIQUE:
Height: Over 6 feet.
Hair: Jet black **Head:** Mallet shaped **Forehead:** ample
Nickname at school: *crow*
whiskers yellowish red.
In old age bald.
Eyes: Hazel. Lively **Nose:** Rather large **Lips:** Upper rather short
Not very large.
Chin: Rather prominent **Skin:** Fresh & ruddy after 40. Earlier yellow.
Digestion: Weak after 60. **Health:** Good **Cause of death:** strangury
Shaking palsy before 60.

AGEING:
1st originality: At 54 the "De Cive" **Best work:** Perhaps *Leviathan* at 63.
Retention of power: Wrote pretty well at 90.

ENVIRONMENT.
Impression on intimates: Bacon was fond of talking to him.

CHARACTER:
Sexuality: Moderate **Music:** Sang to himself when no one could hear
Sympathy: Sight of misery put him in pain **Change of opinions:** Never changed, no matter how absurd.
Sense of duty: Not remarkable. **Courage:** He says himself *Fear* was his twin brother.
Irascibility: Decided & pugnacious Not able to endure contradiction **Conceit:** Swore much. Undervalued all other men. "Irritabile et vanissimum Malmesburiense animal."
Mathematics: Bad **Humor:** Ready wit
Biographies: Three published together by R. B. 1681 and all in Vol. I of *Opera Latina* (Ed. Molesworth) of which one is Autobiog. In Aubrey's papers. Life of Mr. T. H. 1813
Croom Robertson 1886.

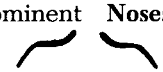
NAME. John Locke

FAMILY. Rank: Grandson Retired Tradesman
His mother a tradesman's daughter
good family
Family ability: Decidedly good. Nothing remarkable **Longevity:** Not? (Not remarkable vitality)
Mother: Pious & affectionate. Nothing more remarkable. Died when John a boy. Father a man of parts. Always mentioned by Locke with great respect & affection.
Hereditary traits: Deficient sexuality.
Parent's children: John eldest. One other only, Thomas, 5 years later. Died childless. **Married when:** Locke never married

CHILDHOOD: 14 years at home with father & brother. 10 years old when Civil War close by. Father joined army (parliamentary) Returned within 2 years.
Education: Taught by father. **Precocity:** No record.

YOUTH: 14 to 20 Westminster school. Stern Dr. Busby. This school always condemned by Locke for verbal learning. 20 to 50 Oxford.
Wealth: Small property, //about $\frac{1}{2}$/not//enough to live on. **Natural destiny:** Church
Circumstances: "I no sooner perceived myself in the world but I found myself in a storm."
Bent: Brought up under books of Burgersdicius(?) Did not study much. Hated scholastic disputation. Discontented with Oxford. Conversation & Correspondence his delight. Read first romances. Later

Descartes. Delighted in his lucidity which encouraged him. Became a sort of Doctor.

PHYSIQUE: Weak
Height: Short **Build trunk:** spare **Set head:** Graceful
neck long
sloping shoulders
Forehead: Peculiar **Eyes:** rather prominent **Nose:** very large pointed
Eyebrows  high origin

Lips: handsome not full **Chin:** good but not large
Health: Always delicate **Cause of death:** Consumption
Asthma.

AGEING: First publication some anonymous articles at 54 years of age.
1st originality: About 57 **1st great work:** *Essay* May 1689 **Best work:** (57)
Retention of power: Complete to last. **Fulfil. Expect. Life:** Died at 72. Too early(?)

ENVIRONMENT. Born 1632. Rebellion
Century: Restauration Revolution **Nation:** England.
Spirit of times: Tussle with tyranny. Lax morals. Awakening science. Descartes writing in Locke's boyhood.

WORK. Hard? Diligent **How?** Methodical in the extreme

GREATNESS, Kind: *Thinker* (with practical tendencies)

CHARACTER: **Muscular exercise:** Riding favorite
Sexuality: Deficient **Sympathy:** Very strong friendships **Generosity:** Charitable Moderate generosity
Society: A man of tender feelings yearning for the society of his friends
Fashion: Always shy of a crowd of strangers **Shrewdness:** At last, has sense of losing skill of dealing with men
Imagination: His verses contain little poetry. **Love of marvel:** Fond of books of travel
Gambling, etc. Nothing more remote from him. Probability was his guiding star.
Change of opinions: Never changed unless perhaps very early. Inconsistent in not tolerating atheists
Generalizing: Extraordinary **Prudence:** Very marked. Shaftesbury ruined by not listening to him Cautious
Acquisitiveness: "Riches may be purchased too dear." **Sense of duty:** Very strong Fraser p. 244
Irascibility: Compares himself to Horace in this respect Choleric, but soon appeased **Religion:** Strong & deep.

Public spirit: Very great **Humor:** Sprightly marked
Wit & irony

Biographies: Le Clerc. *Éloge historique.* 1705 (most important)
Lord King. *Life* 1830
Fox Bourne 1876
Fraser 1890

[Questionnaire Responses for Montaigne, Palissy, Machiavelli, and Lessing]

Item 16 ***MS 488: Fall 1883–Fall 1884***

Montaigne

Parents
5 Noble
8 Father 38
10. Considerable family ability
52 Eleven months
53. Detestation of Medicine & doctors. Hereditary stone in bladder

Br. & sisters
9 Two older brothers. Sisters?
11. Two other brothers & a sister Father left 5 sons 3 dau. 2 sons had died.

Childhood
26 Original opinions but stupid
50. Very peculiar education
51. Sent to school about 6.

Youth
14 Highly fav. circums.
27 No bent
57. Some wild oats

Physique
61 Very small
43 Delicate. Good until 47. Then bad.
60 Weak originally strong
60bis Full face. Agreeable face. Squat
55 No
62.
62bis Brown hair

Effect age
25 Before thirty; disgust for law & cruelty. Says himself his maximum was at 30
47 At 47 first essays
46 At 52 Best essays
19 Died
39 Doubtful
18 59

Progeny
59 Married 33
12 Six daughters (only one lived)
13

Milieu
4. XVI
3 French (English descent)
22 Yes
17. Yes

16	Yes
15	Yes
21	French language
45	Much admired
40	
41	Boetie
23	
24	
28	
7	Yes, countless
	Work
1.	Literary
2.	
20.	The Essay
30	Scepticism. What do I know? Pleasure the only aim
29	No
63.	No
64	No
	Genius
31	No
34	No
48	Intermittent
36	Small
49	Average or less.
37	V. great indeed. Excessively impressionable. Unusually delicate sense of smell. Sensitive to music
38	V. good
33	Very bad indeed
32	
35	No
	———
6	Coward.
6bis	Scepticism. Extreme vanity. Bad mem.
56	Idle cold proud
58	
42.	He dont want his style to be admired but his matter. The amount of it. Thinks his *Essays* not for highest nor for lowest intelligence. Thinks his *Essays* will live 50 years only because French language so changeable a state. But adds that good writings will fix it.
44	

B. Palissy

5.	Noble
8	Unknown
10.	Unknown
52.	Unknown
53.	Father worked in glass Descendents potters?
	———
9	
11	
	———
26	
50	Imperfect
51.	
	———
14.	Unfavorable
27.	
57.	
	———
61	
43.	
60	
60b.	
55	
62.	
62b.	
	———
47.	
46.	
19	
39	
18	
25	

	———
59.	Yes
12.	Many
13.	
	———
4.	XVI
3.	France
22.	
17.	No
16.	Yes
15.	Yes
21.	Italian fayence
45.	Tolerably
40.	
41	
23.	Yes
24.	
28	
7.	Yes
	———
1.	Courage
2.	
20.	Style of pottery
30.	
29	
	———
31	
34.	Yes
48.	Continual
36.	Immense
49.	Great
37.	Considerable
38.	Great
33.	
32.	Unbounded, awful
35.	
	———
6	
6bis	
56	
58	
42	High
44	Very hopeful

Machiavelli

5.	Small noble
8.	
10.	Considerable
52.	
53.	
9.	
11.	
26	
50	
51	
14.	Good
27.	Probably not
57	
61	Middling. Thin
43	Apparently good
60	
60bis	
55	
62	
62bis	olive
47	*Prince* aet. 44
46	Ditto
19	Didn't live old
39	
18	58.
25	About 30 probably
59	Married
12.	4 sons 1 dau.
13	Not known
4.	XVI
3	Italy
22.	Yes to write *Prince*
17.	Rather
16.	Yes
15.	Probably
21.	Villainy
45.	Not highly
40	
41.	No
23.	?
28.	?
7.	No
1	Literary, Perceptive
2.	

20. System of villainy
30. That virtue has no place in politics
29. No
31 No
34. Probably not
48. Continual
36. Good
49 Great
37. V. great

38
33
32 Good
35 No
6. Truckler
6bis
56. Honest but doublefaced
58
42.
44

Lessing

5 Bourgeois
8 Father 36 Mother
10 Ggr. review painter. Marked power rectitude enlightenment
52
53.

———

9.
11 9 Broth. 2 Sis.

———

26. No
50. No
51. No

———

14. Fair
27 Yes
57 Moral but Bohemian. Gambling

———

61 Large
43 Always bad
60 V. strong active
60bis Handsome
55
62 Careful about person
62bis

———

47. *Laocoon* 38, *Nathan* 50
46 Powers failed early

19
39 No
18. 52
25. About 21

———

59. Yes.
12. one
13. Died infant

———

4 XVIII
3. German
22. Yes
17. No
16. Yes?
15 No
21.
45. Very highly
40. Wonderful charm
41.
23. Molière
28
7.

———

1 Literary
2
20. German comedy Lit. criticism
30
29 No
31 No
34.

48	Continual?	35.	Yes.
36	Good	6	Dilatory. Fickle
49	Good	6bis	Unpractical
37.	V. great	56	High
38.	Good	58	Strong
33.	Poor?	42	Modest
32.		44	

[Questionnaire Responses for Short List of 48 Great Men]

Item 17 *MS 489: Fall 1883–Fall 1884*

Milieu

Question 40. What sentiments did he excite in those who surrounded him?
Question 41. Was he greatly beloved by anyone much above the average?

	(40)	(41)
Alexander.	Awe & admiration.	No
Alcibiades.	Admiration & envy.	Socrates
Attila.	? (Commanding presence)	Guess not
Augustus.	Admiration & respect	Caesar when he was a boy Horace?
Barneveldt.	Respect & confidence	Grotius?
Bolivar.	?	?
Bossuet.	Boundless admiration & honor.	?
Copernicus.	Respect	?
Cellini.	Antagonism & admiration	?
Chasles.	Entertained a good deal.	No
Comte.	Not affection.	No
Constantine.	?	No
David.	Admiration & affection.	Jonathan
Euler.		?
Faraday.	Affection & admiration.	[? Graham] (Tyndall) (Dr. Bence Jones)
Galileo.	?	?
Gilbert.	Affable	No record
Homer.	?	?
Helmholtz.	Very much liked. Very pleasant.	Ludwig & ———

	(40)	(41)
Humboldt.	V. kind & amiable but rather vain & given to prosiness.	W. v. Humboldt Agassiz.
Hampden.	?	
Hunter.	Quiet, taciturn	No?
Julian.	Admiration.	[Eusebius?]
Lagrange.	?	No
Lessing.	Wonderful charm	No
Leonidas.		?
Machiavelli.	?	No.
Mencius.	Greatest respect and esteem	? Disciples
Montaigne.	?	La Boétie
Morphy.	Amiable	?
Occam.	?	?
Palissy.	?	No.
Phidias.	?	Pericles
Pythagoras.	Reverence & awe	A number
Rabelais.	Was liked; warm friend	Yes
Rienzi.	Most persuasive	Ariosto
Shakespeare.	?	Probably
Scott.	Jovial	?
Emerson.	Reverence	Carlyle. Alcott. Margaret Fuller Thoreau. Whittier.
Swedenborg.	?	?
Talma.	?	?
Turenne.	?	?
Toussaint.	?	?
Virgil.	Greatest affection & esteem	Horace. Maecenas Augustus
Wellington.	?	The Queen
Young.	?	?
Grotius.	?	Barneveldt
Moltke.	Respect. Taciturn	?

Milieu

Question 28. Did he develope greatness in other men?
Question 7. Followed by imitators?

	(28)	(7)
Alexander.	Yes	No
Alcibiades.	No	No
Attila.	No	No
Augustus.	Yes	No
Barneveldt.	?	No
Bolivar.	(? No)	No
Bossuet.	?	Probably
Copernicus.	No	No
Cellini.	No	?
Chasles.	No?	?
Comte.	Yes?	No
Constantine.	?	No
David.	?	No
Euler.	No.	Hardly
Faraday.	Yes	Tyndall
Galileo.	No	No
Gilbert.	No	No
Homer.	No	?
Helmholtz.	Yes	No
Humboldt.	Yes	Yes
Hampden.	Yes?	No
Hunter.	Yes	Yes

	(28)	(7)
Julian.	?	No
Lagrange.	?	Yes
Lessing.	No.	?
Leonidas.	(Yes.)	No
Machiavelli.	No	No
Mencius.	Yes?	?
Montaigne.	No	Yes
Morphy.	No	?
Occam.	Yes?	No
Palissy.	No	?
Phidias.	Yes	Yes
Pythagoras.	Yes	No
Rabelais.	No	?
Rienzi.	No	No
Shakespeare.	No.	No
Scott.	No	Yes
Emerson.	Yes	Yes
Swedenborg.	No	Hardly
Talma.	?	? Probably
Turenne.	?	?
Toussaint.	?	?
Virgil.	No	Yes
Wellington.	?	Not
Young.	No?	?
Grotius.	?	?
Moltke.	Yes?	Isn't time.

Milieu

Question 66. Natural surroundings in childhood & youth?
Question 67. Were his times especially in youth stormy?

	(66)	(67)
Alexander.	City. Picturesque	Yes
Alcibiades.	Beautiful city. Picturesque	Not specially
Attila.	?	Yes
Augustus.	Rome.	Yes, decidedly
Barneveldt.	Flat country	Yes
Bolivar.	Caracas. Picturesque	Yes?
Bossuet.	?	No
Copernicus.	?	No?
Cellini.	Near Florence beautiful country.	Yes
Chasles.	?	Yes
Comte.	Montpellier	Yes
Constantine.	Rome?	Yes
David.	?	Yes
Euler.	Basel. Swiss.	?
Faraday.	Village	No
Galileo.	Most beautiful cathedral Country not remarkable	No?
Gilbert.	London	No
Homer.	?? Probably picturesque	?
Helmholtz.	Berlin. Ugly city. Ugly country	No
Humboldt.	?	Yes
Hampden.	Lovely country	No?
Hunter.	? Lancashire	No
Julian.	Constantinople. Prob. beautiful	Yes

	(66)	(67)
Lagrange.	Turin (Plain city. Alps in the distance.)	No
Lessing.	?	No.
Leonidas.	City.	Yes
Machiavelli.	Florence. Beautiful city & beautiful country	Yes
Mencius.	?	No
Montaigne.	Pretty place	Yes
Morphy.	?	No
Occam.	Ockham	No
Palissy.	?	No
Phidias.	?	No
Pythagoras.	Picturesque island	No
Rabelais.	?	No
Rienzi.	Born in Rome, brought up in Campagna.	Yes
Shakespeare.	Pretty village	No
Scott.	Very picturesque & mountains	No
Emerson.	Boston & Concord. No beauty Concord tolerably pretty.	No
Swedenborg.	? Stockholm.	No
Talma.	?	No
Turenne.	?	Yes?
Toussaint.	?	No?
Virgil.	?	No
Wellington.	?	No
Young.	?	No
Grotius.	Flat country	Yes
Moltke.	Flat country	Yes

Work

Question 1. Kind of greatness?
Question 2. Degree of greatness?

	(1)	(2)
Alexander.	*Action.* Ruler.	1.7
Alcibiades.	*Action.* General	6.0
Attila.	*Action* General	4.4
Augustus.	*Action* Ruler	2.1
Barneveldt.	*Action* Statesman	3.9
Bolivar.	*Action.* General & statesman	4.0
Bossuet.	*Thought* Religionist & orator	3.5
Copernicus.	*Thought* Astronomer	3.7
Cellini.	*Feeling.* Artist	5.0
Chasles.	*Thought.* Math.	4.8
Comte.	*Thought* Philos.	3.4
Constantine.	*Action* Ruler.	2.8
David.	*Action* Ruler	3
Euler.	*Thought* Math.	2.0
Faraday.	*Thought* Physicist	3.5
Galileo.	*Thought* Physicist	1.5
Gilbert.	*Thought* Physicist	2.7
Homer.	*Feeling.* Poet	1.3
Helmholtz.	*Thought.* Physicist & math. & physiol.	2.9
Humboldt.	*Thought* Physicist	3.1
Hampden.	*Action* Statesman	4.6
Hunter.	*Thought* Biologist	3.0

	(1)	(2)
Julian.	*Action* Ruler	4.0
Lagrange.	*Thought* Math.	3.5
Lessing.	*Feeling* Dramatist & critic	3.6
Leonidas.	*Action* General	5.4
Machiavelli.	*Feeling* Writer	4.8
Mencius.	*Thought* Moralist	2.8
Montaigne.	*Feeling* Writer	4.8
Morphy.	*Thought* Chess player	5.4
Occam.	*Thought* Philosopher & Reformer	2.5
Palissy.	*Action* Personality	3.0
Phidias.	*Feeling* Artist	1.4
Pythagoras.	*Thought* Moralist & Philos. & Math.	1.1
Rabelais.	*Feeling* Writer	4.6
Rienzi.	*Action* Personality	5.5
Shakespeare.	*Feeling* Poet & dram.	1.0
Scott.	*Feeling* Novelist & poet	5.2
Emerson.	*Feeling* Writer & phil. & poet	4.2
Swedenborg.	*Thought* Religionist	4.1
Talma.	*Feeling* Actor	5.2
Turenne.	*Action* General	3.2
Toussaint.	*Action* General	4.3
Virgil.	*Feeling* Poet	3.2
Wellington.	*Action* General & statesman	4.2
Young.	*Thought* Physicist	3.1
Grotius.	*Thought* Statesman & Publicist	4.1
Moltke.	*Action* General & strategist	3.7

Work

Question 20. What did he create?
Question 30. What was his greatest conception?

	(20)	(30)
Alexander.	An empire.	World-state
Alcibiades.		
Attila.	A temporary power	Did not seem to have any? Scourge of God.
Augustus.		
Barneveldt.	Conserved. Did not create	Free & united people
Bolivar.		
Bossuet.	Style of oratory. Éloge	Independent national church.
Copernicus.		
Cellini.	A style in art.	
Chasles.		
Comte.	A philosophy	
Constantine.		
David.	A kingdom.	Building the temple with what that implied.
Euler.		
Faraday.	Theory of electricity.	Lines of force
Galileo.		
Gilbert.	Magnetism.	?
Homer.		
Helmholtz.	The union of physiology with physics & mathematics.	Conservation of force Scientific empiricism
Humboldt.		
Hampden.	Did not create. Spirit of resistance.	
Hunter.		
Julian.	Revival of Hellenism Not a creator. Governor	

	(20)	(30)
Lagrange.		
Lessing.	German drama.	Religion independent of creeds.
Leonidas.		
Machiavelli.	Formulated the immorality of Italian diplomacy.	That good faith merely an ornament of life—while desirable should not be allowed to interfere with serious consideration
Mencius.		
Montaigne.	The modern essay. Abs. scepticism.	Que sais-je?
Morphy.		
Occam.	Nominalism.	
Palissy.		
Phidias.	A style of art. Parthenon	
Pythagoras.		
Rabelais.	Style of writing.	
Rienzi.		
Shakespeare.	Certain plays.	?
Scott.		
Emerson.	A style & a following	Over-soul?
Swedenborg.		
Talma.	A style of acting. Several characters.	?
Turenne.		
Toussaint.	Disciplined government among negroes.	
Virgil.		
Wellington.	Nothing.	
Young.		
Grotius.	Science of jurisprudence.	
Moltke.		

Work

Question 29. Did he make much use of other men?
Question 63. Did he work in bed or with his feet up?

	(29)	(63)
Alexander.	Yes.	?
Alcibiades.		
Attila.	Yes	?
Augustus.		
Barneveldt.	Yes	?
Bolivar.		
Bossuet.	No	?
Copernicus.		
Cellini.	No	No
Chasles.		
Comte.	No	?
Constantine.		
David.	Yes.	?
Euler.		
Faraday.	No	No
Galileo.		
Gilbert.	No	?
Homer.		
Helmholtz.	?	?
Humboldt.		
Hampden.	No	No.
Hunter.		
Julian.	No	?

	(29)	(63)
Lagrange.		
Lessing.	No	?
Leonidas.		
Machiavelli.	No	?
Mencius.		
Montaigne.	No.	No
Morphy.		
Occam.	No	?
Palissy.		
Phidias.	Yes	?
Pythagoras.		
Rabelais.	No	Wrote his book while eating Worked in bed.
Rienzi.		
Shakespeare.	?	?
Scott.		
Emerson.	No	? Went to U. S. Hotel.
Swedenborg.		
Talma.	No	?
Turenne.		
Toussaint.	Yes	?
Virgil.		
Wellington.	Yes	?
Young.		
Grotius.	No.	?
Moltke.		

Work

Question 64. Did he overwork so as to require cessation?
Question 68. At what time of the day did he preferably work?
Question 69. Did he work under stimulants?

	(64)	(68)	(69)
Alexander.	No	?	No
Alcibiades.			
Attila.	?	?	?
Augustus.			
Barneveldt.	No	?	No
Bolivar.			
Bossuet.	No	?	No
Copernicus.			
Cellini.	No	None	No
Chasles.			
Comte.	Yes	?	No
Constantine.			
David.	No	?	?
Euler.			
Faraday.	Yes	Early forenoon	No
Galileo.			
Gilbert.	?	?	?
Homer.			
Helmholtz.	?	?	No?
Humboldt.		Midnight	
Hampden.	No	?	No?
Hunter.			

	(64)	(68)	(69)
Julian.	No	?	No
Lagrange.			
Lessing.	Yes. (Shorted his life)	?	?
Leonidas.			
Machiavelli.	No	?	No
Mencius.			
Montaigne.	No	Daytime?	No
Morphy.			
Occam.	?	?	No
Palissy.			
Phidias.	?	?	?
Pythagoras.			
Rabelais.	No	Mealtime. Noon	No?
Rienzi.			
Shakespeare.	?	?	?
Scott.			
Emerson.	Think so.	?	No
Swedenborg.			
Talma.	?	?	?
Turenne.			
Toussaint.	No	?	?
Virgil.			
Wellington.	No.	Mornings	No
Young.			
Grotius.	No.	?	No?
Moltke.			

Genius

Question 31. Hallucinations
Question 34. Excited by work

	(31)	(34)
Alexander.	?	? Likely
Alcibiades.		
Attila.	?	?
Augustus.		
Barneveldt.	No	No
Bolivar.		
Bossuet.	No	Probably
Copernicus.		
Cellini.	Long account. True hallucinations	Probably
Chasles.		
Comte.	No.	?
Constantine.	Cross in sky	
David.	No	?
Euler.		
Faraday.	No	Enthusiastic
Galileo.		
Gilbert.	?	? (Devoted)
Homer.		
Helmholtz.	?	?
Humboldt.		
Hampden.	No	? Not likely
Hunter.		
Julian.	No	?

	(31)	(34)
Lagrange.		
Lessing.	No	?
Leonidas.		
Machiavelli.	No	No?
Mencius.		
Montaigne.	No	No
Morphy.	Recorded all his games the next morning	
Occam.	No	?
Palissy.		
Phidias.	?	?
Pythagoras.		
Rabelais.	No?	?
Rienzi.		
Shakespeare.	Not unlikely	?
Scott.	True hallucinations	
Emerson.	No	?
Swedenborg.	Had 'em bad.	
Talma.	Voluntarily called up vision of theatre full of skulls	Probably
Turenne.		
Toussaint.	No	?
Virgil.		
Wellington.	No	No
Young.		
Grotius.	No	No
Moltke.		

Genius

Question 48. Work intermittent or continual
Question 36. Strength of will

	(48)	(36)
Alexander.	Continual	Enormous
Alcibiades.		
Attila.	Continual	Great
Augustus.		
Barneveldt.	Contin.	Great
Bolivar.		
Bossuet.	Contin.	Great
Copernicus.		
Cellini.	Intermitt.	V. self-willed
Chasles.		
Comte.	Contin.	Great
Constantine.		Considerable
David.	Continual?	Great
Euler.		
Faraday.	Continual except for ill health	Strong
Galileo.		Weak
Gilbert.	? Cont.?	V. strong
Homer.		
Helmholtz.	Contin.?	?
Humboldt.		Weak
Hampden.	Contin. for few years	Decided
Hunter.		V. strong

	(48)	(36)
Julian.	Contin.	?
Lagrange.		
Lessing.	Continual?	Good
Leonidas.		Strong
Machiavelli.	Continual	Nothing remarkable
Mencius.		Very self-controlled
Montaigne.	Intermittent	Small
Morphy.		
Occam.	?	Great
Palissy.		Enormous
Phidias.	?Contin.?	?
Pythagoras.		Enormous
Rabelais.	?	Strong
Rienzi.		Vacillating
Shakespeare.	?	?
Scott.		
Emerson.	V. intermittent	V. strong (Inspired by an idea)
Swedenborg.		
Talma.	Contin.?	Decided
Turenne.		
Toussaint.	Continual	V. strong
Virgil.		
Wellington.	Contin.	V. great decision
Young.		
Grotius.	Contin.	Average.
Moltke.		

Genius

Question 49. Energy
Question 37. Delicacy of perception
Question 38. Capacity of learning

	(49)	(37)	(38)
Alexander.	Vast	Prob. not remarkable. Good.	Great
Alcibiades.			
Attila.	Vast	?	?
Augustus.			
Barneveldt.	Great.	?	Good
Bolivar.			
Bossuet.	Great deal.	Great	Good
Copernicus.			
Cellini.	Great	Great	Prob. fair
Chasles.			
Comte.	Great	Prob. poor.	May not have been great.
Constantine.			
David.	Great	Prob. good.	V. good
Euler.			
Faraday.	Great	Prob. about average	Almost great
Galileo.			
Gilbert.	Prob. good deal.	?	?
Homer.			
Helmholtz.	Prob. large.	(Sense perceptions good.)	Great
Humboldt.			
Hampden.	Intermittent	?	Nothing to show more than average
Hunter.			

	(49)	(37)	(38)
Julian.	Great	Prob. very good at least	V. good
Lagrange.			
Lessing.	Good	Very great	Good
Leonidas.			
Machiavelli.	Large degree	Very great	Doubtful
Mencius.			
Montaigne.	Average or less	Very great indeed.	Very good.
Morphy.			
Occam.	Large.	?	Prob. great.
Palissy.			Great
Phidias.	Prob. good deal.	Great at least	?
Pythagoras.			
Rabelais.	Large		V. good indeed
Rienzi.			
Shakespeare.	Prob. v. good.	Wonderful.	Great.
Scott.			
Emerson.	Good	Great	Below the average in respect to language.
Swedenborg.			
Talma.	Prob. good deal	Must have been great	?
Turenne.			
Toussaint.	Great	Prob. very good	V. great.
Virgil.			
Wellington.	Great	? Prob. poor	Prob. not above average
Young.			
Grotius.	Large	Not remarkable.	Good
Moltke.			

[Questionnaire Responses for Short List of 24 Great Men]

Item 18 *MS 490: Fall 1883–Fall 1884*

Physique

Question 43. Health?
Question 60. Strength?
Question 60bis. Beauty?

	(43)	(60)	(60bis)
Alexander	good.	strong	rem.
Attila	good	pr. strong	
Barneveldt	prob. good.	pr. strong	
Bossuet	good	pr. strong	
Cellini		strong	
Comte	v. poor.		
David	good.	strong	remark.
Faraday	v. poor	strong	
Gilbert	prob. good.		
Helmholtz			below average
Hampden	prob. good.		
Julian	app. good	weak	Handsome
Lessing	*medium.*	strong	good
Machiavelli	prob. fair	———	ugly.
Montaigne	rather poor.	prob. weak	rather good looker
Ockham			
Phidias			
Rabelais	app. excellent		
Shakespeare	———		average
Emerson	rather poor	prob. weak	below average
Talma			
Toussaint			rather above average
Wellington			Ugly
Grotius			

Physique

Question 62. Other peculiarities?
Question 62bis. Complexion?

	(62)	(62bis)
Alexander	Peculiar odor.	Dark
Attila	Impressive countenance	swarthy
Barneveldt	High broad fore	fair
Bossuet		
Cellini		
Comte		
David		
Faraday		Fair
Gilbert	*tall*	Châtain cheerful complexion
Helmholtz		
Hampden		
Julian		Dark
Lessing		
Machiavelli	olive compl. Meagre.	*olive*
Montaigne	Roman nose.	fair?
Ockham		
Phidias		
Rabelais		
Shakespeare		Fair
Emerson	Very unsymmetric	Fair
Talma		
Toussaint		
Wellington		
Grotius		

Effect of Age

Question 25. At what age did originality first strongly appear?
Question 47. First great work at what age?
Question 46. Best work at what age?

	(25)	(47)	(46)
Alexander	20	20	?
Attila	28	28	40
Barneveldt			45
Bossuet	16	40	50
Cellini	*early*		
Comte	20 or earlier	28	28
David	*young*		
Faraday	?		?47
Gilbert			
Helmholtz			
Hampden	?30		
Julian	26	27	31
Lessing	21	*Laocoon* 38	*Nathan* 50
Machiavelli	30 pre	44	44
Montaigne	?		
Ockham			
Phidias			
Rabelais	28	37	About 55
Shakespeare	25	29	50
Emerson	29		
Talma	25	26	
Toussaint	52	52	52
Wellington	30	30/40	40 46
Grotius	15 or earlier	21	40

Effect of Age

Question 39. Was power of learning a new thing retained to advanced age?
Question 18. Age at death?

	(39)	(18)
Alexander	*Died*	33
Attila	"	47
Barneveldt	∿	71
Bossuet	∿	77
Cellini	Yes	69
Comte		59
David		55??
Faraday	No	75
Gilbert	∿	63
Helmholtz	Yes	
Hampden	died	49
Julian	died	33
Lessing	*No*	52
Machiavelli	*Died*	58
Montaigne	∿	59
Ockham	∿	∿
Phidias	∿	∿
Rabelais	∿	58
Shakespeare	died	52
Emerson	No	78
Talma	∿	63
Toussaint	Yes	60
Wellington	Yes	∿
Grotius	∿	62

Progeny

Question 59. Married? At what age?
Question 12. Number of children?
Question 13. Ability of children?

	(59)	(12)	(13)
Alexander	4 wives	1	died
Attila	47	0	
Barneveldt	Yes	2+	Bright
Bossuet	No	0	
Cellini	No	1	Obscure
Comte	27	0	
David	Yes	2+	great
Faraday	28	0	
Gilbert	No	0	
Helmholtz	Yes	3+?	?
Hampden	25	?	
Julian	30	1 son	died
Lessing	50?	1	died
Machiavelli	Yes	4 s. 1 dau.	Not kn.
Montaigne	33	6 dau.	Ordinary
Ockham	No	0	
Phidias		?	
Rabelais	No	0	
Shakespeare	18	3	No acc.
Emerson	26 (32)	3+	Bright
Talma	Yes *2ce*	3	died
Toussaint	Yes	?	?
Wellington	37	2	Below average
Grotius	25	3 sons 1 dau.	?

Milieu

Question 4. Century?
Question 3. Nation?

	(4)	(3)
Alexander	IV B.C.	*Greek*
Attila	V	*Slav*
Barneveldt	XVI	Dutch
Bossuet	XVII	French
Cellini	XVI	*Ital.*
Comte	XIX	French
David	X B.C.	*Jew*
Faraday	XIX	*Eng.*
Gilbert	XVI	*Eng.*
Helmholtz	XIX	*German*
Hampden	XVII	Eng.
Julian	IV	Ital.
Lessing	XVIII	*German*
Machiavelli	XVI	*Ital.*
Montaigne	XVI	*French*
Ockham	XIV	English
Phidias	V B.C.	Greek
Rabelais	XVI	French
Shakespeare	XVI	*Engl.*
Emerson	XIX	*Am.*
Talma	XVIII	French
Toussaint	XVIII	*Negro*
Wellington	XIX	*Eng.*
Grotius	XVII	Dutch & Fr.

Milieu

Question 17. Did he have any extraordinary good fortune?
Question 16. Did he live in an age particularly given to the sort of thing that distinguished him?
Question 15. Did his immediate predecessors or contemporaries come near to doing the thing that mainly distinguished him?

	(17)	(16)	(15)
Alexander	Yes.	Yes.	No.
Attila	Yes	Yes	Yes.
Barneveldt	No	Yes	Yes
Bossuet	No	Yes	Yes
Cellini	Yes	Yes	Yes
Comte	No	Rather	No
David	Yes.	——	——
Faraday	No	Yes	Yes.
Gilbert	No	Rather	No
Helmholtz	No	Yes	Yes
Hampden	No	Yes	Yes
Julian	Yes	No	No
Lessing	No	Hardly	No.
Machiavelli	No	Yes	Yes
Montaigne	No	Hardly.	No
Ockham	Yes.	Yes	Yes
Phidias	——	Yes	Yes
Rabelais	Yes	Rather	?
Shakespeare	No	Yes	Yes.
Emerson	No	Yes	Yes
Talma	No	[Rather?]	No
Toussaint	Yes	Yes	Yes
Wellington	Yes.	Yes	Yes
Grotius	No	Yes	Yes

Milieu

Question 21. What did he find ready-made to his hand?
Question 45. How appreciated by his contemporaries?

	(21)	(45)
Alexander	Found an army & scheme for subjugation	Worshipped
Attila	Army & nations ready to be conquered	Feared
Barneveldt	Confusion & strong tendency to consol. Great prince	Not fully app.
Bossuet	Gallican church Very fine state of the language Great king.	Fully
Cellini		Fully
Comte		Not app.
David	An opening for a young man	Gr. honored
Faraday	Found an unexplored branch of physics & a great teacher	greatly
Gilbert	Found an unexplored science Found scientific tendencies	Not fully
Helmholtz		Immensely
Hampden		A good deal
Julian		Yes.
Lessing	Theatre, & good actors, audience	Greatly
Machiavelli	Found infinite sphere for application of his views Great readiness in looking at things so Very elegant language.	Consid.
Montaigne	Active state of thought Some elegance of manners	Immensely admired
Ockham		Vast
Phidias		Perhaps fully
Rabelais		Yes.
Shakespeare	Great drama, immense interest, admirable state of language Very lively & interested epoch of thought	Greatly
Emerson	Disciples (People ready for a prophet.)	A good deal
Talma	French drama & theatre	Fully
Toussaint		A good deal
Wellington		Unduly
Grotius		*greatly*

[Great Men: Classifications and Rankings]

Item 19 ***MS 526: Fall 1883–Fall 1884***

[List 1]

Poet	*Artists*	*Writers*	*Writer & Man of action*
Shakespeare 1.0 Homer 1.3	Phidias 1.4 Cellini 5.0 *(Bent)* *Actor* Talma 5.2	Lessing 3.6 *(Bent)* Rabelais 4.5 Montaigne 4.8 Scott 5.2 *(Precocious)*	Bossuet 3.5 *(Precocious)* Macchiavelli 4.8 *Writer & Father of a Religion* Swedenborg 4.1 *(Precocious)*
Moralist	*Philosopher*	*Mathematician*	*Physicist*
Pythagoras 1.1 Mencius 2.8	Ockham 2.5	Euler 2.0 *(Bent)* Lagrange 3.6 Chasles 4.8	Galileo 1.5 Gilbert 2.7 Young 3.1 *(Bent)* Faraday 3.5 Copernicus 3.7
Lawyer	*Linguist*	*Calculator*	*Biologist*
Grotius 4.1 *(Precocious)*		Morphy *(Precocious)*	Hunter 3.0
Statesman	*Ruler*	*Generals*	*Personalities*
Barneveldt 3.9 Bolivar 4.0 Hampden 4.6	Alexander 1.7 *(Precocious)* Augustus 2.1 Julian 4.0	Turenne 3.2 *(Bent)* Toussaint 4.3 Attila 4.4 Alcibiades 6.0	Palissy 3.0 Leonidas 5.4 Rienzi 5.5

*[*List 2*]*

1.0 Shakespeare—Poet
1.1 Pythagoras—Moralist & Mathematician —————— Personality
1.3 Homer—Poet
1.5 Galileo—Physicist
1.4 Phidias—Artist
1.7 Alexander—Conqueror & Ruler
2.0 Euler—Mathematician
2.1 Augustus—Ruler & General
2.5 Ockham—Philosopher & Reformer
2.7 Gilbert—Physicist
2.8 Constantine—Ruler & General
2.8 Mencius—Moralist & Reformer
3.0 Palissy—(Courage & Perseverance) —————— Personality
3.0 King David—Conqueror & Ruler
3.0 John Hunter—Biologist
3.1 Young—Physicist
3.1 Humboldt
3.2 Turenne—General
3.5 Faraday—Physicist
3.5 Lagrange—Mathematician
3.5 Bossuet—Prelate & Writer & Orator
3.6 Lessing—Critic & Writer
3.7 Copernicus—Physicist
3.9 Barneveldt—Statesman & Patriot
4.0 Bolivar—Patriot, General
4.0 Julian—Ruler
4.1 Swedenborg—Mystic
4.1 Grotius—Publicist-Statesman
4.3 Toussaint—Statesman Patriot General
4.4 Attila—Conqueror
4.5 Rabelais—Writer
4.6 Hampden—Statesman Patriot —————— Personality
4.8 Chasles—Mathematician
4.8 Macchiavelli—Statesman Writer
4.8 Montaigne—Writer
5.0 Cellini—Artist —————— Personality
5.2 Scott—Writer
5.2 Talma—Actor
5.4 Leonidas—General —————— Personality
5.5 Rienzi—Demagogue —————— Personality
6.0 Alcibiades—General-Statesman —————— Personality

[List 3]

1.0 Shakespeare
1.1 Pythagoras
1.2
1.3 Homer
1.4 Phidias
1.5 Galileo
1.6
1.7 Alexander
1.8
1.9
2.0 Euler
2.1 Augustus
2.2
2.3
2.4
2.5 Ockham
2.6
2.7 Gilbert
2.8 Constantine, Mencius
2.9
3.0 K. David, John Hunter, Palissy
3.1 A. von Humboldt, Dr. Young
3.2 Turenne
3.3
3.4 Comte
3.5 Bossuet, Faraday, Lagrange
3.6 Lessing
3.7 Copernicus
3.8
3.9 Barneveldt
4.0 Bolivar, Julian
4.1 Swedenborg
4.2
4.3 Toussaint
4.4 Attila
4.5 Rabelais
4.6 Hampden
4.7
4.8 Chasles, Macchiavelli, Montaigne
4.9
5.0 Cellini
5.1
5.2 Talma, Scott
5.3
5.4 Leonidas
5.5 Rienzi
5.6
5.7
5.8
5.9
6.0 Alcibiades

On the Algebra of Logic: Part II

Item 20 ***MS 506: Summer 1884***

I begin by acknowledging some faults in my former paper under this title, published in the third volume of this journal.

The first chapter of that paper developes the reasons for choosing the copula of inclusion, exhibits its formulae, and attempts by means of it to consolidate syllogistic with Boolian algebra. But the study of Professor O. H. Mitchell's important paper "On a New Algebra of Logic" has led me to think that the passage from premiss to conclusion ought not to be considered as the essential and elementary type of logical movement. We have rather two elementary modes of modifying assertions and two corresponding modes of transforming them. The two modes of changing assertions are 1st to drop part of what has been asserted and assert less, and 2nd to add to what has been asserted and assert more. The two modes of transforming assertions are 1st to recognize the partial assertions of which an assertion is composed, 2nd to recognize the alternative assertions of which an assertion is an aggregate. From this point of view, we want no copula except the sign = to unite equivalent assertions. We need, however, a sign of multiplication to join the partial assertions which compose a total assertion, and a sign of addition to join the alternative assertions which make up a disjunctive assertion. Then by the use of an unknown assertion, *X*, we may express the two elementary modes of modifying assertions. Thus,

$$A = B \times x$$

will express that the assertion *A* includes the assertion *B*, and

$$A = B + x$$

will express that the assertion A is included by the assertion B. So that

$$\{A = B \times x\} = \{A \prec B\} = \{B = A + y\}.$$

My main reason for preferring this new view is that the algebra founded upon it is simpler than one founded on the adoption of an unbalanced copula.

The algebra of the copula, as given in my first paper, is adequate to every problem of non-relative logic, and it makes use of but two operational signs, the copula $\prec$ and the sign of negation. Accordingly, an algebra of non-relative logic which contains three signs, say of addition, multiplication, and negation, contains a surplusage of signs. And in point of fact

$$xy = \overline{\bar{x} + \bar{y}}$$
$$x + y = \overline{\bar{x}\bar{y}}$$

showing that the sign of negation with either the sign of addition or that of multiplication would be enough. But negation cannot be expressed by means of the signs of addition and multiplication alone. Indeed, it is plain that an unbalanced, or non-commutative, operation cannot be produced by any combination of balanced operations; although balanced operations may be represented as compounds of unbalanced ones.

[Fragment on the Algebra of Logic]

Item 21 ***MS 507: Summer 1884***

The first system of relationship which logic studies is that of an indefinite collection of units. It may be represented by the schema

| | | | | | | | |

These constitute the universe of discourse. Various names or conventional signs, for which letters may be used, are attached to these in various ways. The study of this schema gives rise to the Boolian calculus. The logic of relatives studies a collection of units arranged in an n-dimensional block, thus:

. . . . .
. . . . .
. . . . .
. . . . .

Distinguishing the different dimensions by the letters $i, j, k,$ etc. we may write

$\Pi_i a,$	for every i has the mark a.
$\Sigma_i a,$	for some i has the mark a.
$\Pi_i \Pi_j l$	Every i is in the relation l to every j.
$\Sigma_i \Pi_j l$	Some i is in the relation l to all j's.
$\Pi_j \Sigma_i l$	To each j some i is in the relation l.
$\Sigma_j \Pi_i l$	To some j all i's are in the relation l.
$\Pi_i \Sigma_j l$	Each i is in the relation l to some j.
$\Sigma_i \Sigma_j l$	Some i is in the relation l to some j.

The order of two Πs or Σs is indifferent. That is

$$\Pi_i \Pi_j = \Pi_j \Pi_i \qquad \Sigma_i \Sigma_j = \Sigma_j \Sigma_i$$

Using the sign $\prec$ to denote that the proposition placed after it follows from that placed before it, we have

$$\Sigma_i \Pi_j \prec \Pi_j \Sigma_i.$$

By virtue of the postulate that every universe contains some object, we have also

$$\Pi \prec \Sigma.$$

The signs of addition and multiplication are used in their usual logical sense. That is to say, to attach to any individual the mark $a + b$ is the same as to say that one or other belongs to it, to attach the mark $a \times b$ is to attach both a and b.

The sign of negation (a mark over the negatived term), namely the negative mark, belongs to every individual to which the positive mark does not belong and to none to which the latter does belong.

The logic of relatives, so understood, coincides with Professor Mitchell's multidimensional logic; and the logic of relatives as De Morgan and I have understood it, is a special case under this broader conception.

In order to use the calculus, we must observe that Π denotes a logical product, Σ a logical sum. Consequently, we have

$$(\Pi_i a)\,(\Pi_i b) = \Pi_i (ab)$$

$$(\Sigma_i a) + (\Sigma_i b) = \Sigma_i (a + b).$$

Also,

$$(\Pi_i a)\,(\Sigma_i b) = (\Pi_i a)\,(\Sigma_i ab)$$

$$(\Pi_i a) + (\Sigma_i b) = \Pi_i (a + b) + (\Sigma_i b).$$

The formula $\Pi \prec \Sigma$ may be written in the form

$$\Pi_i a = (\Pi_i a)\,(\Sigma_i a)$$

The formula

$$\Sigma_i \Pi_j \prec \Pi_j \Sigma_i$$

may be written in the form *[ ]*

On the Algebra of Logic (Second Paper)

Item 22 ***MS 508: Summer 1884***

I begin by briefly restating the principles of the logical algebra, in the light of conceptions derived especially from the study of the important memoir of Professor O. H. Mitchell entitled "On a New Algebra of Logic." When anything signifies or represents any thing or fact, it does so in one or other of three ways: 1st, by its likeness or analogy with the thing signified, 2nd, by being physically connected therewith, or 3rd, by being associated therewith in the mind, say habitual association. It is by this third mode of signifying by which a proposition represents the fact it states; but this mode of signifying cannot exist isolated from the other two. The universe which is the real subject of discourse, can only be pointed out—never described. If you tell me that there is or that there is not a black swan or a white raven, I want to know where you mean; and that you can only show me,—no description will answer the purpose. Such showing or pointing out consists in putting me into a physical connection with the thing signified. Or the qualities that are asserted of the subject can again not be described, but only shown in another way, namely, by exhibiting their like. On the other hand, a proposition cannot be enunciated by the use of these two modes of signifying alone, but both must be subordinated to the third, which depends on natural or habitual association. I point out to you something and then I show you a piece of paper covered with spots, and will mean that the thing pointed at is spotted; but only if you will naturally or by habit catch the idea of connecting my two acts in that way. Any system of notation which is to represent propositions must then have these three elements, the denotative, and the analogical, and the conventional.

From this we may at once draw several important corollaries. In the first place, since the universe of discourse requires to be pointed out, it must be that there is some individual in that universe. Consequently, the distinction between propositions which assert existence and those which assert non-existence is not so absolute as it otherwise would be. Thus, to assert that every griffin breathes fire is as much as to say that in the universe of which we are speaking there is something that coexists only with non-griffins and with things that breathe fire. In the second place, though the analogical element would be a very simple matter if we could confine ourselves to simple propositions involving only simple qualities, yet *[. . .]*

The first system of relationship that logic studies is that of an indefinite collection of disconnected units. It may be represented by this schema.

. . . .

These objects constitute the universe. To these are attached conventional signs: to any object may be attached several signs, and any sign may be attached to several objects, in any arrangement whatever. If letters be used for these signs, they may be conceived to be written under the dots of the schema to which they belong; but we may follow the usual phraseology of logic by saying that the individual (the dot) has the mark a or b, or that it is a or b. Beginning with the consideration of a single individual, let us write a to signify that the individual in question is a. In order to say "If it is a it is b," let us write $a -\!\!< b$. The formulae relating to this symbol $-\!\!<$ constitute what I have called the algebra of the copula. Speaking always of but a single individual, the predication of any mark a can only be wholly true or wholly false. The proposition $a -\!\!< b$ is to be understood as true if either a is false or b is true, and is only false if a is true while b is false. Let v be a mark which is true of the individual considered, and let f be a mark which is false of it. Then of any mark whatever a two propositions are both true, namely $\mathrm{f} -\!\!< a$ and $a -\!\!< \mathrm{v}$; and two other propositions are one or other of them true, namely either $\mathrm{v} -\!\!< a$ or $a -\!\!< \mathrm{f}$. This is so, because it holds whether for a we put v or f; and it is clear that the truth of a general formula may be tested by trying whether it will always hold when either v or f is substituted throughout for each letter. In writing the formulae, it will be well to adopt the rule that a parenthesis extending to the end of the formula or to the end of an exterior parenthesis is to be omitted.

Of formulae of two terms in v and f, only $\mathrm{v} \prec \mathrm{f}$ is false, for $\mathrm{f} \prec \mathrm{f}$, $\mathrm{v} \prec \mathrm{v}$, $\mathrm{f} \prec \mathrm{v}$, are all true. This shows that

$$a \prec a$$

is necessarily true. Of formulae of the form $a \prec b \prec c$ in v and f the only one that is false is $\mathrm{v} \prec \mathrm{v} \prec \mathrm{f}$. It follows that

$$a \prec b \prec b$$

necessarily holds, and also that

$$a \prec b \prec c = b \prec a \prec c.$$

It is easily seen that the rule is that in a formula without a parenthesis the order of all the terms except the last is indifferent. The different rule governing the last letter leads us to use a different symbol for every copula preceding the last without parenthesis. In place of $a \prec b \prec c$ we are led to write $(a \times b) \prec \mathrm{c}$. Then the expression $(a \times b)$ may be defined by the equation

$$(a \times b) = (a \prec (b \prec \mathrm{f})) \prec \mathrm{f}.$$

The definitions which I formerly gave of non-relative addition and multiplication were defective; they failed to yield demonstrations of the propositions

$$(a + b) \times c \prec (a \times c) + (b \times c)$$
$$(a + c) \times (b + c) \prec (a \times b) + c.$$

The true definition of logical multiplication, which my own system, logically developed, would have led me to is contained in the equation just given or in the following, which amounts to the same

$$(a \times b) \prec c = a \prec b \prec c.$$

A parallel definition of addition cannot be given; but I hardly think that this is a defect of the present point of view, for addition is a superfluous operation,—its true definition is that $a + b$ is the negative of the product of the negatives of a and b; and the whole apparent parallelism of the two operations is due to there being one superfluous operation in the algebra. I may remark, however, that we may use the definition

$$(a + b) = (a \prec b) \prec b.$$

This amounts to regarding $a + b$ as $a\bar{b} + b$.

My former treatment of the algebra of the copula and of non-relative logic was much confused by my attempt to exclude the conception of individuals, whereas in truth the whole Boolian calculus is the logic of a single individual,—a remark the meaning of which will presently appear. In consequence of my imperfect mode of considering the subject, I had a difficulty in proving every proposition involving the principle of excluded middle, propositions which will no longer give any difficulty. Thus, defining the negative of a as $(a \prec \mathrm{f})$, the proof of

$$((x \prec \mathrm{f}) \prec \mathrm{f}) \prec x$$

is obvious. It is true if the consequent x is true or if the antecedent $(x \prec \mathrm{f}) \prec \mathrm{f}$ is false. This antecedent (having f for its consequent) is false if its antecedent is true. This second antecedent $(x \prec \mathrm{f})$ is true if its antecedent, x, is false. Therefore, whether x be true or false the proposition is true.

We now pass from the consideration of a single individual to that of the whole universe; and first we have that rude distinction of "some" and "all" which may be said to discriminate logic from mathematics. After the whole Boolian school had for thirty years been puzzling over the problem of how to take account of this distinction in their notation, without any satisfactory result, Mr. Mitchell, by a wonderfully clear intuition, points out that what is needed is to enclose the whole Boolian expression in brackets, and to indicate to what proportion of the universe it refers by exterior signs. Denoting by A any expression such as we have hitherto considered, we may write ΠA to signify that A is true of every individual of the universe, and ΣA to mean that it is true of some individual of the universe. The operational signs $\prec$, $\times$, and $+$ are applicable to these new expressions, and in fact, in virtue of the postulate that something in the universe exists, we have $\Pi A \prec \Sigma A$, or more briefly written

$$\Pi \prec \Sigma.$$

We have also the four formulae

$$(\Pi A) \times (\Pi B) = \Pi(A \times B) \qquad (\Sigma A) + (\Sigma B) = \Sigma(A + B)$$

$$(\Pi A) \times (\Sigma B) \prec \Sigma(A \times B) \qquad \Pi(A + B) \prec (\Pi A) + (\Sigma B).$$

It seems to me that the principles of formal logic, as it is ordinarily understood, are really exhausted, when it is brought to this point, and that there remains nothing of real importance to add to it, except corollaries and applications. De Morgan's numerically definite syllogism affords, however, an example of what is probably an extensive doctrine still to be developed.

Relative logic is distinguished from the non-relative by dealing with a multidimensional universe. If the number of dimensions is two, for instance, the universe may be represented by the schema

$$\begin{array}{ccccccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}$$

In this block no row has any special relation to any particular column, and there is therefore no principal diagonal having peculiar properties. Such a universe is considered by Mr. Mitchell under the head of "Propositions of two dimensions." In highly important cases there is a one-to-one correspondence between the rows and columns, and in that event the schema of the combined universe becomes

$A:A$	$A:B$	$A:C$	$A:D$	$A:E$
$B:A$	$B:B$	$B:C$	$B:D$	$B:E$
$C:A$	$C:B$	$C:C$	$C:D$	$C:E$
$D:A$	$D:B$	$D:C$	$D:D$	$D:E$
$E:A$	$E:B$	$E:C$	$E:D$	$E:E$

In such a case the individuals are of two kinds, those of the form $(A:A)$ and those of the form $(A:B)$. In forming a notation, we may use the same symbols as before, only that we must now distinguish not merely between the different dimensions, but also between, say, two columns which may possibly be different. Thus, supposing the two dimensions represent the one parts of the earth, the other times, we may wish for instance to say that at every instant some parts are in daylight and some in darkness, or that at any time at which any part of the earth is under a total solar eclipse some parts are under a partial eclipse. To mark these distinctions, subjacent letters may be used; and these must be applied not merely to the Πs and Σs, but also to the letters standing for names.

Letter, Peirce to J. E. Hilgard

Item 23 *L 197: 1 October 1884*

Office Weights & Measures
U. S. Coast and Geodetic Survey,
Baltimore Md
October 1, 1884.

Professor J. E. Hilgard
Superintendent

Dear Sir

I have the honor to submit the following sketch of proposed future gravity determinations.

The first consideration in drawing up such a scheme is the use which the determinations are to be put to, which is to ascertain the figure of the level surface. This requires in the first place that the stations shall be well scattered all over the country. It will also be convenient, in the mathematical treatment, that they shall be approximately in some regular lines. I should disapprove of making the computations by a blind and mechanical application of spherical harmonics, and I think that we should rather be guided in this process by positive geographical and geological considerations.

To this end, we ought to crowd our stations where there is geographical and geological diversity and spread them where there is uniformity. We ought to occupy high mountain stations in every important range and valley stations away from the heights. We should occupy oceanic islands, such as the Bermudas, Guadeloupe, etc. It is to be remembered that it is a mathematical requisite for determining the figure of our own territory that we should carry our observations beyond our borders. We should also multiply coast stations, meaning by the coast the neighborhood either of deep water or of extensive deposits horizontally transported from the land.

We should seek to place our stations so as to be between regions of geological disturbance, in the comparatively undisturbed districts.

The second consideration is where these determinations can be well, easily, and cheaply made. The necessity for solid foundations and constancy of temperature which can hardly be secured except in buildings with thick walls and lofty rooms, makes the wild country very unsuitable for this work. The best stations are casemated forts; the next are astronomical observatories, physical laboratories, and the like. Next, come the cellars of great public buildings and churches. It is very convenient to be near a telegraph, so that time need not be observed for. Water communication is highly advantageous. There are, however, stations of the highest importance which can only be occupied with camps. There are a few requiring vessels.

The third question is how much work our scheme ought to include. If I am allowed $5000 a year and a permanent assistant, this amount to be increased when the work goes west of the Mississippi, I could occupy about seven or eight stations a year. The entire variation of gravity not due either to 1, latitude, 2, elevation, or 3 local causes of which we cannot take account is probably over the whole country not over $\frac{1}{10000}$, making a difference of say 5 oscillations daily. The error will not, we hope, exceed one fiftieth of this. But the local variations will probably be considerable, so that the stations must be multiplied to get generalized values. I think they should now for a number of years be placed about 3° from one to the next.

The first thing wanted is to get the values at extreme stations; say at Boston, Key West, Port Townsend, and San Diego. Also, at extremely elevated stations and neighboring valleys. Also, to complete all experimental work as quickly as possible.

After these preliminary labors, I think we should begin at the easternmost range of stations and proceed westward, going north in summer and south in winter, and occupying the more costly stations as we can and passing them by when we must. The island and remote stations must break the regularity of this proceedure; and a great many special considerations must of course be allowed their weight in settling details.

The following is a list of past and proposed stations. Those already occupied are marked by a cross.

Gravity Stations

Easternmost Range

1. Halifax
2. Fort Knox, Penobscot River
X3. Cambridge, Mass
4. Fort Warren, Boston Harbor
5. Nantucket
X6. Hoboken
X7. Baltimore
X8. Washington, Smithsonian
X9. " , Coast Survey
10 " , Observatory
X11. Fort Monroe
12. Fort Macon, Beaufort, N.C.
13. Fort Sumter (or Moultrie)
14. St. Augustine
15. Tampa
16. Key West (Fort Taylor)
17. Havana or Cienfuegos
18. Nassau
19. Kingston Jamaica

Second Range.

19. Quebec
X20. Montreal
21. Fort Montgomery?
22. Adirondack Mountains
23. Mt. Washington
24. Crawford's or Willey's.
X25. Springfield, Mass
X26. Hoosac Tunnel.
X27. Hoosac Camp.
28. Hoosac Summit.
X29. Albany.
30. Ithaca
31. York, Pa
32. Ebensburgh
33. Eliot's Knob, Va
34. Variety Springs, Va.
35. Black Mountains, N.C.
36. Salisbury, N.C.
37. Macon.
38. Apalachicola.

Third Range

39. Fort Niagara
40. Allegheny Observatory
41. Marietta or Chillicothe
42. Lexington or Shelbyville, Ky
43. Chattanooga
44. Selma
45. Ship Island Fort or fort at Rigolets or Fort St. Philip

Fourth Range

45. Detroit.
46. Logansport.
47. Evansville.
48. Corinth or Holly Springs
49. Vicksburg.
50. Baton Rouge?

Fifth Range.

51. Marquette or Wisconsin State Park
52. Père Marquette or White River
53. Dubuque or Milwaukee or between.

54. Springfield, Ill. or thereabouts.
55. In southern Missouri or St. Louis.
56. Little Rock
57. Shreveport
58. Galveston.
59. Matamoros.
60. Tampico
61. Vera Cruz

Sixth Range.

62. Fort Moorhead, Dakota
63. Minneapolis.
64. Des Moines.
65. Kansas City.
66. Fort Smith, Ark
67. Dallas
68. Austin
69. Monterrey
70. Mexico.

Across the Continent

71. Fort Kearney
72. North Platte.
73. Laramie
74. Salt Lake City
75. Eureka
76. Virginia City, Nevada

Pacific Coast

77. Port Townsend, Straits of Juan de Fuca
78. Portland, Oregon.
X79. San Francisco.
80. Santa Barbara.
81. San Diego.

Other Western stations

82. Yellowstone Park
83. Denver
84. Pike's Peak
85. Santa Fé.

Alaska

86. Sitka
87. Point Barrow

Greenland

X88. Lady Franklin Bay
89. Upernavik

Oceanic Stations

91. Reikiavik, Iceland
92. Bermuda
93. Isle of Shoals
94. Fayal.
95. Barbados or other windward island.
96. Galapagos
97. Guadeloupe
X98. Caroline Island
X99. Honolulu
100.
101.
102.
103.

Other foreign Stations

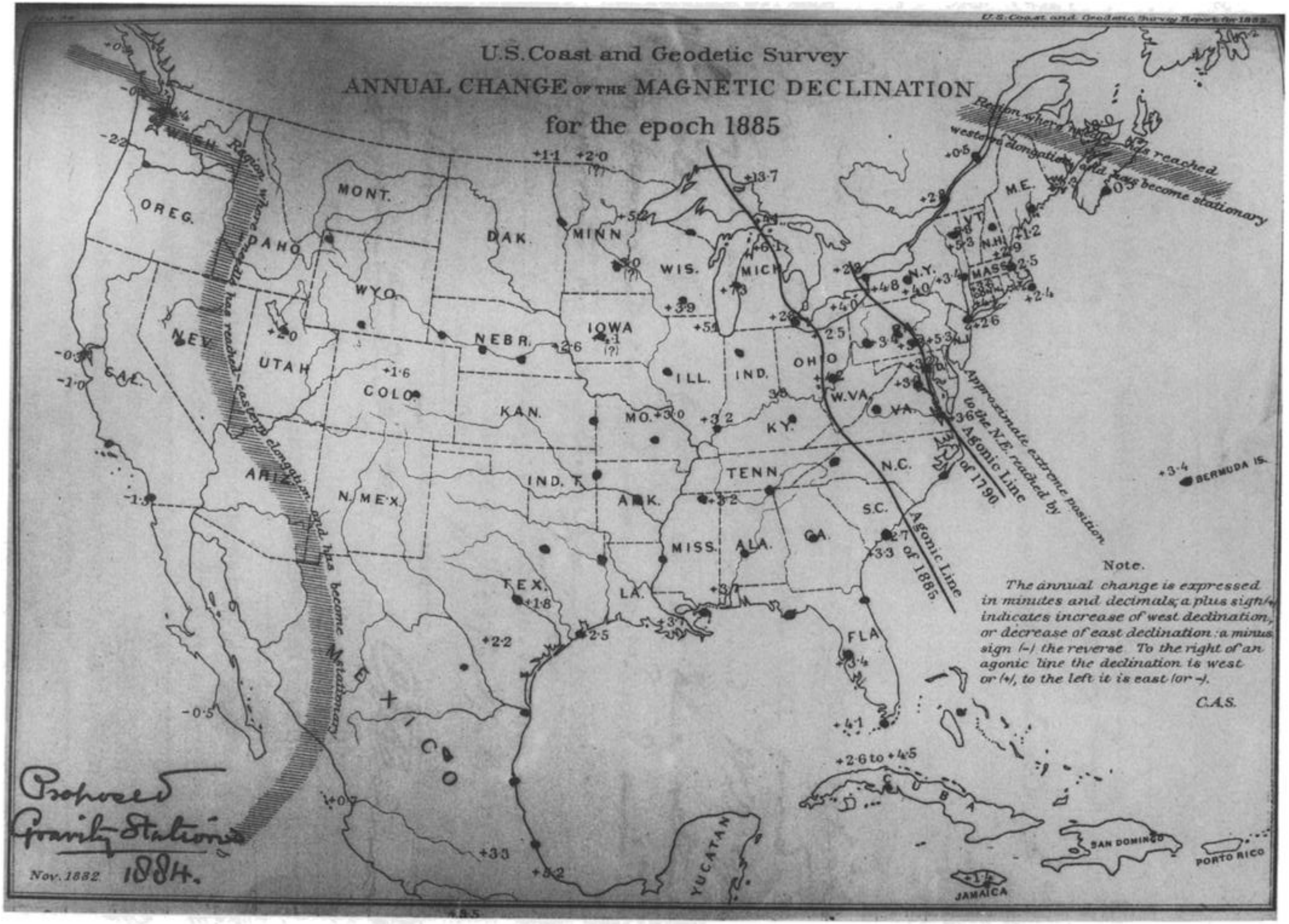
X104. Patagonia
X105. Sydney
X106. Tokio.
107.
109. Hong-Kong
110. Singapore
111.
112.

Stations connecting with foreign series.

X113. Kew
X114. Paris

X115. Berlin
X116. Geneva
117. Rome
118. Madrid
119. St. Petersburg
120. Trinidad
121.
122.
123. Madras.

very respectfully
C S Peirce
Ass't.


U.S. Coast and Geodetic Survey Report for 1882
U.S. Coast and Geodetic Survey
ANNUAL CHANGE OF THE MAGNETIC DECLINATION
for the epoch 1885
Region where the W. elongation has reached eastern elongation and has become stationary
Region where the western elongation has been reached and has become stationary
Approximate extreme position to the N.E. reached by Agonic Line of 1790
Agonic Line of 1885
OREG.
IDAHO
MONT.
DAK.
MINN.
WIS.
MICH.
WYO.
NEV.
UTAH
CAL.
COLO.
NEBR.
IOWA
ILL.
IND.
OHIO
KAN.
MO.
KY.
W.VA.
VA.
N.Y.
VT.
N.H.
ME.
MASS.
ARIZ.
N. MEX.
IND. T.
ARK.
TENN.
N.C.
S.C.
GA.
ALA.
MISS.
LA.
TEX.
FLA.
MEXICO
YUCATAN
CUBA
JAMAICA
SAN DOMINGO
PORTO RICO
BERMUDA IS.
Note.
The annual change is expressed in minutes and decimals; a plus sign (+) indicates increase of west declination, or decrease of east declination: a minus sign (−) the reverse. To the right of an agonic line the declination is west or (+), to the left it is east (or −).
C.A.S.
Proposed Gravity Stations 1884.
Nov. 1882.

On Small Differences of Sensation, by C. S. Peirce and J. Jastrow

Item 24 *P 303:* Memoirs of the National Academy of Sciences *3 (1885): 75–83*
(Presented 17 October 1884)

The physiological psychologists assume that two nerve excitations alike in quality will only produce distinguishable sensations provided they differ in intensity by an amount greater than a fixed ratio. The least perceptible difference of the excitations divided by half their sum is what they call the *Unterschiedsschwelle.* Fechner[1] gives an experiment to prove the fact assumed, namely: He finds that two very dim lights placed nearly in line with the edge of an opaque body show but one shadow of the edge. It will be found, however, that this phenomenon is not a clearly marked one, unless the lights are nearly in range. If the experiment is performed with lateral shifting of one of the lights, and with a knowledge of the effects of a telescope upon the appearance of terrestrial objects at night, it will be found very far from conclusive.

The conception of the psychologists is certainly a difficult one to seize. According to their own doctrine, in which the observed facts seem fully to bear them out, the intensity of the sensation increases continuously with the excitation, so that the least increase of the latter must produce a corresponding increase of the former. And, indeed, the hypothesis that a continuous increase of the excitation

1. *Elemente der Psychophysik,* I, p. 242.

would be accompanied by successive discrete increments of the sensation, gratuitous as it would be, would not be sufficient to account for a constant *Unterschiedsschwelle.* We are therefore forced to conclude that if there be such a phenomenon, it has its origin, not in the faculty of sensation, but in that of comparing sensations. In short, if the phenomenon were established, we should be forced to say that there was a least perceptible difference of sensation—a difference which, though existing in sensation, could not be brought into consciousness by any effort of attention. But the errors of our judgments in comparing our sensations seem sufficiently accounted for by the slow and doubtless complicated process by which the impression is conveyed from the periphery to the brain; for this must be liable to more or less accidental derangement at every step of its progress. Accordingly we find that the frequencies of errors of different magnitudes follow the probability curve, which is the law of an effect brought about by the sum of an infinite number of infinitesimal causes. This theory, however, does not admit of an *Unterschiedsschwelle.* On the contrary, it leads to the method of least squares, according to which the multiplication of observations will indefinitely reduce the error of their mean, so that if of two excitations one were ever so little the more intense, in the long run it would be judged to be the more intense the majority of times. It is true that the astronomers themselves have not usually supposed that this would be the case, because (apart from constant errors, which have no relevancy to the present question) they have supposed this extreme result to be contrary to common sense. But it has seemed to us that the most satisfactory course would be to subject the question to the test of direct experiment. If there be a least perceptible difference, then when two excitations differing by less than this are presented to us, and we are asked to judge which is the greater, we ought to answer wrong as often as right in the long run. Whereas, if the theory of least squares is correct, we not only ought to answer right oftener than wrong, but we ought to do so in a predictable ratio of cases.[2]

2. The rule for finding this ratio is as follows: Divide the logarithm of the ratio of excitations by the probable error and multiply the quotient by 0.477. Call this product t. Enter it in the table of the integral θt, given in most works on probabilities; θt is the proportion of cases in which the error will be less than the difference between the given excitations. In all these cases, of course, we shall answer correctly, and also by chance in one-half of the remaining cases. The proportion of erroneous answers is therefore $(1 - \theta t) \div 2$. In the following table the first column gives the quotient of the

We have experimented with the pressure sense, observing the proportion of errors among judgments as to which is the greater of two pressures, when it is known that the two are two stated pressures, and the question presented for the decision of the observer is, which is which. From the probability, thus ascertained, of committing an error of a given magnitude, the probable error of a judgment can be calculated according to the mathematical theory of errors. If, now, we find that when the ratio of the two pressures is smaller than a certain ratio, the erroneous judgments number one-half of the whole, while the mathematical theory requires them to be sensibly fewer, then this theory is plainly disproved, and the maximum ratio at which this phenomenon is observed is the so-called *Unterschiedsschwelle.* If, on the other hand, the values obtained for the probable error are the same for errors varying from three times to one-fourth of the probable error (the smallest for which it is easy to collect sufficient observations), then the theory of the method of least squares is shown to hold good within those limits, the presumption will be that it extends still further, and it is possible that it holds for the smallest differences of excitation. But, further, if this law is shown to hold good for difference so slight that the observer is not conscious of being able to discriminate between the sensations at all, all reason for believing in an *Unterschiedsschwelle* is destroyed. The mathematical theory has the advantage of yielding conceptions of greater definiteness than that of the physiologists, and will thus tend to improve methods of observation. Moreover, it affords a ready method for determining the sensibility or fineness of perception and allows of a comparison of one observer's results with the results of others; for, knowing the number of errors in a certain number of experiments, and accepting the conclusions of this paper, the calculated ratio to the total excitation of that

logarithm of the ratio of excitation, divided by the probable error, and the second column shows the proportion of erroneous judgments:

0.0	0.50
0.05	0.49
0.1	0.47
0.25	0.43
0.5	0.37
1.0	0.25

To guess the correct card out of a pack of fifty-two once in eleven times, it would be necessary to have a sensation amounting to 0.37 of the probable error. This would be a sensation of which we should probably never become aware, as will appear below.

variation of excitation, in judging which we should err one time out of four, measures the sensibility. Incidentally our experiments will afford additional information upon the value of the normal average sensibility for the pressure sense, which they seem to make a finer sense than it has hitherto been believed to be. But in this regard two things have to be noted: (1) Our value relates to the probable error or the value for the point at which an error is committed half the time; (2) in our experiments there were two opportunities for judging, for the initial weight was either first increased and then diminished, or *vice versa,* the subject having to say which of these two double changes was made. It would seem at first blush that the value thus obtained ought to be multiplied by $\sqrt{2}$ (1.414) to get the error of a single judgment. Yet this would hardly be correct, because the judgment, in point of fact, depended almost exclusively on the sensation of increase of pressure, the decrease being felt very much less. The ratio $\sqrt{2}$ (1.414) would therefore be too great, and 1.2 would perhaps be about correct. The advantage of having two changes in one experiment consists in this: If only one change were employed, then some of the experiments would have an increase of excitation only and the others a decrease only; and since the former would yield a far greater amount of sensation than the latter, the nature of the results would be greatly complicated; but when each experiment embraces a double change, this difference in the amount of sensation caused by an increase and decrease of pressure affects every experiment alike, and the liability to error is constant.[3]

Throughout our observations we noted the degree of confidence with which the observer gave his judgment upon a scale of four degrees, as follows:

0 denoted absence of any preference for one answer over its opposite, so that it seemed nonsensical to answer at all.
1 denoted a distinct leaning to one alternative.
2 denoted some little confidence of being right.
3 denoted as strong a confidence as one would have about such sensations.

We do not mean to say that when zero was the recorded confidence, there was absolutely no sensation of preference for the answer given. We only mean that there was no sensation that the observer noticed when attending to his feelings of this sort as closely as he conve-

3. The number of errors, when an increase of weight was followed by a decrease, was slightly less than when the first change was a decrease of pressure.

niently could, namely, closely enough to mark them on this scale. The scale of confidence fluctuated considerably. Thus, when Mr. Jastrow passed from experiments upon differences of weight of 60, 30, and 15 on the thousand to differences of 20, 10, and 5 on the thousand, although the accuracy of his judgments was decidedly improved, his confidence fell off very greatly, owing to his no longer having the sensation produced by a difference of 60 present to his memory. The estimations of confidence were also rough, and might be improved in future work. The average marks seem to conform to the formula—

$$m = c \log \frac{p}{1-p}$$

where m denotes the degree of confidence on the scale, p denotes the probability of the answer being right, and c is a constant which may be called the index of confidence.

To show that this formula approximates to the truth, we compare it with the average marks assigned to estimates of differences for which more than a hundred experiments were made. Mr. Jastrow's experiments are separated into groups, which will be explained below.

Ratio of pressures.	Peirce, observer.		Jastrow, observer.			
	$c = 1.25.$		First Group. $c = 1.5.$		Second Group. $c = 0.0.$	
	Mean confidence.		Mean confidence.		Mean confidence.	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
1.015.	0.14	0.10	0.30	0.2	0.34	0.27
1.030.	0.30	0.35	0.40	0.42	0.55	0.56
1.060.	0.70	0.70	0.85	0.87	1.02	1.22

Ratio of pressures.	Jastrow, observer.			
	Third Group. $c = 0.25.$		Fourth Group. $c = 0.4.$	
	Mean confidence.		Mean confidence.	
	Observed.	Calculated.	Observed.	Calculated.
1.005.	0.00	0.03	0.00	0.06
1.010.	0.07	0.06	0.05	0.12
1.020.	0.12	0.12	0.50	0.39

The judgments enunciated with any given degree of confidence were more likely to be right with greater differences than with smaller differences. To show this, we give the frequency of the different marks in Mr. Jastrow's second, third, and fourth groups.[4]

4. The result of our observations on the confidence connected with the judgments is as follows:

[Subject, Mr. Peirce.]		
Variations.	Average confidence.	Number of sets of 50.
Grams.		
60	.67	7
30	.28	6
15	.15	5
[Subject, Mr. Jastrow.]		
60	.90	13
30	.51	12
15	.30	12
20	.11	12
10	.06	12
5	.00	10

In 1,125 experiments (subject, Mr. Peirce)—variations 15, 30, and 60 grams—there occurred confidence of 3, 35 times (3 per cent.); of 2, 102 times (9 per cent.); of 1, 282 times (25 per cent.); of 0, 706 times (63 per cent.). In these experiments there were 332 (29 per cent.) errors committed, of which 1 (0.3 per cent.) was made in connection with a confidence 3; 10 (3 per cent.) with a confidence 2; 51 (15 per cent.) with a confidence 1; 270 (81 per cent.) with a confidence 0. From which we find that in connection with a confidence of 3 there occurred 1 error in 35 cases (3 per cent.); with a confidence of 2, 10 errors in 102 cases (10 per cent.); with a confidence of 1, 51 errors in 282 cases (18 per cent.); with a confidence of 0, 270 errors in 706 cases (38 per cent.).

In 1,975 experiments (subject, Mr. Jastrow)—variations 15, 30, and 60 grams—there occurred confidence of 3, 62 times (3 per cent.); of 2, 196 times (10 per cent.); of 1, 594 times (30 per cent.); of 0, 1,123 times (57 per cent.). In these experiments there were 451 (23 per cent.) errors committed, of which 2 (0.4 per cent.) were made in connection with a confidence of 3; 12 (3 per cent.) with a confidence of 2; 97 (22 per cent.) with a confidence of 1, 340 (75 per cent.) with a confidence of 0. Again, in connection with a confidence of 3, errors occurred twice in 62 cases (3 per cent.); with a confidence of 2, 12 times in 196 cases (6 per cent.); with a confidence of 1, 97 times in 594 cases (16 per cent.); with a confidence of 0, 340 times in 1,123 cases (30 per cent.).

In 1,675 experiments (subject, Mr. Jastrow)—variations 5, 10, and 20 grams—there occurred confidences of 3, none; of 2, none; of 1, 115 times (7 per cent.); of 0, 1,560 times (93 per cent.). In these experiments there were 538 (32 per cent.) errors committed, of which 16 (3 per cent.) occurred in connection with a confidence of 1; 522 (97 per cent.) with a confidence of 0. Again, in connection with a confidence of 1, errors occurred 16 times in 115 cases (14 per cent.); with a confidence of 0, 522 times in 1,560 cases (34 per cent.).

Second group.

Ratio of weights.	Mark 0.	Mark 1.	Mark 2.	Mark 3.
1.015	110 right 66 wrong	51 right 17 wrong	3 right 2 wrong	1 right 0 wrong
1.030	106 right 35 wrong	72 right 11 wrong	23 right 1 wrong	2 right 0 wrong
1.060	86 right 8 wrong	75 right 1 wrong	54 right 2 wrong	24 right 0 wrong

Third and fourth groups.
[Marks 2 and 3 do not occur.]

Ratio of weights.	Mark 0.	Mark 1.
1.005	294 right 203 wrong	2 right 1 wrong
1.010	366 right 192 wrong	32 right 30 wrong
1.020	395 right 131 wrong	68 right 6 wrong

The apparatus used was an adaptation of a "Fairbanks" post-office scale; upon the end of the beam of which was fixed a square enlargement (about one-half inch square), with a flat top, which served to convey the pressure to the finger in a manner to be presently described. This was tightly covered with an India-rubber cap, to prevent sensations of cold, &c., from contact with the metal. A kilogram placed in the pan of the balance brought a pressure of one-fourth of its weight upon the finger. The differential pressure was produced by lowering upon the pan of the balance a smaller pan into which the proper weights could be firmly fixed; this little pan had its bottom of cork, and was placed upon a piece of flannel which constantly remained in the pan of the balance. It was lifted off and on by means of a fine India-rubber thread, which was so much stretched by the weight as certainly to avoid any noise or jar from the momentum of the descending pan. A sufficient weight could also be hung on the beam of the balance, so as to take off the entire pressure from the finger at the end of each experiment. This weight could be applied or removed by means of a cam acting upon a lever; and its bearings upon the beam were guarded by India-rubber. It was found that the use of this arrangement, which removed all annoying irregularities of sensation connected with the removal and replacement of the greater (initial) pressure, rendered the results more uniform and diminished the probable error. It also shortened the time necessary for performing the experiments, so that a series of 25 experiments was concluded before the effects of fatigue were noticeable. It may be mentioned that certain causes tended to the constant decrease of the probable error as the experiments went on, these mainly being an increased skill on the part of the *operator* and an education of the sensibility of the *subject.* The finger was supported in such a way as to be lightly but firmly held in position, all the muscles of the arm being relaxed; and the India-rubber top of the brass enlargement at the end of the beam of the balance was never actually separated from the finger. The projecting arm of a filter-stand (the height of which could be adjusted) with some attachments not necessary to detail, gently prevented the finger from moving upwards under the pressure exerted by the weight in the pan. In the case of Mr. Peirce as subject (it may be noted that Mr. Peirce is left-handed, while Mr. Jastrow is strongly right-handed) the tip of forefinger, and in the case of Mr. Jastrow of the middle finger, of the left hand were used. In addition, a screen served to prevent the subject from having any indications whatever of the movements of the operator. It is hardly necessary to say that we were fully on our guard against unconsciously received indications.

The observations were conducted in the following manner: At each sitting three differential weights were employed. At first we always began and ended with the heaviest, but at a later period the plan was to begin on alternate days with the lightest and heaviest. When we began with the heaviest, 25 observations[5] were made with that; then 25 with the middle one, and then 25 with the lightest; this constituted one-half of the sitting. It was completed by three more sets of 25, the order of the weights being reversed. When we began with the lightest, the heaviest was used for the third and fourth sets.

5. At first a short pause was made in the set of 25, at the option of the subject; later this was dispensed with.

In this way 150 experiments on each of us were taken at one sitting of two hours.

A pack of 25 cards were taken, 12 red and 13 black, or *vice versa,* so that in the 50 experiments made at one sitting with a given differential weight, 25 red and 25 black cards should be used. These cards were cut exactly square and their corners were distinguished by holes punched in them so as to indicate the scale of numbers (0, 1, 2, 3) used to designate the degree of confidence of the judgment. The backs of these cards were distinguished from their faces. They were, in fact, made of ordinary playing-cards. At the beginning of a set of 25, the pack was well shuffled, and, the operator and subject having taken their places, the operator was governed by the color of the successive cards in choosing whether he should first diminish the weight and then increase it, or *vice versa.* If the weight was to be first increased and then diminished, the operator brought the pressure exerted by the kilogram alone upon the finger of the subject by means of the lever and cam mentioned above, and when the subject said "change" he gently lowered the differential weight, resting in the small pan, upon the pan of the balance. The subject, having appreciated the sensation, again said "change," whereupon the operator removed the differential weight. If, on the other hand, the color of the card directed the weight to be first diminished and then increased, the operator had the differential weight already on the pan of the balance before the pressure was brought to bear on the finger, and made the reverse changes at the command of the subject. The subject then stated his judgment and also his degree of confidence, whereupon the total pressure was at once removed by the cam, and the card that had been used to direct the change was placed face down or face up according as the answer was right or wrong, and with corner indicating the degree of confidence in a determinate position. By means of these trifling devices the important object of rapidity was secured, and any possible psychological guessing of what change the operator was likely to select was avoided. A slight disadvantage in this mode of proceeding arises from the long runs of one particular kind of change, which would occasionally be produced by chance and would tend to confuse the mind of the subject. But it seems clear that this disadvantage was less than that which would have been occasioned by his knowing that there would be no such long runs if any means had been taken to prevent them.

At the end of each set the results were of course entered into a book.[6]

The following tables show the results of the observations for each day:

Date.	Ratios of pressures. [Subject: Mr. Peirce.]						
	1.100	1.080	1.060	1.050	1.040	1.030	1.015
December 10	2 errors.	. . . .	. . . .	13 errors.	. . . .	. . . .	. . . .
December 13	. . . .	4 errors.	8 errors.	. . . .	15 errors.	. . . .	. . . .
December 17	. . . .	. . . .	11	. . . .	. . . .	20 errors.	. . . .
December 20	. . . .	. . . .	7	. . . .	. . . .	16	21 errors.
January 3	. . . .	. . . .	14	. . . .	. . . .	20	28
January 15	. . . .	. . . .	15	. . . .	. . . .	29	17
January 22	. . . .	. . . .	12	. . . .	. . . .	16	20
January 24	. . . .	. . . .	6	. . . .	. . . .	15	22
Means	2	4	10.4 ± 1.0	13	15	19.3 ± 1.4	21.6 ± 1.1
Calculated from probable error = 0.051	4.6 ± 1.0	7.2 ± 1.6	10.7 ± 0.8	12.7 ± 2.1	14.9 ± 2.2	17.2 ± 0.9	21.0 ± 1.1
Average confidence.							
Observed	1.9	0.9	0.7	0.8	0.3	0.3	0.2
Calculated	1.3	1.0	0.7	0.6	0.5	0.3	0.2

The numbers in the columns show the number of errors in fifty experiments. With the average number of errors in a set of fifty we compare the theoretical value of this average as calculated by the method of least squares. The number .051 thus obtained in this case best satisfies the mean number of errors. The numbers affixed with a sign denote, in the upper row the observed *(a posteriori)* probable error of the mean value as given, in the lower row the calculated *(a priori)* probable error. The last two lines give the average confidence observed and calculated with each variation of the ratios of pressure. It will be seen that the correspondence between the real and theoretical numbers is close, and closest when the number of sets is large. The probable errors also closely correspond, the observed being, as is natural, slightly larger than the calculated probable errors.

The following is a similar table for Mr. Jastrow as subject:

6. In the experiments of December 1883, and January 1884, the method as above described was not fully perfected, the most important fault being that the total weight instead of being removed and replaced by a mechanical device, was taken off by the operator's pressing with his finger upon the beam of the balance.

Date.	Ratios of pressures.									
	1.100	1.080	1.060	1.050	1.040	1.030	1.020	1.015	1.010	1.005
December 10	5	.....	.....	19	.....	.....	.....	.....	.....	.....
December 13	.....	9	15	.....	15	.....	.....	.....	.....	.....
December 17	.....	.....	14	.....	.....	23	.....	.....	.....	.....
December 20	.....	.....	10	.....	.....	17	.....	25	.....	.....
January 3	.....	.....	8	.....	.....	14	.....	24	.....	.....
January 10	.....	.....	7	.....	.....	13	.....	17	.....	.....
January 15	.....	.....	12	.....	.....	6	.....	22	.....	.....
January 22	.....	.....	11	.....	.....	10	.....	16	.....	.....
January 24	.....	.....	4	.....	.....	11	.....	18	.....	.....
February 11	.....	.....	1	.....	.....	7	.....	18	.....	.....
February 17	.....	.....	2	.....	.....	10	.....	17	.....	.....
February 18	.....	.....	2	.....	.....	11	.....	17	.....	.....
February 24	.....	.....	2	.....	.....	8	.....	15	.....	.....
March 4	.....	.....	.....	.....	.....	.....	13	.....	16	.....
March 5	.....	.....	.....	.....	.....	.....	13	.....	17	.....
March 18	.....	.....	.....	.....	.....	.....	14	.....	19	29
March 19	.....	.....	.....	.....	.....	.....	11	.....	21	18
March 23	.....	.....	.....	.....	.....	.....	14	.....	17	18
March 25	.....	.....	.....	.....	.....	.....	12	.....	16	18
March 30	.....	.....	.....	.....	.....	.....	11	.....	16	21
March 31	.....	.....	.....	.....	.....	.....	10	.....	15	21
April 2	.....	.....	.....	.....	.....	.....	11	.....	17	21
April 3	.....	.....	.....	.....	.....	.....	9	.....	18	20
April 6	.....	.....	.....	.....	.....	.....	12	.....	15	21
April 7	.....	.....	0	.....	.....	5	7	14	15	17
Means	5	9	6.6	19	15.0	11.6	11.4	18.9	16.8	20.5

It would obviously be unfair to compare these numbers with any set of theoretical numbers, since the probable error is on the decrease throughout, owing to effects of practice, &c. For various reasons we can conveniently group these experiments into four groups. The first will include the experiments from December 10 to January 22, inclusive; the second from January 24 to February 24, inclusive; the third from March 4 to March 25, inclusive; the fourth from March 30 to the end of the work.

The mean results for the different groups are exhibited in the following tables:

First group.

[Probable error = 0.05]

Ratios of pressures.	Number of sets of 50.	Average number of errors.		Average confidence.	
		Observed.	Calculated from probable error.	Observed.	Calculated.
1.100	1	5	4.4 ±1.4	0.9	1.5
1.080	1	9	7.0 ±1.7	0.9	1.2
1.060	7	11.0 ±0.7	10.4 ±0.7	0.85	0.9
1.050	1	19	12.5 ±2.1	0.35	0.7
1.040	1	15	14.7 ±2.2	0.3	0.6
1.030	6	13.8 ±1.5	17.0 ±0.9	0.5	0.4
1.015	5	20.8 ±1.1	21.0 ±1.1	0.3	0.2

Second group.

[Probable error = 0.0235]

1.060	5	2.2 ±0.3	2.1 ±0.4	1.0	1.2
1.030	5	9.4 ±0.6	9.6 ±0.8	0.55	0.6
1.015	5	17.0 ±0.3	16.6 ±1.0	0.3	0.3

Third group.

[Probable error = 0.02]

Ratios of pressures.	Number of sets of 50.	Average number of errors.		Average confidence.	
		Observed.	Calculated from probable error.	Observed.	Calculated.
1.020	6	12.8 ±0.3	12.5 ±0.8	0.12	0.12
1.010	6	17.7 ±0.6	18.3 ±0.9	0.07	0.06
1.005	4	20.7 ±1.7	21.6 ±1.2	0.00	0.03

Fourth group.

[Probable error = 0.0155]

1.060	1	0	0.8 ±0.6	1.6	....
1.030	1	5	4.8 ±1.4	0.5	0.4
1.020	6	10.0 ±0.5	9.6 ±0.8	0.1	0.2
1.015	1	14	12.8 ±2.1	0.1	0.13
1.010	6	16.0 ±0.3	16.5 ±0.9	0.05	0.12
1.005	6	20.8 ±0.4	20.6 ±1.0	0.00	0.06

The tables show that the numbers of errors follow, as far as we can conveniently trace them, the numbers assigned by the probability curve,[7] and therefore destroy all presumption in favor of an *Unterschiedsschwelle.* The introduction and retention of this false notion can only confuse thought, while the conception of the mathematician must exercise a favorable influence on psychological experimentation.[8]

The quantity which we have called the degree of confidence was probably the secondary sensation of a difference between the primary sensations compared. The evidence of our experiments seems clearly to be that this sensation has no *Schwelle,* and vanishes only when the difference to which it refers vanishes. At the same time we found the subject often overlooked this element of his field of sensa-

7. In the tables of the third and fourth groups, there is a marked divergence between the *a priori* and *a posteriori* probable error, for the average number of errors in 50, making the observed probable error too small. This can only be partly accounted for by the fact that the subject formed the unconscious habit of retaining the number of each kind of experiment in a set and answering according to that knowledge. In point of fact the plus errors and minus errors separately do not exhibit the singular uniformity of their sums, for which we are quite unable to account. Thus in the fourth group we have:

Number of + and − errors.

Date.	1.020	1.010	1.005
March 30	−4, +7	−6, +10	−13, +8
March 31	−7, +3	−5, +10	− 6, +15
April 2	−1, +10	−8, +9	− 8, +13
April 3	−4, +5	−4, +14	−10, +10
April 6	−6, +6	−8, +7	−10, +11
April 7	−5, +9	−8, +7	− 8, +9

8. The conclusions of this paper are strengthened by the results of a series of experiments on the color sense, made with the use of a photometer by Mr. Jastrow. The object was to determine the number of errors of a given magnitude, and compare the numbers thus ascertained with the theoretical numbers given by the probability curve. A thousand experiments were made. Dividing the magnitude of the errors from 0 to the largest error, made into 5 parts, the number of errors, as observed and calculated, that occur in each part are as follows:

Observed	199	181	217	213	190
Calculated	213	197	209	181	200

These numbers would be in closer accordance if the probable error were the same throughout, as it is not owing to the effects of practice, &c. Moreover, the experiments were made on different colors—300 on white and 100 each on yellow, blue, dove, pink, green, orange, and brown. These experiments were not continuous.

tion, although his attention was directed with a certain strength toward it, so that he marked his confidence as *zero.* This happened in cases where the judgments were so much affected by the difference of pressures as to be correct three times out of five. The general fact has highly important practical bearings, since it gives new reason for believing that we gather what is passing in one another's minds in large measure from sensations so faint that we are not fairly aware of having them, and can give no account of how we reach our conclusions about such matters. The insight of females as well as certain "telepathic" phenomena may be explained in this way. Such faint sensations ought to be fully studied by the psychologist and assiduously cultivated by every man.

The Numerical Measure of the Success of Predictions

Item 25 *P 292:* Science *4 (14 November 1884): 453–54*

Suppose we have a method by which questions of a certain kind, presenting two alternatives, can in every case be answered, though not always rightly. Suppose, further, that a large number of such answers have been tabulated in comparison with the events, so that we have given the following four numbers:—

(*aa*), the number of questions for which the answers were the first way and the events the first way;
(*ab*), the number of questions for which the answers were the first way and the events the second way;
(*ba*), the number of questions for which the answers were the second way and the events the first way;
(*bb*), the number of questions for which the answers were the second way and the events the second way.

Then the problem is, from these data to assign a numerical measure to the success or science of the method by which the answers have been produced. Mr. G. K. Gilbert (*Amer. Meteorological Journal,* September, 1884) has recently proposed a formula for this purpose; and I desire to offer another.

I make use of two principles. The first is, that any two methods are to be regarded as equal approximations to complete knowledge, which, in the long-run, would give the same values for (*aa*), (*ab*), (*ba*), and (*bb*). The second principle is, that if the answers had been obtained by selecting a determinate proportion of the questions by chance, to be answered by an infallible witness, while the rest were answered by an utterly ignorant person at random (using *yes* and *no* with determinate relative frequencies), then the approximation to knowledge in the answers so obtained would be measured by the fraction expressing the proportion of questions put to the infallible witness. The second witness may know *how often* he ought to answer "yes"; but I give him no credit for that, because he is ignorant *when* he ought to answer "yes."

Let i be the proportion of questions put to the infallible witness, and let j be the proportion of questions which the ignorant witness answers in the first way. Then we have the following simple equations:—

$$\begin{aligned}
(aa) &= i\,\{(aa)+(ba)\} + (1-i)j\{(aa)+(ba)\},\\
(ab) &= (1-i)j\{(ab)+(bb)\},\\
(ba) &= (1-i)\,(1-j)\{(aa)+(ba)\},\\
(bb) &= i\{(ab)+(bb)\} + (1-i)\,(1-j)\{(ab)+(bb)\}.
\end{aligned}$$

Now, whatever the method of predicting, these equations can always be satisfied by possible values of i and j, unless the answers are worse than if they had been taken at random. Consequently, in virtue of the two principles just enunciated, the value of i obtained by solving these equations is the measure of the science of the method. This value is,

$$\begin{aligned}
i &= \frac{(aa)}{(aa)+(ba)} - \frac{(ab)}{(ab)+(bb)},\\
&= \frac{(aa)}{(aa)+(ba)} + \frac{(bb)}{(ab)+(bb)} - 1,\\
&= \frac{(aa)\,(bb)-(ab)\,(ba)}{\{(aa)+(ba)\}\,\{(ab)+(bb)\}}.
\end{aligned}$$

Mr. Gilbert's formula has the same numerator, but a different denominator. It is, in the present notation,

$$i = 2\frac{(aa)\,(bb)-(ab)\,(ba)}{\{(aa)+(ab)+(ba)+(bb)\}^2-(aa)^2+(ab)^2+(ba)^2-(bb)^2}.$$

For Sergeant Finley's tornado-predictions, $(aa)=28$, $(ab)=72$, $(ba)=23$, $(bb)=2{,}680$. From these data, Mr. Gilbert finds $i=0.216$, while my formula gives $i=0.523$.

If the questions should present more than two alternatives, it would be necessary to assign relative values or measures to the different kinds of mistakes that might be made. I have a solution for this case.

Another problem is to measure the utility of the method of prediction. For this purpose, let p be the profit, or saving, from predicting a tornado, and let l be the loss from every unfulfilled prediction of a tornado (outlay in preparing for it, etc.); then the average profit per prediction would be,

$$\frac{p(aa) - l(ab)}{(aa) + (ab) + (ba) + (bb)}.$$

The "Old Stone Mill" at Newport

Item 26 *P 293:* Science *4 (5 December 1884): 512–14*

Finding myself in Newport lately, I took occasion to make some measurements upon that old circular building about whose origin (whether English or Norse) there has been so much dispute. I have not the slightest title to an opinion upon that subject, in which I have only a metrological concern. The building is circular, and rests upon eight cylindrical pillars. It is of such a size that anyone would say, before measuring it, that the pillars would be circumscribed by a circle of four yards radius, and inscribed by one of three yards radius. The building could not have been erected without a drawing to scale, so that a unit of length must have been employed, and that unit (whether Norsemen or English were the builders) would undoubtedly be a foot. The Icelandic foot was, I take it, the same as Denmark and the Scandinavian countries used up to the adoption of the metric system; that is to say, it coincided with the Prussian foot of 12.36 inches English.

I found the diameters of the structure, measured at the pillars, as follows:—

From outside to outside of the shafts.		Between the inward sides.	
	24 feet 8 inches.		18 feet 6 inches.
	24 " 8 "		18 " 5 "
	24 " 9 "		18 " 4 "
	24 " 7 "		18 " 5 "
Mean	24 feet 8 inches.	Mean	18 feet 5 inches.

I think there can be little question that these lengths were meant to be 24 and 18 of the feet used. But supposing that I ought to have gone, say, farther out for the outer diameter (for instance, as far as the bases of the pillars extend), then I ought to have cut off the internal measure by the same amount; so that the mean of the two measures that I have taken is almost certainly 21 of the original feet. This mean is 21 feet $6\frac{1}{2}$ inches, which, divided by 21, gives 12.31 inches as the length of the foot used. Besides the two lengths just mentioned, I found no other of sufficient magnitude, which I could conveniently measure, except the heights of the pillars. These appear to be intended to be 8 feet from the top of the base to the upper side of the cap-stones. The latter are 6 inches thick, as well as I could judge, leaving $7\frac{1}{2}$ feet for the height between the base and capital. This could readily be measured with a tape-line, and was measured[1] in the insides of the pillars at two places on each pillar,—one at the right, and the other at the left. The following are the results:—

North arch.	East arch.	South arch.	West arch.
7 ft. 7 in.	7 ft. 8 in.	8 ft. 2 in.	7 ft. $7\frac{1}{2}$ in.
7 " 5 "	7 " $8\frac{1}{2}$ "	8 " 2 "	7 " 8 "
North-east arch.	South-east arch.	South-west arch.	North-west arch.
7 ft. $7\frac{1}{2}$ in.	7 ft. 9 in.	8 ft. $0\frac{1}{2}$ in.	7 ft. 6 in.
7 " $8\frac{1}{2}$ "	7 " 8 "	8 " 0 "	7 " $6\frac{1}{2}$ "
East arch.	South arch.	West arch.	North arch.

The mean of these is 7 feet $8\frac{7}{8}$ inches; but the two south-west pillars are so different from the others, that I think it is more satisfactory to adopt the middling heights. Excluding, then, the two highest and two shortest pillars, the others measure

7 feet 8 inches.
7 " $8\frac{1}{4}$ "
7 " $8\frac{1}{2}$ "
7 " $7\frac{3}{4}$ "

Mean . . 7 feet $8\frac{1}{8}$ inches.

1. The tape-line is believed to require about half an inch negative correction for all the measures. This has not been applied, as I have been unable to obtain the tape to verify the correction. In any case, such a correction is negligible in measuring so rough a structure.

We have, then,

	Outer diameter.	Inner diameter.	Height.
Presumed intentional measure,	24 ft. 0 in.	18 ft. 0 in.	7 ft. 6 in.
Same in English feet, if foot used was 12.31 English inches,	24 " 7.4 "	18 " 5.6 "	7 " 8.2 "
Same, if foot used was the Scandinavian foot of 12.36 English inches,	24 " 8.6 "	18 " 6.5 "	7 " 8.7 "
Observed	24 " 8 "	18 " 5 "	7 " 8.1 " (7 " 8.9 ")

I made some other measures, which, though I think them of no value for determining the value of the foot, I proceed to give.

Heights of the pillars on the outside.

7 feet $3\frac{1}{2}$ inches.
7 " 2 "
7 " 4 "
7 " 6 "
7 " 3 "
7 " 10 "
7 " 7 "
7 " 5 "

Mean . . 7 feet $5\frac{1}{16}$ inches.

Diameters of bases of columns (most regular).

3 feet 10 inches.
3 " $9\frac{1}{4}$ "
3 " $9\frac{1}{2}$ "
3 10 "

Mean . . 3 feet $9\frac{3}{4}$ inches.

Fireplace

Breadth . . . 3 ft. 5 in.
Height . . . . 4 " 0 "
Breadth at base, 3 " 7 "

Circumferences of the pillars.

10 feet $1\frac{1}{2}$ inches.
10 " 2 "
10 " 0 "
9 " 9 "
9 " $9\frac{1}{2}$ "

Mean. . 9 feet 11.6 inches,
giving diameter, 3 feet 2.1 inches.

Circumferences of bases.

12 feet 2 inches.
11 " 9 "
12 " 1 "
12 " 2 "

Mean . . 12 feet $0\frac{1}{2}$ inches,
corresponding to diameter, 3 " 10 "

Niche on the right of fireplace.

Breadth . . . 2 ft. $3\frac{1}{4}$ in.
Height . . . 2 " $1\frac{1}{4}$ "

Small niche or socket on south side.

Breadth . . . 1 ft. $6\frac{1}{4}$ in.
Height . . . 1 " 8 "

Sockets for jambs below.

Breadth . . . 0 ft. $4\frac{1}{4}$ in.
Height . . . 0 " 4 "
Depth . . . 0 " $2\frac{1}{3}$ "
Original height of frame, 2 feet.

West window.

Breadth . . . 2 ft. 2 in.

South window.

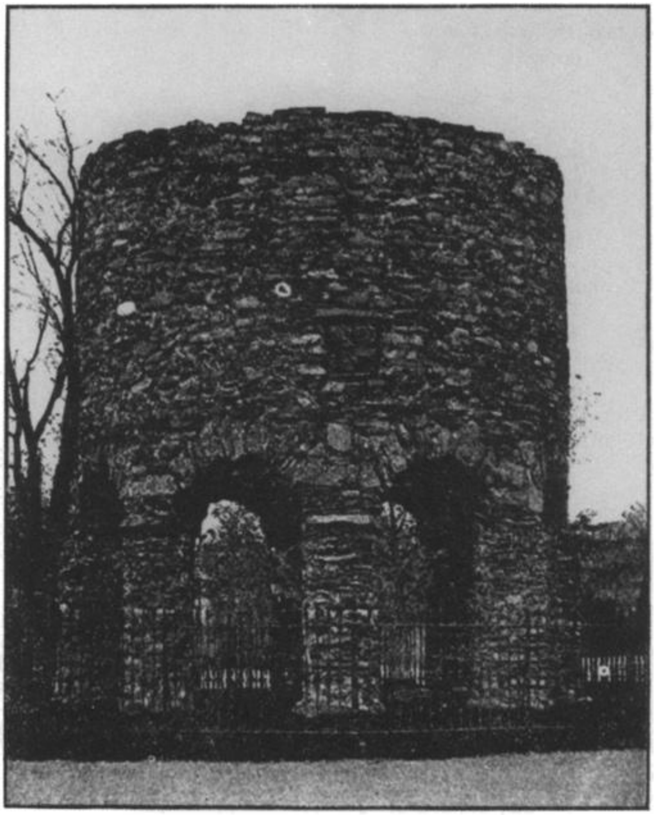
Height outside, 2 ft. $5\frac{1}{4}$ in.
Breadth . . . 2 " 2 "

South-west niche.

Breadth . . . 1 ft. 6 in.
Height . . . . 1 " 3 "

Higher socket.

Breadth . . . 1 ft. 6 in.
Height . . . . 1 " 4 "



My ladder was too short to enable me to measure the upper parts.

Without wishing to express any archeological opinion whatever, I cannot refrain from saying, that, as far as I could perceive, all the rough-cast covering the walls, the smooth mortar in the sockets, etc., were a part of the original mortar. There is a certain amount of later mortar, but it is readily distinguished upon close inspection.

The fireplace has two flues; and the windows formerly had frames, as if for holding glass. The projections of the pillars beyond the upper part of the tower suggest that there might have been a ledge upon which a miller could climb to turn round the axis of the sails of the wind-mill. The two separate flues to the fireplace might prevent the draught from being interfered with by the axle. But would not a fire in a grist-mill be dangerous?

The hearth of the fireplace was elevated above the floor, as in a forge. The building had two stories above the ground. Its total height is about twenty-five feet.

The stones, many of them granite, show no drill-marks and no marks of an axe, but do show marks of the hammer.

The Reciprocity Treaty with Spain

Item 27 *P 279b:* Nation *39 (18 December 1884): 521*

To the Editor of *The Nation:*

Sir: You seem to hold that the ratification of the Spanish treaty would not for a number of years affect the price of sugar "to the consumer," in this country; and that during the gradual decline of importations from non-Spanish ports, the price would be fully maintained. I find this position so difficult to understand, that I beg for some further elucidation of it.

1. Would not the Spanish ports immediately begin sending us more sugar, full 20 per cent. more the first year? Would they not import sugar to send us?

2. If the Spanish ports should send us more, would not one of two things necessarily happen, namely, either that the price would fall, or that the non-Spanish ports would send less?

3. But if the importation from non-Spanish ports were to be diminished by the effect of the treaty (as you seem to admit it would be), would not the sugar withdrawn be the product of those lands which among all those now raising sugar for this country are the worst fitted for this purpose? Would not the result be that the worst of the land then producing sugar for us would be better than the worst of the land now doing so? And would not this state of things, by the operation of competition, work a fall in the price?

C. S. Peirce

Washington, December 15.

[It seems to us a very simple and easily understood proposition that all sellers of sugar in the New York market will ask and obtain the same price for the same grade of sugar, treaty or no treaty. The planter in Manila will receive the same rate per pound as the planter in Cuba. The Manila planter, however, must pay two cents per pound duty before he can reach the market at all, while the Cuban planter need not pay. Now, if Cuba and Porto Rico could at once supply us with all the sugar we consume and something more, then the law of competition among Cuban and Porto Rican planters would force down the price, and the American consumers would get the benefit. But so long as those islands produce something less than the whole amount, a portion of our supply must come from other parts of the world and enter the market loaded with the duty. As there cannot be two prices for the same article at the same place, the market price of sugar in New York under these conditions will be the cost of production in Manila, plus transportation, etc., plus duty. This price the Cuban planter will obtain equally with the planters of Manila, Jamaica, Brazil, and every other country, and of course the American consumer will pay it because the importer must be reimbursed for all his expenses. The situation of the Cuban planter under the operation of the treaty will be precisely the same as that of the Louisiana planter under the tariff. If Louisiana could supply the entire American demand and something more, the law of competition would force down the price more or less, and the consumer would get the benefit.

It has been stated that Cuba and Porto Rico are capable of producing all the sugar consumed in this country. It is possible that if all the land in those islands adapted to sugar-growing were utilized for that purpose, the product might be equal to our present demand. But our demand is not a fixed amount. It grows from year to year. The demand for hardly anything grows more rapidly. It is by no means certain that the annual producing capacity of Cuba and Porto Rico, whose areas are limited, would ever overtake our annual consumption, and if it should not, there would still be an importation of duty-paying sugar, which would, by virtue of the economic law already stated, be the sign and evidence that American consumers were deriving no benefit from the treaty. Since the treaty provides for the introduction free of duty only of sugar *grown* in Cuba and Porto Rico, it would be impossible for them to import sugar to send to us. It was charged at one time that Manila sugar had been imported into Honolulu to be reëxported to San Francisco under the treaty with the Hawaiian Islands, but the charge was not sustained upon investigation. Cuba would undoubtedly import sugar for her own consumption, and send us the corresponding amount of her own growth. This would add to her exporting capacity by whatever amount her present population now use, which is not probably equal to one year's increase of our consumption.

The third question propounded by Mr. Peirce would be relevant if we were the only country buying sugar from non-Spanish ports. The sugar which we now take from them would be diverted to England and other importing countries to whatever extent Cuba increased her supplies to us (our consumption remaining the same), or to whatever extent she increased her proportionate supply. Therefore the difference between best lands and worst lands would not necessarily enter into the problem at all.—Ed. *Nation.*]

The Spanish Treaty Once More

Item 28 ***P 306a:*** **Nation** ***40 (1 January 1885): 12***

To the Editor of *The Nation:*

Sir: I have to express my thanks for your clear explanation of your view that the ratification of the reciprocity treaty with Spain would not affect the price of sugar in this country so long as we continued to import any sugar at all from non-Spanish ports. Cuba, you say, would send us more, but the non-Spanish ports would send just as much less, that trade being diverted to England, etc., to replace the falling off in Cuban sugar there.

But I now object that a great volume of trade will not spontaneously divert itself from one market to another, without any motive. Such an event can only be due either to a fall of price in the first market or to a rise in the second. The sugar which is now sent here is sent because, in the existing state of prices, the owner has found it more advantageous to send here than elsewhere; and here it will continue to come, unless prices change sufficiently to overcome the excess of advantage. If, therefore, the price of sugar were not to fall here on the ratification of the treaty, in England it would have to go up. But an advance in price implies diminished sales—diminished production—somebody forced out of the sugar-growing business. Yet nobody could be forced out of that business if the price had nowhere fallen. How can you escape this dilemma?

You say that the price here would be kept up by the duties that would have to be paid on some of the imported sugar (*i.e.,* by the cost of getting it to market), and that when this sugar, thus sent at a disadvantage, ceased to come, then and only then would the price fall. The principle of this seems to me quite sound—only too sound for your conclusion. For the non-Spanish sugar which we now import comes from various countries very differently situated. Upon some of it there is a considerable profit, while some barely pays the cost of production; upon a part of it there is considerably more profit than if it were sent to England, while for a part it is almost a matter of indifference to which market it is sent. If now the treaty should cause less of this non-Spanish sugar to be sent to this country, that which would be diverted would clearly be that which there is now scarce any inducement to send here. It would follow, I think, according to your own principle, that the price here, being no longer kept up by that very unadvantageously sent sugar, must fall when that should cease to come.

C. S. Peirce

Washington, December 22, 1884.

[We "escape this dilemma" by the use of infinitesimals. One thirty-second of a cent per pound or even less would be a sufficient reduction in price to secure the American market to the Cuban planter for all the sugar he could produce. It would give him all the advantage he needs. One thirty-second of a cent per pound would, therefore, be the maximum gain to the American consumer from the treaty, until (if ever) the Cuban supply could overtake and exceed the American demand. Mr. Peirce's second paragraph, he will permit us to say, carries us into the region of the differential calculus beyond our depth.—Ed. *Nation.*]

[Testimony on the Organization of the Coast Survey]

Item 29 *P 339:* **Miscellaneous Documents of the Senate of the United States** ***82 (1886): 370–78*** **(Given 24 January 1885)**

Mr. CHARLES S. PEIRCE, an assistant in the Coast Survey, appeared and made a statement, as follows:

By Mr. LYMAN:

Question. What position do you hold under the Government?

Answer. I am an assistant in the Coast Survey, and have charge of the Office of Weights and Measures under the Coast Survey. I have charge, also, of the gravimetric survey.

Q. Will you, if you please, give the Commission a short sketch in your own language of your work as the person in charge of the Office of Weights and Measures?

Answer. The Office of Weights and Measures at present is a very slight affair, I am sorry to say. It only exists by law, because Congress many years ago directed the Secretary of the Treasury to supply the different States and Territories and so forth with standard weights and measures, and that provision was afterwards extended to the agricultural schools, so that for that purpose it has been necessary to have standards and balances made and the States and schools have been supplied with these articles. We have our office there to keep up the supply of these various things and we take occasion to verify any standard that is referred to us.

But an office of weights and measures in the sense in which it exists in every other country, namely, an office which should be prepared to make exact verifications of all sorts of standards and to certify officially to them, does not exist in the United States. We have had within a week or so an application from a surveyor who sent on a couple of yards which he wanted verified better than that could be done at any private institution. We have had standard gauges to verify. We have had micrometrical scales, whose errors were to be determined. So far as we can do those things we do them, and if any work involving expense is required we have that done outside. In far too many instances we want the means of executing the verifications asked of us. Thus what we do in the Office of Weights and Measures is to supply the different persons whom we are directed to supply, and then in addition do as much of the usual work of an office of weights and measures as our very narrow circumstances will permit. But we are not prepared to fulfill the real functions of such an office and we do not do what the public demands of us, or what we are constantly asked to do.

Q. Will you tell us what your idea is of what an office of weights and measures should be—such an organization as would fulfill reasonably the public demands for such work. Supposing you had *carte blanche* for such an organization, how would you proceed?

A. I think the proper functions of weights and measures in this country are these: In the first place the legal units of length and of weight, say the yard, the metre, the pound, and kilogram, we should be prepared to verify. That is, if anybody sends in copies of those weights and measures and wishes to have them compared with our standards, we should be authorized to send them back an official certificate as to the length or weight of the copies sent and that certificate should be valid in the courts. We now do these things, but our certificate has no legal value of a different nature from that of any expert testimony. In the second place, we should be prepared to speedily verify, for people making proper application, the multiples and submultiples of the units of mass and length. Messrs. Pratt & Whitney, of Hartford, Messrs. Brown & Sharp, of Providence, Messrs. Sellers & Co., of Philadelphia, and others are engaged in making gauges of great precision, and it is highly important and to the interest of all parties that their gauges should exactly correspond. This exact uniformity can only be secured by all their gauges measuring precisely what they purport to measure in inches. It is therefore desirable that our Office should be prepared to speedily ascertain for such parties whether a given gauge is really, say an inch and a half, or what its true measure is. And to this we should certify officially.

The same thing is true with regard to weights about which we have constant applications; and, so with the capacity measures.

In the third place, there was a conference of electricians in Philadelphia last summer, under the auspices of the Government, and those gentlemen passed a unanimous resolution that the Government ought to supply units for electrical measurements. I think that that was a proper demand, because there exists precisely the same reason why Congress should establish electrical units, as why Congress should establish units of weight and measures of length, namely, important and very numerous contracts are made about electricity. Electricity is an article of sale, it is something offered in the market, and the interpretation of these contracts depends upon the units of electrical quantities. Now you cannot define how much you have got of such an article any more than how much you have of any material substance which is to be weighed, except by reference to some arbitrary standard, some particular body, which is to be its equivalent; and the only way of producing any uniformity in regard to an arbitrary unit is for the Government to intervene and declare what the standard shall be. That is, Government must declare, that whatever bears the stamp or certificate of a certain office shall be respected in the courts as being entitled to the designation, say of an ohm or a weber. This is what the electricians wish to have done, and it appears that Congress is authorized to do it under the terms of the Constitution, and that it is a reasonable demand that the Office of Weights and Measures shall be put in a condition to give those people the ohms, &c., that they require.

Another function that I think the Office should fulfill is that we should have a scientific control over all operations of weighing and measuring in the different bureaus of the Treasury. I do not mean that we should be constantly interfering, but rather that the fact should be recognized that those offices may throw upon the Office of Weights and Measures the responsibility of regulating their methods of procedure. Take the Mint, for instance; the Mint is not, in a narrow use of the term, a scientific office; and yet, it carries on one operation at least, that of weighing the gold and silver, where scientific exactitude is highly desirable, if not necessary. In weighing gold it makes a very considerable difference if all your coins turned out of a certain mint are even a little too heavy or too light; and it appears to me that it would be better for the mints and better for the country, if there were a scientific control of the operation of weighing, more than the annual inspection gives, if there were somebody who was responsible for the methods they use being correct.

Congress has declared a certain brass weight, which is deposited in the Philadelphia mint, to be the pound of the United States for purposes of coinage. Copies of that brass weight are made and sent to the different mints in the United States, and a piece of gold is there put upon one side of an ordinary balance and this brass weight on the other side, and they are balanced, the one against the other. But, owing to the buoyancy of the air, which affects brass rather considerably, but gold hardly any at all, gold being so dense, the piece of gold which is put on the pan of the balance will really not weigh quite so much as the brass. That is, it would not balance quite so much brass *in vacuo* as it does in air. The brass is slightly buoyed up. If this buoyancy were equal at all times it would be a matter of indifference, but when they take that brass weight up to the Carson or Denver mints a considerable part of the atmosphere is left below the level of those places, and they get too much metal into their gold coins.

By the CHAIRMAN:

Q. Too much at those points?

A. More than is in the coins coined at the Philadelphia mint. You will see that that is the necessary result of the thing.

Now that is an example of how it is needful that scientific operations, like weighing and measuring, when they are performed in bureaus that have not otherwise a scientific organization, should be controlled by an office of weights and measures. We ought to have such an office to fulfill these functions toward the public and towards the other bureaus of the Government. It need not enter upon the business of inspecting commercial standards, because that is done already by the States in a satisfactory way. They come to us, too. For instance, it was only the other day that the city of New York applied to our office to see that the office in the City Hall had the proper weights and measures for the verifications of the weights and measures used in the markets of the city. And that is a satisfactory way of dealing with ordinary commercial interests; but when it comes to exact verifications, the ordinary State officers of inspection are not prepared to do the work. The national office lacks both authority and means. Neither does it fulfill the reasonable demand of the electricians, that we should supply them with physical units.

By Mr. LYMAN:

Q. Has this brass weight, that you mentioned as being in the Philadelphia mint, ever been weighed *in vacuo* in this country?

A. No, sir; it has never been weighed *in vacuo* at all. It was obtained in 1827 under the presidency of John Quincy Adams. Mr. Gallatin was our minister at London, and it was made by Captain Kater, who was at that time a great authority on the subject; but it is a weight which would now be deemed badly made, because it is hollow, and we do not use hollow weights for measures of precision, because their buoyancy cannot be accurately ascertained. The mint pound was not weighed *in vacuo* by Kater and it has never been weighed since. The Government has, in point of fact, never had any balance that could weigh the pound *in vacuo.*

Q. So that, as I understand you, the pound used in the determination of all our balances is one whose weight we do not know?

A. We do not know it, sir, though it is held that that pound is the pound of the United States. Yet we at our office cannot tell how much that pound is. We do not know, and, consequently, when a person comes to us and asks us to verify a pound weight, and to tell what the correction of it is, we tell them what the correction of it is in terms of the British pound.

Now, the British pound is a pound which was made about 1855, in consequence of the destruction of the old pound and after the system of weights in England had been reformed and modified in 1818. This weight is supposed to be nearly a copy of the old avoirdupois pound. As a standard it has never had any authority conferred upon it by our Congress, and it could not have had any authority at the time when we separated from the mother country; so that I cannot see how that can be legally our pound, except upon the belief that it is of just the same weight as the old pound which was destroyed. Yet our office gives the people the pound in terms of that pound. We verify pounds brought to us and give the errors in terms of that pound. We do so, because that is a pound, the true weight of which we can ascertain, while the weight of the one in the Philadelphia mint we cannot.

Q. When you say "that pound" you mean the British pound which you have imported from England, do you?

A. I was speaking, not of the pound we have imported from England, but of the pound that is deposited there in England; for, once in a while, our pound gets sent over there for reverification; and there is no doubt about that pound and its relation to our copies of it.

By the CHAIRMAN:

Q. You have a copy of that pound in your office here?

A. Yes; an excellent copy.

Q. Is a copy of that pound in the mint, or is that in the mint a different pound?

A. At the time when the pound in the mint was constructed the standard pound of England was the old troy pound of 1758, which was afterwards destroyed at the burning of the houses of Parliament. After that, when new standards were made in consequence of the destruction of those old standards, the English ceased to take the troy pound as their standard. They established the avoirdupois pound, which avoirdupois pound was assumed to have a certain relation to the troy pound, that is, to weigh 7,000 grains, of which the troy pound contained 5,760. The pound at the mint at Philadelphia is a troy pound, a copy of that old pound; yet when people bring us avoirdupois pounds and ask us what their errors are we are compelled to tell them in terms of the present British pound.

Q. Of the avoirdupois and not the troy?

A. Yes; but that is not quite the point. You might convert it from the avoirdupois to the troy by multiplying by $\frac{7000}{5760}$. But the question is about the grain, or fraction of a grain by which the British avoirdupois pound differs from $\frac{7000}{5760}$ of the Philadelphia pound.

Q. How does the mint in Philadelphia give an exact copy to the mint in Denver?

A. Although our Office is in this state of nonentity, having hardly any legal existence, the mints and other bureaus are in the habit of coming to us and asking us to make copies of the pound and to verify them, and we have sent to Philadelphia and have placed the mint pound upon one beam of a balance and another troy pound upon the other side, and have ascertained that the former balances a certain piece of brass in the air; but, inasmuch as the mint pound is hollow, and we cannot tell whether the air leaks in and leaks out with the changes of the barometer or not, the whole proceeding is very unsatisfactory. The question asked brings me to another desideratum to put this Office upon a proper footing. As I say, the English reformed and modified their system of weights and measures in 1818, so that ours no longer coincides with theirs. For instance, a gallon and a quart are very different in the two countries, about in the ratio of 4 to 5, and apothecaries' weights are so different—about 10 per cent.—that one who had got a prescription from an English physician in Montreal and then came and got it put up in New York, would get an appreciably different dose.[1]

Q. We get more, or less?

A. More in this country; for everything in England now goes by avoirdupois weight, while we have retained the old weights and measures. It is highly desirable, it appears to me, that England, and America, and Russia, which are the three countries which use the English system of weights and measures, should establish the weights and measures they do use on an international basis, and at present there is no international basis.

By Mr. HERBERT:

Q. Have you ever compared the pound which you have verified with the mint pound in Philadelphia?

A. That comparison was made by Mr. Lane in 1876.

Q. Was there any difference, and if so, how much?

A. It is believed that there is quite a considerable difference, so great a difference that we are a little in a quandary when people come to us for verifications. The city of New York sent to us recently for avoirdupois and troy weights both, and they send weights to us which are in true relation between troy and avoirdupois weights, and yet our practice has been to verify the avoirdupois weights by the English avoirdupois pound and the troy weights by the pound at Philadelphia, and there is an appreciable difference between those two.

Q. You have a standard troy pound here, have you not?

A. We have no standard troy pound except one derived from that pound at the mint. We can also ascertain what a troy pound would be if obtained from the avoirdupois pound by multiplying by the necessary fraction—a theoretical fraction.

1. 1 British gallon = 1.201 American gallon; 1 British fluid ounce = 0.960 American fluid ounce; 1 British ounce = 0.911 apothecaries' ounce.

Q. Without regard to the mode of obtaining it, my question is, What is the difference, if any, between the troy pound in the mint at Philadelphia and what you understand as the true troy pound—however you may obtain it? What is the difference between that troy pound at the mint in Philadelphia and the true troy pound?

A. The mint pound is declared by Congress to be the true troy pound.

Q. Well, what is the value of a troy pound?

A. It would be a fraction of a grain, between four and five hundredths of a grain, in air different from $\frac{5760}{7000}$ of the British pound.

Q. Is that more or less?

A. That I do not remember. I cannot carry the figures in my mind.

By the CHAIRMAN:

Q. I take it that the troy pound of the mint in Philadelphia having been copied by your office and verified, they are exactly equal in air, are they not? That is, if you have taken a true copy of the Philadelphia brass piece, they must be exactly equal when put upon the scale, or else they would not be equal.

A. They are undoubtedly exactly equal when weighed at that point of air.

Q. But you think if you were to reduce the difference of the troy pound from the avoirdupois pound which you have that they would not coincide with the mint pound at Philadelphia?

A. Precisely.

By Mr. HERBERT:

Q. Is there any metal other than brass that you would suggest, which would avoid this difficulty about the difference in weight resulting from the buoyancy of the atmosphere according to the altitude?

A. Yes.

Q. What metal would be the same *in vacuo* as in air?

A. A different set of weights should be employed to weigh silver and weigh gold. In weighing silver nothing could be made better than a weight made of sterling silver, and in weighing gold, since a gold weight would be rather too soft so that it might wear down, I would prefer an alloy of platinum and iridium, which is considerably used for weights in other countries.

Q. Would that obviate the difficulty?

A. That would obviate the difficulty after we had once ascertained how much to call a pound; but it seems to me as one of the things greatly wanted that the English system of weights and measures should be put upon an international basis, and it is plain that a bureau having a distinct legal standing would be in a better condition to make the necessary *pourparlers* than ours in its present state can be. I think a reorganization of the Office might bring about an international understanding. Those who are opposed to the metric system certainly ought to desire that the English system should be brought into as perfect a condition as possible, so as to give it every chance in the struggle for existence which intrinsically belongs to it. The reasons scientific men oppose the English system of weights and measures are capable of being in great measure obviated by some slight modifications of that system.

By the CHAIRMAN:

Q. It is claimed that the metric system is more accurate, is it not? What is the scientific view of that point?

A. Well, sir, I do not myself believe that the metric system is so accurate. What I mean to say is this: that a yard is capable of being ascertained and known more accurately than a metre. When you speak of a yard you know exactly what length that is, better than you do now when you speak of a metre, because the yard means the distance between two lines drawn upon a certain bar in London, when that bar is at the temperature of 62° Fahrenheit, which is about the temperature at which measurements are generally made. Therefore there is usually but a small correction to be applied to the length of a yard bar to bring it to its standard temperature. But the metre is the distance between the ends of a certain bar at the freezing point of water. It is forbidden to touch those ends or abut up against them, and therefore it is next to impossible to ascertain what is the true distance between them. That difficulty will be obviated in a few years because the International Commission is making new metres which will be defined by lines. But another difficulty will still remain that the standard of temperature of the bar is the freezing point. Now you may ascertain that another bar is of just the same length as that at the freezing point, but then you have got to correct all the way from the temperature of the freezing point to the temperature at which measures are made; and this will always be a subject of very great difficulty; and, therefore, in my opinion, if you come to a question of exactitude the yard is susceptible of being determined with greater exactitude than the metre, and the balance of utility is, therefore, if anything, upon the side of the yard.

I must say this in regard to the metric system: That before I was appointed to the Office of Weights and Measures I was very strongly in favor of that system, and I thought people who were opposed to it were standing in the way of the progress of the world. But since I have been in a more responsible and more practical relation to the subject my opinion has changed very much, and I now think that it is worth while to see whether opinion could not be united upon a reformed English system, which, with slight changes, could have substantially all the advantages of the metrical system; because, if opinion could be united upon such a system in England and in this country, then I think it would be perfectly feasible for us, considering that we are the nations which perform the accurate measurements on a great scale, that we are the ones that have the accurate machine-shops (there being very few, comparatively, in other countries), and that we are, too, the commercial peoples, I think that even at this date it would not be too late to bring about the universal adoption of such a reformed English system.

Q. Then you think an international arrangement between the United States, Russia, and England very desirable?

A. Yes, sir; but that could not be brought about until some one could go to the English and say, "Now, we have a certain authority and influence here, and we feel confident that certain things could be done if you could do certain other preliminary things." The preliminaries cannot be effected unless we have some existence as an office of weights and measures.

By Mr. HERBERT:

Q. Will you state definitely what authority you think Congress ought to confer on your Office in addition to what it possesses now?

A. I think we ought to be authorized to give certificates. We are not authorized at present to give any certificates. The weight of authority in courts ought to be given to our certificates, and those certificates ought to relate to the units of weight and units of length, together with their multiples and submultiples, and also to the units derived from the units of weight, length, and time. That would include the electrical and other physical units.

Q. In addition to giving those certificates, what power is necessary or proper, in your opinion, to be conferred upon your Office?

A. We should exercise certain authority, which is now exercised by the Commissioner of Internal Revenue, with reference to methods of gauging and weighing in the internal revenue, and we should have similar responsibility for the methods of gauging and weighing in the custom-houses and in the mints.

By the CHAIRMAN:

Q. Do you not verify the gauges and other instrumentalities used by the Internal Revenue Bureau now, or are they actually independent of you?

A. They are independent of us, but the methods of gauging were, I believe, settled by a commission established some years ago.

Q. In 1868?

A. Yes.

By Mr. LYMAN:

Q. Would you extend it to such things as the test of sugar by the spectroscope?

A. No; I had not intended that we should do anything further than weighing and measuring. That is a matter of chemical analysis. Yet I would not object to being called upon for such a thing as that.

By Mr. HERBERT:

Q. Do you think you ought to verify the polariscope?

A. My definition, I suppose, would not include it. My definition would take in length and units derived from units of weight, length, and time, thus including units of force, velocity, electrical resistance, &c.

Q. The custom-houses throughout the country ought to put themselves in communication with you, ought they not?

A. They ought to do so.

Q. Do they?

A. I do not know what methods of gauging are employed in the custom-houses; they may, from time to time, have consulted our Office.

Q. Do you mean to say that your Bureau ought to have power to go into these other different departments of the Government and say to them, "Your weights and measures are incorrect and ought to be corrected"; or do you mean to say, simply, that you ought to have the power to give certificates to them when they make application to you?

A. Well, I think the latter would be satisfactory enough; still I think it would be better to require us to report, at stated periods, to the Secretary of the Treasury upon the methods of weighing and measuring used in those different Bureaus.

By the CHAIRMAN:

Q. As to whether their methods are proper?

A. Proper.

By Mr. LYMAN:

Q. What force do you think would be required for an efficient and economical administration of such a Bureau? Of course you cannot tell exactly, but give us an estimate.

A. I should think a dozen persons would be sufficient.

Q. At present about how many can you command in case of necessity?

A. At present we have three persons who could be used in verifications.

Q. With a dozen persons you think you could cover the ground of the electrical ohm and other measures, do you?

A. With regard to the electrical measurements, I think a great deal of that work ought to be done outside. I think a Bureau of the Government cannot very properly be expected to be doing original scientific work. Its natural functions are to do routine work. It can rise above that to a certain extent now and then, but you cannot expect the highest kind of scientific work to result from any mere work of organization. That can only come from individual genius.

If you find that the head of a Bureau is disposed to take up scientific questions and treat the business of his office with a little unnecessary scientific thoroughness, I should not think it was advisable to come down upon him and say, "You are to work simply for the $4,500 or $5,000 that you are receiving, and are to take no interest in the scientific perfection of your work beyond the point at which it saves money." I would not frown upon scientific aspirations; and yet it is hardly to be expected that scientific investigation undertaken incidentally by a Bureau of the Government, should, in the long run, be of the very highest character.

By the CHAIRMAN:

Q. Here is the original provision [reading]: "For construction and verification of standard weights and measures, including metric standards for the custom-houses, other offices of the United States, and for the several States, and mural standards of length in Washington, District of Columbia," &c. So it seems to contemplate, at least if called upon, that you are bound to furnish the custom-houses with verifications of weights?

A. You are undoubtedly right about that. Since I have been in charge of the office we have not happened to be called upon to supply custom-houses.

Adjourned to meet at the call of the chair.

On the Algebra of Logic: A Contribution to the Philosophy of Notation

Item 30 *P 296:* American Journal of Mathematics *7 (1885): 180–202*

I.—*Three kinds of Signs.*

Any character or proposition either concerns one subject, two subjects, or a plurality of subjects. For example, one particle has mass, two particles attract one another, a particle revolves about the line joining two others. A fact concerning two subjects is a dual character or relation; but a relation which is a mere combination of two independent facts concerning the two subjects may be called *degenerate,* just as two lines are called a degenerate conic. In like manner a plural character or conjoint relation is to be called degenerate if it is a mere compound of dual characters.

A sign is in a conjoint relation to the thing denoted and to the mind. If this triple relation is not of a degenerate species, the sign is related to its object only in consequence of a mental association, and depends upon a habit. Such signs are always abstract and general, because habits are general rules to which the organism has become subjected. They are, for the most part, conventional or arbitrary. They include all general words, the main body of speech, and any mode of conveying a judgment. For the sake of brevity I will call them *tokens.*

But if the triple relation between the sign, its object, and the mind, is degenerate, then of the three pairs

sign	object
sign	mind
object	mind

two at least are in dual relations which constitute the triple relation. One of the connected pairs must consist of the sign and its object, for if the sign were not related to its object except by the mind thinking of them separately, it would not fulfil the function of a sign at all. Supposing, then, the relation of the sign to its object does not lie in a mental association, there must be a direct dual relation of the sign to its object independent of the mind using the sign. In the second of the three cases just spoken of, this dual relation is not degenerate, and the sign signifies its object solely by virtue of being really connected with it. Of this nature are all natural signs and physical symptoms. I call such a sign an *index*, a pointing finger being the type of the class.

The index asserts nothing; it only says "There!" It takes hold of our eyes, as it were, and forcibly directs them to a particular object, and there it stops. Demonstrative and relative pronouns are nearly pure indices, because they denote things without describing them; so are the letters on a geometrical diagram, and the subscript numbers which in algebra distinguish one value from another without saying what those values are.

The third case is where the dual relation between the sign and its object is degenerate and consists in a mere resemblance between them. I call a sign which stands for something merely because it resembles it, an *icon*. Icons are so completely substituted for their objects as hardly to be distinguished from them. Such are the diagrams of geometry. A diagram, indeed, so far as it has a general signification, is not a pure icon; but in the middle part of our reasonings we forget that abstractness in great measure, and the diagram is for us the very thing. So in contemplating a painting, there is a moment when we lose the consciousness that it is not the thing, the distinction of the real and the copy disappears, and it is for the moment a pure dream,—not any particular existence, and yet not general. At that moment we are contemplating an *icon*.

I have taken pains to make my distinction[1] of icons, indices, and tokens clear, in order to enunciate this proposition: in a perfect system of logical notation signs of these several kinds must all be employed. Without tokens there would be no generality in the statements, for they are the only general signs; and generality is

1. See *Proceedings of the American Academy of Arts and Sciences*, Vol. VII, p. 294, May 14, 1867.

essential to reasoning. Take, for example, the circles by which Euler represents the relations of terms. They well fulfil the function of icons, but their want of generality and their incompetence to express propositions must have been felt by everybody who has used them. Mr. Venn has, therefore, been led to add shading to them; and this shading is a conventional sign of the nature of a token. In algebra, the letters, both quantitative and functional, are of this nature. But tokens alone do not state what is the subject of discourse; and this can, in fact, not be described in general terms; it can only be indicated. The actual world cannot be distinguished from a world of imagination by any description. Hence the need of pronouns and indices, and the more complicated the subject the greater the need of them. The introduction of indices into the algebra of logic is the greatest merit of Mr. Mitchell's system.[2] He writes F_1 to mean that the proposition F is true of every object in the universe, and F_u to mean that the same is true of some object. This distinction can only be made in some such way as this. Indices are also required to show in what manner other signs are connected together. With these two kinds of signs alone any proposition can be expressed; but it cannot be reasoned upon, for reasoning consists in the observation that where certain relations subsist certain others are found, and it accordingly requires the exhibition of the relations reasoned with in an icon. It has long been a puzzle how it could be that, on the one hand, mathematics is purely deductive in its nature, and draws its conclusions apodictically, while on the other hand, it presents as rich and apparently unending a series of surprising discoveries as any observational science. Various have been the attempts to solve the paradox by breaking down one or other of these assertions, but without success. The truth, however, appears to be that all deductive reasoning, even simple syllogism, involves an element of observation; namely, deduction consists in constructing an icon or diagram the relations of whose parts shall present a complete analogy with those of the parts of the object of reasoning, of experimenting upon this image in the imagination, and of observing the result so as to discover unnoticed and hidden relations among the parts. For instance, take the syllogistic formula,

2. *Studies in Logic. By Members of the Johns Hopkins University.* Boston: Little, Brown, & Co., 1883.

All M is P
S is M
$\therefore$ S is P.

This is really a diagram of the relations of S, M, and P. The fact that the middle term occurs in the two premises is actually exhibited, and this must be done or the notation will be of no value. As for algebra, the very idea of the art is that it presents formulae which can be manipulated, and that by observing the effects of such manipulation we find properties not to be otherwise discerned. In such manipulation, we are guided by previous discoveries which are embodied in general formulae. These are patterns which we have the right to imitate in our procedure, and are the *icons par excellence* of algebra. The letters of applied algebra are usually tokens, but the x, y, z, etc. of a general formula, such as

$$(x + y)z = xz + yz,$$

are blanks to be filled up with tokens, they are indices of tokens. Such a formula might, it is true, be replaced by an abstractly stated rule (say that multiplication is distributive); but no application could be made of such an abstract statement without translating it into a sensible image.

In this paper, I purpose to develope an algebra adequate to the treatment of all problems of deductive logic, showing as I proceed what kinds of signs have necessarily to be employed at each stage of the development. I shall thus attain three objects. The first is the extension of the power of logical algebra over the whole of its proper realm. The second is the illustration of principles which underlie all algebraic notation. The third is the enumeration of the essentially different kinds of necessary inference; for when the notation which suffices for exhibiting one inference is found inadequate for explaining another, it is clear that the latter involves an inferential element not present to the former. Accordingly, the procedure contemplated should result in a list of categories of reasoning, the interest of which is not dependent upon the algebraic way of considering the subject. I shall not be able to perfect the algebra sufficiently to give facile methods of reaching logical conclusions: I can only give a method by which any legitimate conclusion may be reached and any fallacious one avoided. But I cannot doubt that others, if they will take up the subject, will succeed in giving the notation a form in which it will be highly useful in mathematical work. I even hope that what I have done may prove a first step toward the resolution of one of the main problems of logic, that of producing a method for the discovery of methods in mathematics.

II.—*Non-relative Logic.*

According to ordinary logic, a proposition is either true or false, and no further distinction is recognized. This is the descriptive conception, as the geometers say; the metric conception would be that every proposition is more or less false, and that the question is one of amount. At present we adopt the former view.

Let propositions be represented by quantities. Let v and f be two constant values, and let the value of the quantity representing a proposition be v if the proposition is true and be f if the proposition is false. Thus, x being a proposition, the fact that x is either true or false is written

$$(x - \mathrm{f})(\mathrm{v} - x) = 0.$$

So

$$(x - \mathrm{f})(\mathrm{v} - y) = 0$$

will mean that either x is false or y is true. This may be said to be the same as "if x is true, y is true." A hypothetical proposition, generally, is not confined to stating what actually happens, but states what is invariably true throughout a universe of possibility. The present proposition is, however, limited to that one individual state of things, the Actual.

We are, thus, already in possession of a logical notation, capable of working syllogism. Thus, take the premises, "if x is true, y is true," and "if y is true, z is true." These are written

$$(x - \mathrm{f})(\mathrm{v} - y) = 0$$
$$(y - \mathrm{f})(\mathrm{v} - z) = 0.$$

Multiply the first by $(\mathrm{v} - z)$ and the second by $(x - \mathrm{f})$ and add. We get

$$(x - \mathrm{f})(\mathrm{v} - \mathrm{f})(\mathrm{v} - z) = 0,$$

or dividing by $\mathrm{v} - \mathrm{f}$, which cannot be 0,

$$(x - \mathrm{f})(\mathrm{v} - z) = 0;$$

and this states the syllogistic conclusion, "if x is true, z is true."

But this notation shows a blemish in that it expresses propositions in two distinct ways, in the form of quantities, and in the form of equations; and the quantities are of two kinds, namely those which must be either equal to f or to v, and those which are equated to *zero.* To remedy this, let us discard the use of equations, and perform no operations which can give rise to any values other than f and v.

Of operations upon a simple variable, we shall need but one. For there are but two things that can be said about a single proposition, by itself; that it is true and that it is false,

$$x = \mathrm{v} \quad \text{and} \quad x = \mathrm{f}.$$

The first equation is expressed by x itself, the second by any function, φ, of x, fulfilling the conditions

$$\varphi \mathrm{v} = \mathrm{f} \qquad \varphi \mathrm{f} = \mathrm{v}.$$

The simplest solution of these equations is

$$\varphi x = \mathrm{f} + \mathrm{v} - x.$$

A product of n factors of the two forms $(x - \mathrm{f})$ and $(\mathrm{v} - y)$, if not zero equals $(\mathrm{v} - \mathrm{f})^n$. Write P for the product. Then $\mathrm{v} - \dfrac{P}{(\mathrm{v} - \mathrm{f})^{n-1}}$ is the simplest function of the variables which becomes v when the product vanishes and f when it does not. By this means any proposition relating to a single individual can be expressed.

If we wish to use algebraical signs with their usual significations, the meanings of the operations will entirely depend upon those of f and v. Boole chose $\mathrm{v} = 1$, $\mathrm{f} = 0$. This choice gives the following forms:

$$\mathrm{f} + \mathrm{v} - x = 1 - x$$

which is best written $\bar{x}$.

$$\mathrm{v} - \frac{(x - \mathrm{f})(\mathrm{v} - y)}{\mathrm{v} - \mathrm{f}} = 1 - x + xy = \overline{x\bar{y}}$$

$$\mathrm{v} - \frac{(\mathrm{v} - x)(\mathrm{v} - y)}{\mathrm{v} - \mathrm{f}} = x + y - xy$$

$$\mathrm{v} - \frac{(\mathrm{v} - x)(\mathrm{v} - y)(\mathrm{v} - z)}{(\mathrm{v} - \mathrm{f})^2} = x + y + z - xy - xz - yz + xyz$$

$$\mathrm{v} - \frac{(x - \mathrm{f})(y - \mathrm{f})}{\mathrm{v} - \mathrm{f}} = 1 - xy = \overline{xy}.$$

It appears to me that if the strict Boolian system is used, the sign $+$ ought to be altogether discarded. Boole and his adherent, Mr. Venn (whom I never disagree with without finding his remarks profitable), prefer to write $x + \bar{x}y$ in place of $\overline{\bar{x}\,\bar{y}}$. I confess I do not see the advantage of this, for the distributive principle holds equally well when written

$$\overline{\bar{x}\;\bar{y}}\,z = \overline{\overline{xz}\;\overline{yz}}$$
$$\overline{\overline{xy}\;\bar{z}} = \overline{\bar{x}\;\bar{z}} \cdot \overline{\bar{y}\;\bar{z}}.$$

The choice of v $= 1$, f $= 0$, is agreeable to the received measurement of probabilities. But there is no need, and many times no advantage, in measuring probabilities in this way. I presume that Boole, in the formation of his algebra, at first considered the letters as denoting propositions or events. As he presents the subject, they are class-names; but it is not necessary so to regard them. Take, for example, the equation

$$t = n + hf,$$

which might mean that the body of taxpayers is composed of all the natives, together with householding foreigners. We might reach the signification by either of the following systems of notation, which indeed differ grammatically rather than logically.

Sign.	Signification. 1st System.	Signification. 2nd System.
t	Taxpayer.	He is a Taxpayer.
n	Native.	He is a Native.
h	Householder.	He is a Householder.
f	Foreigner.	He is a Foreigner.

There is no *index* to show who the "He" of the second system is, but that makes no difference. To say that he is a taxpayer is equivalent to saying that he is a native or is a householder and a foreigner. In this point of view, the constants 1 and 0 are simply the probabilities, to one who knows, of what is true and what is false; and thus unity is conferred upon the whole system.

For my part, I prefer for the present not to assign determinate values to f and v, nor to identify the logical operations with any special arithmetical ones, leaving myself free to do so hereafter in the manner which may be found most convenient. Besides, the whole system of importing arithmetic into the subject is artificial, and modern Boolians do not use it. The algebra of logic should be self-developed, and arithmetic should spring out of logic instead of reverting to it. Going back to the beginning, let the writing of a letter by itself mean that a certain proposition is true. This letter is a *token.* There is a general understanding that the actual state of things or some other is referred to. This understanding must have been established by means of an *index,* and to some extent dispenses with the need of other indices. The denial of a proposition will be made by writing a line over it.

I have elsewhere shown that the fundamental and primary mode of relation between two propositions is that which we have expressed by the form

$$\mathrm{v} - \frac{(x-\mathrm{f})(\mathrm{v}-y)}{\mathrm{v}-\mathrm{f}}.$$

We shall write this

$$x \prec y,$$

which is also equivalent to

$$(x-\mathrm{f})(\mathrm{v}-y) = 0.$$

It is stated above that this means "if x is true, y is true." But this meaning is greatly modified by the circumstance that only the actual state of things is referred to.

To make the matter clear, it will be well to begin by defining the meaning of a hypothetical proposition, in general. What the usages of language may be does not concern us; language has its meaning modified in technical logical formulae as in other special kinds of discourse. The question is what is the sense which is most usefully attached to the hypothetical proposition in logic? Now, the peculiarity of the hypothetical proposition is that it goes out beyond the actual state of things and declares what *would* happen were things other than they are or may be. The utility of this is that it puts us in possession of a rule, say that "if A is true, B is true," such that should we hereafter learn something of which we are now ignorant, namely that A is true, then, by virtue of this rule, we shall find that we know something else, namely, that B is true. There can be no doubt that the Possible, in its primary meaning, is that which may be true for aught we know, that whose falsity we do not know. The purpose is subserved, then, if, throughout the whole range of possibility, in every state of things in which A is true, B is true too. The hypothetical proposition may therefore be falsified by a single state of things, but only by one in which A is true while B is false. States of things in which A is false, as well as those in which B is true, cannot falsify it. If, then, B is a proposition true in every case throughout the whole range of possibility, the hypothetical proposition, taken in its logical sense, ought to be regarded as true, whatever may be the usage of ordinary speech. If, on the other hand, A is in no case true, throughout the range of possibility, it is a matter of indifference whether the hypothetical be understood to be true or not, since it is useless. But it will be more simple to class it among true propositions, because the cases in which the antecedent is false do not, in any other case, falsify a hypothetical. This, at any rate, is the meaning which I shall attach to the hypothetical proposition in general, in this paper.

The range of possibility is in one case taken wider, in another narrower; in the present case it is limited to the actual state of things. Here, therefore, the proposition

$$a \prec b$$

is true if a is false or if b is true, but is false if a is true while b is false. But though we limit ourselves to the actual state of things, yet when we find that a formula of this sort is true by logical necessity, it becomes applicable to any single state of things throughout the range of logical possibility. For example, we shall see that from $x \,\overline{\prec}\, y$ we can infer $z \prec x$. This does not mean that because in the actual state of things x is true and y false, therefore in every state of things either z is false or x true; but it does mean that in whatever state of things we find x true and y false, in that state of things either z is false or x is true. In that sense, it is not limited to the actual state of things, but extends to any single state of things.

The *first icon* of algebra is contained in the formula of identity

$$x \prec x.$$

This formula does not of itself justify any transformation, any inference. It only justifies our continuing to hold what we have held (though we may, for instance, forget how we were originally justified in holding it).

The *second icon* is contained in the rule that the several antecedents of a *consequentia* may be transposed; that is, that from

$$x \prec (y \prec z)$$

we can pass to

$$y \prec (x \prec z).$$

This is stated in the formula

$$\{x \prec (y \prec z)\} \prec \{y \prec (x \prec z)\}.$$

Because this is the case, the brackets may be omitted, and we may write

$$y \prec x \prec z.$$

By the formula of identity

$$(x \prec y) \prec (x \prec y);$$

and transposing the antecedents

$$x \prec \{(x \prec y) \prec y\}$$

or, omitting the unnecessary brackets

$$x \prec (x \prec y) \prec y.$$

This is the same as to say that if in any state of things x is true, and if the proposition "if x, then y" is true, then in that state of things y is true. This is the *modus ponens* of hypothetical inference, and is the most rudimentary form of reasoning.

To say that $(x \prec x)$ is generally true is to say that it is so in every state of things, say in that in which y is true; so that we may write

$$y \prec (x \prec x),$$

and then, by transposition of antecedents,

$$x \prec (y \prec x),$$

or from x we may infer $y \prec x$.

The *third icon* is involved in the principle of the transitiveness of the copula, which is stated in the formula

$$(x \prec y) \prec (y \prec z) \prec x \prec z.$$

According to this, if in any case y follows from x and z from y, then z follows from x. This is the principle of the syllogism in *Barbara.*

We have already seen that from x follows $y \prec x$. Hence, by the transitiveness of the copula, if from $y \prec x$ follows z, then from x follows z, or from

$$(y \prec x) \prec z$$

follows

$$x \prec z,$$

or

$$\{(y \prec x) \prec z\} \prec x \prec z.$$

The original notation $x \prec y$ served without modification to express the pure formula of identity. An enlargement of the conception of the notation so as to make the terms themselves complex was required to express the principle of the transposition of antecedents; and this new *icon* brought out new propositions. The third *icon* introduces the image of a chain of consequence. We must now again enlarge the notation so as to introduce negation. We have already seen that if a is true, we can write $x \prec a$, whatever x may be. Let b be such that we can write $b \prec x$ whatever x may be. Then b is false. We have here a *fourth icon*, which gives a new sense to several formulae. Thus the principle of the interchange of antecedents is that from

$$x \prec (y \prec z)$$

we can infer

$$y \prec (x \prec z).$$

Since z is any proposition we please, this is as much as to say that if from the truth of x the falsity of y follows, then from the truth of y the falsity of x follows.

Again the formula

$$x \prec \{(x \prec y) \prec y\}$$

is seen to mean that from x we can infer that anything we please follows from that thing's following from x, and *a fortiori* from everything following from x. This is, therefore, to say that from x follows the falsity of the denial of x; which is the principle of contradiction.

Again the formula of the transitiveness of the copula, or

$$\{x \prec y\} \prec \{(y \prec z) \prec (x \prec z)\}$$

is seen to justify the inference

$$x \prec y$$
$$\therefore \bar{y} \prec \bar{x}.$$

The same formula justifies the *modus tollens,*

$$x \prec y$$
$$\bar{y}$$
$$\therefore \bar{x}.$$

So the formula

$$\{(y \prec x) \prec z\} \prec (x \prec z)$$

shows that from the falsity of $y \prec x$ the falsity of x may be inferred.

All the traditional moods of syllogism can easily be reduced to *Barbara* by this method.

A *fifth icon* is required for the principle of excluded middle and other propositions connected with it. One of the simplest formulae of this kind is

$$\{(x \prec y) \prec x\} \prec x.$$

This is hardly axiomatical. That it is true appears as follows. It can only be false by the final consequent x being false while its antecedent $(x \prec y) \prec x$ is true. If this is true, either its consequent, x, is true, when the whole formula would be true, or its antecedent $x \prec y$ is false. But in the last case the antecedent of $x \prec y$, that is x, must be true.[3]

3. It is interesting to observe that this reasoning is dilemmatic. In fact, the dilemma involves the fifth icon. The dilemma was only introduced into logic from rhetoric by the humanists of the *renaissance;* and at that time logic was studied with so little accuracy that the peculiar nature of this mode of reasoning escaped notice. I was thus led to suppose that the whole non-relative logic was derivable from the principles of the ancient syllogistic, and this error is involved in Chapter II of my paper in the third volume of this Journal. My friend, Professor Schröder, detected the mistake and showed that the distributive formulae

$$(x + y)z \prec xz + yz$$
$$(x + z)(y + z) \prec xy + z$$

could not be deduced from syllogistic principles. I had myself independently discovered and virtually stated the same thing. (*Studies in Logic,* p. 189.) There is some disagreement as to the definition of the dilemma (see Keynes's excellent *Formal Logic,* p. 241); but the most useful definition would be a syllogism depending on the above distribution formulae. The distribution formulae

$$xz + yz \prec (x + y)z$$
$$xy + z \prec (x + z)(y + z)$$

are strictly syllogistic. De Morgan's added moods are virtually dilemmatic, depending on the principle of excluded middle.

From the formula just given, we at once get

$$\{(x -\!\!< y) -\!\!< \alpha\} -\!\!< x,$$

where the α is used in such a sense that $(x -\!\!< y) -\!\!< \alpha$ means that from $(x -\!\!< y)$ every proposition follows. With that understanding, the formula states the principle of excluded middle, that from the falsity of the denial of x follows the truth of x.

The logical algebra thus far developed contains signs of the following kinds:

1st, Tokens; signs of simple propositions, as t for "He is a taxpayer," etc.

2nd, The single operative sign $-\!\!<$; also of the nature of a token.

3rd, The juxtaposition of the letters to the right and left of the operative sign. This juxtaposition fulfils the function of an index, in indicating the connections of the tokens.

4th, The parentheses, subserving the same purpose.

5th, The letters α, β, etc. which are indices of no matter what tokens, used for expressing negation.

6th, The indices of tokens, x, y, z, etc. used in the general formulae.

7th, The general formulae themselves, which are *icons,* or exemplars of algebraic proceedings.

8th, The fourth *icon* which affords a second interpretation of the general formulae.

We might dispense with the fifth and eighth species of signs—the devices by which we express negation—by adopting a second operational sign $\overline{-\!\!<}$, such that

$$x \mathrel{\overline{-\!\!<}} y$$

should mean that $x = \mathrm{v}$, $y = \mathrm{f}$. With this, we should require new indices of connections, and new general formulae. Possibly this might be the preferable notation. We should thus have two operational signs but no sign of negation. The forms of Boolian algebra hitherto used, have either two operational signs and a special sign of negation, or three operational signs. One of the operational signs is in that case superfluous. Thus, in the usual notation we have

$$\overline{x + y} = \bar{x}\bar{y}$$
$$\bar{x} + \bar{y} = \overline{xy}$$

showing two modes of writing the same fact. The apparent balance between the two sets of theorems exhibited so strikingly by Schröder, arises entirely from this double way of writing everything. But while the ordinary system is not so analytically fitted to its purpose as that here set forth, the character of superfluity here, as in many other cases in algebra, brings with it great facility in working.

The general formulae given above are not convenient in practice. We may dispense with them altogether, as well as with one of the indices of tokens used in them, by the use of the following rules. A proposition of the form

$$x \prec y$$

is true if $x = \mathrm{f}$ or $y = \mathrm{v}$. It is only false if $y = \mathrm{f}$ and $x = \mathrm{v}$. A proposition written in the form

$$x \overline{\prec} y$$

is true if $x = \mathrm{v}$ and $y = \mathrm{f}$, and is false if either $x = \mathrm{f}$ or $y = \mathrm{v}$. Accordingly, to find whether a formula is necessarily true, substitute f and v for the letters and see whether it can be supposed false by any such assignment of values. Take, for example, the formula

$$(x \prec y) \prec \{(y \prec z) \prec (x \prec z)\}.$$

To make this false we must take

$$(x \prec y) = \mathrm{v}$$
$$\{(y \prec z) \prec (x \prec z)\} = \mathrm{f}.$$

The last gives

$$(y \prec z) = \mathrm{v}, \quad (x \prec z) = \mathrm{f}, \quad x = \mathrm{v}, \quad z = \mathrm{f}.$$

Substituting these values in

$$(x \prec y) = \mathrm{v} \quad (y \prec z) = \mathrm{v}$$

we have

$$(\mathrm{v} \prec y) = \mathrm{v} \quad (y \prec \mathrm{f}) = \mathrm{v},$$

which cannot be satisfied together.

As another example, required the conclusion from the following premises. Any one I might marry would be either beautiful or plain; any one whom I might marry would be a woman; any beautiful woman would be an ineligible wife; any plain woman would be an ineligible wife. Let

m be any one whom I might marry,
b, beautiful,
p, plain,
w, woman,
i, ineligible.

Then the premises are

$$m \prec (b \prec \mathrm{f}) \prec p,$$
$$m \prec w,$$
$$w \prec b \prec i,$$
$$w \prec p \prec i.$$

Let x be the conclusion. Then,

$$[m \prec (b \prec \mathrm{f}) \prec p] \prec (m \prec w) \prec (w \prec b \prec i) \prec (w \prec p \prec i) \prec x$$

is necessarily true. Now if we suppose $m = \mathrm{v}$, the proposition can only be made false by putting $w = \mathrm{v}$ and either b or $p = \mathrm{v}$. In this case the proposition can only be made false by putting $i = \mathrm{v}$. If, therefore, x can only be made f by putting $m = \mathrm{v}$, $i = \mathrm{f}$, that is if $x = (m \prec i)$, the proposition is necessarily true.

In this method, we introduce the two special tokens of second intention f and v, we retain two indices of tokens x and y, and we have a somewhat complex *icon*, with a special prescription for its use.

A better method may be found as follows. We have seen that

$$x \prec (y \prec z)$$

may be conveniently written

$$x \prec y \prec z;$$

while

$$(x \prec y) \prec z$$

ought to retain the parenthesis. Let us extend this rule, so as to be more general, and hold it necessary *always* to include the antecedent in parenthesis. Thus, let us write

$$(x) \prec y$$

instead of $x \prec y$. If now, we merely change the external appearance of two signs; namely, if we use the vinculum instead of the parenthesis, and the sign $+$ in place of $\prec$, we shall have

$$\begin{array}{lcl} x \prec y & \text{written} & \bar{x} + y \\ x \prec y \prec z & " & \bar{x} + \bar{y} + z \\ (x \prec y) \prec z & " & \overline{\bar{x} + y} + z, \text{ etc.} \end{array}$$

We may further write for $x \overline{\prec} y$, $\overline{\bar{x} + y}$ implying that $\bar{x} + y$ is an antecedent for whatever consequent may be taken, and the vinculum becomes identified with the sign of negation. We may also use the sign of multiplication as an abbreviation, putting

$$xy = \overline{\bar{x} + \bar{y}} = \overline{x \prec \bar{y}}.$$

This subjects addition and multiplication to all the rules of ordinary algebra, and also to the following:

$$\begin{array}{ll} y + x\bar{x} = y & y(x + \bar{x}) = y \\ x + \bar{x} = \mathrm{v} & \bar{x}x = \mathrm{f} \end{array}$$
$$xy + z = (x + z)(y + z).$$

To any proposition we have a right to add any expression at pleasure; also to strike out any factor of any term. The expressions for different propositions separately known may be multiplied together. These are substantially Mr. Mitchell's rules of procedure. Thus the premises of *Barbara* are

$$\bar{x} + y \text{ and } \bar{y} + z.$$

Multiplying these, we get

$$(\bar{x} + y)(\bar{y} + z) = \bar{x}\bar{y} + yz.$$

Dropping $\bar{y}$ and y we reach the conclusion $\bar{x} + z$.

III.—*First-intentional Logic of Relatives.*

The algebra of Boole affords a language by which anything may be expressed which can be said without speaking of more than one individual at a time. It is true that it can assert that certain characters belong to a whole class, but only such characters as belong to each individual separately. The logic of relatives considers statements involving two and more individuals at once. Indices are here required. Taking, first, a degenerate form of relation, we may write $x_i y_j$ to signify that x is true of the individual i while y is true of the individidual j. If z be a relative character, z_{ij} will signify that i is in that relation to j. In this way we can express relations of considerable complexity. Thus, if

$$\begin{matrix} 1, & 2, & 3, \\ 4, & 5, & 6, \\ 7, & 8, & 9, \end{matrix}$$

are points in a plane, and l_{123} signifies that 1, 2, and 3 lie on one line, a well-known proposition of geometry may be written

$$l_{159} \prec l_{267} \prec l_{348} \prec l_{147} \prec l_{258} \prec l_{369} \prec l_{123} \prec l_{456} \prec l_{789}.$$

In this notation is involved a *sixth icon*.

We now come to the distinction of *some* and *all*, a distinction which is precisely on a par with that between truth and falsehood; that is, it is descriptive, not metrical.

All attempts to introduce this distinction into the Boolian algebra were more or less complete failures until Mr. Mitchell showed how it was to be effected. His method really consists in making the whole expression of the proposition consist of two parts, a pure Boolian expression referring to an individual and a Quantifying part saying what individual this is. Thus, if k means "he is a king," and h, "he is happy," the Boolian

$$(\bar{k} + h)$$

means that the individual spoken of is either not a king or is happy. Now, applying the quantification, we may write

$$\text{Any } (\bar{k} + h)$$

to mean that this is true of any individual in the (limited) universe, or

$$\text{Some } (\bar{k} + h)$$

to mean that an individual exists who is either not a king or is happy. So

$$\text{Some } (kh)$$

means some king is happy, and

$$\text{Any } (kh)$$

means every individual is both a king and happy. The rules for the use of this notation are obvious. The two propositions

$$\text{Any } (x) \qquad \text{Any } (y)$$

are equivalent to

$$\text{Any } (xy).$$

From the two propositions

$$\text{Any } (x) \qquad \text{Some } (y)$$

we may infer

$$\text{Some } (xy).^4$$

Mr. Mitchell has also a very interesting and instructive extension of his notation for *some* and *all,* to a two-dimensional universe, that is,

4. I will just remark, quite out of order, that the quantification may be made numerical; thus producing the numerically definite inferences of De Morgan and Boole. Suppose at least $\frac{2}{3}$ of the company have white neckties and at least $\frac{3}{4}$ have dress coats. Let w mean "he has a white necktie," and d "he has a dress coat." Then, the two propositions are

$$\frac{2}{3}\,(w) \text{ and } \frac{3}{4}\,(d).$$

These are to be multiplied together. But we must remember that xy is a mere abbreviation for $\overline{\bar{x} + \bar{y}}$, and must therefore write

$$\overline{\overline{\tfrac{2}{3}w} + \overline{\tfrac{3}{4}d}}.$$

Now $\overline{\frac{2}{3}w}$ is the denial of $\frac{2}{3}w$, and this denial may be written $(>\frac{1}{3})\bar{w}$, or more than $\frac{1}{3}$ of the universe (the company) have not white neckties. So $\overline{\frac{3}{4}d} = (>\frac{1}{4})\bar{d}$. The combined premises thus become

$$\overline{(>\tfrac{1}{3})\bar{w} + (>\tfrac{1}{4})\bar{d}}.$$

Now $(>\frac{1}{3})\bar{w} + (>\frac{1}{4})\bar{d}$ gives

$$\text{May be } (\tfrac{1}{3} + \tfrac{1}{4})\,(\bar{w} + \bar{d}).$$

Thus we have

$$\overline{\text{May be } (\tfrac{7}{12})\,(\bar{w} + \bar{d})},$$

and this is

$$(\text{At least } \tfrac{5}{12})\,(\overline{\bar{w} + \bar{d}}),$$

which is the conclusion.

to the logic of relatives. Here, in order to render the notation as iconical as possible we may use Σ for *some,* suggesting a sum, and Π for *all,* suggesting a product. Thus $\Sigma_i x_i$ means that x is true of some one of the individuals denoted by i or

$$\Sigma_i x_i = x_i + x_j + x_k + \text{etc.}$$

In the same way, $\Pi_i x_i$ means that x is true of all these individuals, or

$$\Pi_i x_i = x_i x_j x_k, \text{ etc.}$$

If x is a simple relation, $\Pi_i \Pi_j x_{ij}$ means that every i is in this relation to every j, $\Sigma_i \Pi_j x_{ij}$ that some one i is in this relation to every j, $\Pi_j \Sigma_i x_{ij}$ that to every j some i or other is in this relation, $\Sigma_i \Sigma_j x_{ij}$ that some i is in this relation to some j. It is to be remarked that $\Sigma_i x_i$ and $\Pi_i x_i$ are only *similar* to a sum and a product; they are not strictly of that nature, because the individuals of the universe may be innumerable.

At this point, the reader would perhaps not otherwise easily get so good a conception of the notation as by a little practice in translating from ordinary language into this system and back again. Let l_{ij} mean that i is a lover of j, and b_{ij} that i is a benefactor of j. Then

$$\Pi_i \Sigma_j l_{ij} b_{ij}$$

means that everything is at once a lover and a benefactor of something; and

$$\Pi_i \Sigma_j l_{ij} b_{ji}$$

that everything is a lover of a benefactor of itself.

$$\Sigma_i \Sigma_k \Pi_j (l_{ij} + b_{jk})$$

means that there are two persons, one of whom loves everything except benefactors of the other (whether he loves any of these or not is not stated). Let g_i mean that i is a griffin, and c_i that i is a chimera, then

$$\Sigma_i \Pi_j (g_i l_{ij} + \bar{c}_j)$$

means that if there be any chimeras there is some griffin that loves them all; while

$$\Sigma_i \Pi_j g_i (l_{ij} + \bar{c}_j)$$

means that there is a griffin and he loves every chimera that exists (if any exist). On the other hand,

$$\Pi_j \Sigma_i g_i (l_{ij} + \bar{c}_j)$$

means that griffins exist (one, at least), and that one or other of them loves each chimera that may exist; and

$$\Pi_j \Sigma_i (g_i l_{ij} + \bar{c}_j)$$

means that each chimera (if there is any) is loved by some griffin or other.

Let us express: every part of the world is either sometimes visited with cholera, and at others with small-pox (without cholera), or never with yellow fever and the plague together. Let

c_{ij}	mean	the place	i has	cholera	at the	time j.
s_{ij}	"	"	"	small-pox	"	"
y_{ij}	"	"	"	yellow fever	"	"
p_{ij}	"	"	"	plague	"	"

Then we write

$$\Pi_i \Sigma_j \Sigma_k \Pi_l (c_{ij} \bar{c}_{ik} s_{ik} + \bar{y}_{il} + \bar{p}_{il}).$$

Let us express this: one or other of two theories must be admitted, 1st, that no man is at any time unselfish or free, and some men are always hypocritical, and at every time some men are friendly to men to whom they are at other times inimical, or 2nd, at each moment all men are alike either angels or fiends. Let

u_{ij}	mean	the man	i is	unselfish	at the	time j,
f_{ij}	"	"	"	free	"	"
h_{ij}	"	"	"	hypocritical	"	"
a_{ij}	"	"	"	an angel	"	"
d_{ij}	"	"	"	a fiend	"	"
p_{ijk}	"	"	"	friendly	"	"
		to the man k,				

e_{ijk} the man i is an enemy at the time j to the man k;
1_{jm} the two objects j and m are identical.

Then the proposition is

$$\Pi_i \Sigma_h \Pi_j \Sigma_k \Sigma_l \Sigma_m \Pi_n \Pi_p \Pi_q (\bar{u}_{ij} \bar{f}_{ij} h_{hj} p_{kjl} e_{kml} \bar{1}_{jm} + a_{pn} + d_{qn}).$$

We have now to consider the procedure in working with this calculus. It is far from being true that the only problem of deduction is to draw a conclusion from given premises. On the contrary, it is fully as important to have a method for ascertaining what premises will yield a given conclusion. There are besides other problems of transformation, where a certain system of facts is given, and it is required to describe this in other terms of a definite kind. Such, for example, is the problem of the 15 young ladies, and others relating to synthemes. I shall, however, content myself here with showing how, when a set of premises are given, they can be united and certain letters eliminated. Of the various methods which might be pursued, I shall here give the one which seems to me the most useful on the whole.

1st. The different premises having been written with distinct indices (the same index not used in two propositions) are written together, and all the Πs and Σs are to be brought to the left. This can evidently be done, for

$$\Pi_i x_i \cdot \Pi_j x_j = \Pi_i \Pi_j x_i x_j$$
$$\Sigma_i x_i \cdot \Pi_j x_j = \Sigma_i \Pi_j x_i x_j$$
$$\Sigma_i x_i \cdot \Sigma_j x_j = \Sigma_i \Sigma_j x_i x_j.$$

2nd. Without deranging the order of the indices of any one premise, the Πs and Σs belonging to different premises may be moved relatively to one another, and as far as possible the Σs should be carried to the left of the Πs. We have

$$\Pi_i \Pi_j x_{ij} = \Pi_j \Pi_i x_{ij}$$
$$\Sigma_i \Sigma_j x_{ij} = \Sigma_j \Sigma_i x_{ij}$$

and also

$$\Sigma_i \Pi_j x_i y_j = \Pi_j \Sigma_i x_i y_j.$$

But this formula does not hold when the i and j are not separated. We do have, however,

$$\Sigma_i \Pi_j x_{ij} \prec \Pi_j \Sigma_i x_{ij}.$$

It will, therefore, be well to begin by putting the Σs to the left, as far as possible, because at a later stage of the work they can be carried to the right but not to the left. For example, if the operators of the two premises are $\Pi_i \Sigma_j \Pi_k$ and $\Sigma_x \Pi_y \Sigma_z$, we can unite them in either of the two orders

$$\Sigma_x \Pi_y \Sigma_z \Pi_i \Sigma_j \Pi_k$$
$$\Sigma_x \Pi_i \Sigma_j \Pi_y \Sigma_z \Pi_k,$$

and shall usually obtain different conclusions accordingly. There will often be room for skill in choosing the most suitable arrangement.

3rd. It is next sometimes desirable to manipulate the Boolian part of the expression, and the letters to be eliminated can, if desired, be eliminated now. For this purpose they are replaced by relations of second intention, such as "other than," etc. If, for example, we find anywhere in the expression

$$a_{ijk}\bar{a}_{xyz},$$

this may evidently be replaceable by

$$(n_{ix} + n_{jy} + n_{kz})$$

where, as usual, n means not or other than. This third step of the process is frequently quite indispensible, and embraces a variety of processes; but in ordinary cases it may be altogether dispensed with.

4th. The next step, which will also not commonly be needed, consists in making the indices refer to the same collections of objects, so far as this is useful. If the quantifying part, or Quantifier, contains Σ_x, and we wish to replace the x by a new index i, not already in the Quantifier, and such that every x is an i, we can do so at once by simply multiplying every letter of the Boolian having x as an index by x_i. Thus, if we have "some woman is an angel" written in the form $\Sigma_w a_w$ we may replace this by $\Sigma_i (a_i w_i)$. It will be more often useful to replace the index of a Π by a wider one; and this will be done by adding $\bar{x}_i$ to every letter having x as an index. Thus, if we have "all dogs are animals, and all animals are vertebrates" written thus

$$\Pi_d \alpha_d \qquad \Pi_a v_a,$$

where a and α alike mean animal, it will be found convenient to replace the last index by i, standing for any object, and to write the proposition

$$\Pi_i (\bar{\alpha}_i + v_i).$$

5th. The next step consists in multiplying the whole Boolian part, by the modification of itself produced by substituting for the index of any Π any other index standing to the left of it in the Quantifier. Thus, for

$$\Sigma_i \Pi_j l_{ij},$$

we can write

$$\Sigma_i \Pi_j l_{ij} l_{ii}.$$

6th. The next step consists in the re-manipulation of the Boolian part, consisting, 1st, in adding to any part any term we like; 2nd, in dropping from any part any factor we like, and 3rd, in observing that

$$x\bar{x} = \mathrm{f}, \qquad x + \bar{x} = \mathrm{v},$$

so that

$$x\bar{x}y + z = z \qquad (x + \bar{x} + y)z = z.$$

7th. Πs and Σs in the Quantifier whose indices no longer appear in the Boolian are dropped.

The fifth step will, in practice, be combined with part of the sixth and seventh. Thus, from $\Sigma_i \Pi_j l_{ij}$ we shall at once proceed to $\Sigma_i l_{ii}$ if we like.

The following examples will be sufficient.

From the premises $\Sigma_i a_i b_i$ and $\Pi_j (\bar{b}_j + c_j)$, eliminate b. We first write

$$\Sigma_i \Pi_j a_i b_i (\bar{b}_j + c_j).$$

The distributive process gives

$$\Sigma_i \Pi_j a_i (b_i \bar{b}_j + b_i c_j).$$

But *we always have a right to drop a factor or insert an additive term.* We thus get

$$\Sigma_i \Pi_j a_i (b_i \bar{b}_j + c_j).$$

By the third process, we can, if we like, insert n_{ij} for $b_i \bar{b}_j$. In either case, we identify j with i and get the conclusion

$$\Sigma_i a_i c_i.$$

Given the premises

$$\Sigma_h \Pi_i \Sigma_j \Pi_k (\alpha_{hik} + s_{jk} l_{ji})$$

$$\Sigma_u \Sigma_v \Pi_x \Pi_y (\epsilon_{uyx} + \bar{s}_{yv} b_{vx}).$$

Required to eliminate s. The combined premise is

$$\Sigma_u \Sigma_v \Sigma_h \Pi_i \Sigma_j \Pi_x \Pi_k \Pi_y (\alpha_{hik} + s_{jk} l_{ji})(\epsilon_{uyx} + \bar{s}_{yv} b_{vx}).$$

Identify k with v and y with j, and we get

$$\Sigma_u \Sigma_v \Sigma_h \Pi_i \Sigma_j \Pi_x (\alpha_{hiv} + s_{jv} l_{ji})(\epsilon_{ujx} + \bar{s}_{jv} b_{vx}).$$

The Boolian part then reduces, so that the conclusion is

$$\Sigma_u \Sigma_v \Sigma_h \Pi_i \Sigma_j \Pi_x (\alpha_{hiv}\epsilon_{ujx} + \alpha_{hiv} b_{vx} + \epsilon_{ujx} l_{ji}).$$

IV.—*Second-intentional Logic.*

Let us now consider the logic of terms taken in collective senses. Our notation, so far as we have developed it, does not show us even how to express that two indices, i and j, denote one and the same thing. We may adopt a special token of second intention, say 1, to express identity, and may write 1_{ij}. But this relation of identity has peculiar properties. The first is that if i and j are identical, whatever is true of i is true of j. This may be written

$$\Pi_i \Pi_j \{\bar{1}_{ij} + \bar{x}_i + x_j\}.$$

The use of the general index of a token, x, here, shows that the formula is iconical. The other property is that if everything which is true of i is true of j, then i and j are identical. This is most naturally written as follows: Let the token, q, signify the relation of a quality, character, fact, or predicate to its subject. Then the property we desire to express is

$$\Pi_i \Pi_j \Sigma_k (1_{ij} + \bar{q}_{ki} q_{kj}).$$

And identity is defined thus

$$1_{ij} = \Pi_k (q_{ki} q_{kj} + \bar{q}_{ki} \bar{q}_{kj}).$$

That is, to say that things are identical is to say that every predicate is true of both or false of both. It may seem circuitous to introduce the idea of a quality to express identity; but that impression will be modified by reflecting that $q_{ki} q_{kj}$ merely means that i and j are both within the class or collection k. If we please, we can dispense with the token q, by using the index of a token and by referring to this in the Quantifier just as subjacent indices are referred to. That is to say, we may write

$$1_{ij} = \Pi_x (x_i x_j + \bar{x}_i \bar{x}_j).$$

The properties of the token q must now be examined. These may all be summed up in this, that taking any individuals i_1, i_2, i_3, etc., and any individuals, j_1, j_2, j_3, etc., there is a collection, class, or predicate embracing all the i's and excluding all the j's except such as are identical with some one of the i's. This might be written

$$(\Pi_\alpha \Pi_{i_\alpha})(\Pi_\beta \Pi_{j_\beta})\Sigma_k(\Pi_\alpha \Sigma_{i'_\alpha})\Pi_l \quad q_{ki_\alpha}(\bar{q}_{kj_\beta} + q_{li'_\alpha} q_{lj_\beta} + \bar{q}_{li'_\alpha}\bar{q}_{lj_\beta}),$$

where the i's and the i''s are the same lot of objects. This notation presents indices of indices. The $\Pi_\alpha \Pi_{i_\alpha}$ shows that we are to take any collection whatever of i's, and then any i of that collection. We are then to do the same with the j's. We can then find a quality k such that the i taken has it, and also such that the j taken wants it unless we can find an i that is identical with the j taken. The necessity of some kind of notation of this description in treating of classes collectively appears from this consideration: that in such discourse we are neither speaking of a single individual (as in the non-relative logic) nor of a small number of individuals considered each for itself, but of a whole class, perhaps an infinity of individuals. This suggests a relative term with an indefinite series of indices as $x_{ijkl\ldots}$. Such a relative will, however, in most, if not in all cases, be of a degenerate kind and is consequently expressible as above. But it seems preferable to attempt a partial decomposition of this definition. In the first place, any individual may be considered as a class. This is written

$$\Pi_i \Sigma_k \Pi_j \; q_{ki}(\bar{q}_{kj} + 1_{ij}).$$

This is the *ninth icon.* Next, given any class, there is another which includes all the former excludes, and excludes all the former includes. That is,

$$\Pi_l \Sigma_k \Pi_i (q_{li}\bar{q}_{ki} + \bar{q}_{li} q_{ki}).$$

This is the *tenth icon.* Next, given any two classes, there is a third which includes all that either includes, and excludes all that both exclude. That is

$$\Pi_l \Pi_m \Sigma_k \Pi_i (q_{li} q_{ki} + q_{mi} q_{ki} + \bar{q}_{li}\bar{q}_{mi}\bar{q}_{ki}).$$

This is the *eleventh icon.* Next, given any two classes, there is a class which includes the whole of the first and any one individual of the second which there may be not included in the first and nothing else. That is,

$$\Pi_l \Pi_m \Pi_i \Sigma_k \Pi_j \{q_{li} + \bar{q}_{mi} + q_{ki}(q_{kj} + \bar{q}_{lj})\}.$$

This is the *twelfth icon.*

To show the manner in which these formulae are applied let us suppose we have given that everything is either true of i or false of j. We write

$$\Pi_k (q_{ki} + \bar{q}_{kj}).$$

The tenth icon gives

$$\Pi_l \Sigma_k (q_{li}\bar{q}_{ki} + \bar{q}_{li}q_{ki})(q_{lj}\bar{q}_{kj} + \bar{q}_{lj}q_{kj}).$$

Multiplication of these two formulae gives

$$\Pi_l \Sigma_k (q_{ki}\bar{q}_{li} + q_{lj}\bar{q}_{kj}),$$

or, dropping the terms in k

$$\Pi_l (\bar{q}_{li} + q_{lj}).$$

Multiplying this with the original datum and identifying l with k, we have

$$\Pi_k (q_{ki}q_{kj} + \bar{q}_{ki}\bar{q}_{kj}).$$

No doubt, a much more direct method of procedure could be found.

Just as q signifies the relation of predicate to subject, so we need another token, which may be written r, to signify the conjoint relation of a simple relation, its relate and its correlate. That is, $r_{j\alpha i}$ is to mean that i is in the relation α to j. Of course, there will be a series of properties of r similar to those of q. But it is singular that the uses of the two tokens are quite different. Namely, the chief use of r is to enable us to express that the number of one class is at least as great as that of another. This may be done in a variety of different ways. Thus, we may write that for every a there is a b, in the first place, thus:

$$\Sigma_\alpha \Pi_i \Sigma_j \Pi_h \{\bar{a}_i + b_j r_{j\alpha i}(\bar{r}_{j\alpha h} + \bar{a}_h + 1_{ih})\}.$$

But, by an icon analogous to the eleventh, we have

$$\Pi_\alpha \Pi_\beta \Sigma_\gamma \Pi_u \Pi_v (r_{u\alpha v} r_{u\gamma v} + r_{u\beta v} r_{u\gamma v} + \bar{r}_{u\alpha v}\bar{r}_{u\beta v}\bar{r}_{u\gamma v}).$$

From this, by means of an icon analogous to the *tenth,* we get the general formula

$$\Pi_\alpha \Pi_\beta \Sigma_\gamma \Pi_u \Pi_v \{r_{u\alpha v} r_{u\beta v} r_{u\gamma v} + \bar{r}_{u\gamma v} \ (\bar{r}_{u\alpha v} + \bar{r}_{u\beta v})\}.$$

For $r_{u\beta v}$ substitute a_u and multiply by the formula the last but two. Then, identifying u with h and v with j, we have

$$\Sigma_\alpha \Pi_i \Sigma_h \Pi_h \{\bar{a}_i + b_j r_{j\alpha i}(\bar{r}_{j\alpha h} + 1_{ih})\}$$

a somewhat simpler expression. However, the best way to express such a proposition is to make use of the letter c as a token of a one-to-one correspondence. That is to say, c will be defined by the three formulae,

$$\Pi_\alpha \Pi_u \Pi_v \Pi_w (\bar{c}_\alpha + \bar{r}_{u\alpha v} + \bar{r}_{u\alpha w} + 1_{vw})$$
$$\Pi_\alpha \Pi_u \Pi_v \Pi_w (\bar{c}_\alpha + \bar{r}_{u\alpha w} + \bar{r}_{v\alpha w} + 1_{uv})$$
$$\Pi_\alpha \Sigma_u \Sigma_v \Sigma_w (c_\alpha + r_{u\alpha v} r_{u\alpha w} \bar{1}_{vw} + r_{u\alpha w} r_{v\alpha w} \bar{1}_{uv}).$$

Making use of this token, we may write the proposition we have been considering in the form

$$\Sigma_\alpha \Pi_i \Sigma_j \; c_\alpha (\bar{a}_i + b_j r_{j\alpha i}).$$

In an appendix to his memoir on the logic of relatives, De Morgan enriched the science of logic with a new kind of inference, the syllogism of transposed quantity. De Morgan was one of the best logicians that ever lived and unquestionably the father of the logic of relatives. Owing, however, to the imperfection of his theory of relatives, the new form, as he enunciated it, was a down-right paralogism, one of the premises being omitted. But this being supplied, the form furnishes a good test of the efficacy of a logical notation. The following is one of De Morgan's examples:

> Some X is Y,
> For every X there is something neither Y nor Z;
> Hence, something is neither X nor Z.

The first premise is simply

$$\Sigma_a x_a y_a.$$

The second may be written

$$\Sigma_\alpha \Pi_i \Sigma_j \; c_\alpha (\bar{x}_i + r_{j\alpha i} \bar{y}_j \bar{z}_j).$$

From these two premises, little can be inferred. To get the above conclusion it is necessary to add that the class of X's is a finite collection; were this not necessary the following reasoning would hold good (the limited universe consisting of numbers); for it precisely conforms to De Morgan's scheme.

Some odd number is prime;
Every odd number has its square, which is neither prime nor even;
Hence, some number is neither odd nor even.[5]

Now, to say that a lot of objects is finite, is the same as to say that if we pass through the class from one to another we shall necessarily come round to one of those individuals already passed; that is, if every one of the lot is in any one-to-one relation to one of the lot, then to every one of the lot some one is in this same relation. This is written thus:

$$\Pi_\beta \Pi_u \Sigma_v \Sigma_s \Pi_t \{\bar{c}_\beta + \bar{x}_u + x_v r_{u\beta v} + x_s(\bar{x}_t + \bar{r}_{t\beta s})\}.$$

Uniting this with the two premises and the second clause of the definition of c, we have

$$\Sigma_a \Sigma_\alpha \Pi_\beta \Pi_u \Sigma_v \Sigma_s \Pi_i \Sigma_j \Pi_t \Pi_\gamma \Pi_e \Pi_f \Pi_g x_a y_a c_\alpha (\bar{x}_i + r_{j\alpha i}\bar{y}_j\bar{z}_j)$$
$$\{\bar{c}_\beta + \bar{x}_u + x_v r_{u\beta v} + x_s(\bar{x}_t + \bar{r}_{t\beta s})\}(\bar{c}_\gamma + \bar{r}_{e\gamma g} + \bar{r}_{f\gamma w} + 1_{ef}).$$

We now substitute α for β and for γ, a for u and for e, j for t and for f, v for g. The factor in i is to be repeated, putting first s and then v for i. The Boolian part thus reduces to

$$(\bar{x}_s + r_{j\alpha s}\bar{y}_j\bar{z}_j)c_\alpha x_a y_a r_{a\alpha v} x_v r_{j\alpha v}\bar{y}_j\bar{z}_j\, 1_{aj}$$
$$+ r_{j\alpha s}\bar{y}_j\bar{z}_j x_s \bar{x}_j(\bar{x}_v + r_{j\alpha v}\bar{y}_j\bar{z}_j)(\bar{r}_{a\alpha v} + \bar{r}_{j\alpha v} + 1_{aj}),$$

which, by the omission of factors, becomes

$$y_a\bar{y}_j 1_{aj} + \bar{x}_j\bar{z}_j.$$

Thus we have the conclusion

$$\Sigma_j \bar{x}_j \bar{z}_j.$$

5. Another of De Morgan's examples is this: "Suppose a person, on reviewing his purchases for the day, finds, by his counterchecks, that he has certainly drawn as many checks on his banker (and maybe more) as he has made purchases. But he knows that he paid some of his purchases in money, or otherwise than by checks. He infers then that he has drawn checks for something else except that day's purchases. He infers rightly enough." Suppose, however, that what happened was this: He bought something and drew a check for it; but instead of paying with the check, he paid cash. He then made another purchase for the same amount, and drew another check. Instead, however, of paying with that check, he paid with the one previously drawn. And thus he continued without cessation, or *ad infinitum.* Plainly the premises remain true, yet the conclusion is false.

It is plain that by a more iconical and less logically analytical notation this procedure might be much abridged. How minutely analytical the present system is, appears when we reflect that every substitution of indices of which nine were used in obtaining the last conclusion is a distinct act of inference. The annulling of $(y_a \bar{y}_j 1_{aj})$ makes ten inferential steps between the premises and conclusion of the syllogism of transposed quantity.

Notes on the Algebra of Logic

Item 31 ***MS 538: Spring 1885***

On the General Rule of Inference in Relative Logic.

Under the 3rd step (p. 183), an example was given which is really a general formula of elimination. Namely, we have

$$a_{ijk\text{ etc.}}\,\bar{a}_{xyz\text{ etc.}} \prec \bar{1}_{ix} + \bar{1}_{jy} + \bar{1}_{kz} + \text{etc.},$$

and conversely,

$$1_{ix}1_{jy}1_{zk}\text{ etc.} \prec a_{ijk\text{ etc.}} + \bar{a}_{xyz\text{ etc.}}$$

The principles of contradiction and of excluded middle might be considered as mere special cases of these formulae. Namely, the latter give

$$a_i\bar{a}_i \prec \bar{1}_{ii}$$

$$1_{ii} \prec a_i + \bar{a}_i$$

But the definition of identity is

$$\mathrm{v} \prec 1_{ii} \qquad \bar{1}_{ii} \prec \mathrm{f};$$

whence

$$a_i\bar{a}_i \prec \mathrm{f} \qquad \mathrm{v} \prec a_i + \bar{a}_i.$$

Under the head of the third step belongs the frequently necessary development of the Boolian by means of distribution-formulae. [This proceeding, and indeed the whole of the third step might more properly have been made to follow the fourth.] The fundamental formulae of distribution are

$$x(y + z) = xy + xz$$

$$x + yz = (x + y)(x + z).$$

The general development formulae thence resulting are

$$\begin{aligned} Fx &= xF1 + \bar{x}F0 \\ &= (x + F0)(\bar{x} + F1). \end{aligned}$$

The following formulae, which find continual application, are deducible from the above:

$$\begin{aligned} &a + b = a + \bar{a}b \\ &xy + \bar{x}\,\bar{y} = (x + \bar{y})(\bar{x} + y) \\ &(a + b)c \prec a + bc \\ &(a + x)(b + y) \prec a + b + xy. \end{aligned}$$

The *fifth step* is composed of two kinds of operations, the involution of the whole expression, and the identification and discrimination of indices.

It is plain that *any algebraical expression of a proposition may be multiplied into itself any number of times without ceasing to be true;* and it will be found that such involution is essentially necessary in all difficult modes of inference. Consider, for example, the last formula but one of those last given,

$$(a + b)c \prec a + bc.$$

This is not itself a distribution-formula, but only an association-formula, and therefore the deduction of a distribution-formula from it is not a matter of the very utmost simplicity. If, however, we square the antecedent we have

$$(a + b)c = (a + b)c(a + b)c = c(a + b)c,$$

and then by the application of the association-formula twice over we get

$$c(a + b)c \prec (ca + b)c \prec ca + bc;$$

so that we have proved

$$(a + b)c \prec ac + bc,$$

which is the main distribution-formula, with the aid of which all the others are readily obtained.

Other much more striking examples of the utility of involution will present themselves in the course of this paper.

In multiplying a proposition by itself, we have the right to choose any point from the beginning to the end of the quantifier and to identify in two factors all the indices to the left of this point while diversifying all to the right of it. When there is but one index, this is plainly true (see the formulae in the middle of p. 182); for

$$\Pi_i a_i = (\Pi_i a_i)\,(\Pi_i a_i) = (\Pi_i a_i)\,(\Pi_j a_j)$$
$$\Sigma_i a_i = (\Sigma_i a_i)\,(\Sigma_i a_i) = (\Sigma_i a_i)\,(\Sigma_j a_j).$$

Now

$$(\Pi_i a_i)\,(\Pi_j a_j) = (a_1 a_2 a_3 \text{ etc.})\,(a_1 a_2 a_3 \text{ etc.})$$

while

$$\begin{aligned}\Pi_i \Pi_j a_i a_j = {} & a_1 a_1 \cdot a_1 a_2 \cdot a_1 a_3 \cdot \text{etc.}\\ & \times a_2 a_1 \cdot a_2 a_2 \cdot a_2 a_3 \cdot \text{etc.}\\ & \times a_3 a_1 \cdot a_3 a_2 \cdot a_3 a_3 \cdot \text{etc.}\\ & \times \text{etc.}\end{aligned}$$

and

$$\Pi_i a_i a_i = a_1 a_1 \cdot a_2 a_2 \cdot a_3 a_3 \cdot \text{etc.}$$

But since $aa = a$ all these are equivalent.
Also

$$(\Sigma_i a_i)\,(\Sigma_j a_j) = (a_1 + a_2 + a_3 + \text{etc.})\,(a_1 + a_2 + a_3 + \text{etc.})$$

while

$$\begin{aligned}\Sigma_i \Sigma_j a_i a_j = {} & a_1 a_1 + a_1 a_2 + a_1 a_3 + \text{etc.}\\ & + a_2 a_1 + a_2 a_2 + a_2 a_3 + \text{etc.}\\ & + a_3 a_1 + a_3 a_2 + a_3 a_3 + \text{etc.}\\ & + \text{etc.}\end{aligned}$$

and

$$\Sigma_i a_i a_i = a_1 a_1 + a_2 a_2 + a_3 a_3 + \text{etc.}$$

The first two expressions are equivalent by the distribution-principle. The third is equivalent to $\Sigma_i a_i$ because $aa = a$, and thus all three are equivalent to one another.

The theorem enunciated having thus been proved true of every proposition having a single index, it only remains to show that if it be true of every proposition having n indices, it is true for every proposition having $n + 1$ indices.

Now let Φ and * stand either for Σ and $+$ or for Π and $\times$ respectively, so that

$$\Phi_i u_i = u_1 * u_2 * u_3 * \text{etc.}$$

This may represent any proposition in $n + 1$ indices of which i is the first. When i is fixed in a certain individual as in u_1, u_2, etc. u becomes a proposition in n indices. Let u^2 represent any legitimate square of u. Then the square of the whole expression with identification of i in the two factors is

$$\Phi_i u_i^2 = u_1^2 * u_2^2 * u_3^2 * \text{etc.}$$

and since by hypothesis

$$u_1 \prec u_1^2 \qquad u_2 \prec u_2^2 \qquad u_3 \prec u_3^2 \qquad \text{etc.}$$

it follows that

$$\Phi_i u_i \prec \Phi_i u_i^2.$$

Thus it is shown that the rule holds for every proposition in $n + 1$ indices, if the first of them is identified in the two factors. But according to the rule, the first cannot be diversified unless all are diversified; and all may be diversified, by an obvious extension of the formulae relating to propositions in a single index. The theorem is therefore proved.

If the proposition is raised to a higher power, the diversifications toward the right may be represented as branchings from a stem, thus.

After the involution has once been performed, further identification and diversification of indices may be effected by applying the following formulae.

For identifications.	*For diversifications.*
$\Pi_i \Pi_j l_{ij} \prec \Pi_i l_{ii}$	$\Pi_i l_{ii} \prec \Pi_i \Sigma_j l_{ij}$
$\Sigma_i \Pi_j l_{ij} \prec \Sigma_i l_{ii}$	$\Sigma_i l_{ii} \prec \Sigma_i \Sigma_j l_{ij}$

Besides these formulae, the following are occasionally useful

$$\Pi_i m_i = \Pi_i \Pi_j m_i m_j \qquad \Sigma_i m_i = \Sigma_i \Sigma_j (m_i + m_j).$$

The following example shows the utility of changing the indices, even without involution. From the premise

$$\Pi_i \Pi_j (m_i l_{ij} + \bar{m}_i \bar{l}_{ij})$$

let it be required to eliminate *m*. The immediate dropping of this token would only yield an identical proposition. But by separating the Boolian into factors and then changing the indices, we have

$$\begin{aligned} &\Pi_i \Pi_j (m_i l_{ij} + \bar{m}_i \bar{l}_{ij}) \\ &= \Pi_i \Pi_j (m_i + \bar{l}_{ij})\,(l_{ij} + \bar{m}_i) \\ &= \Pi_i \Pi_j (m_i + \bar{l}_{ij}) \Pi_j (l_{ij} + \bar{m}_i) \\ &= \Pi_i \Pi_j (m_i + \bar{l}_{ij}) \Pi_k (l_{ik} + \bar{m}_i) \\ &= \Pi_i \Pi_j \Pi_k (m_i + \bar{l}_{ij})\,(l_{ik} + \bar{m}_i) \\ &= \Pi_i \Pi_j \Pi_k (m_i l_{ik} + \bar{m}_i \bar{l}_{ij}) \end{aligned}$$

Now dropping *m* we have

$$\Pi_i \Pi_j \Pi_k (l_{ik} + \bar{l}_{ij})$$

i.e. whatever is in the relation l to anything is in that relation to everything.

An artifice which I have not included among the regular steps of the inferential proceedure but which is occasionally useful, consists in taking the latter part of the quantifier away from its position at the head of the proposition and putting it before the part which alone it concerns. The formulae governing this operation are

$$\Pi_j a_i b_{ij} = a_i \Pi_j b_{ij} \qquad \Sigma_j a_i b_{ij} = a_i \Sigma_j b_{ij}$$
$$\Pi_j (a_i + b_{ij}) = a_i + \Pi_j b_{ij} \qquad \Sigma_j (a_i + b_{ij}) = a_i + \Sigma_j b_{ij}.$$

This transformation will generally be used in connection with the formulae

$$a \prec a + b$$

and

$$a + xb = a\bar{x} + xb + a\,x\bar{b}.$$

6th Step. The step numbered 5th on page 183 may more conveniently be separated into two. The first of these somewhat resembles the one just considered, being a sort of development or setting forth in detail of the proposition. It is not, however, founded upon a distribution-formula, but consists in raising the whole premise to a power or multiplying it into itself any number of times. It will be seen in the sequel that most difficult inferences involve this proceeding. We have to consider what disposition is to be made of the indices of the different factors of the power. Confining our attention for a moment to the first index, the proposition is, plainly, either of the form $\Pi_i a_i$ or of the form $\Sigma_i a_i$. We have

$$\begin{aligned}
\Pi_i a_i &= (\Pi_i a_i)^2 = \Pi_i a_i a_i = (a_1 a_2 a_3 \text{ etc.})\,(a_1 a_2 a_3 \text{ etc.})\\
&= \Pi_i a_i \cdot \Pi_j a_j = \Pi_i \Pi_j a_i a_j\\
\Sigma_i a_i &= (\Sigma_i a_i)^2 = \Sigma_i a_i a_i = a_1 a_1 + a_2 a_2 + a_3 a_3 + \text{etc.}\\
&= (a_1 + a_2 + a_3 + \text{etc.})\,(a_1 + a_2 + a_3 + \text{etc.})\\
&= \Sigma_i a_i \cdot \Sigma_j a_j = \Sigma_i \Sigma_j a_i a_j.
\end{aligned}$$

The first index may thus remain the same or be made different in the different factors. The same is obviously true of any index, provided all that go before it remain unchanged. For instance,

$$\Pi_i \Sigma_j \Pi_k a_{ijk} = \Pi_i \Sigma_j (\Pi_k a_{ijk})$$

and since, as we have just seen

$$\Pi_k a_{ijk} = \Pi_k a_{ijk} a_{ijk} = \Pi_k \Pi_l a_{ijk} a_{ijl}$$

we have

$$\Pi_i \Sigma_j \Pi_k a_{ijk} = \Pi_i \Sigma_j \Pi_k a_{ijk} a_{ijk} = \Pi_i \Sigma_j \Pi_k \Pi_l a_{ijk} a_{ijl}$$

The same is also true of any index preceded only by others quantified as it is, whether these others remain unchanged in the different factors or not. Thus,

$$\begin{aligned}
\Sigma_i \Sigma_j l_{ij} &= \Sigma_j \Sigma_i l_{ij} = \Sigma_i \Sigma_k \Sigma_j l_{ij} l_{kj}\\
&= \Sigma_j \Sigma_i \Sigma_k l_{ij} l_{kj} = \Sigma_i \Sigma_j \Sigma_k \Sigma_l l_{ij} l_{kl}.
\end{aligned}$$

Any index may be made different in the different factors by the formulae of the 5th step; and by a formula to be given under the 7th step an index universally quantified (quantified by a Π) may be made identical in the different factors. It, therefore, only remains to consider under what circumstances indices particularly quantified (quantified by Σs) can be identified in the different factors.

The fact that an index is preceded by a particularly quantified index not identified in the different factor will not prevent it from being identified. Thus, consider

$$\Sigma_i \Pi_j \Sigma_k b_{ijk}.$$

This is equivalent to

$$\Sigma_i \Sigma_l \Pi_j \Sigma_k b_{ijk} b_{ljk}.$$

And it is plain that it will always be so because

$$\Sigma_i a_{ii} \prec \Sigma_i \Sigma_j a_{ij}.$$

But an index particularly quantified, which is preceded by one universally quantified, cannot be identified in the different factors unless the latter is so. Thus, from

$$\Pi_i \Sigma_j l_{ij}$$

we cannot infer

$$\Pi_i \Pi_k \Sigma_j l_{ij} l_{kj}\,;$$

for this would imply that there were two objects i and k in the relation l to the same correlate j.

Second-intentional logic, resumed.

The special peculiarity of ordinary algebra has given us the false notion that inverse operations are the general means of solving algebraical problems. But the study of general algebra shows that inverse operations lead to determinate results only in special cases,—that what is called "the general case" is in truth a mere form of speciality,—and that the truly general method of elimination is by performing a direct operation which will give a constant result whatever the value of the variable. Thus, in ordinary algebra, it happens to be the case that every quantity has a reciprocal so that

$$\frac{1}{x}\cdot x = 1.$$

So in logical algebra, the only way of eliminating any token is by means of the properties of special terms of second intention such as

$$a_{ij}\bar{a}_{xy} \prec \bar{1}_{ix} + \bar{1}_{jy} \qquad a_i\bar{a}_i \prec 0.$$

In order to bring these formulae to bear it may be necessary to multiply the premise into itself, to manipulate the indices, and use processes of association and of development by distribution; but whatever cannot be eliminated by these means cannot be eliminated at all.

The universes of marks to which the tokens q, r, and others like these refer are, in reference to the combinations of objects to which they are attached, unlimited universes. [Compare the Thomist doctrine of angelic natures.] That is to say, every lot of objects has some quality common and peculiar to the objects composing it; every lot of pairs has some relation subsisting between the first and second members of each one of those pairs and between no others. In other regards, the universes are not unlimited; of characters familiar to us there is quite a limited number; of colours definable by Newton's diagram not all can exist. In like manner, the universes of quantity, position, etc. of mathematics are unlimited universes. Of the objects of such a universe everything is true, which can be true; every proposition is true from which the unlimited universe cannot be eliminated without yielding a true proposition. [As a general rule, every proposition in $\Sigma_\alpha q_\alpha$ and $\Sigma_\alpha r_\alpha$ is true; but there are many exceptions.]

To illustrate the application of this principle, consider the ninth icon, page 186. This is written

$$\Pi_a \Sigma_k \Pi_b q_{ka}(\bar{q}_{kb} + 1_{ab})$$

Now k can only be eliminated from this without involution in two ways; 1st, eliminating q_{ka} and q_{kb} independently, we have

$$\Pi_a \Pi_b (\bar{1}_{ab} + 1_{ab}),$$

an identical proposition; 2nd, identifying b with a, we get

$$\Pi_a 1_{aa},$$

another identical proposition. We next proceed to square the proposition; and we have

$$\Pi_a \Sigma_k \Pi_b \Pi_c q_{ka}(\bar{q}_{kb} + 1_{ab})\,(\bar{q}_{kc} + 1_{ac}).$$

We must identify k in the factors or we should reach no result; and this forces us to identify a. But it is plain that we cannot from this square eliminate in any new way, and therefore we could not from any higher power, and consequently the original proposition is proved true.

Suppose, however, we had written that proposition

$$\Sigma_k \Pi_a \Pi_b q_{ka}(\bar{q}_{kb} + 1_{ab})$$

Squaring this we have

$$\Sigma_k \Pi_a \Pi_b \Pi_c \Pi_d q_{ka} q_{kc}(\bar{q}_{kb} + 1_{ab})\,(\bar{q}_{kd} + 1_{cd})$$

Now identify d with a, c with b, and we have

$$\Pi_a \Pi_b 1_{ab},$$

which is absurd.

By the same principle, we can if we please solve the example of p. 187 as follows. Given the premise

$$\Pi_k(q_{ki} + \bar{q}_{kj})$$

We multiply the square of this by an identical proposition, thus,

$$\Pi_\lambda \Pi_k \Pi_\mu (q_{\lambda i} + \bar{q}_{\lambda i})\,(q_{\lambda j} + \bar{q}_{\lambda j})\,(q_{ki} + \bar{q}_{kj})\,(q_{\mu i} + \bar{q}_{\mu j})$$

Identifying μ with λ, we get

$$\Pi_\lambda \Pi_k\ (q_{\lambda i} + \bar{q}_{\lambda i})\,(q_{\lambda j} + \bar{q}_{\lambda j})\,(q_{\lambda i} + \bar{q}_{\lambda j})\,(q_{ki} + \bar{q}_{kj}).$$

We now introduce q_{ki} and q_{kj} into the identical proposition wherever we please so long as they cannot be eliminated without making the proposition otherwise than identical; and this condition will be fulfilled so long as we do not introduce q_{ki} or $\bar{q}_{kj}$. Only Π_k must be changed to Σ_k. We get then

$$\Pi_\lambda \Sigma_k (q_{\lambda i}\bar{q}_{ki} + \bar{q}_{\lambda i})\,(q_{\lambda j} + q_{kj}\bar{q}_{\lambda j})\,(q_{\lambda i} + \bar{q}_{\lambda j})\,(q_{ki} + \bar{q}_{kj})$$

This gives successively

$$\Pi_\lambda \Sigma_k (q_{\lambda i}\bar{q}_{ki} q_{\lambda j}\bar{q}_{kj} + \bar{q}_{\lambda i} q_{kj}\bar{q}_{\lambda j} q_{ki})$$

and

$$\Pi_\lambda (q_{\lambda i} q_{\lambda j} + \bar{q}_{\lambda i}\bar{q}_{\lambda j}).$$

But the same conclusion can be reached much more easily by identifying q_k with 1_j, which we have a right to do on account of the universal quantifier. Thus from

$$\Pi_i \Sigma_j \Pi_k (q_{ki} + \bar{q}_{kj})$$

we get

$$\Pi_i \Sigma_j (1_{ji} + \bar{1}_{jj}) = \Pi_i \Sigma_j 1_{ji}.$$

Hence the proposition

$$\Pi_\lambda (q_{\lambda i} q_{\lambda j} + \bar{q}_{\lambda i} \bar{q}_{\lambda j})$$

holds, because it becomes identical when i is substituted for j.

Let us now consider some examples of a somewhat more difficult kind. Given the proposition

$$\Sigma_k \Pi_a \Sigma_b \Pi_c \{\bar{x}_a + r_{kab} y_b (\bar{r}_{kcb} + 1_{ac})\},$$

that is, there is a relation k such that whatever object be taken, either this is not an x or it stands in the relation k to some object which is a y and to which nothing except that x stands in that relation. Required to find all the propositions deducible from this by elimination of k. The simple omission of factors gives

$$\Pi_a \Sigma_b (\bar{x}_a + y_b),$$

that is, there either is no x or there is a y. We get nothing further by identifying c with a. There is therefore no further conclusion without involution. Squaring, with identification of k (without which we should plainly reach nothing new) we have

$$\Sigma_k \Pi_a \Sigma_b \Pi_c \Pi_d \Sigma_e \Pi_f \{\bar{x}_a + r_{kab} y_b (\bar{r}_{kcb} + 1_{ac})\} \{\bar{x}_d + r_{kde} y_e (\bar{r}_{kfe} + 1_{df})\}$$

The application to the Boolian of the common formula

$$(a + x)\ (b + y) \prec a + b + xy$$

gives

$$\bar{x}_a + \bar{x}_d + r_{kab} r_{kde} y_b y_e (\bar{r}_{kcb} + 1_{ac})\ (\bar{r}_{kfe} + 1_{df}).$$

We now identify f with a; when the first and last factors of the last term become

$$r_{kab} (\bar{r}_{kae} + 1_{ad}).$$

But we have

$$r_{kab} \bar{r}_{kae} \prec \bar{1}_{be}.$$

Thus, we reach the conclusion

$$\Pi_a \Sigma_b \Pi_d \Sigma_e (\bar{x}_a + \bar{x}_d + 1_{ad} + y_b y_e \bar{1}_{be});$$

that is, there are either not two x's different from one another or there are two y's different from one another. A little examination will show that no other conclusion could be reached by elimination from the square.

Cubing the original proposition, we reach in a similar way

$$\Sigma_k \Pi_a \Sigma_b \Pi_c \Pi_d \Sigma_e \Pi_f \Pi_g \Sigma_h \Pi_i$$
$$\bar{x}_a + \bar{x}_d + \bar{x}_g + y_b y_e y_h r_{kab} r_{kde} r_{kgh} (\bar{r}_{kcb} + 1_{ac}) (\bar{r}_{kfe} + 1_{df}) (\bar{r}_{kih} + 1_{gi})$$

Identifying f with a, i with d, g with c, the last term of the Boolian becomes

$$y_b y_e y_h r_{kab} r_{kde} r_{kch} \, (\bar{r}_{kcb} + 1_{ac}) (\bar{r}_{kae} + 1_{ad}) (\bar{r}_{kdh} + 1_{cd})$$

whence

$$y_b y_e y_h (\bar{1}_{bh} + 1_{ac}) (\bar{1}_{be} + 1_{ad}) (\bar{1}_{eh} + 1_{cd}),$$

whence again

$$y_b y_e y_h \bar{1}_{bh} \bar{1}_{be} \bar{1}_{eh} + 1_{ac} + 1_{ad} + 1_{cd},$$

so that we have the conclusion

$$\Pi_a \Sigma_b \Pi_c \Pi_d \Sigma_e \Sigma_h \bar{x}_a + \bar{x}_d + \bar{x}_c + 1_{ac} + 1_{ad} + 1_{cd} + y_b y_e y_h \bar{1}_{be} \bar{1}_{bh} \bar{1}_{eh},$$

that is, there either are not three x's all different from one another or there are three y's all different from one another.

It is plain that by raising to the $n!$ power we should get any proposition of this form, and that no others could be obtained.

Let us now seek all the propositions deducible, by elimination of k, from

$$\Pi_k \Sigma_a \Pi_b \Sigma_c u_a (\bar{w}_b + \bar{r}_{kab} + r_{kcb} \bar{1}_{ac})$$

We have at once

$$\Sigma_a u_a.$$

The following proposition is universally true

$$\Pi_\alpha \Pi_\beta \Sigma_\gamma \Pi_i \Pi_j \{ \bar{r}_{\gamma ij} (\bar{r}_{\alpha ij} + \bar{r}_{\beta ij}) + r_{\gamma ij} r_{\alpha ij} r_{\beta ij} \}.$$

To prove this, note first that if we eliminate γ at once, we have an identical proposition. If we raise the whole to a power, there will be no additional mode of elimination, unless i, j, and γ be identified in different factors; but then all the indices must be alike and nothing will be changed. As a special case of this formula we put u_i for $r_{\beta ij}$. This we can do, because

$$\Sigma_\beta \Pi_i \Pi_j \{u_i r_{\beta ij} + \bar{u}_i \bar{r}_{\beta ij}\}$$

which is true by the same reasoning as that just used. The product of these two formulae gives

$$\Pi_\alpha \Sigma_\gamma \Pi_i \Pi_j \{\bar{r}_{\gamma ij}(\bar{r}_{\alpha ij} + \bar{u}_i) + r_{\gamma ij} r_{\alpha ij} u_i\}.$$

Multiplying this twice into the first proposition, identifying k with γ, i with a, and j with b, in one factor, and k with γ, i with c, j with b in the other, we have

$$\Pi_\alpha \Sigma_\gamma \Sigma_a \Pi_b \Sigma_c u_a(\bar{w}_b + \bar{r}_{\gamma ab} + r_{\gamma cb}\bar{1}_{ac})\{\bar{r}_{\gamma ab}(\bar{r}_{\alpha ab} + \bar{u}_a)$$
$$+ r_{\gamma ab} r_{\alpha ab} u_a\}\{\bar{r}_{\gamma cb}(\bar{r}_{\alpha cb} + \bar{u}_c) + r_{\gamma cb} r_{\alpha cb} u_c\}$$

This gives

$$\Pi_\alpha \Sigma_a \Pi_b \Sigma_c u_a(\bar{w}_b + \bar{r}_{\alpha ab} + r_{\alpha cb} u_c \bar{1}_{ac}).$$

We have universally

$$\Sigma_\delta \Pi_m \Pi_n (r_{\delta mn} 1_{mn} + \bar{r}_{\delta mn}\bar{1}_{mn}).$$

Multiplying this twice into the last proposition, identifying α with δ, and n with b, in both factors, m with a in one and with c in the other, we have

$$\Sigma_a \Pi_b \Sigma_c u_a(\bar{w}_b + \bar{1}_{ab} + 1_{cb} u_c \bar{1}_{ac})$$

or since

$$1_{cb}\bar{1}_{ac} \prec \bar{1}_{ab}$$
$$\Sigma_a \Pi_b u_a(\bar{w}_b + \bar{1}_{ab})$$

and identifying b with a

$$\Sigma_a u_a \bar{w}_a$$

Again, we have universally

$$\Sigma_\delta \Pi_m \Pi_n r_{\delta mn}$$

Multiplying this twice into the proposition

$$\Pi_\alpha \Sigma_a \Pi_b \Sigma_c u_a(\bar{w}_b + \bar{r}_{\alpha ab} + r_{\alpha cb} u_c \bar{1}_{ac})$$

with the same identifications as before, we get

$$\Sigma_a \Pi_b \Sigma_c u_a (\overline{w}_b + u_c \overline{1}_{ac}),$$

that is, there is a u, and if there be a w there is a second u. This last proposition shows, by the formula

$$\Sigma_a \Sigma_c x_a x_c \overline{1}_{ac} \prec \Pi_a \Sigma_c (\overline{x}_a + x_c \overline{1}_{ac})$$

that the original proposition may be written in the form

$$\Pi_a \Sigma_a \Pi_b \Sigma_c u_a \{\overline{w}_b + u_c \overline{1}_{ac} (\overline{r}_{aab} + r_{acb})\}$$

Studies in Logical Algebra

Item 32 *MS 539: 20–25 May 1885*

1885 May 20.

Every proposition relating to q_α and r_α which has Σ_α as the last term of the quantifier except quantifiers of things and which gives a true proposition when α is eliminated is true.

Method of Elimination

Before the fifth step in my rule (Am. Jour. Math vii. 198) I should insert a step to use as many separate indices as possible to express the meaning.

For example, suppose we have the premise

$$\Pi_i \Pi_j (m_i l_{ij} + \overline{m}_i \bar{l}_{ij}).$$

The token m cannot be directly eliminated but we have the above

$$\begin{aligned} &= \Pi_i \Pi_j (m_i + \bar{l}_{ij})\,(l_{ij} + \overline{m}_i) \\ &= \Pi_i \{\Pi_j (m_i + \bar{l}_{ij}) \Pi_j (l_{ij} + \overline{m}_i)\} \\ &= \Pi_i \Pi_j \Pi_k (m_i + \bar{l}_{ij})\,(l_{ik} + \overline{m}_i) \end{aligned}$$

and now m can be eliminated giving

$$\Pi_i \Pi_j \Pi_k (l_{ik} + \bar{l}_{ij}).$$

Use of General Formulae in q_α and r_α

Suppose we have the premise

$$\Pi_k (q_{ki} + \bar{q}_{kj})$$

Now take the identical formula

$$\Pi_l (q_{li} + \bar{q}_{li})\,(q_{lj} + \bar{q}_{lj}).$$

Introduce into this q_{mi} and q_{mj} in any way thus

$$\Pi_l \Sigma_m (q_{li}\bar{q}_{mi} + \bar{q}_{li}q_{mi})\ (q_{lj}\bar{q}_{mj} + \bar{q}_{lj}q_{mj})$$

Multiply by the original premise

$$\Pi_l \Sigma_m \Pi_k (q_{li}\bar{q}_{mi} + \bar{q}_{li}q_{mi})\ (q_{lj}\bar{q}_{mj} + \bar{q}_{lj}q_{mj})\ (q_{ki} + \bar{q}_{kj})$$

By the fifth step

$$\begin{aligned}
&\Pi_l \Sigma_m \Pi_k \\
&(q_{li}\bar{q}_{mi} + \bar{q}_{li}q_{mi})\ (q_{lj}\bar{q}_{mj} + \bar{q}_{lj}q_{mj})\ (q_{mi} + \bar{q}_{mj})\ (q_{li} + \bar{q}_{lj})(q_{ki} + \bar{q}_{kj}) \\
&= \Pi_l \Sigma_m \Pi_k (q_{li}q_{lj}\bar{q}_{mi}\bar{q}_{mj} + \bar{q}_{li}\bar{q}_{lj}q_{mi}q_{mj})\ (q_{ki} + \bar{q}_{kj})
\end{aligned}$$

Eliminating m and k

$$\Pi_l (q_{li}q_{lj} + \bar{q}_{li}\bar{q}_{lj})$$

Now instead of introducing q_m we can proceed as follows. The premise

$$\Pi_k (q_{ki} + \bar{q}_{kj})$$

is to be multiplied by any identical equation and Step 5 may be performed.

We may then introduce into the identical proposition $\bar{q}_{ki}$ and q_{kj} wherever we please so long as they are eliminable with restoration of the identical equation but Π_k in the quantifier must be changed to Σ_k.

For example the steps are

$$\begin{aligned}
&\Pi_k (q_{ki} + \bar{q}_{kj}) \\
&\Pi_l \Pi_k (q_{li} + \bar{q}_{li})\ (q_{lj} + \bar{q}_{lj})\ (q_{ki} + \bar{q}_{kj}) \\
&\Pi_l \Pi_k (q_{li} + \bar{q}_{li})\ (q_{lj} + \bar{q}_{lj})\ (q_{li} + \bar{q}_{lj})\ (q_{ki} + \bar{q}_{kj}) \\
&\Pi_l \Sigma_k (q_{li}\bar{q}_{ki} + \bar{q}_{li})\ (q_{lj} + q_{kj}\bar{q}_{lj})\ (q_{li} + \bar{q}_{lj})\ (q_{ki} + \bar{q}_{kj}) \\
&\Pi_l \Sigma_k (q_{li}\bar{q}_{ki}q_{lj}\bar{q}_{kj} + \bar{q}_{li}q_{kj}\bar{q}_{lj}q_{ki}) \prec \Pi_l (q_{li}q_{lj} + \bar{q}_{li}\bar{q}_{lj})
\end{aligned}$$

May 21.

Given that there is not a y for every x, required to prove that there is an x for every y.

$$\left.\begin{aligned}
&\Pi_i \Sigma_a \Sigma_b \Sigma_c \Sigma_d \Pi_e \\
&(r_{iac}r_{ibc}x_a x_b y_c \bar{1}_{ab} + x_d \bar{r}_{ide} + \bar{y}_e)
\end{aligned}\right\}\ \text{The premise}$$

That is we wish to prove

$$\Sigma_i\Sigma_j\Pi_a\Sigma_b\Pi_c\Pi_d\Sigma_e\Pi_f\bar{x}_a + r_{iab}y_b(\bar{r}_{icb} + 1_{ac}) + \bar{y}_d + r_{ide}x_e(\bar{r}_{ife} + 1_{df})$$

But one does not see why the following is not true by the rule that eliminating the r's gives a true proposition

$$\Sigma_i\Pi_a\Sigma_b\Pi_c\{\bar{x}_a + r_{iab}y_b(\bar{r}_{icb} + 1_{ac})\}.$$

There must then be some undiscovered method of elimination.

The proposition remains false or contingent even if any other relative be substituted for 1. Still, any such relation implies identity. However, if another relative be substituted, elimination shows the false (or contingent) character of the proposition. Thus

$$\Sigma_i\Pi_a\Sigma_b\Pi_c\{\bar{x}_a + r_{iab}y_b(\bar{r}_{icb} + l_{ac})\}$$

gives

$$\Pi_a\Sigma_b\{\bar{x}_a + y_b l_{aa}\}$$

This distinguishes the case where 1 is put for l.

Now substitute for 1 its definition. That is

$$1_{pq} = \Pi_k(q_{ka}q_{kb} + \bar{q}_{ka}\bar{q}_{kb})$$

If we write Π_k before Σ_i

$$\Pi_k\Sigma_i\Pi_a\Sigma_b\Pi_c\{\bar{x}_a + r_{iab}y_b(\bar{r}_{icb} + q_{ka}q_{kc} + \bar{q}_{ka}\bar{q}_{kc})\}$$

the proposition is necessarily true.

If we write Π_k after Σ_i where it belongs, the rule heretofore given (p 204) is not violated.

Therefore that rule should contain the proviso that no logical truth is to be recognized when merely expressed by a token. It should be set forth by an icon.

We now return to the longer proposition above. As the proposition is not true of two infinite classes (though I dont feel quite sure of this) it is to be expected that when r is eliminated it would give the definition of a finite class. [At any rate, propositions might be constructed having that property.] But the difficulty is that when the definition of identity is substituted there is a Π_k after the Σ_i and Σ_j. It suggests itself that that q_k may be there because it belongs in the definition of the finite class.

That definition runs that

$$\Pi_i\Sigma_a\Pi_f\Sigma_b\Sigma_c\Sigma_k\Pi_d\Sigma_e\{x_a x_c\bar{r}_{iaf} + x_a x_b x_c r_{iab}r_{icb}q_{ka}\bar{q}_{kc} + \bar{x}_d + x_e r_{ied}\}$$

Here arises a difficulty viz:—Apparently q_k cannot be eliminated without yielding an identical proposition

But plainly strictly speaking the proposition is not identical when it has Σs in the indicator & if we disregard that circumstance we may identify a with c and so eliminate q in a way not to give an identical or a true proposition.

But owing to k having the quantifier Σ, it would seem that the suggestion on p 206 must be wrong.

Probably it is necessary to proceed by the usual mathematical reasoning. Still, it seems highly desirable to be able to perform the elimination suggested on p. 206.

On the whole it does not seem that either class need be finite. Suffice it that one is exhaustible that is subject to Fermatian reasoning, not ultrainfinite.

Required then to define such a class. This I have done in my paper on the Logic of Number. Not having that at hand I make another definition

$$\Sigma_i\Sigma_a\Pi_j\Sigma_b\Sigma_c\Pi_d(x_bx_cq_{jb}r_{ibc}\bar{q}_{jc}+\bar{q}_{ja}+\bar{x}_d+q_{jd})$$

Usual method of proving the proposition above—

$$\Pi_a\Pi_b\Sigma_i\Pi_j\Pi_c\Pi_d\{\bar{x}_a+\bar{y}_b+r_{iab}((q_{ja}q_{jc}+\bar{q}_{ja}\bar{q}_{jc})\,(q_{jb}q_{jd}+\bar{q}_{jb}\bar{q}_{jd})+\bar{r}_{icb})\}$$

Now we multiply this by the following (i remaining the same)

$$\Pi_e\Pi_f\Sigma_{i'}\Pi_g\Pi_h\{\bar{x}_e+\bar{y}_f+r_{ief}+r_{i'ef}(1_{ge}1_{hf}+r_{igh}r_{i'gh}+\bar{r}_{igh}\bar{r}_{i'gh})\}$$

Or to abridge the expressions

$$\Pi_a\Pi_b\Sigma_i\Pi_c\Pi_d\{\bar{x}_a+\bar{y}_b+r_{iab}(\bar{r}_{icd}+1_{ac}1_{bd})\}$$
$$\Pi_j\Pi_e\Pi_f\Sigma_g\Sigma_h\Sigma_k\,\Pi_m\Pi_n$$
$$\bar{x}_e+\bar{y}_f+r_{jeh}+r_{jgf}+r_{kef}(r_{jmn}r_{kmn}+\bar{r}_{jmn}(\bar{r}_{kmn}+1_{me}1_{nf}))$$

The indices are now so numerous, it will be well to use numbers for indices of things and Greek letters for those of characters.

Multiplying the two last propositions by the definition of non-ultrainfinite we have

(Postpone this.)

I am not satisfied with any reason yet given why

$$\Sigma_\alpha\Pi_1\Sigma_2\Pi_3\{\bar{x}_1+r_{\alpha12}y_2(\bar{r}_{\alpha32}+1_{13})\}$$

is not necessary; while

$$\Pi_1 \Pi_2 \Sigma_\alpha \Pi_3 \Pi_4 \{\bar{x}_1 + \bar{y}_2 + r_{\alpha 12}(\bar{r}_{\alpha 34} + 1_{13} 1_{24})\}$$

is so.

But, after all, there seems to be no difficulty. The whole story is that the elimination from the first gives

$$\Pi_1 \Sigma_2 (\bar{x}_1 + y_2)$$

If there is an x there is a y. The elimination from the second gives an identical proposition.

And the whole proposition which we seek to prove (the long one of p 206) is true simply because the elimination reduces it to

$$\Pi_a \Sigma_b \Pi_d \Sigma_e (\bar{x}_a + y_b + \bar{y}_d + x_e)$$

which is an identical proposition.

But if this be the case, it should hold even for ultrainfinites.

It is most important to get a conclusive demonstration of the theorem of p. 204.

What is true is that objects may be taken in pairs, triplets, etc. in any way and these may be collected into groups, as the individuals may, in any way. When we state that any kind of relation (defined by the characters of its members) exists, or would exist under any circumstances this can only be false by its implying some numbers or numerical relations among classes of things or things in definite relations. If this numerical relation is incompatible with the existence or non-existence of any class or explicit relation, then the elimination of q_i or r_i will show it. We have only to examine the cases in which the numerical relation is not thus incompatible.

Take for instance the proposition that there either are no x's or there are at least as many as there are of y's.

$$\Pi_1 \Sigma_\alpha \Pi_2 \Sigma_3 \Pi_4 \, \bar{x}_1 + \bar{y}_2 + r_{\alpha 23} x_3 (\bar{r}_{\alpha 43} + 1_{24})$$

May 22.

From this elimination will give

$$\Pi_1 \Sigma_2 \Sigma_3 \Pi_4 \Pi_5 \, \bar{x}_1 + x_2 x_3 \bar{1}_{23} + \bar{y}_4 + \bar{y}_5 + 1_{45}$$

May 22.

By introducing 1's such elimination may evidently be performed in every case, for every numerical relation must exclude some combination of numbers of the classes.

By what rule such elimination is to be performed is the next question

$$\begin{aligned}&\Pi_1\Sigma_\alpha\Pi_2\Sigma_3\Pi_4\Sigma_\beta\Pi_5\Sigma_6\Pi_7\\ &\bar{x}_1+\{\bar{y}_2+r_{\alpha23}x_3(\bar{r}_{\alpha43}+1_{24})\}\{\bar{y}_5+r_{\beta56}x_6(\bar{r}_{\beta76}+1_{57})\}\\ -\!\!\!<\ &\bar{x}_1+\bar{y}_2+\bar{y}_5+r_{\alpha23}r_{\beta56}x_3x_6\bar{r}_{\alpha43}\bar{r}_{\beta76}\bar{1}_{24}\bar{1}_{57}+1_{24}1_{57}\end{aligned}$$

Best repeat Boolian identifying 4 with 2 and 2 with 4 and putting 6 for 3 in repetition

$$\begin{aligned}&\bar{x}_1+\{\bar{y}_2+r_{\alpha23}x_3(\bar{r}_{\alpha43}+1_{24})\}\{\bar{y}_4+r_{\alpha46}x_6(\bar{r}_{\alpha26}+1_{42})\}\\ -\!\!\!<\ &\bar{x}_1+\bar{y}_2+\bar{y}_4+1_{24}+r_{\alpha23}r_{\alpha46}\bar{r}_{\alpha43}\bar{r}_{\alpha26}x_3x_6\end{aligned}$$

☞ Note the rule $\quad q_{i1}\bar{q}_{i2}-\!\!\!<\bar{1}_{12}$.

Now the question is, is the eliminatory process permissible?

Except in one feature, that of the introduction of the 6, it is only the 5th Step. But can that never go wrong?

$$\Pi_1\Sigma_2\Pi_3(l_{11}+b_{12}x_2)(l_{33}+b_{32}\bar{x}_2)$$

I think that is proved in my paper (pp. 183–84). Now can I, in the repetition, substitute 2 for 4 and 4 for 2 at the same time?

$$\Pi_i\Pi_j l_{ij}-\!\!\!<\Pi_i\Pi_j l_{ij}l_{ji}$$

This seems proof.

Can I change the 3 to 6? There seems no doubt of this.

One line of inquiry would be how to perform the elimination in general.

Also how to recognize a proposition from which elimination will yield only an identical proposition.

But it seems more attractive to inquire whether there be not a way of starting, say, with the definition of a non-ultrainfinite and getting propositions true of such quantity.

Let φx be the statement that x is a non-ultrainfinite. Then

$$\overline{\varphi x}+\varphi x$$

is an identical equation and let $\varphi' x$ be what φx becomes when complicated by eliminible r's and q's. Then if we have to deal with a non-ultrainfinite we have

$$\varphi x(\overline{\varphi \mathrm{x}}+\varphi' \mathrm{x})$$

or

$$\varphi x \varphi' x$$

but $\varphi' x$ implies φx and therefore

$$\varphi' x$$

may be assumed at once.

The same applies whatever the meaning of φx and shows that any proposition involving Σ_α later than any Π_β is true provided it is so when α is eliminated, and provided α is defined only by the relates between which it subsists.

Another thing I might do would be to form a systematic list of all such propositions.

To prove the transitiveness of as small as.

$$\Pi_\alpha \Sigma_1 \Pi_2 \Sigma_3 x_1 (\bar{r}_{\alpha 12} + \bar{y}_2 + r_{132} \bar{1}_{13})$$

states that x is not as small as y.

$$\Sigma_\gamma \Pi_\alpha \Sigma_1 \Pi_2 \Sigma_3 \Pi_\beta \Sigma_4 \Pi_5 \Sigma_6 \Pi_7 \Sigma_8 \Pi_9 \, x_1 (\bar{r}_{\alpha 12} + \bar{y}_2 + r_{\alpha 32} \bar{1}_{13})$$
$$+ y_4 (\bar{r}_{\beta 45} + \bar{z}_5 + r_{\beta 65} \bar{1}_{46}) + \{\bar{x}_7 + r_{\gamma 78} z_8 (\bar{r}_{\gamma 98} + 1_{79})\}$$

Now we *could* eliminate by first identifying α and β with γ and then dropping factors; and we should have

$$\Sigma_1 \Sigma_4 \Pi_7 \Sigma_8 \Pi_9 x_1 + y_4 + \bar{x}_7 + z_8$$

which is identical. But how to recognize that there is no *other* elimination which would not yield an identical proposition?

To try how propositions look implying various existences

$$\Sigma_\alpha \Sigma_1 \, q_{\alpha 1}$$

This only gives

$$\Sigma_1 1_{11} \qquad \text{or something exists.}$$

It seems impossible to produce a false proposition unless some of the things have a particular quantification.

The only example of a true proposition with such quantification was where on elimination we had

$$\Pi_a \Sigma_e (\bar{x}_a + x_e).$$

May 23

$$\Sigma_\gamma \Pi_\alpha \Sigma_1 \Pi_2 \Sigma_3 \Pi_7 \Sigma_8 \Pi_9 x_1(\bar{r}_{\alpha 12} + \bar{y}_2 + r_{\alpha 32}\bar{1}_{13}) + \bar{x}_7 + r_{\gamma 78} z_8(\bar{r}_{\gamma 98} + 1_{79})$$

{Ought to give $\Pi_a \Pi_b \Sigma_c(\bar{x}_a + \bar{y}_b + z_c)$}

Put $r_\alpha = 1$ and drop tokens in γ.

$$\begin{aligned} &\Sigma_1 \Pi_2 \Pi_7 \Sigma_8 x_1(\bar{1}_{12} + \bar{y}_2) + \bar{x}_7 + z_8 \\ &= \Sigma_1 \Pi_7 \Sigma_8(x_1 \bar{y}_1 + \bar{x}_7 + z_8) \end{aligned}$$

which is not an identical proposition; showing that the first proposition is false (or contingent.)

To prove a proposition true in this way, show that its negative is false. Thus the negative of the proposition of p 210 is

$$\Pi_\gamma \Sigma_7 \Pi_8 \Sigma_9 \Sigma_\alpha \Pi_1 \Sigma_2 \Pi_3 \Sigma_\beta \Pi_4 \Sigma_5 \Pi_6$$
$$\{\bar{x}_1 + r_{\alpha 12} y_2(\bar{r}_{\alpha 32} + 1_{13})\}\{\bar{y}_4 + r_{\beta 45} z_5(\bar{r}_{\beta 65} + 1_{46})\} x_7(\bar{r}_{\gamma 78} + \bar{z}_8 + r_{\gamma 98}\bar{1}_{79})$$

This gives

$$\Pi_1 \Sigma_2 \Pi_4 \Sigma_5 \Sigma_7(\bar{x}_1 + y_2)(\bar{y}_4 + z_5) x_7 \bar{z}_7$$

It ought to give some absurdity. Write the quantifier

$$\Sigma_\beta \Sigma_\alpha \Pi_\gamma \Sigma_7 \Pi_1 \Sigma_2 \Pi_3 \Pi_4 \Sigma_5 \Pi_6 \Pi_8 \Sigma_9.$$

Identify 8 with 5, 1 with 7, 4 with 2, 6 and 3 with 9 and $r_{\gamma 15}$ with $\Sigma_i r_{\alpha 1 i} r_{\beta i 5}$

$$\begin{aligned} &r_{\alpha 12} y_2(\bar{r}_{\alpha 32} + 1_{13}) r_{\beta 25} z_5(\bar{r}_{\beta 65} + 1_{26}) x_1(\bar{r}_{\gamma 15} + r_{\gamma 95}\bar{1}_{19}) \\ &y_2(\bar{r}_{\alpha 92} + 1_{19}) z_5 1_{29} x_1 r_{\gamma 93} r_{\beta i 5} \bar{1}_{19} \\ &y_2 z_5 x_1 1_{29} \bar{1}_{19} \bar{r}_{\alpha 92} \end{aligned}$$

Cant see how it yields absurdity.

Consider

$$\Sigma_\gamma \Pi_7 \Sigma_8 \Pi_9 \{\bar{x}_7 + r_{\gamma 78} z_8(\bar{r}_{\gamma 98} + 1_{79})\}$$

Elimination gives at once

$$\Pi_7 \Sigma_8(\bar{x}_7 + z_8)$$

The contradictory of this is

$$\Sigma_9 \Pi_0 x_9 \bar{z}_0$$

Consider then

$$\Sigma_\gamma \Pi_7 \Sigma_8 \Pi_9 \Pi_0 \{\bar{z}_0 + \bar{x}_7 + r_{\gamma 78} z_8(\bar{r}_{\gamma 98} + 1_{79})\}$$

This must give

$$\Pi_a\Pi_b\Sigma_c\Sigma_d\Pi_e(\bar{x}_a + \bar{x}_b + 1_{ab} + z_c z_d \bar{1}_{cd} + \bar{z}_e)$$

For this purpose we change the quantifier to $\Sigma_\gamma\Pi_7\Pi_9\Sigma_8\Pi_0$ and repeat Boolian putting 7 for 9 and 9 for 7 and 1 for 8. We thus have

$$\Sigma_\gamma\Pi_7\Pi_9\Sigma_8\Sigma_1\Pi_0\{\bar{z}_0 + \bar{x}_7 + r_{\gamma 78}z_8\,(\bar{r}_{\gamma 98} + 1_{79})\}$$
$$\{\bar{z}_0 + \bar{x}_9 + r_{\gamma 91}z_1(\bar{r}_{\gamma 71} + 1_{79})\} \prec \bar{z}_0 + \bar{x}_7 + \bar{x}_9 + z_1 z_8 r_{\gamma 78}\bar{r}_{\gamma 71} + 1_{79}$$

The contradictory of this is

$$\Sigma_e\Sigma_a\Sigma_b\Pi_c\Pi_d(x_a x_b \bar{1}_{ab} z_e(\bar{z}_c + \bar{z}_d + 1_{cd})$$

Consider then

$$\Sigma_\gamma\Sigma_1\Sigma_2\Pi_7\Sigma_8\Pi_9\Pi_0\Pi_3\Pi_4$$
$$x_1 x_2 \bar{1}_{12}(\bar{z}_3 + \bar{z}_4 + 1_{34}) + \bar{z}_0 + \bar{x}_7 + r_{\gamma 78}z_8(\bar{r}_{\gamma 98} + 1_{79})$$

This must give

$$\Sigma_a\Sigma_b\Sigma_c\Pi_d\Pi_e\Pi_f\Pi_g z_a z_b z_c \bar{1}_{ab}\bar{1}_{bc}\bar{1}_{ac} + \bar{x}_d + \bar{x}_e + \bar{x}_f + 1_{def} + \bar{z}_g$$

To get this we must cube the Boolian

$$\bar{x}_7 + \bar{x}_{7'} + \bar{x}_{7''} + 1_{79} + 1_{7'9'} + 1_{7''9''}$$
$$+ r_{\gamma 78}r_{\gamma'7'8'}r_{\gamma''7''8''}z_8 z_{8'} z_{8''}\bar{r}_{\gamma 98}\bar{r}_{\gamma'9'8'}\bar{r}_{\gamma''9''8''}$$
$$7' = 9 \quad 7'' = 9' \; 7 = 9''$$

$$r_{\gamma 78}r_{\gamma 7'8'}r_{\gamma 7''8''}$$
$$\bar{r}_{\gamma 78''}\bar{r}_{\gamma 7'8}\bar{r}_{\gamma 7''8'}$$

Eureka!

All the inferences then are obtained by raising the Boolian to some power and identifying.
For example

$$\Sigma_\alpha\Sigma_\beta\Pi_1\Pi_4\Sigma_2\Sigma_5\Pi_3\Pi_6\bar{x}_1 + r_{\alpha 12}y_2(\bar{r}_{\alpha 32} + 1_{13}) + \bar{y}_4 + r_{\beta 45}x_5(\bar{r}_{\beta 65} + 1_{46})$$

to the n^{th} power gives

$$\Sigma_\alpha\Sigma_{\alpha'} \text{ etc } \Pi_1\Pi_{1'} \text{ etc } \Sigma_2\Sigma_{2'} \text{ etc } \Pi_3\Pi_{3'} \text{ etc}$$
$$S\bar{x}_1^P\,\bar{y}_4^Q\,x_5^R\,y_2^{N\text{-}P\text{-}Q\text{-}R}$$

Principles of raising to a power

$$\Sigma_1 \Pi_2 \Sigma_3 b_{123}$$

There is someone who betrays everyone to someone

$$\Sigma_1 \Sigma_4 \Pi_2 \Pi_5 \Sigma_3 \Sigma_6 b_{123} b_{456}$$

We have a right to identify the 1 and 4 and the 2 and 5 but not generally the 3 and 6.

$$\begin{aligned}
\Sigma_1 \Pi_2 l_{12} &= l_{11} l_{12} l_{13} \text{ etc} + l_{21} l_{22} l_{23} + \text{etc} \\
\Sigma_1 \Sigma_3 \Pi_2 \Pi_4 l_{12} l_{34} &= l_{11} l_{12} \text{ etc} \\
&\quad + l_{11} l_{12} \text{ etc } l_{21} l_{22} \text{ etc} \\
&\quad + l_{21} l_{22} \text{ etc} \\
\Pi_1 \Sigma_2 l_{12} &= (l_{11} + l_{12} + \text{etc})\,(l_{21} + l_{22} + \text{etc})
\end{aligned}$$

Relatives of Second Intention

Of absolute terms we have

1_a or a exists

and

0_a or a does not exist

Of dual relatives

1_{ab} or a and b are identical
0_{ab} or a and b are not identical

Of triple relatives

$$\begin{array}{l}
1_{ab} + 1_{ac} + 1_{bc} \\
1_{ab} 1_{ac} \\
0 \text{ etc}
\end{array}$$

$$\begin{array}{ll}
1_a x_a = x_a & \\
\Sigma_b 1_{ab} x_{bc} = x_{ac} & \Pi_b(\bar{1}_{ab} + l_{bc}) = l_{ac} \\
x_{bc} \bar{x}_{ac} \prec \bar{1}_{ab} & 1_{ab} \prec x_{bc} + \bar{x}_{ac} \\
x_a \bar{x}_a \prec 0_a &
\end{array}$$

May 24

Step 5 considered

$$\begin{aligned}
\Pi_i m_i &= \Pi_i \Pi_j \Pi_k \text{ etc } m_i m_j m_k \text{ etc.} \\
\Sigma_i m_i &= \Sigma_i \Sigma_j \Sigma_k \text{ etc } m_i m_j m_k \text{ etc.} \\
\Pi_i \Sigma_j l_{ij} &= \Pi_i \Pi_k \Sigma_j \Sigma_l l_{ij} l_{kl}
\end{aligned}$$

$$= \Pi_i \Pi_k \Sigma_j l_{ij} l_{kj}$$

$$\Sigma_i \Pi_j l_{ij} = \Sigma_i$$

$$\Sigma_i \Pi_j \Sigma_k \; l_{ijk}$$

Raising to a power one step. Identification another.

$$\Pi_i \Pi_k \Sigma_j l_{ij} l_{kj}$$

Notes on the General Rule of Proceedure

3rd Step. This depends on the formula

$$a_{ijk \text{ etc}} \, \bar{a}_{xyz \text{ etc}} \prec \bar{1}_{ix} + \bar{1}_{jy} + \bar{1}_{kz} + \text{etc}$$

whence

$$1_{ix} 1_{jy} 1_{kz} \text{ etc} \prec a_{ijk \text{ etc}} + \bar{a}_{xyz \text{ etc.}}$$

A special case of this, is the principle of contradiction

$$a_i \bar{a}_i \prec 0_i \qquad \text{That is, } 0_i = \bar{1}_{ii}$$

and excluded middle

$$1_{ii} \prec a_i + \bar{a}_i.$$

4th Step *(bis)* Introduction of separate indices as above.
5th Step. This consists of two distinct parts. First, multiplying Boolian (or any factor of it) by itself with introduction anywhere or everywhere of new indices. Second, identification by the rule that

$$\Pi_i \Pi_j l_{ij} \prec \Pi_i l_{ii} \qquad \Sigma_i \Pi_j l_{ij} \prec \Sigma_i l_{ii}$$

$\Pi^{[n]}$ and Σ^m

$\Pi'_i x_i$ means everything except perhaps *one* is x
$\Pi''_i x_i$ " " " " *two* is x
etc
$\Sigma^2_i x_i$ means there are *two* x's at least
$\Pi_i \Pi'_j 1_{ij}$ everything loves every object but one
$\Pi'_j \Pi_i 1_{ij}$ every object but one is loved by everything
$\Pi'_j \Pi_i \prec \Pi_i \Pi'_j$
$\Sigma_i \Sigma^2_j l_{ij}$ something loves two things

$\Sigma_j^2 \Sigma_i l_{ij}$ there are two things loved by something (not necessarily by the same)

$\Sigma_i \Sigma_j^2 \prec \Sigma_j^2 \Sigma_i$

$\Sigma_i''' \Sigma_j^3 l_{ij}$ every object but three loves 3 things

$\Sigma_j^3 \Pi_i''' l_{ij}$ there are 3 things loved by every object but 3.

$\Pi_i^{(n)} \Sigma_j^m l_{ij}$ Every man but n loves m women

$\Sigma_j^m \Pi_i^{(n)} l_{ij}$ There are m women loved by every man but n

Suppose total number of men N
" " " " women M
First prop. says $N - n$ men love each m women
Second says m women are loved each by $N - n$ men
First prop. How many men are the m women each loved by? m is the smallest number women who can be loved

If the number of women M is as great as $(N - n)m$ each may be loved by only one man

If $(N - n)m > M$
One woman at least must be loved by 2 men

If $(N - n)m > M + 1$
Either one woman must be loved by 3 men or 2 women must be loved by 2 men etc

If $M < (N - n - 1)m + 1$
2 women must be loved by 2 men.

Sum

If every x has the relation i to one z and to one only, and if every y has the relation j to one z and to one only, and if the same z is not $(i+j)$'d by two different objects, then the z's i'd by x's or j'd by y's are the sum of the x's and y's.

To write that the sum of the x's and y's is as small as the w's.

$$\Pi_z \Pi_i \Pi_j \Sigma_1 \Pi_2 \Sigma_3 \Sigma_4 \Sigma_5 \Sigma_6 \Pi_7 \Sigma_8 \Sigma_9 \Sigma_0 \Sigma_{(11)} \Sigma_{12} \Sigma_{13} \Sigma_k \Pi_{14} \Sigma_{15} \Pi_{16}$$
$$x_1 \ (\bar{r}_{i12} + \bar{q}_{z2}) + x_3 r_{i34} r_{i35} \bar{1}_{45} q_{z4} q_{z5}$$
$$+ y_6(\bar{r}_{j67} + q_{z7}) + y_8 r_{j89} r_{j80} \bar{1}_{90} q_{z9} q_{z0}$$
$$+ q_{z(11)} \ (r_{i.12.11} + r_{j.12.11}) \ (r_{i.13.11} + r_{j.13.11}) \bar{1}_{12.13}$$
$$+ \bar{q}_{z.14} + r_{k.14.15} w_{15} (\bar{r}_{k.16.15} + 1_{14.16})$$

Before undertaking to work with such a formula, let us see if we cannot find a direct method of passing from premise to conclusion. For instance

$$\Pi_\alpha \Sigma_1 \Pi_2 \Sigma_3 x_1 \ (\bar{r}_{\alpha 12} + \bar{y}_2 + r_{\alpha 32} \bar{1}_{13})$$

That is the x's are more than the y's. To prove (2)

$$\Sigma_\beta \Pi_4 \Sigma_5 \Pi_6 \{\bar{y}_4 + r_{\beta 45} x_5 (\bar{r}_{\beta 65} + 1_{46})\}$$

The first may be written

$$\Pi_\alpha \Sigma_1 \Pi_2 \Sigma_3 \{\bar{y}_2 x_1 + x_1 \ (\bar{r}_{\alpha 12} + r_{\alpha 32} \bar{1}_{13})\}$$

This might be written for short

$$\Pi_\alpha \Sigma_1 \Pi_2 x_1 (\bar{y}_2 + m_{\alpha 12})$$

Then we should have to develope the properties of the relative m.

The conclusion, (2), only implies

$$\Pi_4 \Sigma_5 (\bar{y}_4 + x_5)$$
$$\Pi_{41} \Pi_{42} \Sigma_{51} \Sigma_{52} \bar{y}_{41} + \bar{y}_{42} + 1_{41.42} + x_{51} \cdot x_{52} \bar{1}_{51.52}$$
$$\Pi_{41} \Pi_{42} \Sigma_{43} \Sigma_{51} \Sigma_{52} \Sigma_{53} \bar{y}_{41} + \bar{y}_{42} + \bar{y}_{43} + 1_{41.42} + 1_{42.43} + 1_{41.43} + x_{51} \cdot x_{52} \cdot x_{53} \bar{1}_{51.52} \bar{1}_{51.53} \bar{1}_{52.53}$$

etc.

Now take the premise

$$\Pi_\alpha \Sigma_1 \Pi_2 \Sigma_3 \{\bar{y}_2 x_1 + x_1 \ (\bar{r}_{\alpha 12} + r_{\alpha 32} \bar{1}_{13})\}$$

The mere dropping of factors gives

$$\Sigma_1 \Pi_2 \{\bar{y}_2 + x_1\} \prec \Pi_2 \Sigma_1 \{\bar{y}_2 + x_1\}$$

The square gives

$$\Pi_\alpha \Pi_{\alpha'} \Sigma_{11} \Sigma_{12} \Pi_{21} \Pi_{22} \Sigma_{31} \Sigma_{32} \{\bar{y}_{21} x_{11} + \bar{y}_{22} x_{12} + \bar{x}_{11} x_{12} \\ (\bar{r}_{\alpha.11.21} + r_{\alpha.31.21} \bar{1}_{11.31})(\bar{r}_{\alpha'.12.22} + r_{\alpha'.32.22} \bar{1}_{12.32})\}$$

My difficulty here shows I have not got to the bottom of the implications of Π_α

The square is to be multiplied by

$$r_{\alpha.11.21} r_{\alpha'.12.22} \ (\bar{r}_{\alpha.11.41} + 1_{11.41}) \ (\bar{r}_{\alpha'.12.42} + 1_{12.42})$$

and this gives it.

The amount of it is that a proposition in Π_α can be multiplied by any true proposition in Σ_a, or any number of such propositions

And note this. When we have $\Pi_\alpha r_{\alpha 13}$ we can multiply and have $\Pi_\alpha x_1 r_{\alpha 13}$. This needs development. Therefore before squaring

$$\Pi_2 \Sigma_1 \Sigma_3 (\bar{y}_2 + x_1 x_3 \bar{1}_{13})$$

and a fortiori

$$\Pi_2 \Pi_4 \Sigma_1 \Sigma_3 (\bar{y}_2 + \bar{y}_4 + 1_{14} + x_1 x_3 \bar{1}_{13})$$

Go back then to the premise

$$\Pi_\alpha \Sigma_1 \Pi_2 \Sigma_3 \{\bar{y}_2 x_1 + x_1 \ (\bar{r}_{\alpha 12} + r_{\alpha 32} \bar{1}_{13})\}$$

We have at once

$$\Sigma_1 x_1 \quad \text{and a fortiori} \quad \Pi_2 \Sigma_1 \ (\bar{y}_{2'} + x_1)$$

We have

$$\Sigma_\alpha \Pi_1 \Pi_2 \ (\bar{x}_1 \bar{r}_{\alpha 12} + x_1 r_{\alpha 12})$$

This is obviously true.

Multiplying then by this we have

$$\Sigma_\beta \Pi_\alpha \Sigma_1 \Pi_2 \Sigma_3 \Pi_4 \Pi_5 \{\bar{y}_2 x_1 + x_1 \ (\bar{r}_{\alpha 12} + r_{\alpha 32} \bar{1}_{13})\} \ (\bar{x}_4 \bar{r}_{\beta 45} + x_4 r_{\beta 45})$$

Multiply again by the last factor identifying 5 with 2 in both factors and 4 in one factor with 1 and in the other with 3. Identify α with β. We get by omission of factors

$$\Pi_2 \Sigma_1 \Sigma_3 (\bar{y}_2 + x_1 x_3 \bar{1}_{13})$$

Now take the square of the premise

$$\begin{aligned}
&\Pi_\alpha \Pi_{\alpha'} \Sigma_{11} \Sigma_{12} \Pi_{21} \Pi_{22} \Sigma_{31} \Sigma_{32} \\
&\{\bar{y}_{21} x_{11} + \bar{y}_{22} x_{12} + x_{11} x_{12} (\bar{r}_{\alpha.11.21} + r_{\alpha.31.21} \bar{1}_{11.31}) \\
&(\bar{r}_{\alpha'.12.22} + r_{\alpha'.32.22} \bar{1}_{12.32})\}
\end{aligned}$$

Now we have

$$\begin{aligned}
&\Pi_4 \Pi_5 \Pi_6 \Pi_7 \Sigma_i \\
&\{\bar{x}_4 + \bar{x}_5 + 1_{45} + \bar{y}_6 + \bar{y}_7 + 1_{67} + r_{i46} r_{i57} \ (\bar{r}_{i56} + 1_{45})(\bar{r}_{i47} + 1_{54})\}
\end{aligned}$$

But by the last result $\Pi_2 \Sigma_1 \Sigma_3 (\bar{y}_2 + x_1 x_3 \bar{1}_{13})$ the first three terms are eliminated. Then 6 and 7 are identified with 21 and 22.

All this is not very clear. There seems to be a need of introducing again the last result.

But, after all, why not proceed thus

$$\Pi_\alpha \Sigma_1 \Pi_2 \Sigma_3 x_1(\bar{r}_{\alpha 12} + \bar{y}_2 + r_{\alpha 32}\bar{1}_{13}) \prec \Pi_\alpha \Pi_2 \Sigma_1 \Sigma_3 \{\bar{y}_2 + x_1(\bar{r}_{\alpha 12} + r_{\alpha 32}\bar{1}_{13})\}$$

Now $(\bar{r}_{\alpha 12} + r_{\alpha 32}\bar{1}_{13})$ expresses a relation between 1 and 2 and $\bar{r}$ may be so specialized that nothing but 2 has that relation to 1. Hence

$$\Sigma_\beta \Pi_2 \Sigma_1 \Pi_4 \{\bar{y}_2 + x_1 r_{\beta 12}(\bar{r}_{\beta 14} + 1_{24})\}$$

The conclusion.

$$\Sigma_\alpha \Pi_2 \Pi_1 \Pi_3 \Pi_4 \{r_{\alpha 14}(\bar{r}_{\alpha 34} + 1_{13}) + 1_{24}\}$$

It is absurd to say that 1 is thus related to 2 alone. However

$$\Sigma_\beta \Pi_2 \Pi_1 \Pi_4 (\bar{r}_{\beta 12} + \bar{r}_{\beta 14} + 1_{24})$$

is obviously true and this gives the result.

The excessive length of the formulae, from which the method suffers so much, ought to be capable of reduction and splitting up the work by the introduction of equations, and of subsidiary tokens such as *m* on p 216 above.

Restatement

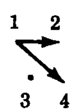
$$\Pi_\alpha \Sigma_1 \Pi_2 \Sigma_3 x_1(\bar{r}_{\alpha 12} + \bar{y}_2 + r_{\alpha 32}\bar{1}_{13})$$

$$\prec \Pi_\alpha \Pi_2 \Sigma_1 \Sigma_3 \{\bar{y}_2 + x_1\ (\bar{r}_{\alpha 12} + r_{\alpha 32}\bar{1}_{13})\}$$

For $\Sigma_3\ (\bar{r}_{\alpha 12} + r_{\alpha 32}\bar{1}_{13})$ put $r_{\beta 12}$?

$$\Sigma_\alpha \Pi_1 \Pi_2 \Pi_4 \Sigma_3 \{r_{\alpha 12}\ (\bar{r}_{\alpha 32} + 1_{13}) + r_{\alpha 14}\ (\bar{r}_{\alpha 34} + 1_{13}) + 1_{24}\}$$

No this is absurd.



It seems impossible to prove the proposition in this way.

May 25

Suppose we square the premise of p 216 in its original form.

$$\Pi_\alpha \Pi_{\alpha'} \Sigma_{11} \Sigma_{12} \Pi_{21} \Pi_{22} \Sigma_{31} \Sigma_{32}$$
$$x_{11}x_{12}\ (\bar{r}_{\alpha.11.21} + \bar{y}_{21} + r_{\alpha.31.21}\bar{1}_{11.31})\ (\bar{r}_{\alpha'.12.22} + \bar{y}_{22} + r_{\alpha'.32.22}\bar{1}_{12.32})$$

We need to assume the former result

$$\Pi_1 \Sigma_2 \Sigma_3\ (\bar{y}_1 + x_2 x_3 \bar{1}_{23})$$

Now

$$\Sigma_\beta \Pi_4 \Pi_5 \Pi_6 \Pi_7 \Sigma_8 \Sigma_9 \Sigma_{10} \Sigma_{20} \Pi_{30} \Pi_{40} \bar{y}_4 + \bar{y}_5 + 1_{45} + \bar{x}_6 + \bar{x}_7 + 1_{67}$$
$$+ y_8 y_9 \bar{1}_{89} x_{10} x_{20} \bar{1}_{10.20} r_{\beta 8.10} r_{\beta 9.20} (\bar{r}_{\beta.30.10} + 1_{8.30}) (\bar{r}_{\beta.40.20} + 1_{9.40})$$

The product of last two equations will annul the three terms in 6.7 in last (i.e. $\bar{x}_6 + \bar{x}_7 + 1_{67}$)

From

$$\Pi_\alpha \Sigma_1 \Pi_2 \Sigma_3 x_1 \ (\bar{y}_2 + \bar{r}_{\alpha 12} + r_{\alpha 32} \bar{1}_{13})$$

to prove

$$\Pi_\alpha \Sigma_1 \Pi_2 \Sigma_3 x_1 \ (\bar{y}_2 + \bar{r}_{\alpha 12} + x_3 r_{\alpha 32} \bar{1}_{13}).$$

The premise may be developed thus

$$\Sigma_a \Pi_b \Sigma_e \Pi_\alpha \Sigma_f \Pi_\beta \Sigma_c \Pi_d \Sigma_i \Pi_\gamma \Pi_g \Sigma_h \Sigma_j$$
$$x_a \bar{y}_b + y_e(p_\alpha + \bar{p}_\alpha) + y_f\{\bar{p}_\beta + x_c(\bar{y}_d + \bar{r}_{acd})\} + y_i(\bar{x}_g + \bar{p}_\gamma + r_{\gamma gh})(r_{\gamma jh} \bar{1}_{gj})$$

$$\Pi_1 \Pi_2 \{(\bar{r}_{\alpha 12} + \bar{x}_1) + r_{\beta 12}\} \ \{r_{\alpha 12} x_1 + \bar{r}_{\beta 12}\}$$

$$\Pi_\alpha m_\alpha \prec \Pi_\beta \Pi_\alpha \Sigma_1 \Sigma_2 \{r_{\alpha 12} x_1 \bar{r}_{\beta 12} + (\bar{r}_{\alpha 12} + \bar{x}_1) r_{\beta 12} + m_\beta\}$$

$$\Pi_\beta \Pi_\alpha \Sigma_a \Sigma_b \Sigma_1 \Pi_2 \Sigma_3$$

$$r_{\alpha ab} x_a \bar{r}_{\beta ab} + (\bar{r}_{\alpha ab} + \bar{x}_a) r_{\beta ab} + x_1 \ (\bar{y}_2 + \bar{r}_{\beta 12} + r_{\beta 32} \bar{1}_{13})$$

$$\Pi_\beta \Pi_\alpha [\Sigma_a \Sigma_b r_{\alpha ab} x_a \bar{r}_{\beta ab} + (\bar{r}_{\alpha ab} + \bar{x}_a) r_{\beta ab} + \Sigma_1 \Pi_2 \Sigma_3 \Pi_a \Pi_b$$
$$\{\bar{r}_{\alpha ab} + x_a + r_{\beta ab}\} \ \{r_{\alpha ab} x_a + \bar{r}_{\beta ab}\} x_1 (\bar{y}_2 + \bar{r}_{\beta 12} + r_{\beta 32} \bar{1}_{13})]$$

Identify *a* with 3, *b* with 2

$$\Pi_\beta \Pi_\alpha [\Sigma_a \Sigma_b \{r_{\alpha ab} x_a \bar{r}_{\beta ab} + (\bar{r}_{\alpha ab} + \bar{x}_a) r_{\beta ab}\}$$
$$+ \Sigma_1 \Pi_2 \Sigma_3 (\bar{r}_{\alpha 32} + \bar{x}_3 + r_{\beta 32})(r_{\alpha 32} x_3 + \bar{r}_{\beta 32}) x_1 (\bar{y}_2 + \bar{r}_{\beta 12} + r_{\beta 32} \bar{1}_{13})]$$

multiply again by this factor with *a* identified with 1. This immediately gives by omission of factors

$$\Pi_\beta \Pi_\alpha [\Sigma_a \Sigma_b \{r_{\alpha ab} x_a \bar{r}_{\beta ab} + (\bar{r}_{\alpha ab} + \bar{x}_a) r_{\beta ab}\}$$
$$+ \Sigma_1 \Pi_2 \Sigma_3 x_1 (\bar{y}_2 + \bar{r}_{\alpha 12} + x_3 r_{\alpha 32} \bar{1}_{13})]$$

Now make first factor disappear by

$$\Pi_\alpha \Sigma_\beta \Pi_a \Pi_b (\bar{r}_{\alpha ab} + \bar{x}_a + r_{\beta ab}) \ (r_{\alpha ab} x_a + \bar{r}_{\beta ab})$$

Having thus shown the theorem p 219 to be true, I return to the problem of p 218–19

It is now solved at once by the method of p 217, where observe 1_{45} being an integrant term, its negative may multiply any other factor. Identify 4 with 11, 5 with 12.

Now the question arises would not the theorem of p 219 enable us to proceed as on pp 217–18.

Numbers

A universe limited only by general conditions.
e next larger. e_{ij} j is next larger than i.

$$\Pi_A \Sigma_B e_{AB}$$
$$\Pi_A \Pi_B \Sigma_C \Sigma_D (e_{CA} + e_{DB} + 1_{AB})$$
$$\Pi_A \Pi_B \Pi_C (\bar{e}_{AC} + \bar{e}_{BC} + 1_{AB})$$
$$\Pi_A \Pi_B \Pi_C (\bar{e}_{CA} + \bar{e}_{CB} + 1_{AB})$$
$$\Sigma_A \Pi_B \bar{e}_{BA}$$
$$\Pi_\alpha \Pi_A \Sigma_B \Sigma_C \Sigma_D \Pi_E (\bar{q}_{\alpha A} + e_{BA} + q_{\alpha C} \bar{q}_{\alpha D} e_{CD} + q_{\alpha E})$$

Counting

The numbers used in counting any class α, are such that there is a relation which every object has to a number used and every number to an object, no object to two numbers, no two objects to one number, and every number next smaller than a number used is used.

$$\Pi_\alpha \Sigma_i \Pi_A \Pi_a \Sigma_B \Sigma_b \Pi_C \Pi_c \bar{q}_{\alpha a} + \bar{c}_{\alpha A} + r_{iaB} r_{ibA} (\bar{r}_{iaC} + 1_{BC})(\bar{r}_{icA} + 1_{bc})$$
$$\Pi_\alpha \Pi_A \Pi_B \bar{c}_{\alpha A} + \bar{e}_{BA} + c_{\alpha B}$$

A difficult proposition to prove

$$\Sigma_\alpha \Pi_a \Sigma_b \Pi_c \Pi_d \Sigma_e \Pi_f [\bar{y}_a + x_b r_{\alpha ba} (\bar{r}_{\alpha cd} + 1_{ad} + \bar{1}_{bc})] \bar{r}_{\alpha ef} x_e$$
$$\prec \Pi_\beta \Sigma_1 \Pi_2 \Sigma_3 \{x_1 (\bar{r}_{\beta 12} + \bar{y}_2 + r_{\beta 32} \bar{1}_{31})\}$$

This would be proved by raising to powers. The antecedent gives at once

$$\Sigma_e x_e$$

and identifying f with a

$$\Pi_a \Sigma_b \Sigma_e \bar{y}_a + x_b x_e \bar{1}_{be}$$

Squaring. Identifying a' with d, taking out 1_{ad} as a term by itself multiplying the rest by $\bar{1}_{ad}$. Identifying c with b', we get

$$\Pi_a \Pi_d \Sigma_b \Sigma_c \Sigma_e \bar{y}_a + \bar{y}_d + 1_{ad'} + x_b x_c x_e \bar{1}_{bc} \bar{1}_{be} \bar{1}_{ce}$$

An American Plato: Review of Royce's *Religious Aspect of Philosophy*

Item 33 ***MS 541: Summer 1885***

When I meet with an opinion held by sane and instructed men, which is so radically different from my own that it seems strange and incomprehensible to me how men can believe such a thing, I for my part (and I suppose others have the same curiosity) am always tempted to examine the grounds of that opinion with a good deal of care, and am not satisfied until I have either become a convert to it or at least can fully understand and feel how it is that others hold to it. Such an opinion the scientific man finds in Hegelianism; not so much in its conclusions, which relate for the most part to matters upon which science does not touch, as in the extraordinary modes of reasoning by which it professes to reach those conclusions. But, as every one knows, the system of Hegel is very difficult to understand. A real mastership of it is what neither Hegel himself nor any of his followers or critics has possessed. It contains many distinct elements; and the study of logic has not yet been carried far enough to estimate precisely the value of all of them.

Dr. Royce has produced a work which will form a good introduction to Hegel. His language and his thought are equally lucid and within the capacity of ordinary minds; his style is animated and readable, and in passages rises without effort to true philosophical eloquence. His method is a dialectic one; that is to say, it proceeds by the criticism of opinions, first, destructively to absolute scepticism, and then finds hidden in that scepticism itself the highest truth. It differs, however, very decidedly from the dialectic of Hegel; and in its simplicity and general tone reminds us rather of the reasoning of Plato.

But before we examine the method, let us glance at the philosophical upshot of the book. This is, that the reality of whatever really exists consists in the real thing being thought by God. Ordinary people think that things exist by the *will* of God; and if thought be taken in so wide a sense as to include volition, they have no difficulty in admitting the proposition which Dr. Royce has borrowed from Hegel and Schelling. But ordinary people say that not merely the real but all that can possibly enter into the mind of man must be within the thought of God in some sense; so that it must be some particular kind of divine thought which constitutes reality; and that particular kind of thought must be distinguished by a volitional element. In short, ordinary people make, at once, the very same criticism that the profoundest students of philosophy have made, namely, that the Hegelian school overlooks the importance of the will as an element of thought.

A certain writer has suggested that reality, the fact that there is such a thing as a true answer to a question, consists in this: that human inquiries,—human reasoning and observation,—tend toward the settlement of disputes and ultimate agreement in definite conclusions which are independent of the particular stand-points from which the different inquirers may have set out; so that the real is that which any man would believe in, and be ready to act upon, if his investigations were to be pushed sufficiently far. Upon the luckless putter-forth of this opinion Dr. Royce is extremely severe. He will not even name him (perhaps to spare the family), but refers to him by various satirical nick-names, especially as *"Thrasymachus,"*—a foolish character introduced into the *Republic* and another dialogue of Plato for the purpose of showing how vastly such an ignorant pretender to philosophy is inferior to Socrates (that is, to Plato himself) in every quality of mind and heart, and especially in good manners. But I must with shame confess that if I understand what the opinion of this poor, Royce-forsaken Thrasymachus is, I coincide with it exactly. I ask any man, Suppose you could be miraculously assured that a certain answer to any question that interests you would be the one in which, were your life and mental vigor to be indefinitely prolonged, you must eventually rest, would you not cease all inquiries at once, and be content with that answer now, as being the very thing you had been striving after? This question Dr. Royce answers explicitly in the negative. "No barely possible judge," he says, "who *would* see the error, *if* he were there, will do for us." Yet if I were to represent Dr. Royce as preferring to believe for a little while that which a certain Being—no matter who—imagines, rather than to come at once to the belief to which investigation is destined at last to carry him, I should probably be doing him injustice; because I suppose he would say that the thing which God imagines, and the opinion to which investigation would ultimately lead him, in point of fact coincide. If, however, these two things coincide, I fail to understand why he should be so cruel to the childish Thrasymachus; since after all there is no real difference between them, but only a formal one,—each maintaining as a theorem that which the other adopts as a definition. As was just remarked, the Hegelian school do not sufficiently take into account the volitional element of cognition. Dr. Royce admits in words that belief is what a man will act from; but he does not seem to have taken the truth of this proposition home to him, or else he would see that the whole end of inquiry is the settlement of belief; so that a man shall not war against himself, nor undo tomorrow that which he begins to do today. Dr. Royce's main argument in support of his own opinion, to the confusion of Thrasymachus, is drawn from the existence of error. Namely, the subject of an erroneous proposition could not be identified with the subject of the corresponding true proposition, except by being completely known, and in that knowledge no error would be possible. The truth must, therefore, be present to the actual consciousness of a living being. This is an argument drawn from Formal Logic, for formal logic it is which inquires how different propositions are made to refer to the same subject, and the like. German metaphysics has, since Kant, drawn its best arguments from formal logic; and it is quite right in doing so, for the conceptions which are proved to be indispensible in formal logic, must have been already rooted in the nature of the mind when reasoning first began, and are, so far, *à priori.* But one would surely have supposed that when the German philosophers were thus drawing their arguments from formal logic, they would have postponed their venturesome flights into the thin air of theology and the vacuum of pure reason, until they had carefully tried the strength of every part of that logical machine on which they were to depend. Instead of that, they have left the great work of creating a true system of formal logic to English authors, who, while they have done most excellent work, have (with the insignificant exception of the present writer) been quite indifferent to the transcendental bearings of their results. Kant gives a half dozen only of his brief pages to the development of the system of logic upon which his whole philosophy rests; and though many valuable treatises on the science have appeared in Germany, there is hardly one of them which is not more or less marred by some arrant absurdity, acknowledged to be so by all the others; Grassmann and Schroeder alone pursuing the one method which will yield positive results properly secured against error. We must not, therefore, wonder that Dr. Royce's argument from formal logic overlooks one of the most important discoveries that have lately resulted from the study of that exact branch of philosophy. He seems to think that the real subject of a proposition can be denoted by a general term of the proposition; that is, that precisely what it is that you are talking about can be distinguished from other things by giving a general description of it. Kant already showed, in a celebrated passage of his cataclysmic work, that this is not so; and recent studies in formal logic[1] have put it in a clearer light. We now find that, besides general terms, two other kinds of signs are perfectly indispensible in all reasoning. One of these kinds is the *index,* which like a pointing finger, exercises a real physiological *force* over the attention, like the power of a mesmerizer, and directs it to a particular object of sense. One such index at least must enter into every proposition, its function being to designate the subject of discourse. Now observe that Dr. Royce does not merely say that there are no means by which an erroneous proposition can be produced; what he says is that the conception of an erroneous proposition (without an actual including consciousness) is absurd. If the subject of discourse had to be distinguished from other things, if at all, by a general term, that is, by its peculiar characters, it would be quite true that its complete segregation would require a full knowledge of its characters and would preclude ignorance. But the index, which in point of fact alone can designate the subject of a proposition, designates it without implying any characters at all. A blinding flash of lightning forces my attention and directs it to a certain moment of time with an emphatic "Now!" Directly following it, I may judge that there will be a terrific peal of thunder, and if it does not come I acknowledge an error. One instant of time is, in itself, exactly like

1. Mitchell in *Studies in Logic. By Members of the Johns Hopkins University,* and Peirce in *The American Journal of Mathematics,* vol. vii.

any other instant, one point of space like any other point; nevertheless dates and positions can be approximately distinguished. And how are they so distinguished? By *intuition* says Kant; perhaps not in so many words; but it is because of this property that he distinguishes Space and Time from the general conceptions of the understanding and sets them off by themselves under the head of intuition. But I should prefer to say that it is by volitional acts that dates and positions are distinguished. The element of feeling is so prominent in sensations, that we do not observe that something like Will enters into them, too. You may quarrel with the word *volition* if you like; I wish I had a more general one at my hand. But what I mean is that that strong, clear, and voluntary consciousness in which we act upon our muscles is nothing more than the most marked variety of a kind of consciousness which enters into many other phenomena of our life, a consciousness of duality or dual consciousness. Feeling is simple consciousness, the consciousness that can be contained within an instant of time, the consciousness of the excitation of nerve-cells; it has no parts and no unity. What I call volition is the consciousness of the discharge of nerve-cells, either into the muscles, etc., or into other nerve-cells; it does not involve the sense of time (i.e. not of a continuum) but it does involve the sense of action and reaction, resistance, externality, otherness, pair-edness. It is the sense that something has hit me or that I am hitting something; it might be called the sense of collision or clash. It has an outward and an inward variety, corresponding to Kant's outer and inner sense, to will and self-control, to nerve-action and inhibition, to the two logical types *A*:*B* and *A*:*A*. The capital error of Hegel which permeates his whole system in every part of it is that he almost altogether ignores the Outward Clash.[2] Besides the lower consciousness of feeling and the higher consciousness of nutrition, this direct consciousness of hitting and of getting hit enters into all cognition and serves to make it mean something real. It is formal logic which teaches us this; not that of a Whately or a Jevons, but formal logic in its new development, drawing nutriment from physiology and from history without leaving the solid ground of logical forms.

An objection different from that of Dr. Royce might be raised. Namely it might be asked *how* two different men can know they are

2. "We must be in contact with our subject-matter," says he in one place, "whether it be by means of our external senses, *or, what is better,* by our profounder mind and our innermost self-consciousness."

speaking of the same thing. Suppose, for instance, one man should say a flash of lightning was followed by thunder and another should deny it. How would they know they meant the same flash? The answer is that they would compare notes somewhat as follows. One would say, "I mean that very brilliant flash which was preceded by three slighter flashes, you know." The second man would recognize the mark, and thus by a probable and approximate inference they would conclude they meant the same flash.

Dr. Royce in describing the opinion of Thrasymachus has selected the expression "a *barely* possible judge." Is there not an ambiguity in this mode of speech which is unfair to Thrasymachus? The final opinion which would be sure to result from sufficient investigation may possibly, in reference to a given question, never be actually attained, owing to a final extinction of intellectual life or for some other reason. In that sense, this final judgment is not *certain* but only possible. But when Dr. Royce says "bare possibility is blank nothingness," he would seem to be speaking of mere logical possibility, and not a possibility which differs but by a hair's-breadth from entire certainty. Let us consider what probability there is that a given question, say one capable of being answered by *yes* or *no,* will never get answered. Let us reason upon this matter by inductive logic. Dr. Royce and his school, I am well aware, consider inductive reasoning to be radically vicious; so that we unhappily cannot carry them along with us. [They often deny this, by the way, and say they rest entirely on experience. This is because they so overlook the Outward Clash, that they do not know what experience is. They are like Roger Bacon, who after stating in eloquent terms that all knowledge comes from experience, goes on to mention spiritual illumination from on high as one of the most valuable kinds of experiences.] But they will not succeed in exploding the method of modern science; and there is no reason why those who believe in induction at all, should not be willing to apply it to the subject now in hand. In the first place, then, upon innumerable questions, we have already reached the final opinion. How do we know that? Do we fancy ourselves infallible? Not at all; but throwing off as probably erroneous a thousandth or even a hundredth of all the beliefs established beyond present doubt, there must remain a vast multitude in which the final opinion has been reached. Every directory, guide-book, dictionary, history, and work of science is crammed with such facts. In the history of science, it has sometimes occurred that a really wise man has said concerning one question or another that there was reason to believe it never would be answered. The proportion of these which have in point of fact been conclusively settled very soon after the prediction has been surprizingly large. Our experience in this direction warrants us in saying with the highest degree of empirical confidence that questions that are either practical or could conceivably become so are susceptible of receiving final solutions provided the existence of the human race be indefinitely prolonged and the particular question excite sufficient interest. As for questions which have no conceivable practical bearings, as the question whether force is an entity, they mean nothing, and may be answered as we like, without error. We may take it as certain that the human race will ultimately be extirpated; because there is a certain chance of it every year, and in an indefinitely long time the chance of survival compounds itself nearer and nearer to zero. But, on the other hand, we may take it as certain that other intellectual races exist on other planets,—if not of our solar system, then of others; and also that innumerable new intellectual races have yet to be developed; so that on the whole, it may be regarded as most certain that intellectual life in the universe will never finally cease. The problem whether a given question will ever get answered or not is not so simple; the number of questions asked is constantly increasing, and the capacity for answering them is also on the increase. If the rate of the latter increase is greater than that of the former the probability is unity that any given question will be answered; otherwise the probability is *zero.* Considerations too long to be explained here lead me to think that the former state of things is the actual one. In that case, there is but an infinitesimal proportion of questions which do not get answered, although the multitude of unanswered questions is forever on the increase. It plainly is not fair to call a judgment which is certain to be made a "barely possible" one. But I will admit (if the reader thinks the admission has any meaning, and is not an empty proposition) that some finite number of questions, we can never know which ones, will escape getting answered forever. Nor must I forget that I have not given the reader my proof that out of the questions asked at any time the proportion that will never be answered is infinitesimal; so that he may be in doubt upon this point. That is not a thing to be regretted; for scepticism about the reality of things,—provided it be genuine and sincere, and not a sham,—is a healthful and growing stage of mental development. Let us suppose, then, for the sake of argument, that some questions eventually get settled, and that some others, indistinguishable from the former by any marks, never do. In that case, I should say that the conception of reality was rather a faulty one, for while there is a real so far as a question that will get settled goes, there is none for a question that will never be settled; for an unknowable reality is nonsense. The non-idealistic reader will start at this last assertion; but consider the matter from a practical point of view. You say that real things are manifested by their effects. True; for example, if the timbers of my house are inwardly rotting, it will some day fall down, and thus there will be a practical effect for me, whether I know the beams be rotten or not. Well, but if all the effects consistently point to the theory that the beams are rotting, it will come to be admitted at last that they are so; and if nothing is ever settled about the matter, it will be because the phenomena do not consistently point to any theory; and in that case there is a want of that "uniformity of nature" (to use a popular but very loose expression) which constitutes reality, and makes it differ from a dream. In that way, if we think that some questions are never going to get settled, we ought to admit that our conception of nature as absolutely real is only partially correct. Still, we shall have to be governed by it practically; because there is nothing to distinguish the unanswerable questions from the answerable ones, so that investigation will have to proceed as if all were answerable. In ordinary life, no matter how much we believe in questions ultimately getting answered, we shall always put aside an innumerable throng of them as beyond our powers. We shall not in our day seek to know whether the centre of the sun is distant from that of the earth by an odd or an even number of miles on the average; we shall act as if neither man nor God could ever ascertain it. There is, however, an economy of thought, in assuming that it is an answerable question. From this practical and economical point of view, it really makes no difference whether or not all questions are actually answered, by man or by God, so long as we are satisfied that investigation has a universal tendency toward the settlement of opinion; and this I conceive to be the position of Thrasymachus.

If there be any advantage to religion in supposing God to be omniscient, this sort of scepticism about reality can do no practical harm. We can still suppose that He knows all that there is of reality to be known. On the theory of Dr. Royce, the real existence of God would consist in his imagining or positing Himself; it would thus be, according to him, of the same nature as the reality of anything else. For my part, I hold another theory, which I intend to take an early opportunity of putting into print. I think that the existence of God, as well as we can conceive of it, consists in this, that a tendency toward ends is so necessary a constituent of the universe that the mere action of chance upon innumerable atoms has an inevitable teleological result. One of the ends so brought about is the development of intelligence and of knowledge; and therefore I should say that God's omniscience, humanly conceived, consists in the fact that knowledge in its development leaves no question unanswered. The scepticism just spoken of would admit this omniscience as a regulative but not a speculative conception. I believe that even that view is more religiously fruitful than the opinion of Dr. Royce.

Let us now turn to the examination of Dr. Royce's peculiar method of reasoning; for that is always the most important element in every system of philosophy. His work is divided into a brief introduction and two books, the first entitled "The search for a moral ideal"; the second "The search for a religious truth." These titles seem to me to point, at the outset, to a fault of method. The pursuit of a conscience, if one hasn't one already, or of a religion, which is the subjective basis of conscience, seems to me an aimless and hypochondriac pursuit. If a man finds himself under no sense of obligation, let him congratulate himself. For such a man to hanker after a bondage to conscience, is as if a man with a good digestion should cast about for a regimen of food. A conscience, too, is not a theorem or a piece of information which may be acquired by reading a book; it must be bred in a man from infancy or it will be a poor imitation of the genuine article. If a man has a conscience, it may be an article of faith with him that he should reflect upon that conscience, and thus it may receive a further development. But it never will do him the least good to get up a make-believe scepticism and pretend to himself not to believe what he really does believe. In point of fact, every man born and reared in a christian community, however little he may believe the dogmas of the Church, does find himself believing with the strongest conviction in the moral code of christendom. He has a horror of murder and incest, a disapproval of lying, etc., which he cannot escape from. The modern dialectician (if he will pardon a touch of exaggeration) would have such a man say to himself, Now I am going to be sceptical, but only provisionally so, in order to return to my faith with renewed conviction! But the whole history of thought shows that men cannot doubt at pleasure or merely because they find they have no positive reason for the belief they already hold. Reasons concern the man who is coming to believe, not the man who believes already. It has often been remarked that metaphysics is an imitation of mathematics; and it may be added that the philosophic doubt is an imitation of the absurd proceedure of elementary geometry, which begins by giving worthless demonstrations of propositions nobody ever questions. When Hegel tells me that thought has three stages, that of naïve acceptance, that of reaction and criticism, and that of rational conviction; in a general sense, I agree to it. And a down-right living scepticism, without *arrière-pensée,* may be beneficial. It is not perhaps easy to see why an imaginary scepticism might not sometimes serve the same purpose; but experience shows that in questions of magnitude men haven't imagination enough to put themselves in the true doubter's shoes. But be that as it may, the idea that the mere reaction of assent and doubt, the mere play of thought, the heat-lightning of the brain, is going to settle anything in this real world to which we appertain,—such an idea only shows again how the Hegelians overlook the facts of volitional action & reaction in the development of thought. I find myself in a world of forces which act upon me, and it is they and not the logical transformations of my thought which determine what I shall ultimately believe.

Dr. Royce seems to hold that at least in the philosophy of morals and religion a mere contemplation of our own crude beliefs will lead us to absolute scepticism and that then a mere contemplation of our own absolute scepticism will lead us back to rational conviction. Neither I nor the readers of the *Popular Science Monthly* can possibly believe that, in advance. But let us see how the method will work when applied to the discussion of ethics.

The moral stand-point from which every man with a christian training sets out, even if he be a dogmatic atheist, is pretty nearly the same. He has a horror of certain crimes and a disapproval of certain lesser sins. He is also more or less touched with the spirit of christian love, which he believes should be his beacon, and which in point of fact, by its power in his heart, shall and will govern him in all questions of disputed morals. More or less, in all of us, this sentiment replaces and abolishes conscience; like Huckleberry Finn, we act from christian charity without caring very much whether conscience approves of the act or not.

This is the state of mind of the ordinary man or woman who will open Dr. Royce's book. And now Dr. Royce proposes that this person shall ask himself the question what validity or truth is there in the distinction of right and wrong. To me, it plainly appears that such a person, if he have a clear head, will at once reply, right and wrong are nothing to me except so far as they are connected with certain rules of living by which I am enabled to satisfy a real impulse which works in my heart; and this impulse is the love of my neighbor elevated into a love of an ideal and divine humanity which I identify with the providence that governs the world. But Dr. Royce says that different people will answer the question in different ways; some will take the position of the "moral realist" and say that moral distinctions are founded on some matter of fact (say a decree from Sinai), while others will take the position of the "moral idealist" and say that these distinctions are founded on an inward sentiment,—an ideal. Two such persons come into collision; they find by mutual criticism that both positions are unsatisfactory; external fact can only determine what *is*, not what *ought* to be; while inward sentiment cannot be a resting-place, because it is only individual caprice and has no authority for another man. From this criticism the only outcome is ethical scepticism.

This is a fair specimen of Dr. Royce's logical method, which is a mere apotheosis of the dilemma, as the great instrument of thought. As compared with syllogistic method of the middle ages (which survives in certain quarters, yet) it is certainly wonderfully superior; but as compared with the mathematical reasoning upon which modern science is built, it is inefficacious and restricted.

In the particular case in hand, it appears to me that the ordinary christian does not find himself caught in Dr. Royce's dilemma at all. He is a moral idealist; yet far from being shaken by the spectacle of different men having different passions, he feels that every man may come to the same passion which animates him by a mere enlargement of his horizon, and that his is the only sentiment in which all others may be reconciled. For altruism is but a developed egoism; that same sensitiveness which in its lowest state is selfishness, first transforms itself into *esprit de corps* or collective selfishness; then, passing from feeling for others collectively to feeling for them individually, it becomes philanthropy, pity, sympathy tossed hither and thither rudderless on the ocean of human misery; finally, steadying itself by the conception of an ideal humanity and a divine providence, it passes into christian charity, which gathers up all selfishnesses and all pities and is ready to give each its due measure.

The author having stated the above argument with admirable clearness, fills a hundred pages with a perhaps not altogether necessary, though a charmingly written and highly interesting elaboration and illustration of it. He here passes in review a goodly number of the ethical theories which have been proposed at different times. After the Sophists, Plato, Aristotle, and the Stoics, he criticizes what he conceives to be the ethics of Jesus. Every christian will tell him that he makes the mistake of viewing that as a *theory* or speculation which is really a spiritual *experience;*—another example of his neglect of the volitional element. For instance, he asks, "If I feel not the love of God, how prove to me that I ought to feel it?" The answer to that need not be pointed out.

In what he says about Herbert Spencer, he seems to forget that Mr. Spencer is not addressing a body of moral sceptics but readers animated by the sentiments which, in our day, animate every man who reads at all.

At last he takes up the thread of his argument as follows. The conflict between moral realism and idealism can only lead to moral scepticism. Now what is this scepticism? It is the contemplation of two opposing aims. Here he adduces the testimony of modern psychologists to show that we cannot think of willing without actually willing. (But for all that, I fancy I notice a difference sometimes in cold weather between thinking of willing to take my morning dash of cold water and actually willing to take it.) Scepticism, then, shares at once these opposing aims, or strives to share them. It has thus itself an aim, namely, to reconcile opposing aims. So absolute moral scepticism is self-destructive. "Possibly this result may be somewhat unexpected," says our author. Not at all unexpected to one who does not believe in the dialectical method. You started with a hypochondriac hankering after an aim; and now you have acquired it. *Eurekas!* Well, what is it, this aim which you have at last got? Why, to have an aim! But that is nothing but the old nonsensical longing with which you set out. Like Kant's dove, you have been winging a vacuum, without remarking that you never advanced an inch. I do not misrepresent the author. "For behold," he says, "made practical, brought down from its lonesome height, my Ideal very simply means the Will to direct my acts *towards* the attainment of universal Harmony." But this, I must insist, was obviously implied in the original fantastic desire to have an aim. When I say that this is a fantastic desire, I do not of course mean to deny that there may be such an operation as the *choice of an aim,* if by that aim be meant a secondary or derived one; but I do say that it is absurd to speak of choosing an original and ultimate aim. That is something which if you haven't it, you have nothing to do but wait till the grace of God confers it on you. I should think, however, that were it once admitted to be a rational performance to go a-hunting for an ultimate aim or end, the first preliminary would be to recognize the axiom that such an end must have unity, after which the hunt might begin. But Royce, calling this axiom the "ideal of ideals," as it certainly is, in a sense, exclaims "Here I have the aim I wanted, and the hunt is over." If one might be permitted to enliven a dry subject with a little folly, I should say that it reminded me of the surveyor Phoenix, who after purchasing 365 solar compasses and a vast amount of other paraphernalia, in order to ascertain the distance between San Francisco and the Mission of Dolores, stepped into a grocery and inquired how far it was, and returned "much pleased at so easily acquiring so much valuable information." If Dr. Royce merely means that it can be shown that a man who fancies he has no moral ideal really has one, I heartily grant it; and I will further admit that dialectic is the proper instrument to show this. But then a very lowly kind of dialectic will do; and a rather more definite ideal may be pointed out.

The rest of Book I is occupied, as it seems to me, with illicitly slipping some content into an empty formula. Much of this part of the book is splendidly said. But other passages seem to me to preach, in a way quite uncalled for by the premises, an ethics of the evil eye. "It is well that we should feel . . . joy whenever pride has a fall. . . . In all such ways . . . we must show *no mercy."* "When the hedonist gives us his picture of a peaceful society, where, in the midst of good humor, his ideal, the happiness of everybody concerned, is steadfastly pursued, we find ourselves disappointed and contemptuous. . . . Who cares whether that wretched set . . . think themselves happy or not?" "The appearance of anybody who pretends to be content with himself must be the signal not for admiration at the sight of his success, but for a good deal of contempt." etc. Some of the students to whom this ethics is taught at Harvard may upon reflection think that christian charity is not so much lower a frame of mind after all.

In Book II Dr. Royce undertakes by the same dialectic procedure to establish the existence of a God. [S]pace does not permit me to enter into a criticism of the second book; nor is it necessary, for it consists only of an application of the same method to a subject to which dialectics is far less suited. Besides, to the reader who has had the kindness and the resolution to follow me to this point I can say, "You are the man to enjoy Dr. Royce's own book, which I can promise you you shall find, in comparison with harsh and crabbed matter you have been reading here, to be 'as musical as is Apollo's lute.' "

[Notes on the Categories]

Item 34 *MS 545: Summer–Fall 1885*

[Segment 1]

[. . .] or disjunctive, and either problematical, assertorical or apodictic. Then he reasoned: since these distinctions are essential in logic and thus implied in all reasoning, the conceptions they employ must be involved in the constitution of the mind, or at least be prior to reasoning, and cannot have been learned by the aid of reasoning from the observation of nature. Kant's list of categories is no longer believed in, and I shall not defend it. Indeed, it is hard to understand how he could have persuaded himself that the slight examination he made of his so-called "Functions of Judgment" was sufficient to establish them as the essential distinctions of formal logic. But still, Kant's general method of determining the categories was well chosen. If the distinctions of judgments in his list had really been as essential in logic as he thought, so that no reasoning could be carried on without them, I think it must be admitted that the conceptions entering into these distinctions would have been truly proved to have, in some sense, an *à priori* character, to be distinguished from other conceptions in their origin, at least comparatively, and to be at all events of exceptional importance, whatever role they might be found to play in other branches of philosophy.

A certain precaution is necessary in using Kant's method owing to the circumstance that the categories do not present themselves in formal logic in their simplicity and direct fulness. They are there used in a formal way, with an understanding that they may not be meant. Thus, the relation of subject and predicate is undoubtedly the relation of substance and accident; yet we do not generally mean that the subject and predicate are really in that relation. It is merely a convenient fiction; as when we say, "Mercy is an attribute to God." So the relation of antecedent and consequent is the relation of cause and effect; but by a convenient fiction we say "If there is smoke, there must be fire," thus stating the effect as if it were cause. So the whole series of abstract nouns are fictions representing characters as if they were things. We must be careful therefore to restore to the conceptions their primitive objective senses before ranking them as categories.

It is remarkable that although the system of formal logic upon which Kant founded his list of categories was extremely imperfect, yet his categories themselves are at least highly important conceptions. They form four triads, and each triad involves the conceptions of first, second, and third. This shows that even when the method is not followed out with perfect accuracy, its results are not utterly without value. If in place of distinctions absolutely essential in logic we take those which approach that character, the categories obtained though not quite *à priori* will be highly primitive conceptions.[1]

These three conceptions are involved in the notions of First, Second, Third. The question whether there be other fundamental categories forming distinct sets, will be passed over for the present. It is easy to see that no other conceptions can be thought separated from these. But just as in Kant's categories, these three conceptions are mixed with those of Quantity, quality, relation, and modality; so it may be that there is another order of conceptions equally essential

1. It must be said, however, that Kant's categories do not always correspond with the functions of judgment. Under quantity, the functions of judgment are Universal, particular, singular; while the categories are totality, plurality, unity. These categories are nearly mine; for plurality supposes *two* but not necessarily three, while totality involves synthesis or the bringing together of two at least under a third. But the particular judgment does not involve the idea of plurality at all; it differs from the universal in stating that a given description of object is found while the universal states that a given description of object is not found in the universe. The latter contains the idea of *Law,* the former that of spontaneous occurrence or Chance. The kind of judgment which really makes a triad by mediating between these two is the statistical judgment which contains the idea of chance approaching to law. As for the singular judgment it does not introduce the idea of a universe at all, and the conception which it employs is not that of unity but that of individuality. Under quality, the kinds of judgment given by Kant are the affirmative, negative, and infinitated. But the judgment which really mediates between the affirmative and negative is the probable judgment. Kant's categories of quality are reality, negation, and degree. Degree corresponds better with the probable than with the infinitated judgment, but the other two which make a triad with this are the absolute minimum and maximum of degree, and reality is a poor name for the latter.

in formal logic and perhaps equally important in other branches of philosophy.

The assurance against error afforded by Kant's method of investigating the categories has not been appreciated. This is natural because he himself applied it so //badly/ill//. Hegel thought there was no need of studying the categories through the medium of formal logic and professed to evoke them by means of their own organic connections. Certainly, if it be true that *first, second, third* are the three categories, it could hardly be that the first three conceptions called up by Hegel's method should fail of containing at least these elements. But there is nothing in Hegel's method to guard against mistakes, confusions, misconceptions; and the list of categories given by him has the coherence of a dream. The method of Kant, properly carried out, would require, first, the invention of a perfectly exact, systematic, and analytic language in which all reasoning could be expressed and be reduced to formal rules; and, second, the analysis of the signs of that language so as to make a table of all their varieties. This project is certainly difficult to carry out, and it will perhaps be impossible to be ever assured that it has been brought to completion. Still, we can easily satisfy ourselves by the working of the logical calculus, that the work done is right so far as it goes; and such positive conclusions as we may reach will be beyond *[. . .]*

*[*Segment 2*]*

Now modern logic enables us to show that three conceptions are really essential in formal logic; so that they are three fundamental categories of thought. Furthermore, reasons can be given for holding that these three conceptions are due to the three fundamental faculties of the mind, these again to three fundamental functions of the nerves; and finally these to three elementary constituents of the physical universe. To set forth all this is the object of the present contribution.

I cannot impart to the reader a correct apprehension of the new categories; that he must achieve by his own efforts. Yet I hope that by the time he has read this paper he will have come to understand them as well as I do myself, perhaps better. At the outset, such a description of these conceptions as I can give may be of some aid. Perhaps it is not right to call these categories conceptions; they are so intangible that they are rather tones or tints upon conceptions. In my first attempt to deal with them,[2] I made use of three grades of separability of one idea from another. In the first place, two ideas may be so little allied that one of them may be present to the consciousness in an image which does not contain the other at all; in this way we can imagine *Red* without imagining Blue, and *vice versa;* we can also imagine Sound without Melody, but not Melody without Sound. I call this kind of separation *Dissociation.* In the second place, even in cases where two conceptions cannot be separated in the imagination, we can often suppose one without the other, that is we can imagine data from which we should be led to believe in a state of things where one was separated from the other. Thus, we can suppose uncolored space, though we cannot dissociate space from color. I call this mode of separation Prescission. In the third place, even when one element cannot even be supposed without another, they may ofttimes be distinguished from one another. Thus we can neither imagine nor suppose a Taller without a Shorter, yet we can distinguish the taller from the shorter. I call this mode of separation *Distinction.* Now, the categories cannot be dissociated in imagination from each other, nor from other ideas. The category of First can be prescinded from Second and Third, and Second can be prescinded from Third. But Second cannot be prescinded from First nor Third from Second. The categories may, I believe, be prescinded from any other one conception, but they cannot be prescinded from some one and indeed many elements. You cannot suppose a First unless that first be something definite and more or less definitely supposed. Finally, though it is easy to distinguish the three categories from one another, it is extremely difficult accurately and sharply to distinguish each from other conceptions so as to hold it in its purity and yet in its full meaning. The category of First ought to be the easiest to seize: it is the mode of thought of a child. Yet I believe that to an educated man, it is the most difficult of the three. The word First fails to express it, for this word suggests the question of a Second. Imagine something definite, which is quite new and fresh, yet not aggressively so; something unlike and unrelated to a second, not merely to your thought, but in itself; something without genesis, flourishing in spontaneous and pristine freedom. Think what should have been the first idea in the consciousness of Adam at the moment

2. "On a New List of Categories." *Proceedings Am. Academy of Arts & Sciences,* May 1867.

of his creation, when he found a lively feeling which had not become defined. Remember, however, that every description of the First must be false. I use the word *free,* for example, to describe it. But it is not a freedom that can be asserted, but a freedom that is neither questioned nor thought of that it possesses. This is nonsense, I grant; but it gives a hint of the conception, which the reader must seize for himself. The greatest fault of the description, after all, is that it conveys the suggestion that the conception is to be attained by straining; whereas the whole difficulty of seeing it is that it lies too immediately in the foreground, beyond the margin of any representation.

The category of Second has been so prominent in the thought of our age, that it is decidedly the easiest to isolate. A definite object or event whose existence is dependent upon something else, something determined by limitation, in real relation to something, a clash, a constraint, an effort, a force, an end,—these are familiar experiences involving the Second or correlate. This mode of thought is absolutely opposed to the First; it says that the first is but a reflex second.

Third; medium; representation; synthesis (and analysis); resultant; absorption; evolution.

The Third is the medium between the first and the second, the beginning and the end. It is what actually is, while the First and the Second are merely its limits. The first and second are hard and solid; that is why they are easy to understand. The Third flows from one to the other. It is the representation by which anything is carried out of itself into another. It is that which brings about the pair by establishing a relationship. It is the synthesis (and equally analysis or reversed synthesis). It is the whole which absorbs the parts and makes them belong to one another by belonging to it. It feeds; and it reproduces. It is the whole process by which from the first is evolved the second.

*[*Segment 3*]*

I may pass briefly over the application of the Categories in Formal Logic, since I have treated of this in other works. They appear in the three genera of logical forms, the Term, Proposition, and Argumentation. The most rudimentary kind of inference is exemplified by the *Cogito, ergo sum.* If this be an inference at all, it must recognize itself as proceeding according to some rule,—for that is precisely what the *ergo* implies,—and this rule may be stated as "Si cogito, sum." Thus the inference is brought to the form of a *Consequentia* or Modus ponens.

Si cogito, sum.
Cogito.
Ergo, sum.

Here there are three propositions. *Cogito* is the fact originally observed and *quasi* free, that is, unnecessitated, undeduced; therefore the *first.* Sum is the fact that follows, that is necessitated, the *Second.* Si cogito sum is the fact which brings the other two into relation, the medium or *Third.* Moreover, three kinds of words or signs are necessary in stating this argument. First, signs for single facts, *cogito* and *sum;* Second, a sign having the force of the word *si,* which establishes a relation between *two* facts; and Third a sign having the force of the word *Ergo,* which connects a conclusion with a premise or premises and a rule of inference and so connects three propositions, at least. Here, then, the three categories reappear.

*[*Segment 4*]*

Logic as ordinarily and conveniently presented is full of *dichotomies* or dual distinctions. Some of these are true; but most of them arise from rough and inexact conceptions. Thus, it is said that every proposition is either true or false. This may be so about such propositions as $5 + 3 = 7$ and $5 + 3 = 8$; but these seem to be mere formulae without real content. But propositions about real things and events can never be freed from ambiguity. We say that the mean distance of the sun from the earth is so many miles. But a mile is a certain number of yards; and a yard is the distance between two lines traced on a certain bar. These lines have not absolutely sharp edges. Besides the distance is undoubtedly variable even at a given temperature. Besides that, the temperature is not, and perhaps cannot be, exactly defined. Then, we must take a *mean* distance. But the mean may not in its nature be susceptible of exact definition. In like manner the mean distance of the earth from the sun may be somewhat vague and variable. If then the margin of uncertainty of the defining distance expressed in miles wholly includes the margin of uncertainty of the distance defined, that of the sun from the earth, the statement may be called true,—and if the first margin wholly excludes the second, the statement is false. But if the first margin partly includes and partly excludes the second, the proposition is in a predicament intermediate between truth and falsehood. Propositions are again distinguished as affirmative and negative; but intermediate between the two are statements of probability. Thus we have the forms

Affirmative. The probability of P is *unity.*
Probable. The probability of P is p.
Negative. The probability of P is *zero.*

It may be asked why unity and zero should be separated from other grades of probability. The answer is that the probability of a proposition may change in consequence of information and reasoning; from being indefinitely near to *zero* it can change to being indefinitely near to *unity,* and the reverse; but no information can ever render a probable proposition absolutely certain, nor a certain proposition uncertain. Again, propositions are divided into the universal and the particular; but intermediate between these are the statistical. We have the forms

Universal. Any S is P.
Statistical. The proportion ρ of the S's are P's.
Particular. Some S is P.

Ordinary logic is the logic of absolute assertion, hard and changeless. That is why it altogether overlooks one of the important distinctions of propositions. Namely, a proposition may in the first place state something as a fact freely chosen or taken at hazard, within stated limits. In the second place a proposition may be stated as a result brought about by the circumstances. In the third place a proposition may be stated as the governing rule of an experience by which a fact freely chosen is made to give birth to a result.

One, Two, Three: Fundamental Categories of Thought and of Nature

Item 35 — *MS 546: Summer–Fall 1885*

Kant, the king of modern thought, it was who first remarked the frequency in logical analytics of *trichotomies* or three-fold distinctions. It really is so; I have tried hard and long to persuade myself that it is only fanciful, but the facts will not countenance that way of disposing of the phenomenon. Take any ordinary syllogism:

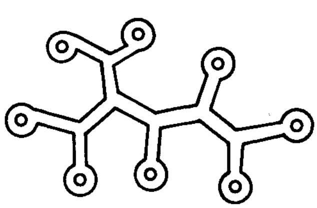
All men are mortal,
Elijah was a man;
Therefore, Elijah was mortal.

There are here three propositions, namely, two premises and a conclusion; there are also three terms, *man, mortal,* and *Elijah.* If we transpose one of the premises with the conclusion, denying both, we obtain what are called the indirect figures of syllogism; for example

All men are mortal,
But Elijah was not mortal;
Therefore, Elijah was not a man.

Elijah was not mortal,
But Elijah was a man;
Therefore, some men are not mortal.

Thus, there are three figures of ordinary syllogism. It is true there are other modes of inference which do not come under any of these heads; but that does not annul the fact that we have here a trichotomy. Indeed, if we examine by itself what is by some logicians called the fourth figure we find that it also has three varieties related to one another as the three figures of ordinary syllogism. There is an entirely different way of conceiving the relations of the figures of syllogism; namely by means of the conversion of propositions. But from that point of view also the same classes are preserved. De Morgan has added a large number of new syllogistic moods which do not find places in this classification. The reasoning in these is of a peculiar character and introduces the principle of dilemma. Still, regarding these dilemmatic reasonings by themselves, they fall into three classes in a precisely similar manner. Again, I have shown that the probable and approximate inferences of science must be classified on the very same principles, being either Deductions, Inductions, or Hypotheses. Other examples of threes in logic are statements of what is actual, what is possible, and what is necessary; the three kinds of forms, Names, Propositions, and Inferences; affirmative, negative, and uncertain answers to a question. One very important triad is this: it has been found that there are three kinds of signs which are all indispensible in all reasoning; the first is the diagrammatic sign or *icon,* which exhibits a similarity or analogy to the subject of discourse; the second is the *index,* which like a pronoun demonstrative or relative, forces the attention to the particular object intended without describing it; the third is the general name or description which signifies its object by means of an association of ideas or habitual connection between the name and the character signified. But there is one triad in particular which throws a strong light on the nature of all the others. Namely, we find it necessary to recognize in logic three kinds of characters, three kinds of facts. First there are *singular* characters which are predicable of single objects, as when we say that anything is white, large, etc. Secondly, there are dual characters which appertain to pairs of objects; these are implied by all relative terms as "lover," "similar," "other," etc. Thirdly, there are plural characters, which can all be reduced to triple characters but not to dual characters. Thus, we cannot express the fact that *A* is a benefactor of *B* by any descriptions of *A* and *B* separately; we must introduce a relative term. This is requisite, not merely in English, but in every language which might be invented. This is true even of such a fact as *A* is taller than *B.* If we say "*A* is tall, but *B* is short," the conjugation "but" has a relative force, and if we omit this word the mere collocation of the two sentences is a relative or dual mode of signifying. We must, however, recognize two kinds of relative characters, those which can be implied in such a mere collocation and those, like *A*'s benefitting *B*, which cannot. The mathematicians say that two straight lines constitute a *degenerate* conic. To borrow this expression we may say that the mere collocation of two facts about *A* and *B* constitutes a degenerate relation. In this case, the two facts are seen by the mind to be in relation to one another. As the old logicians said, relations are divisible into real relations or relations of reason. Let us now consider a triple character, say that *A* gives *B* to *C*. This is not a mere congeries of dual characters. It is not enough to say that *A* parts with *C*, and that *B* receives *C*. A synthesis of these two facts must be made to bring them into a single fact; we must express that *C* in being parted with by *A* is received by *B*. If, on the other hand, we take a quadruple fact, it is easy to express as a compound of two triple facts. Thus, suppose *A* gives *B* to *C* in exchange for *D*. Then, if we take the word "sale" to include the parting with a thing in exchange not merely for money but for any object in barter, we may express the fact in question thus: "*A* makes a sale, *E*, to *C*, and *E* is a sale of *B* in exchange for *D*." We are here able to express the synthesis of the two facts into one, because a triple character involves the conception of synthesis. Analysis involves the same relations as synthesis; so that we may explain the fact that all plural facts can be reduced to triple facts in this way. A road with a fork in it is the analogue of a triple fact, because it brings three termini into relation with one another. A dual fact is like a road without a fork; it only connects two termini. Now, no combination of roads without forks can have more than two termini; but any number of termini can be connected by roads which nowhere have a knot of more than three ways. See the figure, where I have drawn the termini as self-returning roads, in order to introduce nothing beyond the road itself. Thus, the three essential elements of a network of roads are *road about a terminus*, *roadway-connection*, and *branching;* and in like manner, the three fundamental categories of fact are, fact about an object, fact about two objects (relation), fact about several objects (synthetic fact).



We have seen that the mere coexistence of two singular facts constitutes a degenerate form of dual fact; and in like manner there are two orders of degeneracy in plural facts, for either they may consist in a mere synthesis of facts of which the highest is dual, or they may consist in a mere synthesis of singular facts. This explains why there should be three classes of *signs;* for there is a triple connection of *sign, thing signified, cognition produced in the mind.* There may be a mere relation of reason between the sign and the thing signified; in that case the sign is an *icon.* Or there may be a direct physical connection; in that case, the sign is an *index.* Or there may be a relation which consists in the fact that the mind associates the sign with its object; in that case the sign is a *name.* Now consider the difference between a logical *term,* a *proposition,* and an *inference.* A term is a mere general description, and as neither *icon* nor *index* possess generality, it must be a name; and it is nothing more. A proposition is also a general description, but it differs from a term in that it purports to be in a real relation to the fact, to be really determined by it; thus, a proposition can only be formed of the conjunction of a name and an index. An inference, too, contains a general description and purports to really represent the fact; but it does more, it professes to give in its premises such a representation of the fact that by the contemplation of them something else may be learned about the thing; this it can only do by presenting an analogue or *icon* of the fact; and thus we see why all three kinds of signs are requisite for inference.

By such sort of synthesis, the whole organism of logic may be mentally evolved from the three conceptions of first, second, and third, or more precisely, An, Other, Medium.

But if these three conceptions enter as we find they do as elements of all conceptions connected with reasoning, they must be virtually in the mind when reasoning first commences. In that sense, at least, they must be innate ideas; and consequently they must be capable of explanation, psychologically;—there must be in the consciousness three faculties corresponding to these three categories of logic. Now, we know that the introspective psychologists have for a long time told us that the mind has three fundamental faculties, the feeling of pleasure and pain, volition and desire, and cognition. Let us see, then, if we cannot modify this statement so that without being less true to the phenomena of introspective consciousness,—even more true, if possible,—it may be put into a form in which it will serve as the psychological explanation, which must surely lie somewhere, of the new facts which we bring from the study of logic.

The ordinary doctrine is open to a variety of objections from the very point of view from which it was first delineated. 1^{st}. Desire certainly includes an element of pleasure quite as much as of will. Wishing is not willing; it is a speculative variation of willing mingled with a speculative and anticipatory feeling of pleasure. Desire should therefore be struck out of the definition of the second faculty leaving it mere volition. But volition without desire is not voluntary; it is mere activity. Consequently, all activity, voluntary or not, should be brought under the second faculty. Thus attention is a kind of activity which is sometimes voluntary and sometimes not so. 2^{nd}. Pleasure and pain can only be recognized as such in a judgment; they are general predicates which are attached to feelings rather than true feelings. But mere passive feeling, which does not act and does not judge, which has all sorts of qualities but does not itself recognize these qualities, because it does not analyze nor compare,—this is an element of all consciousness to which a distinct title ought to be given. 3^{rd}. Every phenomenon of our mental life is more or less like cognition. Every emotion, every burst of passion, every exercise of will is like cognition. But modifications of consciousness which are alike have some element in common. Cognition, therefore, has nothing distinctive and cannot be regarded as a fundamental faculty. If, however, we ask whether there be not an element in cognition which is neither feeling, sense, nor activity, we do find something; the faculty of learning, acquisition, memory-and-inference, synthesis. 4^{th}. Looking once more at activity, we observe that the only consciousness we have of it is the sense of resistance. We are conscious of hitting or of getting hit, of meeting with a *fact*. But whether the activity is within or without we know only by secondary signs and not by our original faculty of recognizing fact.

It seems, then, that the true categories of consciousness are, 1^{st}, Feeling, the consciousness which can be included with an instant of time, passive consciousness of quality, without recognition or analysis; 2^{nd}, Consciousness of an interruption into the field of consciousness, sense of resistance, of an external fact, of another something; 3^{rd}, synthetic consciousness binding time together, sense of learning, thought.

If we accept these as the fundamental elementary modes of consciousness, they afford a psychological explanation of the three logical conceptions of quality, relation, and synthesis or mediation. The conception of quality, which is absolutely simple in itself and yet viewed in its relations is seen to be full of variety, would arise whenever feeling or the singular consciousness becomes prominent. The conception of relation comes from the dual consciousness or sense of action and reaction. The conception of mediation springs out of the plural consciousness or sense of learning.

Three such fundamental elements of consciousness must be capable of a physiological explanation from three fundamental properties of the nervous system. Physiologists have not, it is true, reduced the properties of the nerves to three. Consequently, we are obliged to leave the solid basis of ascertained fact, and guess what those properties may hereafter perhaps be ascertained to consist in. We know, however, that the contents of nerve-cells may be set into a state in which they are disengaging energy, and there can be no doubt that this is what is going on when we have a feeling. We, also, know that discharges of nervous energy take place, both onto nerve-cells and onto muscular and other cells exterior to the nervous system; and it seems quite reasonable to believe that the sense of shock,—the dual consciousness,—is due to these nerve-currents. It is more difficult to explain the synthetic consciousness. Looking upon it as a sense of learning, and remembering that nerves have the capacity of acquiring habits, we are inclined to say that this is the property of the nerves which we are in quest of. But physiologists do not usually consider habit as one of the primordial properties of the nerves; and on the other hand there is such a property, the most important of all, namely the power of nutrition, growth, development. Without knowing exactly how a nerve-cell grows, it is difficult to make sure that the faculty of habit is an incident to its faculty of growth. It would seem, however, very natural to suppose, that the sense of learning is connected with the process of recuperation of energy within a cell, and the storing of it up to be expended when the moment of excitation comes.

We have here, at any rate, a reasonable hypothesis, affording a possible physiological explanation of the different elements of consciousness; and physiology will soon be able either to confirm or to refute it. I would like, if the reader will listen to such rashness, to push speculation one step further yet. I cannot easily believe that life is so special a phenomenon as to depend upon certain secondary contingencies in nature. Although a cell appears to be a very particular sort of arrangement; I cannot help guessing that it may contain all the fundamental elements of the uni-*[. . .]*

[Measurement Scales and the Absolute]

Item 36 *MS 547: Summer–Fall 1885*

[. . .] scale. Colors, too, may be marked upon a scale; they are frequently expressed by the wavelengths of the parts of the spectrum which most nearly match them. In such a scale of numbers, all that is absolutely essential is that each possible variety of the quality measured shall have a single number attached to it and that each number shall be attached to a single variety of the quality. If then we consider two different scales for the measurement of the same quality, each number of either must correspond to a single number of the other, and nothing more is necessary. There will be some rule for the conversion of one scale into the other, and that rule may be expressed as an algebraical equation between x, the grade of any variety of quality on the first scale, and y, the grade of the same quality on the other scale. If in this equation we assign any particular value to x, the equation will show the single value of y that corresponds to it. It must, therefore, be an equation of the first degree in y, because it is only equations of the first degree that always yield one single solution. For the same reason, the equation must also be of the first degree in x. It must, then, be of the form

$$axy + bx + cy + d = 0,$$

where a, b, c, d are any values whatever. Now we notice that there can at most be but two qualities for which the grade on one scale is the same as the grade on the other scale; because if we put $x = y$, which equation expresses that the grades on the two scales are the same, the equation just written reduces to

$$ax^2 + (b + c)x + d = 0$$

which is an equation of the second degree and therefore has two roots, which may however be equal or imaginary.

Let us now suppose an infinite series of scales, such that any one of them is connected with the first by an equation

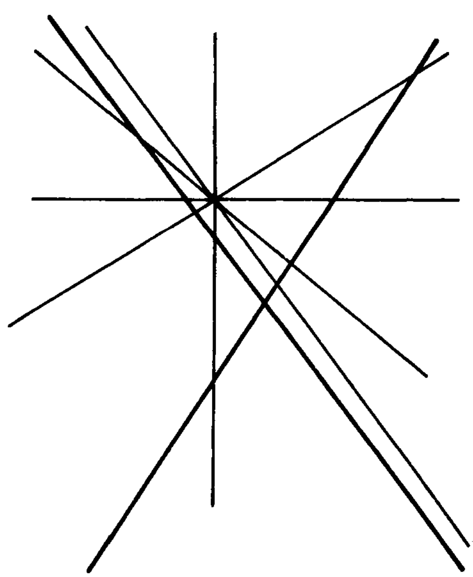
$$axz + b'x + c'z + d = 0$$

where z is the grade on one of these scales of the quality which has the grade x on the first, and where

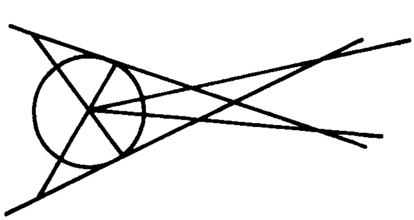
$$b' + c' = b + c.$$

Then all of this infinite series of scales will represent two certain qualities by the same pair of numbers (real, imaginary, or coincident). This pair of qualities is called the Absolute for this series of scales of measurement.

All such series of scales are of three kinds, according as the absolute consists of two distinct points, only one point (the two coinciding), or is fictitious (two distinct imaginary points). As an example of the first kind we have the measurement of the probability of our knowledge. Here, the *zero* of the scale represents certainty that an event will not happen, *unity* certainty that it will happen, the intermediate numbers various degrees of uncertainty. Now as our knowledge increases, these intermediate numbers change, but whatever is once absolutely certain remains so forever, and nothing which is uncertain can ever



become absolutely certain.[1] Here, the absolute has two distinct moments; absolute certainty against and absolute certainty for a proposition. An example of the second kind is obtained in the following way. Let AB and CD be two straight lines in a plane crossing at X. Let the eye of an observer be stationed at E and let him make a perspective picture on CD of the points on AB. That is to say, draw lines from E intersecting both AB and CD and then regard the point of intersection of each of these lines with CD as the picture of its point of intersection with AB. If now E is shifted to any other point on the line through E and X the positions of all these pictures or projections will be shifted with the single exception of X which must of course always be its own projection, and is *absolutely* fixed. There is one thing to be noticed about this example, which is familiar to all who are acquainted with the principles of perspective. It is this. Suppose that we draw through E a line parallel to AB. This will of course intersect with CD in a single point but it will not intersect AB at all. The intersection on CD is conceived by mathematicians, as well as by artists, to be the picture or projection of all the points on AB at an infinite distance, so that from this point of view there appears to be but a single point on AB at infinity, and this point is no more at one end than at the other. This remark enables us to understand how linear measurement affords another example of a single absolute. Suppose we have an infinitely long line upon which we can measure by means of an infinitely long scale, which we can shove backwards and forwards as far as we please along this line. Then in any two different positions of the scale any point on the fixed line comes against two different points on the scale, except only the single point at infinity which is always at infinity on the scale, and is therefore the single absolute of linear measurement. The third case may be illustrated by the following construction. Imagine a circle with two tangents, one fixed, the other capable of rolling round the circle. Let the centre of the circle be taken as the centre of projection. Then the projection



1. This case probably does not come strictly within the above formula, yet it evidently represents a generalization of that formula; and would be a valuable conception, if put into a precise form.

upon the movable tangent of any point whatever on the fixed one will be at different points on the former tangent for all different positions of that tangent. In this there is no absolute; or rather, as mathematicians say, it consists of two imaginary points. It was the great mathematician Arthur Cayley who first gave the name of the Absolute to this mathematical conception, which it is easy to see presents more than an analogy with the metaphysical absolute. If the universe never advances or developes, at all, or if there is merely an alternate flow and ebb of progress then there is no absolute in creation. If, as the pessimists maintain, the character of its growth is such that it tends in the infinite future to come out at the same non-entity from which it has been proceeding since an infinite past, that infinitely distant state of *Nirwana* is the single absolute. But if, as evolutionists believe, it has been since infinity leaving a state of absolute homogeneity and is tending forever toward another different state of absolute organized heterogeneity, there is a double absolute.

Types of Third Degenerate in the Second Degree

Item 37 *MS 548: Summer–Fall 1885*

A	3 separate things in comparison
B	3 " " two resembling
C	3 " " all resembling
D	2 separate things one related to self & compared
E	2 " " " " " " " resembling
F	2 separate things compared & character of one
G	2 " " resembling " " " "
H	1 thing in double relation to itself
I	1 thing related to self and the relation
J	1 " " " " " its character
K	1 thing its character and relation of possession.

A	Philadelphia between Washington & New York
	Centaur is mixture of man & horse Mermaid } Approaches B
B	*X* wants the character in which *Y* and *Z* resemble one another.
	A is not in the line through *B* and *C*.
	Washington free from selfishness of Caesar & Napoleon.
C	Friedrich had the egotism of royalty.
D	Detaille tries to make himself unlike Vernet.
E	Attempt to imitate
F	*A* greater than *B* in respect to character *C*
F′	*A* has character *C* which makes it unlike *B*

G	*A* like *B* in respect to *C*
G′	*A* has character *C* which makes it like *B*
H	A person educates himself to control himself
I	A person has the relation *R* to himself
J	A person gives himself the character *C*
K	*A* has to the character *C* the relation of possession *P*.

[*Clifford's* The Common Sense of the Exact Sciences]

Item 38 *P 307:* Nation *41 (3 September 1885): 203*

The Common Sense of the Exact Sciences. By the late William Kingdon Clifford. New York: Appletons. [International Scientific Series.]

It was in 1875, when Clifford was in fairly good health, that he dictated the whole of three chapters and part of another for a projected book to be entitled "The First Principles of the Mathematical Sciences Explained to the Non-Mathematical." Three years later, shortly before his death, he expressed the wish that the book should be published only after very careful revision, and that the title should be changed. It has certainly not received the sort of revision that Clifford desired; for as published it abounds in errors, and contains several quite anti-Cliffordian views. For instance, he says that if a point on the surface of a sphere is brought into contact with a point on the flat face of a cube, "we cannot move the sphere ever so little without separating these points." This is erroneous, because we can spin the surface about the point of contact; but although the passage has passed under the hands of two successive mathematical editors, neither has seen, what the course of reasoning shows, that Clifford in dictating said "move" when he meant *roll.* He wanted to show that all surfaces would fit together at any points where they are not broken by edges or corners, much as a ball may fit into a cup, only that the fitting is confined to a single point. Now surfaces that fit together may or may not be capable of being slipped or spun one on the other, but they cannot be rolled one on the other. A rolling motion, therefore, was the only one which had to be considered.

Again, he defines a surface as the boundary between two portions of space which it separates absolutely. Now, without speaking of spirals, which obviously do not separate space into two parts, the most familiar of all surfaces, the plane, does not do so (according to the conception of the modern geometrician). Two planes will separate space, and one of these may be the plane at infinity; but a single plane does not. For if a point (say the focal point of a lens) be carried off with sufficient acceleration from one side of a plane, it will come back on the other side. Every surface may, it is true, form a part of the boundary between two regions of space. But even so modified, the definition is hardly satisfactory; for the calculus requires us to suppose that a solid body may approach indefinitely near to being a surface, which it certainly could not do were the two objects essentially disparate in their nature. Clifford here says:

> The surface of a thing is something that we constantly observe. We see it and feel it, and it is a mere common-sense observation to say that this surface is common to the thing itself and to the space surrounding it.
>
> The important thing to notice is that we are not here talking of ideas or imaginary conceptions, but only making common-sense observations about matters of every-day experience.

But, as the editor, "K. P.," remarks, "we are compelled to consider the surface of the geometer as an idea or imaginary conception, drawn from the apparent (not real) boundaries of physical objects." The truth is, that the geometrical conception of space itself is a fiction. The geometer thinks of space as an individual thing or (as Mr. F. E. Abbot expresses it) a receptacle of things having an existence as something individual. If this were so, absolute position in space (independent of other bodies) and absolute velocity would have a meaning; but, in fact, they appear to have none. What is true is, that rigid bodies in their displacements are subject to certain laws which are the principles of geometry; and we have an instinctive acquaintance with these positional laws, which makes it easy for us to imagine the fictitious receptacle in which these laws are embodied. Thus, space only exists under the form of general laws of position; there is really nothing individual about it. And easy as is the geometer's conception, it is by no means born in us. The natural man knows of space only as a synonym for "air." Kant is responsible for the perpetuation of the erroneous conception of space which Leibnitz had escaped. It is impossible to have clear ideas concerning the non-

Euclidean geometry, space of *n* dimensions, and such matters, without a proper understanding of this.

The main fault of the whole plan of the book is, that while it gives no adequate explanation of many mathematical conceptions interesting to a large body of non-mathematical minds—such as the square root of the negative, multiple algebra, space of *n* dimensions, the mathematical conception of the Absolute, non-Euclidean space, invariants, Riemann's surfaces, etc., conceptions perfectly susceptible of clear and interesting explanation, without too severely taxing the powers of the non-mathematical—it does suppose a reader whose interest in the logical *enchaînement* of mathematics is exceptionally great. Nine persons out of ten will read the chapter on number and exclaim, "This is nothing but what we learned at school," thus missing the whole argument, which will fly over their heads unperceived. The book has something of Clifford's style and traces of his power, but only faint ones. It will be of some service, but not very much. The parts added by "K. P.," one chapter and a half, bear comparison with those written by Clifford; it is a pity that the revision of the latter has not been more minute and accurate.

[Perrin's The Religion of Philosophy*]*

Item 39 *P 308:* Nation *41 (19 November 1885): 431*

The Religion of Philosophy or The Unification of Knowledge: A comparison of the chief philosophical and religious systems of the world, made with a view to reducing the categories of thought, or the most general terms of existence, to a single principle, thereby establishing a true conception of God. By Raymond S. Perrin. G. P. Putnam's Sons. 1885.

Six pages would have been ample to set forth the doctrine here diluted to six hundred. Motion is the only existence; time and space merely its phases. Time is identical with force; space with matter. God is the universal principle of motion. In place of arguing these propositions, the author tags them incongruously to sketches of the history of philosophy—sketches nil as arguments, and as history rambling, feeble, and ill-proportioned. Some healthy sentiments about morality and religion are expressed in an easy and pleasing style, but the philosophical conceptions seem to be nebulous, and the method of presenting them unsuccessful.

[Kant's *Introduction to Logic*]

Item 40 *MS 555: Fall–Winter 1885*

Kant's Introduction to Logic, and his Essay on the Mistaken Subtilty of the Four Figures: Translated by Thomas Kingsmill Abbott, B. D., Fellow and Tutor of Trinity College, Dublin: with a few Notes by Coleridge. London: Longmans, Green, & Co. 1885.

Kant's whole philosophy turns upon his logic. He gives the name of logic to the greater part of his *Critic of the Pure Reason,* and it is a result of the great fault of his logical theory that he does not extend that name to the whole work. This greatest fault was at the same time the greatest merit of his doctrine: it lay in his sharp discrimination of the intuitive and the discursive processes of the mind. The distinction itself is not only familiar to everybody but it had long played a part in philosophy. Nevertheless, it is on such obvious distinctions that the greatest systems have been founded, and Kant saw far more clearly than any predecessor had done the whole philosophical import of this distinction. This was what emancipated him from Leibnizianism, and at the same time turned him against sensationalism. It was also what enabled him to see that no general description of existence is possible, which is perhaps the most valuable proposition that the *Critic* contains. But he drew too hard a line between the operations of observation and of ratiocination. He allows himself to fall into the habit of thinking that the latter only begins after the former is complete; and wholly fails to see that even the simplest syllogistic conclusion can only be drawn by observing the relations of the terms in the premises and conclusion. His doctrine of the *schemata* can only have been an afterthought, an addition to his system after it was substantially complete. For if the *schemata* had been considered early enough, they would have overgrown his whole work.

Kant's brochure on the *Mistaken Subtilty of the Four Syllogistic Figures* was published before he saw the full consequences of the distinction just mentioned. Yet the main *[ ]*

[Fiske's *The Idea of God*]

Item 41 ***MS 556: Fall–Winter 1885***

The Idea of God as Affected by Modern Knowledge. By John Fiske. Boston and New York: Houghton, Mifflin and Co.

Mr. John Fiske formerly chose to entitle his own religious theory "Cosmic theism, as opposed to Anthropomorphic theism"; and was thereupon supposed, not unnaturally, to have pantheistic leanings. In his *Idea of God,* however, he succeeds in making it clear that he has never wavered in his belief in a personal deity. He almost makes it too clear; for he seems to have been at all times so entirely satisfied with the orthodox conception, that one wonders whether he can ever have sufficiently felt in his own person the religious difficulties suggested by scientific ideas, to be of much service in furthering the perhaps destined fusion of religion and science. Just as it was requisite for the Saviour to feel the force of sin, so it seems that to teach the world the truth of religion one must have felt the force of Doubt. It seems now that Mr. Fiske's positivism can never have been more than skin-deep; for all that he has written since 1869 has ever been regarded by the author as so many "wayside studies" preliminary to a religious book to be entitled "Jesus of Nazareth and the Founding of Christianity." It would seem that at the time of writing the *Outlines of Cosmic Philosophy* the author could not have appreciated the veritable nature of the doctrine of evolution especially in the special form of it advocated by Mr. Spencer. Spencer makes the evolution of the world depend exclusively upon the principles of mechanics; while according to other evolutionists there are two factors, force and the effect of accidental variations, probability acting upon high numbers of elements,—to these two elements the whole development of the world is attributed. But Mr. Fiske now says "The events of the universe are not the work of chance, neither are they the outcome of blind necessity." This is consistent with no variety of evolutionism, unless Hegelianism is called evolutionism. All the minds which have been formed under the influence of physical science do believe that the events of this world are brought about by force and chance, and freedom is either not allowed to deflect a molecule from its necessitated path, or at least freedom is restricted within extremely narrow limits.

On the Use of the Noddy for Measuring the Amplitude of Swaying in a Pendulum Support

Item 42 *P 315:* **Coast Survey Report 1884,** *475–82*

The "Noddy" is an instrument invented by Thomas Hardy, a well-known English clockmaker of the early part of this century. It was employed by him and others to detect any oscillatory swaying of a pendulum support, but I use it to measure the amplitude of such swaying. It consists of a little pendulum supported, like an ordinary clock pendulum, from a reed or spring; but instead of hanging down it stands erect, so that gravity acts against the spring and causes the pendulum, although quite short, to oscillate with the same natural period as the gravity pendulum, which hangs from and sways the support. The instrument that was constructed after my design by the late M. Breguet is firmly attached to a brass bed-plate, resting on three screw-feet. The reed is 1 centimetre in length, and from it springs the staff of the noddy, consisting of a steel wire 1 millimetre in diameter and about 9 centimetres long. From the lower part of the staff, just above the attachment of the reed, two short wires extend at right angles to the axis of the staff and in the plane of the oscillation; they have screw-threads cut upon them, and carry short brass cylinders through the axes of which they pass, and which make the principal weight of the noddy. These can be screwed along the wires so as to adjust the equilibrium and alter the radius of gyration about an axis through the centre of mass. Another weight, spherical in form, slides upon the staff of the noddy, and serves to adjust the height of the centre of mass. There are several sets of cylindrical weights and several sliding weights. At the top of the staff is fixed a small oblong frame carrying a glass scale of tenths of millimetres, the lines being vertical and the scale running in the direction of the oscillatory motion. The scale is 10 centimetres from the attachment of the staff to the reed. A pillar attached to the bed-plate, with its axis in the vertical plane of the reed, carries a horizontal microscope directed toward the scale on the noddy, which is illuminated from an adjustable reflector behind. The microscope is focused with a ratchet; it is furnished with a draw-tube, and carries in the focus of the eye-piece a horizontal scale on glass, each division of which is equivalent to about $0^{\text{mm}}_{.}03$ in the focus of the objective as ordinarily used. The noddy is protected from currents of air by being inclosed in a tight metallic cylinder, furnished with two plate-glass windows opposite to one another at the level of the micrometer scale. This cylinder carries the reflector. It is also furnished with a stop-cock, so that the air can be exhausted if desired. The upper ends of the screws of the screw-feet are pointed like the lower ends, so as to serve as feet; and there is a wooden stand with rests for these feet, upon which the whole apparatus can be placed upside down to permit the observation of the period of oscillation of the pendent noddy.

We may first consider the case of the free oscillation of the noddy, and for the present we may neglect all resistance to its motion. Let us assume a system of rectangular co-ordinates having its origin at the root or fixed attachment of the reed, the axis of y being directed vertically upwards and that of x being in the plane of oscillation. The axis of z is then parallel to the axis of rotation, but the motion will be assumed to be in the plane of xy. Let s be the distance of any particle of the reed from the root; θ, the inclination of the reed to the vertical at the distance s from the root; S, the whole length of the reed; θ_S, the value of θ when $s = \mathrm{S}$; θ_0, the value of θ when $s = 0$, so that $\theta_0 = 0$; h, the distance of the centre of mass of the noddy from the attachment of the reed; x and y, the co-ordinates of this centre of mass; γ, the radius of gyration of the noddy about an axis parallel to that of z and passing through the centre of mass; ϵ, the elasticity of the reed at any point, and we suppose this to be constant throughout the length of the reed; g, the acceleration of gravity; M, the mass of the noddy (that of the reed being neglected); E, the kinetic energy; Υ, the kinetic potency or positional energy. Then

$$x=\int_0^{S}\sin\theta\cdot ds+h\sin\theta_s \qquad y=\int_0^{S}\cos\theta\cdot ds+h\cos\theta_s$$

$$D_t x=\int_0^{S}\cos\theta\cdot D_t\,\theta\cdot ds+h\cos\theta_s\cdot D_t\,\theta_s$$

$$D_t y=-\int_0^{S}\sin\theta\cdot D_t\,\theta\cdot ds-h\sin\theta_s\cdot D_t\,\theta_s.$$

Let θ' be a quantity which is the same function of a variable s' that θ is of s. Then we have

$$\frac{E}{M}=\frac{1}{2}\int_0^{S}\{ds'\int_0^{S}\cos(\theta-\theta')\cdot D_t\theta\cdot D_t\theta'\cdot ds\}+h\;D_t\theta_s$$
$$\cdot\int_0^{S}\cos(\theta-\theta_s)\cdot D_t\theta\cdot ds+\frac{1}{2}(h^2+\gamma^2)(D_t\theta_s)^2$$
$$\frac{T}{M}=\frac{\epsilon}{M}\int_0^{S}(D_s\theta)^2\,ds+g\int_0^{S}\cos\theta\;\cdot ds+gh\cos\theta_s.$$

In the first approximation we neglect the fourth power of θ in comparison with the second, and with this simplification we proceed to form the Lagrangian equations, according to the formula

$$D_t\frac{\partial E}{\partial D_t\theta}+\frac{\partial T}{\partial\theta}=0.$$

The partial differential coëfficients are to be taken on the hypothesis of a change in the value of θ corresponding to a single value of s, all other values remaining unchanged, so that

$$\frac{\partial\int F\theta\cdot ds}{\partial D_t\theta}=ds\cdot\frac{\partial F\theta}{\partial D_t\theta}.$$

The partial differential coëfficient of the first term of $\frac{E}{M}$ is

$$\frac{1}{2}\,ds'\int_0^{S}D_t\theta\cdot ds+\frac{1}{2}\,ds\int_0^{S}D_t\theta'\cdot ds'=ds\int_0^{S}D_t\theta\cdot ds.$$

This does not, however, apply to $\theta=\theta_s$; in that case the whole effect is given by the second and third terms of $\frac{E}{M}$. The partial differential coëfficient of the first term of $\frac{T}{M}$ is most clearly deduced as follows: Let s_{i-1}, s_i, s_{i+1} be the distances of successive particles of the reed from the root, and let $\theta_{i-1}, \theta_i, \theta_{i+1}$ be the corresponding values of θ. We have

$$s_{i+1}-s_i=s_i-s_{i-1}=ds.$$

Let us write

$$\frac{\epsilon}{ds} = \eta.$$

Then that part of the first term of $\frac{\mathrm{T}}{\mathrm{M}}$ which involves θ_i is

$$\frac{1}{2}\frac{\eta}{\mathrm{M}}(\theta_{i+1} - \theta_i)^2 + \frac{1}{2}\frac{\eta}{\mathrm{M}}(\theta_i - \theta_{i-1})^2$$

and the differential coëfficient of this, relatively to θ_i is

$$\frac{\eta}{\mathrm{M}}(-\theta_{i+1} + 2\theta_i - \theta_{i-1}) = -\frac{\epsilon}{\mathrm{M}}\mathrm{D}_s^2\,\theta\cdot ds.$$

But when $s = \mathrm{S}$, the first term of the binomial expressing the part of $\frac{\mathrm{T}}{\mathrm{M}}$ to be considered is to be struck off, because there is no particle of the reed further from the root; consequently the differential coëfficient then becomes

$$\frac{\eta}{\mathrm{M}}(\theta_s - \theta_{s-1}) = \frac{\epsilon}{\mathrm{M}}\,\mathrm{D}_s\theta.$$

We now see that the simplified Lagrangians are

$$\mathrm{D}_t^2 \int_0^{\mathrm{S}} \theta\cdot ds + h\ \mathrm{D}_t^2\,\theta_s - \frac{\epsilon}{\mathrm{M}}\,\mathrm{D}_s^2\,\theta - g\ \theta = 0$$

$$h\ \mathrm{D}_t^2 \int_0^{\mathrm{S}} \theta\cdot ds + (h^2+\gamma^2)\ \mathrm{D}_t^2\,\theta_s + \frac{\epsilon}{\mathrm{M}}\,\mathrm{D}_s\,\theta_s - gh\ \theta_s = 0.$$

Differentiating the first of these equations relatively to s, we have

$$-\frac{\epsilon}{\mathrm{M}}\,\mathrm{D}_s^3\,\theta - g\ \mathrm{D}_s\,\theta = 0.$$

If we write σ for $\sqrt{\frac{\epsilon}{\mathrm{M}g}}$, which has the dimension of a line, the solution to the last equation is

$$\theta = \Theta_1\ \sin\frac{s}{\sigma} + \Theta_2\ \cos\frac{s}{\sigma} + \Theta_3$$

where Θ_1, Θ_2, Θ_3 are arbitrary functions of the time, independent of s. Since $\theta_0 = 0, 0 = \Theta_2 + \Theta_3$; so that

$$\theta = \Theta_1\ \sin\frac{s}{\sigma} + \Theta_2\left(\cos\frac{s}{\sigma} - 1\right).$$

It thus appears that the figure of the reed is a curve of sines, or a part of such a curve, the wave-length being $2\pi\sigma$.

We now form from the last equation expressions for θ_s and for $\int_0^S \theta \cdot ds$, as well as for those terms of the two Lagrangians which involve θ and its derivatives; and from this we eliminate Θ_1 and Θ_2, so as to make the Lagrangians linear equations in θ_s and $\int_0^S \theta \cdot ds$. And here it will be convenient to introduce the abbreviations

$$\chi = \frac{\int_0^S \theta \cdot ds}{\sigma} \qquad \vartheta = \theta_s \qquad \psi = \frac{S}{\sigma}$$

$$p = \sigma \cos \frac{S}{\sigma} - h \sin \frac{S}{\sigma} \qquad q = h \cos \frac{S}{\sigma} + \sigma \sin \frac{S}{\sigma}.$$

The expression just found for θ then gives us

$$-\vartheta + \Theta_1 \sin \psi - \Theta_2 (1 - \cos \psi) = 0$$

$$-\chi + \Theta_1 (1 - \cos \psi) - \Theta_2 (\psi - \sin \psi) = 0$$

$$\frac{\epsilon}{M} D_s^2 \theta + g\theta = -\Theta_2 g \qquad \frac{\epsilon}{Mh} D_s \theta_s = \Theta_1 \frac{g}{h} \sigma \cos \frac{S}{\sigma} - \Theta_2 \frac{g}{h} \sigma \sin \frac{S}{\sigma}$$

$$-g \theta_s = -\Theta_1 \frac{g}{h} h \sin \frac{S}{\sigma} + \Theta_2 \frac{g}{h} (h - h \cos \frac{S}{\sigma}).$$

Eliminating Θ_1 and Θ_2 first from the first three of these equations, and afterward from the first two and the sum of the last two, we have

$$\begin{vmatrix} \frac{\epsilon}{M} D_s^2 \theta + g\theta, & 0, & g \\ \vartheta, & \sin \psi, & (1 - \cos \psi) \\ \chi, & (1 - \cos \psi), & \psi - \sin \psi \end{vmatrix} = 0$$

$$\begin{vmatrix} -\frac{\epsilon}{Mh} D_s \theta_s + g\theta_s, & -\frac{g}{h} p, & g - \frac{g}{h} q \\ \vartheta, & \sin \psi, & (1 - \cos \psi) \\ \chi, & (1 - \cos \psi), & \psi - \sin \psi \end{vmatrix} = 0$$

We have only to replace the first element of each of these determinants by its value as given by one of the Lagrangians, namely

$$\frac{\epsilon}{M} D_s^2\theta + g\theta = hD_t^2\vartheta + \sigma D_t^2\chi$$

$$-\frac{\epsilon}{Mh} D_s\theta_s + g\theta_s = \left(h + \frac{\gamma^2}{h}\right) D_t^2\vartheta + \sigma D_t^2\chi$$

to obtain the Lagrangians freed from indeterminate values of s and θ. The two Lagrangians may now be embraced in a single expression by the introduction of an indeterminate number, n. Namely, we multiply the first by $(1 - nh)$ and the second by nh, and add, when we get

$$\begin{vmatrix} (h + n\gamma^2)D_t^2\vartheta + \sigma D_t^2\chi, & -ngp, & g - ngq \\ \vartheta & , \sin\psi & , 1 - \cos\psi \\ \chi & , 1 - \cos\psi, & \psi - \sin\psi \end{vmatrix} = 0$$

The abscissa, x_r, of a particle on the staff of the noddy, at a distance r above the centre of mass is

$$x_r = \sigma\chi + (h + r)\vartheta.$$

Let ρ be the value of r for a particle so situated that it has a single harmonic motion. Then x_ρ being the abscissa,

$$D_t^2 x_\rho = -\frac{\pi^2}{T^2} x_\rho$$

where T is the period of oscillation. We may give n such a value that the equation combining the Lagrangians becomes identically equal to this, that is, to

$$(h + \rho)\, D_t^2\vartheta + \sigma D_t^2\chi + \frac{\pi^2}{T^2}(h + \rho)\vartheta + \frac{\pi^2}{T^2}\sigma\chi = 0.$$

This gives

$$\rho = n\gamma^2$$

$$\frac{\pi^2}{T^2}(h + \rho) = g\frac{\begin{vmatrix} 1 - \cos\psi, & \psi - \sin\psi \\ - \quad np, & 1 - nq \end{vmatrix}}{\begin{vmatrix} \sin\psi, & 1 - \cos\psi \\ 1 - \cos\psi, & \psi - \sin\psi \end{vmatrix}} = g\frac{1 - \cos\psi + n(h - q + p\psi)}{\psi\sin\psi - 2(1 - \cos\psi)}$$

$$\frac{\pi^2}{T^2}\sigma = g\frac{\begin{vmatrix} -np, & 1 - nq \\ \sin\psi, & 1 - \cos\psi \end{vmatrix}}{\begin{vmatrix} \sin\psi, & 1 - \cos\psi \\ 1 - \cos\psi, & \psi - \sin\psi \end{vmatrix}} = g\frac{n(\sigma - p) - \sin\psi}{\psi\sin\psi - 2(1 - \cos\psi)}$$

I have carefully performed the elimination of ρ and n from these equations, and have thus obtained the quadratic

$$\frac{\pi^4}{g^2T^4}\gamma^2\sigma[2(1-\cos\psi)-\psi\sin\psi]$$
$$-\frac{\pi^2}{gT^2}[(h^2+\gamma^2+\sigma^2)\sin\psi-\sigma p\psi]+p=0.$$

The proper adjustment of the noddy requires p to be a very short length. The coëfficient of the first term ought also to be small. Then the solution of the quadratic is

$$\frac{\pi^2}{gT^2}=\frac{1}{2}\,\frac{(h^2+\gamma^2+\sigma^2)\sin\psi-\sigma p\psi}{\gamma^2\sigma(2\text{ ver sin }\psi-\psi\sin\psi)}$$
$$\left(1\pm\sqrt{1-4\frac{\gamma^2\sigma p(2\text{ ver sin }\psi-\psi\sin\psi)}{\{(h^2+\gamma^2+\sigma^2)\sin\psi-\sigma p\psi\}^2}}\right)$$

and the approximate values of the roots are

$$\frac{\pi^2}{gT_1^2}=\frac{(h^2+\gamma^2+\sigma^2)\sin\psi-\sigma p\psi}{\gamma^2\sigma\,(2\text{ ver sin }\psi-\psi\sin\psi)}$$
$$=\frac{h^2+\gamma^2+\sigma^2+Sh-S\sigma\cot\psi}{\gamma^2\,(2\sigma\tan\frac{1}{2}\psi-S)}$$
$$\frac{\pi^2}{gT_2^2}=\frac{p}{(h^2+\gamma^2+\sigma^2)\sin\psi-\sigma p\psi}$$
$$=\frac{\frac{\sigma}{h}\cot\psi-1}{\frac{h^2+\gamma^2+\sigma^2}{h}+S-S\frac{\sigma}{h}\cot\psi}.$$

The latter root represents the principal component of the oscillation. The corresponding values of ρ are

$$\rho_1=-\frac{(h^2+\sigma^2)\sin\psi-\sigma p\psi}{\sigma-p}=-\frac{h+\frac{\sigma^2}{h}-S\left(\frac{\sigma}{h}\cot\psi-1\right)}{1+\frac{\sigma}{h}\tan\frac{1}{2}\psi}$$
$$\rho_2=\gamma^2\frac{(h^2+\gamma^2+\sigma^2)\sin\psi-2\sigma p\tan\frac{1}{2}\psi}{(h^2+\gamma^2+\sigma^2+Sh-S\sigma\cot\psi)\,(\sigma-p)}.$$

For any fixed value of ψ, the first component oscillation will be infinitely rapid when

$$2\sigma \tan \frac{1}{2} \psi - S = 0$$

that is, when

$$\tan \frac{S}{2\sigma} = \frac{S}{2\sigma}$$

and the second component oscillation will have a period of infinite length when

$$\frac{\sigma}{h} \cot \psi - 1 = 0$$

that is, when

$$\tan \frac{S}{\sigma} = \frac{\sigma}{h}.$$

This affords a means of determining σ, by measuring h when the adjustment is such as to give this condition of things.

The amplitudes of the two component oscillations depend upon the manner in which the noddy is set into motion, but the second will usually be the principal one and the first will be insensible; the noddy will consequently rotate about a fixed point determined by the value of ρ_1.

When the noddy is in the pendent position the vertical co-ordinates may be taken to increase downwards. Then, those terms of $\frac{T}{M}$ which involve g will have their signs reversed. The equation to determine the figure of the reed will accordingly be

$$\frac{\epsilon}{M} D_s^3 \theta - g D_s \theta = 0.$$

The solution of this contains Gudermannian instead of trigonometric functions, and may be written

$$\theta = -\Theta_1 \sinh \frac{s}{\sigma} + \Theta_2 \cosh \frac{s}{\sigma} + \Theta_3$$

and since as before

$$\theta_0 = 0 = \Theta_2 + \Theta_3$$

this takes the form

$$\theta = -\Theta_1 \sinh\frac{s}{\sigma} + \Theta_2\left(\cosh\frac{s}{\sigma} - 1\right).$$

This gives us

$$\begin{gathered}
\vartheta + \Theta_1 \sinh\psi - \Theta_2(\cosh\psi - 1) = 0 \\
\chi + \Theta_1(\cosh\psi - 1) - \Theta_2(\sinh\psi - \psi) = 0 \\
+\frac{\epsilon}{M} D_s^2\theta - g\,\theta - g\,\Theta_2 = 0 \\
-\frac{\epsilon}{Mh} D_s\,\theta_s - g\,\theta_s - \Theta_1\frac{g}{h}p' + \Theta_2\left(+\frac{g}{h}q' - g\right) = 0
\end{gathered}$$

where

$$\begin{aligned}
p' &= h\sinh\psi + \sigma\cosh\psi \\
q' &= h\cosh\psi + \sigma\sinh\psi.
\end{aligned}$$

Thus the Lagrangians become

$$\begin{vmatrix} h\,D_t^2\,\vartheta + \sigma D_t^2\chi, & 0 & , +g \\ \vartheta & , \sinh\psi & , \cosh\psi - 1 \\ \chi & , \cosh\psi - 1 & , \sinh\psi - \psi \end{vmatrix} = 0$$

$$\begin{vmatrix} \left(h + \frac{\gamma^2}{h}\right) D_t^2\,\vartheta + \sigma D_t^2\chi, & -\frac{g}{h}p' & , -\frac{g}{h}q' + g \\ \vartheta & , \sinh\psi & , \cosh\psi - 1 \\ \chi & , \cosh\psi - 1, & \sinh\psi - \psi \end{vmatrix} = 0$$

and their combination is

$$\begin{vmatrix} (h + n\gamma^2)\,D_t^2\vartheta + \sigma D_t^2\chi, & -ngp' & , g - ngq' \\ \vartheta & , \sinh\psi & , \cosh\psi - 1 \\ \chi & , \cosh\psi - 1 & , \sinh\psi - \psi \end{vmatrix} = 0$$

The equations to determine ρ and T are

$$\rho = n\gamma^2,$$

$$\frac{\pi^2}{T^2}(h + n\gamma^2) = g\frac{\begin{vmatrix} \cosh\psi - 1, & \sinh\psi - \psi \\ -np' & , 1 - nq' \end{vmatrix}}{\begin{vmatrix} \sinh\psi & , \cosh\psi - 1 \\ \cosh\psi - 1, & \sinh\psi - \psi \end{vmatrix}} = g\frac{\cosh\psi - 1 + n(-h + q' - p'\psi)}{2(\cosh\psi - 1) - \psi\sinh\psi}$$

$$\frac{\pi^2}{T^2}\sigma = g\frac{\begin{vmatrix} -np' & , 1 - nq' \\ \sinh\psi & , \cosh\psi - 1 \end{vmatrix}}{\begin{vmatrix} \sinh\psi & , \cosh\psi - 1 \\ \cosh\psi - 1, & \sinh\psi - \psi \end{vmatrix}} = g\frac{-\sinh\psi + n(p' - \sigma)}{2(\cosh\psi - 1) - \psi\sinh\psi}$$

The elimination of n gives

$$\frac{\pi^4}{g^2 T^4}\gamma^2\sigma\{2\,(\cosh\psi - 1) + \psi\sinh\psi\}$$
$$-\frac{\pi^2}{gT^2}\{(h^2+\gamma^2-\sigma^2)\sinh\psi + \sigma p'\psi\} + p' = 0$$

and the approximate values of the roots are

$$\frac{\pi^2}{gT_1^2} = \frac{(h^2+\gamma^2-\sigma^2)\sinh\psi + \sigma p'\psi}{\gamma^2\sigma[2\,(\cosh\psi - 1) + \psi\sinh\psi]}$$
$$\frac{\pi^2}{gT_2^2} = \frac{p'}{(h^2+\gamma^2-\sigma^2)\sinh\psi + \sigma p'\psi}$$
$$= \frac{1+\frac{\sigma}{h}\coth\psi}{\frac{h^2+\gamma^2-\sigma^2}{h} + S + S\frac{\sigma}{h}\coth\psi}.$$

When the noddy stands on the support of a gravity pendulum oscillating in the same plane, we may neglect the influence of the former upon the latter. Then if ξ be the horizontal displacement of the support, we have

$$D_t x = \int_0^S \cos\theta\cdot D_t\,\theta\cdot ds + h\,\cos\vartheta\cdot D_t\,\vartheta + D_t\xi.$$

Consequently $\frac{E}{M}$ is increased by the terms

$$D_t\xi\int_0^S \cos\theta\cdot D_t\theta\cdot ds + h\,\cos\vartheta\cdot D_t\vartheta\cdot D_t\xi + \frac{1}{2}\,(D_t\xi)^2.$$

The first Lagrangian heretofore considered will be increased by $D_t^2\xi$, and the second by $hD_t^2\xi$. The figure of the reed will not be affected, and the combination of the Lagrangians will simply have $D_t^2\xi$ added to it. We will now write

$$\xi = \Xi\,\cos\frac{t}{T'}\pi$$

where Ξ is the constant amplitude of oscillation of the support and T′ is the period of the gravity pendulum. Thus, the differential equation for x_ρ becomes

$$D_t^2[(h+\rho)\vartheta + \sigma\chi] + \frac{\pi^2}{T^2}[(h+\rho)\vartheta + \sigma\chi] - \frac{\pi^2}{T'^2}\,\Xi\,\cos\frac{t}{T'}\pi = 0.$$

This will add to the motion of x_ρ a harmonic component, having the period T′, so that it will be

$$(h+\rho)\vartheta+\sigma\chi=\mathrm{X}\cos\frac{t-t_0}{\mathrm{T}}\pi-\mathrm{Q}\cos\frac{t}{\mathrm{T'}}\pi.$$

To determine Q we take the second derivative:

$$\begin{aligned}\mathrm{D}_t^2[(h+\rho)\vartheta+\sigma\chi]&=-\frac{\pi^2}{\mathrm{T}^2}\mathrm{X}\cos\frac{t-t_0}{\mathrm{T}}\pi+\frac{\pi^2}{\mathrm{T'}^2}\mathrm{Q}\cos\frac{t}{\mathrm{T'}}\pi\\&=-\frac{\pi^2}{\mathrm{T}^2}[(h+\rho)\vartheta+\sigma\chi]+\frac{\pi^2}{\mathrm{T'}^2}\Xi\cos\frac{t}{\mathrm{T'}}\\&=-\frac{\pi^2}{\mathrm{T}^2}\mathrm{X}\cos\frac{t-t_0}{\mathrm{T}}\pi+\pi^2\left(\frac{\mathrm{Q}}{\mathrm{T}^2}+\frac{\Xi}{\mathrm{T'}^2}\right)\cos\frac{t}{\mathrm{T'}}\pi.\end{aligned}$$

Thus we have

$$\frac{\mathrm{Q}}{\mathrm{T'}^2}=\frac{\mathrm{Q}}{\mathrm{T}^2}+\frac{\Xi}{\mathrm{T'}^2}$$

or

$$\mathrm{Q}=\frac{\mathrm{T}^2}{\mathrm{T}^2-\mathrm{T'}^2}\Xi.$$

But the noddy has no oscillation to begin with. This fact is represented by the equations

$$t_0=0\qquad \mathrm{X}=\mathrm{Q}.$$

Hence

$$\begin{aligned}(h+\rho)\,\vartheta+\sigma\chi&=\frac{\mathrm{T}^2\,\Xi}{\mathrm{T}^2-\mathrm{T'}^2}\left(\cos\frac{t}{\mathrm{T}}\pi-\cos\frac{t}{\mathrm{T'}}\pi\right)\\&=\Xi\,\frac{2\mathrm{T}^2}{\mathrm{T}^2-\mathrm{T'}^2}\sin\frac{\mathrm{T}-\mathrm{T'}}{2\mathrm{TT'}}t\pi\sin\frac{\mathrm{T}+\mathrm{T'}}{2\mathrm{TT'}}t\pi.\end{aligned}$$

This equation shows that the noddy would oscillate with a period, a harmonic mean between its natural period and that of the gravity pendulum. The amplitude of oscillation would increase from nothing at an initial rate nearly independent of the value of (T − T′), and the increase would continue until it reached its maximum, when

$$t = \frac{TT'}{T - T'}.$$

At the beginning the noddy would be a quarter of a phase behind the gravity pendulum; at the maximum oscillation of the noddy it would be in opposition to the pendulum; and when it was reduced to rest again, it would be a quadrant in advance. It would then start up as before.

In considering the influence of the gravity pendulum upon the noddy, however, it is essential to take account of the resistance to the motion of the latter, owing to the internal friction of the spring and to the viscosity of the air. The dissipation produced by the former cause will be

$$\frac{1}{2}\mu \int_0^S (D_t\,\theta)^2\, ds$$

where μ is a constant. This will add the term

$$\frac{\mu}{M} D_t \theta$$

to the first Lagrangian. It will slightly change the figure of the spring, and the equation to determine this will be a partial differential equation, showing that the wave-length will not be constant. But this effect will be very small and may be neglected. Neglecting also the effect of the resistance upon the period of the motion, we find that if the natural motion of the noddy is

$$(h + \rho)\,\vartheta + \sigma\chi = \Theta\, \epsilon^{\frac{t}{2T}B\pi} \cos \frac{t}{T}\pi$$

then its motion under the influence of the pendulum is

$$(h + \rho)\,\vartheta + \sigma\chi = \frac{\Xi}{R}\left\{ \sin\omega \cdot \epsilon^{-\frac{t}{2T}B\pi} \cos \frac{t}{T}\pi + \sin\left(\frac{t}{T'}\pi - \omega\right) \right\}$$

where

$$\tan\omega = \frac{A}{B} \qquad R = \sqrt{A^2 + B^2} \qquad A = 1 - \frac{T'^2}{T^2}.$$

It will be seen that the natural period and rate of decrement of the arc of the noddy have to be observed, and that weighings and measures of its parts have to be made so as to calculate ρ_1 and ρ_2.

Then, it is necessary to observe, while the gravity pendulum is swinging, the relative amplitude and phase of the motion of the noddy.

I have made considerable use of the instrument, and find it gives results agreeing within a few *per cent.*, and that it affords on the whole a tolerably satisfactory way of determining the amount of swaying of a pendulum support.

Note on the Effect of the Flexure of a Pendulum upon its Period of Oscillation

Item 43 *P 316:* **Coast Survey Report 1884,** ***483–85***

In determining the acceleration of gravity, it is necessary to consider the effect of the flexure of the pendulum during its oscillation.

Suppose, first, that a pendulum, otherwise rigid, has an elastic joint parallel to the knife-edge and at a distance r vertically below it in the position of repose. Let m_0 be the mass of the piece of the pendulum connected with the knife, m that of the piece jointed to it; h_0 the distance of the centre of mass of the first piece below the knife-edge in the position of repose, h that of the second piece below the joint; γ_0 and γ the radii of gyration of the two pieces about axes through their centres of mass parallel to the knife-edge; θ_0 and $\theta = \theta_0 + \delta\theta$ the angular displacements of the two pieces about their centres from the position of repose of the whole pendulum. Let ϵ be the coëfficient of elasticity of the joint. With this notation, the kinetic potency[1] is written

$$U = gm_0h_0\ (1 - \cos\theta_0) + gmr\ (1 - \cos\theta_0) + gmh\ (1 - \cos\theta) + \frac{1}{2}\epsilon\ (\delta\theta)^2.$$

1. The term *potential energy* grates upon the ear of a student of Aristotelian philosophy, because both words derive their whole standing in language from that philosophy, and in their proper meanings are directly contradictory of one another. "Energy" means actuality, and "potential" means not yet actualized, so that potential energy is unactual actuality. The conception of half the *vis viva* as an actuality or energy of which the negative of the potential is the corresponding potency or power, seems preferable to the contrary conception that the latter is the real "work" or performance, and the former merely the "power" or *vis* which develops it; because the negative of the potential may subsist for any length of time, but always tends to produce *vis viva,* while the *vis viva* may increase or diminish and does not particularly tend to do either rather than the other. I shall, therefore, venture to call $\frac{1}{2}\ \Sigma mv^2$ the kinetic act or kinetic energy and the negative of the potential, the kinetic power or kinetic potency. For the sum of the two I can think of no better term than *motivity* or *kinesis.*

If the differential equations were formed, were made linear by the omission of terms of higher degrees, and were then resolved, the motion would be seen to consist of two harmonic components, the amplitudes of which are arbitrary constants. It is, therefore, possible for one of these to vanish, and we may assume such an equation between θ and θ_0 as to bring about this result. This equation must express the condition that the kinetic potency shall be a minimum. It is, therefore,

$$gmh \sin\theta + \epsilon\cdot\delta\theta = 0$$

or

$$\delta\theta = -\frac{gmh}{\epsilon}\sin\theta.$$

The elasticity of the joint might be measured by holding the first piece of the pendulum firmly in the horizontal position, while the second piece stood out straight, and measuring the angular flexure at the joint. Denoting this by ψ, the last equation gives

$$\psi = \frac{gmh}{\epsilon}.$$

Neglecting the square of θ (and *a fortiori* of ψ)

$$\delta\theta = -\psi\theta = -\psi\theta_0 \qquad \theta = (1-\psi)\theta_0.$$

To find the kinetic energy, we assume a system of rectangular co-ordinates having the origin at the middle of the knife-edge, the axis of y being directed vertically downwards, and that of x sideways, perpendicular to the knife-edge. Then, writing the co-ordinates of the centre of mass of the first piece x_0 and y_0, those of that of the second piece x and y, we have

$$x_0 = h_0 \sin\theta_0 \qquad y_0 = h_0\cos\theta_0$$
$$x = r\sin\theta_0 + h\sin\theta \qquad y = r\cos\theta_0 + h\cos\theta$$
$$(\mathrm{D}_t x_0)^2 + (\mathrm{D}_t y_0)^2 = h_0^2\,(\mathrm{D}_t\theta_0)^2$$
$$(\mathrm{D}_t x)^2 + (\mathrm{D}_t y)^2 = r^2\,(\mathrm{D}_t\theta_0)^2 + 2\,rh\cos(\theta-\theta_0)\cdot\mathrm{D}_t\theta_0\cdot\mathrm{D}_t\theta + h^2(\mathrm{D}_t\theta)^2.$$

The kinetic energy is, therefore,

$$\mathrm{E} = [\tfrac{1}{2}\,m_0\,(h_0^2+\gamma_0^2) + \tfrac{1}{2}\,mr^2]\,(\mathrm{D}_t\theta_0)^2 + mrh\cos\delta\theta\cdot\mathrm{D}_t\theta_0\cdot\mathrm{D}_t\theta$$
$$+\tfrac{1}{2}\,m\,(h^2+\gamma^2)\,(\mathrm{D}_t\theta)^2.$$

Neglecting the square of θ, the differential equations are

$$[m_0\ (h_0^2 + \gamma_0^2) + mr^2]\ \mathrm{D}_t^2\theta_0 + (gm_0h_0 + gmr + \epsilon)\ \theta_0$$
$$+\ mrh\,\mathrm{D}_t^2\ \theta - \epsilon\theta = 0$$
$$mrh\mathrm{D}_t^2\theta_0 - \epsilon\theta_0 + m(h^2 + \gamma^2)\ \mathrm{D}_t^2\theta + (gmh + \epsilon)\ \theta = 0.$$

Using the notation

$$\mathrm{A} = m_0\,(h_0^2 + \gamma_0^2) + mr^2 \qquad \mathrm{B} = gm_0h_0 + gmr + \epsilon$$
$$\mathrm{C} = mrh \qquad \mathrm{E} = -\epsilon \qquad \mathrm{G} = m\,(h^2 + \gamma^2)$$
$$\mathrm{H} = gmh + \epsilon \qquad \mathrm{L} = \mathrm{AG} - \mathrm{C}^2$$
$$\mathrm{M} = \mathrm{AH} + \mathrm{BG} - 2\mathrm{CE} \qquad \mathrm{N} = \mathrm{BH} - \mathrm{E}^2$$

we have the two approximate values

$$\mathrm{D}_t^2 = -\frac{\mathrm{N}}{\mathrm{M}}\ \text{or}\ -\frac{\mathrm{M}}{\mathrm{L}}.$$

Substituting the above value of ψ we get

$$\mathrm{B} = g\ (m_0h_0 + mr + mh\ \psi^{-1}) \qquad \mathrm{E} = -gmh\ \psi^{-1}$$
$$\mathrm{H} = gmh\,(1 + \psi^{-1}).$$

Then

$$\mathrm{M} = gmh\,[m_0\,(h_0^2 + \gamma_0^2) + mr^2]\ (1 + \psi^{-1})$$
$$+\ gm\,(h^2 + \gamma^2)\ (m_0\,h_0 + mr + mh\ \psi^{-1}) + 2\,gm^2rh^2\ \psi^{-1}$$

$$\psi\ \mathrm{M} = gmh\ [m_0\,(h_0^2 + \gamma_0^2) + m\ (h + r)^2 + m\gamma^2]$$
$$+\ gmh[m_0\,(h_0^2 + \gamma_0^2) + mr^2 + \frac{h^2 + \gamma^2}{h}(m_0\,h_0 + mr)]\psi$$

$$\mathrm{N} = g^2mh\,(m_0h_0 + mr + mh\psi^{-1})\ (1 + \psi^{-1}) - g^2m^2h^2\psi^{-2}$$

$$\psi\mathrm{N} = g^2mh\ [m_0h_0 + m\ (h + r)] + g^2mh\ (m_0h_0 + mr)\psi$$

$$\mathrm{L} = m\ (h^2 + \gamma^2)\ [m_0\,(h_0^2 + \gamma_0^2) + mr^2]$$
$$-\ m^2r^2h^2 = m\ (h^2 + \gamma^2)\ m_0\,(h_0^2 + \gamma_0^2) + m^2\gamma^2r^2.$$

If we now write

$$\mathfrak{M} = m_0 + m$$
$$\mathfrak{M}\mathfrak{H} = m_0h_0 + m\ (h + r)$$
$$\mathfrak{M}\mathfrak{H}\mathfrak{L} = m_0\ (h_0^2 + \gamma_0^2) + m\,[(r + h)^2 + \gamma^2]$$

and if we denote the period of oscillation by $T + \Delta T$, then when ψ vanishes

$$T = \pi\sqrt{\frac{L}{g}}$$

and

$$(T + \Delta T)^2 = \frac{\pi^2}{g}\,\frac{\mathfrak{M}\mathfrak{H}\mathfrak{L} + \left\{\mathfrak{M}\mathfrak{H}\mathfrak{L} + \left(m_0\frac{h_0}{h} - m\right)(h^2+\gamma^2) + mr\left(\frac{\gamma^2}{h} - h\right)\right\}\psi}{\mathfrak{M}\mathfrak{H} + [\mathfrak{M}\mathfrak{H} - mh]\,\psi}$$

$$\frac{\Delta T}{T} = \frac{1}{2}\left\{\frac{\left(m_0\frac{h_0}{h} - m\right)(h^2+\gamma^2) + mr\left(\frac{\gamma^2}{h} - h\right)}{\mathfrak{M}\mathfrak{H}\mathfrak{L}} + \frac{mh}{\mathfrak{M}\mathfrak{H}}\right\}\psi.$$

If there are a number of stiff joints, the sum of their separate effects gives their combined effect.

It appears, therefore, that the effect of the flexure of a pendulum upon its period of oscillation is virtually to lengthen it by a quantity which is generally of the same order of magnitude as the amount by which one extremity of the pendulum would sag if the other end were held rigidly horizontal. This must be quite a considerable quantity for all the reversible pendulums which have ever been constructed.

Two Letters, Peirce to F. E. Abbot

Item 44 ***L 1: 31 December 1885 and 11 January 1886***

Ithaca N.Y. 1885 Dec 31

My dear Abbot

I have just begun your book, in which I am sure to find much to admire, and to the leading doctrines of which I strongly hope to be able to assent or nearly so; for "universal endocosmic teleology" describes my own opinion.

I now write to ask a few questions about your Relationism. You say the scholastic realists were wrong in making universals inhere in individuals rather than in groups. (This is not your phrase exactly, but I use it, not knowing what you mean by an *individual as a group.*) Please say what you mean by a group. Do you mean any two of the subjects of the universal? That is, do you deny that any one thing apart from others is, say, *blue;* while you say that *blueness* inheres in any pair of blue things, and is wholly in each pair? Or do you mean that the universal exists only in the total collection of things of which it is predicable? Or, do you mean that it exists only in the relation between the thing & the mind, which appears to me to be Kant's view?

Next, I wish you would elucidate this point following. Take a dual relation, predicable of a pair of objects. Do you mean that this relation really inheres in each pair or only in groups of pairs?

As far as I have got (the first two sections of the Introduction) you draw no distinction between different kinds of relations, say relations of reason and real relations. If a fellow were to come up to me in the street and suddenly give me a fist-blow in the eye, my mode of cognition would be such that I should readily admit a real relation subsisting between the two individuals concerned. What I have designated as Dual Consciousness, or the sense of action & reaction, which has nothing general about it, would come in there in its first and unreflective modification. But when I speak of the right & left sides of a theatre, or of the first, second, and third persons, or of one thing as *bluer* than another, is the case quite the same? Above all, when I say that your being an American & my being an American, constitutes a relation between us, although each of us can be so all by himself, the other's being so giving neither help nor hindrance, shall I say that *this* relation inheres in the pair of individuals apart from the mind's thinking? Dual consciousness enters here only in a second, reflex, modification.

I do not undertake to argue the questions, but merely seek to know your position.

1st What do you mean by the *group* in which a universal inheres?

2nd Does a relation inhere in a single pair (if a dual relation, etc) or in a group of pairs?

3rd. Do you draw no distinction between relations, as to their independence of thought?

Yours faithfully
C. S. Peirce

Address as above

Ithaca 1886 Jan 11

My dear Abbot

I have this day received from you a presentation copy of your book with a pleasant inscription. I value both highly. I have already carefully read the book. The religious part is very fine; but it is, as it seems to me, only an anticipation of a scientific theism which cannot really come for very many years.

The philosophy at least indicates a terrible lacuna in philosophy-as-it-is. I am myself not only phenomenalist, but also idealist. I do not quarrel with the idealism of Hegel because it goes too far; but only because it is too simple an account of a more complicated matter. Your arguments in your chapter entitled Phenomenism do not impress me. I do not think the theory is self-contradictory; and I do not think that physics has any pretension to have got down to the bottom facts, the absolute subjects of appearances at all. I am more struck with your remark that the intelligibility of the world consists in its having an immanent relational constitution. For the whole motive to idealism is to make the world cognizable.

Being an idealist, of course, I cannot yet accept the objectivity of relations in the sense in which you mean it. But that relations are as real as anything in the world,—much *more* real, according to my notion, than being dead things-in-themselves would make them,—to this, I fully subscribe. Philosophy has too often half-forgotten this, as Hegel only half apprehends the second movement of his dialectic. I said something about this in a note I wrote you a few days ago.

As to the *perception* of relations, I am one of a good many who have called attention to the importance of this. In the *American Journal of Mathematics* for January 1885. p 182. I say "Reasoning consists in the *observation* that where certain relations subsist certain others are found . . . It has long been a puzzle how it could be that, on the one hand, mathematics is purely deductive in its nature, and draws its conclusions apodictically, while on the other hand, it presents as rich and apparently unending a series of surprizing discoveries as any observational science. Various have been the attempts to solve the paradox by breaking down one or other of these assertions, but without success. The truth, however, appears to be that all deductive reasoning, even simple syllogism, involves an element of observation; namely, deduction consists in constructing an icon or diagram the relations of whose parts shall present a complete analogy with those of the parts of the object of reasoning, of experimenting upon this image in the imagination, and of observing the result so as to discover unnoticed and hidden relations among the parts. Etc."

I grant you one cannot perceive relations without a perceptive understanding. I suppose the work of Helmholtz etc. on vision would lead about to that, since they admit we must have some special innate faculty in consequence of which when objects present themselves in spatial relations we can recognize those relations.

It would be very rash to pronounce any decided opinion on your two main positions. We now want a full statement of your whole system, with arguments *mainly such as do not depend on the failure of other systems,* I mean with little historical & critical matter.

I will conclude this letter with a few quotations from Occam which you may not have seen.

Quidam ponunt quod relatio non est alia res extra animam distincta realiter et totaliter a re absoluta et a rebus absolutis. Et de illa opinione reputo fuisse aristotelem et alios sequentes ipsum. Alii autem ponunt quod relatio est quaedam res quae non plus est res absoluta quam homo est asinus, sed est distincta realiter et totaliter a re absoluta et a rebus absolutis; et de illa opinione sunt multi theologi, quam aliquando credidi esse opinionem aristotelis. [This means he formerly held it himself. It is a thoroughly nominalistic opinion. As he says Secundum illam opinionem nihil est in *genere* relationis nisi nomen mentale vocale vel scriptum.]

Et sciendum est quod hoc nomen "opposita" significat tam res extra animam quam in anima quam signa rerum; sed omnes res extra animam quae non sunt signa, si sint oppositae, non opponuntur nisi contrarie, vel secundum unam opinionem relative opponuntur aliquae. . . . Relative autem opponuntur nomina relativa quae non possunt de eodem respectu ejusdem verificari et hoc verum est sive res extra animam aliquae opponuntur relative sive non. Nec propter hoc quod dico "nomina relativa" nego relationem extra animam, quia relativum potest dici tam de nomine rei quam de re.

Occam of course frequently speaks of the universal as a thing, *res*, but not as *res extra animam.*

Omnes propositiones et consequentiae (inferences) et termini mentales sunt entia rationis et tamen sunt vere et realiter existentia in rerum natura, et entia perfectiora et realiora quam qualitates quaecumque corporales.

Your loving classmate
C. S. Peirce

Fundamental Properties of Number

Item 45 ***MS 567: 5 January 1886***

Ithaca, 1886 Jan. 5

1. Numbers can be distinguished in a systematic way; and therefore a class of characters can be found, such that any two numbers that differ at all differ in respect to these characters. I shall call these characters essential characters.

2. Numbers form a complete system; and therefore the system of essential characters may be so chosen in advance that any two that differ at all are distinguished by some number possessing the one and wanting the other. I shall adopt as essential characters "not more than 1," "not more than 2," "not more than 3," etc.

3. Numbers form a linear system: that is to say, taking any two numbers and any two essential characters, either the first number possesses the first character, or the second number the second character, or the first number wants the second character, or the second number the first character.

4. The system of number is discrete throughout; so that given any essential character that is possessed by any number, there will always be two numbers that differ in regard to no essential character but this, and given any number that has any essential character, there will always be two essential characters which disagree only in that one belongs to this number and the other not. We thus have the ideas of a next following number and of a next preceding essential character; and we are led to take into our system of essential characters "not more than 0" which belongs to no number at all.

5. The system of positive number is limited at one extremity, so that there is an essential character that belongs to no number at all,

and there is a number that possesses every essential character except that one.

6. The system of number is unlimited at the other extremity, so that there is no number that has no essential character, and no essential character that belongs to every number but one.

7. But there is no number that cannot be reached by counting; and therefore whatever is true of the number next following each number of which it is true is true of every number which [wa]nts any essential character of any number of which it is true.

Dr. F. E. Abbot's Philosophy

Item 46 *P 326:* Nation *42 (11 February 1886): 135–36*

Organic Scientific Philosophy: Scientific Theism. By Francis Ellingwood Abbot, Ph.D. Boston: Little, Brown & Co. 1885.

Dr. Abbot is one of the many thinkers who believe that science is destined to produce a theism, and he belongs also to the smaller number who think that it is already possible to say what that doctrine shall be. Considerably more than half of his *Scientific Theism* is taken up with the proof that the world is intelligible; but this lengthy and metaphysical argumentation will convince nobody for whom very simple considerations would not have sufficed. How is it that one who believes he has the message of a new religion to announce to humanity should choose so roundabout a way of setting it forth? The following is one of the author's own summaries of his line of argument:

1. Because the universe is in some measure actually known in human science, it must be in itself both absolutely self-existent and infinitely intelligible: that is, it must be a noumenon because it is a phenomenon.
2. Because it is infinitely intelligible, it must be likewise infinitely intelligent.
3. Because it is at the same time both infinitely intelligible and infinitely intelligent, it must be an infinite subject-object or self-conscious intellect.
4. Because it is an infinitely intelligible object, it must possess throughout an immanent relational constitution.
5. Because it possesses an infinitely intelligible relational constitution, it must be an absolutely perfect system.
6. Because it is an absolutely perfect system, it cannot be an infinite machine, but must be an infinite organism.
7. Because it is an infinite organism, its life principle must be an infinite immanent Power, acting everywhere and always by organic means for organic ends, and subordinating every event to its own infinite life: in other words, it must be infinite Will directed by infinite Wisdom.

8. Because it is an infinite organism, its exient organic end disappears as such, but reappears as infinite Love of itself and infinite Love of the finite.

9. Because it is an infinite organism, its immanent organic end appears as the eternal realization of the ideal, and therefore as infinite Holiness.

10. Because, as an infinite organism, it thus manifests infinite Wisdom, Power, and Goodness, or thought, feeling, and will in their infinite fulness, and because these three constitute the essential manifestations of personality, it must be conceived as Infinite Person, Absolute Spirit, Creative Source, and Eternal Home of the derivative finite personalities which depend upon it, but are no less real than itself.

If this last conclusion really follows from the original premise, why need the proof have been so long? It is not like a geometrical demonstration, where there is a complicated diagram, every part of which has to be separately considered. In this case the premise is as simple a fact as can be—that something is known; the conclusion that the universe is an infinite person is also not very complex, and the intricacy of the argument to connect them affords ground for a suspicion that there is a fallacy somewhere. It would be a flattery of metaphysics to say that its history gives any warrant for holding that no more than one deduction in ten as plausible as the above turns out to be fallacious; and therefore the probability that there is no fallacy in the whole of the above chain of ten consequences is only $\frac{9}{10}$ths to the tenth power, which is about $\frac{1}{3}$. In advance of the verdict of posterity, then, the odds are two to one against Dr. Abbot's argument being sound. The subtlety of Nature, as Bacon says, far exceeds that of the human mind, and has a way of eluding our *must-bes.* To look no further than Dr. Abbot's first consequence, may it not be that nature is sufficiently intelligible to account for the degree of success that natural science has met with, without being necessarily *infinitely* intelligible?

The religion of the book seems to be only an appendage to a system of metaphysics. Whether true or false, this system is certainly valuable as presenting Objectivism, or the doctrine of an existence over against thought, in its extremest form. Its most striking philosophical characteristic is an energetic dualism. It makes the fundamental doctrines of philosophy consist in distinctions, crystalline, sharp, and unyielding; and the oppositions of things to which these distinctions refer go down to the bottom of being. The appearance and the thing are sundered by an impassable gulf, and the element of concrete outward reaction in sense and volition is much more emphasized than in other philosophical theories. The same spirit affects the author's whole style of thought and writing, which is clear and hard, and impels him to destroy every opposing tendency of thought "root and branch," instead of imitating other recent revolutionizers of philosophy in wishing to show that the error need only to receive complete development in order to be turned to the truth. Everything like uniting the members of his main distinctions by insensible gradations, by a deeper underlying unity, or by any mediating cause, except the Divine Mind which creates the relations but not the related things, is foreign to his idea.

Dr. Abbot holds that things, as they are known to physical science, possess absolute existence in themselves, not relative to or dependent upon thought of any kind. He holds that the relations of these things are hard facts, equally independent of all thought. There seems, however, to be some vagueness in his theory of relations, for on page 28 he seems to say that relations are something over and above the related things—"things and relations constitute two great distinct orders of objective reality"; while on page 63 we are told that "the affirmation of the objectivity of the relation [must not be misconceived] as an affirmation that the relation is an entity apart from the things it relates." He holds that relations inhere in groups; but whether the existence of these groups consists in the existence of the relations, or the existence of the relations consists in that of the groups, or whether groups form a third order of reality distinct alike from things and relations, he does not inform us. And it will be one of his difficulties that his system, from the nature of it, at once opens a multitude of questions of this sort, the consideration of which cannot be shirked. The author is so remarkably loath to admit mediation that he will not admit there is any such thing as a symbolical conception (p. 141):

> The universal notion, or concept proper, is a pure thought-system of relations, reproducing only the objective system of relations of resemblance among many individuals—never the image or mental picture of one individual.

The doctrine seems to be that the relations are reproduced, without being embodied in any diagram, as "concepts of relations, dropping out of consideration the things related." The knowledge of relations depends upon a special "perceptive use of the understanding." This view, although it is not adequately set forth, is the centre of all that is original in the book, and is sure to excite a fruitful discussion of the question of the mode of our discernment of relations. Of all the sciences—at least of those whose reality no one disputes—mathematics is the one which deals with relations in the abstractest form; and it never deals with them except as embodied in a diagram or construction, geometrical or algebraical. The mathematical study of a construction consists in experimenting with it; after a number of such experiments, their separate results suddenly become united in one rule, and our immediate consciousness of this rule is our discernment of the relation. It is a strong secondary sensation, like the sense of beauty. To call it a perception may perhaps be understood as implying that to discern each special relation requires a special faculty, or determination of our nature. But it should not be overlooked that we come to it by a process analogous to induction.

The one great argument which Doctor Abbot uses to support his "noumenism," as he calls it, is that the existence of natural science supposes it. But the physicist always talks and thinks of phenomena or appearances, and makes not the slightest pretension to have anywhere got down to the noumena, bottom facts, or ultimate subjects of appearances. He discovers, for instance, that air is viscous, and viscosity is a non-conservative force. It is a reality; but yet, according to the physicist, only a phenomenal reality. Matter in itself is not viscous: but this phenomenon is due to the air being composed of countless molecules moving very rapidly in nearly rectilinear paths. These molecules themselves are not necessarily the bottom subjects; they may be mere systems of atoms, which in turn may be merely phenomena due to the vortex-motions of an underlying fluid. This fluid may come to be studied in time, and physicists will be quite prepared to learn that it again is only phenomenal. The physicist certainly holds that he reaches real facts, which no more depend upon anybody's thought of them for their existence than the coach in the fable depended on the fly for its motion. For example, he holds this to be true of the laws of the mixture of colors. These laws are realities, which remain what they are whatever our opinions about them may be. But to say this, is not to say that the colors themselves are anything more than appearances. Further, although science must hold the facts it discovers to be independent of the opinion of any person or persons, it by no means follows that it need insist on their being independent of the final upshot of sufficient investigation, nor that it need hold them to be independent of the creative thought of the Deity. As yet, science does not decide either for or against any of the current systems of philosophy. Some are undoubtedly more in harmony with its spirit than others; but we can hardly reckon among the former a theory so averse to the conceptions of the differential calculus, and so prone to hard and discrete distinctions, as the one we have noticed. It is, however, a strongly characterized and scholarly piece of work, doing honor to American thought; and it is much to be desired that the world should see the system developed in its entirety.

This page intentionally left blank

*[*ONE, TWO, THREE*]*

One, Two, Three: Kantian Categories

Item 47 *MS 572: Summer 1886*

This is the day for doubting axioms. With mathematicians, the question is settled; there is no reason to believe that the geometrical axioms are exactly true. Metaphysics is an imitation of geometry, and with the geometrical axioms the metaphysical axioms must go too.

We have no reason to think that the sum of the three angles of a triangle is exactly equal to two right-angles. All that we can say is that the excess or defect is proportional to the area of the triangle, and that it is excessively minute even for the most enormous triangles of astronomy. The sum of the three angles of a triangle of unit size is a physical constant nearly equal to 180 degrees; but its exact value is unknown to us.

Since we have no reason to think that this constant is exactly equal to 180 degrees, and there is an infinite multitude of other values that it can equally well have, the odds are at present infinity to one against its being exactly 180 degrees, so that that hypothesis ought to be entirely dismissed from our minds.

It is difficult for us to believe that any physical constant, any finite quantity in nature, is primordial. It may be so, but we cannot help at least asking how it came to have the precise value that it has. Especially do we feel the need of an explanation when the quantity in question comes very near to unity, zero, or any other remarkable number. For then it is suggested that there must have been some cause tending to change the value of the constant and to bring it nearer and nearer to the number that it nearly equals. In such a case, therefore, we have a positive reason for thinking that the quantity is not primordial.

Thus the principles of logic require us to think that space did not always have its present simple construction, but that this has been brought about by some gradual process. This is not quite a correct statement, however, because space, as an individual receptacle of things, is a fiction. Were it otherwise, absolute position and absolute velocity in space would mean something, which we have no reason to think that they do. What is true is that there are certain general laws of position, but not that there is a receptacle which accounts for those laws. This is a fiction of geometry.

The same reasoning applies to the axiom that every thing that happens is completely determined by exact laws. We have no reason to think that the accordance of phenomena with formulae is absolutely exact. Whenever we attempt to verify the accordance of fact with law, we find discrepancies which we rightly enough attribute to errors of observation. But we cannot be sure that there are not similar, though much smaller, aberrations in the events themselves. Since we have no reason to think that the mean aberration of phenomena from law is equal to zero, it is infinitely more probable that it is not. We must therefore suppose an element of absolute chance, sporting, spontaneity, originality, freedom, in nature. We must further suppose that this element in the ages of the past was indefinitely more prominent than now, and that the present almost exact conformity of nature to law is something that has been gradually brought about. We have to suppose that in looking back into the indefinite past we are looking back towards times when the element of law played an indefinitely small part in the universe.

If the universe is thus progressing from a state of all but pure chance to a state of all but complete determination by law, we must suppose that there is an original, elemental, tendency of things to acquire determinate properties, to take habits. This is the Third or mediating element between chance, which brings forth First and original events, and law which produces sequences or Seconds. Now the tendency to take habits is something essentially finite in amount, an infinitely strong tendency of this sort [unlike an absolute conformity to law] is inconceivable and self-contradictory. Consequently this tendency must itself have been gradually evolved; and it would evidently tend to strengthen itself.

Here then is a rational physical hypothesis, which is calculated to account, or all but account for everything in the universe except pure originality itself. The next step in order would be to attempt to verify this hypothesis by seeing how far it would account for and explain the observed characteristics of the laws of nature. But I postpone that to another chapter in order now to sketch the remainder of the theory of which this hypothesis is but a part.

One, Two, Three

Item 48 *MS 573: Summer–Fall 1886*

Contents

Trichotomic is the art of making threefold divisions. It depends on the conceptions of 1^{st}, 2^{nd}, 3^{rd}.

I.

I am going in this book to propound a hypothesis about the constitution of the universe. The hypothesis is constructed from three elementary conceptions,—or perhaps I should rather say three mental tones,—the ideas of First, Second, Third.

By the First, I do not at all mean the philosophical notion of the One, as a synthetizing unity or Whole. I mean simply what presents itself as first, fresh, immediate, free, spontaneous. An object at first blush, unreflectively taken. Undifferentiated. What Adam on first opening his eyes thought of the world before examining it.

The Second, last or term or end is in the fact of otherness, relation, force (not in the abstract, but as it feels when one gets hit), effect, dependence, occurrence, reality, upshot.

The Third is the medium, or that which mediates between the absolute first and absolute last. Continuity. Process. Flow of time. Sympathy. Comparison, Exchange. Modification, compromise (an outward shallow sort of third). Sign, representative. Combination, mixture. Half-breed. Pin. Coherence. Whole. Ordering & legislation but Law established & active force = 2^{nd}.
Means = 3^{rd} but End = 2^{nd}. Thread of life
But cut the thread = 2^{nd}
Class. Generality. But conformity = 2^{nd}
Rule.
Fork in the road (Road = 2^{nd} Place = 1^{st})
Triple for very. Plurality (but mere manifold = 1^{st})
Growth, change. Generation Plasticity
Curve = 3^{rd} Broken line = 2^{nd} Straight line = 1^{st}
Accelerative force = 3^{rd} Impulsive force = 2^{nd}

II.

Logic treats of signs. A sign is a third.
Three kinds of signs
Icons, Indices, Tokens.
Terms, Propositions, Argumentations.
The divisions of propositions are dual.
Deduction, Induction, Hypothesis.
Three figures of Syllogism.
Different kinds of terms.

IIa.

The very first philosophical conception that appeared in early Greece was that of primal matter. What is the world made of? was the question of Thales and the other Ionian philosophers. "Arche," was their word,—the beginning. They sought the First. Inquiry had sprung out of the wonder of awakening intelligence at the variety, the inexhaustible wealth of manifoldness in the heavens and the earth. How did this come about? Tracing back of phenomena step by step to their causes by Baconian Induction, even if it had occurred to them, those inexperienced minds could not possibly have appreciated the need of doing, nor the possibility of it. For them, indeed, it would have been impossible. Besides, each step would only have carried them to another question of the same sort in an endless regress. If they were to account for the world it seemed to them that they must first determine where their account was to begin. To explain heterogeneity they must start from an indeterminate homogeneity, that is from the material out of which the world was formed. Herbert Spencer knows no better at this day. That was their conception of their problem.

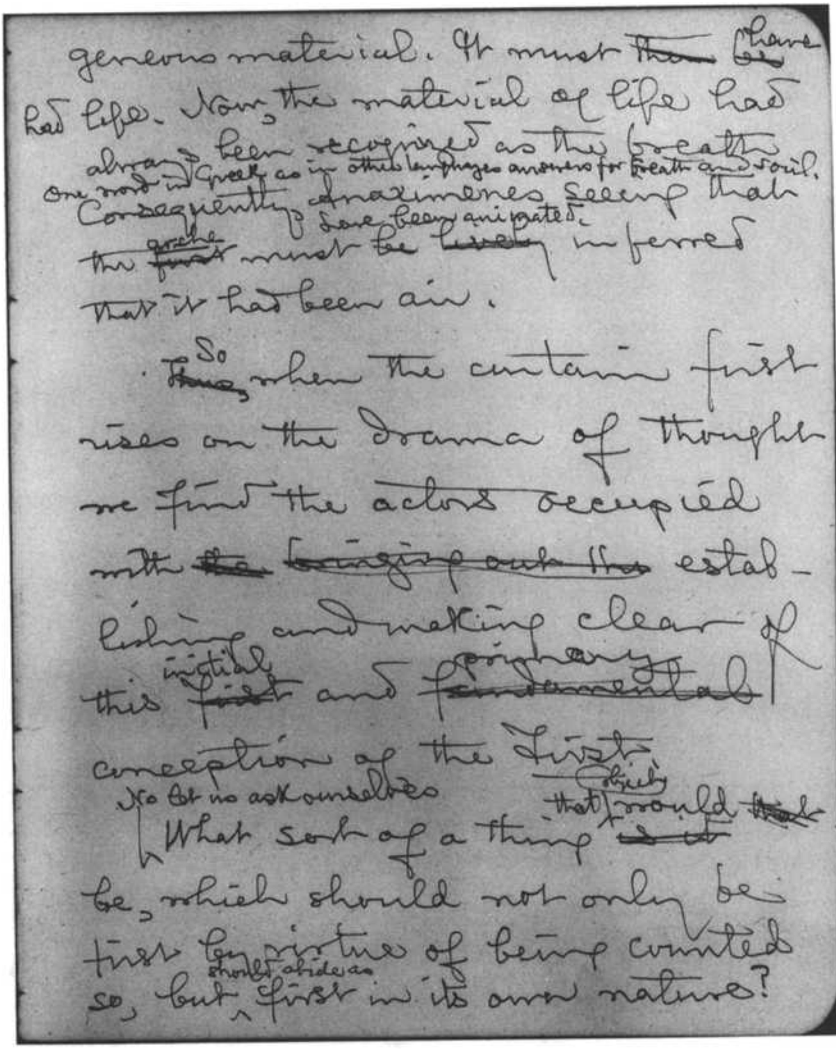
Now every intellectual undertaking must in its inception strike out with an original ejaculation of thought. A guess has always to be made, and Thales guessed at water as the first material of things; because the sea seemed to breed so many new forms.

This hypothesis was improved by Anaximenes, his line of reflection easily traced. The aim of speculation was to show how the variety of the world had come about, that is, to show how it had sprung from something homogeneous. But how is it possible that sameness should give birth to variety? Plainly not at all, unless by a fecund principle of spontaneity in that original homogeneous material. It must have had life. Now, the material of life had always been recognized as the breath. One word in Greek as in other languages answers for breath and soul. Consequently, Anaximenes seeing that the *arche* must have been animated inferred that it had been air.

So when the curtain first rises on the drama of thought we find the actors occupied with establishing and making clear of this initial and primary conception of the *First.*

Now let us ask ourselves what sort of a thing that object would be, which should not only be first by virtue of being counted so, but should abide as first in its own nature?

Anaximenes was not in possession of the three laws of motion nor of the dynamical principle of the "persistence of force" or conservation of energy, which would have sufficed to demonstrate the impregnable strength of his position. But he had the perspicacity to see the axiom which Spencer is blind to, that no dead force, no hard mechanical causation by fixed laws, could possibly suffice to bring organized heterogeneity out of unorganized homogeneity. To effect that, the homogeneous material must have life, or else some other principle of spontaneity if other can be found. Freedom must be of the essence of the material. No peculiarity in the forces acting on the matter but distinct from it would answer the purpose, for such a



generous material. It must ~~than~~ have ~~be~~ had life. Now, the material of life had always been recognized as the breath. One word in Greek as in other languages answers for breath and soul. Consequently Anaximenes seeing that the ~~first~~ archē must be ~~living~~ have been animated, inferred that it had been air.

~~Thus~~ So when the curtain first rises on the drama of thought we find the actors occupied with ~~the~~ ~~bringing out the~~ establishing and making clear of this ~~first~~ initial and ~~fundamental~~ primary conception of the First. No let us ask ourselves What sort of a thing ~~is it~~ that Firsts would ~~that~~ be, which should not only be first by virtue of being counted so, but should abide as first in its own nature?

Holograph page from MS 573
(Harvard Peirce Papers MS 905)

peculiarity in the forces would itself be a heterogeneity requiring to be explained and referred back to a homogeneity. This is the clear utterance of philosophical perception; and no "science," as that word is understood by scientific men (which does not include such speculations as Spencer's) will or can gainsay it. No, though the theory may not go deep enough, the inference was logical that—for the main conception of a primal homogeneous material may be crude, but granted that—the primal matter must itself be living.

III.

In Psychology

Kantian division of the faculties: Feeling, Knowing, & Willing. Generally recognized as having some truth but inexact.

Association philosophers have 3 intellectual phenomena—Sensation, Association & Comparison.

Plato's ideas come from Pythagoras

Aristotle's Vegetative, Sensitive, & Rational Souls.

One, Two, Three: An Evolutionist Speculation

Item 49 ***MS 575: Summer–Fall 1886***

CHAPTER I—First, Second, Third.

This chapter is not written to enunciate any opinion, but only to interpret the significations I propose to attach to the words First, Second, Third. Abstract definitions would not answer the purpose; the notions are extremely broad,—vague, if you like; at any rate, it is not by any means minute accuracy which is needed to distinguish them, but the knack of attending to elements so universal as often to escape notice. It is therefore preferable to elucidate the ideas by setting various illustrations side by side; and I hope that if the reader will consent to put himself in a receptive attitude for the time, he will soon be able to gather my meaning, & so be in a situation to assent to or dissent from the doctrines contained in the following chapters.

The First is that which has its being or peculiarity within itself. The Second is that which is what it is by force of something else. The third is that which is as it is owing to other things between which it mediates.

The First presents itself as original, immediate, fresh, unsubjected to anything that went before or stands behind, and is therefore spontaneous and free. Of these words, fresh, perhaps, nearest expresses the idea in its virgin purity. Some of the others seem to imply a series of which the first is the beginning, or otherwise to refer to something not first. They attain the notion by a circumlocution, as what is not yet second, instead of apprehending the first in its original positiveness. The instant that freshness is distinctly asserted, it has lost its characteristic innocence. The idea of prime precedes all assertion, all differentiation. There is no synthetical unity in it, no wholeness nor consistency; it is the sheer wonder and manifold of first impressions. In itself, however, it is not manifold; it has no parts; but because it has no wholeness nor consistency, the understanding analyzes it into an infinitely varied manifold. Kant talks inaccurately of the manifold of sense; in fact the first impression has no parts, any more than it has unity or wholeness; yet it may be allowed to be potentially a manifold, if we say that all that the intellect evolves from it lies involved within it. The pure First is essentially vivid, present, and conscious; for that which is dead or remote is as it is only for him who may perceive it. What the world was to Adam on the day he opened his eyes to it, before he had drawn any distinctions, or had become conscious of his own existence,—that is first.

The Second, last, term, or end is found in such facts as Another, Relation, Force [not in the abstract, but as it presents itself to one who gets hit], Effect, Dependence, Occurrence, Reality, Upshot. Another is a second; relations subsist between pairs; force really couples things and constitutes their real relations; and effect is that which follows or comes second to a cause; dependence is the character of what is secondary. An occurrence meets us, runs up against us; and *ob* or against expresses the idea of second with peculiar distinctness. The Real, the stubborn fact, is that which is as it is, whatever I may opine about it, and which I must accordingly conceive as beside my thought, as a second or object over against me as subject or first. In the living feeling, the idea of first is prominent; in the hard fact of distinguishing assertion, that of second. The first and fresh is not purely so unless it is conscious; the completely second is dead. With what firstness

> The scarfed bark puts from her native bay;

with what secondness

> doth she return,
> With over-weathered ribs and ragged sails.

Fate with its sure result is second; *chance* with its irregular manifoldness is first. Conflict involves a second; *duel* is dual. So with negation, as a kind of conflict; and lying as a negation of truth [*duplicity*]. The deed, as implying action and reaction, implies a second; while the thought is restrained within the first.

There are two kinds of second, the External or normal, and the Internal or degenerate. For example, all relation implies a second, but identity is a kind of relation which makes a thing second to itself. To say one dines with Duke Humphrey, is to count oneself as company. We speak of motives or allurements as forces, as if I were under compulsion from within. So with duty, and the voice of conscience. An echo is my own voice coming back to answer itself. All likeness is mere internal secondness,—an identity in the characters of the resembling things. Relations which are mere comparisons, the members being related only in virtue of characters which each could equally well have were the other annihilated, were called relations of reason by the old logicians, in contrast to real relations. They are the seconds of the internal type.

By the Third, I understand the medium which has its being or peculiarity in connecting the more absolute first and second. The end is second, the means third. A fork in the road is third, it supposes three ways. A straight road [considered merely as a fact of connection, and not as passing through intermediate places, or being anything in itself] is a second. No putting of straight roads end on end will make a fork, but repeated forkings will give any number of branches. Position is first, velocity or the relation of two successive positions supposes a second, acceleration or the relation of three successive positions involves a third. But velocity, so far as it is continuous, likewise involves a third. The first and second are hard, absolute, and discrete, like *yes* and *no;* the perfect third is plastic, relative, and continuous. Every process, and whatever is continuous, involves thirdness. Moderation is of that sort. The positive degree of an adjective is first, the superlative second, the comparative third. All exaggerated language, "supreme," "utter," "matchless," "root and branch," is the furniture of minds which think of seconds but forget thirds. Action is second, but conduct third. Law as an active force is second, but order and legislation third. Sympathy, flesh and blood, that by which I feel my neighbor's feelings, contains thirdness. Every kind of sign, representative, or deputy, everything which for any purpose stands instead of something else, whatever is helpful, or mediates between a man and his wish, is a Third.

Thirds are capable of two degrees of degeneracy. The undegenerate third is that which mediates between its extremes by virtue of a character which it could not possess were either of the extremes to be removed. Such, for example, are comparisons, gifts, compacts, things exchanged. Such, too, is a proposition, in that it attaches a predicate to a subject. A third degenerate in the first degree is one which mediates between its extremes by virtue of two characters, one of which alone it would lose if one extreme were removed, the other if the other extreme were removed. Such is a pin which fastens things together. Such is a mixture (not a compound but a mere aggregate) of different substances. We may call these *accidental* thirds. " 'How did I slay thy son?' asked the merchant, and the genie replied, 'When thou threwest away the date-stone, it smote my son, who was passing at the time, on the breast, and he died forthright.' " A third degenerate in the second degree is one which mediates between its extremes by virtue of a character which it would possess even were both extremes destroyed. Thus, Philadelphia lies between New York and Washington simply because it has a geographical position which it would have if the other cities were burnt down. In like manner, a right angle is intermediate between an acute and an obtuse angle, and zero is intermediate between positive and negative quantity. We may name these *comparative thirds.*

Fourths, fifths, etc. are not essentially different from thirds. They simply mediate between a larger number of terms. The singular, dual, and plural are all the essentially different forms of number.

The conceptions which I have thus very imperfectly described are, of course, far from being new: everybody makes use of them a thousand times a day, though usually without a very full apprehension of them. They have also played a great part in philosophy, particularly in that of Hegel. The account which I give of them, however, is very different from his, most of all in my noting two forms of second, while he chiefly recognizes an internal second, the more characteristic external force hardly playing any part in his logic. This important difference sufficiently accounts for my pursuing in what follows a method as unhegelian as possible.

[First, Second, Third]

Item 50 *MS 578: Summer–Fall 1886*

I am going to try to dash off, in intervals snatched from breadwinning, as well as my tired brain will, some poor sketch of ideas which have interested me intensely, and which I hope may interest some readers. Perhaps it may serve as a hint to some thinker more happily circumstanced than I, out of which he may work something of utility to mankind. I offer among other things a highly speculative theory of the origin of things, a theory of the development of the universe, which I believe that every intelligent man will say is worth considering, and this I suppose will receive some notice, though nothing else in the book may. Yet this and all the rest is but the application of certain conceptions which as it seems to me ought to reign over every department of philosophy.

First, I will try to convey some preliminary rough notion of these guiding conceptions. They are not my discovery; in special and unphilosophical forms, they are familiar enough. They are well-known in philosophy; and have formed the basis of more than one famous system, already. But I have my way of apprehending them, which it is essential to bring to the reader's mind. They are nothing but the three conceptions which rule in the majority of three-fold divisions. Now triads have at all times recommended themselves to all minds. There are psychological attractions for other numbers than three. Two is the number of hard common sense, of the stern moralist, of the practical man. "Yes or no? Answer me categorically," says such a man. Heaven and hell, right and wrong, truth and fiction, gain and loss, agent and patient, living or dead,—on such distinctions our practical life turns. Fours play a remarkable part, too; music is mostly built out of fours, there are four cardinal points, etc. Seven, twelve, and sixty, are other numbers to which there have been strong leanings. In regard to these higher numbers I express no opinion. It only concerns my purpose to develope the ideas of one, two, three. For what mystic reason should there be three dimensions to space? Why do the actions of forces depend on three orders of differentials, the displacements, the velocities, or rates of change of the displacements, and the accelerations, or the rates of change of the velocities? Why are there three laws of mechanics,—I do not mean those rather artificial ones of Newton, but the laws of the preservation of mass, of momentum, and of energy? Why are there three fundamental units of measurement in physics, the units of mass, of length, and of time? Several religions have trinities, several philosophies have triads, both those that are true, and still more those that are false. For fiction even more than truth is inclined to triads. The three brothers in so many stories, three trials, three graces, fates, and furies, three apples of the Hesperides etc. point to a significant psychological fact.

Let me then try to show the reader how to grasp in their fullness and purity the conceptions of First, Second, and Third. These ideas are so very much broader than any others, that no training in ordinary thinking, nor even in mathematical analysis is much aid to the apprehension of them. The power of drawing accurate distinctions is not what is needed to draw ideas so broad that they might be called moods or tones of thought rather than conceptions. It would be useless to try to express them by abstract definitions; that would be the certain way to miss them. We must mainly rely on examples, wholly inadequate, but from which the reader may get some induction; together with remarks however exaggerated and absurd, which shall point in the right direction.

First, Second, and Third, are the beginning, end, and middle. The first is that which has its being or peculiarity within itself. The Second is that which is what it is by force of something to which it is second. The third is that which is as it is owing to things between which it mediates and which it brings into relation.

The first has nothing to do with the One of Parmenides, nor with the unity of the I think, nor with any of the other unities that are talked about in philosophy; for these are really unities of totality, and so imply plurality. But the First presents itself as unsubjected and unrelated to anything that goes before or stands behind, and therefore as present, immediate, fresh, new, initiative, original, spontaneous, free. Of these words fresh, perhaps, nearest expresses the idea in its virgin purity. Some of the others seem to imply a series of which the first is the beginning, or otherwise to refer to something not first. They attain something like the notion by a circumlocution, as what is not second, instead of apprehending the First in its original positiveness. Let freshness be distinctly asserted, it has lost already its characteristic innocence. Stop to think of it and it has flown. If we could hold the living present and arrest the flow of time,—or rather if we could bring back the beginning of all time and keep it steadily present,—silly as it is to talk of such a thing, the notion shows the impossibility of actually grasping the conception of the First. The idea of prime precedes all assertion, all differentiation. There is no synthetical unity in it, no wholeness nor consistency; it is the sheer wonder and manifold of first impressions. In itself, however, it is not manifold; it has no parts; but because it has no wholeness nor consistency, the understanding analyzes it into an infinitely varied manifold. Kant talks inaccurately of the manifold of sense; in fact, the first impression has no parts, any more than it has unity or wholeness; yet it may be allowed to be potentially manifold, by conceiving whatever the intellect evolves from it to lie involved within it. The pure First is essentially vivid, present, and conscious; for that which is dead or remote is as it is only to him that may perceive it. The livingness of consciousness is Firstness. What the world was to Adam on the day he opened his eyes to it, before he had drawn any distinctions or had become conscious of his own existence,—that is first.

The Second or last is found in such facts as Another, Relation, Compulsion (not in the abstract, but as it presents itself to him who gets hit), Effect, Dependence, Independence, Occurrence, Reality, Result. An other is a second; relations subsist between pairs; force really couples things and constitutes their real relations; an effect is that which follows or comes second to its cause; dependence is the character of what is secondary; the independent is also a second. An occurrence meets us, runs up against us, and *ob* or against expresses the idea of second in tolerable purity. The Real, the stubborn fact, is that which is as it is, whatever I may opine about it, and which I must accordingly acknowledge as beside my thought, as a second or object over against me, the subject or first. In the living feeling the idea of first is prominent, in the hard fact of distinguishing assertion, that of second. As the first and fresh is not purely so unless it is conscious, so the completely second is dead. With what firstness,

> The scarfed bark puts from her native bay;

with what secondness,

> doth she return,
> With over-weathered ribs and ragged sails.

Fate, both as a final result, and as something sure and determinate, is second; chance, with its irregular manifoldness, and with its uncausedness or uncertainty, is first. Conflict involves a second, duel is dual. So with negation as a kind of conflict; and lying as a negation of truth (duplicity). The deed as involving action and reaction, implies a second; while what we only dream of is restrained within the first. On the whole, the Second must be reckoned as an easy conception. It is the main lesson of life. The first is too tender an idea to be laid hold of without destroying it; the slightest admixture of the idea of second renders it no longer the positive first. But the second is hard and tangible, and an admixture of firstness, far from destroying it, is a necessary ingredient of it. Yet familiar as the notion of second is in our mouths and our actions, after all we can only be conscious of the first. We are forced to acknowledge our limitations, but in the drunken dream of our immediate feeling we remain gods. Secondness, limitation, death, is something that we must admit but can never realize.

The only conception of the series which is quite realizable is that of the third. The Third is the medium whose function is to bring the more absolute First and Second into relation. The beginning is first, the end second, the means third. *A* and *B* the parties of the first and second part make a contract; that contract, *C*, is what brings them into relation, and herein lies its essence. A sign is a third; it brings the mind and the thing signified into relation, and if it should fail to do so it would fail to be a sign. Any kind of a representative or deputy which stands for another thing for any purpose mediates between that thing and the purpose. Any means, any process, anything that brings anything about is a third. Continuity involves thirdness in an eminent degree. All measure, all quantity is thirdness. A qualitative theory is one which makes only dual distinctions, distinguishes only between having a character and not having it. A quantitative theory makes plural distinctions.

It //might/may// be asked why stop, at the third? Why not go on to new concepts involved in the higher numbers, 4, 5 etc. indefinitely. But it is easy to show that fourness is only a complication of threeness, while three is not to be brought about by twoness alone. A combination necessarily supposes three terms, the two terms at least to be united and the uniting whole. Therefore, combinations of duals are impossible without the idea of threeness coming in. But the idea of combination once had, *A* and *B* can first be combined to make a whole *Z*, and then this *Z* can be combined with a new term *C* to make a new whole *Y*, and then this *Y* can be combined with a new term *D* to make a new whole *X*, and so on ad infinitum. Therefore the Greek language is more profoundly true than ours in its Singular, Dual, and Plural numbers. If there is such a thing as a priori grammar, it is as much violated in using the plural for the dual as it would be in using the singular for the plural, as we do in point of fact find it convenient to do in algebra. A relation between two objects if not regarded as an object, but only modally thought, is not fully a third; it is only the pairedness of those two things. The third is an object uniting two objects. An object uniting two objects and capable of being made the term of a similar junction, leads on to a combination of any order. A straight road between two places, if not regarded itself as a place, is not a third place but only the pairedness of the two places it connects. But a forking road involves a third place. Now no number of straight roads put end on end will ever have more than two ends after all; but forking roads put end on end will ramify into any number of ends. In like manner, in chemistry, were there no atoms but univalent ones, that is such as are capable of pairing only, there could be no combinations but binary combinations. Whereas bivalent atoms, or those capable of uniting with two others, which are therefore thirds, might give rise to combinations of any number of atoms. But bivalent atoms may be considered as involving only secondness in respect to having only two free bonds, and consequently they can only unite two univalent atoms however they may be arranged and multiplied. While trivalent atoms because they have three free bonds will serve to unite any number of univalent atoms.

If *B* is second to *A*, then reciprocally *A* is second to *B;* but it may happen that one is more appropriately regarded as first and the other as second. At the beginning there is freedom and indeterminacy; at the end matters are hardened and irrevocably fixed. The agent is free, the patient forced. In most cases the reason for the preference is sufficiently obvious; in others it is not quite so. Thus, of many pairs of correlative ideas one is derived from the other, and is usually considered as second. Thus negatives are second to positives. We could not have had the idea of a transparent body, if it were not for the opaque ones. Yet truth is a notion attained by experience of falsity, and yet we put truth first. And that though falsehood is free while truth is constrained. But we put truth first, because we identify it with the reality. That seems, therefore, to be a false point of view. Truth is rather the finality.

So in a triad any one of the six orders of taking the terms is in a certain sense a true one. But one arrangement is more appropriate than any other. We usually adopt an order different from that which I use, calling the middle second and the end third. This is the order of time, while my order is that of the simplicity of conceptions. The Latin language arranges its sentence in the way that seems to me rational, putting the subject first, the object second, and the verb which brings them into relation third. But the English language babbles on like a brook, the stick began to beat the dog, the dog began to worry the cat, the cat began to catch the mouse, the mouse began to nibble the corn, etc., is more sensuously true.

I now wish to call attention to certain important modifications of these conceptions. Secondness is of two grades, 1st, the normal and genuine and external secondness, where one thing really acts upon another, which I call external secondness; and a degenerate secondness (to borrow a term from the geometry of curves) where there is no pairing in the fact itself, but only in thought. There are furthermore two types of degenerate second. The first is where the things are related only in so far as they are compared; the second is where something acts upon itself. These types may be still further subdivided. When two things are in relation only because the mind compares them,—when to use the scholastic phrase, there is a relation of reason between them—what are directly in relation are not the things themselves but the ideas of the things. Now this relation between the ideas may be so far not degenerate, that there is a real first and second between them as when one has a positive character in a higher degree than the other. Or this relation may be one of mere identity, constituting a similarity between the objects. When a thing acts upon itself, one part commonly acts upon another part, so that in truth there is a real first acting upon a second external to itself. Thus when a man drowns himself, the man that jumps overboard seems to be distinguished by quite a different purpose from the fellow struggling in the water. When the relate and correlate are absolutely indistinguishable, as in consciousness, the designation internal secondness seems particularly appropriate.

Note on a Device for Abbreviating Time Reductions

Item 51 *P 334:* **Coast Survey Report 1885,** *503–8*

The simple suggestion that I wish to make is that by multiplying the observed times of transit (after applying a provisional correction and subtracting the right ascension) by the cosine of the declination, the field-reduction is much facilitated, and the labor of the least-square reduction is decidedly lessened. This arises from two circumstances: First, that the azimuth, level, and collimation coëfficients are much quicker taken out, and for northern stars quicker used; and second, that the weights to be assigned to the observations have a small range and are readily applied, while in a rough reduction they may be omitted without disadvantage.

The following are the values of the coëfficients:

	Old way.	New way.
Azimuth.	$\sin \zeta \sec \delta$	$\sin \zeta$
Level.	$\cos \zeta \sec \delta$	$\cos \zeta$
Collimation.	$\pm \sec \delta$	± 1

The old coëfficients have to be taken out of a table of double-entry (which is always disagreeable), and this table extends over eight pages quarto at least. The new coëfficients are taken instantly and with greater accuracy from a single-page three-place table of sines and cosines. The old table is of no use for any other purpose, but the little table of trigonometric functions is invaluable for a thousand purposes, and ought to be at every observer's elbow, whether this way of reducing time-observations is used or not. The old coëfficients have often three significant figures, even when only two places of decimals are used; but the new ones never exceed unity, and a two-by-two place multiplication table, such as Waldo's, is all that is required with them.

The weights of times of transit of stars of different declinations, according to the researches of Mr. Schott, agreeing with the formula of Dr. Albrecht, are given by the formulæ

$$p = \frac{1}{1 + \left(\frac{36}{63}\right)^2 \tan^2 \delta} \text{ for the C. S. large instruments,}$$

$$p = \frac{1}{1 + \left(\frac{63}{80}\right)^2 \tan^2 \delta} \text{ for the C. S. small instruments.}$$

The weights for $D = d \cos \delta$, or the residual times multiplied by the cosines of the declinations, are the above multiplied by $\sec^2 \delta$, which gives

$$P = \frac{1}{0.663 + 0.337 \cos^2 \delta} \text{ for the large instruments,}$$

$$P = \frac{1}{0.81 + 0.19 \cos^2 \delta} \text{ for the small instruments.}$$

Thus, for the large instruments, the weight ranges only from 1 at the equator to 3.06 at the pole, and for the small instruments from 1 to 1.6. The old way of reducing observations, as if all the times had equal weight, is entirely inadmissible even in a rough field-reduction. But after multiplying by $\cos \delta$, observations with the small instruments may be treated as of equal weight; while with large instruments, in a not very refined reduction, we may use the rule that $P = 1$ from the equator to 41° of declination, $P = 2$ from 41° to 70°, and $P = 3$ from 70° to the pole. It will never be worth while to use more than one place of decimals for the weights, which are given in the following table:

Declination.	P	.7 P	Declination.	P	.7 P
°			°		
0	1.00	0.70	50	1.65	1.15
5	1.01	0.71	55	1.82	1.27
10	1.03	0.72	60	2.02	1.41
15	1.05	0.74	65	2.24	1.57
20	1.09	0.76	70	2.47	1.73
25	1.14	0.80	75	2.69	1.88
30	1.20	0.84	80	2.88	2.02
35	1.28	0.90	85	3.02	2.11
40	1.38	0.97	90	3.06	2.14
45	1.51	1.06			

In a field-reduction, the unknown chronometer correction being small, the azimuth will be determined in each position by a north and a south star, and the residual D calculated for each star. This is corrected for aberration, and then the simple half difference of the mean residuals in the two positions of the instrument is the collimation, while the half sum (taking only high stars) is the chronometer error multiplied by the cosine of the latitude.

In reducing by least squares, we have the same quantities to calculate as by the old way; but these are obtained in a different manner, as shown by the following formulæ. The multiplications are generally easier by the new method.

Quantities used in least-square work.

$$p = \mathrm{P} \cos^2 \delta$$
$$p\mathrm{A} = \mathrm{P} \cos \delta \sin \zeta$$
$$p\mathrm{C} = \pm \mathrm{P} \cos \delta$$
$$pd = \mathrm{PD} \cos \delta$$
$$p\mathrm{A}^2 = \mathrm{P} \sin^2 \zeta$$
$$p\mathrm{AC} = \pm \mathrm{P} \sin \zeta$$
$$p\mathrm{A}d = \mathrm{PD} \sin \zeta$$
$$p\mathrm{C}^2 = \mathrm{P}$$
$$p\mathrm{C}d = \mathrm{PD}$$
$$\sqrt{p} \cdot d = \sqrt{\mathrm{P}} \cdot \mathrm{D}.$$

The following is an example: On 1885, April 5, I observed time with transit No. 5 (large size), at Key West, latitude + 24° $33\frac{1}{2}'$. The correction of the chronometer had been found to be

April 3, 10^{h} sid. t.	$+12^{s}.07$
April 4, 10^{h} sid. t.	$+11^{s}.34$.

It was, therefore, supposed to be about $+10^{s}.60$ on the 5^{th} at 10^{h}, with a diminution of $0^{s}.01$ every twenty minutes. The level was observed as follows:

			Corrected for pivots.		C
	h. m.	*d.*	*d.*	s.	s.
Clamp E.	9 18	− 19.1	− 20.7	= − 0.420	} − 0.372
	9 48	− 14.3	− 15.9	= − 0.323	
Clamp W.	9 52	− 22.4	− 20.8	= − 0.422	} − 0.402
	10 48	− 20.4	− 18.8	= − 0.382	

The stars observed, with their constants, were as follows:

Star.	Approx. time.		δ		cos δ	cos ζ	sin	*p.*
Clamp E.	h.	m.	°	°				
ι Argus.	9	14	− 58.8	+ 83.3	.518	.117	+ .994	2.0
θ Ursæ Majoris.		25	+ 52.2	− 27.6	.613	.886	− .463	1.7
10 Leonis Minoris.		28	+ 36.9	− 12.3	.800	.977	− .213	1.3
ο Leonis.		35	+ 10.4	+ 14.2	.984	.969	+ .245	1.0
ε Leonis.		39	+ 24.3	+ 0.3	.911	1.000	+ .005	1.1
μ Leonis.		46	+ 26.5	− 1.9	.895	.999	− .033	1.2
Clamp W.								
υ^2 Hydræ.		59	− 12.5	+ 37.1	.976	.798	+ .601	1.0
γ^1 Leonis.	10	13	+ 20.4	+ 4.2	.937	.998	+ .073	1.1
9 H. Draconis.		25	+ 76.3	− 51.7	.237	.620	− .785	2.7
41 Leonis Minoris.		37	+ 23.8	+ 0.8	.915	1.000	+ .014	1.1
β Ursæ Majoris.		55	+ 57.0	− 32.4	.545	.844	− .536	1.9

We begin the computation with two zenith stars observed in the two positions. The following gives the data and computation:

	Clamp E. ε Leonis.	*Clamp W.* 41 Leon. Min.
	h. m. s.	h. m. s.
Chron. time.	9 39 10.65	10 37 01.63
Assumed correction.	+ 10.61	+ 10.58
t	9 39 21.26	10 37 12.21
a	9 39 20.92	10 37 11.47
$t-a$	+ 0.34	+ 0.74
$(t-a)\cos\delta$	+ 0.31	+ 0.68
$b\cos\zeta$	− 0.37	− 0.37
	− 0.06	+ 0.31
Approx. azimuth.	.00	− 0.01
	− 0.06	+ 0.30
Corrected for aberration.	− 0.08	+ 0.28

The mean between the final figures or $\frac{1}{2}(-0^s\!.08 + 0^s\!.28) = +0^s\!.10$ is the chronometer error multiplied by the cosine of the latitude. Multiplying by the secant, or 1.1, we get $+0^s\!.11$ as the first approximation to the clock error. We therefore assume that instead of $+10^s\!.60$ at 10^h, the error was $+10^s\!.49$, with a diminution of $0^s\!.01$ every 17 minutes.

We now proceed to find the azimuth. The data and computations are as follows:

CLAMP E.

	ι Argus.	θ Ursæ Majoris.
	h. m. s.	h. m. s.
Chron. time.	9 13 53.76	9 25 0.55
Correction.	+ 10.52	+ 10.51
t	9 14 04.28	9 25 11.06
a	9 14 2.98	9 25 11.25
$t-a$	+ 1.30	− 0.19
$(t-a)\cos\delta$	+ 0.67	− 0.12
$b\cos\zeta$	− 0.05	− 0.33
	+ 0.62	− 0.45
$\sin\zeta$	+ 0.994	− 0.463

The difference between $+0\overset{s}{.}62$ and $-0\overset{s}{.}45$ or $+1\overset{s}{.}07$ is due to the difference between the two sines or $+1.457$. Hence, $a = \frac{1.07}{1.457} = +0\overset{s}{.}734$. Similarly, in the other position, we have the following data and calculations:

CLAMP W.

	υ^2 Hydræ.	9 H. Draconis.	β Ursæ Majoris.
	h. m. s.	h. m. s.	h. m. s.
Chron. time.	9 59 23.91	10 25 9.92	10 54 45.70
Corr.	+10.49	+10.47	+10.46
t	9 59 34.40	10 25 20.39	10 54 56.16
a	9 59 33.51	10 25 21.17	10 54 55.93
$t-a$	+0.89	−0.78	+0.23
$(t-a)\cos\delta$	+0.87	−0.19	+0.13
$b\cos\zeta$	−0.32	−0.25	−0.34
	+0.55	−0.44	−0.21
$\sin\zeta$	+0.601	−0.785	−0.536

Combining with υ^2 Hydræ, first, 9 H. Draconis, we get

$$a = \frac{+0\overset{s}{.}99}{+1.386} = 0\overset{s}{.}714$$

and second, β Ursæ Majoris, we have

$$a = \frac{+0\overset{s}{.}76}{+1.137} = 0\overset{s}{.}668.$$

As the former star has a weight of 3 and the latter of 2, we conclude

$$a = +0\overset{s}{.}696.$$

Applying this value of the azimuth to the high stars, we get the following results:

	Chron. time.	Corr.	Seconds of *a*	*t*−*a*	(*t* − *a*) cos δ	Level.	Az.	
Clamp E.								
	h. m. s.		s.					
10 Leonis Minoris.	9 28 01.73	+10.51	12.17	+0.07	+0.06	−0.36	+0.16	−0.14
ο Leonis.	34 52.22	+10.51	02.31	+0.42	+0.41	−0.36	−0.18	−0.13
ε Leonis.	39 10.65	+10.50	20.92	+0.23	+0.21	−0.37	0.00	−0.16
μ Leonis.	46 04.68	+10.50	14.91	+0.27	+0.24	−0.37	+0.02	−0.11
							Mean.	−0.14
							Corrected for aberration.	−0.16
Clamp W.								
γ^1 Leonis.	10 13 29.71	+10.48	39.58	+0.61	+0.57	−0.40	−0.05	+0.12
41 Leonis Minoris.	37 01.63	+10.47	11.47	+0.63	+0.58	−0.40	−0.01	+0.17
							Mean.	+0.14
							Corrected for aberration.	+0.12

It therefore appears that the collimation is $+0^{\mathrm{s}}.14$ and the additional chronometer correction $+0^{\mathrm{s}}.02$, making the correction at 10^{h}, $+10^{\mathrm{s}}.51$. We now apply these corrections to all the stars, with the following results:

Star.	*t*−*a*	(*t*−*a*) cos δ	Az.	Level.	Coll. and Ab.	D	P
Clamp E.							
ι Argus.	+1.32	+0.68	−0.73	−0.04	+0.12	+0.03	2
θ Ursæ Majoris.	−0.17	−0.10	+0.34	−0.33	+0.12	+0.03	2
10 Leonis Minoris.	+0.09	+0.07	+0.16	−0.36	+0.12	−0.01	1
ο Leonis.	+0.44	+0.43	−0.18	−0.36	+0.12	+0.01	1
ε Leonis.	+0.25	+0.23	+0.00	−0.37	+0.12	−0.02	1
μ Leonis.	+0.29	+0.26	+0.02	−0.37	+0.12	+0.03	1
Clamp W.							
υ^2 Hydræ.	+0.91	+0.89	−0.42	−0.32	−0.16	−0.01	1
γ^1 Leonis.	+0.63	+0.59	−0.05	−0.40	−0.16	−0.02	1
9 H. Draconis.	−0.76	−0.18	+0.55	−0.25	−0.16	−0.04	3
41 Leonis Minoris.	+0.65	+0.59	−0.01	−0.40	−0.16	+0.02	1
β Ursæ Majoris.	+0.25	+0.14	+0.37	−0.34	−0.16	+0.01	2

Taking the weighted means of D on the two sides, we see that the collimation has to be reduced by $0^{\mathrm{s}}.01$, while the chronometer correction remains unchanged. We then have for D

Clamp E.	Clamp W.
+0.02	0.00
+0.02	−0.01

−0.02	−0.03
0.00	+0.03
−0.03	+0.02
+0.02	

This concludes the field-reduction, and the magnitudes of the residuals illustrate its superiority over old methods. Finally, we will calculate further corrections by least squares. The quantities for the normal equations are calculated as follows:

P $\cos^2\delta$	P $\cos\delta$ $\sin\zeta$	$\pm$P $\cos\delta$	PD $\cos^2$ δ	P $\sin^2$ ζ	$\pm$P sin ζ	PD $\cos\delta$ sin ζ	$\pm$PD cos δ
Clamp E.							
0.5	+1.0	+1.0	+0.01	2.0	+2.0	+0.02	+0.02
0.6	−0.5	+1.0	+0.01	0.4	−0.8	−0.01	+0.02
0.8	−0.2	+1.0	−0.02	0.1	−0.3	0.00	−0.02
1.0	+0.2	+1.0	0.00	0.1	+0.3	0.00	0.00
0.9	0.0	+1.0	−0.03	0.0	0.0	0.00	−0.03
0.9	0.0	+1.0	+0.02	0.0	0.0	0.00	+0.02
4.7	+0.5	+6.0	−0.01	2.6	+1.2	+0.01	+0.01
Clamp W.							
1.0	+0.6	−1.0	0.00	0.4	−0.6	0.00	0.00
1.0	+0.1	−1.0	−0.01	0.0	−0.1	0.00	+0.01
0.2	−0.5	−0.6	0.00	1.6	+2.1	−0.02	+0.02
0.9	0.0	−1.0	+0.03	0.0	0.0	0.00	−0.03
0.6	−0.6	−1.0	+0.01	0.5	+1.0	−0.01	−0.02
3.7	−0.4	−4.6	+0.03	2.5	+2.4	−0.03	−0.02

The normal equations themselves are:

$$
\begin{array}{rrrrl}
 & & & & \text{s.} \\
8.4t & +0.5a & -0.4a' & +\ 1.4c & = +0.02 \\
0.5 & +2.6 & & +\ 1.2 & = +0.01 \\
-0.4 & & +2.5 & +\ 2.4 & = -0.03 \\
1.4 & +1.2 & +2.4 & +16 & = -0.01.
\end{array}
$$

The solution is

$$t = +.001 \quad a = +.003 \quad a' = -.012 \quad c = +.001.$$

So that the field-reduction cannot be improved.

I will add that it seems to me that this mode of reduction will be much more easily understood by inexperienced computers than the old one.

On the Influence of a Noddy on the Period of a Pendulum

Item 52 *P 335:* **Coast Survey Report 1885,** ***509–10***

Suppose a noddy, adjusted to accord with a reversible pendulum, remain on the pendulum-support throughout the experiments to determine gravity. How much can the results be affected by this circumstance?

Let us use this notation:

l and l', the lengths of the simple pendulums corresponding to the pendulum and noddy, respectively; that is, in each case the square of the radius of gyration divided by the distance between the centre of mass and centre of rotation;

μ and μ', the ratio of any linear displacement of the support to the angular displacement of the pendulum or noddy required to produce it;

τ and τ', the natural periods of pendulum and noddy;

T, the period of either harmonic constituent of the motion.

Then, the formula, easily derived from my paper on two pendulums on one support, is:

$$\mathrm{T}^2 = \frac{1}{2}\left\{\left(1+\frac{\mu}{l}\right)\tau^2 + \left(1+\frac{\mu'}{l'}\right)\tau'^2\right\}$$

$$\pm\sqrt{\frac{1}{4}\left\{\left(1+\frac{\mu}{l}\right)\tau^2 - \left(1+\frac{\mu'}{l'}\right)\tau'^2\right\} + \frac{\mu\mu'}{l\,l'}\tau^2\,\tau'^2}.$$

Any increase of τ' always produces an increase of T; and of the two values of T^2, one is always smaller, the other greater than

$$\left(1+\frac{\mu}{l}\right)\tau^2.$$

Consequently, the greatest effect is produced when one value of T^2 is as much greater as the other is less than

$$\left(1+\frac{\mu}{l}\right)\tau^2$$

that is, when

$$\left(1+\frac{\mu'}{l'}\right)\tau'^2=\left(1+\frac{\mu}{l}\right)\tau^2.$$

In this case,

$$T^2=\left(1+\frac{\mu}{l}\right)\tau^2\pm\sqrt{\frac{\mu\mu'}{l\,l'}\tau^2\tau'^2}.$$

Denote by M and M′ the masses of the pendulum and noddy, respectively, and by h and h' the distance in each between the centre of mass and the centre of rotation. Then

$$\mu\tau^2:\mu'\tau'^2=\frac{Mh}{l}:\frac{M'h'}{l'}$$

and

$$\sqrt{\frac{\mu\mu'}{ll'}\tau^2\tau'^2}=\frac{\mu}{l}\tau^2\sqrt{\frac{\mu'}{\mu}\frac{\tau'^2}{\tau^2}\frac{l}{l'}}=\frac{\mu}{l}\tau^2\frac{l}{l'}\sqrt{\frac{M'h'}{M\,h}}.$$

Assuming

$$\frac{M'}{M}=\frac{1}{100}\qquad\frac{h'}{h}=\frac{1}{36}$$

for heavy end down, $\frac{h'}{h}=\frac{1}{12}$ for heavy end up, and $\frac{l}{l'}=20$, it would follow that the effect of the noddy might be as great as $\frac{1}{3}$ of the flexure with heavy end down, and as $\frac{1}{\sqrt{3}}$ times the flexure with heavy end up. But it could not produce a sensible effect in both positions.

On the Effect of Unequal Temperature upon a Reversible Pendulum

Item 53 *P 336:* **Coast Survey Report 1885,** ***511–12***

The upper part of a pendulum swinging in a room not very lofty is always warmer than the lower part. Let us suppose that the temperature varies uniformly along the length of the pendulum.

A reversible pendulum consists of two parts, (first) the part symmetrical about the middle of the pendulum, and (second) the load, however distributed. We may distinguish letters referring to these two parts by the subjacent indices 1 and 2. Letters without an index may refer to the whole pendulum.

Considering, first, the symmetrical part, it is plain that the inequality of temperature cannot sensibly alter the radius of gyration of this about its centre of mass, which depends on the mean temperature only, since the centre of mass of this part is at the centre of the pendulum. The distance of the centre of mass from the supporting knife-edge will, however, be increased, say, by σ. Then, if M stand for mass, l for the length of the corresponding simple pendulum, and h for the distance of the centre of mass from the point of support, the moment of inertia will be (neglecting σ^2)

$$\mathrm{M}lh + \mathrm{M}_1 l\sigma,$$

that is, it will be multiplied by

$$1 + \frac{\mathrm{M}_1}{\mathrm{M}}\frac{\sigma}{h}.$$

But the moment of gravity will be

$$\mathrm{M}h + \mathrm{M}_1\sigma,$$

so that it will also be multiplied by

$$1 + \frac{M_1}{M}\frac{\sigma}{h}$$

and the period of oscillation, which depends on the ratio of the moment of inertia to the moment of gravity, will not be affected.

Let us now consider the load, which we may suppose to be symmetrical about its middle. Denote the distance of its centre from that of the pendulum by η. Let $\delta\tau$ be the excess of the temperature of the upper knife above that of the lower one. Then $\frac{\eta}{l}\delta\tau$ will be the difference of temperature of the centre of the load from the mean temperature of the pendulum. It will have the *minus* sign with heavy end down, the *plus* sign with heavy end up. Let k be the coëfficient of expansion. Then, every dimension of the load will, owing to the inequality of temperature, be multiplied by

$$1 \mp \frac{\eta}{l}k\,\delta\tau,$$

where the double sign corresponds to the two positions of the pendulum. The radius of gyration about the centre of the load (which we may denote by γ_2) will be multiplied by the same amount; and consequently the moment of inertia and the square of the period of oscillation will be multiplied by

$$1 \mp \frac{M_2}{M}\frac{\gamma_2^2}{hl}\frac{\eta}{l}k\,\delta\tau.$$

The middle of the distance from the supporting knife-edge to the centre of the load will be at a distance

$$\mp\frac{1}{2}\eta + \frac{1}{4}l$$

above the centre of the pendulum in the two positions, and this distance will be

$$\pm\eta + \frac{1}{2}l.$$

The inequality of temperature will therefore in both positions raise the centre of the load by

$$\frac{1}{2}\frac{\eta^2 - \frac{1}{4}l^2}{l}k\,\delta\tau.$$

The moment of inertia will therefore be multiplied by

$$1 \mp \frac{M_2}{M}\frac{\eta + \frac{1}{2}l}{l}\frac{\eta^2 - \frac{1}{4}l^2}{hl}k\,\delta\tau$$

the moment of gravity will be multiplied by

$$1 \mp \frac{M_2}{M}\frac{1}{2}\frac{\eta^2 - \frac{1}{4}l^2}{hl}k\,\delta\tau$$

and the square of the period of oscillation will be multiplied by

$$1 \mp \frac{M_2}{M}\frac{\eta}{l}\frac{\eta^2 - \frac{1}{4}l^2}{hl}k\,\delta\tau.$$

In order to make these effects as small as possible, the load should be concentrated as much as possible about one knife-edge. For this purpose, the symmetrical part might be an unloaded tube about $\sqrt{3}$ times the distance between the knife-edges in length. This construction would also have the advantage of eliminating the error due to the flexure of the pendulum staff.

In the case of the Peirce metre pendulums, we have

$$\gamma_2^2 = \frac{1}{4}\,(3\ cm)^2 + \frac{1}{12}\,(16.03\ cm)^2 = 23.66\ (cm)^2$$

$$M_2 = 4000 \text{ grams} \qquad \eta = 65.63\ cm \qquad M = 10500 \text{ grams}$$

$$h_d = 75\ cm \qquad h_u = 25\ cm \qquad l = 100\ cm \qquad k = .00001863 \qquad T = 1^{s}.003.$$

And the correction to the time of 5000 oscillations with heavy end up, or 15000 with heavy end down, per degree centigrade of difference of temperature between the knife-edges is

$$\pm 0^{s}.00856.$$

In the case of the Peirce yard pendulum,

$$\gamma_2^2 = \frac{1}{4}(3\ cm)^2 + \frac{1}{12}(15.19\ cm)^2 = 21.48\ (cm)^2$$

$$M_2 = 3770 \text{ grams} \qquad \eta = 60.68\ cm \qquad M = 10000 \text{ grams}$$

$$h_d = 68.58 \qquad h_u = 22.86 \qquad l = 91.44 \qquad k = .00001863 \qquad T = 0^s\!.9594$$

and the same correction is

$$\pm 0^s\!.00863.$$

Qualitative Logic

Item 54 ***MS 582: Fall–Winter 1886***

Preface.

This book is not intended to guide children in their first attempts to use their minds, nor does it address itself particularly to persons of great experience, who, while they may still correct their tendencies to be a little too credulous or too sceptical concerning any kind of evidence, have long since past the time when any consideration of the theory of drawing inferences could influence their practice;—although such persons may find something to interest them in these pages. But there is an age, between boyhood and manhood, when there is a natural tendency to look at life in a rather theoretical way and when such speculation is of real use and service. The native force of intellect is at that age perhaps as great as it is ever destined to be, but want of skill in handling the reason and inexperience of the deceptions to which it is subject render the effective power of the mind very inferior to that which is later developed. If a young man at that time of life will only acquire a distinct preliminary conception of the methods of thought and of inquiry by which alone the truth can be ascertained, though this theoretical knowledge will not make him a powerful thinker, it will form an admirable foundation upon which to build habits of effective thinking.

To such a young man, I offer an outline sketch of the whole method of reason. If he is disposed to accept what I say with implicit belief, I shall at least try not to abuse his confidence, so that he may not some day wake up to find that it was all an idle and delusive dream. If he feels strong enough to subject what I say to a critical examination, so much the better; I shall endeavor to make my statements and to give my reasons in such a way as to facilitate his investigation.

The name of the study which forms the subject of this volume is Logic. But logic as it is set forth in most books upon the subject, is a study far worse than useless. It tends to make a man captious about trifles, and neglectful of weightier matters. It condemns every inference which is really sound, and admits only those which are really childish. The reason is that this logic has been handed down from the Middle Ages, the ages of Faith, i.e. of Unreason, before modern science, physical and moral, had begun. The lore of those days was laborious nonsense, their law was regulated warfare, their medicine fetichism, their history fables, their astronomy astrology, their chemistry alchemy. They never, by any chance, themselves reasoned rightly. How then can they teach us to do so, or how can their theory of reasoning be of the slightest value?

This traditional logic did not, however, originate in the middle ages, but with Aristotle at a comparatively early period in the history of the Greek mind, when it was fully developed upon the artistic side, but before it had done much in the way of discovering truth. Aristotle's logic was a mere first essay; it did not fairly represent Aristotle's own methods of thought,—far less those of the early Greek mathematicians. It did not enjoy, in antiquity, the immense renown which has since attached to it; although it did faithfully mirror the distrust which the ancients generally entertained of Observation as a foundation for scientific truth. Those who did believe in observation, the Epicureans,—like Roger Bacon in the 13th Century,—were hostile to the Aristotelean logic.

In modern times, logic has very naturally had a bad reputation.

First, there was in the 16^{th} Century a pretended reform of the science in the interest mainly of literary elegance, by Peter Ramus. He imported the Dilemma from rhetoric into logic. Early in the 17^{th} Century came two important books, the *Discours de la Méthode* of Descartes, and the *Novum Organum* of Francis Bacon. The former represents, in a very vague sketch, but with great perspicuity, the methods of thinking of modern metaphysicians, of those who draw their convictions from within. The latter is an eloquent and majestic assertion of the dignity of nature and the littleness of man, who can only attain knowledge by observation. All the greatest steps in the progress of modern science have involved improvements in the art of reasoning. Harvey's discovery of the circulation of the blood, Kepler's researches on the orbits of the planets, Galileo's development of the principle of inertia, and many other such works contained lessons in logic,—which were perhaps even more valuable to the world and contributed even more to the progress of civilization than their more special teachings. This has been particularly the case with discoveries in mathematics. Pure mathematics, indeed, is nothing but an art of drawing conclusions of a particular description. But the great discoveries in mathematics have carried with them very important improvements in the methods of thinking concerning nearly every subject. In ancient times, the discovery by Euclid that the whole of Geometry can be deduced from a few elementary principles suggested the idea of metaphysics and profoundly modified all subsequent science and philosophy. In modern times, several mathematical discoveries have added even more momentous consequences. The Coördinates of Descartes are now constantly applied to every subject of reasoning with great advantage; while the ideas of the infinitesimal calculus have penetrated everywhere, forming, for example, an important not to say the principal factor in Ricardo's political economy.

The doctrine of chances is a direct contribution made by modern mathematicians to the general principles of logic. It may be called the modern logic for it decidedly outweighs all that was known of reasoning before this invention.

The present century has produced important treatises upon the theory of logic, which have all come either from Germany or from Great Britain. Two different schools have prevailed in the two countries, which have not in the least understood one another. The German school, of which Hegel is the type, have approached the subject from the side of theology and metaphysics; the English school, represented, for example, by Mill, have viewed the matter from the side of modern science. But there is good reason to trust that the breach between those two schools, which is but the continuation of a dispute existing almost since the birth of philosophy, is now in the process of being closed and healed.

Chapter I
The Association of Ideas.

In the absence of external impressions, thoughts chase one another through the mind in a sort of Bacchic train. Each suggests another. After a while, the clear train of thought is broken, the ideas remain scattered for a time, and then reconcentrate in another train. Psychologists recognize that the suggestion of one idea by another may take place according to either one of two different principles; for an idea may suggest another like it, or it may suggest another which has been connected with it in experience. Thus, the thought of Niagara may suggest a hero or anything else that is grand, and so similar to the cataract, or it may suggest a crowd of importunate hack-drivers, which is connected with the place in every visitor's experience.

Association of the latter kind, association by *contiguity* as it is called, is the more typical. In it the characteristics of mental association are more strongly marked. Association by similarity is related to association by contiguity somewhat as our inward consciousness is related to outward experience; the one association is due to a connection in outward experience, the other to a connection in our feelings. Many psychologists have proposed to reduce association by similarity to a special kind of association by contiguity; few have been inclined to reduce the latter principle to the former.

Suggestions of these two kinds characterize not merely dreams and dreamy meditations, but also thoughts referred to the real world, or in technical language *categorical judgments.* Association is the only force which exists within the intellect, and whatever power of controlling the thoughts there may be can be exercized only by utilizing these forces; indeed, the power, and even the wish, to control ourselves can come about only by the action of the same principles. Still, the force of association in its native strength and wildness is seen best in persons whose understandings are so little developed that they can hardly be said to reason at all. Believing one thing puts it into their heads to believe in another thing; but they know not how they come by their beliefs, and can exercize no control over the inferential process. These unconscious and uncontrolled reasonings hardly merit that name; although they are very often truer than if they were regulated by an imperfect logic, showing in this the usual superiority of instinct over reason, and of practice over theory. They take place like other mental suggestions according to the two principles of similarity and connection in experience.

Inference from connection in experience is most rudimentary of all reasoning. The lower animals plainly reason in this way. The dog, when he hears the voice of his master runs expecting to see him, and if he does not find him will manifest surprize, or at any rate perplexity. This is as good an example of inference from connection in experience as could easily be given.

Inference from resemblance probably implies a higher degree of self-consciousness than any of the brutes possess. It involves a somewhat steady attention to qualities as such; and this must rest on the capacity for language, if not on language itself. Primitive man, however, reasons in this way; for mythology is built of such inferences. Our ancestors saw something manlike in the sun, and could even tell what kind of a man the sun-god was.

But we need not go to the lower animals nor to savages for examples of associational determinations of belief. Our daily life is full of such phenomena. We have the naive idea that our beliefs are principally determined by the exercise of our conscious intellect; but it is not so. I converse with a man and learn how he is thinking: I fancy that he has told me, but he has not. I hear a new slang word, but I need not ask for a definition of it; I understand its meaning much better without a definition, I know not how, than I should with one. In riding a horse, I understand him and he understands me; but how we understand one another I know hardly better than he.

All such inferences are, of course, beyond the jurisdiction of criticism. It is the part of psychology to explain their processes as it can; but as long as they are out of consciousness, they are out of our control and it is idle to call them good or bad. We may, however, say that the ordinary business of life is best conducted without too much self-criticism. Just as our respiration, circulation, and digestion are far better carried on by involuntary than they could possibly be by voluntary actions, so the countless little reasonings which we are continually making,—although they may often be defective,—are nevertheless much better performed unconsciously than they would be if we were to try to interfere with them by a captious and hypochondriac logic. It is very different with the actions which we undertake in order to carry out our grander purposes. Here, all must be voluntary, thoroughly conscious, and based upon the most critical reflection. Here logic is wanted, to pull to pieces our inferences, to show whether they are good or bad, how they can be strengthened, and by what methods they ought to proceed. Intermediate between the lesser and the greater inferences, lies an intermediate class which are best governed by habits, yet by habits formed or corrected under conscious criticism. Within a man's own special profession, his habits of thinking will have been subjected to a good deal of criticism (not necessarily the most intelligent, but based on experience); outside of that, if he has not studied logic, his habits of thinking will have been carelessly formed, or if he has studied the traditional logic, they will have been formed under an influence truly baleful.

Chapter II.
The Simple Consequence

Reasoning unconsciously can hardly be called reasoning at all. As long as I simply find myself seized with a belief, without being able to give any account of how I came by it, logic has nothing to say except to warn me of the extreme danger that I shall err.

Reasoning proper begins when I am conscious that the judgment I reach is the effect in my mind of a certain judgment which I had formed before. The judgment which is the cause is called the *premise,* that which is the effect is called the *conclusion.* When I am aware that a certain conclusion which I draw is determined by a certain premise, there are three things which I have more or less clearly in my mind. First, I have a peculiar sense of constraint to believe the conclusion, connected with a sense that that constraint comes from the premise; second, I have a conception that there is a whole class of possible analogous inferences (though I may not be able to define the class) in which a similar constraint would be felt by me; and third I have a present belief that all of these inferences, or at least the great body of them would be true.

The lowest kind of conscious reasoning is where I know what the premise is from which my belief in the conclusion follows, and I feel that it follows upon some principle,—technically called the *leading principle* of the inference,—but I do not distinctly know what that principle is. Such an inference is called a *simple consequence.*

Such inferences are common enough. Uneducated people seldom reason in a higher way; and educated people reason so very often. Since the object of reasoning is merely to arrive at the truth, if the leading principle of our inference be really true, it is not necessary for us to know it, for in that case the mode of inference based upon it can in no case carry us away from a true premise to a false conclusion. Such a mode of inference is *valid,* that is, its leading principle is true, but it is only *materially valid,* that is, valid because as a matter of fact the leading principle happens to be true, it is not *logically valid,* that is, the leading principle might be false. Although the simple consequence may be valid, logic condemns it. A conclusion may be true, though the inference by which it was reached was invalid; that is, it may chance to be true. But the inference is condemned because other conclusions similarly drawn may be false. In the same way, a simple consequence may happen to be valid; but since the leading principle is not recognized, there is no security for its validity, and the next simple consequence drawn, though indistinguishable from the first (the leading principle being unrecognized), may very likely fail.

Simple consequences have occasionally been introduced into philosophy. The most remarkable instance is the *Cogito, ergo sum* of Descartes, who wished every philosopher to begin by doubting everything without exception. But even in doubting everything he must, says Descartes, be aware that he doubts, that he thinks. Now from this belief that he thinks he is led by a blind but irresistible constraint to believe in his own existence. This inference, "I think, therefore I am," is a simple consequence, for as long as the philosopher doubts everything he can have no fuller reason to give. He cannot, for example, say that thought supposes a thinker, for he is bound as a Cartesian to doubt that among other things at the outset. Such is the doctrine of Descartes. Of course, were there nothing to check the absolutely resistless force of a belief, logic must be silent. *Leges silent inter armes.* But in point of fact, no belief is found thus absolutely irresistible. There is always room for the reflection that an error may have been committed.[1]

1. Absolute doubt is also impossible.

Chapter III.
The Modus Ponens.

When the validity of the simple consequence, "*P*, therefore *C*," is challenged, we are led to search for the principle upon which the inference proceeds; and perhaps we may come to recognize this in the judgment that if *P* is true, *C* is true. It would be a mistake to suppose that is always even tacitly assumed in the simple consequence; for it may be the manner in which the premise, *P*, presents itself to my notice, and not its bare truth alone, which causes my judgment that *C* is true. In any case, in drawing the simple consequence, "*P*, therefore *C*," I do not consciously judge that "If *P*, then *C*." But in the after criticism of the inference, I shall be apt to recognize this proposition as its principle; and from that moment my reasoning ceases to be a simple consequence, because the proposition "If *P*, then *C*," now becomes a second premise. In short, I now reason in the form

If *P* is true, *C* is true;
But *P* is true;
Hence, *C* is true.

This form of inference is called the *modus ponens* (or positing mode, to distinguish it from the *modus tollens*, or removing mode, which will be noticed hereafter). *P* is called the *antecedent*, *C* the *consequent*. The hypothetical proposition is called the *consequence*.

For example, a little girl might reason,

If I am good, my mamma will love me;
Now, I will be good;
And so my mamma will love me.

We have seen that even in drawing a simple consequence, I have a vague consciousness of a body of possible inferences to which the inference actually drawn belongs. In the *modus ponens*, the conception of the Possible emerges more clearly. We have here, as one of the premises, a judgment, "If *P*, then *C*," which is not *categorical*, that is, does not relate to the real world alone, but is *hypothetical*, that is, is meant to apply to a whole universe of possibilities. To say that a fact is *possible* means primarily that it is not known not to be true, that our knowledge leaves room for it. But there are, besides, kinds of possibility, which are determined not by the knowledge which we happen just now to have, but by every conceivable state of information. Thus, we say, Napoleon did not win the battle of

Waterloo, but he might possibly have done so. We use the past tense, *might,* because the fact supposed is now no longer consistent with what we have learned; but we mean that a knowledge of all the previous conditions, capable of being known beforehand, would leave room for such an event. We say that it is physically possible that a needle should be so balanced as to rest upright upon its point. We mean that if we knew only the laws of physics and not various familiar facts, we should not know that this did not happen. We say that it is impossible for anything in nature to happen otherwise than it does happen. We mean that if we knew all past facts and all the laws of nature and all that could be deduced therefrom we should know everything that was about to happen. There is therefore a sense in which only the actual is possible. At the furthest extreme from that is the logically possible. A proposition is logically possible when if we knew no facts at all, but only the meanings of words, we could not reject the proposition. It is true that such a state of knowledge is itself in a certain sense impossible, like a geometrical line or surface; but it is none the less a very useful conception.

A supposed fact which would be true in *some* or *all* the states of things for which an assumed condition of knowledge leaves room is said to be *possible;* one which is true in none of these states is said to be *impossible;* one which is true in all is said to be *necessary;* one which is true in a part or none is said to be *contingent.*

To say that "If P is true, C is true," means that in an assumed condition of information, every possible state of things in which P would be true is a state of things in which C would be true.

Every addition or improvement to our knowledge, of whatsoever kind, comes from an exercize of our powers of perception. In necessary inference my observation is directed to a creation of my own imagination, a sort of diagram or image in which are portrayed the facts given in the premises; and the observation consists in recognizing relations between the parts of this diagram which were not noticed in constructing it. Different persons no doubt construct their logical diagrams in different ways; many probably very oddly; but every person must construct some kind of a diagram or its equivalent, or he could not perform necessary reasoning, at all. It is a part of the business of logic to teach useful ways of constructing such diagrams. The whole range of logical possibility may be represented by an imaginary sheet of paper occupying the whole field of vision. Every point on this sheet is to represent some conceivable state of things, of which the real state of things is one, undistinguished from the rest. Everything learned cuts off and removes from this sheet some part and leaves a range of possibility less than the whole range of logical possibility, but still containing the unknown point which represents the actual state of things. To form a diagram of the truth of the hypothetical proposition "If *P* is true, *C* is true," we must suppose that all the points which represent states of things in which *P* is true are gathered together in one area, and that a line is drawn around this area; and that the same is done with the points representing states of things in which *C* is true. If then the boundary of the area of the truth of *P* is imagined to lie wholly within the area of the truth of *C*, the hypothetical proposition is represented as true. If this happens before the original sheet has been cut down at all, the proposition is represented as true by logical necessity; but if the sheet has been cut down it is only represented to be true for some degree of knowledge not defined.

Let us now suppose that, in addition to the hypothetical proposition, we are certain that the antecedent, *P*, is true. Then, in our state of knowledge it is not possible for *P* to be false; and to represent our knowledge, we must cut down the sheet of possibility so as to leave only a part lying wholly within the area of *P*. We perceive that this will also lie within the area of *C*, and therefore we shall be certain that the consequent is true.

[Variant Fragment 1]

It is quite essential to apprehend these distinctions of terms clearly. *Necessary* reasoning is not necessarily true or *valid* reasoning; nor is it necessarily *complete*. If a hypothetical proposition is true, whether it be known to be so or not (in a different state of information from that which determines its truth), the inference from antecedent to consequent is *valid*. But to be *logically valid* is quite another thing. The following scheme may assist the reader:

Necessary inference is either
- Complete, and this either
  - Logically valid,
  - or
  - Invalid.
- or
- Incomplete, and this either
  - Materially, not logically, valid,
  - or
  - Invalid.

For many pages, we shall study only necessary inference.

An inference is not rendered invalid because its conclusion is false; it is only a false conclusion following from true premises which is inconsistent with the validity of the reasoning. In necessary reasoning, at least, it is too much to expect that the conclusion should come out true, irrespective of the truth of the premises; that would be a magical logic. If it is possible for the premises to be true while the conclusion is false, the inference (supposing it necessary) is *invalid.* But if this is not the case, the inference must be allowed to be valid. That is to say, the inference is valid, if in each possible case either the premises are false or the conclusion true. Similar rules hold for logically valid inference. It follows that every inference is logically valid the conclusion of which is logically necessary, and every inference is logically valid the premise of which is logically impossible. These statements may surprize the reader. Take such an argument as this:

> Every whale is a fish,
> Therefore, a chimera (if there be one) is a chimera.

I fully admit that such an argument would be a bad way to convince anybody of the truth of the conclusion. But, nevertheless, the inference is logically valid. It may be thought that premise and conclusion have no logical connection, but this is not so; and the proof is that no system of logical rules can be devised which shall allow all acknowledged inferences to pass as good, and not allow such as these. This shall be shown, further on. The logically necessary may in fact be defined as that which logically follows from every premise; the logically impossible as that from which every conclusion logically follows. The materially necessary is that which is a consequent to every antecedent, the materially impossible that which is an antecedent to every consequent. Truth and falsehood are but special varieties of necessity and impossibility; namely such as are determined by omniscience. For to omniscience, the universe of possibility is obviously no wider than the actual fact. There is nothing which may be for aught omniscience knows, except what is. Accordingly, to say that the proposition P is true, is of the nature of making it the consequent of a hypothetical proposition; and to say that it is false is of the nature of making it the antecedent of a hypothetical proposition, and may be written "If P, then X." By the *modus ponens,* the following inference is logically valid:

If *P*, then *X*;
But *P*;
Hence, *X*.

This is the same as to say that for *P* to be false (If *P*, then *X*) and at the same time true, is logically impossible (that is, anything, *X*, logically follows from it). This is called the *principle of contradiction.*

[Variant Fragment 2]

To say that "If *P* is true, *C* is true," means that in some state of true knowledge (not explicitly defined) every possible state of things in which *P* is true is a state of things in which *C* is true.

When I draw a simple consequence, I have a vague conception of a vast number of possible inferences similar to the inference I am making, but I do not define the particular state of knowledge which determines these to be possible. I believe, however, that among all these possible inferences, out of those which have true premises, either, 1st, every one without exception, or, 2nd, the greater number, have true conclusions. In the first case, my inference is a *necessary* one,[2] in the second, a *probable* one. If this belief is really true, the inference is *valid.* The *modus ponens* is always a *necessary* inference, because in making it I believe that every possible such inference having true premises would have a true conclusion. It is, besides, a *complete* argumentation, because the kind of possibility which I contemplate in making it is logical possibility. That is to say, I think it not only is necessary in the present constitution of the universe but would be seen to be necessary if we were indefinitely ignorant of how the universe were constructed, so long as the words retained their meanings. It is, also, a *valid* argument because that belief which makes it a necessary inference is a belief in something really true. It is a *logically valid* argument, because it is both complete and valid, that is, because the principle believed would be true throughout the whole universe of logical possibility.

The *modus ponens* is the kind of reasoning which is to be used in the direct application of a general rule, of any kind. Suppose, for instance, a command to be given "If such an occasion arises, such and such action is to be taken"; the event does arise, and, by the *modus ponens,* that action is to be taken. So if implicit faith is put in any

2. The necessity is believed in but the belief may be erroneous.

authority. "If the Bible says so and so, it must be true; the Bible does say it, therefore it must be true." This mode of inference is thus suited to a childish state of mental development. It is the commonest form of argument in books which have been handed down to us from the "Ages of Faith." It is not without its use, but it takes the very humblest place among all modes of complete and valid inference.

Chapter IV.
The Traditional Syllogistic.
*[*Version 1*]*

One kind of reasoning a little more complicated than the *modus ponens,* is the following:

If *A* is true, *B* is true;
If *B* is true, *C* is true;
Hence, if *A* is true, *C* is true.

This involves the principle that the relation of antecedent to consequent is in technical language a *transitive relation.* By a transitive relation is meant one such that if *A* is in that relation to *B* and *B* in the same relation to *C,* then *A* is in that relation to *C.* Thus, *greater than* is a transitive relation; for if *A* is greater than *B,* and *B* is greater than *C,* then *A* is greater than *C.*

This form of inference is properly termed a hypothetical syllogism. The name is frequently applied to the *modus ponens,* but improperly because that inference does not imply the transitiveness of the illative relation. By the *illative* relation, is meant in technical language the relation of premise to conclusion. This is essentially the same as the relation of antecedent to consequent. To say that it is transitive is to say that if from *A* we infer *B* and then from *B* we infer *C,* we have indirectly inferred *C* from *A.* A judgment-inferred is a judgment following another in time (under certain conditions), and thus the transitiveness of the illative relation arises from the transitiveness of the temporal relation; to say that *C* is subsequent to something subsequent to *A* is to say that *C* is subsequent to *A.* Time is nothing but a regularity among existences. To say that time flows is to say that existences evolve themselves or are developed. Thus, the principle of syllogism is traced back to its origin in the condition that existences arrange themselves in order.

An inference depending mainly on the transitive principle may properly be called a *syllogism.* The best known kind of syllogism is the categorical syllogism, of which the following is an example:

> All men die,
> All the patriarchs (including Enoch and Elijah) were men,
> Therefore, these patriarchs had to die.

The only difference between a hypothetical and a general categorical proposition is that the former relates to a universe of possibilities, the latter to a universe of real objects. The hypothetical proposition says that the whole of one class of possibilities is contained among another class of possibilities. The general categorical proposition says that the whole of one class of objects lies within another class of objects. In the hypothetical proposition the smaller class of possibilities is expressed by the antecedent, the larger class by the consequent. The general categorical proposition may be expressed in the form

Every S is a P,

where S and P are two nouns; S, the name of the Smaller class of objects, is called in the technical language of logic the subject; P the name of the larger class is called the predicate of the proposition. The verb *is,* is called the *copula.* The categorical syllogism depends on the transitiveness of the copula.

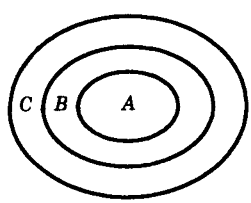
We may here note a rather important point. In ordinary language, some general categorical propositions are intended to assert the existence of something while others are not so meant. Thus we may say *a salamander is an animal which lives in the fire.* We do not mean that there is such a thing but only that, *if there be* any such thing, it so lives. In the language of logic it is understood that a general categorical proposition never asserts that anything really exists, nor a hypothetical proposition that anything is possible.

We will make an immediate application of this principle. An inference is said to be a *valid, necessary* inference, when every logically possible state of things in which the premises are true is a state of things in which the conclusion is true. But this does not imply that there is any logically possible state of things in which the premises are true. Consequently, from an *absurd,* or logically impossible, premise there is a valid necessary inference to any proposition whatever. But from the truth of nothing possible can its falsity be inferred; and thus to say that a proposition is absurd and to say that from it any proposition whatever follows by a valid necessary inference are identical statements. In like manner, a hypothetical proposition is true if every possible case of the antecedent being true is a case of the consequent being true. But this does not assert that the antecedent can possibly be true, from an impossible antecedent any consequent whatever follows. Hence, to say that a proposition is impossible and to say that if it is true every proposition is true are one and the same. So a general categorical proposition is true when every individual denoted by the subject is denoted by the predicate. But this is not to say that there really is any individual of the kind denoted by the subject, of a non-existent individual anything may be truly predicated; and to say that an individual does not exist and that it re-unites all predicates are one and the same assertion.

Chapter IV.
The Syllogism.
[Version 2]

If from A as a premise we conclude that B is true, and then taking B as a premise conclude from it that C is true, we do indirectly conclude C from A. Consequently if B follows from A (logically or materially), and C from B, then necessarily C follows in the same sense from A.

The diagram to illustrate this shows three areas lying one within another; for to say that one proposition follows from another is to say that the area of the truth of the latter lies within that of the former.



This principle is called the principle of the transitiveness of the illative relation. By the *illative relation* is meant the relation of conclusion to premise. A relation is said to be transitive, when A being in that relation to B, and B in that relation to C, makes A in that relation to C. The principle is also stated in the form *Nota notae est nota rei ipsius,* the mark of a mark is a mark; for by a mark is meant a predicate or consequent. The principle is often referred to as the *"nota notae."* Essentially the same principle is stated in the *Dictum de omni et de nullo,* which is that what is true or false of all is true or false of each one. These last two statements have in view the general categorical proposition, "Every *S* is a *P,*" which only differs from the hypothetical proposition "If *S* is true, *P* is true," by referring to a universe of real things instead of to a universe of possibilities. It says that the *S*'s are among the *P*'s, instead of saying that the cases of the possibility of *S* are among the cases of the possibility of *P.* The class-name or common noun *S* is called in logical terminology the *Subject,* and the class-name, common noun, or adjective, *P,* is called the *Predicate* of the categorical proposition. The verb *is* is called the copula. The word *every* or whatever other modifying word may be used is called a *syncategorematic term.* But it is quite needless in logic to consider Categorical propositions separately from hypotheticals. A categorical is merely a particular variety of hypothetical, to say that "Every *S* is a *P*" is to say that if any object is an *S,* then that object is a *P.*

The principle of the transitiveness of the illative relation gives the following form of inference, which is called the *Direct Syllogism:*

If *S,* then *M;* If *M,* then *P;*
Hence, If *S* then *P.*

The categorical form is,

Every *S* is an *M;* Every *M* is a *P;*
Hence, Every *S* is a *P.*

As a concrete example take the following:

Every man has to die;
Every patriarch (including Enoch & Elijah) was a man;
Hence, every patriarch had to die.

In the case of the categorical syllogism, the class-names, *S, M, P,* are called the terms, and the same nomenclature may be extended to the hypothetical form. The term whose area of truth is the smallest is called the *minor term;* that whose area is the largest is called the *major term;* these two are called the *extremes,* the third term is called the *middle,* or *middle term.* The premise containing the minor term is called the *minor premise;* that containing the major term, the *major premise.*

Taking the major premise of the syllogism as a part of the conclusion, we have the following form of inference called *contraposition.*

If S then M;
Hence from, If M then P, it logically follows that If S then P.

Using other letters, to avoid confusion

If A then B;
Hence, from If B then Q, it would logically follow that If A, then Q.

Now, for A, substitute "If S, then M"; and for B, substitute "If S, then P," and we find the following:

Supposing that from "If S then M," it would follow that "If S then P," (which is the case in the direct syllogism owing to If M, then P). Then from the premise "From if S then P, would follow Q" we can conclude "From if S then M, would follow Q."

It follows that the following form of inference, called the *Major indirect syllogism,* is valid:

If M, then P;
But from "If S, then P" would follow Q;
Hence, from "If S, then M" would follow Q.

As a concrete example take the following:

Every man has to die;
But if every patriarch had to die, the Bible errs;
Hence, if every patriarch was a man, the Bible errs.

In a similar way, by contraposing the major premise and conclusion of the direct syllogism, we get the following *Minor indirect syllogism:*

From if S then P, would follow Q;
But S is M;
Hence, from if M then P, would follow Q.

In a concrete example:

If every patriarch had to die, the Bible errs;
But every patriarch was a man;
Hence, if every man has to die, the Bible errs.

The following are obvious complications of the three forms of syllogism:

Complex Direct Syllogism.

If *S*, then *M;* If both *M* and *N* are true, then *P;*
Hence if both *S* and *N* are true, then *P.*

Complex major indirect syllogism.

If both *M* and *N* are true, then *P;*
But from "If both *S* and *N* are true, then *P,*" would follow *Q;*
Hence, from "If *S* then *M*" would follow *Q.*

Complex minor indirect syllogism.

From "If both *S* and *N* are true, then *P*" would follow *Q;*
But if *S*, then *M;*
Hence, from "If both *M* and *N* are true, then *P,*" would follow *Q.*

To form concrete examples of these forms put

for *S*, prophet,
N, swallowed by a whale,
P, die,
M, ordinary man,
Q, the Bible errs.

We thus get

Complex direct syllogism.

Every prophet is an ordinary man;
If an ordinary man were to be swallowed by a fish, he would die.
∴ If a prophet were to be swallowed by a fish, he would die.

Complex major indirect syllogism.

If an ordinary man were to be swallowed by a fish, he would die;
But if every prophet swallowed by a fish would die, the Bible errs;
Hence, if every prophet is an ordinary man, the Bible errs.

Complex minor indirect syllogism.

If every prophet swallowed by a fish would die, the Bible errs;
But every prophet is an ordinary man;
Hence, if every ordinary man swallowed by a fish would die, the Bible errs.

Still more complicated forms could easily be given; but we already begin to find ourselves hampered by the imperfection of ordinary language, the syntax of which is not adequate to the clear setting forth of such complicated relations. The whole thing can be rendered much more intelligible by the use of the sign $\dashv$ to express the illative relation. For instance, let us write

$$\overline{S} \dashv P$$

to mean If S, then P, whether S follows from P logically or materially. We might, if we chose, distinguish a logical consequence by putting a dot under the down-stroke of the sign of consequence; thus

$$\overline{S} \underset{\cdot}{\dashv} P.$$

And different kinds of material consequence could be distinguished by other diacritical marks. Such signs, however, are hardly necessary; and it would be better to distinguish material consequence, where necessary, by writing the universe of possibility as an antecedent. For example, it is true as a physical consequence, that if a sane man were to throw himself over Niagara, he would be killed; but granting that no sane man would kill himself or could fail to know what would kill him, then as a moral consequence (supposing some doubt as to the physical necessity) if a sane man were to throw himself into Niagara, it would be because he knew he would not be killed. Let N denote that the sane man throws himself into Niagara, K, that he would be killed, F, false, P, that the laws of physics are true, M, that a sane man would not kill himself; then the first proposition might be written as follows,

$$\overline{P} \dashv (\overline{N} \dashv K)$$

while the second would be

$$\overline{M} \dashv \{\overline{N} \dashv (\overline{K} \dashv F)\}.$$

It is simpler in writing such expressions to discard the parentheses. By $\overline{A} \dashv \overline{B} \dashv C$, we will agree to mean $\overline{A} \dashv (\overline{B} \dashv C)$, everything following a sign of consequence constituting the consequent. But we must carefully distinguish $\overline{A} \dashv \overline{B} \dashv C$ from

$$\overline{\overline{A} \dashv B} \dashv C$$

that is, from $(\overline{A} \dashv B) \dashv$ C, which means that C is a consequent from the antecedent that B is a consequent from A. The proposition $\overline{A} \dashv \overline{B} \dashv C$ means that within the area of the truth of A, the area of the truth of B is everywhere included within that of C. But $\overline{A \prec B} \prec C$ means that the whole area within which A is included in B (that is, the whole field of possibility except where A extends beyond B) is included in the area of C. We may read these formulae thus:

$\overline{A} \prec \overline{B} \prec C$. If A then if B then C.
$\overline{A \prec B} \prec C$. If if A then B then C;

but the spoken words can hardly be understood without the simultaneous exhibition of the formula.

The *principle of identity* is that we are always authorized to write

$$\overline{A} \prec A,$$

whatever A may be.

The *principle of the commutation of antecedents* is that $\overline{A} \prec \overline{B} \prec C$ is the same as to say that C follows from A taken jointly with B.

The principle of the transitiveness of *illation* is that if we have $\overline{A} \prec B$ and $\overline{B} \prec C$, we are authorized to write $\overline{A} \prec C$.

The principle of identity gives

$$\overline{\overline{S} \prec P} \prec \overline{S} \prec P.$$

From this, the principle of commutation of antecedents gives as a valid inference,

$$\begin{array}{l} \overline{S} \prec P \\ S \\ \therefore P \end{array}$$

the *modus ponens.*

By the *modus ponens,* we have

$$\overline{S} \prec \overline{\overline{S} \prec M} \prec M$$

and

$$\overline{M} \prec \overline{\overline{M} \prec P} \prec P.$$

Consequently, by the transitiveness of illation

$$\overline{S} \prec \overline{\overline{S} \prec M} \prec \overline{\overline{M} \prec P} \prec P$$

or

$$\overline{\overline{S}\prec M}\prec\overline{\overline{M}\prec P}\prec\overline{S}\prec P,$$

which gives as a valid inference

$$\begin{array}{l} \overline{S}\prec M \\ \overline{M}\prec P \\ \therefore \overline{S}\prec P, \end{array}$$

the *direct syllogism.* In this formula if we substitute for S, $\overline{A}\prec B$, and for M, $\overline{A}\prec C$, we have, as a valid inference

$$\begin{array}{l} \overline{\overline{A}\prec B}\prec\overline{A}\prec C \\ \overline{\overline{A}\prec C}\prec P \\ \therefore \overline{\overline{A}\prec B}\prec P, \end{array}$$

or

$$\begin{array}{l} \overline{\overline{A}\prec B}\prec\overline{A}\prec C \\ \therefore \overline{\overline{\overline{A}\prec C}\prec P}\prec\overline{\overline{A}\prec B}\prec P \end{array}$$

But by syllogism

$$\overline{\overline{B}\prec C}\prec\overline{\overline{A}\prec B}\prec\overline{A}\prec C;$$

consequently by the transitiveness of illation, we have as a valid inference

$$\begin{array}{l} \overline{B}\prec C \\ \therefore \overline{\overline{\overline{A}\prec C}\prec P}\prec\overline{\overline{A}\prec B}\prec P \end{array}$$

or

$$\begin{array}{l} \overline{B}\prec C \\ \overline{\overline{A}\prec C}\prec P \\ \therefore \overline{\overline{A}\prec B}\prec P, \end{array}$$

which is the *major indirect syllogism.* The further working out of these forms is left as an exercize for the student.

It will be seen that what syllogism enables us to do is to trace out the remoter consequences of simple rules and of combinations of such rules. General rules are useful, indeed indispensible in many ways. Every conscious resolution or habit is a rule. Besides that we learn by conversation with other men, that they will follow certain rules; and by scientific research we find that nature follows certain laws. To apply our own general resolutions or to find how other men or nature will behave in particular cases we have to syllogize. This is the only way of applying rules. But the rules themselves cannot be ascertained by syllogism, except when they happen to be consequences of known rules; nor can we find out the causes of events in this way. The common saying is that syllogism does not add to our knowledge but only explicates what we already know. This is true in the sense that syllogism only traces rules to their consequences. But strictly speaking syllogism does add to our knowledge. As I was one day walking by the Niagara River above the falls, I saw two men in a boat at a point from which I had been informed that no man could reach the shore. I, therefore, knew in advance that these men would be carried over the falls. I could not know that except by syllogism, so that the syllogism did add to my knowledge. Syllogism is such an easy and obvious mode of reasoning, that Stuart Mill would not admit that it deserved the name of reasoning at all. As soon as the two premises are put together, the conclusion is known; not, however, until by an act of thought or inward observation the relation between those premises has been perceived. Locke and others have gone so far as to say that syllogism is bad reasoning. The reason they give is that the premises are no better known than the conclusion; but this only shows how easy and perfect the inference is. If syllogism is bad reasoning, then general rules are of no use; for they cannot be used unless they are applied syllogistically; and without the consciousness of general rules, the mind of man would be no better than those of the brutes.

At the same time, it is quite true that syllogism is a very low order of reasoning. Positive knowledge cannot be enlarged in this way; nothing new, not a mere result of what is already known, can thus be discovered. Not only so, but even the theorems of mathematics, although they are necessary consequences from the premises, cannot be deduced by syllogism.

In the middle ages, no higher kind of reasoning than this was ever consciously employed; and science had to remain at a stand-still, as long as that was the case. No change that has ever been brought about in the form of a plant by cultivation has been more wonderful than the development of human reason since the time of the Plantagenets. The intellect of a modern mathematician rises as high above that of a Scholastic Theologian as his does above that of a dog. When we have discussed the syllogism, almost the whole of logic still lies before us.

Is it not, then, a shameful thing that in most of our colleges logic still means little more than the study of the methods of thought used in the middle ages, before the development of modern science, before modern geometry, before the infinitesimal calculus, before the theory of probability, before the conceptions of dynamics, the statistical and the historical methods, or the ideas of evolution, had ever been heard of? And that in most of the text-books now in use, the syllogism still occupies the most prominent place? It would be bad enough if the traditional logic did no worse than merely fail to teach any higher reasoning; but what it actually does, is to teach that all reasoning higher than the syllogism is bad. The teachers of this baleful doctrine, very estimable persons, for the most part, though necessarily strangers to all that gives our age its greatness, have fortunately no masterly grasp even of their own wretched philosophy, and fail to influence the minds of their pupils. The consequence is that most educated men are but little the worse for having studied logic. Somewhat worse they are, and educated men do not usually reason so well as uneducated men and women. But wherever there has been a teacher of more power, or an unusually acute and attentive student, there disastrous consequences have followed the old teaching. Captious reasoners have been produced, raising all sorts of empty distinctions, carping at all fruitful methods, and falling back into exploded errors.

But the fact that the ancient logic is so rife, renders an acquaintance with it useful, in order to understand the ideas of those who employ it and to enable us to meet them upon some common ground of thought. I therefore proceed to give a brief sketch of the traditional syllogistic and doctrine of fallacies and I shall in the course of this work, notice other doctrines belonging to the old systems of logic.

The *modus ponens,* although it does not involve the principle of transitiveness, which is the essence of syllogism, is in the old system called a hypothetical syllogism. There is besides a negative hypothetical syllogism, the *modus tollens,* which has the form

> If *S*, then *P*;
> But *P* is false;
> Hence, *S* is false.

This is, in fact, a direct syllogism. Categorical syllogism is treated separately. Categorical propositions have four forms, denoted by the four vowels, A, E, I, O. (A and I are the first two vowels of *affirmo,* E and O of *nego.*)

A. *Universal Affirmative.* Every S is a P.
E. *Universal Negative.* No S is a P.
I. *Particular Affirmative.* Some S is a P.
O. *Particular Negative.* Some S is not a P.

Affirmative and negative propositions are said to differ in *Quality;* universal and particular in *Quantity.* The particular negative is the *contradictory* or precise denial of the universal affirmative; and the particular affirmative of the universal negative. It follows, since the particular propositions certainly assert the existence of something, that the universal propositions should be considered as merely asserting non-existence. The universal negative is so understood. To say No S is a P, does not mean that any S or any P exists. But in the case of the Universal Affirmative, the logicians of the middle ages were led astray by the usage of ordinary language. The medieval logician considered it to be a part of his business to show the precise meaning of every form of ordinary speech; the modern logician supposes every man to know what is in his own mind without the aid of logic. According to the traditional doctrine, then, some universal affirmatives do not imply existence (giving rise to a complicated doctrine of Modals), but ordinarily existence is implied in this form. Accordingly, the particular affirmative is considered to be *subalternate* to, that is, to be implied by, the universal affirmative. Moreover, the particular negative is considered as subalternate to the universal negative. This is a downright error; because the particular asserts existence, which the universal does not. The error does not, however, happen to affect the doctrine of the syllogism. The varieties or *moods* (modes) of syllogism belong to three (or by a later doctrine, *four*) *figures.* In the *first figure,* the middle term is the subject of one and the predicate of the other premise; in the *second figure,* it is the predicate of both; in the *third figure,* the subject of both. An *indirect syllogism* is one in which either the major or the minor extreme appears both as subject and as predicate; a *direct syllogism* is where one extreme appears only as subject, the other only as predicate. All the moods of the second and third figures, and part of the moods of the first figure, are *indirect.* The indirect moods of the first figure are considered by other authors to constitute a *fourth figure.* The direct moods with their names are as follows:—

Barbara.
Every M is P,
Every S is M;
$\therefore$ Every S is P.

Celarent.
No M is P,
Every S is M;
$\therefore$ No S is P.

Darii.
Every M is P,
Some S is M;
$\therefore$ Some S is P.

Ferio.
No M is P,
Some S is M;
$\therefore$ Some S is not P.

The names of the moods date from the first half of the thirteenth century, the age of St. Thomas Aquinas, the age of the Early English and other chaster forms of Gothic Architecture. They appear in the celebrated *Summulae Logicales,* of Petrus Hispanus (who became Pope John XXI), the ordinary text-book of logic in the two following centuries, and until displaced by that of Ramus. The three vowels of each of these names stand for the major premise, minor premise, and conclusion of the syllogism; and show their quantity and quality. The consonants are merely euphonic except the initial letters which are B, C, D, F, the first four consonants of the alphabet. But the consonants in the names of the indirect moods are generally significant as will be seen. *Barbara* is my direct syllogism, *Celarent* a case of the complex direct syllogism, *Darii* a complex minor indirect syllogism, *Ferio* a complex major indirect syllogism.

The Second Figure has four moods, which with their names are as follows:—

Cesare.
No M is a P,
Every S is a P;
$\therefore$ No S is an M.

Camestres.
Every M is a P,
No S is a P;
$\therefore$ No S is an M.

Festino.
No M is a P,
Some S is a P;
$\therefore$ Some S is not an M.

Baroko.
Every M is a P,
Some S is not a P;
$\therefore$ Some S is not an M.

Here, *Cesare* and *Camestres* are the same as *Celarent,* otherwise stated. *Festino* is the same as *Ferio. Baroko* is a particular case of Major indirect syllogism.

The names *Cesare* and *Camestres* begin with *C*, because these moods are to be *reduced to*, that is to say deduced from, *Celarent.* For similar reason, *Festino* has the initial of *Ferio*, and *Baroko* of *Barbara.* The *s* in *Cesare* shows that the reduction of that mood is to be effected by the inference called Simple conversion applied to the major premise. *Conversion* is an immediate inference transposing the subject and predicate of a proposition. *Simple conversion* is where the conclusion has the same quantity and quality as the conclusion. There are two forms of simple conversion, that of E, and that of I. The simple conversion of E runs:

No A is B,
∴ No B is A.

The simple conversion of I is,

Some A is B,
∴ Some B is A.

By simple conversion, the major premise of *Cesare* becomes No P is M, and then the conclusion follows from the other premise by *Celarent.* The two letters *s* in *Camestres* show that the inference of simple conversion is to be applied to the minor premise and conclusion; while the *m* signifies that the major premise of *Camestres* becomes the minor premises of *Celarent* and *vice versa.* Thus the minor premise of *Camestres* gives by conversion No P is an S; then by *Celarent*

No P is an S,
Every M is a P;
∴ No M is an S.

Finally, the simple conversion of the conclusion gives the conclusion of *Camestres. Festino* is reduced like *Cesare.* The *k* in *Baroko* shows that this is to be reduced *per impossibile,* that is to say, as follows. It must be true that Some S is not an M, for otherwise Every S would be an M, and then, since Every M is a P, by *Barbara,* Every S would be a P. Now this is not so, since Some S is not a P.

The moods of the Third Figure, with their names, are

Darapti	*Disamis*	*Datisi*
Every S is a P,	Some S is a P,	Every S is a P,
Every S is an M;	Every S is an M;	Some S is an M;
∴ Some M is a P.	∴ Some M is a P.	∴ Some M is a P.

Felapton	*Bocardo*	*Ferison*
No S is a P,	Some S is not a P,	No S is a P,
Every S is an M;	Every S is an M;	Some S is an M;
∴ Some M is a P.	∴ Some M is not a P.	∴ Some M is not a P.

In fact, *Disamis* and *Datisi* are mere verbal variations of *Darii; Ferison* of *Ferio. Bocardo* is a particular case of *Minor indirect syllogism. Darapti* and *Felapton* are not valid unless the universal affirmative be understood to assert existence. In that case by means of the inference from *all* to *some, Felapton* is reduced to *Ferison,* and *Darapti* to either *Disamis* or *Datisi.* In the names *Felapton* and *Darapti,* the p signifies conversion *per accidens,* that is, the inference

Every A is B,
∴ Some B is A.

The following are the indirect moods of the first figure with their names, under each of which is written the most usual form of its name when it is considered to belong to a fourth figure. But the last names are modern and not very well-settled.

Baralipton.	*Dabitis.*	*Celantes.*
(Bramantip.)	*(Dimaris.)*	*(Camenes.)*
Every M is P,	Every M is P,	No M is P,
Every S is M;	Some S is M;	Every S is M;
∴ Some P is S.	∴ Some P is S.	∴ No P is S.

Fapesmo.	*Frisesomorum.*
(Fesapo.)	*(Fresison.)*
Every M is P,	Some M is P,
No S is M;	No S is M;
∴ Some P is not S.	∴ Some P is not S.

Celantes, Dabitis, and *Frisesomorum* are respectively equivalent to *Celarent, Darii,* and *Ferio. Baralipton* and *Fapesmo* are only valid if the Universal affirmative implies existence and then *Baralipton* reduces to *Dabitis, Fapesmo* to *Frisesomorum.*

The list of the moods is given in the ancient verses:

Barbara: Celarent: Darii: Ferio: Baralipton.
Celantes: Dabitis: Fapesmo: Frisesomorum.
Cesare: Camestres: Festino: Baroko: Darapti.
Disamis: Datisi: Felapton: Bokardo: Ferison.

The moods of syllogism are subject to general canons, of which various different systems are given by different authors. The following (which supposes the universal affirmative not to assert existence) will be found applicable beyond the limits of the ordinary forms of propositions.

Canons of Syllogism.

1. A syllogism has three propositions, and three terms.
2. The middle term must be *distributed* once and once only. A term is said to be *distributed* when a change in the character of a single thing denoted by the term would change the proposition from true to false. The subject of a universal proposition is distributed and so is the predicate of a negative one.
3. The extremes must each be distributed or undistributed in the conclusion as in the premise.
4. The conclusion from two universal propositions is universal, from a universal and a particular is particular, from two particulars is doubly particular or *spurious.* As an example of the last rule, take the following:

Some Shakespeare is a poet,
Some William is not a poet;
∴ Some William is not some Shakespeare.

5. The conclusion from an odd number of negatives with or without affirmatives is negative, from an even number or from no negatives is affirmative.

The *reductio ad absurdum* is a method of reasoning in which something is provisionally supposed to be true in order to show that a false consequent would follow from it, so that the thing supposed must be false. This may be regarded as a *modus tollens,* and therefore a direct syllogism. But usually, it will involve the dilemmatic inference, that if one hypothesis is false the opposite one is true. We say

S is either true or false
But if S is false, M.
M is false
∴ S is true.

This is therefore a kind of reasoning, which may be conceived either as syllogism or as dilemma. It is a method to which much objection is made, because it does not show clearly from what antecedent the conclusion follows as a consequent, but it is perfectly valid reasoning.

The whole doctrine of false reasonings, or *paralogisms*, exhibits the intellectual condition of Greece in Aristotle's time in a strange light, and shows the barbarism of the middle ages and the incompetency of later logicians who have been satisfied with the doctrine. Paralogism is either Fallacy, if unintentional, or Sophism, if intended to deceive. It is under the latter aspect particularly that Aristotle considers false reasoning. Fallacies are either fallacies of words (ἔλεγχοι παρὰ τὴν λέξιν, *fallaciae dictionis, sophismata in voce*) or *fallacies in things* (οἱ ἔξω τῆς λέξεως, *fallaciae extra dictionem, sophismata in rebus*). Fallacies in speech are of six kinds as follows:

1. *Equivocation* is when the deceit consists in the doubtfulness of some one word having divers significations.

2. *Amphibology* is when the double meaning resides, not in a single word, but in a phrase.

3. *Composition* or *conjunction* is the joining of things that should be severed. As, two and three are odd and even, five is two and three; hence, five is odd and even. It is impossible for a man to sit and stand; John is sitting; therefore, John cannot stand. Every animal is either rational or irrational; but not every animal is rational; hence, every animal is irrational.

4. *Division.* The opposite fault. All the planets are seven; the sun and moon are planets; hence, the sun and moon are seven.

5. *Accent* is a pun used sophistically.

6. *Figure of speech,* is where two words being analogous in form are assumed to be analogous in meaning.

Paralogisms *extra dictionem* are of seven kinds as follows.

1. The *fallacia accidentis* arises when the distinction between what is true of a thing and what is true of a predicate of that thing is neglected. Common examples are these. Man is a species; Socrates is a man; therefore Socrates is a species. A mouse gnaws cheese, but mouse is a syllable; therefore a syllable gnaws cheese. Brass is a natural product, the statue is brass; therefore the statue is a natural product. Do you know who that is who is coming? No. Well, it is your father; therefore, you do not know your father. Man is an animal, and a horse is an animal; therefore a man is a horse. That man is a white man, and he is a friar; therefore he is a white friar. That dog is a father, and he is yours; therefore he is your father.

2. *Fallacia a dicto secundum quid ad dictum simpliciter.* A predicate is *dictum simpliciter* when it is applied to its subject in its natural and full sense, without limitation; it is *dictum secundum quid*

when it is applied in a peculiar sense or respect, or under certain limitations.

3. *Ignoratio elenchi* is when a professed refutation of an opinion is really directed against another opinion which is not maintained. Of course, the reasoning itself may be good.

4. *Fallacia consequentis,* or *non sequitur,* is reasoning from consequent to antecedent. For instance, when Kepler inferred that the planet Mars moved in an ellipse because that hypothesis would satisfy its longitudes and latitudes (its distances from the earth not being capable of observation), his reasoning according to the traditional logic was unqualifiedly bad, and would be sententiously condemned by any school-boy, in the days when logic was taught to school-boys, as a *non sequitur,* though it was really the most admirable intellectual achievement which humanity had up to that time performed.

5. *Petitio principii,* or begging the question, is when the conclusion is assumed in the premises, or when the premises are not better known than the conclusion. This is necessarily good reasoning, logically. It may or may not be a bad way to convince a man of a truth to get him to virtually admit it at the outset; but when he has once implicitly admitted it, he is logically bound to admit it explicitly.

6. *Fallacia secundum non causa ut causa,* occurs in a *reductio ad absurdum* or indirect reasoning to prove the falsity of a certain assumption. In this mode of argumentation, the proposition to be disproved is assumed as true, and it is shown that something false follows as a consequence. When the false conclusion is deduced independently of the proposition in question, the fallacy of *non propter hoc* or *non causa pro causa* is committed.

7. *Fallacy of multiple question,* occurs when different questions are joined so that a person is led to answer them all together, although one may require an affirmative, another a negative answer.

Substantially the whole of the above doctrine of syllogism and of fallacies is due to Aristotle. There have been many attempts to improve upon it; but most of these have merely resulted in a useless complication of the doctrine. The quantification of the predicate, an idea originating with the botanist George Bentham, was for some years in vogue in this country and in England. It was developed with a singular defect of clear thought by Sir Wm. Hamilton, and more ably by Stanley Jevons. The idea consisted in regarding the copula as a sign of identity,—so that in place of writing, "Every *S* is a *P*" meaning that all the *S*'s are included among the *P*'s, the advocates of the quantification of the predicate wrote "All the *S*'s are (that is, are identical with) some *P*'s." Thus, the distinction of *all* and *some* was extended to the predicate. Aristotle notices this conception merely to discard it; for, he says, it is never true that "Every *S* is every *P*," if there are more *S*'s than one. If the theory of syllogism had been extended to relative terms, the quantification of the predicate might have been useful; but in the hands of Sir William Hamilton it only led to a complicated syllogistic, full of blunders; in the hands of Jevons, it stood in the way of a better development of a meritorious system of formal logic.

The syllogistic of De Morgan better deserves attention. Instead of extending the distinctions of quantity to the predicate, he extends the distinctions of quality to the subject. That is to say, he adds to the four forms of categorical propositions noticed above, these others:

Every not-*S* is *P*, or	Everything is either *S* or *P*.
Every not-*S* is not-*P*, or	Every *P* is *S*.
Some not-*S* is *P*, or	Some *P* is not-*S*.
Some not-*S* is not-*P*, or	Something is neither *S* nor *P*.

He makes use of the following terminology, to which in one or two cases I note objections.

When every *S* is a *P*, he calls the class *S* a *species* of the class *P*, and *P* a *genus* of *S*. He, therefore, regards every class as a *species* and a *genus* of itself, which does not seem very objectionable. He also calls the attribute of *P* an *essential* of that of *S*, and *S* a *dependent* of *P*. The term essential is properly applied not to all consequents, but only to logical consequents.

In the opposite case, when some *S* is not *P*, he calls the class *S* an *exient* of *P*, the class *P* a *deficient* of *S*. He calls the attribute *P* *inessential* of *S*, and the attribute of *S* *independent* of *P*.

When no *S* is *P*, he calls each class the *external* of the other, and the two he calls *coexternal*. He calls each attribute *repugnant* of the other.

In the opposite case when some *S* is *P*, he calls each class a *partient* of the other, and the two *copartients;* each attribute *irrepugnant* of the other. The term *concomitant* might be suggested.

When everything is either *S* or *P*, he calls each class a *complement* of the other; and each attribute an *alternative* of the other.

In the opposite case, when something is neither *S* nor *P*, he calls each class a *coinadequate* of the other; each attribute an *inalternative* of the other.

He also has the following names for complex relations.

When every *S* is *P*, and every *P* is *S*, the classes are called *identical* (better, *coextensive*); the attributes *specific inherent* (better, *coinherent*).

When every *S* is *P*, but some *P* is not *S*, the class *S* is called a deficient species or *subidentical* of *P*, *P* an exient genus or *superidentical* of *S*; the attribute of *P* is independent essential, or *generic inherent* of *S*, that of *S* inessential dependent or *specific accident* or *generic non-accident* of *P*.

When no *S* is *P* and everything is either *S* or *P*, he calls the classes *contraries*, the attributes repugnant alternatives or *specific excludents*. This use of the term contrary is quite inconsistent with established logical terminology and must be condemned. *S* and *P* are properly called *negatives* or *contradictories*.

When no *S* is *P* and there is something neither *S* nor *P*, he calls the classes *subcontrary*, the attributes inalternative repugnants or *generic excludents*. The term inalternative repugnant is the best of these.

When some *S* is *P* and everything is either *S* or *P*, he calls the classes *supercontrary*, the attributes irrepugnant alternatives, *generic accidents*, or *specific non-accidents*. The term irrepugnant (or concomitant) alternative is the best. Subcontrary and supercontrary must be rejected with the word contrary.

De Morgan commits the error of making every proposition assert the existence of its subject and predicate, as well as of their negatives.

He is also inconsistent in not recognizing any but the first figure and that although he considers the following to be two different syllogisms:

Some *X*'s are *Y*'s,	Some *Y*'s are *X*'s,
All *Y*'s are *Z*'s;	Some *Z*'s are all the *Y*'s;
∴ Some *X*'s are *Z*'s.	∴ Some *Z*'s are *X*'s.

He has eight universal syllogisms. Namely, starting with *Barbara*, he can substitute independently for each of the three terms its negative, giving $2^3 = 8$ moods. In the same way, starting with *Darii* he has 8 minor-particular moods, or moods having the minor premise particular. The contraposition of all the propositions of these give 8 major-particular moods. He has also 8 moods derived from *Darapti*.

Some of the major-particular syllogisms do not come under any of the forms which I have given. They depend upon the principle of excluded middle, and ought not to be considered as syllogistic inferences at all, but as dilemmatic. See the next chapter. Such for instance is the following

> Everything is either X or Y,
> Something is neither Y nor Z;
> Hence, some X is not Z.

The advantage of the view taken by me is that it reduces syllogistic to the smallest number of moods, while it represents all truly syllogistic reasoning, including some forms of inference overlooked by other logicians. It combines simplicity with completeness.

Chapter V
Dilemmatic Reasoning.
*[*Version 1*]*

A mode of inference which differs essentially from syllogism is the following:

> If S is not true, P is true;
> But if S is true, P is true;
> Hence, P is true.

This depends upon the *principle of excluded middle,* so called because it implies that there is no medium between truth and falsehood. This principle is usually stated: "everything is either true or false." It may be stated: "what is not not true is true." But perhaps the best statement of it is that "whatever follows both from the truth and from the falsity of any hypothesis is true."

The precise meaning of the term dilemma has never been settled.[3] The standard example given in all the old books, and taken from Aulus Gellius is as follows: "It is not good to marry a wife; for if she be fair, she will be common; if not fair, then loathsome." The best definition is that it is an inference, in one of the premises of which alternatives are given, from all of which, as the other premises show, the same consequent follows.

The dilemma was imported into logic from rhetoric by Peter Ramus, whose *Dialectica* drove the *Summulae logicales* of Petrus

3. See Keynes, *Formal Logic.* Chap. VIII, §214.

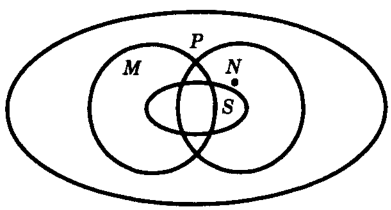
Hispanus out of the schools, soon after the revival of learning. Dilemmatic reasoning was not known to the medieval logicians, nor was it employed (unless very rarely) by any writers during the middle ages. Since the reformation, it has had a place in all logical treatises, but usually with some disparagement. Even so enlightened a writer as Jevons thinks most dilemmas are fallacious. It is a kind of reasoning which two persons in conversation or amicable discussion are very likely to strike out. It has, therefore, always been a favorite method with writers who cultivate a lively style. Modern German metaphysicians, also, who have sought to develop philosophy in historical continuity and to exhibit each his own system as the inevitable result of the discussion which had gone before, have made dilemmatic inference their chief instrument. Hegel, it is true, denies the principle of excluded middle, which however he does not clearly distinguish from the principle of contradiction. "Neither in heaven and earth," he says, "neither in the world of mind nor of nature, is there anywhere such an abstract, 'Either-or' as the understanding maintains. . . . Contradiction, above all things is what moves the world: and it is ridiculous to say that contradiction is unthinkable. The correct point in that statement is that contradiction is not the end of the matter, but cancels itself." The movement of Hegel's logic has two parts. In the first, the conception of the first stage of thought reduces itself to an absurdity. In the second movement, it is shown that the two alternatives, each reducing itself to the other, have a resultant like two forces (this at least is Hegel's own favorite comparison), or rather there is a positive conception wherein they can be held together. The analogy with the reaction of forces is not exact; for forces that are directly opposed have no resultant, the weaker only partially destroying the stronger. When two forces are not directly opposed, their mode of reaction depends upon the proposition that any two forces having the same point of application can be conceived as each a sum of two components, one of which is the same in both forces, the others equal and opposite, that is to say, any two forces having the same point of application can be represented as $A + B$ and $A - B$; the B's cancel and the A's are added together to give the resultant. In the Hegelian logic, the reacting conceptions have in the first place no magnitude, so that the analogy with forces can be but general. But the third stage of the Hegelian logic contains the whole of the first two in their entirety; nor are the first two conceptions conceived as compounded of one common element and two opposite ones of which one conception contains one and the other the other. The movement is a synthesis consisting in drawing that conclusion which the contrast of the two alternatives makes clear to us as being equally the consequent from either.

Chapter V.
The Dilemma.
[Version 2]

The simplest kind of dilemma proceeds as follows:

If S then either M or N;
If M, then P;
If N, then P;
Hence, if S, then P.

The diagram is like that for the syllogism except that the middle term is replaced by two areas which may partly overlap.



To say that either M or N is true, is the same as to say that if M is false N is true. Now syllogisms occur involving the term "false"; but in these, it is sufficient to put any other consequent in place of "false," and the conclusion follows equally well. Now this is not so in the dilemma. Writing the first premise

If S is true, and if from M, X follows, then N is true,

the third premise gives

If S is true, and if from M, X follows, then P is true,

or what is the same thing

If from M, X follows, then from S, P follows.

From this and the second premise, it is plain the conclusion does not follow, unless we consider the peculiar nature of the consequent X.

All inferences of this sort depend upon the *principle of excluded* *middle,* which is that every proposition is either true or false, there is no third alternative. This is proved as follows; To say that A is false is to say that every proposition follows from it, when the universe of possibility includes only what is actually true. For something not to follow from it, that consequent must be false, and even then it follows unless P is true. Hence, if P is not false, it is true. But this reasoning is itself essentially dilemmatic, because it assumes that either everything follows from P or something does not follow from it.

From the principle of excluded middle, we can infer from "If not-S, then P," to "If not-P, then S." For, first, by the principle of contraposition, we infer "If P is false, then not-S is false," and by the principle of excluded middle "If not-S is false, then S is true."

Returning to the dilemma, we have seen how the first and third premises give us,

If S is true, then from the falsity of M would follow the truth of P.

But, by the mode of inference just established, this is the same as,

If S is true, then from the falsity of P would follow the truth of M.

From this and the second premise we have by direct syllogism

If S is true, then from the falsity of P would follow the truth of P.

But by the principle of contradiction P cannot be at once false and true. Since, therefore, if S is true, P cannot be false without being also true, it follows that it cannot be false at all, and therefore by the principle of excluded middle it must be true. Thus, the conclusion, "If S is true, P is true," is reached.

No precise meaning for the word dilemma has ever been agreed upon. The following is the standard example of this kind of reasoning:

It is not well to marry. For a wife will either be handsome or not; if she is handsome, she will be unfaithful, and so make her husband unhappy; if she is not handsome, she will not attract her husband, and so again will make him unhappy. Either way, therefore, a wife will cause her husband unhappiness.

This differs from syllogism by involving the principle of excluded middle; and I propose to call every inference resting mainly on that principle *dilemmatic* reasoning.

The dilemma was imported into logic from rhetoric by Peter Ramus, whose *Dialectica* drove the *Summulae logicales* of Petrus Hispanus out of the schools soon after the revival of learning. Dilemmatic reasoning was not known to the medieval logicians, nor was it employed (unless very rarely) by any writers during the middle ages. It is a kind of inference which two persons in conversation or rational discussion are very likely to make. It has, therefore, always been a favorite method with writers who cultivate a lively style. Modern German metaphysicians, who have sought to develop philosophy in historical continuity and to exhibit each his own system as the inevitable result of the discussion which has gone before, have made dilemmatic inference their chief instrument. Hegel introduced a particular kind of dilemma, in which the principle of excluded middle seems to be denied; but we shall find on closer examination in another chapter that this is not really the case.

The dilemma is a very valuable instrument of reason; it is of a far greater order than the syllogism, and the prevalence of it indicates a stage of intellectual development much beyond the dogmatism of the middle ages. But it does not rise to the level of the simplest mathematical reasoning, far less to the living thought of the natural sciences.

Chapter VI
Logical Extension and Comprehension.
*[*Excursus*]*

This chapter ought perhaps to be called an Excursus. It is not, like the previous ones, to be devoted to the consideration of a general form of inference, but to the exposition of certain conceptions which have been found useful in thinking about reasoning.

The hierarchical plan of classification which is used in Zoology and Botany, which divides, for example, created things into kingdoms, each kingdom into Types, each type into classes, each class into orders, each order into families, each family into genera, and each genus into species, is,—as far as its general idea goes,—the idea of Aristotle. In other branches of science we usually find cross-divisions. Thus, the chemical elements are arranged in an oblong table, all those in one column forming a class, and all those in one row forming a class. But in zoology and botany, cross classification is usually considered to be illusory and false. Whether there is any foundation in nature for this prejudice or not, whether or not the classes of living beings, having arisen from variations in generation, really do arrange themselves in the hierarchical fashion, I am confident that as a historical fact this conception of classification was taken from the Aristotelian philosophy. Let me explain this.

That wherein a number of things agree is, as we regard the matter, a consequent. Mammals, reptiles, birds, and fishes all have backbones. We can therefore reason from the fact that an object is a reptile—(say, a tortoise) to the fact that it has a backbone. From that point of view, the common fact or character, is a consequent. But Aristotle thinks that while this is so in the order of knowledge, yet in the order of being the reverse is the case. The general is prior, the particular comes after. Aristotle was a naturalist, an embryologist, and a sort of evolutionist. An egg growing into an animal, was always before his mind as the type of the order of nature. In the egg, the animal is *in posse;* in the egg, we have the unformed matter which takes on successive specializations. Thus, Aristotle regards what is general as matter, what is special as form. An object is first a material body; then it becomes an animal; then, a vertebrate; then, a mammal; then, a biped; last, a man. He has not precisely this classification; but this is his general idea. In each of these steps of development the object takes on a specialization, which in technical language is called a *specific difference.* And Aristotle lays it down that the *differences of different genera are different,* thus prohibiting all cross classification.

It will be observed that the species as compared with the genus, embraces fewer individuals, but it has more characters, namely, first, the specific difference, and then various properties and accidents which are consequences either logical or material of that difference. The individuals embraced in a genus or species were called by the scholastics its *substantial* or *subjective parts;* the characters which go to make it up were called its *essential parts.* They also distinguished between what a noun denotes or *names;* that is to say, the subjects of which it may be predicated (its antecedents), and what it *signifies,* that is, the predicates of which it is the subject (its consequents).

When the Aristotelian doctrine which prohibited cross-division was banished from logic, the conception of the sum of the subjects to which a name is applicable, and the sum of predicates which are applicable to it, as being *two quantities* connected with the name, took on a more abstract, distinct, and almost mathematical character. It was in the Port Royal Logic, entitled *L'Art de Penser,* by Arnauld and Nicole, a clever but by no means powerful work, that the idea first appears clearly developed. It is so far beyond anything else in the book that it seems as if Pascal must have suggested it.

Chapter VI.
The Logical Algebra of Boole.

The actual state of things is represented by a single point in the field of possibility; and the reasonings above treated have been concerned with this single point. Categorical propositions, it is true, in one sense refer to whole classes of individuals, but they only take these classes *distributively,* that is to say, they only consider characters as belonging to any one or some one individual of such a class, not as relations between several individuals.

Every qualitative reasoning about single individuals may be analyzed into syllogisms and dilemmas; but it is beyond the power of ordinary language to state complicated arguments of this kind with perspicuity or precision. In the case of such complicated reasoning,—and indeed wherever the logic of our inferences requires to be analyzed,—there is a great saving of trouble and gain of accuracy in employing a logical "algebra," or perfectly systematic written language or body of symbols in which the premises being expressed, the conclusion may be obtained by transformations of the expressions according to formal rules.

I speak of a body of symbols, but in point of fact, syllogism and dilemma,—every qualitative reasoning about individuals may be expressed by the use of one single symbol besides the expressions for the facts whose relations are examined. This symbol must signify the relation of antecedent to consequent. In the form I would propose for it, it takes the shape of a cross placed between antecedent and consequent with a sort of streamer extending over the former. Thus, "If *a,* then *b,*" would be written

$$\overline{a}\rceil\!\!\!+ b.$$

"From *a,* it follows that if *b* then *c,*" would be written

$$\overline{a}\rceil\!\!\!+ \overline{b}\rceil\!\!\!+ c.$$

"From 'if *a,* then *b,*' follows *c,*" would be written

$$\overline{\overline{a}\rceil\!\!\!+ b}\rceil\!\!\!+ c.$$

To say that *a* is false, is the same as to say that from *a* as an antecedent follows any consequent we like. This is naturally shown by leaving a blank space for the consequent, which may be filled in at pleasure. That is, we may write "*a* is false"

$$\overline{a} \mathrel{\rceil\!\!\!\!+} ,$$

implying that from a every consequence may be drawn without passing from a true antecedent to a false consequent, since a is not true. Instead of the blank space, we might use a special symbol, say a circle, and write

$$\overline{a} \mathrel{\rceil\!\!\!\!+} \mathrm{O}.$$

By means of this sign of illation, or *copula,* we may express the most complicated relations without the slightest ambiguity. Thus, we may write

$$\overline{\overline{\overline{b \mathrel{\rceil\!\!\!\!+} c} \mathrel{\rceil\!\!\!\!+} m} \mathrel{\rceil\!\!\!\!+} n} \mathrel{\rceil\!\!\!\!+} \overline{\overline{\overline{a \mathrel{\rceil\!\!\!\!+} b} \mathrel{\rceil\!\!\!\!+} l} \mathrel{\rceil\!\!\!\!+} m} \mathrel{\rceil\!\!\!\!+} \overline{\overline{a \mathrel{\rceil\!\!\!\!+} c} \mathrel{\rceil\!\!\!\!+} l} \mathrel{\rceil\!\!\!\!+} n$$

Let, for instance, a mean that any object is either Enoch or Elijah, b that that object is a man, c that that object is mortal, l that Enoch and Elijah were not snatched up to heaven, m that the Bible errs, n falsity. Then what is written signifies that if it be false that all men's being mortal would imply that the Bible errs, and if the Bible would err were the fact of Enoch and Elijah being men to imply that they were not snatched up to heaven, then it is false that the fact of Enoch and Elijah being mortal implies that they were not snatched up to heaven. Thus, by some //effort/strain// we can express such a proposition in ordinary language. But to express the *general* proposition about a, b, c, l, m, n, we should have to say

If { If[If(If b, then c), then m], then n },
then (if { if[if(if a, then b), then l], then m },
then if[if(if a, then c), then l], then n.)

But when we write the algebraic formula and desire to pronounce it, we may do so in this way,—the rule being to pronounce "if" at the beginning of each streamer and "then" at each cross. The braces, brackets, and parentheses, are quite unnecessary; and the *ifs* and *thens* may be pronounced rapidly and without pause, because the symbol exhibits the meaning.

Here then we have a written language for relations of dependence. We have only to bear in mind the meaning of the symbol $\rceil\!\!\!\!+$ (not by translating it into *if* and *then,* but by associating it directly with the conception of the relation it signifies), in order to reason as well in this language as in the vernacular,—and, indeed, much better.

So far, we have a *language,* but still no *algebra.* For an algebra is a language with a code of formal rules for the transformation of expressions, by which we are enabled to draw conclusions without the trouble of attending to the meaning of the language we use.

Our algebraic rules must enable us to prove the two propositions,

If $\overline{a \dashv}\, b$, then if a then b; and
If from a follows b, then $\overline{a \dashv}\, b$.

Any rules which will prove these propositions will evidently enable us to prove every conclusion. But that is not enough; for we require that the rules should enable us to dispense with all reasoning in our proofs except the mere substitution of particular expressions in general formulae. We do not as yet demand rules which shall enable us to dispense with difficult reasoning in discovering the truth and in inventing modes of proof,—that would be demanding more than an *algebra,* namely a *calculus,*—but we do require that in the proofs themselves nothing but simple substitutions shall be called for. The first of the above propositions, however, namely, that if a and $\overline{a \dashv}\, b$ then b, being nothing but the *modus ponens,* might stand as a fundamental rule. The second proposition may be divided into two, each of which follows from it while it follows from them dilemmatically. They are

If not a, then $\overline{a \dashv}\, b$; and
If b, then $\overline{a \dashv}\, b$.

In making our code of laws, we shall first want a rule to show what substitutions can be made; next, we must give the properties of denial; and finally we need a general test of necessary truth.

1. The general rule of substitution is that if $\overline{m \dashv}\, n$, then n may be substituted for m under an even number of streamers (or under none), while under an odd number m may be substituted for n.

The rule may be proved true by the peculiar kind of reasoning invented by Fermat. Any expression in which m is under an even number of streamers may be written in one of the forms of the two infinite series here exhibited

m	$\overline{s \dashv}\, m$
$\overline{\overline{m \dashv}\, A \dashv}\, a$	$\overline{\overline{\overline{s \dashv}\, m \dashv}\, A \dashv}\, a$
$\overline{\overline{\overline{\overline{m \dashv}\, A \dashv}\, a \dashv}\, B \dashv}\, b$	$\overline{\overline{\overline{\overline{\overline{s \dashv}\, m \dashv}\, A \dashv}\, a \dashv}\, B \dashv}\, b$
etc.	etc.

The rule holds for the first form of the first series by the *modus ponens,* and for the first form of the second series by syllogism. If in either series the rule holds from the first member up to any form for which we may write M, then let this form after the substitution of n for m become N. Then according to the rule, $\overline{M}\prec N$. The next form of the series after M will be say $\overline{\overline{M}\prec I}\prec i$. Now by dilemmatic reasoning, we have

$$\overline{M}\prec N$$
$$\overline{\overline{M}\prec I}\prec i$$
$$\text{Hence,}\quad \overline{\overline{N}\prec I}\prec i.$$

But this conclusion is the result of substituting n for m in $\overline{\overline{M}\prec I}\prec i$. Thus we see that the rule is true of the first form of each series, and if true of any form true of the next in the same series; consequently, it is true of all forms which can be built up in this way.

In precisely the same way, any proposition in which n is under an odd number of streamers, may be written in one of the forms

$\overline{n}\prec p$	$\overline{\overline{s}\prec n}\prec p$
$\overline{\overline{\overline{n}\prec p}\prec A}\prec a$	$\overline{\overline{\overline{\overline{s}\prec n}\prec p}\prec A}\prec a$
$\overline{\overline{\overline{\overline{\overline{n}\prec p}\prec A}\prec a}\prec B}\prec b$	$\overline{\overline{\overline{\overline{\overline{\overline{s}\prec n}\prec p}\prec A}\prec a}\prec B}\prec b$
etc.	etc.

The rule holds of the first form of the first series by syllogism. It holds of the first form of the second series, because

$$\overline{\overline{s}\prec m}\prec\overline{s}\prec m$$

is true identically; and taking this as a premise of a syllogism of which the other is $\overline{m}\prec n$, we have the conclusion

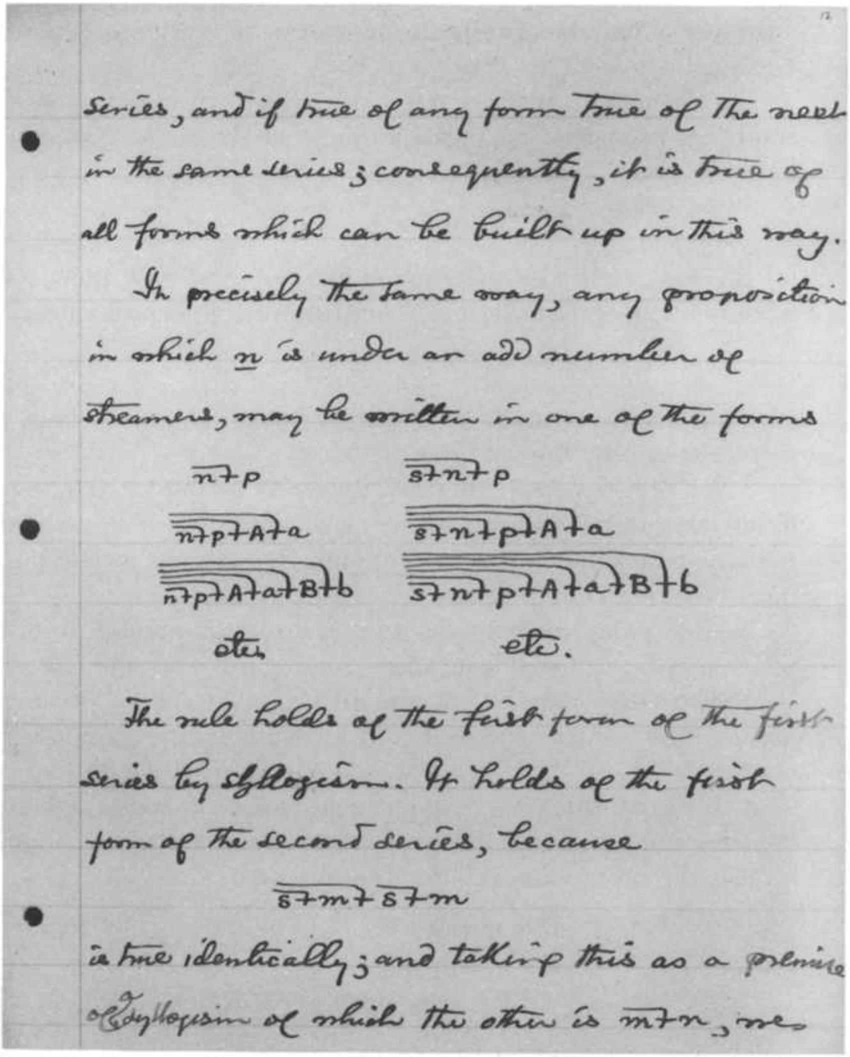
$$\overline{\overline{s}\prec m}\prec\overline{s}\prec n,$$

and this taken with $\overline{\overline{s}\prec n}\prec p$ gives by another syllogism

$$\overline{\overline{s}\prec m}\prec p.$$

If the rule hold for any form N, let this after the substitution be M, and we have by the rule $\overline{N}\prec M$. The next form of the same series is $\overline{\overline{N}\prec I}\prec i$ and by the same dilemma as before we prove that $\overline{\overline{M}\prec I}\prec i$, and consequently that the rule is correct.

2. For the circle which is the symbol of falsity (or that all propositions are true), we have the general formula



series, and if true of any form true of the next in the same series; consequently, it is true of all forms which can be built up in this way.

In precisely the same way, any proposition in which n is under an odd number of streamers, may be written in one of the forms

$\overline{n \prec p}$ $\qquad$ $\overline{s \prec n \prec p}$

$\overline{\overline{\overline{n \prec p \prec A \prec a}}}$ $\qquad$ $\overline{\overline{\overline{s \prec n \prec p \prec A \prec a}}}$

$\overline{\overline{\overline{\overline{\overline{n \prec p \prec A \prec a \prec B \prec b}}}}}$ $\qquad$ $\overline{\overline{\overline{\overline{\overline{s \prec n \prec p \prec A \prec a \prec B \prec b}}}}}$

etc. $\qquad$ etc.

The rule holds of the first form of the first series by syllogism. It holds of the first form of the second series, because

$\overline{s \prec m \prec s \prec m}$

is true identically; and taking this as a premise of a syllogism of which the other is $m \prec n$, we

Holograph page from MS 582
(Harvard Peirce Papers MS 736)

$$\overline{\bigcirc \prec}\, a$$

whatever a may be. The falsity of a, usually written $\overline{a \prec}$ is really equivalent to $\overline{a \prec}\, \bigcirc$.

The formula of identity $\overline{a \prec}\, a$, for which we may write ∞ for short, has properties conjugate to those of the circle. Namely, we have the general formula

$$\overline{b \prec}\, \overline{a \prec}\, a \quad \text{or} \quad \overline{b \prec}\, \infty,$$

for this is a case under the general principle that if B, then $\overline{A \prec}\, B$, since $\overline{a \prec}\, a$ is necessarily true. The truth of b may be written

$$\overline{\infty \prec}\, b;$$

for if b is true, so is this by the general principle just cited, and if this is true, so is b by the *modus ponens*.

3. If a proposition is true when the circle is substituted for one of its letters or terms, and also when ∞ is substituted for the same term, then the proposition is true in its original form; for the term replaced must be either true or false.

Besides these fundamental rules, there are a number of others which may be deduced from them. Among these are the following:

4. The circle may be substituted for any term under an odd number of streamers, and ∞ for any term under an even number or none.

5. If a proposition has its antecedent false or its consequent true, it is true; and conversely, if its antecedent is true while its consequent is false, the proposition is false. That is to say,

If b, then $\overline{a \prec}\, b$
If $\overline{a \prec}$, then $\overline{a \prec}\, b$
If $\overline{a \prec}\, b$, then either $\overline{a \prec}$ or b.[4]

The first proposition follows by the rule of substitution from $\overline{a \prec}\, \infty$ and $\overline{\infty \prec}\, b$, which are true by Rule 2. The second proposition follows from $\overline{a \prec}\, \bigcirc$ and $\overline{\bigcirc \prec}\, b$. The third may be proved by Rule 3, as follows. First, put $a = \infty$, $b = \infty$, and the proposition becomes

4. It must be carefully borne in mind that all our discourse in this part of the book is about individuals. Give $\overline{a \prec}\, b$ a categorical form, and the proposition stated seems false. Namely, it does not follow that because all men are mortal, therefore either every object is a non-man or else that every object is mortal; but it does follow that each single individual is either not a man or is mortal.

$$\text{If } \overline{\infty}\dashv\infty, \text{ then either } \overline{\infty}\dashv \text{ or } \infty, \text{ i.e. } \overline{a}\dashv a$$

Now $\overline{a}\dashv a$ is proved by Rule 3, for both

$$\overline{\infty}\dashv\infty \quad\text{and}\quad \overline{\mathrm{O}}\dashv\mathrm{O}$$

are true by Rule 2. Second, put $a = \mathrm{O}$, $b = \mathrm{O}$, and the proposition becomes

$$\text{If } \overline{\mathrm{O}}\dashv\mathrm{O}, \text{ then either } \overline{\mathrm{O}}\dashv \text{ (that is, } \overline{\mathrm{O}}\dashv\mathrm{O}) \text{ or } \mathrm{O},$$

and the first alternative is true, by Rule 2. Third, put $a = \mathrm{O}$, $b = \infty$, and the proposition becomes

$$\text{If } \overline{\mathrm{O}}\dashv\infty, \text{ then either } \overline{\mathrm{O}}\dashv\mathrm{O} \text{ or } \overline{a}\dashv a.$$

Fourth, put $a = \infty$, $b = \mathrm{O}$. In this case, the proposition becomes

$$\text{If } \overline{\infty}\dashv\mathrm{O}, \text{ then either } \overline{\infty}\dashv\mathrm{O} \text{ or } \mathrm{O}.$$

Now if $\overline{\infty}\dashv\mathrm{O}$ then $\overline{\infty}\dashv\mathrm{O}$ is a case under $\overline{a}\dashv a$.

6. We have necessarily

$$\overline{\overline{a}\dashv\overline{b}\dashv c}\dashv\overline{b}\dashv\overline{a}\dashv c,$$

so that, by the *modus ponens,*

$$\overline{a}\dashv\overline{b}\dashv c = \overline{b}\dashv\overline{a}\dashv c$$

and antecedents can be transposed. This is proved by Rule 3, for

$\overline{\overline{\mathrm{O}}\dashv\overline{\mathrm{O}}\dashv c}\dashv\overline{\mathrm{O}}\dashv\overline{\mathrm{O}}\dashv c$ is true, since $\overline{\mathrm{O}}\dashv c$ is true;
$\overline{\overline{\mathrm{O}}\dashv\overline{\infty}\dashv c}\dashv\overline{\infty}\dashv\overline{\mathrm{O}}\dashv c$ is true for the same reason;
$\overline{\overline{\infty}\dashv\overline{\mathrm{O}}\dashv c}\dashv\overline{\mathrm{O}}\dashv\overline{\infty}\dashv c$ is true, since $\overline{\mathrm{O}}\dashv\overline{\infty}\dashv c$ is true;
$\overline{\overline{\infty}\dashv\overline{\infty}\dashv c}\dashv\overline{\infty}\dashv\overline{\infty}\dashv c$ is true by identity.

7. We have necessarily

$$\overline{\overline{\overline{\overline{a}\dashv b}\dashv c}\dashv d}\dashv\overline{a}\dashv\overline{\overline{b}\dashv c}\dashv d,$$

so that by the *modus ponens,*

$$\text{If } \overline{\overline{\overline{a}\dashv b}\dashv c}\dashv d, \text{ then } \overline{a}\dashv\overline{\overline{b}\dashv c}\dashv d,$$

and the ends of two streamers terminating together can be cut off to any point together. This is conveniently proved by a *reductio ad absurdum* by means of Rule 5. The proposition can only be false if

(1) $\overline{\overline{\overline{a} \prec b} \prec c} \prec d$ is true, while
(2) $\overline{a} \prec \overline{\overline{b} \prec c} \prec d$ is false.

The second can be false only if

(3) a is true, while
(4) $\overline{\overline{b} \prec c} \prec d$ is false.

The fourth can be false only if

(5) $\overline{b} \prec c$ is true, while
(6) d is false.

But if (1) is true while d is false,

(7) $\overline{\overline{a} \prec b} \prec c$ is false, and therefore
(8) $\overline{a} \prec b$ is true, while
(9) c is false.

But if c is false, while (5) is true,

(10) b is false.

And if b is false, while (8) is true, a is false, which contradicts (3).

8. Suppose two propositions such that when any term or terms are replaced by the circle, one proposition becomes false, while when the same terms are replaced by identity, $\overline{a} \prec a$, the other proposition becomes true, then the latter follows from the former, both in their original forms. For let A be the proposition which becomes $\bigcirc$, and let A' be what it becomes when identity is substituted. Let B be the proposition which becomes true, and let B' be what it becomes when the circle is substituted. Then the proposition $\overline{A} \prec B$, becomes in the two cases $\overline{\bigcirc} \prec B'$ and $\overline{A'} \prec \infty$, both of which are true; so that $\overline{A} \prec B$ is true by Rule 3.

9. We have necessarily

$$\overline{\overline{A} \prec \overline{B} \prec C} \prec \overline{\overline{C} \prec L} \prec \overline{A} \prec \overline{B} \prec L,$$

so that from

$$\overline{A} \prec \overline{B} \prec C \quad \text{follows} \quad \overline{\overline{C} \prec L} \prec \overline{A} \prec \overline{B} \prec L.$$

For if either A or B is replaced by the circle the proposition is true. If both are replaced by ∞ and C by the circle, $\overline{A} \prec \overline{B} \prec C$ is false, and again the proposition is true. If C is ∞ and L is $\bigcirc$, $\overline{C +}\, L$ is false and the proposition is true, while if L is true the proposition is true.

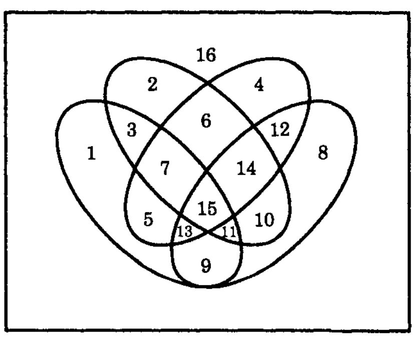
10. Any two premises a and b may at once be united in the form $\overline{\overline{a +}\; \overline{b +}\; +}$. For if the two premises are true, this form becomes $\overline{\overline{\infty +}\; \overline{\infty +}\; \bigcirc +}\; \bigcirc$ which is true because $\overline{\infty +}\; \overline{\infty +}\; \bigcirc$ is false.

[Alternate Set of Rules]

Rule I.

If a and b, then $\overline{\overline{a +}\; \overline{b +}\; +}$.

This is the *rule of combination.* It is plain that we cannot reason, unless we can combine different premises; and $\overline{\overline{a +}\; \overline{b +}\; +}$ expresses no more than that a and b are both true at once. To show this, we use a diagram invented by Mr. Venn.



The rectangle represents the whole field of possibility. The oval containing the odd numbers represents the area of possibility of a. The oval containing the even numbers not divisible by 4 and the odd numbers one less than 4 (2, 6, 10, 14, and 3, 7, 11, 15) represents the area of possibility of b. The oval containing the numbers 4 to 7 and 12 to 15 inclusive represents the area of possibility of another fact c; and finally the oval containing the numbers from 8 to 15 inclusive represents the area of possibility of a fourth fact d. The proposition $\overline{b +}\, c$ is true wherever b is not true or c is true, that is everywhere except in the compartments 2, 3, 10, 11. The proposition $\overline{a +}\; \overline{b +}\, c$ is true wherever $\overline{b +}\, c$ is true or a is not true, that is, everywhere except in compartments 3 and 11. Finally, the proposition

$\overline{\overline{a \dashv}\,\overline{b \dashv}\,c \dashv}\,d$ is true wherever $\overline{a \dashv}\,\overline{b \dashv}\,c$ is not true or w is true, that is, in compartment 3 and all those numbered from 8 to 15 inclusive. Now the last expression $\overline{\overline{a \dashv}\,\overline{b \dashv}\,c \dashv}\,d$ becomes $\overline{\overline{\overline{a \dashv}\,\overline{b \dashv}}\,\dashv}$ on substituting O for c and d. That is, we must erase the ovals c and d. Then there remain only the compartments 1, 2, 3, 16, of which 3 is that where a and b are both true, and also that where $\overline{\overline{a \dashv}\,\overline{b \dashv}\,\dashv}$ is true.

Rule II.

$\overline{a \dashv}\,\overline{b \dashv}\,b$ is necessarily true.

This is the *rule of identity.* To prove it, we note as above that in the diagram the proposition $\overline{a \dashv}\,\overline{b \dashv}\,b$ is true everywhere except in the compartments 3 and 11. Now make $c = b$, that is, erase all compartments where either extend beyond the other, that is, compartments 2 to 5 and 10 to 13. Then we have erased 3 and 11 and consequently all where $\overline{a \dashv}\,\overline{b \dashv}\,c$ is not true. So that $\overline{a \dashv}\,\overline{b \dashv}\,b$ is true everywhere.

Rule III.

$\overline{a \dashv}\,\overline{b \dashv}\,c = \overline{b \dashv}\,\overline{a \dashv}\,c.$

This is the *rule of commutation.* We have seen that $\overline{a \dashv}\,\overline{b \dashv}\,c$ is true everywhere except in compartments 3 and 11, which are symmetrically placed with reference to the ovals of a and b. Consequently, a and b can be transposed in the proposition without change of meaning.

Rule IV.

If $\overline{\overline{a \dashv}\,\overline{\overline{a \dashv}\,c \dashv}\,\dashv}$, then c.

This is the *rule of the modus ponens.* The proof of the rule of combination, by putting $\overline{a \dashv}\,c$ in place of b, shows that $\overline{\overline{a \dashv}\,\overline{\overline{a \dashv}\,b \dashv}\,\dashv}$ expresses the conjoint truth of a and $\overline{a \dashv}\,c$. Consequently, by the *modus ponens, c* follows from it.

Rule V.

If $\overline{\overline{\overline{a \dashv}\,b \dashv}\,\overline{\overline{b \dashv}\,c \dashv}\,\dashv}$, then $\overline{a \dashv}\,c$.

This is the *rule of syllogism.* It is proved similarly to rule IV.

Rule VI.

If $\overline{b}+$, then $\overline{b}+c$.

This is the *rule of contradiction.* It serves to state that negation is equivalent to saying that every consequent follows from the fact denied. It is proved by the diagram which shows that $\overline{b}+c$ is true over the whole field of possibility except a part of the area of b. If therefore the actual state of things is a point outside of b, it is within the area of the truth of $\overline{b}+c$.

Rule VII.

If $\overline{\overline{\overline{\overline{a}+b}+c+\overline{\overline{a}+c}+}}+$, then c.

This is the *rule of dilemma.* It is proved like rules V and VI.

We now have a complete algebra for qualitative reasoning concerning individuals. But it is not yet a very commodious calculus. To render it so, we introduce certain abbreviations which make it identical with the logical algebra of Boole as modified by Jevons and Mitchell.[5] Namely we first separate the streamer of the sign of illation from the cross, and in place of

$$\overline{a}+b \quad \text{write} \quad \bar{a}+b.$$

Second, wherever the sign of illation is followed by a blank we omit the cross, and thus

in place of $\overline{a}+$, write $\bar{a}$.

Third, as the sign of the simultaneous truth of a and b, instead of writing $\overline{\overline{a}+\overline{b}+}+$, or $\overline{\bar{a}+\bar{b}}$, we write simply ab. This last is a superfluous sign, adopted for the sake of abbreviation. Our Seven Rules now take the following forms.

I. *Rule of combination.* If a and b, then $\overline{\bar{a}+\bar{b}}=ab$.
II. *Rule of identity.* $\bar{b}+\bar{a}+a$.
III. *Rule of commutation.* $\bar{a}+\bar{b}+c=\bar{b}+\bar{a}+c$, or $\bar{a}+\bar{b}=\bar{b}+\bar{a}$.
IV. *Rule of the modus ponens.* If $a(\bar{a}+c)$, then c.
V. *Rule of Syllogism.* If $(\bar{a}+b)\,(\bar{b}+c)$, then $\bar{a}+c$.
VI. *Rule of contradiction.* If $\bar{b}$, then $\bar{b}+c$.
VII. *Rule of dilemma.* If $(a\bar{b}+c)\,(\bar{a}+c)$, then c.

5. Other modifications by Mr. Mitchell relate to the logic of relatives.

The Logic of Relatives: qualitative and quantitative

Item 55 *MS 584: 1886*

§1. My purpose is to give as elementary an account as possible of the logic of relations, while carrying the subject considerably beyond the previous bounds of our knowledge. I shall employ the same algebraical notation used in my last paper in Vol. vii of the *American Journal of Mathematics.* This requires an acquaintance with the Boolian algebra in the form given to it by Professor Mitchell. Accordingly, my paper will consist of three parts, on the non-relative logic, relative logic, and quantitative logic.

Part I.

§2. In all reasoning, we pass from belief in one proposition, say A, to belief in another, say B. In demonstrative reasoning, we hold that if A is true, B is true. What should it mean to say that if A is true, B is true? Every kind of state of things which we do not know not to be the actual state of things is said to be *possible.* The possible is that which is not known to be non-existent, or what is either existent or unknown in respect to existence. To know that if A is true, B is true, is to have such knowledge that should we come to know A to be true, we shall at once know B is true. That is to say, among the states of things our knowledge admits, there is none in which A is true while B is false, but in every possible case either A is false or B is true. It may be that A is false or B true without our *knowing* that if A is true, B is true; it may also be that we may know A to be false or B true, while the ordinary usage of language does not permit us to say that if A is true, B is true. But in logic, we must follow a perfectly consistent and rational system of phraseology, and according to such a system the proposition "If A, then B," is true if A is false or B true and is false only if A is true while B is false.

But in order to take one difficulty at a time, we will begin by supposing that the possible is limited to a single state of things, the actually existing state.

§3. Let us now ask the meaning of such propositions as "If A is true, then it is true that if B is true C is true," and "If it be true that if A is true B is true, then C is true." To thread these mazes, and others far more complicated, a notation is called for. Let us then write

$$\overline{A} \dashv B$$

to signify that if A is true, B is true, that is, that either A is false or B true, or to deny that A is true and B false, at once.

Let

$$\overline{A} \dashv \overline{B} \dashv C$$

mean that if A is true $\overline{B} \dashv C$ is true; that is, that either A is false or $\overline{B} \dashv C$ true, or again that either A is false or B is false or C is true. This interpretation shows that

$$\overline{A} \dashv \overline{B} \dashv C = \overline{B} \dashv \overline{A} \dashv C.$$

Let

$$\overline{\overline{A} \dashv B} \dashv C$$

mean that if $\overline{A} \dashv B$ is true, then C is true, that is, that either $\overline{A} \dashv B$ is false or C true, or again that either A is true and B false or C is true.

This completely describes the notation. It does not yield a convenient calculus, but it has the logical merit of doing everything that the Boolian algebra does without any superfluous symbols.

By way of exercise, let us interpret a few propositions expressed in this system. Let F be a proposition known to be false. Then $\overline{A} \dashv \mathrm{F}$ will mean that either A is false or F true, that is, that A is false. $\overline{\overline{A} \dashv A} \dashv B$ will mean that either A is true and A false or B is true, that is, that B is true. $\overline{\overline{A} \dashv B} \dashv A$ will mean that either A is true and B false or A is true, that is, that A is true. $\overline{\overline{B} \dashv A} \dashv A$ will mean that either B is true and A false or A is true, that is, that either A or B is true. $\overline{\overline{A +}\, \overline{B +}\, \mathrm{F} +}\, \mathrm{F}$ will mean that $\overline{A +}\, \overline{B +}\, \mathrm{F}$ is false, that is, that A and B are true and F false, that is, that A and B are true.

§4. Let us now ask, supposing we had given a complicated proposition in this notation, such as

$$\overline{\overline{\overline{\overline{A +}\, B +}\, \overline{C +}\, D +}\, \overline{\overline{E +}\, \mathrm{F} +}\, \overline{G +}\, H +}\, \overline{\overline{\overline{I +}\, J +}\, \overline{K +}\, L +}\, \overline{\overline{M +}\, N +}\, \overline{O +}\, P$$

what transformations should we have a right to operate upon it without impairing its truth?

We need first a little terminology. Let us call the lines extending over the letters "streamers," and let us say that the letters under them are covered by them. Thus A in the above proposition is covered by four streamers, and B by three. Let the streamer with its $+$ on the right be called a "sign." Let a proposition of the n^{th} order, where n is any number, be one in which the highest number of streamers covering any letter is n. Let the "immediate components" of a proposition be all those parts into which its expression can be separated by vertical cuts not severing any streamer. Thus, the immediate components of $\overline{\overline{A +}\, B +}\, \overline{C +}\, D$ are $\overline{\overline{A +}\, B +}$, $\overline{C +}$, D, $\overline{\overline{A +}\, B +}\, \overline{C +}$, $\overline{C +}\, D$, and $\overline{\overline{A +}\, B +}\, \overline{C +}\, D$. Let the "constituent parts" of a proposition consist of its immediate components, of every expression obtained by removing a sign from over the whole of an immediate component, and of every expression obtainable by such a decomposition of the proposition. Thus the constituent parts of $\overline{\overline{A +}\, B +}\, \overline{C +}\, D$, besides its immediate components, are $\overline{A +}\, B$, C, $\overline{A +}$, B, and A. But $\overline{\overline{A +}\, B +}\, C$ is not a constituent part.

I now give the principal rules of transformation, with the proofs of them.

Rule I. The immediate components of any constituent part of a proposition can be transposed among one another in any way.

Proof. I shall first show that if the rule holds good for every proposition of the n^{th} order or lower, it holds also for every proposition of the $(n + 1)^{\text{th}}$ order; and then I shall show that it holds good for every proposition of the first order. This will prove the rule for every proposition we can write down; for in writing down a proposition we build up its order by successive increments of unity.

Let $\overline{A +}\, \overline{B +}\, \overline{C +} \ldots Z$ represent a proposition of the $(n + 1)^{\text{th}}$ order, where A, B, C, etc. are propositions of orders not higher than the n^{th}. Then, if the rule is true of the first order, the immediate components of the whole proposition can be transposed in any way among one another. If the rule is true up to the n^{th} order, and if A' represents the result of any transposition of the immediate components of any constituent part of A, then if A is true, A' is true, and if A' is true, A is true. Hence, A and A' represent the same possible state of things. Now the proposition of the $(n+1)^{\text{th}}$ order may be written

$$\overline{A \dashv}\, W$$

and if this is true, evidently

$$\overline{A' \dashv}\, W$$

is also true. Now the constituent parts of the proposition of the $(n + 1)^{\text{th}}$ order consist of its immediate components and of the constituent parts of A, B, etc. The rule has been proved to hold of both these kinds of constituent parts, provided it holds of every proposition of an order not higher than the n^{th}.

I now prove that the rule holds for the first order. To do this, I first show that it holds when the number of elementary components is three; and then that if it holds when the number of elementary components is n, it also holds when this number is $n + 1$. We have already seen that it holds for $\overline{A \dashv}\,\overline{B \dashv}\, C$, that is, when there are three elementary components. Let $\overline{A \dashv}\,\overline{B \dashv}\, W$ represent any proposition of the first order having $(n + 1)$ elementary components, W standing for a similar proposition having $(n - 1)$ elementary components. Then

$$\overline{A \dashv}\,\overline{B \dashv}\, W = \overline{B \dashv}\,\overline{A \dashv}\, W,$$

for this may be regarded as having but three elementary components. If, then, every transposition of the components of $\overline{B \dashv}\, W$ and $\overline{A \dashv}\, W$ is allowable, every transposition of the components of $\overline{A \dashv}\,\overline{B \dashv}\, W$ is allowable.

The proof is now complete, except that we have not referred to displacements of the last elementary component of a constituent part. We have adopted no rule for the interpretation of such an expression as $\overline{A \dashv}\, C\, \overline{B \dashv}$. But we plainly could not pass through such an expression to anyone that should be interpretable and unallowable, and therefore we may extend the rule to these cases. In fact, so long as it is evident that no result could be reached by an indirect application of the rules which could not be reached directly, or that no interpretable result could be reached through an uninterpretable one which could not be reached without passing through an uninterpretable stage, we need not concern ourselves with uninterpretable results.

Rule II. We may insert any expression under an even number of streamers, or erase any expression under an odd number.

Thus, if we have given $\overline{A + B +}\, C$, we may insert $\overline{D +}$ under two streamers, giving $\overline{\overline{D + A + B +}}\, C$, or under none, giving $\overline{D +}\,\overline{A + B +}\, C$; or we may erase $\overline{A +}$, which is under one streamer, leaving $\overline{B +}\, C$.

Proof. The application of the remark just made shows that we need not concern ourselves with uninterpretable results. Thus, if we add an expression not under a single sign, we get an uninterpretable expression, because there will then be a constituent part having two elementary components under even numbers of signs; but this cannot, by any rules thus far enounced, lead to any interpretable expression, for nothing can be erased as the component of an expression to which a component could be inserted. For a similar reason the erasure of an expression covered by an even number of signs cannot lead to anything interpretable.

I now proceed to show, first, that the rule holds for propositions of the lowest orders, and then that if it holds for all propositions up to the n^{th} order, inclusive, it holds for all those of the $(n+1)^{\text{th}}$ order.

Let A be a proposition of the *zero* order. The only even number of streamers covering any part of this is *zero*. We may insert $\overline{X +}$, for $\overline{X +}\, A$ is true if either X is false or A true, *à fortiori* therefore if A is true. There is nothing under an odd number of streamers to be erased.

Let $\overline{A +}\, B$ be a proposition of the first order. The only insertion giving an interpretable result is $\overline{X +}\,\overline{A +}\, B$ which is manifestly true. The only erasure following the rule gives an uninterpretable result.

Let

$$\overline{W_1 +}\,\overline{W_2 +}\,\overline{W_3 +} \ldots W_n$$

be a proposition of the second order, where

$$W_1 = \overline{A_1 +}\,\overline{B_1 +}\,\overline{C_1 +} \ldots T_1,$$
$$W_2 = \overline{A_2 +}\,\overline{B_2 +}\,\overline{C_2 +} \ldots T_2,$$
etc.

The only erasures giving interpretable results would be erasures of elementary components of W's, not T's, such as A_1. Put

$$\begin{aligned} U_1 &= \overline{B_1} + \overline{C_1} + \ldots T_1 \\ V &= \overline{W_2} + \overline{W_3} + \ldots W_n \end{aligned}$$

Then the proposition takes the form

$$\overline{\overline{A_1} + U_1} + V.$$

It is evident that $\overline{A_1} +$ can be erased, giving

$$\overline{U_1} + V,$$

for if $\overline{\overline{A_1} + U_1} + V$ is true, either A_1 is true and U_1 is false, or V is true; hence, either U_1 is false or V is true; hence, if U_1 is true, V is true, or

$$\overline{U_1} + V.$$

The rule is thus proved true for the propositions of lowest degrees.

Now let

$$\overline{W_1} + \overline{W_2} + \overline{W_3} + \ldots W_n$$

be a proposition of the $(n' + 1)^{\text{th}}$ order, W_1, W_2, W_3, etc. being of the n^{th} order at most. Suppose that according to the rule, which holds good up to the n^{th} order inclusive, $\overline{X} +$ could be inserted in W_1, and let Y be the result. Then, it follows from the rule that $\overline{X} +$ can be erased from

$$\overline{Y} + \overline{W_2} + \overline{W_3} + \ldots W_n,$$

giving

$$\overline{W_1} + \overline{W_2} + \overline{W_3} + \ldots W_n$$

Put

$$Z = \overline{W_2} + \overline{W_3} + \ldots W_n$$

then the rule only says that if from W_1 follows Y, then from $\overline{Y} + Z$ follows $\overline{W_1} + Z$; or supposing these premises are true

If W_1, then Y
If Y, then Z;

then this conclusion is true,

If W_1, then Z.

But this is obviously true.

Next suppose according to the rule $\overline{X}+$ can be erased from W_1 giving V. Then the purport of the rule is that $\overline{X}+$ can be inserted in

$$\overline{V}+Z$$

giving

$$\overline{W_1}+Z$$

which is evident as before.

Rule III. Consider a constituent part of a proposition lying under an even number of streamers. Any direct component of this constituent can be inserted again under any odd number of streamers in any other component of the same constituent. And in like manner, we can insert as a direct component of any constituent under an odd number of streamers any expression found under an even number of streamers as a component of a constituent of that constituent.

Thus, if we have given $\overline{A}+\overline{B}+C$, we can write $\overline{\overline{B}+A}+\overline{B}+C$; and if we have given $\overline{\overline{\overline{A}+B}+C}+D$ we can write $\overline{\overline{\overline{A}+B}+\overline{A}+C}+D$.

An Elementary Account of the Logic of Relatives

Item 56 ***MS 585: 1886***

Signs, or representations, are of three kinds: Icons, Indices, and Tokens. The token represents its object in consequence of a mental association, and depends upon a habit. Such signs are abstract and general, because habits are general rules to which the organism has become subjected. They are, for the most part, conventional and arbitrary. All common nouns are tokens. Indeed, all words, insofar as they are conventional, partake of this character.

Indices are signs which stand for their objects in consequence of a real relation to them. An index is a sign which stands for its object in consequence of having a real connection with it. A pointing finger is its type. Of this sort are all natural signs and physical symptoms. The index has no generality in itself. It does not depend on a mental association, but upon a real reaction between the mind and the external world at the moment when the index acts. The index asserts nothing; it only says "There!" It takes hold of our eyes, as it were, and forcibly directs them to a particular object, and there it stops. For example, I wanted one day to know whether it was raining. I looked out of the window, and could not be sure that I saw any rain. However, I noticed a number of people with their umbrellas up, and concluded it was raining. Now so far as the open umbrella asserts rain, it does so by virtue of a mental association, and is a token, but so far as it assures us of the thenness and thereness of its indication, it had nothing at all to do with association, but acted directly on my faculty of influencing-and-being-influenced-by-the-external-world. Relative pronouns are indices, for though their meaning is to a certain extent conventional, yet what a relative pronoun stands for in any particular case depends upon the observed position of the words in the sentence. Demonstrative pronouns are also indices, because we have to make out what they denote by an observation of the circumstances of //the/each// case. The letters on a geometrical diagram are indices, and so are the subscript numbers which, in algebra, distinguish one root of an equation from another. You cannot say how large anything is without making use of an index, for if the Euclidean system of geometry is true, there are no geometrical properties depending on size. You cannot, therefore, describe a size; you can only say as large as this here. You cannot distinguish an actual occurrence from a hypothetical one, or make your discourse refer to the reality, except by an index. For actuality has no general characters; we can only cognize it by our consciousness of influencing-and-being-influenced-by-the-external-world.

The icon represents its object by virtue of resembling it. It thus depends upon simple feeling. Mental association has nothing to do with it. The icon has no generality, because it does not analyze the character it exhibits. There is thus no more generality in the icon than there is in its object. Nor has the icon anything to do with the sense of contact with the world, nor with the actual existence of its object. It is a mere dream. Icons comprehend all pictures, imitations, diagrams, and examples. Every algebraical formula, in so far as it shows the letters connected by signs in a manner analogous to the relations between the quantities those letters denote, is an icon: q^q. No quality or character of any kind can be conveyed or made known, except by means of an icon. With reference to qualities of feeling this is evident. But it is equally true with regard to relations. If a person did not know what it was for two objects to be connected together, how could it possibly be explained to him, except by an example? The analysis of a complex character may be represented by means of tokens, but how the elements will appear when they are put together, only an icon can show.

It is, of course, obvious that most signs, if not all, partake more or less of the characters of the three varieties. Thus pictures, though their main mode of representation is iconical, yet depend very much upon conventions. That is why new methods of painting are always unpopular; people cannot see, at first, that a picture resembles nature until they have become familiar with the conventions of the method. Nevertheless, it will be found that in any instance of the use of a sign, there is one of the three characters which completely overshadows the others; and that the division is practically most important.

Every inquiry is carried on by means of experimentation, External or Internal. The chemist mounts an apparatus of flasks and tubes, places certain substances in the flasks, lights a Bunsen burner underneath, and watches to see what the result will be. The mathematician constructs a geometrical diagram according to a certain prescription which describes the relations of the parts sufficiently for the purpose, and then looks out for new relations, not thought of in the construction. The chemist relies on the laws of nature; the mathematician on the associations and laws of the mind. Whatever distinctive names you apply to the two proceedings, they should receive one general designation, to indicate their essential resemblance. When, in 1877, I wrote, "Lavoisier made of his alembics and cucurbits instruments of thought, giving a new conception of reasoning, as something which was to be done with one's eyes open *[ ]*

QUALITATIVE DEDUCTION

Though my main purpose is to explain the logic of relatives, yet as that requires an account of the Boolian algebra, I think it will be useful to make this account clear enough to serve the purpose of a beginner. The Boolian algebra is called Boolian after the English mathematician, George Boole, who invented it about 1846 (though it has been greatly improved since), and is called algebra because it employs the nomenclature and notation of addition and multiplication, but with meanings having only a general resemblance to their original arithmetical meanings.

The letters of the alphabet are used to express propositions. Thus, we might agree to regard the letter j standing by itself as asserting that no man ever made the sun stand still, and e standing by itself as meaning that the Bible contains an error. Logical addition is aggregation of logical breadth; that is to say, whatever propositions may be signified by x and y, $x + y$ will signify that proposition which is true if either x or y is true, and is false only if x and y are both false. Thus, $j + e$ would assert that either no man ever made the sun stand still or else the Bible errs; it is true if either or both of these propositions are true, and is false if both are false. Logical multiplication is combination in logical depth; that is to say, xy signifies that proposition which is true only if x and y are both true, and is false if either is false. Thus, je would assert that no man ever made the sun stand still and that the Bible errs; it is true only if both these propositions are true, and is false if either is false.

To say that two propositions are equivalent, to write, for example, $x = y$, means that the propositions are either both true or both false. The reader may object to this, rightly holding that two propositions ought not to be considered as equivalent unless every possible state of things in which either would be true would be one in which the other was true. But we shall see below that it is necessary to make a special attachment to a Boolian expression to enable it to take account of more than one state of things. At present we can only consider what actually is, and not the circumstances under which a proposition would be true, and from this point of view there is no distinction between propositions unless that one is true and another false. This is somewhat difficult to rightly apprehend. The propositions that no man ever made the sun stand still and that the Bible errs are certainly not identical because we certainly can conceive of either being true while the other is false. But we represent these propositions by j and e, and this we do for the purpose of saving ourselves trouble by thinking of //j and e/the letters// instead of these propositions. We shall find need of enlarging the Boolian algebra so that it can take account of those conceivable cases in which j should be true while e was false or vice versa, independent of what actually occurs; but so far, it is restricted to the actual state of things, and in so far, if j and e are both true or are both false, they are identical. I do not say that the propositions they are used to represent are identical, because these propositions refer to conceivable states of things possibly different from the actual state; but j and e can, from the nature of the Boolian algebra, at present only represent those propositions in their relations to the actual state of things, and thus are equivalent under the circumstances mentioned.

Thus, every true proposition has one value, and every false one another. Every algebra which recognizes only two possible values in the system of quantity is substantially the same as the Boolian algebra. Let T be the value of the true, and F the value of the false. Then any algebra in which every quantity, x, is subjected to the equation $(x - \mathrm{T})(x - \mathrm{F}) = 0$, so that either $x = \mathrm{T}$ or $x = \mathrm{F}$ may be used for the same purpose as the Boolian algebra; and is converted into it by operating upon every quantity in a certain way. But in order to preserve the above definitions of addition and multiplication, we may make $\mathrm{F} = 0$ while for T we may take an infinite of the zero order and positive sign which we may denote by \$. For we have, as the definitions of addition and multiplication require,

$$0+0=0, \quad 0+\$=\$, \quad \$+0=\$, \quad \$+\$=\$,$$
$$00=0, \quad 0\$=0, \quad \$0=0, \quad \$\$=\$.$$

From the eight equations last written as premises all the general equations expressing the properties of logical addition and multiplication can be deduced. These general equations may be stated as follows.

1st, Logical addition and multiplication are both commutative; that is to say,

$$x+y=y+x \qquad xy=yx$$

whatever propositions x and y may signify. This is evident, because the sum is equal to \$ if either of the additive terms is \$, and otherwise is equal to 0. And the product is equal to 0 if either of the factors is equal to 0, and otherwise is equal to \$. Thus, the order of the letters has nothing to do with the value.

2nd, Logical addition and multiplication are both associative; that is to say,

$$(x+y)+z=x+(y+z) \qquad (xy)z=x(yz)$$

This is proved in the same way.

3rd, Multiplication is distributive with reference to addition, as in ordinary algebra; and furthermore, addition is distributive with reference to multiplication. That is to say, not only do we have in every case,

$$(a+b)\,(x+y)=ax+ay+bx+by$$

but also,

$$ab+xy=(a+x)\,(a+y)\,(b+x)\,(b+y)$$

4th, We have the two peculiar equations

$$x+x=x \qquad xx=x.$$

Besides the transformations sanctioned by these equations, we are also at liberty to add any term we please to any accepted proposition or to any factor of that proposition. Thus, if we have admitted je, we can admit $(j+x)e$ whatever may be the signification of x. On the other hand, we are at liberty to delete any factor of any accepted proposition or of any additive term of such proposition. If we have admitted *je,* we have virtually admitted *j.* But we must not delete an entire additive term or entire accepted proposition, unless we leave \$ in place of it.

One more sign is necessary, before we can make much use of the Boolian algebra. Let $\bar{x}$ signify the denial of the proposition x. That is, let the drawing of a line over any expression amount to the denial of the proposition signified by the expression. The properties of the negative are all deducible from the two propositions that $\bar{0} = \$$, $\bar{\$} = 0$. The most useful rules are as follows:

1st, $\bar{\bar{x}} = x$.

2nd, we have a right to delete $x\bar{x}$ as an additive term, and also to insert $(x + \bar{x})$ as a factor.

3rd, the negative of a sum is the product of the negatives of its terms, and the negative of a product is the sum of the negatives of its factors.

I will now show by a few examples the use of the Boolian algebra. When a number of premises are given, they must be multiplied together with the application of the formula

$$(x + uy)\ (v\bar{y} + z) = xv\bar{y} + uyz.$$

This gives a conclusion precisely equivalent to the premises. After this, such a course is to be pursued as the nature of the question calls for.

Example 1. Given these premises. If the truth of your discovery would certainly carry with it the falsity of the proposition that man is but a protoplasmic organism, you would either be a benefactor of humanity or a charlatan. But if your discovery is true, some men are not doomed to perish; and you are certainly not a charlatan. Does it follow that if every protoplasmic organism is doomed to perish, you are a benefactor of humanity?

Let a mean that your discovery is true.

b	that you are a benefactor of humanity.
c	that you are a charlatan.
m	that any given object is a man.
o	that that organism is a protoplasmic organism.
p	that that object is doomed to perish.

Then $\bar{m} + o$ will mean that any given object is either not a man or is a protoplasmic organism, that is, that every man is a protoplasmic organism. The negative of this is $m\bar{o}$, that is, a given object is a man without being a protoplasmic organism. Then $\bar{a} + m\bar{o}$ means that either your discovery is not true or a given object is a man without being a protoplasmic organism; that is, the truth of your discovery would certainly carry with it the falsity of the proposition that man is but a protoplasmic organism. The negative of this is $a(\overline{m} + o)$ and therefore $a(\overline{m} + o) + b + c$ represents with sufficient accuracy the first premise. Next $\overline{m} + p$ means that a given object is either not a man or is bound to perish, and $m\bar{p}$ means that a given object is a man but is not bound to perish, and $\bar{a} + m\bar{p}$ means that either your discovery is not true or a given object is a man without being bound to perish, which sufficiently represents the second premise. Finally $\bar{c}$ is the third premise. The product of the premises is

$$(\bar{a} + m\bar{p})\ [a(\overline{m} + o) + b + c]\ \bar{c} = m\bar{p} + a\bar{c} + (\bar{a} + m\bar{p})b\bar{c}$$

By the deletion of factors, this gives $o\bar{p} + b$, or a given object is a protoplasmic organism without being doomed to perish or else you are a benefactor of humanity. In other words, if every protoplasmic organism is doomed to perish, you are a benefactor of humanity. The premises cannot be precisely expressed without certain additions to the Boolian algebra which will be considered below; still, with caution, the above proceeding may be followed.

Example 2. The members of a board were all of them either bondholders or shareholders, but no member was bondholder and shareholder at once; and the bondholders, as it happened, were all on the board. What was the relation between the classes of bondholders and shareholders? Let a mean that any given person was a member of the board, b that he was a bondholder, c that he was a shareholder. Then, $\bar{a} + b + c$ will mean that any given person was either not a member of the board or was a bondholder or was a shareholder; which is the first premise. And $\bar{a} + \bar{b} + \bar{c}$ will mean that any given person was either not a member of the board or not a bondholder or not a shareholder; which is the second premise. And $\bar{b} + a$ will mean that any given person was either not a bondholder or was a member of the board. The product of the first and third premises, which are the most easily multiplied, (one easily recognizes this, after some practice) is

$$(\bar{b} + a)\ (\bar{a} + b + c) = \bar{a}\ \bar{b} + ab + ac.$$

To multiply this by the second premise, we recognize that this simply requires that one of the three letters should be negatived in each additive term. We therefore write out the product at once, as follows: $\bar{a}\,\bar{b} + ab\bar{c} + a\bar{b}c$. Now deleting a, this becomes $\bar{b} + b\bar{c} + \bar{b}c = \bar{b} + \bar{c}$ or no person is at once bondholder and shareholder.

Example 3. A young king, on coming to the throne, found himself attacked by a powerful neighbor, and thereupon made the following reflections. I shall either prove myself to be a great man and conquer in the first campaign or a prudent man and trust implicitly in my chancellor or a fool and ruin the dynasty by headstrong rashness. If I am a great man or popular with the army, my soldiers will stand by me. If I am a prudent man or have good councillors, I shall obtain the assistance of some of the neighboring princes. If I am a fool, or behave like a coward, I shall alienate my people. If my soldiers stand by me, and I conquer in the first campaign, I shall reduce my enemy to vassalage, unless I ruin everything by headstrong rashness in spite of good councillors. If I obtain the assistance of a neighboring prince and if I trust implicitly in my chancellor I shall be in no danger of acting the part of a coward nor of ruining everything by headstrong rashness. If I am not popular with my soldiers, of course I shall not conquer in the first campaign. All these things I am profoundly convinced of, and they afford me a legitimate ground of confidence; for I have only to resolve that I will do nothing tending to alienate my people unless it assures my popularity with the army, and that if I do not have good councillors I will not trust implicitly to my chancellor, and I shall thus be assured of reducing my enemy to vassalage. Was this conclusion well-drawn?

Let a mean that the king was a great man, b that he was a prudent man, c that he was a fool, x that he would conquer in the first campaign, y that he would trust implicitly to his chancellor, z that he would ruin everything by headstrong rashness, u that he would be popular with the army, v that he would have good councillors, w that he would act like a coward, p that his soldiers would stand by him, q that he would gain the assistance of a neighboring prince, r that he would alienate his people, and t that he would reduce his enemy to vassalage. It will also be convenient to use the abbreviations $A = ax$, $B = by$, $C = cz$, $A' = \bar{a}\,\bar{u}$, $B' = \bar{b}\,\bar{v}$, $C' = \bar{c}\,\bar{w}$. Then $AA' = 0$, $BB' = 0$, $CC' = 0$. The premises are as follows: $A + B + C$, 2nd, $A' + p$; 3rd, $B' + q$; 4th, $C' + r$; 5th, $\bar{p} + \bar{x} + t + zv$; 6th, $\bar{q}\,\bar{y} + \bar{w}\,\bar{z}$; 7th, $u + \bar{x}$; 8th, $\bar{r} + u$; 9th, $\bar{y} + v$.

As we wish to eliminate every term except t, we may begin by eliminating all that only occur in two premises. These are *a, b, c, w, p,* and *q,* which only occur in the first six premises and the eighth. We may first eliminate p from the 2^{nd} and 5^{th} premises, which gives $\bar{a}\bar{u} + x + t + zv$. We may next eliminate q between the 3^{rd} and 6^{th} premises, thus getting, $\bar{b}\bar{v} + y + w\bar{z}$. We may next eliminate r between the 4^{th} and 8^{th} premises, thus getting $\bar{c}\bar{w} + u$. We now multiply together the last three propositions together with the first premise $ax + by + cz$; and we then eliminate *a, b,* and *c.* This is done very easily, in fact at sight, by taking the three additive terms of the first premise separately. We thus get

$$(t + zv)\ (\bar{v} + \bar{y} + w\bar{z})\ (\bar{w} + v)x + (\bar{u} + \bar{x} + t + zv)(\bar{w} + u)y$$
$$+\ (\bar{u} + \bar{x} + t + v)\ (\bar{u} + y)uz.$$

If we now arbitrarily erase the terms in t, and then multiply in the 7^{th} and 9^{th} premises, we shall by the simple application of the distributive principle and the principle of contradiction find that the proposition reduces to zero; which proves that t can be inferred from it as it stands.

[Words in E for the *Century Dictionary*]

Item 57 ***MS 586: 1886***

E, in logic, signifies a universal negative proposition. See A.

EARTH. Under 2, strike out all after the first sentence, and substitute:

Its figure is that of a sphere slightly compressed in the direction of its polar axis. Its equatorial diameter is 12756506 metres or 7926 statute miles and 1041 yards, its polar diameter is 12713042 metres or 7899 statute miles and 1023 yards. The mean density of the earth is 5.6 and its interior is probably metallic. The force of gravity expressed in terms of the velocity acquired in one second is 9.83 metres at the equator and is greater by the 191st part at the poles. In latitude 45° a body falls 16 feet 2 inches in a second. The earth revolves upon its axis once in 23 hours 56 minutes 4.191 seconds. It revolves about the sun in one tropical year 20 minutes and 23.17 seconds or 365 days, 6 hours, 9 minutes, 9.40 seconds. The earth's equator is inclined to the plane of its orbit by 23°27′31.″83 and the inclination is diminishing at the rate of 0.″47 per annum.

EASTER. For the sentence beginning "Easter is the first" read:

Easter, according to the Gregorian calendar, is the Sunday following the 14th day of that moon whose 14th day next follows the 21st of March. But the moon spoken of is not the real moon but a fictitious moon placed for convenience in advance of the real moon and called the ecclesiastical moon. In the Greek church, which adheres to the Julian calendar, Easter is usually a week earlier or later than in the western churches.

EAU DE COLOGNE. For the last sentence, put:

It consists of strong spirits of wine mixed with small quantities of the essential oils of orange, lemon, and the like.

EBB. Note the phrase "on the ebb."

ECCEITY, or Haecceity. (L. ecce, lo.) A peculiar metaphysical principle, supposed by Duns Scotus, as that the possession of which constitutes individuality and distinguishes an individual from a general essence.

ECCENTRIC, a. Eccentric angle, in geom., an angle connected with an ellipse and defined as follows. Let ABDE be an ellipse. Upon the transverse axis AB as a diameter erect the circle ABFG. Then, taking any point on the ellipse, as H, let fall the perpendicular HK upon the transverse axis AB, and continue this perpendicular until it cuts the circle at the point L on the same side of the transverse axis AB. Join L with the common centre C of the ellipse and circle. Then the angle BCL reckoned from one determinate end B of the transverse axis is called the eccentric angle of the point H. Eccentric anomaly, in astron., the eccentric angle of a planet in its elliptical orbit.

ECCENTRIC, n. For 1 put:

In anc. astron., a circle having its centre away from the earth and carrying an epicycle which in its turn was supposed to carry a planet.

Give the example from Milton. Then let 3b follow, with the introductory words "In modern astron."

ECCENTRICITY. For 2 substitute:

2. In geom. the distance between the foci of a conic divided by the transverse diameter. It is equal to the secant of half the angle between the asymptotes. In the figure, AB:GE is the eccentricity, while $2(FC - CE):(FC + CE)$ is the ellipticity. Angle of eccentricity, the angle whose sine is equal to the eccentricity of an ellipse. In anc. astron., eccentricity is the distance of the centre of the eccentric from that of the earth.

ECCLESIASTICAL. Ecclesiastical moon, or calendar moon, a fictitious moon used in determining the date of Easter.

ECHIGUEN. The constellation Pisces.

ECHO. 6. The 60th planetoid, discovered by Ferguson in 1860.

ECLECTIC, a. Add the words, "also to many modern philosophers, especially Schelling and Cousin."

ECLIPTIC. For 1 substitute:

1. A great circle of the heavens, the path of the centre of the sun in its mean annual (and of course only apparent) motion among the stars. The inclination of the ecliptic is the angle between the planes of the ecliptic and the earth's equator. It is equal in 1850 to 23°27′ 31.″83 and is diminishing at the rate of 17″ per century.

Strike out the definition of plane of the ecliptic, *[. . .]*

EFFORT. For the definition substitute:

An exertion of the will, consciously directed toward the performance of any action, external or internal, and usually prepared by a psychological act of gathering or coordination of the powers. A voluntary action, not requiring such preparation is said to be performed without effort. Effort of nature, a phrase introduced by Sydenham to express the concurrence of physiological processes tending toward the expulsion of morbific matter from the system. In mech., when a force is resolved into two components, one in the direction and the other perpendicular to it, the former is sometimes, but not advantageously, called effort.

"The system of our patient" said the Doctor, "has sustained a shock, from which it can only hope to rally by a great and strong—"

"And vigorous," murmured the family practitioner.

"Quite so," assented the Doctor— "and vigorous effort. . . . Nature must be called upon to make a vigorous effort in this instance; and if our interesting friend the Countess of Dombey—I beg your pardon; Mrs. Dombey—should not be—"

"Able," said the family practitioner.

"To make that effort successfully," said Doctor Parker Peps, "then a crisis might arise which we should both sincerely deplore." Dickens.

EGERIA. The 13th planetoid, discovered by De Gasparis in 1850.

EGO. The I, that which feels, acts, and thinks. Any person's self, considered as essentially the same as any other person.

Retain the citation, but attribute it to its true author, Hamilton, not Reid. Also, after the word philosophers insert "simply." The former compiler borrowed the citation from Fleming, who omits the same word, and who refers to Reid's *Works.* It is incredible that the compiler was so ignorant as to suppose that Reid could have written it.

EGOISM. Put the second meaning first. For the definition now standing first, substitute:

2. The opinion that no matter exists and only one mind, that of the individual holding the opinion. A doctrine said to have been held by a person or persons in Paris, early in the XVIIIth century. They naturally wrote no books.

AEGINA. The 91st planetoid, discovered in 1866.

AEGLE. The 96th planetoid, discovered by Coggia in 1868.

EGOTISTICAL. 3. Pertaining to the Ego, or person's self. Egotistical object, a mode of consciousness regarded as an object. Egotistical idealism, the opinion that nothing exists but the mind and its modifications. Egotistical representationism, the doctrine that the external world is known to us by means of representative ideas and that these are modifications of consciousness.

EGRESS. To 2 add:

Also, the reappearance of a star which has been occulted.

EIGHT. To 1 add:

One more than seven.

ELABORATIVE. Elaborative faculty. Add to the definition that the term was introduced by Hamilton.

ELASTIC. Strike out the sentence defining modulus of elasticity. For the definition of elastic curve, substitute:

Elastic curve, the figure assumed by a thin elastic plate acted upon by a force and a couple. See the figure. Perfectly elastic, having the property of resisting a given deformation equally, however that deformation may have been applied, whether slowly or suddenly, etc. All bodies, however, have different elasticities at different temperatures, and if the deformation is so sudden as to change the temperature of the body and so alter its resistance to deformation, this is not considered as making it imperfectly elastic.

ELASTICITY. Under 1 add:

Elasticity of bulk, resistance to change of bulk. Elasticity of shape, resistance to change of shape. Limit of elasticity, an amount of deformation which if applied to a body, it will spring back when released, but such that under any greater deformation it will not completely spring back. Modulus of elasticity, the ratio of stress to strain. Young's modulus of elasticity, the ratio of the pressure upon the end of a uniformly thick bar to the compression expressed in terms of the original length. Coëfficient of elasticity, as usually used, the reciprocal of the modulus, but sometimes used to mean simply the modulus. Axis of elasticity, a direction in a solid body, with respect to which some kind of symmetry exists in the relation of strains and stresses. Axis of direct elasticity, a direction in a solid body such that a longitudinal strain in that direction produces a stress precisely opposed to the strain. Fresnel's surface of elasticity, a surface whose radii vectores are proportional to the square roots of the elastic forces which, upon Fresnel's theory of light, are exerted in the directions of those radii round any point of a crystalline body.

ELECTION. For 1, put:

1. Deliberate act of choice of one among two or more alternatives, particularly among means for accomplishing a given end.

Strike out the third meaning, and attach the example from Daniel to the first.

ELEATIC. Add: The main Eleatic doctrines are developments of the conception that the One, or Absolute, alone is real.

ELECTRA. 1. The name of an ancient sophism. Electra is asked who the man before her is. Not recognizing him she replies she does not know. "Then you do not know who your own brother Orestes is, for this man is he."

2. One of the Pleiades, 20 Tauri.

3. The 130th planetoid, discovered by Peters in 1873.

ELECTRO-CHRONOGRAPH. Strike out sentence beginning "Called."

ELEMENT. (L. From eleo, a supposed form of oleo, to grow. Thus, elementum would mean an increment, a first increment, a seed. It is used to translate the Gr. stoicheion, which originally meant one of a row, but was the earliest word ever used in a philosophical sense, and meant the same as our "element.")

1. In math. one of a number of objects arranged in a symmetrical or regular figure. The elements of a determinant, the quantities arranged in a square block or matrix, the sum of whose products forms the determinant.

2. As defined by Aristotle, that of which anything is compounded, which exists in it, and which is itself not decomposable into parts of a different kind. Thus, the sounds of the letters are the elements of speech, for these sounds are not decomposable into sounds different in kind from themselves. The sound of O, for example, may be divided in time, but each part will sound as a short O. According to Aristotle, the elements of a substance are its matter and form. In the calculus, an element of a figure is an infinitesimal part of it.

3. A kind of matter undecomposable into other kinds. The four elements of Empedocles (still called so though the doctrine has long been exploded, see below) are Fire, Water, Earth, and Air. The old chemists (XVth century and later) recognized three elements, Sulphur, Mercury, and Salt. The modern chemists make no pretense to have discovered the ultimate elements, but enumerate 70 undecomposed bodies, which they call elements. The following is a list of them, with their symbols and atomic weights. They are arranged in series and groups according to the accepted views of Mendelejeff. The number of the series precedes the name. The atomic weight follows the symbol.

First Group.

1. Hydrogen	H	1	2. Lithium	Li	7
3. Sodium	Na	23	4. Potassium	K	39
5. Copper	Cu	63	6. Rubidium	Rb	85
7. Silver	Ag	108	8. Caesium	Cs	133
11. Gold	Au	196 (197)			

Second Group.

			2. Beryllium	Be	9
3. Magnesium	Mg	24	4. Calcium	Ca	40
5. Zinc	Zn	65	6. Strontium	Sr	87 88
7. Cadmium	Cd	112	8. Barium	Ba	137
			10. Thulium		
11. Mercury	Hg	200			

Third Group.

			2. Boron	B	11
3. Aluminium	Al	27	4. Scandium	Sc	44
5. Gallium	Ga	69	6. Yttrium	Y	90
7. Indium	In	113 114	8. Lanthanum	La	139
			10. Ytterbium	Yb	173
11. Thallium	Tl	204			

Fourth Group.

			2. Carbon	C	12
3. Silicon	Si	28	4. Titanium	Ti	50
5. Germanium	Ge	72	6. Zirconium	Zr	89 90
7. Tin	Sn	118	8. Cerium	Ce	140 (141)
			10. Decipium		
11. Lead	Pb	206 (207)	12. Thorium	Th	233 234

Fifth Group.

			2. Nitrogen	N	14
3. Phosphorous	P	31	4. Vanadium	V	51
5. Arsenic	As	75	6. Columbium	Cb	94
7. Antimony	Sb	120	8. Didymium	Di	147 (145)
9. Erbium	Er	166	10. Tantalum	Ta	182 183
11. Bismuth	Bi	208	12. Philippium		

Sixth Group.

			2. Oxygen	O	16
3. Sulphur	S	32	4. Chromium	Cr	52
5. Selenium	Se	79	6. Molybdenum	Mo	96
7. Tellurium	Te	128	8. Terbium	Tr	148
			10. Tungsten	W	184
			12. Uranium	U	238 239

Seventh Group.

			2. Fluorine	Fl	19
3. Chlorine	Cl	35	4. Manganese	Mn	54
5. Bromine	Br	80	6.		
7. Iodine	I	127	8. Samarium	Sm	150

Eighth Group.

4.	Iron	Fe	56	Nickel	Ni	58	Cobalt	Co	59
6.	Ruthenium	Ru	104	Rhodium	Rh	104	Palladium	Pd	106
10.	Osmium	Os	198 199	Iridium	Ir	193	Platinum	Pt	194 195

4. One of the four things Fire, Water, Earth, and Air. See above. Water as an element consists of all that is in the rain, the rivers, the sea, etc.; fire, of lightning, the sun, etc.; these together with the air and earth make up the matter of nature. The elements or element, in poetry, generally means the sky.

> My Ariel, chick,
> That is thy charge: then to the elements
> Be free, and fare thou well. Shak.

> Witness, you ever-burning lights above,
> You elements that clip us round about. Shak.

5. The environment of anything, that in which it lives, as water is the element of fish, air of birds. The horizon, or sphere of experience of a person. The class of persons with whom one naturally associates.

> We are simple men; we do not know what's brought to pass under the profession of fortune-telling. She works by charms, by spells, by the figure, and such daubery as this is, beyond our element: we know nothing. Shak.

> You are idle, shallow things: I am not of your element. Shak.

6. An ingredient, especially of the temperament. "There's little of the melancholy element in her, my lord." Shak.

7. Those things by the synthesis of which anything is constructed. The elements of geometrical figures are points, lines, and planes.

8. pl. The rudimentary principles of any science. Euclid's *Elements* (Gr. stoicheia) is a work setting forth in an orderly and logical way the simple and fundamental propositions of geometry.

9. One of the quantities necessary to be known in calculating the place of a planet (perhaps, because the planets were called elements). These are six, viz.: the longitude of the ascending node, the inclination of the orbit to the ecliptic, the longitude of the perihelion, the mean distance, and the mean longitude at any epoch. Hence, a datum required for the solution of any problem.

10. The bread and the wine used in the celebration of the Lord's supper are the two elements of the eucharist.

ELEMENTAL. Meanings 2 and 3 need not be distinguished. Add: Elemental law of thought, a first principle, a fundamental belief.

ELEMENTARY. Elementary logic, the science of the general use of reason, containing the necessary rules of thought, without which we cannot reason at all, and treating of the intellect without regard to the differences of the objects to which it may be directed. Elementary proposition, a self-evident and indemonstrable proposition.

ELENCH, ELENCHUS. 1. A refutation, confutation, an argumentation concluding the falsity of something maintained.

Reprehension or elench is a syllogism which gathereth a conclusion contrary to the assertion of the respondent. Blundeville, 1619.

The sophistical elenchus or refutation, being a delusive semblance of refutation which imposes on ordinary men and induces them to accept it as real, cannot be properly understood without the theory of elenchus in general; nor can this last be understood without the entire theory of the syllogism, since the elenchus is only one variety of syllogism. The elenchus is a syllogism with a conclusion contradictory to or refutative of some enunciated thesis or proposition. Accordingly we must understand the conditions of a good and valid syllogism, before we study those of a valid elenchus; these last, again, must be understood, before we enter on the distinctive attributes of the pseudo-elenchus—the sophistical, invalid, or sham, refutation. Grote.

Sometimes erroneously to mean a false elench.

ELEVATION. For 7 put:

7. In topography, generally used for height, the vertical distance above the sea-level or other surface of reference. Also, for the angle by which anything is raised above a horizontal direction, as a telescope.

ELEVEN, a. For the definition, put: One more than ten.

ELEVEN, n. 1. The number one greater than ten.

These may be thought insignificant changes, but the idea of addition is subsequent in logical order to that of number, while the mere increase by one is involved in number and does not involve the whole conception of addition.

ELIMINANT. A function of the coëfficients of any number of homogeneous equations among the same number of unknown quantities, such that the vanishing of it is the necessary and sufficient condition of the equations being consistent with one another. (The word was introduced by De Morgan. Many writers continue to use Bezout's word resultant.)

ELIMINATE. 4. To remove (a quantity) from a system of equations by the reduction of the number of them. Thus, if we have two equations expressing respectively the rates at which an orange growing on a tree increases in bulk and in weight, we can combine them so as to eliminate the time and so obtain an equation expressing the relation between the bulk and the weight.

ELL-AND-YARD. The westernmost of the three stars in the belt of Orion.

ELLIPSE, ELLIPSIS. (Gr. elleipsis, defect, being less.) 1. A conic that does not extend to infinity, or having its intersections with the line at infinity imaginary. The name ellipse is derived from the following construction. Take a line AB called the latus rectum. At its extremity erect the perpendicular AD called the transverse axis. Connect BD, and complete the rectangle DABK. From any point L, on the line AD, erect the perpendicular LZ, cutting BK in Z and BD in H. Draw a line HG completing the rectangle ALHG. There are now two points E and E′ on the line LZ such that the square on LE is equal to the rectangle ALHG. The locus of all such points found by taking L at different places on the line AD forms an ellipse. In a parabola on the contrary, the square on EL is equal to the rectangle ALZB. The ellipse is so called because EL is for it *too small* to fulfil the condition defining the parabola. It easily follows that any ellipse can be placed within the concavity of any parabola, and this is because it has no real points at infinity. An ellipse may also be regarded as a flattened circle, that is as a circle all the chords of which parallel to a given chord have been shortened in a fixed ratio, by cutting off equal lengths from the two extremities. It may also be defined as the locus of points the sum of whose distances from two fixed points is constant. These fixed points are called the foci. The two lines from the foci to any point of an ellipse make equal angles with the tangent at that point.

ELLIPSOID. Add: Strain ellipsoid, or ellipsoid of expansion, the ellipsoid into which any strain transforms any infinitesimal sphere in a body. Reciprocal ellipsoid of expansion, the surface of which each radius vector is inversely proportional to the square root of the linear expansion in the same direction. Poinsot's momental ellipsoid or the inverse ellipsoid of inertia, a surface of which every radius vector is equal to the radius of gyration of the body about that radius vector as an axis. Central ellipsoid, that momental ellipsoid whose centre is at the centre of mass of the body.

ELLIPTIC. Strike out the words "oblong with rounded ends." Add: Elliptic coordinates, a system of coordinates for defining curves upon an ellipsoid by means of the intersections of two systems of confocal hyperboloids. Elliptic space, a kind of space having properties different from those assumed in the usual axioms of geometry, inasmuch as two straight lines in a plane which do not intersect may either of them be rotated through a finite angle in the plane without making them intersect, that is to say, at any finite distance. All propositions about intersections in elliptic geometry (or the geometry of elliptic space) will be the same as for the ordinary, or Euclidean geometry, but measures of distances, angles, areas, etc., will generally be different. At present, we measure by means of rigid bars, which we assume do not change their length when they are carried about,—an assumption justified by the fact that one bar thus used tells the same story as another in regard to the relative distances of points. We might however make the arbitrary and unreasonable, but still perfectly consistent assumption that all bodies expand as they are carried away from a certain fixed point in space, so that certain distances which as measured by rods appear finite are really infinite. If we thus assume that all the points on a certain ellipse are at an infinite distance, we have an elliptic geometry. Elliptic function, a doubly-periodic function analogous to a trigonometrical function, and the inverse of an elliptic integral. Elliptic integral, an integral expressing the length of the arc of an ellipse. Elliptic integrals are of three kinds, which are of the following forms:

Elliptic Integral of the first kind	$\int \frac{d\varphi}{\sqrt{1 - k^2(\sin\varphi)^2}}$
Elliptic Integral of the second kind	$\int d\varphi \cdot \sqrt{1 - k^2(\sin\varphi)^2}$
Elliptic Integral of the third kind	$\int \frac{d\varphi}{(1 + n^2 (\sin\varphi)^2)\sqrt{1 - k^2(\sin\varphi)^2}}$

ELLIPTICITY. For the ellipticity of the earth, put $\frac{1}{293}$. Add: Although the ellipticity is usually defined as above it might without appreciable error be taken as twice the difference divided by the sum of the two axes.

ELONGATION. Under 5 add:
Also, the angular distance of a satellite from its primary.

ELSA. The 182^{nd} planetoid, discovered by Palisa in 1878.

EMANANT, n. The result of operating any number of times upon a given homogeneous function of any number of variables, x, y, z, etc., with the operative symbol

$$(x'\mathrm{d} : \mathrm{d}x + y'\mathrm{d} : \mathrm{d}y + z'\mathrm{d} : \mathrm{d}z + \text{etc.}).$$

EMANATION. 4. In alg., the process of obtaining an emanant.

Regnault's chemical principle of substitution and the algebraical one of emanation are identical. Sylvester.

EMANATIVE. For the definition substitute:
Relating to the production of an effect by the mere existence of a cause, without any particular activity of the latter.

It sometimes happens that a cause causes the effect by its own existence, without any causality distinct from its existence; and this by some is called emanative: which word though feigned with repugnancy to the analogy of the Latin tongue, yet is it to be used upon this occasion till a more convenient can be found out. Burgersdicius, tr. by a Gentleman.

EMENDATION. Historical, conjectural, emendation, an attempt to restore the true reading of a passage upon historical grounds, external or internal, by means of conjecture or circumstantial evidence.

EMERGE. To come into existence, to pass from being in causes to being in act.

Contrary opposition emerges, when a plurality of propositions can severally deny the original enouncement. Hamilton.

ELICIT, a. For the definition substitute:

1. Immediately directed to an end; opposed to imperate, which see. The words are found in Aquinas.

To give alms is an elicit act of charity. Bp. Taylor.

2. Applied to an immediate act of the will. Fleming borrows this definition from Chauvin.

The schools dispute whether in morals the external action superadds anything of good or evil to the internal elicit act of the will. South.

ELICITATION. Add this example:

That elicitation which the schools intend is a deducing of the power of the will into act: that drawing which they mention is merely from the appetibility of the object. Bp. Bramhall.

EMOTION. The compiler derives this from "emotio," a word which as far as I know and I am pretty well read in hog-latin is entirely imaginary. For derivation and definition put:

(L. emotus, pp. of emoveo, to expel, agitate.)

1. A secondary sensation, or feeling excited by an idea, that is accompanied by a bodily commotion, like blushing, blenching, trembling, weeping, etc.

2. Physical excitement, as emotion of the pulse induced by exercise.

EMPEIREMA, EMPIREMA. A proposition grounded upon experience.

EMPIRICAL. Consult D. Stewart vol. i.

EMPTY. 2. In Kantian philos., devoid of matter, having no object in experience.

A concept is to be considered as empty and as referring to no object, if the synthesis which it contains does not belong to experience. Kant, tr. by Mueller.

EMULATION. Put the clause beginning "desire" first.

ENCOUNTER. Under 1 add:

In the kinetical theory of gases, the coming of two molecules within the sphere of each other's action.

END. 8. In phil., traditionally defined as that for the sake of which anything is or is done; more precisely, that result toward which the actions of anything tend, so that if one of these is prevented from attaining the result in question another calculated to do so will be substituted.

The end of which is that which the efficient desires. For which, for whose sake the efficient desires such an end. Burgersdicius, tr. by a Gentleman.

The principal or chief end is that which is mainly intended. A secondary or succedaneous end is some additional object to be attained. Subordinate end, that which is aimed at as a means to some further end. Ultimate end, that which is referred to no further end. Subjective or relative end, that to which some particular impulses tend. Objective or absolute end, or end in itself, in Kantian phil., that which is the condition of the possibility of all other ends.

ENDOSCOPIC, a. In alg., involving the conception of the coëfficients of a quantic as composed of roots or other elements.

ENERGETICS. Strike out the definition and substitute:

The science of the general laws of energy.

ENERGY. (Gr. energeia, action.)

1. A fact of acting or actually being. (Rare.)

All verbs that are strictly so called denote energies. Harris.

The last of a series of cognate terms are act, operation, energy. They are all mutually convertible, as all denoting the present exertion or exercise of a power, a faculty, or a habit. Hamilton.

2. Actuality; opposed to power or potentiality. First energy, the state of an acquired habit. Second energy, the exercise of a habit. Hermogenes when he has learned to sing is a singer in first energy, when he is singing he is a singer in second energy. See ACT.

3. Activity, readiness to put forth strength or power, physical or mental.

It is the essence of the pure energetic temperament still to energise and not to enjoy even the fruits of energy. Bain.

4. Force, efficacity. In rhetoric, the quality of awakening the imagination of the reader or hearer, and bringing the meaning of what is said home to him. (So used since Aristotle.)

Beg the blessed Jesus to give an energy to your imperfect prayers, by his most powerful intercession. Smalridge.

> Waller was smooth; but Dryden taught to join
> The varying verse, the full resounding line,
> The long majestic march, and energy divine. Pope.

5. In dynamics, the power of performing work, considered as a quantity and measured by the total amount of that work. Potential energy or energy of position, that due to the proximity of two repelling bodies or the distance of two attracting bodies. Actual or kinetic energy or energy of motion, vis viva, or energy due to the velocity of a body. (The term energy was introduced into mechanics by Young, early in this century, as a synonym of vis viva. It was extended to its present signification by Wm. Thomson, and Rankine applied to it the distinguishing adjectives actual and potential. These adjectives put in a strong light the departure of the mechanical use of the term from the Greek, for they seem to speak of an actual actuality and a potential actuality. From this point of view, the term power, meaning power of doing work, is far preferable.) Conservation or persistance of energy, the principle that energy is never produced nor destroyed, but only transformed. The discovery of this law is the greatest that science has ever made, and nothing that can be discovered hereafter (unless it be of a supernatural kind) can equal it in importance.

SYN. Action, operation, exercise, activity, violence, impetuosity, spirit, dash, life, vivacity, force, strength, power, cogency, efficiency, efficacity, might.

ENGINE-TURNING. The wood-cuts are disgraceful.

ENIF. (Ar. enf, nose.) A second magnitude star in the muzzle of Pegasus, epsilon Pegasi.

ENLARGEMENT. In the calculus of finite differences, the operation of changing a function by adding unity to the variable. It is denoted by the letter E. Thus, $E \log x = \log(x + 1)$.

ENLIGHTENMENT. 2. Translation of the German Aufklaerung, thinking for one's self, independence of thought, rationalism, especially the rationalism of the eighteenth century.

This enlightenment Hegel had received at first in its sober German form,—in the dry analysis and superficial criticism of the post-Wolffian age; but at the university he came to know it in its more intensive French form, which was to the German enlightenment as wine to water. Caird.

ENNEAD. To 1, add: "a system of nine objects."

ENNEADIC, a. Pertaining to an ennead.

ENNUI. For the definition put:

In French, more properly a sorrow arising from the loss or absence of something, also an annoyance. In French and in English, a painful state of mind due to the want of any object of interest, tedium.

Retain the fine example.

ENS. (L. As if pp. of verb esse, to be. Translates the Gr. on.)

1. That which in any sense is, an object, something that can be named and spoken of. In Latin (bad Latin) ens may be taken as a participle, and in that case to say for example "Innocence is ens," means that innocence is actually attained, but if ens is taken as a noun, as it properly is, then the sentence means absolutely nothing except that innocence (whether it is to be found or not, whether it is possible or not) has received a name. Ens of reason (ens rationis) a product of mental action. Real ens, anything whose characters are independent of what any person, or any number of persons may think them to be. Dependent ens, that which is caused by another, opposed to independent ens. Necessary ens, that the non-existence of which involves contradiction, opposed to contingent ens. Fictitious ens, a product of the inventive imagination. Apparent or intentional ens, a real, but unsubstantial, appearance, as a rainbow. Complex ens, a fact, as that Columbus discovered America. Most perfect ens, that whose essence involves all perfections, including

existence (an absurd idea, because existence is not a general character).

ENTELECHY, n. (Gr. entelecheia, word invented by Aristotle, from en telei echon, having attained the end.)

Literally, attainment, realization; opposed to power, potentiality, and nearly the same as energy or act (actuality). The idea of entelechy is connected with that of form, the idea of power with that of matter. Iron is potentially in its ore, which to be made iron must be worked. When this is done, the iron exists in entelechy. The passage from power to entelechy takes place by means of change (kinesis). This is the imperfect energy, the perfected energy is the entelechy. First entelechy is being in working order, second entelechy is being in action. The soul is said to be a first entelechy, that is, a thing precisely like a man in every respect, except that it would not feel, would be a body without a soul; but a soul once infused is not lost whenever the man is asleep. This is the Aristotelian sense, but Cudworth and others have used entelechy and first entelechy somewhat differently. Cudworth calls his plastic nature or vital principle the first entelechy, and Leibniz terms a monad an entelechy.

ENTHYMEME. (Gr. enthymema, an inference.) In Aristotle's logic, an inference from likelihoods and signs, which with Aristotle is the same as a rhetorical syllogism. Owing to a change in the conception of a rhetorical argument, with the Roman writers (Quintilian, etc.) it began to take the sense it now bears, namely that of a syllogism with one of the premises unexpressed. Enthymeme of the first (second) order, a syllogism with only the major (minor) premise expressed.

ENTITATIVE. (L. entitativus.) Pertaining to actually existing objects. Entitative act, actuality, that which distinguishes an actually existing thing or fact from one that has mere potential being. Entitative power, a power which is not realized by the existence of its subject, as a power of doing or becoming something. Entitative being, existence, real being; opposed to intentional or objective being. Entitative is usually opposed to objective.

ENTITATIVELY. (L. entitative.) Intrinsically, taken in itself, apart from extrinsic circumstances.

ENTITY. (L. entitas.) In scholastic phil., the abstract noun corresponding to the concrete ens, or so to speak "being-ness," also a mode or element of "being-ness." The metaphysics of the realists as developed by Scotus was distinguished by supposing different entities to concur in one being. In modern times, the word entity is hardly used except by persons of nominalistic tendencies in attempting to characterize the opinions of realists, to whom they unconsciously attribute their own individualism. So used, entity means something existing in the sense in which an individual substance exists.

> When first thou gav'st the promise of a man,
> When th'embrion speck of entity began. Hart.

The multiplication of entities had gone on with immense fecundity in the schools. Lewes.

ENTROPY. For the totally incorrect definition substitute:

1. As originally used by Clausius, that part of the energy of a system which cannot be converted into mechanical work without communication of heat to some other body or change of volume.

2. As used by Tait (who wrote for the purpose of discrediting Clausius) the available energy, that part of the energy which is not included under the entropy, as properly used.

ENUMERATE. For the definition put:

1. To tell over, to state in detail, in a list.

2. To count, as one of a lot of objects, in determining the total number.

Schedule number one shall contain inquiries as to the relation of each person enumerated to the head of the family. U. S. Census Act, for 1880.

ENUMERATION. Transpose the first and second meanings. The one now put first is more properly termed denumeration. Add this example:

I will make a true and exact enumeration of all the inhabitants within the subdivision assigned to me. Enumerator's oath, U. S. Census of 1880.

Add: 4. In logic, a kind of argument which enumerates all the different possibilities, and then successively excludes all but one, abscissio infiniti, the method of exclusions, disjunctive syllogism.

ENUMERATOR. To Britain, add the U. S.

ENUNCIATION. For 3 and 4 put:

3. In logic, a proposition, that which is subject to truth and falsity, especially when set forth in words.

> An enunciation is an oration, form of speech, or declaration, in which something true or false is pronounced of another. Burgersdicius, tr. by a Gentleman.

Simple enunciation, one consisting of a subject and predicate, a categorical proposition, opposed to a composite enunciation which states some relation between facts described in dependent clauses. A composite enunciation is either copulative, hypothetical, disjunctive, adversative, or relative, according to the nature of the conjunctions uniting the clauses. Modal (contradistinguished from a pure) enunciation is one which states some fact to be possible or impossible, necessary or contingent. Exponible enunciation, one which has to be replaced by another form of speech before applying the rules of syllogism, etc. Exclusive enunciation, one which contains an exclusive particle, as "The elect only shall be saved." Exceptive enunciation, one which contains an exceptive expression, as "All mankind were drowned except Noah and his family." Restrictive enunciation, one which contains a restrictive expression, as "Christ, in respect to his divine nature, is omnipresent." See PROPOSITION.

ENUNCIATIVE. For the definition, put:

Declaring something as true, declarative.

ENVELOPE, n. For 4, put:

4. In astron., a shell partly surrounding the nucleus of a comet on the side next the sun and away from the tail, and appearing like a semicircular arch. Large comets generally show several of these, under the telescope. They successively rise from the nucleus and disappear.

ENVELOPE, n. Add:

In geom., a curve or surface touching a continuous series of curves or surfaces. Suppose a plane curve to undergo a continuous change in its shape and position. Then, the curve as it is at any instant is intersected by the curve as it is at any subsequent instant, and the closer the second instant follows after the first, the closer do these intersections approach certain positions on the first curve. These positions are points on the envelope, and in this way all the points on the envelope are determined. Caustics and evolutes are examples of envelopes.

ENVY. Add this example:

> Envy is an uneasiness of mind caused by the consideration of a good we desire, obtained by one we think should not have had it before us. Locke.

EOLOTROPIC, a. (Gr. aiolos, nimble, and tropos, a way.) Possessing different properties in different directions. Said of an individual body and of a kind of substance.

EOLOTROPY, n. The character of being eolotropic.

EPACT, n. (Gr. epakte, epact, intercalary.) The age of the calendar moon at the beginning of the year. The epact was used at an early age for calculating the date of Easter. At the time of the invention of the Gregorian calendar, it had fallen into disuse, but was then revived by Lilius and embodied in the new calendar. The so-called Julian epact was invented at that time, as a means of calculating the Gregorian epact. This had led to the erroneous statement that Lilius invented the epact.

(I never before heard of a menstrual epact, and as the rest of the article is quite erroneous, I infer that the compiler has misunderstood some hypothetical account of the origin of the epact.)

EPAGOGE. Strike out the whole article. This is simply the Greek for induction, and there is no such word in use in English. Writers of course have introduced the Greek word in the course of an English composition, but not often enough to warrant its being inserted into an English dictionary.

EPAGOGIC. For the definition, put:

Pertaining to induction, which in Greek is epagoge. (The word is not peculiarly rhetorical.)

EPHEMERIS. In place of the clause beginning "such as" read:

The chief publications of this sort are, the French *Connaissance des temps* (from 1679), the British *Nautical Almanack* (from 1766), the Berlin *Astronomisches Jahrbuch* (from 1776), and the *American Ephemeris* (from 1855).

EPICHEIREMA. For the words "In logic and rhetoric," put:

1. As used by Aristotle, a reasoning based on premises generally admitted but open to doubt.
2. As commonly used,

EPICTETIAN. Add: Epictetus taught that we should not allow ourselves to be dependent upon good things not within our own power, and that we should worship our consciences.

EPICURE. Under 1, strike out all after "Epicurus" and substitute: See EPICUREAN.

EPICUREAN, a. Under 1, after Epicurus, insert: the great sensualistic philosopher of antiquity, whose logical and ethical doctrines (totally misrepresented by his opponents) were similar to those of Bentham and Mill. He held to the theory of atoms, and is reckoned the best physical philosopher of the Greeks.

EPICYCLE. Add: Copernicus also made use of epicycles, which were only banished by Kepler. In fact, the elliptical motion might be represented by a deferent and epicycle.

EPICYCLOID. For the definition put:

A curve generated by the motion of a point on the circumference of a circle, which rolls upon the convex side of a fixed circle. These curves were invented by the Danish astronomer Roemer in 1674.

EPICYCLOIDAL, a. For the definition put:

In the form of an epicycloid, depending upon the properties of the epicycloid. Epicycloidal teeth, teeth for gearing cut in the forms of epicycloids.

EPISYLLOGISM. A syllogism having for one of its premises the conclusion of another syllogism.

EPITROCHOID. (This definition is right, because taken from a more reliable work: it states virtually that his definition of epicycloid was wrong.)

EPOCH. Under 2, (a) and (b) seem to be indistinguishable. Strike out (a) and add: Used also in defining other periodic motions.

EQUAL. Strike out the whole of 1, and substitute:

1. Having the same measure. Two collections of objects are equal in number when the operation of counting applied to either ends with the same number. Two lengths are equal when either will cover the other. Two stars appear of equal brightness, when the eye can detect no difference between them in this respect. Quantities of more than two dimensions are equal, only when they are equal in each dimension separately. Thus, two vectors are not necessarily equal because they are equal in length. It is necessary that they should also be parallel. It is therefore preferable not to speak of two forces (or anything else capable of representation by vectors) as equal, unless they be parallel. Nevertheless, the usage is to call two such things equal when their tensors or moduli are equal. On the other hand, common usage presents an opposite inconsistency in refusing to call geometrical figures (particularly triangles) equal, unless they can be superposed. Euclid and some modern geometers make it an axiom that figures that can be superposed are equal; but others define equal figures as such as can be superposed. According to this, two unequal figures may be capable of being each cut into two parts which are equal, those of the one to those of the other.

(The examples from Jefferson and Milton seem to belong under 7.)

Equal propositions are propositions which state the same fact.

EQUALITY. Strike out 4 as a separate number and put:

Sign of equality, the sign $=$ which the mathematician writes between the symbols of two quantities the equality of which he wishes to assert.

EQUANT. For the definition put:

In the Ptolemaic system of astronomy, a circle about the centre of which the centre of the epicycle of a planet was supposed to describe equal angles in equal times.

EQUATION. For 3s = 36d read 3 lb = 48oz. Before the sentence beginning "An equation" insert:

Identical equation, one which holds whatever be the values assigned to the quantities denoted by letters.

At the end of 2 add:

Algebraical equation, one in which the variables or unknown quantities only enter as factors or as terms or as factors of terms. Transcendental equation, one in which the unknowns enter in a more complicated way than in algebraic equations. Simultaneous equations, two or more equations which are true at the same time. Binomial equation, an equation having two terms. Numerical equation, an equation having all its coëfficients individual numbers. Irreducible equation, an equation which, when all the terms are transposed onto one side, admits no commensurable divisor. Abelian equation, an equation all the roots of which are rational functions of one of them. Functional equation, an equation in which the unknown is not a quantity but a functional operator. Such, for example, is the equation $F^2 = 1$, which means that the operation F is such that the result of performing it twice is to restore the original operand. Equation of differences, a form of functional equation between a function and its successive differences or enlargements. Differential equation, an equation involving differentials. Theory of equations, the doctrine of the methods of solving algebraical equations. Equation to a curve, surface, etc., an equation defining the shape and position of the curve, surface, etc.

After the definition of numerical equation given above, I should have inserted the following:

Reciprocal equation, an equation which is satisfied by the reciprocal of the unknown quantity.

Under 2, it seems to me best to put the special cases of that meaning, and not to relegate them to the end. The following are such cases:

Equation of the centre, the correction for the first inequality or inaequalitas soluta, which is the principal one affecting the time at which a planet comes to opposition with the sun. This is called the equation of the centre, because in the Ptolemaic system, it defines the position of the centre of the epicycle. Equation of the argument or of the orbit, the correction for the second inequality, or difference between the place of the planet and the centre of the epicycle, referred to the zodiac. In the case of a superior planet, it is merely the reduction from the heliocentric to the geocentric longitude. In the case of an inferior planet, it is the elongation of the planet from the sun. In the case of the sun it is the difference between the mean and true anomaly. Equation of time, the reduction from apparent solar time to mean solar time. (Equation of equinoxes, omit.) Personal equation, a correction which must be applied to one person's observations (especially of time) to make them accord with another person's. Absolute personal equation, the correction which has to be applied to a time observed by any given person to get (in the mean) the true time. This correction is partly due to the time occupied by the nerve-currents in passing between the brain and periphery, partly to that taken up by cerebral processes, and partly to anticipations of events which are seen to be approaching.

EQUATOR and EQUATORIAL are well defined, for a wonder.

EQUIANHARMONIC, a. A term applied to the situation of four points or other elements (one of which at least must be imaginary) whose anharmonic ratio is a cube root of unity.

EQUIANHARMONICALLY, adv. In an equianharmonic situation.

EQUIDISTANT. For the whole definition put: Equally distant.

EQUIANGULAR. Equiangular spiral, the logarithmic spiral, a curve making everywhere the same angle with its radius vector.

EQUILIBRANT, n. A system of forces which would bring a given system of forces to equilibrium.

EQUILIBRATE. Insert:

Equilibrated arch or rib, one whose thrust is tangent to the curve.

EQUILIBRIUM. Strike out 2 and for 1 put:

A situation of a body in which the forces acting on it balance one another: also a determination of forces such that they balance one another, so that their resultant vanishes. When a heavy body rests on a table, the weight and the elastic forces which the weight evokes are in equilibrium, that is, are precisely equal and opposite. A man dancing the tight-rope usually carries a stick to aid him in preserving his equilibrium, that is, in keeping his centre of gravity over the rope, so that his weight and the spring of the rope may act in the same vertical line.

(The definitions of the different forms of equilibrium may stand as in the text.) In place of 4, 5, 6, 7 put:

2. The state of balance of any causes, powers, or motives, so that no effect is produced.

The balance is turned, and wherever this happens there is an end of the doubt or equilibrium. Sharp.

(Give also the example from Arbuthnot.)

EQUINOCTIAL. Equinoctial colure, the great circle passing through the poles and equinoctial points. Equinoctial time, time reckoned from the instant at which the sun passes the vernal equinox, a method of reckoning time independent of the longitude, invented by Sir John Herschel.

EQUINOX. The moment when the (unperturbed) sun crosses the plane of the earth's equator. Vernal equinox, that one which falls in the spring, namely on the 21st of March according to the Gregorian calendar, as well as according to the Julian calendar at the time of the council of Nice, A.D. 325, but falls one day earlier every 128 years according to that calendar, so that it now falls on the 9th of March. Autumnal equinox, that one which falls in the autumn, namely on the 22nd of September according to the Gregorian calendar. The term equinox is also loosely applied to the equinoctial points.

EQUIPARANCE, n. Identity of reciprocal relations. Thus, cousins are said to be relates of equiparance, because if A is cousin to B, then B is equally cousin to A.

EQUIPARANT, n. A relate of equiparance.

EQUIPOLLENCY. For 2 put:

An identity of meaning of two or more propositions.

As to the equipollency or like value of enunciations in sense which differ in words, which the author says something of, I thought it of little significancy to be mentioned. Tr. of Burgersdicius.

The immediate inference of equipollence is merely the grammatical translation of an affirmation into a double negation, or of a double negation into an affirmation. Hamilton.

Let him study in equipollence,
And let lies and fallaces
If that he would deserve our graces. Chaucer.

EQUIPOLLENT. For 2 put:
2. In logic, having the same meaning, as two propositions.

EQUIPOTENTIAL, a. Connected with a single value of the potential. Equipotential surface, a surface over which the potential has the same value: better termed a level surface.

EQUISEGMENTAL, a. In math., applied to lines such that to any segment of the one corresponds an equal segment of the other.

EQUITANGENTIAL, a. Having equal tangents. Equitangential curve, one upon whose tangents a fixed line (called the directrix) intercepts equal distances from the points of tangency.

EQUIVOCAL, a. (L. aequivocus, a word invented by Boethius to translate the Gr. homonymos, from homos, same, and onoma, name.) 1. Having the same name, but not the same nature. Also, applied to words alike in sound, but essentially different in meaning. Equivocal terms are so either by chance, when there is no reason for their bearing the same names, or by consimilarity, when there is such a reason.

2. Equivocal is used to limit a noun used in a secondary or improper sense. Equivocal generation, generation improperly so called, being a spontaneous evolution from something of a different kind. The true reason why spontaneous generation is called equivocal having been forgotten, the phrase has sometimes been used in a different sense, as in the example from Pope.

(Here insert the examples from Pope and Harris.)

Equivocal cause, a principal cause which is of a different nature from and better than its effect.

The effect is said to be contained in the cause either formally or eminently. When eminently, or the cause by a nobler sort of virtue produces the effect, it is said to be equivocal, and is better than its effect; as when light produces heat, or the architect an house. Burgersdicius, tr. by a Gentleman.

3. Of doubtful nature or significance. Equivocal test, an inconclusive test. Equivocal symptom, one which may arise from several different diseases.

4. Open to dishonorable suspicion, as equivocal birth. See the quotation from Pope, above. Equivocal reputation, that of a person who is generally looked upon with grave suspicion, but maintains some pretension to respectability.

(Under 1, above, insert: Equivocal words are sometimes called equivocant, equivocal things equivocate. Also, insert the example from Sir Th. Browne.)

EQUIVOCANT, a. Having like sounds but different significations.

EQUIVOCATE, a. Having the same name but different natures.

EQUIVOCATION. 2. In logic, a fallacy depending upon the double signification of some one word. Distinguished from amphibology which depends upon the doubtful interpretation of a whole sentence.

EQUIVOQUE. Under 1, add: Also, things of different nature signified by one name are termed equivoques.

Equivokes be such things as have one self name, and yet be divers in substance or definition; as a natural dog and a certain star in the firmament are both called by one name in Latin, Canis, yet they be nothing like in substance, kind, or nature. Blundeville, 1619.

EQUULEUS. For 1, put:

1. An ancient northern constellation, supposed to represent a horse's head. It lies west of the head of Pegasus and its brightest star is of the fourth magnitude. Equuleus pictoris, generally called Pictor, is a southern constellation invented by Lacaille. It lies south of the dove and west of Canopus, and its brightest star is of the fourth magnitude.

ERATO. 2. The 62nd planetoid, discovered by Foerster and Lesser in 1860.

ERECT, v. a. 7. To draw, as a figure after an initial line has been given. As to erect a horoscope, to erect a circle on a given line as semidiameter, etc.

ERG. For the definition, put:

The unit of work, in the centimetre-gramme-second system. It is equal to twice the work required to give a mass of one gramme a velocity of one centimetre per second.

(The definition in the text seems to imply it makes a difference how fast this velocity is communicated, which is not so.)

One foot-pound is about thirteen and a half million ergs, and one theoretical horse-power is about 7460 million ergs per second.

ERGO. Add: Used technically in logic to introduce the conclusion of a complete and necessary syllogism.

ERGOMETER, n. An instrument for measuring work, generally called a dynamometer. Watt's indicator diagram is an example of an ergometer.

ERIDANUS. The ancient southern constellation of the river. It is situated south of Taurus, and contains the first magnitude star Achernar, which is however invisible in Europe, and barely visible in Alexandria. In the United States, it can be seen in winter anywhere south of Savannah.

ERIGONE, n. The 163rd planetoid, discovered by Perrotin in 1876.

ERISTIC. Add: Eristic science, logic.

EROTEMATIC, a. Proceeding by means of questions. Erotematic method, a method of instruction in which the teacher asks questions, whether catechetical or dialogical.

ERRAI. (Ar. ar-ra'i, a shepherd.) A third magnitude star on the left foot of Cepheus. Twenty-three centuries hence Errai will be the pole-star.

ERROR. For the definition put:

1. A wandering, going astray.

(Give the examples from Spenser and Dryden.)

2. A discrepancy between what is thought to be true and what is true: also the false proposition itself. An unintentional positive falsity.

(Give the example from Brougham, though it is not particularly good considering its length.)

3. An inaccuracy, oversight, something done different from what was intended, especially in speaking, writing, and printing. Clerical error, an error of printing. This strange expression is said to have its origin in the fact that the first printing-press in England was set up by Caxton in a chapel in Westminster, in consequence of which a number of clerical expressions became applied to printers.

(Refer this to the philologist.)

4. A wrong-doing, a moral fault, a sin, generally not very heinous.

(Give examples from the Psalms and from Tennyson.)

5. The observed or otherwise determined value of a physical quantity less the true value; also called the true error. By the error is often meant the error according to some possible theory. Thus, the rule of the physicists is said to be to make the sum of the squares of the errors a minimum; that is that theory is adopted according to which the sum of the squares of the errors of the observations is represented to be less than according to any other theory. The error of an observation is separated into two parts, the accidental error and the constant error. The accidental error is that part of the total error which would entirely disappear from the mean of an indefinitely large series of observations taken under precisely the same circumstances; the constant error is that error which would still affect such a mean. The law of error is a law connecting the relative magnitudes of errors with their frequency. The law is that the logarithm of the frequency is proportional to the square of the error. This law only holds for the accidental part of the error, and only for certain kinds of observations, and to those only when certain observations affected by abnormal errors have been struck out. Probable error, a magnitude which one half the accidental errors would in the long run exceed; this is a well-established but rather unfortunate expression for that magnitude which exceeds one half of the accidental errors in the long run, of observations similar to given observations. Mean

error, the quadratic mean of the errors of observations similar to given observations.

6. (Here give the legal signification.)

ESOTERIC. Originally applied, by Cicero and others, to certain writings of Aristotle (embracing no doubt all that have come down to us) of a scientific, as opposed to a popular, character. Then, applied to the secret or acroamatic teachings of Pythagoras. The inner or higher teaching of a philosopher, not communicated to the many.

There grew up, in the minds of some commentators, a supposition of exoteric doctrine as denoting what Aristotle promulgated to the public, contrasted with another secret or mystic doctrine reserved for a special few, and denoted by the term esoteric; though this term is not found in use before the days of Lucian. I believe the supposition of a double doctrine to be mistaken in regard to Aristotle; but it is true as to the Pythagoreans, and is not without some colour of truth even as to Plato. Grote.

(Strike out all the examples given.)

ESSE. (L., to be.) Being, in the sense which this word has when it is an abstract noun, "being-ness."

ESSENCE, n. (L. essentia, a word invented to translate the Gr. ousia.) In Greek, a subject in esse, something whose mode of being corresponds to that of a subject, as distinguished from a predicate, in speech. But while this is the original conception, the word essence, even in Latin, usually carries a different sense. The essence is rather the idea of a thing, the law of its being, that which makes it the kind of thing that it is, that which is expressed in its definition. In regard to artificial things the conception of an essence is usually tolerably clear; thus, the essence of a bottle is that it should be a vessel with a tubular orifice. Those philosophers who speak of the essences of natural things hold that natural kinds are regulated by similar ideas. Nominalists hold that definitions do not belong to things but to words; and accordingly they speak of the essences of words, meaning what is directly implied in their definitions.

First, essence may be taken for the being of anything, whereby it is what it is. And thus the real internal, but generally in substances unknown, constitution of things, whereon their discoverable qualities depend, may be called their essence. . . .

Secondly, . . . But it being evident, that things are ranked under names into sorts or species, only as they agree to certain abstract ideas, to which we have annexed those names; the essence of each genus or sort comes to be nothing but that abstract idea which the general or sortal (if I may have leave so to call it from sort as I do general from genus) name stands for. And this we shall find to be that which the word essence imports in its most familiar use. These two sorts of essences, I suppose, may not unfitly be termed, the one the real, the other the nominal essence. Locke.

(Give the example from Ferrier.)

Man, proud man,
Most ignorant of what he's most assured
His glassy essence. Shak.

2. Any kind of matter which being an ingredient or constituent of some better known substance, gives it its peculiar character; as Liebig's essence of beef. Especially an oil distilled at a comparatively low temperature, from a plant in which it already exists, as essence of peppermint. Also applied to solutions of such oils in alcohol, to strong alcoholic tinctures, etc.

Nor let the imprisoned essences exhale. Pope.

3. An elementary ingredient or constituent; anything uncompounded. The fifth essence, or element, in the philosophy of Aristotle, is ether, the other four being in their order, earth, water, air, and fire. Hence, essence is anything whose substance is etherial, pure, heavenly.

Uncompounded in their essence pure. Milton.

Heavenly essences. Milton.

(Give the example from Bacon.)

4. The leading idea of any discourse.

(I do not believe it ever means existence simply. I think that in such examples as that from Sidney, it means the soul or life, as that which constitutes man.)

ESSENCE-BOTTLE. A bottle for holding an essence. Boyle.

ESSENTIAL, a. 1. Involved in the essence or definition of a kind of thing or of a word, as essential character, quality, perfection, power. Essential difference, distinction, diversity, one given in the definitions of the things distinguished. Essential definition, a strict definition stating the true constitutive essence of the definitum. Essential whole, that whose parts are matter and form.

2. Constituting or making that which is characteristic and mainly noticeable about a thing, as "an essential feature of Shakespeare's style." Essential oil, a volatile oil occurring in a plant, and giving its characteristic odour. Essential oils are either distilled or expressed, they are mostly hydrocarbons, many of them have precisely the same chemical composition, and though they are distinguished by various physical characters, their excellence can only be determined by the sense of smell.

3. Needful, indispensible, important.

> Judgment is more essential to a general than courage. Denham.

4. In medicine, idiopathic, not symptomatic merely.

ESSENTIALLY. Add this example:

> Causes are said to be either essentially or accidentally subordinated. Essentially are those of which one depends upon another when and inasmuch as it causes. Burgersdicius, tr. by a Gentleman.

ETAMIN. (Ar. ras-el-tannin, the dragon's head.) A second magnitude star above the head of the dragon, gamma draconis. It is the zenith star of the Greenwich observatory, where it has always been used for determinations of aberration.

ETANIN. I never met with this form of the name.

ETEOPOLYMORPHISM, n. (Gr. eteos, genuine.) True polymorphism.

ETERNAL. Add under 5: having no relation to time.

ETHER. (Strangely enough, the non-scientific meaning is omitted. Insert it as the first meaning, as follows:)

1. The upper air, the blue heavens. It was supposed by Aristotle to extend from the fixed stars down to the moon.

> E'en like the passage of an angel's tear
> That falls through the clear ether silently. Keats.

(Strike out the quotation from Dryden.)

ETHICAL, a. Add: Ethical truth is the agreement of what is said with what is really believed, veracity, opposed to lying. Ethical virtue, the subordination of appetite to reason.

ETHICS. Under 1 add:

Kant distinguishes between pure morals, which is the science of the necessary moral laws of a free will, and ethics properly speaking, considers those laws as under the influence of sentiments inclin *[. . .]*

Letter, Peirce to A. Marquand

Item 58 ***L 269: 30 December 1886***

36 W 15$^{\text{th}}$ St.
New York, 1886 Dec 30.

My dear Marquand:

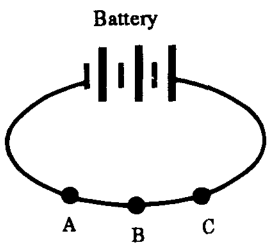
I am as deeply touched by your generosity, as if you had made me a present; and doubtless you think it somewhat problematical when you see your money again. Nevertheless, I have no intention of remaining so poor as I am, and if there is no other way, if the world does not care to pay for my philosophy, I will abandon that and apply my logic to private ends. I mean at once to advertise that I will give lessons by correspondence in the art of reasoning. If this idea is as successful as I hope, I shall not keep you long waiting. If not, I have others.

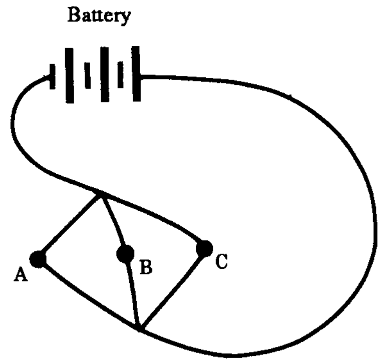
I am very sorry to have missed you. I want to talk to you about my great Idea in philosophy. I would gladly go to Princeton to do so.

You spoke, when I saw you, as if disappointed with the reception your machine had met with. I wish I could see it. My impression is that it has two defects; first, I believe it only extends to four simple terms instead of to six as it should; and second, I believe it does not reduce the solution to its simplest expression. It ought to perform 4 operations, or 3 at least. First it should develope any expression as a into $abcdef + abcde\bar{f} + abcd\bar{e}f +$ etc. Second it should reduce expressions; for instance

$$\begin{array}{ll} & abcdef + abcde\bar{f} + abcd\bar{e}f \\ \text{into} & abcde + abcdf. \end{array}$$

Third, it should multiply two developed polynomials, if not any two. Fourth, though not absolutely required, it would be well to have it capable of adding. I think you ought to return to the problem, especially as it is by no means hopeless to expect to make a machine for really very difficult mathematical problems. But you would have to proceed step by step. I think electricity would be the best thing to rely on.

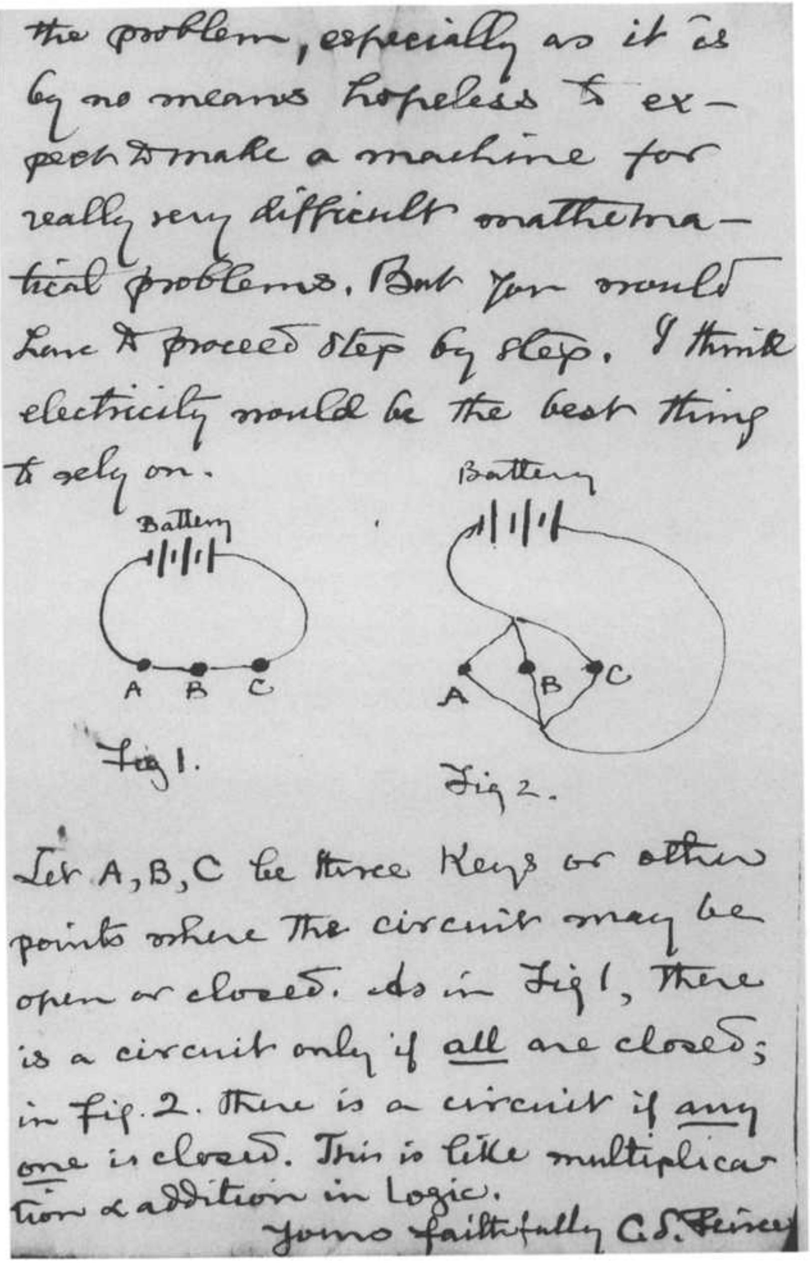




Let A, B, C be three keys or other points where the circuit may be open or closed. As in Fig 1, there is a circuit only if *all* are closed; in Fig. 2. there is a circuit if *any one* is closed. This is like multiplication & addition in Logic.

Yours faithfully
C. S. Peirce

P.S. If you will send me a copy of your last paper on your machine, I will act as Devil's Advocate, by attacking it.



the problem, especially as it is by no means hopeless to expect to make a machine for really very difficult mathematical problems. But you would have to proceed step by step. I think electricity would be the best thing to rely on.

Battery

A B C

Fig 1.

Battery

A B C

Fig 2.

Let A, B, C be three keys or other points where the circuit may be open or closed. As in Fig 1, there is a circuit only if all are closed; in Fig. 2. there is a circuit if any one is closed. This is like multiplication & addition in Logic.

yours faithfully C.S. Peirce

Holograph page from Peirce's 30 December 1886 letter to Marquand (Allan Marquand Papers. With permission of Princeton University Library)

This page intentionally left blank

Notes

The functions of these notes, which are keyed to page and line numbers, are generally self-explanatory. The notes identify obscure proper names not found in the standard reference works for our edition or, in cases of possible confusion, give the full name of a person to whom Peirce refers only by last name or by a descriptive phrase. Our standard reference works are the *Dictionary of American Biography*, the *Dictionary of National Biography*, the *Dictionary of Scientific Biography*, the 15th edition of the *Encyclopedia Britannica*, *The Encyclopedia of Philosophy*, the *National Cyclopedia of American Biography*, and the *New Century Cyclopedia of Names.*

The notes identify the source of quotations, direct and indirect, and paraphrases. Every effort is made to cite the editions Peirce is known to have owned or had available to him. When we cannot provide such information or when the edition he used was not available to us, we cite one that was accessible to him. Although quotations are allowed to stand as Peirce gives them even if they differ from the originals, such differences (both in substantives and accidentals) are explained. Unless otherwise indicated, all translations are by the editors.

Finally, the notes explain some difficult or obscure passages and identify some philosophical and scientific terms; provide the historical background for, as well as Peirce's own commentary and estimation of, some of his writings; quote the commentary and estimation of others; and give diplomatic transcriptions (with occasional corrections in italic brackets) of some of Peirce's annotations in offprints, of some relevant manuscript pages not incorporated into the text, of correspondence relevant to his work, and of published summaries and abstracts of his work and that of others.

Citations are given in shortened form; complete bibliographic information appears in the Bibliography of Peirce's References. (For works referred to by the editors, full bibliographic information is provided in the notes.) References to the first four volumes of the present edition appear as W1, W2, W3, or W4.

1.17–18 "Measurements. . . ."] See W4:79–144.

1n.1–2 The. . . .] The *American Ephemeris for the year 1879* gives the latitude of Allegheny as 40°26′36″.

4.12 Very] Frank Washington Very (1852–1927), American astronomer and chief assistant for 10 years to Samuel P. Langley at the Allegheny Observatory.

6.4–6 The experiments. . . .] See "On the flexure of Pendulum Supports," W4:515–28; the part referred to is not reprinted in W4.

6.11–12 page 72 . . . 1876)] The reference is to page 72 in the separately published offprint; see "Gravity at Initial Stations," W4:116.

6n.1–2 See W4:136 (and the preceding note).

7.18–19 as . . . 463] See "Gravity at Paris," W4:151.

7.28 my paper . . . 445] See "On the Deduction of the Ellipticity of the Earth," W4:534.

8.12 Beers] D. G. Beers, cartographer and publisher in Philadelphia.

8.18 Baker] Marcus Baker (1849–1903), American geographer and an assistant in the Coast Survey.

8.21 Farquhar] Henry Farquhar, Coast Survey employee and Peirce's occasional assistant.

8.35–9.3 Darling, Brown, and Sharpe . . . Stackpole & Brothers . . . Byrne] Instrument makers.

10.5 Appendix 14] See "On the flexure of Pendulum Supports," W4: 515–28.

10.8 page 388] This page is not reprinted in W4.

10.16 Smith] Edwin Smith (1851–1912), American astronomer and geodesist, and a Coast Survey employee.

13.27 Rogers] William Augustus Rogers, astronomer at Harvard College Observatory, 1870–86.

13.32 Schott] Charles Anthony Schott.

13.34 Breguet] Maison Breguet (also Bréguet), French engineering firm established by the Swiss watchmaker Abraham Louis Breguet.

17.12–14 The. . . .] See "On the flexure of Pendulum Supports," W4:515–28; the pages referred to are not reprinted in W4.

23.16 Appendix No. 15] See note 7.28.

26–106 These pages represent one of the earliest studies of great men (persons) based on statistical reasoning. Peirce called his work, begun in 1883 with students at Johns Hopkins, a study in comparative biography. It is not clear whether his primary interest, at the outset, was the study of greatness or the application of statistics, but the study of greatness continued to be of considerable interest to him throughout his life. The list that follows contains, in alphabetical order, the names of everyone who appears in any of the lists in items 2–19. The names in parentheses are spelled as they appear in the critical text. (For more on this study, see the Essay on Editorial Method and the Introduction.)

Abel, Niels Henrik
Abelard, Peter
Aeschylus
Aesop
Agassiz, Louis
Alcibiades
Alembert, Jean Le Rond d'
Alexander III
Alfieri, Vittorio
Alfred (the Great)
Ambrose (Père)
Apelles
Apollonius of Perga
Archimedes
Aristophanes
Aristotle
Arkwright, Richard

Artevelde, Jacob van
Artevelde, Philip van
Athanasius
Attila
Augustine
Augustus
Bach, Johann Sebastian
Bacon, Francis
Bacon, Roger
Baer, Karl Ernst von
Balzac, Honoré de
Barneveldt, Jan van Olden
Baxter, Richard
Bayard, Pierre Terrail
Becket, Thomas
Beethoven, Ludwig van
Belisarius
Bellini, Giovanni
Bentham, Jeremy
Bentley, Richard
Berkeley, George
Bernard of Clairvaux
Bernoulli, Jakob
Bernoulli, Johann
Berzelius, Jöns Jakob
Bhaskara
Bichat, Marie François Xavier
Bismarck, Otto von
Boccaccio, Giovanni
Bolívar, Simón
Boole, George
Bopp, Franz
Borromeo, Carlo
Bossuet, Jacques Bénigne
Botticelli, Sandro
Boyle, Robert
Brahe, Tycho
Browning, Robert
Bruce, James
Bruno, Giordano
Buch, Christian Leopold von
Buddha
Buffon, Georges Louis Leclerc de
Burke, Edmund
Burns, Robert
Byron, George Gordon
Caesar, Gaius Julius
Calderón de la Barca, Pedro
Calvin, John
Camoëns, Luiz Vaz de
Canova, Antonio
Carlyle, Thomas
Cauchy, Augustin Louis
Cavour, Camillo Benso di
Cecil, Robert
Cellini, Benvenuto
Cervantes Saavedra, Miguel de
Champollion, Jean-François
Charlemagne
Charles Martel
Charles V
Charles XII
Chasles, Michel
Chaucer, Geoffrey
Chopin, Frédéric François
Chrysostom, John
Churchill, John (/Duke of/ Marlborough)
Cicero, Marcus Tullius
Cid, The
Clausius, Rudolf Julius Emanuel
Cleopatra
Clive, Robert
Colbert, Jean-Baptiste
Coleridge, Samuel Taylor
Columbus, Christopher
Comte, Auguste
Confucius
Constantine I
Cook, James
Copernicus, Nicolaus
Corday, Charlotte
Cortés, Hernando
Coulomb, Charles Augustin de
Crichton, James
Cromwell, Oliver
Cuvier, Georges
Cyrus
Dalton, John
Dante
Danton, Georges Jacques
Darius I
Darwin, Charles
David
Davy, Humphry
De Witt, Jan
Democritus
Descartes, René
Deschapelles, Louis Honoré Lebreton
Dickens, Charles
Diderot, Denis
Diez, Friedrich Christian
Dirichlet, Peter Gustav Lejeune
Drake, Francis
Dryden, John
Duns Scotus, John
Dürer, Albrecht
Ehrenberg, Christian Gottfried
Eliot, George
Elizabeth (Queen)
Emerson, Ralph Waldo
Epaminondas
Epicurus
Erasmus, Desiderius
Erigena, John Scotus
Euclid
Euler, Leonhard
Euripides
Eyck, Jan van
Fabricius, David
Faraday, Michael
Farragut, David Glasgow
Fénelon, François de Salignac de La Mothe-
Fermat, Pierre de
Fibonacci, Leonardo
Fichte, Johann Gottlieb
Fielding, Henry
Fox, Charles James
Franklin, Benjamin
Frederick II (the Great)
Fresnel, Augustin Jean
Froissart, Jean
Galen
Galilei, Galileo
Gama, Vasco da
Gambetta, Léon Michel
Garcilaso de la Vega
Garibaldi, Giuseppe
Garrick, David
Gauss, Karl Friedrich
Germanicus
Gilbert, Humphrey

Mitscherlich, Eilhardt
Molière
Moltke, Helmuth von
Monge, Gaspard
Montaigne, Michel Eyquem de
Montesquieu
More, Thomas
Morphy, Paul Charles
Moses
Mozart, Wolfgang Amadeus
Müller, Johannes Peter
Müller, Max
Munkácsy, Mihály von
Murat, Joachim
Murray, William (Lord Mansfield)
Napier, John
Napoléon I (Bonaparte)
Necker, Jacques
Nelson, Horatio
Newton, Isaac
Niebuhr, Barthold Georg
Ockham, William of
Oersted, Hans Christian
Ohm, Georg Simon
Omar I
Omer Pasha
Origen
Osman
Oxenstiern, Axel Gustafsson
Palestrina, Giovanni
Palissy, Bernard
Palladio, Andrea
Paracelsus
Pascal, Blaise
Pasteur, Louis
Patrick, St.
Paul, St.
Peel, Robert
Penn, William
Pericles
Perugino, Il
Peter I (the Great)
Peter the Hermit
Petrarch
Phidias
Philip II (of Macedon)
Pindar
Pisano, Nicola
Pitt, William
Pizarro, Francisco
Plato
Poe, Edgar Allan
Polo, Marco
Polycarp
Poniatowski, Józef Antoni
Pope, Alexander
Porson, Richard
Porta, Giambattista della
Praxiteles
Priestley, Joseph
Ptolemy
Pym, John
Pyrrhus
Pythagoras
Rabelais, François
Rachel, Mlle.
Racine, Jean
Raleigh, Walter
Ramus, Petrus
Raphael
Ray, John
Rembrandt
Reynolds, Joshua
Ricardo, David
Richelieu
Richter, Jean Paul Friedrich
Riemann, Georg Friedrich Bernhard
Rienzi, Cola di
Ristori, Adelaide
Rollo
Rousseau, Jean-Jacques
Rubens, Peter Paul
Saadi
Saladin
Sarto, Andrea del
Savonarola, Girolamo
Saxe, Hermann Maurice de
Scheele, Karl Wilhelm
Schiller, Johann Friedrich von
Schopenhauer, Arthur
Scipio Africanus
Scott, Walter
Servetus, Michael
Shakespeare, William
Shelley, Percy Bysshe
Sherman, William Tecumseh
Siddons, Sarah
Sidney, Philip
Sixtus V
Smith, Adam
Smith, Joseph
Sobieski, Jan
Socrates
Solomon
Solon
Sophocles
Sorel, Agnès
Spenser, Edmund
Spinoza, Baruch
Steiner, Jakob
Stephenson, George
Stevens, Thaddeus
Sully, Maximilien de
Swedenborg, Emanuel
Swift, Jonathan
Sydenham, Thomas
Sylvester, James Joseph
Tacitus
Talleyrand-Périgord, Charles Maurice de
Talma, François Joseph
Tamerlane (Timour)
Tancred
Tasso, Torquato
Tecumseh
Tennyson, Alfred
Thales
Themistocles
Theodoric
Theodosius I
Thiers, Louis Adolphe
Thomas à Kempis
Thomas Aquinas
Thompson, Benjamin ([Count] Rumford)
Thorvaldsen, Bertel
Thucydides
Tintoretto, Il
Titian
Toussaint L'Ouverture
Trajan
Troyon, Constant
Turenne, Vicomte de
Turgenev, Ivan Sergeevich
Turgot, Anne Robert Jacques
Turner, Joseph Mallord William
Vauban, Sébastien Le Prestre de
Vega, Lope de
Velásquez, Diego Rodríguez de Silva
Veronese, Paolo
Vesalius, Andreas

29(c3).4–5 Massasoit . . . Philip)] Peirce conflates the names of Massasoit, a Wampanoag Indians chief, and of his son, known as King Philip.

29(c3).7 Müller] Max Müller (1823–1900), German philologist.

29(c3).15 Niccola] Nicola Pisano (c. 1220–1278 [or 1284]), Italian sculptor and architect.

31(c3).2 William of Sens] A twelfth-century mason from northern France, and architect of Canterbury Cathedral.

39.12 Poisson] Siméon Denis Poisson.

39.18 Crelle] August Leopold Crelle.

39.25 The reference is to Lagrange's memoir on maxima and minima, in the first volume of the proceedings of the Turin Academy of Sciences (which he helped to establish).

40.3 Duke of Brunswick] Karl Wilhelm Ferdinand, to whom Gauss dedicated his first masterpiece, *Disquisitiones arithmeticae.*

40.4 Kästner] Abraham Gotthelf Kästner.

40.8 Gauss is credited with saying that mathematics is the queen of the sciences, but arithmetic the queen of mathematics (while Leopold Kronecker said that God made integers, and all else is the work of man). Even in his own time, Gauss was called *Mathematicorum princeps* (Prince of Mathematicians).

40.19 Pfaff] Johann Friedrich Pfaff.

40.19–21 The question was asked of Laplace by Alexander von Humboldt, who supported Gauss for the directorship of the Göttingen Observatory. In another version of the anecdote, Laplace said that Gauss was the greatest mathematician of the world.

40.22 Comet] The reference is to The Great Comet whose parabolic orbit Gauss calculated; the Comet was regarded then as an omen portending the burning of Moscow and the defeat of Napoleon.

40.23–24 Gauss supervised the field work on a polygonal line between the observatories in Göttingen and Altona; his yearly reports were published in 1828 as *Determination of the Difference in Latitude between the Observatories in Göttingen and Altona.* A heliotrope is a sun-reflecting apparatus invented by Gauss for use in geodetic triangulations.

40.27–28 With Wilhelm Eduard Weber, Gauss constructed and used in the measurement of terrestrial magnetism the bifilar magnetometer, in which the magnet is suspended from two threads. See his *Intensitas, Atlas,* and "Zur Bestimmung der Constanten."

40.28 Theory . . . magnetism] See Gauss's *Atlas des Erdmagnetismus.*

40.30 Gauss's last publication was a new edition of his 1818 *Theorematis . . . demonstrationes,* which he prepared for the 1849 golden anniversary celebration of the granting of his doctorate.

42–44 The biographical sketch is based on Legge's *Life and Works of Mencius,* while quotations and paraphrases of Mencius are taken from *The Chinese Classics.*

42.22–26 See *The Chinese Classics,* bk. 5, pt. 1, ch. 5, §§6–8.

42.27–29 "The people. . . ."] Ibid., bk. 7, pt. 2, ch. 14, §1; "in a nation" is italicized in the original.

42.29 A bad. . . .] Ibid., bk. 7, pt. 2, ch. 14, §3.

43.1–6 Who . . . righteousness.] Ibid., bk. 1, pt. 2, ch. 8.

43.9 The aims. . . .] Ibid., bk. 3, pt. 1, ch. 3, §3.

43.20–21 He had. . . .] Ibid., bk. 3, pt. 1, ch. 3, §3.

43.26–31 "The tendency. . . ."] Ibid., bk. 6, pt. 1, ch. 2, §§2–3. In the original the passage reads:

> The tendency of man's nature to good is like the tendency of water to flow downwards. There are none but have this tendency to good, *just as* all water flows downwards.
>
> Now by striking water and causing it to leap up, you may make it go over your forehead, and, by damming and leading it, you may force it up a hill;—but are such movements according to the nature of water? It is the force applied which causes them. When men are made to do what is not good, their nature is dealt with in that way.

43.33–35 Ibid., bk. 2, pt. 1, ch. 6, §6.

43.39 Ibid., bk. 4, pt. 2, ch. 12.

43.40–44.6 Ibid., bk. 3, pt. 2, ch. 2, §3.

45.19–20 "He . . . myself"] See Vasari, "Michelagnolo Buonarroti," p. 229.

46.1–2 Ibid., p. 228; the person addressed is Giorgio Vasari.

46.13 Cardiere] Andrea detto il Cardiere (fl. 1490), lute-player and improvisatore.

46.23 Aldovrandi's] Gianfrancesco Aldovrandi, a nobleman who was Michelangelo's patron in Bologna.

47.22–23 The quotations . . . Virgilio] Machiavelli is believed to have learned Latin from Marcello Virgilio di Adriano Berti who taught at the University of Florence.

48.6–8 In his remarks on style, Peirce paraphrases Macaulay, *Critical and Historical Essays,* p. 42.

48.16 Machiavelli's comedies had not been translated into English in Peirce's time.

54–56 The earliest versions of the Great Men Questionnaire are no longer extant but some fragments remain, such as the following page from MS 485. The consecutive numbering of the questions and the occasional correspondence with question numbers in item 12 (for example, nos. 57 and 58) suggest that some early "canonical" version supplied the numbers for the reorganized questionnaire of item 12. Question numbers are omitted in the latest versions (items 13 and 14).

31. Was he very patient?
32. Was his power of memory remarkable?
33. Was he greatly excited by his work?
34. Was he urged by a great longing to do that which he mainly accomplished?
35. What was the strength of his will?
How great was his delicacy of feeling?
37. What was his capacity for learning a new thing?
38. Did he preserve this faculty to a late age?
39. What sentiments did he excite in those who immediately surrounded him?
40. Was he greatly beloved by any person much above the average?
41. Self-estimate what?
42. Good health?
43. Opinions about great men
44. Appreciation by his contemporaries — Definitions of
45. Best work at what age. — greatness
46. First great work — ----------
47. Work whether intermittent.
48. Estimate of energy.
49. Was his education peculiar
50. Was he brought up with other children and youths?
51. Was he born after regular time of gestation
52. Hereditary peculiarities
53. Influence on other children
54. Whether left-handed.
55. Moral character
57 Wild oats.
58 Religious charac[t]er
59 Whether married
60 Whether very strong
61 Whether large or sm
62. Anything peculiar about physique or appearance.

66.26–28 If my. . . .] See Vasari, p. 228.

66.31 "He. . . ."] Not located.

67.26–36 Peirce's description of Hobbes's physique is largely based on Aubrey's "Life."

68.8–10 "Irritabile. . . ."] Not located.

68.12–15 Peirce draws here on Robertson's *Hobbes,* p. 2, who describes three accounts of Hobbes's life written in Latin and first published together in 1681 by Richard Blackbourne, a friend of Hobbes's admirer, John Aubrey; the accounts are Hobbes's autobiography, a biography written by Blackbourne on the basis of Aubrey's notes, and another autobiography written by Hobbes in elegiac couplets at the age of 84.

68.33 Busby] Richard Busby (1606–1695), headmaster of Westminster (1638–95).

68.38–39 "I no sooner. . . ."] Fraser, *Locke,* p. 14.

69.12 articles] Several works published between 1670 and 1685, formerly attributed to Locke, are now known to be either spurious or only partially written by him. *Méthode nouvelle de dresser des Recueils* is now regarded as the first work published by Locke. It appeared in Le Clerc's *Bibliothèque Universelle* in 1686 and was translated into English as *A New Method of making Common-Place Books* (London: Greenwood, 1706).

69.13 *Essay*] *An Essay Concerning Human Understanding,* published in 1689 when Locke was 57.

69.31–32 His verses. . . .] Fox Bourne quotes some of Locke's early poetry in his *Life,* 1:52.

69.37–38 Shaftesbury ruined] The reference is to Lord Ashley, Earl of Shaftesbury (1621–1683), an English statesman with whom Locke was closely associated and who was tried for treason in 1681.

69.40–41 "Riches. . . ."] Fraser, *Locke,* p. 244.

70.3 *Éloge historique.*] Published in vol. 6 of Le Clerc's *Bibliothèque choisie.*

71(c2).14–15 Says . . . 30] See Montaigne's essay "Of Age," bk. 1, ch. 57.

71(c2).16–17 The first two books of *Essays* were published in 1580; the third, with numerous additions, appeared in 1587.

72(c1).15 What do I know?] The famous skeptical *Que sais-je?* asked by Montaigne in his "An Apology of Raymond de Sebonde," bk. 2, ch. 12.

72(c2).15–24 See Montaigne's essay "Of Vanity," bk. 3, ch. 9.

73(c1).43 *Prince*] *The Prince* was written in 1513.

76(c3).25 Graham] Thomas Graham.

76(c3).25 Tyndall] John Tyndall.

76(c3).26 Jones] Henry Bence Jones (1814–1873), Faraday's biographer.

76(c3).30 Ludwig] Karl Friedrich Wilhelm Ludwig.

77(c1).4 Julian] Julian the Apostate.

77(c3).3 Agassiz] Louis Rodolphe Agassiz.

77(c3).5 Eusebius] Eusebius of Nicomedia.

77(c3).11 La Boétie] Etienne de la Boétie (1530–1563), French writer and close friend of Montaigne, whose essay "Of Friendship" (bk. 1, ch. 27) was written in memory of La Boétie.

77(c3).21 Alcott] Amos Bronson Alcott.

107–108 This is one of several beginnings of a continuation of the 1880 "Algebra of Logic" (W4:163–209); see also items 21 and 22 below. Following the lead of his Johns Hopkins student Oscar Howard Mitchell (1851–1889), Peirce here describes a system of logic in which the only logical connectives are negation and disjunction (logical summation) or conjunction (logical product), and the only rules of inference are elimination (simplification) and amplification (addition). Peirce's (and Mitchell's) system anticipates many important features of Gentzen's system of natural deduction.

107.4 my . . . paper] The reference is to "Algebra of Logic," W4:163–209.

107.9–10 Mitchell's . . . Logic"] The paper appeared in *Studies in Logic,* pp. 72–106 (Boston, 1883), which was edited by Peirce.

107.23–108.3 Then by. . . .] Here, as in other parts of his work, the influence of linear and multilinear algebras on Peirce's logic is quite prominent. If, for example, A is a matrix whose entries are coefficients of variables of a system of simultaneous equations, B is a matrix of constants which are solutions of those equations, and X is the matrix whose entries are the unknowns of those equations, then the equation $A = B \times x$ can be used to solve the system (by the formula $A^{-1} \times B = X$, when the inverse A^{-1} of A is obtained.)

107.26–108.3 $A = B \times x$] Peirce here treats the "$\prec$" mainly as a sign of partial ordering (he was a pioneer in the theory of partially ordered sets, if not its founder); he effectively gives a definition of partial ordering broader than the one ordinarily used, which is

$$A \leq B \text{ is defined as } A = B \wedge A.$$

Peirce employs the indeterminate x in his definition where the contemporary definition uses A. Also he uses this sign (and "$=$") more loosely than is ordinary in today's mathematical usage: he uses these signs (particularly the "$\prec$") for *functions* (i.e., on the same linguistic level as logical sum and product, for example) and also for "relations."

108.7–17 Although he does not use the term, Peirce is explicitly relying upon the concept of a *basis,* which is the smallest set of logical connectives necessary and sufficient for expressing all logical relations. (See also "A Boolian Algebra," W4:218–21.)

108.18–21 Prima facie, Peirce's doctrine here, that non-commutative operations cannot be produced by any combination of commutative operations, might seem to be contradicted by the fact that all truth-functional operators can be defined in terms of the Sheffer stroke and dagger functions. Yet, the fact that Peirce had already explored such functions ("A Boolian Algebra," W4:218–21) suggests that his doctrine is more complicated than its mere statement would indicate. In a letter dated 23 September 1905, Peirce vigorously defended his doctrine against a technical attack upon it made by Josiah Royce. In the letter, which can be found in the Royce Papers deposited in Harvard University's Pusey Library, Peirce says: "So what you say is that inequality can result from equality and nothing else. You cannot be surprised if I say that this is too Hegelian."

109 *[diagrams]*] Peirce's use of spatial (iconic) diagrams as models of the universe of discourse is an approach he will develop in detail a decade later in his existential graphs.

109.5 universe of discourse] See "Algebra of Logic," W4:170 and the related note 170.9.

109.9 n-dimensional block *[and following diagram]*] In this n-dimensional array we get a glimpse of an n-ary relation treated as a set of ordered n-tuples; this modern view of relations is reflected even more strongly in later papers. In matrix theory, a *block* is a submatrix of a matrix. The term derives from the fact that for a matrix of finite submatrices, the matrix is seen as divided into "blocks" forming an array of matrices. The term "matrix" was first used by Sylvester (*Phil. Mag.* 37 [1850]: 363–70; reprinted in CP 1.145–51).

110.30 The missing formula must be $\Sigma_i \Pi_j l = (\Sigma_i \Pi_j l)\ (\Pi_j \Sigma_i l)$. The claw symbol in the preceding context means logical implication and the equality symbol means logical equivalence. Peirce is saying that the fact that

$(\exists x)(y) Lxy \therefore (y)(\exists x) Lxy$ is a valid argument may be expressed by saying that $(\exists x)(y) Lxy$ and $(\exists x)(y) Lxy$ & $(y)(\exists x) Lxy$ are logically equivalent.

111–115 Like items 20 and 21, item 22 is an incomplete preliminary version of item 30.

111.7–11 When. . . .] In this, and the discussion that follows, Peirce returns to his theory of signs, which is more fully developed in sec. 1 of item 30.

112 *[diagram]*] See note 109.

112.29–30 Let v. . . .] This passage may represent the first straightforward use of two truth-values in modern logic. See item 30, p. 166 (and note 166.11–20), for a more developed view of the meaning of v and f.

112.36–38 In writing. . . .] This stipulation amounts to "right-to-left associativity." Thus $(a \prec b \prec c)$ is to be read as $(a \prec (b \prec c))$ and so forth.

113.6 Here Peirce uses "$\prec$" as he most commonly does, as a "function" (on the same linguistic level as "$\supset$", for example—as opposed to "$\vdash$", or to the "$\leq$" of modern algebra).

113.16–20 See "Algebra of Logic," W4:183, for the relevant definitions; for a discussion of his "failure" to provide a demonstration for the distribution formula shown in the last two lines, see item 30, 173n.1–19 and note 184.3 in W4.

113.24 This law is now usually known as the law of "exportation."

113.25–31 Peirce here appears to presuppose the conception of a *basis*, but less explicitly than in item 20 (see note 108.7–17).

113.28–30 the whole. . . .] In 1867, Peirce had exhibited the "perfect balance" between logical addition and logical multiplication ("Boole's Calculus of Logic," W2:12–23), and in 1879 he had incorporated a principle of duality into the basis of his algebraic logic ("Algebraic Principles of Logic," W4:21–37), but by early 1880 ("Algebra of Logic," W4:163–209) he had retreated somewhat in his acceptance of duality (the parallelism of logical addition and logical multiplication) as of fundamental importance for logic.

114.8–9 defining . . . $(a \prec \mathrm{f})$] Unfortunately the manuscript breaks off before Peirce reaches a discussion of the definition of negation after he introduces quantifiers, but there is reason to believe he would have opted for a quantified expression of this sort: $\bar{a} \prec (\Pi a)\,(a \prec \mathrm{f})$.

114.21–24 Mitchell. . . .] See "On a New Algebra of Logic," p. 75.

115.2–3 De Morgan's . . . syllogism] See his *Syllabus*, pp. 27–30.

115.8–10 In this. . . . *[and preceding diagram]*] See notes 109 and 109.9.

115.9 principal diagonal] The *principal diagonal* of a matrix (sometimes called "main diagonal") is the set of entries on the diagonal of the matrix running from the upper right corner to the lower left. A *diagonal matrix* is a square matrix all of whose non-zero elements are in the principal diagonal. If, in addition, all the diagonal elements are equal, the matrix is a *scalar matrix*. An *identity matrix* (*unit matrix*) is a diagonal matrix all of whose elements on the principal diagonal are the identity element (unity). Thus, the principal diagonal of a matrix characterizes the matrix in special ways.

115.10–11 Such. . . .] See "On a New Algebra of Logic," p. 87.

115.23–28 Thus. . . .] The concept of quantification over "times" is often employed by Peirce as a means of dealing with *varying universes of discourse.* This anticipation of "possible worlds" appears as early as 1880 and continues in later writings.

116.21–117.2 Peirce believed the U.S. gravity network should consist of about a thousand stations; it had surpassed that number by 1949.

117.9–12 The best. . . .] An example of a casemated fort where Peirce swung pendulums is Fort Monroe, VA, of which he wrote to Hilgard, "I consider this the best station I ever occupied" (25 August 1884). An example of an observatory is the Kew Observatory in England; of a physical laboratory, Cornell University in Ithaca, NY; and of a cellar, the northeast corner of the cellar of the Smithsonian in Washington, DC.

122.10–15 *Unterschiedsschwelle. . . .*] "Differential threshold." Fechner's shadow experiment is described as follows in the *Elements of Psychophysics,* translated by H. E. Adler (New York: Holt, Rinehart and Winston, 1966), 1:203.

> One places two lamps next to each other in front of an object that will throw a shadow. Each of the lamps throws a shadow which is illuminated by the other lamp, whereas the surrounding background is illuminated by both lamps. If one now turns down the wick of one of the lamps or moves it away from the object that is throwing the shadow, the shadow is seen to become weaker and weaker, while the surrounding illumination becomes less and less different from it. Finally the shadow disappears, apparently absorbed by the general illumination of the background, in spite of the fact that both light sources are still present. I was most astonished when my attention was first drawn to the sight of two lights throwing but one shadow. Both lamps were clearly lit, but only one shadow was seen. In a word, when the difference between the illumination of the shadow and that of the surrounding background falls below a certain amount, the difference in sensations disappears completely and can no longer be perceived by any means.

122n.1 The term *Unterschiedsschwelle* appears on p. 242 and p. 243.

123.26–33 But. . . .] The main result of this experiment is that there is no minimum perceptible difference in sensation, and therefore Fechner's theory that there is an *Unterschiedsschwelle* is wrong. The evidence suggests that we perceive smaller and smaller differences in sensation with no definite cut-off value and, as a secondary result, perceive smaller differences than we are conscious of perceiving. These results support Peirce's claim that cognition is continuous, *not* beginning with *first impressions of sensation.*

136–38 This article may have been influenced by Boole's *Laws of Thought* (ch. 19).

136.19–21 Gilbert . . . ;] See "Finley's Tornado Predictions," p. 169.

136.22 I make. . . .] It may appear, in what follows, that Peirce is making use of Bayes's theorem, which states (according to Peirce's definition in the *Century Dictionary*) that "the probability of a cause is equal to the probability that an observed event would follow from it divided by the sum of the corresponding probabilities for all possible causes"; but Peirce went on to call it "fallacious rule."

136.29 relative frequencies] In statistics, when a collection of data is divided into several categories, the number of items in a given category is the absolute, or class, frequency. Relative frequency is the absolute frequency divided by the total number of items.

137.26–28 Gilbert, "Finley's Tornado Predictions," pp. 167 and 171.

139–143 A few days after this paper appeared in *Science, The Nation* published a short unsigned review in its 12 December issue, as well as a longer letter to the editor in its 25 December issue. After a brief summary of Peirce's conclusions, the review voices this small criticism: "Fault might be found with [Peirce's] assumption that 'the building could not have been erected without a drawing to scale,' whence it is concluded that 'a unit of length must have been employed'" (39:522). The letter, written by a Caroline H. Dall and dated "Washington, December 20, 1884," is more negative in its tone, as in the following excerpt:

> A person who lives quite a moderate number of years has occasion to see two or three generations struggling with the same problems, and falling successively into the same errors. It is a pity that there is not a point on the earth's surface where a gigantic tablet, marking "Errors finally corrected," could be mounted. There is no reason why Mr. Chas. S. Peirce or any other gentleman should discuss the measurements of the "Old Stone Mill." Its history and use were settled beyond appeal by a pamphlet published by the Rev. Charles T. Brooks at Newport in 1851, thirty-three years ago. . . . It is well known that on the east coast of Ireland and the west coast of England there are several round stone mills resembling this at Newport. . . . At the time this pamphlet was printed there were aged persons living who could give hearsay evidence as to the designation of Newport mill as far back as 1699. It had originally been a "powder mill," with a circular roof, and when the wind required that wings and roof should be turned around, it took a yoke of oxen to do it. The will of Benedict Arnold, first Charter Governor of Rhode Island, calls it, in 1677, "my stone-built wind-mill," and there are pretty good reasons for supposing that Arnold himself built it in place of a wooden mill which blew down in 1675. The great grandson of Arnold always spoke of the mill as built by the Governor, and of the field in which it stood as the "Mill Field". . . . The chimney of Arnold's house was pulled down in 1780, and proved to be of stone and hard-shell cement identical with that used in building the mill. Arnold's granddaughter married Edward Pelham and inherited the old mill. . . . The architecture of the mill . . . does not resemble that of any Norse buildings . . . but resembles exactly the English stone mills of the twelfth century. As I have seen an allusion to Mr. Peirce's communication to *Science* in your columns, I have taken the trouble to write these few lines to prevent other scientific investigators from wasting time and brains on a delusive quest. . . . (39:544)

140.14–20 Each of the eight sets of heights are the measurements taken on either side of one of the eight pillars. For example, the pillar between the southeast and south arches measured 7 ft. 9 in. on the southeast arch side and 7 ft. 8 in. on the south arch side.

140.23–24 two highest . . . pillars] The excluded measurements are those of the southwest arch pillars (8 ft. 2 in., 8 ft. 2 in.; 8 ft. $\frac{1}{2}$ in., 8 ft. 0 in.) and the north arch pillars (7 ft. 6 in., 7 ft. $6\frac{1}{2}$ in.; 7 ft. 7 in., 7 ft. 5 in.).

141(c4).7 (7 " 8.9 ") This is the observed mean height of the pillars if all sixteen measurements (two measures for each of the eight pillars) are included. The mean given at 140.21, is "7 feet $8\frac{7}{8}$inches."

149.8 Lyman] Theodore Lyman (1833–1897), American politician and biologist.

149.10–11 I am. . . .] In a letter dated 1 July 1884, Peirce was appointed to the Office of Weights and Measures, which he headed from 1 October 1884 to 22 February 1885.

150.31–33 Pratt . . . Sellers] Instrument makers.

153.6 Gallatin] Albert Gallatin.

153.7 Kater] Henry Kater (1777–1835), English geodesist.

155.18 Herbert] Hilary Abner Herbert.

155.21 Lane] Jonathan Homer Lane (1819–1880), American physicist.

156.35–37 I would. . . .] In a letter of 16 January 1879 to his father (then consulting geometer of the Survey), Peirce remarked that the Berlin platinum meter was the best practical standard. On 5 April 1893, the standards of mass in the U.S. became kilograms numbers 4 and 20 obtained from the International Bureau of Weights and Measures (and consisting at 90% platinum and 10% iridium). The National Bureau of Standards, which was created on 3 March 1901, kept the same standards.

162.10–12 but. . . .] In terms of quantificational logic, a dual relation $R(x, y)$ is *degenerate* if and only if it is logically equivalent to $A(x)$ & $B(y)$ for some A, B.

162.15–163.32 Peirce's effort here is to identify tokens (later, symbols), indices, and icons by (in part) the logical decomposability or indecomposability of the triadic sign relation in each respective case. Thus (he maintains) we have a *token* if and only if the sign relation S(s, o, m) is non-degenerate (in the sense of the context). We have an *index* if and only if S(s, o, m) is logically equivalent either to

1. N(s, o) & M(o, m) for some N, M; or to
2. N(s, o) & O(s, m) for some N, O; or to
3. N(s, o) & M(o, m) & O(s, m) for some N, M, O; *and furthermore that* N(s, o) is non-degenerate.

Finally, we have an *icon* if and only if S(s, o, m) is logically equivalent to one of the formulas 1, 2, 3 *and furthermore that* N(s, o) is logically equivalent to P(s) & P(o), for some P, so that N(s, o) is a dual degenerate relation and in fact degenerate in such a way that s and o bear a point of similarity: the predicate P. For another discussion of the three types of signs, see item 56, pp. 379–80.

162.16 degenerate species] In this context, Peirce employs a concept of a *degenerate plural relation* that is somewhat different from the one he often employs elsewhere. For instance, he typically means by a degenerate triple relation a relation (to use quantificational logic) $R(x, y, z)$ that is logically equivalent to one of the following:

1. $A(x)$ & $B(y)$ & $C(z)$ for some A, B, C;
2a. $A(x)$ & $F(x, y)$ for some A, F;
2b. $A(y)$ & $F(x, z)$ for some A, F;
2c. $A(z)$ & $F(y, z)$ for some A, F.

In this typical sense of a degenerate triadic relation, there are two fundamental types: the type equivalent to the form 1 and the type equivalent to one of the forms 2a–2c. In the present context, however, Peirce means by a degenerate triple relation a relation (to use quantificational logic) $R(x, y, z)$ that is logically equivalent to one of the following:

1a. $F(x,y)$ & $G(x,z)$ for some F, G;
1b. $F(x,y)$ & $G(y,z)$ for some F, G;
1c. $F(x,z)$ & $G(x,z)$ for some F, G;

In this sense, there is only one type of degenerate triadic relation.

163n.1–2 See "New List of Categories," W2:56.

164n.1–2 "On a New Algebra of Logic," p. 74; see also note 107.9–10.

166.6–10 Peirce's characterization of the metric conception of logic can be seen as the catalyst for development of many-valued logics, and especially, in combination with Lobachewsky's non-Euclidean "imaginary" geometry, for N. A. Vasiliev's non-Aristotelian "imaginary" logic, or paraconsistent logic (that is, many-valued logic in which any truth-value $\neq 0$ is understood as $=$ t). This passage is reported to have been frequently cited by Łukasiewicz in his logic lectures.

166.11–20 Let. . . .] According to Alonzo Church (*Introduction to Mathematical Logic I* [Princeton 1956] p. 25, note 67), this is the first explicit use of two truth-values (for Frege's discussion did not appear until six years later in his *Funktion und Begriff*).

166.16 In this notation, it is important to read the horizontal dash as a *minus* sign. Thus, for example, "$x - \mathrm{f}$" is to be read "(the quantity (of) x minus (the quantity) f." This quantity will be zero, of course, if and only if (the quantity (of) x is equal to (the quantity) f, that is to say if and only if (the proposition) x is false. It is important also to understand the multiplication at hand as an actual multiplication of quantities. One problem with this notation, as Peirce quickly shows, is that it requires three quantity values, f, v, and 0.

169.3–5 The algebra. . . .] This is sometimes taken as evidence of logicist sympathy, but Peirce may only be pointing out that logic provides the analytical tools and methods (for example, mathematical inference) needed for foundational work—not that logic provides the theoretical foundations for arithmetic.

169.20–22 But. . . .] Peirce is speaking of the conditional we now call the material conditional, not a conditional (such as, strict implication, the counterfactual conditional, the semi-factual conditional) that would require reference to other "possible worlds."

169.23–24 To make. . . .] The "hypothetical proposition, in general" as defined here is broader than the conditional *de inesse* (the $a \prec b$ of the next paragraph) which Peirce examines mathematically in this paper. As he makes clear in the present paragraph, the *hypothetical proposition in general* involves a *range* of "states of things"—effectively, of "possible worlds." The hypothetical in this sense and the conditional *de inesse* are related as third to second.

171.8 Here and in the following formulas, "right-to-left associativity" must be understood to operate: $(y \prec (x \prec z))$ is the meaning, not $((y \prec x) \prec z)$.

172.6–10 The original. . . .] Peirce here alludes to his double use of "$-\!\!<$" that he originally employed as a sign of partial order ("to express the pure formula of identity"); in this role, however, the sign can occur just once in an expression. Thus, he enlarges "the conception of the notation so as to make the terms themselves complex [making possible, for example, expression of] the principle of the transposition of antecedents" (his *second icon*). Although he moves fairly casually between these uses of "$-\!\!<$", the passage makes clear that he was well aware of the distinction.

173.10–11 The first five icons constitute a complete (though not minimal) basis for the Classical Propositional Calculus.

173.15 This principle, known since Łukasiewicz as Peirce's Law, is necessary for rendering the positive implicational calculus classically complete.

173.16 This . . . axiomatical] This is not the technical sense of "axiom" current today; the term "axiomatical" as used here seems to approximate something like "self-evident." Peirce's demonstration, which follows, introduces the method of truth-function analysis, although he does not make the kind of distinction between semantics and proof theory taken for granted today. See note 175.15–18 below.

173n.6 this error] In the margin of an offprint (MS 558) Peirce wrote: "But it was not an error!!! See my original demonstration in marginal note." The marginal note has not been found. See note 184.3 in W4.

173n.6–7 Chapter II] See "On the Algebra of Logic," W4:181–93.

173n.7–11 Schröder. . . .] The reference is to his "Exposition."

173n.12 *Studies* . . . 189] See "Note B," W4:455.

175.15–18 Accordingly. . . .] Peirce suggests here a "devil's advocate" strategy for establishing "necessary truth," which anticipates the strategy employed today in methods such as semantic tableaux and consistency trees; more straightforwardly, it is what is today referred to as the "short" or "indirect" truth-table technique.

177.20 These. . . .] See Mitchell's "On a New Algebra of Logic," p. 84 (and note 107.9–10).

178.16–19 His method. . . .] Ibid., p. 75.

179.10–180.3 Mitchell. . . .] Ibid., §2.

180.8 Peirce's style of presentation makes it necessary that we sometimes infer from his formulas principles which he certainly recognizes, but which he may not state explicitly (perhaps because he believes them to be obvious). One such would be "universal instantiation": $\Pi_j(\Pi_i x_i -\!\!< x_j)$ which follows from the present line and the (non-relative) $x_i x_j x_k$ etc. $-\!\!< x_l$. The derivability of this principle, and others given in the next item in the present volume, indicates that Peirce had a complete basis for quantification theory at this time.

180.27–31 Let . . . all] Peirce's English is slightly inaccurate here. What he says would be the English for $\Pi_j\ (\bar{c}_j + \Sigma_i g_i l_{ij})$. The symbolism that Peirce here translates would be more exactly expressed as: "There is something such that if there are any chimeras then that something is a griffin that loves them all."

183.3–13 It is. . . .] Here Peirce is appealing to what in quantificational logic is handled under the topic of identity. The point he is making is that from

$$A(x,y,z) \text{ and not } -A(t,u,v)$$

we may validly derive

$$x \neq t \text{ or } y \neq u \text{ or } z \neq v.$$

This is a standard result.

184.16 Peirce might have made his meaning clearer if he had *first* written $(\Sigma_i a_i b_i)\ (\Pi_j(\bar{b}_j + c_j))$ and *then* by rearranging derived $\Sigma_i \Pi_j a_i b_i (\bar{b}_j + c_j)$.

186.9–15 The necessity. . . .] Here (and in the previous discussion) Peirce is laying out an extremely general Leibnizian account of identity, which would require defining identity via substitutivity of identicals in *any* well-formed formula (to use the language of quantificational logic), no matter how many free variables the formula contained. Peirce has previously discussed unary relations (or predicates) and binary relations; here he wants to extend the idea of Leibniz's Law to include relations of arbitrary adicity. This may be missed if the reader concentrates on the fact that Peirce does not seem in this essay to distinguish clearly between open formulas and their closures (in the language of quantificational logic).

187.14 This and the previous twelve lines provide an ingenious proof of the identity of two entities i, j from the claim that everything (that is, every property) is either true of the one (i) or false of the other (j). It relies on the Leibnizian theory of identity and several elementary facts about classes, for example the existence of a complement class. Peirce's extremely cautious account of class existence does not commit him to a universal class. As with Zermelo's *Aussonderungsaxiom,* Peirce's principle of class existence presupposes a previously-given class of individuals.

187.24–26 Peirce's argumentation from this point until the end of the piece is complex, sufficiently so to warrant an exposition beyond the scope of a mere explanatory note. In the present context, however, the following must suffice.

Peirce's notation "q_{xy}" is to be read as either $X(y)$ or as $y \epsilon x$. Similarly, his notation "$r_{y\alpha x}$" is to be read as either $\alpha\ (x,y)$ or as $(x,y) \epsilon \alpha$. Thus, his definition of "for every a there is a b"—let us use capitals and say "For every A there is a B"—means:

There is some binary relation α such that:

(1) $(x)\ \{Ax \supset (\exists y)\ [By\ \&\ \alpha xy\ \&\ (z)\ (\{Az\ \&\ \alpha zy\} \supset x = z)]\}$.

From (1) Peirce takes himself to be deriving an equivalent, namely

(2) $(x)\ \{Ax \supset (\exists y)\ [By\ \&\ \alpha xy\ \&\ (z)\ (\alpha zy \supset x = z)]\}$.

Unfortunately, in fact (2) is not equivalent to (1) and does not derive from (1). It is stronger than (1) in that from (2) we can derive (1). (2) does derive from (1) *plus* the added assumption that $(x)Ax$, but unfortunately Peirce is not assuming that $(x)Ax$.

The difficulty here can, however, be overcome by realizing that Peirce in this context is attempting to define, in terms of logic, the idea that the number of the B's is at least as great as the number of the A's. In order to do so, moreover, he is attempting to flesh out the idea in exactly the same way as is standardly done in set theory, for example in the ideas underlying the Schröder-Bernstein Theorem and the definition of Cardinal Number. That is to say, Peirce's *understanding* of "For every A there is a B" is: There is a *one-one function* α *(an injection)* α from A to B. With the understanding that α is a (one-one) function from A to B, the explicit conjunction of A_z with α_{zy} is unnecessary because α_{zy} will imply that A_z. Peirce is undoubtedly assuming that α is a subset of the Cartesian Product $A \times B$; and with this assumption added, (2) does follow from (1). It is in light of these remarks that Peirce's analysis of De Morgan's syllogism of transposed quantity should be understood (see note 188.23–25).

188.14–16 In an appendix. . . .] See De Morgan's "On the Structure of the Syllogism [No. I]," pp. 406–408.

188.21–25 The following. . . .] See "On the Syllogism, No. IV," p. 353. The example given here and attributed to De Morgan might seem not to be a good example of a syllogism of transposed quantity; for it might seem not to allow the inference even in *finite* cases. Let the X's be two in number, say a and b. Let a be both X and Y (never mind whether it is Z or not). And let b be X but neither Y nor Z. Then the premisses of the argument would seem to be true. The conclusion, however, would seem to say that either (a is not X and a is not Z) or (b is not X and b is not Z). The conclusion would then seem obviously false.

The foregoing analysis, however, fails to take into consideration that for Peirce "For every A there is a B" means that there is an injective function from A to B. Thus, "For every X there is something neither Y nor Z" means that there is an injective function α from X to $\bar{Y}\bar{Z}$.

189n.1–6 "Suppose. . . ."] *Formal Logic,* p. 168.

195.19 Peirce's techniques would also permit derivations of the even stronger conclusions: $\Pi_i\Pi_j\Pi_k\ (l_{ik} + \bar{l}_{ij})\ (\bar{l}_{ik} + l_{ij})$ and $\Pi_i\Pi_j\Pi_k\ (l_{ij}l_{ik} + \bar{l}_{ij}\bar{l}_{ik})$. In quantificational logic, these derivations would be (in essence) (x) (y) $(Mx \equiv Lxy)$ to (x) (y) (z) $(Lxy \equiv Lxz)$. These results are standard and well-known.

195.27–28 These are from an important class of quantificational formula; in particular, we may note $\Pi_j(a_i + b_{ij}) = a_i + \Pi_j b_{ij}$. Since Peirce's intent here has to be that j not be free in a, this formula easily yields $\Pi x(a \prec b) \prec (a \prec \Pi x b)$, where x is not free in a which principle is employed in well-known axiomatizations of quantification theory.

196.2 Peirce's corrected equation is proved as follows:

$$\begin{aligned} a + xb &= a(x + \bar{x}) + xb \\ &= ax + a\bar{x} + xb \\ &= ax(b + \bar{b}) + a\bar{x} + xb \\ &= axb + ax\bar{b} + a\bar{x} + xb \\ &= ax\bar{b} + a\bar{x} + (axb + xb) \\ &= a\bar{x} + xb + ax\bar{b}. \end{aligned}$$

196.3–197.20 MS 538 contains the following variant of step 6 (bracketed page numbers refer to item 30):

6^{th} Step. The step numbered 5^{th} on page 198 */183/* may more conveniently be separated into two. The first of these somewhat resembles the last; it is a sort of development or setting forth in detail the premise, but instead of being founded upon a distribution formula it consists in raising the whole premise to a power or multiplying it into itself. At each such multiplication any of the indices may be changed to new ones, their order in the quantifier being determined by the rules of the 2^{nd} step. To prove that this can be done we begin by confining our attention to the first index of the quantifier. The proposition is then either of the form $\Pi_i a_i$ or $\Sigma_i a_i$. We have obviously

$$\Pi_i a_i = a_1 a_2 a_3 \text{ etc} = \Pi_i \Pi_j a_i a_j$$
$$\Sigma_i a_i = a_1 + a_2 + a_3 + \text{etc}$$
$$= (a_1 + a_2 + a_3 + \text{etc}) (a_1 + a_2 + a_3 + \text{etc})$$
$$= \Sigma_i \Sigma_j a_i a_j .$$

Next consider the first two indices. The proposition is of one of the four forms

$$\Pi_i \Pi_j a_{ij} \qquad \Sigma_i \Pi_j a_{ij} \qquad \Pi_i \Sigma_j a_{ij} \qquad \Sigma_i \Sigma_j a_{ij} .$$

For the first and last of these, we have only to apply the formulae just obtained, with the first two of p. 197 */182.22–23/*. That is,

$$\Pi_i \Pi_j a_{ij} = \Pi_j \Pi_i a_{ij} = \Pi_i \Pi_k \Pi_j a_{ij} a_{kj}$$
$$= \Pi_j \Pi_i \Pi_k a_{ij} a_{kj} = \Pi_j \Pi_l \Pi_i \Pi_k a_{ij} a_{kl} .$$
$$\Sigma_i \Sigma_j a_{ij} = \Sigma_j \Sigma_i a_{ij} = \Sigma_i \Sigma_k \Sigma_j a_{ij} a_{kj}$$
$$= \Sigma_i \Sigma_j \Sigma_k \Sigma_l a_{ij} a_{kl} .$$

For the other two forms the proceeding is not more difficult. By the formulae for a single index

$$\Sigma_i (\Pi_j a_{ij}) = \Sigma_i \Sigma_k (\Pi_j a_{ij}) (\Pi_j a_{kj}) = \Sigma_i \Sigma_k \Pi_j a_{ij} a_{kj}$$
$$\Pi_i (\Sigma_j a_{ij}) = \Pi_i \Pi_k (\Sigma_j a_{ij}) (\Sigma_j a_{kj})$$

197.30 every quantity] Strictly speaking, every *non-zero quantity* has a reciprocal.

198.9–10 Thomist doctrine] See Thomas Aquinas, *Summa theologica,* pt. 1, qu. 50–64.

198.30 This means that we have $\Pi x 1 = 1$ where $1 =$ *verum;* thus

If A is provable, so is $\Pi x A$

is a principle of inference in Peirce's algebra; together with "universal instantiation" and the principle noted at p. 195.27–28, this gives a basis for classical quantification theory, when subjoined to the classical logic of propositions we know to be there.

199.14–15 Peirce's exposition is elliptical. The actual steps taken here are first, squaring to get $\Pi_k \Pi_u (q_{ki} + \bar{q}_{kj}) (q_{ui} + \bar{q}_{uj})$; second, multiplying by the identical proposition $\Pi_\lambda (q_{\lambda i} + \bar{q}_{\lambda i}) (q_{\lambda j} + \bar{q}_{\lambda j})$; and third, applying Peirce's prenexing formulas to obtain the result Peirce writes down.

200.5 because] The word "because" here is misleading, since Peirce has already *derived* this formula. Its meaning is (essentially) that for any binary relational predicate Q we have at our disposal formulas of the type (x) $[(Q(x,y) \,\&\, Q(x,z)) \equiv (Q(x,y) \,\&\, Q(x,z))]$. This, of course, is hardly surprising as a result.

202.23–24 Peirce's commentary is elliptical. Actually several steps are taken. First, from $\Sigma_a \Pi_b u_a(\overline{w}_b + \bar{l}_{\mathrm{ab}})$ we obtain $\Sigma_a u_a(\overline{w}_a + \bar{l}_{\mathrm{aa}})$ by what Peirce calls "identification" and which is basically a universal instantiation, performed without first dropping previous quantifiers. Next we may eliminate the addend $\bar{l}_{\mathrm{aa}}$ since it is self-contradictory. This yields Peirce's result: $\Sigma_a u_a \overline{w}_a$.

204.18 See note 195.19.

205.4 See note 199.14–15.

205.27–220 Here Peirce's writing becomes exploratory and directed toward the discovery of truth rather than the exposition of already-discovered truth. For this reason his train of thought is difficult to follow, and a full explication of these pages would be quite lengthy.

Peirce is dealing with a conglomerate of issues as he attempts to construct both a set theory and an underlying logic that is adequate for dealing with this set theory. He is, moreover, trying to build both a logic and a set theory in the context of attempting to clarify arguments similar to De Morgan's syllogism of transposed quantity. This task in turn forces him to consider whether principles of inference such as mathematical induction (which he calls "Fermatian Inference") apply only to finite sets or also to transfinite sets (which he calls "ultrainfinites").

Peirce's discussion of these complex issues is further complicated by two features of his basic approach. First, he is building a second-order logic (which he calls a logic of "second intentionality") in order to deal with his problems, rather than—as is typical nowadays—using a first-order logic coupled with the device of introducing ordered n-tuples through the Axiom of Pairs. Second, although he wants—and clearly must have—an exposition in which he can discuss the properties of relations and functions, he never clearly defines the notions nowadays called "domain" and "range." For this reason, certain ambiguities in his ideas—such as the one discussed in note 187.24–26—continue to vex Peirce.

206.21 204] 204.13–18.

206.25 We now. . . .] 206.2.

206.28–29 definition. . . .] See the discussion and formulas on pp. 188–90.

207.8 206] 206.31–32.

207.11 206] 206.3–6.

207.15 my paper] See W4:299–309.

207.32–33 At this point Peirce begins to use his devices for second-order logic—viz. his "general-indices-of-tokens" q and r—slightly differently from their earlier use. He now begins to use Greek letters to indicate the unary or binary relation over which he quantifies. Also, for r in particular, the binary relation quantified over is now indicated by the *first*, rather than the *second*, of the three subscripts of r. These changes—together with the immediately prior note, "(Postpone this.)"—suggest that Peirce had set aside his (largely inconclusive) explorations for a moment (perhaps to "let his head clear" or to gain new perspective) before resuming them again.

208.9–12 This reduction of the long proposition does not prove it to be true. The reduction merely shows that "elimination" does not produce a false or contingent proposition that would thus refute the logical validity of the long proposition. Peirce clearly realizes this as he shows in his subsequent remarks.

208.10 206] 206.2.

208.15 204] 204.4–6.

209.15 proved. . . .] The reference is to item 30; but see also item 31, 192.15–194.

211.10 210] 210.15–16.

211.28 Probably this expression should read $\Sigma_7 \Pi_8 x_7 \bar{z}_8$.

215.27 are] Peirce should have said: "are equinumerous with."

218.13 216] 216.9–11.

218.22 216] 216.4.

219.28 219] 219.6–9.

219.29 218–19] 218.21–219.5.

219.30 217] 217.19–27.

220.1 219] 219.6–9.

220.2 217–18] 217.30–218.9.

222.4–5 the reality. . . .] *Religious Aspect,* especially bk. 2, ch. 12.

222.9 Hegel. . . .] The reference is to Hegel's absolute idealism (especially in his *Phänomenologie des Geistes*) and to Schelling's pantheistic idealism (in his *Vorlesungen*).

222.18 A certain writer] Peirce refers to himself. (See "How to Make our Ideas Clear," W3:271–76; also CP 5.311, 7.186, and 7.336n11.)

222.26–33 He will. . . .] See *Religious Aspect,* p. 426; the rhetor Thrasymachus of Chalcedon appears in Books 1 (336–354) and 5 (450a) of *The Republic* and is also mentioned in *Phaedrus* (261c, 267e, 269e, and 271a).

222.33–35 But. . . .] In an earlier version of this manuscript, Peirce started the defense of Thrasymachus (that is, of himself) with a different intention, that of bringing the matter of the categories, especially of his three categories, to the forefront of the discussion, and he wrote several pages to that effect. He dropped that part of his argument in the final version of the review, even though it clearly pervades it. The dropped part (from MS 540) reads as follows:

> We may suppose, then that Thrasymachus set out, as wiser men have done, from the standpoint of Kant. Kant's monumental work, the "Critics of the Pure Reason," is neither more nor less than a treatise upon logic. In this book it is shown that certain fundamental conceptions, Cause, Reality, and the like, of which a list are given are essentially presupposed in the ordinary logical divisions of propositions. For example, the relation of substance and accident is plainly the same as that of the subject and predicate of a categorical proposition; the relation of cause and effect does not essentially differ from that of the antecedent and consequent of a hypothetical proposition, etc. If we assume then that the logical distinctions of propositions be necessarily involved in reasoning and take their origin in the nature of the human mind, then so also do these conceptions, *cause, reality,* etc. which are essentially presupposed in those distinctions. The main purpose of the Critics of the Pure Reason is to develope this idea and trace out its consequences, and most important these are shown to be from several points of view. Thus, the whole system of Kant depends upon the truth and necessity of the system of formal logic which

furnishes these distinctions of propositions. If the latter system is artificial, the Kantian philosophy must fall to the ground; yet even then it would seem that there must be in place of that a true system which would be based in a similar way upon the correct analysis of formal logic. Animated by this idea, Thrasymachus devotes his life to the dry study of formal logic, in the hope that those who come after him may, upon the foundation thus laid, erect the glorious structure of a true and scientific philosophy. He finds before very long that the traditional distinctions of propositions rest nearly all of them upon mere accidents of language. But the positive result which he does reach in his plodding way after long years of plodding, —a result which Plato would have seen with one glance of his transcendental eye, —is that there are three conceptions which enter necessarily into formal logic at every turn and under a thousand shapes; namely, the ideas of First, Second, and Third; or more accurately expressed An, Other, and Medium. This may be shown in many ways. For instance, logic supposes a belief, and a belief has consistency, wherein is the conception of *unity.* But logic also supposes doubt, wherein is the conception of *two* alternatives. And logic further supposes a passage from doubt to belief through some *means,* or third element.

The conception of Other can in no way be constructed from that of One; not, for instance, by repetition, because repetition at once involves the idea of a second. So the conception of Third cannot be constructed by the addition of one to a pair; because addition involves the idea of a Medium. On the other hand, the conception of a Third supposes that of a Second, which in its turn supposes that of a First. The conceptions of four, five, etc. are, however, at once built up by addition from Three. All this is vague and unconvincing when thus abstractly stated; it is perceived with certainty to be true by the help of the concrete instances of formal logic.

223.1–2 "No. . . ."] *Religious Aspect,* p. 427.

223.15 Royce . . . ;] Ibid., p. 4.

223.17–19 the whole. . . .] This is the argument of "Fixation of Belief" (W3:242–57).

223.33 German philosophers] Among those Peirce had in mind were probably Schelling, Fichte, and Hegel.

224.2–4 Kant . . . ;] See the chapter on transcendental logic in *Critique* A50–64 (B74–88).

224.8 method] Peirce is apparently referring to algebraical logic, the most exact logic so far developed. See p. 237.20–21.

224.15–17 Kant . . . ;] Peirce's reference is probably to *Critique* A71ff. (B96ff.) or A598–99 (B626–27).

224.19–22 One. . . .] This conception of the index is anticipated in "Consequences of Four Incapacities" (W2:224ff.), "On Representations" (W3:62ff.), and "Methods of Reasoning" (W4:251).

224n.1–2 See Mitchell's "On a New Algebra of Logic" and Peirce's "On the Algebra of Logic," item 30.

225.3–6 By *intuition.* . . .] The reference is to Kant's conception of space and time as forms of sensible intuition, in *Critique* A22–36 (B37–53).

225.10–11 I wish. . . .] Peirce later used the word "molition" which, in a 17 December 1909 letter to William James, he defined as "volition minus all desire and purpose, the mere consciousness of exertion of any kind" (CP 8.303).

225.25–27 Kant's. . . .] See *Critique* A22–23 (B37–38).

225.33 Whately] Richard Whately was the author of *Elements of Logic,* the first logic book Peirce ever read (see W1:xviii).

225n.1–3 Hegel, *Encyclopädie,* §7, p. 10; the translation is Peirce's.

226.10 "a *barely*. . . ."] *Religious Aspect,* p. 427; "No barely" in the original.

226.11–15 The final. . . .] See "How to Make our Ideas Clear," W3:273–76.

226.16–17 "bare . . . ,"] *Religious Aspect,* p. 430.

226.26–29 Roger Bacon. . . .] Peirce refers here most likely to the *Compendium,* ch. 1.

227.37–40 That is. . . .] See the distinction between feigned and real doubt in "Consequences of Four Incapacities" (W2:212). Also see "Practical Logic" (W2:354–56), "Logic, Truth, and the Settlement of Opinion" (W3:14), "Chapter 2." (W3:23), and "Fixation of Belief" (W3:248).

228.2–6 In that. . . .] In 1905 Peirce remarked that such questions, "concerning which the pendulum of opinion never would cease to oscillate," are not *real* questions (CP 5.461).

228.16 "uniformity of nature"] The reference is to Mill's *System of Logic,* bk. 3, ch. 3, §1.

229.2–3 I hold. . . .] Not long before, on 13 May 1884, Peirce had delivered a paper entitled "Logic of Religion" to the Johns Hopkins Metaphysical Club (see MS 505 and W4:lxvi–lxvii). In 1893 he more fully expressed his views on religion in "Evolutionary Love" (CP 6.287–317) and, in 1908, in "A Neglected Argument" (CP 6.452–93).

230.8–10 When . . . ;] See *The Logic of Hegel,* §§79–82.

231.12 "moral realist"] Royce has "ethical realist" (*Religious Aspect,* p. 21).

231.22–27 About the dilemma, see item 54, pp. 355–59.

232.12–13 "If . . . ?"] *Religious Aspect,* p. 48; in the original the clauses are separated with "the objector will say."

232.15–18 Royce was concerned with Spencer's "Data of Ethics" in *Principles of Ethics* I, Part I (1879); see *Religious Aspect,* pp. 70 and 82.

232.22–23 psychologists] Royce mentions Alexander Bain's conception of memory and Francis Galton's investigation of word associations (Ibid., pp. 135 and 136).

232.29–30 "Possibly . . . ,"] Ibid., p. 138.

232.35 Kant's dove] This is an allusion to the following passage in the *Critique* A5 (B8–9):

> The light dove, piercing in her easy flight the air and perceiving its resistance, imagines the flight would be easier still in empty space. It was thus that Plato left the world of sense, as opposing so many hindrances to our understanding, and ventured beyond on the wings of his ideas into the empty space of pure understanding. He did not perceive that he was making no progress by these endeavours.

232.37–39 "For behold. . . ."] *Religious Aspect,* pp. 140–41.

233.11 ideal of ideals] Ibid., p. 144.

233.11–12 "Here. . . ."] Apparently a stylized paraphrase of Royce's remarks on pp. 144–46 of his book.

233.14–19 Phoenix. . . ."] John Phoenix was the pseudonym of George

Horatio Derby, American soldier and humorist. The story (and quotation) comes from "Official Report of Professor John Phoenix, A. M., Of a Military Survey and Reconnaissance of the Route from San Francisco to the *Mission of Dolores,*" in his *Phoenixiana,* p. 21. Only 184 solar compasses are mentioned in the original.

233.28–29 "It is. . . ."] *Religious Aspect,* pp. 181–82; "no mercy" is not italicized in the original.

233.29–34 "When . . . ?"] Ibid., p. 187; the original reads "of universal good humor" and "really wretched."

233.34–36 "The . . . contempt"] Ibid., pp. 196–97; the original reads "anybody else" and "*him*self."

234.7 "'as musical. . . .'"] Milton, *Comus,* l. 477. The entire passage (ll. 475–79) reads:

> How charming is divine Philosophy!
> Not harsh and crabbed, as dull fools suppose,
> But musical as is Apollo's lute,
> And a perpetual feast of nectared sweets,
> Where no crude surfeit reigns.

See also Shakespeare's *Love's Labour's Lost,* 4.3.341–42: "as sweet and musical / As bright Apollo's lute."

235–41 Peirce apparently wrote this text shortly after he finished the Royce review (item 33), wishing to develop the abandoned discussion of the categories he originally intended to incorporate in the review (see note 222.33–35). Item 34 is the first text in which Peirce returns explicitly to the matter of categories since his 1867 paper "On a New List of Categories."

235.3–237.22 Segment 1 discusses Kant's categories, their relation to Peirce's categories, and Hegel's method of deriving his categories. A loose page in the manuscript contains the following disconnected footnote, perhaps related to this segment (or to item 33): "It may be well to criticize in detail the Categories of Kant."

235.9–14 Kant's. . . .] Peirce often relates his conception of categories to that of Kant, as in the early "Logic Chapter I" (W1:351–56). For Kant's doctrine of categories, see his *Critique,* especially bk. 1 of "Transcendental Analytic."

235.12 "Functions of Judgment"] Ibid., A70 (B95).

237.5–8 Hegel. . . .] See his *Encyclopädie,* §§40–44.

237.23–239.30 In Segment 2, Peirce discusses his own method of deriving the categories (prescission) and presents his three categories, particularly the first.

237.24–31 This outline closely matches the plan followed in item 35, and already announces the table of contents of the 1887–88 "A Guess at the Riddle."

238.13 I call. . . .] Prescission is treated in various places in Peirce's writings; for earlier treatments, see "Searching for the Categories" (W1:518–19) and "New List of Categories" (W2:50–51).

238n.1–2 See W2:49–59.

239.31–240.16 Segment 3 is a short treatment of the application of the three categories to formal logic.

240.17–241.27 In Segment 4, Peirce argues *against* the use of dichotomic distinctions in logic and *for* trichotomic distinctions, and he applies the latter to different kinds of propositions.

242–47 Not long after item 34, Peirce undertook a more systematic essay on the categories, setting forth the exact extent of their universality. Item 35 follows closely the outline presented in the first paragraph of Segment 2 of item 34: it begins by detecting the categories in logic (which takes a good half of the text) and then turns to the categories in psychology and in physiology. The last (incomplete) paragraph suggests that Peirce wrote a few more pages about the categories as they appear in the physical universe. Along with Segment 2 of item 34, item 35 is a clear anticipation of Peirce's 1886 "One, Two, Three" project and his soon-to-follow "A Guess at the Riddle."

242.5–7 Kant. . . .] In his *Critique,* Kant remarks that "in view of the fact that all *a priori* division of concepts must be by dichotomy, it is significant that in each class the number of categories is always the same, namely three. Further, it may be observed that the third category in each class always arises from the combination of the second category with the first" (B110). In a footnote at the end of the Introduction to his *Critique of Judgment,* he explains that threefold divisions in pure Philosophy are due to the "nature of the thing": "If there is to be an *a priori* division it must be either *analytical,* . . . which is always twofold . . . ; or it is synthetical. And if in this latter case it is to be derived from *a priori concepts* . . . the division must necessarily be trichotomy."

243.5–7 De Morgan. . . .] See "On the Structure of the Syllogism [No. I]," p. 381.

243.10–13 I have. . . .] See "Deduction, Induction, and Hypothesis," W3:323–38.

243.16–18 One . . . ;] For more on the classification of signs, see pp. 162–66 and 224, and also "Consequences of Four Incapacities" (W2:224ff.), "On Representations" (W3:62ff.), and "Methods of Reasoning" (W4:251).

244.7–8 As the. . . .] Peirce refers here to the Scholastic logic and the distinction between first and second intentions; see also his "Intention" in Baldwin's *Dictionary of Philosophy and Psychology* (New York: Macmillan and Co., 1901–1902).

245.25 An. . . .] See "Logic of Relatives" (W2:365) and note 222.33–35.

245.32–34 the introspective. . . .] The reference may be to the "faculty psychology" of such philosophers as Christian Wolff and William Hamilton. But Peirce often attributed influential triadic theories of mind to Kant and Johann Nicolaus Tetens, as well as to Plato and others.

248–251 Exactly how item 36 fits in with the philosophical discussion undertaken in item 35 can best be understood by consulting Peirce's "The Architecture of Theories" (especially CP 6.27–28); see also "A Guess at the Riddle" (especially CP 1.362).

248.6–249.13 In such. . . .] In the *Century Dictionary,* for which Peirce wrote the entire set of definitions for the word "absolute," the following mathematical definition is given:

> In math., a locus whose projective relation to any two elements may be considered as constituting the metrical relation of these elements to one another. All measurement is made by successive superpositions of a unit upon parts of the quantity to be measured. Now, in all shiftings of the standard of measurement, if this be supposed to be rigidly connected with an unlimited continuum superposed upon that in which lies the measured quantity, there will be a certain locus which will always continue unmoved, and to which, therefore, the scale of measurement can never be applied. This is the absolute. In order to establish a system of measurement along a line, we first put a scale of numbers on the line in such a manner that to every point of the line corresponds one number, and to every number one point. If then we take any second scale of numbers related in this manner to the points of the line, to any number, x, of the first scale, will correspond just one number, y, of the second. If this correspondence extends to imaginary points, x and y will be connected by an equation linear in x and linear in y, which may be written thus: $xy + ax + by + c = 0$. The origin of the scale will thus be shifted from $x = 0$ to $y = 0$ or $x = -c/a$. In this shifting, two points of the scale remain unmoved, namely, those which satisfy the equation $x^2 + (a + b)x + c = 0$. This pair of points, which may be really distinct, coincident, or imaginary, constitute the absolute. For a plane, the absolute is a curve of the second order and second class. For three-dimensional space it is a quadric surface. For the ordinary system of measurement in space, producing the Euclidean geometry, the absolute consists of two coincident planes joined along an imaginary circle, which circle is itself usually termed the absolute.

249.20–34 As an. . . .] See item 34, p. 241.

251.4–6 It was . . . ,] See Cayley's "A Sixth Memoir," pp. 82 and 84.

251.10–16 If. . . .] The following parallel passage appears in "A Guess at the Riddle" (CP 1.362):

> If you think the measurable is all there is, and deny it any definite tendency whence or whither, then you are considering the pair of points that makes the absolute to be imaginary and are an Epicurean. If you hold that there is a definite drift to the course of nature as a whole, but yet believe its absolute end is nothing but the Nirwana from which it set out, you make the two points of the absolute to be coincident, and are a pessimist. But if your creed is that the whole universe is approaching in the infinitely distant future a state having a general character different from that toward which we look back in the infinitely distant past, you make the absolute to consist in two distinct real points and are an evolutionist.

252–53 Peirce treated the degeneracy of the categories elsewhere, including his 1903 lectures on pragmatism and his letters to Lady Welby. For further treatment in the present volume, see p. 301 and the relevant notes.

254.4–6 *Common Sense* was edited and completed by Karl Pearson, whose initials are given at 255.21 and 256.17.

254.9–10 "The First. . . ."] Clifford, *Common Sense,* p. v.

254.17–18 "we cannot. . . ."] Ibid., p. 65.

255.1–2 he defines. . . .] Ibid., pp. 48–49.

255.15–20 Ibid., p. 50. In the original, the second sentence begins: "We can see it." Peirce has reversed the paragraphs.

255.21–23 "we are. . . ."] Ibid., p. 48; the original reads: "We are, I think, compelled to consider the surface of the geometer as an 'idea or imaginary conception,' drawn from the *apparent* (not real) boundaries of physical objects."

255.25–27 The. . . .] See Abbot's, "Philosophy of Space and Time," pp. 91 and 115. Francis Ellingwood Abbot (1836–1903) was a Harvard classmate of Peirce and member of the Cambridge Metaphysical Club. He is often remembered as the object of Royce's apparent jealousy and hatred after Abbot had filled in for Royce at Harvard in 1888.

255.37–38 Kant. . . .] The reference is to Kant's conception of space as the a priori form of intuition; Leibniz considered space as ideal and abstract and derived from enduring concrete things.

256.7 the . . . Absolute] See note 248.6–249.13.

257.7 Perrin] Raymond St. James Perrin (1849–1915), American philosopher, writer, and manufacturing executive.

258–59 There is no clear indication by T. K. Abbott that the *Logic* is largely the work of Kant's former student, Gottlob Benjamin Jäsche (who became a professor at the University of Dorpat). In "On the Textual Authenticity of Kant's *Logic*" (*History and Philosophy of Logic* 9 [1988]: 193–203), T. Boswell has shown that the notes in Kant's copy of Georg Friedrich Meier's *Auszug aus der Vernunftlehre* (Halle, 1752), from which Jäsche took the annotations that form the basis of his edition of Kant's *Logic*, are scattered and often fragmentary. Boswell has used the philological evidence to conclude that, although Kant's annotations form the basis of the work, the *Logic* is largely Jäsche's rather than Kant's. Coleridge's "Notes" are taken from his copy of John Richardson's translation of the *Logic* published in London in 1819.

258.7 Kant's. . . .] See item 33 (224.2–4), item 34 (236.9–12), and note 222.33–35.

258.19–20 general . . . existence] The reference is to Kant's famous argument that existence is not a predicate; see *Critique* A598–99 (B626–27).

260–61 Peirce's study of Fiske and Spencer and the brief review here may have stimulated his metaphysical speculations that culminated in his *Monist* series of the early 1890s, where he offered an account of his view of evolution, which includes the hypothesis that chance plays a substantive role within a larger teleological framework. John Fiske (1842–1901) was an American historian and evolutionary philosopher, and a part-time member of the Cambridge Metaphysical Club.

260.6 "Cosmic . . . theism"] Fiske, *Outlines*, p. 425; the original reads: "Cosmic Theism, in contrast with the Anthropomorphic Theism."

260.19 "wayside studies"] *Idea of God*, p. xxxi.

260.20–21 "Jesus. . . ."] Ibid., p. xxx.

260.26–28 according . . . elements,] The "other evolutionists" are Charles Darwin (for the variation factor) and, probably, the American geologist Clarence King (for the force factor). See King's 1877 address to the Sheffield Scientific School of Yale College (on its thirty-first anniversary) entitled "Catastrophism and the Evolution of Environment."

260.29–261.2 "The events. . . ."] *Idea of God*, p. 166.

262.5 Hardy] See "Flexure of Pendulum Supports," W4:515 and 526–28.

262.8–9 I use. . . .] At many stations there is no firm foundation upon which to mount a microscope in a sufficiently stable position to measure the flexure of a pendulum stand. For such cases Peirce introduced the method of using a noddy to measure the flexure which gives a correction, Peirce says, within two microns. The theory of the noddy is a special case of the theory of the double pendulum.

265.22 Both the pendulum and the inverted pendulum (noddy) obey the equation of motion for a simple harmonic oscillator. This equation is essentially $y = \mathrm{A} \sin \mathrm{k}x$. The period, T, is the time for the angle kx to run from 0 to 2.

$$\mathrm{kT} = 2\Pi$$

$$\text{so } \mathrm{T} = \frac{2\Pi}{\mathrm{k}}$$

In the case of the equation at 265.22, the period is

$$\mathrm{T} = \frac{2\Pi}{1/\sigma} \qquad \mathrm{T} = 2\Pi\sigma.$$

The wavelength is the same as the period of the trigonometric function representing the motion, therefore the wavelength is $\lambda = 2\Pi\sigma$. This inversion happens essentially because the restoring force in the case of a noddy is the elasticity (a constant) as opposed to gravity.

275–78 In their *Bestimmung der Absoluten Grösze der Schwerkraft zu Potsdam mit Reversionspendeln* (Berlin, 1906), Kühnen and Furtwängler refer to this paper by Peirce and give credit to him and Lorenzoni for having been the first to work on the problem of the flexure of the pendulum, but remark that their results were insufficient (Introduction, p. xiv). The amount of flexure of the pendulum staff is significant and must be corrected for in determinations of absolute gravity.

275n.14 *motivity*] As pointed out by Peirce in the *Century Dictionary,* this term was used by Locke to mean "the active power of moving" (*Essay,* bk. 2, ch. 23, §28).

279.5 your book] The reference is to *Scientific Theism,* which is reviewed in item 46 (pp. 285–89).

279.7 "universal . . . teleology"] Not found verbatim, but close to "The immanent life-principle of this cosmical organism is endocosmic and monistic teleology" (ibid., p. 202).

279.24–26 you draw. . . .] On relations of reason and real relations, see pp. 244 and 300 of the present volume, as well as notes 300.18–30 and 301.26–27. The distinction parallels Peirce's account of the first and second categories. A relation of reason may be a degenerate second; a real relation is an instance of genuine secondness.

281.12 182] 164.

281.29 Helmholtz] The reference is to his *Handbuch.*

281.39–282.23 The quotations are from Ockham's *Summa logicae.* The following translations are taken from Michael J. Loux, *Ockham's Theory of Terms* (Notre Dame, IN: University of Notre Dame Press, 1974).

281.39–282.5 Quidam. . . .] "Some hold that relation is not something outside the mind really and completely distinct from one or more absolute things. I think that Aristotle and the philosophers following him held this view. Others, however, claim that while relation is not an absolute thing in the way that a man or a donkey is, it is really distinct from one or more absolute things. Many theologians hold this view. I used to think that Aristotle held it also" (p. 158).

282.7–8 Secundum. . . .] "Nothing is to be placed in the genus of relation except those mental, vocal and written names" (p. 171).

282.9–13 Et sciendum. . . .] "It should be noted that the term 'opposite' signifies not only things (both those that are in the soul and those outside) but also signs of things. But things outside the soul, things that are not signs, can only be opposed as contraries; or, according to one view, they can also be opposed as relatives" (p. 115).

282.13–17 Relative. . . .] "Opposition in terms of relatives obtains among names which cannot be affirmed of the same thing with respect to the same thing. This holds true whether things outside the soul are opposed as relatives or not. It is not, however, because I deny that relations are things outside the mind that I speak of names as relatives. One could use the term apropos of things as well as names of things" (p. 120).

282.20–23 "All propositions, inferences, and mental terms are beings of the reason. Nonetheless, they really exist in nature and are beings more perfect and more real than any corporeal qualities" (p. 127).

283.18–26 In this condition Peirce is capturing the idea that is today expressed by saying that the positive integers are *well-ordered* (from Cantor's *wohlgeordnet*). This concept is one of the two keys to unlocking the employment of transfinite induction; the other is the claim that every set can be well-ordered, a claim that is equivalent to the Axiom of Choice.

285.15–286.10 Abbot, *Scientific Theism,* pp. 208–9.

285.15 some measure] The original has "some small measure."

285.16–17 intelligible:] The original has a semicolon.

285.30 life:] The original has a comma and dash.

286.8 Source,] There is no comma in the original.

286.25–26 The subtlety. . . .] Peirce refers here to *Novum Organum,* bk. 1, aph. X, p. 158: "The subtlety of Nature far exceeds the subtlety of sense and intellect: so that these fine meditations, and speculations, and reasonings of men are a sort of insanity, only there is no one at hand to remark it."

287.4 root and branch] Abbot, *Scientific Theism,* p. 79.

287.17–18 great distinct] There is a comma in the original.

287.19–21 "the affirmation. . . ."] The original reads: "But the relation must not be misconceived as a 'thing,' nor the affirmation of the objectivity of the relation as an affirmation that the relation is an entity apart from the things it relates."

287.33 individuals—] In the original, a comma precedes the dash.

287.38–39 "perceptive. . . ."] Abbot, *Scientific Theism,* p. 135.

288.16 noumenism] Ibid., p. 109.

292.4 This is. . . .] The relative inexactitude of axioms is an important recurrent topic in Peirce from this time onward. See CP 1.354 and 400–402 ("A Guess at the Riddle"), CP 6.29–30 and 1.130–32 ("The Architecture of Theories"), and CP 1.137.

293.18 We. . . .] This was the hypothesis Peirce had proposed in his January 1884 Johns Hopkins Metaphysical Club lecture entitled "Design and Chance" (W4:544–54). See also "The Doctrine of Necessity Examined" (CP 6.35–65) and "The Logic of Continuity" (CP 6.185–213).

293.37 Here . . . hypothesis] The hypothesis set out in the previous paragraph is an early statement of Peirce's guess at the riddle, and it serves as a preview of his evolutionary cosmology that will be further developed in his five *Monist* papers of 1891–93.

294.1 another chapter] The phrase shows that this paper was to be part of a book on the three categories. The topic discussed here will eventually be treated in chapter 7 of "A Guess at the Riddle."

295.27 This is a statement Peirce always qualified, as he did in Segment 4 of item 34 (pp. 240–41) and in the table of contents of "A Guess at the Riddle" (Ch. 2, point 4, in CP 1.354). The division of propositions Peirce had in mind here was probably the "descriptive conception" (item 30, p. 166) that divides all propositions into those that are true and those that are false. According to his "metric conception," a clear-cut dual division of propositions cannot be assured.

296.1 Baconian Induction] Francis Bacon's conception of induction relies on the compilation of three tables: the table of presence (or affirmation), which is a list of all known instances of a phenomenon that share a same character; the table of absence (or negation), which lists all known cases where the investigated character is absent; and the table of degrees (or comparison), which calls for the study of variation in different phenomena to find certain correlations.

296.9–10 Spencer. . . .] The reference may be to Spencer's *First Principles* or *Principles of Biology.*

298.11 Kantian division] See *Critique of Judgment,* sec. 3 of the Introduction.

298.13 Association philosophers] Locke, Berkeley, and Hume laid the foundations for the two doctrines of sensationism and associationism. David Hartley and Thomas Brown in the eighteenth century, and James Mill, John Stuart Mill, and Alexander Bain in the nineteenth century, made important contributions to associationism.

298.16 Aristotle, *De Anima,* bk. I.

299.25–26 manifold of sense] Kant, *Critique* A77–79 (B102–5).

300.9–12 The quotation is from *The Merchant of Venice,* 2.6.15–18.

300.18–30 MS 577 contains a few relatively rough pages in which Peirce discusses the degeneracy of the categories. About degenerate secondness, he wrote the following:

Besides genuine secondness, there is a degenerate order of secondness, which does not exist as such, but is only so conceived. Degenerate seconds are of two families; for either there are two independent objects not coupled together in existence but only by the mind, or there is but one object which is second to itself or to which an abstraction is made a second. The medieval logicians (following a hint of Aristotle) distinguished between *real relations* and *relations of reason.* A real relation subsists in virtue of a fact which would be totally impossible were either of the related objects destroyed, while a relation of reason subsists in virtue of two facts one of which only would disappear on the annihilation of either of the relates. Such are all resemblances. Any two objects in nature resemble one another, and in themselves just as much as any other two; it is only in reference to our senses and needs that one resemblance counts for more than another. Rumford and Franklin were related to one another by virtue of both being Americans; but either would have been just as much an American if the other had never lived. On the other hand, the fact that Cain killed Abel is not a mere aggregate of the two facts that Cain killed somebody and that Abel was killed. The first kind of degenerate secondness then is the relation of reason.

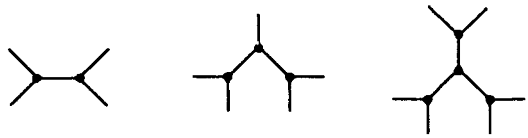
Identity is the relation that everything bears to itself. Lucullus dines with Lucullus. We speak of allurements and motives in the language of forces, as though a man were under compulsion from within. So with the voice of conscience. We observe ourselves and our own feelings, by a reflective sense. An echo is my own voice coming back to answer itself. We may call these Internal seconds. There is, in fact, nothing but a first, which the mind finds it more or less convenient to regard as second to itself. In like manner, we may take the abstract quality of any object,— say one which is blue, and regard the blueness as a second object related in a certain way to the subject or first.

300.29 old logicians] See note 244.7–8.

300.31–38 By. . . .] The following is an alternate rendition of this passage, from MS 577, in which Peirce extends his discussion to chemical valency. (Note 1 is to Sylvester's "On an Application of the New Atomic Theory"; see also "Logic of Plural Relatives," W4:210–11. For note 2, see "Logic of Relatives," W2:425.)

The third is the medium which brings the more absolute first and second into relation to one another. The end is second, the means third. A gift is a third between the giver and the receiver. Anyone of the three may be taken as third, but the gift is more appropriately so considered, while the active giver is first and the passive receiver second. In some cases, however, there is no such ground of choice.

A road that forks is a third to its first and second branches; for by running into each of these it makes them run into one another; and the three termini are by the same fact made first, second, and third to one another. The termini of a straight road are first and second; A place without a road is first. Chemists distinguish atoms as *monads, dyads, triads, tetrads, pentads,* and *hexads* according to the number of bonds or attachments by which they can be united to others. The bond of a monad is first, the two bonds of a dyad are first and second, the three bonds of a triad are first, second, and third, without any choice that we know of as to which shall be taken as third. Now note this: a road that never forks can have but two termini, but repeated forkings will produce *any number;* From monads and dyads alone only dyad groups at highest can be built, but from triads, we can form tetrads, pentads, hexads, etc indefinitely.[1]



The reason is that *three* involves the element of mediation not contained in *two;* but there is nothing in any higher degree of plurality essentially different from thirdness. Thus, Greek grammar is quite philosophical in distinguishing the singular, dual, and plural numbers.[2]

Position etc.

[1]The special fact about chemistry was remarked by Prof. Sylvester in 1878; but the general principle concerning relations, of which this is but an obvious application was fully explained in my Logic of Relatives, 1870, p 58.

[2]It is true, the dual is not indispensible; but neither is the plural. Algebra does not know it.

301.16 Thirds. . . .] See pp. 252–53. The following passage comes after this sentence in the alternate version in MS 577:

The first degree is where there is in the fact itself no thirdness or mediation but where there is true duality; the second degree is where there is not even genuine secondness in the fact itself. Each of these degrees of degeneracy exists in several different forms.

301.26–27 We. . . .] In MS 577, Peirce gives examples of two further types of thirds degenerate in the first degree, the *associating* and the *reflective* thirds:

We may next notice the case in which, instead of two independent facts of secondness, there is but one in which A and B are similarly related to the third C. An engineer and fireman are killed in one railway accident. The accident, which is here the third, by virtue of a real fact, brings the engineer and fireman into relation; yet the relation into which they are brought is a mere resemblance or relation of reason. In such a case we may call the medium an *associating third.*

Finally there is the case in which a thing is brought into relation to itself by the mediation of something else to which it stands in a real relation. Thus, a man looks at himself in a looking glass; and Lydia Languish used to write herself anonymous letters. We may call these cases of *reflective thirds.* Any dual relation may be regarded as involving such a third. If A loves B, then by the medium of B, A is a co-lover with himself. Only in some cases such a way of regarding the fact leads to some result, while in others it has no importance nor significance.

Finally, if A and B are in a dual relation to one another, the relation itself, taken abstractly as an object may be regarded as mediating between the first and second. If A loves B, B is second to A owing to the fact of loving as third.

301.27–29 How. . . .] The following explanation is found in MS 577:

Here there were two independent facts, first that the merchant threw away the date-stone, and second that the date stone struck the genie's son. Had the date-stone been aimed at the genie's son, the case would be different; for the fact of aiming could not take place without an actual aimer and an object aimed at. Nor can aiming take place without a weapon to aim; and an accidental thirdness plus a relation between the first and second will never build up an undegenerate or genuine mediation.

301.30–32 A third. . . .] See item 37.

301.37 We. . . .] In MS 577, Peirce gives an example of a third whose degree of degeneracy is not very clear.

> Very much of the same sort [of dual relations] are cases where a resemblance between a first and second is constituted by their real relations to a third. The diamond and corundum are alike in that both scratch glass.
> This example is second order.

And:

> Of thirds degenerate in the second degree, we may first notice a case which perhaps should be regarded as of the first degree. A diamond and a piece of corundum have this in common, that they will both scratch glass.

302.4–5 They. . . .] Peirce often compared his work with that of Hegel. See, for example, CP 5.79 and 6.218.

305.12–15 See note 300.9–12.

309–16 In a letter to Coast Survey Superintendent F. M. Thorn (received 8 April 1886), Peirce acknowledged having had his paper on "Reduction of Time Observations" returned for revision for publication in the 1885 *Report*, and he responded in some detail to an accompanying memorandum from C. A. Schott in which a number of objections had been raised against his adoption of a new method of reduction. Peirce briefly described his new method of reduction "as the treatment of the observed distances in the field of the telescope as the observed quantities instead of treating the times as the observed quantities." He then defended his method against Schott's claim that it yielded no appreciable advantage over the old method, and he concluded by recommending that Schott choose a test case to compare the relative merits of the different methods of reduction.

310.7 Waldo's] The reference is to the *Multiplication and Division Table* by Leonard Waldo (1853–1929), an American engineer.

310.10 according . . . Schott] Starting in 1854, Schott regularly published results of his geodetic research in the Coast Survey *Reports.* But here Peirce may be referring to results Schott mentioned in his memorandum on item 51 (see note 309–16).

317–18 See "Flexure of Pendulum Supports" (W4:515 and 526–28) and item 42 above (pp. 262–74).

319–22 F. R. Helmert refers to this paper in his *Beiträge zur Theorie des Reversionspendels* (Potsdam, 1898) p. 92n, where Peirce's results are said to be essentially the same as Helmert's, though they were reached by other means. Temperature corrections for Peirce pendulums nos. 3 and 4 are given in a letter to F. M. Thorn (10 November 1888).

323–71 The table of contents suggests a rather traditional treatment of standard logical topics; what Peirce actually does, however, is decidedly non-traditional. The announced chapter topics are used as a platform for discussions of some of the underlying bases of Peirce's (deductive) logical theory; of special note is the role of possible universes of discourse, to which chapters 2 and 3 devote considerable attention.

324.34 mathematicians] The early Greek mathematicians to whom Peirce alludes probably are pre-Euclidean.

325.3 He. . . .] Ramus imported into logic, not the dilemma properly speaking, but the "division by dichotomy," also known as the "dichotomy by contradiction." The Ramean Tree is an illustration of it: one starts from a given subject, divides it into two contradictory parts, applies the same division to each part, and so on. See p. 173n.2–4 above on the importance of dilemmatic reasoning.

326.7–328.17 The first draft of Chapter 1 shows that Peirce did not originally intend to dwell upon the topic of the association of ideas at any length, on account of its connection to psychology. He quickly changed his mind, split the chapter into two chapters and rewrote both. The paragraph preceding the split in the first draft reads as follows:

> We here drop this subject. A large volume might well be filled with what has been made out concerning our natural ways of thinking. This is an important branch of psychology. But the proper object of logic is not to consider how we naturally think nor how we cannot help thinking, but where we have a choice between thinking in different ways to find out what is the best way of thinking.

328.26–27 The. . . .] The following explanation comes after this sentence in the second draft of chapter 2:

> When I say that the premise, P, *causes* the conclusion, C, what I mean is, 1^{st}, that the judgment C follows after the judgment, P, and not only that, but, 2^{nd}, follows after it according to some principle, rule, or law (I need not know what law) which so acts within me that I should usually be led to a conclusion like C, if I were to hold to a belief like P.

329.1–2 *leading* . . . inference] See "Algebra of Logic," W4:168.

329.36 *Leges*. . . .] The original reads *Silent enim leges inter arma* (Cicero, *Pro Milone,* IV, 11): "When arms speak, the laws are silent."

329.37–38 There. . . .] Following this sentence, Peirce wrote the end of the first draft of Chapter II:

> My thoughts are more or less bound together, and this binding together of my thoughts constitutes some kind of an existence for me; but such a consideration is no longer the simple consequence of Descartes.

330.31–34 We. . . .] This represents a progression toward a more definite conception of possibility and possible worlds than that connected with the simple consequence. Elaboration of this conception is a principal thrust in the present section.

331.27–332.22 The diagrammatic treatment of logic here is suggestive of Peirce's later graphical treatments but is more akin to his entitative than to his existential graphs. Note the emphasis on inclusion of the antecedent in the consequent of a true hypothetical proposition. What Peirce says here brings to mind a treatment of entailment in terms of (empirically) *confirming evidence: P* entails *C* if and only if the set of (possible) evidence which confirms *P* is included in the set of (possible) evidence which confirms *C*. Peirce's account of "If *P* then *C*" is an account of strict implication and not of mere material implication.

332.16–22 The following passage comes from a set of three variant pages that are related to this part of the chapter:

> *[. . .]* presents what is possible; and we can see that in this cutting everything outside of the area of the truth of C is equally cut away. Thus, it is a positive perception and not a blind constraint which carries us from the premises to the conclusion of the *modus ponens*.
>
> This method of reasoning is then logically perfect whenever it can be applied; but when can it be applied, how can it increase our knowledge? The example of this inference given above supposes it to be used by a child: it is a mode of inference well suited to intellectual capacities and needs of children. The example also supposes it used in reference to the attainment of an end, and it is adapted to considerations leading to immediate action. Every man is in the habit of making resolutions. A resolution is a hypothetical judgment. "If so and so happens I will do so and so." When the antecedent does occur, he proceeds to put the consequent in execution, and the immediate act of will is deduced by an inference in the *modus ponens*.
>
> This is also the kind of reasoning required to enable us to obey any order or rule. The rule is "If such an event occurs, you are to act in a certain way." The obedient servant need do nothing till the event does happen, and then he has to act in that way. The same is the case where the order is to believe a certain thing, and the *modus ponens* therefore applies where we have to fram*[*e*]* our beliefs so as to be consistent with an infallible authority. It was the most usual mode of reasoning in the middle ages. It is the same where the problem is to be self-consistent, and therefore this kind of reasoning is useful in argumentation or dispute where the object is not the truth, but to lead an opponent to admit a predetermined *[. . .]*

335.10–20 Here and in the following, Peirce may be thought to be conflating—as he typically does in employing his "claw" symbol—a binary *relation* of propositions with a binary *operation* on propositions. Whether the resultant conflations of propositions used and propositions mentioned has a deleterious effect on Peirce's systems of logic is a disputable point.

337.22–24 See note 331.27–332.22.

337.29–30 *Nota . . . ipsius*] This is the logical principle that the predicate of the predicate is the predicate of the subject. In his *"Nota notae,"* in Baldwin's *Dictionary of Philosophy and Psychology* (New York: Macmillan, 1901–1902), Peirce traces the principle back to Aristotle, although the Latin phrase first occurred in Kant.

338.3 *Dictum . . . nullo*] This is the principle on which the classical theory of syllogism is based and which states that whatever can be said of a class, can be said of its members. It is often attributed to Aristotle but, as the principle of syllogism, is said to have been first recognized by Boethius.

341.1–343.25 The use of the "streamer" notation here may indicate that this part of Chapter IV was composed after Chapter VI was written. Peirce's notation (see also pp. 361–71 and item 55) bears a close resemblance to that developed by George Spencer-Brown in his *Laws of Form* (1972). There are interesting commonalities between Peirce's streamer-logic and Spencer-Brown's logical calculus, although Peirce's later existential graphs may be even closer to Spencer-Brown's system.

341.5–6 by. . . .] On page 337, Peirce had explained that by the illative relation he meant the relation of conclusion to premiss. Here he introduces the sign $-\!\!<$ to express the illative relation but immediately proceeds to use it as a sign of implication. (See note 335.10–20.)

341.7 This may be thought of as a Peircean version of what is known as

Polish notation. The location of the left tip of the "streamer" makes it a prefixive (and parenthesis-free) notation.

341.9–11 If we read $\overline{S \dashv}\, P$ as the strict implication, then we should read $\overline{S} \dashv P$ as what nowadays is usually written $\{S\} \vdash P$, and would be even more accurately written $\{\ulcorner S \urcorner\} \vdash \ulcorner P \urcorner$, where the turnstile is a metasymbol indicating that from the proposition S the proposition P is a logical consequence, given the deductive system at hand.

341.23 *F*, false] The proposition F, the logically false proposition, may of course be used in the typical fashion to define negation, and thus all the other standard connectives. Thus we can define

$$\sim P \text{ as } \overline{P \dashv}\, F$$
$$P \vee Q \text{ as } \overline{\overline{P \dashv}\, F \dashv}\, Q$$

and

$$P \,\&\, Q \text{ as } \overline{\overline{\overline{\overline{P \dashv}\, F \dashv}\, F \dashv}\, \overline{Q \dashv}\, F \dashv}\, F$$

etc.

Later in his career, Peirce called F "the pseudoproposition."

341.28–32 The parentheses are, of course, redundant. Note that in this paragraph Peirce distinguishes between $A \supset (B \supset C)$ and $(A \supset B) \supset C$, giving his notational equivalents of the respective latter-day Polish formulas *CaCbc* and *CCabc*.

341.33 $(\overline{A \dashv}\, B) \dashv C$] This should probably be written as $(\overline{\overline{A \dashv}\, B) \dashv}\, C$.

344.18–19 Locke. . . .] See his *Essay,* bk. 4, ch. 17, §4.

347.13–350.3 For the historical presentation of the mnemonics as given by Peter of Spain (and his successors), see Peirce's "Mnemonic Verses and Words" in Baldwin's *Dictionary* (in CP 2.584), where he remarks that the mnemonic verses were "older, perhaps very much older" than Petrus Hispanus; see also I. M. Bochenski, *A History of Formal Logic* (Notre Dame Univ. Press, 1961), pp. 210–17. Bochenski (pp. 210–11) cites evidence that the use of the mnemonics that gave the names to the moods of syllogisms pre-dates Peter of Spain, that "this is a very primitive technique, [. . . and] that the highly developed terminology of Peter of Spain had antecedents in Scholasticism itself," for example, in a manuscript discovered by L. Minio-Paluello which dates "at the latest from 1200."

347.15–18 They. . . .] *Summulae,* 4.17; the Ramus reference is to *Dialecticae libri duo.*

351.1–352.30 The doctrine comes from *De Sophisticis Elenchis* 165b23–25, the last book of Aristotle's *Topics,* which contains a theory of fallacious inferences on which the Scholastic doctrines of fallacies were based. Peirce's presentation follows closely the exposition in Petrus Hispanus's *Summulae* (tract. 7, "De fallaciis").

352.34–38 The quantification. . . .] The idea of quantification of the predicate first occurred in George Bentham's 1827 *Outline of a New System of Logic* (ch. 8, "On Propositions") before Bentham became a botanist. During the bitter controversy concerning the priority of the authorship of the idea of quantification, William Hamilton accused De Morgan of plagiarism without acknowledging that he himself encountered it first in Bentham's book. Peirce's references are to Hamilton's "New Analytic of Logical Forms" (*Lectures on Logic,* app. 5) and to Jevons's *Studies in Deductive Logic,* ch. 18.

353.4–5 it is. . . .] Aristotle, *De interpretatione* 17b13–16.

353.13–18 he adds. . . .] De Morgan, "On the Structure of the Syllogism [No. I]," p. 381.

353.21–354.25 Peirce's presentation of De Morgan's terminology follows closely "On the Syllogism, No. III," pp. 199–202.

354.34 He. . . .] See note 353.13–18.

355.24–26 But. . . ."] The following passage occurs after this sentence in a different version (Peirce's use of the streamer may indicate that parts of Chap. VI were written before Chap. V was finished):

To adapt the notation of the last chapter to this reasoning, we have to add the formula

$$\overline{\overline{\overline{S +}\, M +}\, P +}\, \overline{\overline{S +}\, P +}\, P$$

The diagram to prove this represents S and M as having intersecting areas of truth.

Now, to say that $\overline{\overline{S +}\, M +}\, P$ or If "If S then M," then P is represented by making the area of P cover the whole field of possibility except where S extends beyond M. But $\overline{S +}\, P$ is represented by making the area of S covered by P. When both these are true, the area of P covers the whole field of possibility.

355.29–30 "It is. . . ."] Aulus Gellius, *Noctium Atticarum,* bk. 5, ch. 11; the translation is Peirce's.

355.34–356.1 The. . . .] Ramist logic was a sine qua non for Protestant humanist philosophers who rejected Aristotelianism and medieval scholasticism during the sixteenth and seventeenth centuries.

356.1–2 Dilemmatic . . . logicians] See p. 173n.2–4.

356.5–6 Even. . . .] See Jevons's *Elementary Lessons,* ch. 19.

356.15–21 "Neither. . . ."] The quotation comes from *The Logic of Hegel,* §119, p. 192; the original reads "in heaven nor in earth." A different comment on this quotation, found on a variant page, reads as follows:

But this statement properly understood is as much as to say that the world of thought and of existence moves by the dilemma. For Hegel's idea is that the distinction of true and false is too crude and coarse to represent the true logic, that there is continuity everywhere, that the positive and negative are perpetually blending and producing a resultant, and that in fact this is the formula of all action of force. But in the parallelogram of motion, the resultant is that wherein the two component forces concur; the conclusion of the dilemma is in the same way the resultant of the two alternatives. The truth is that the whole matter of Hegel's logic consists [of] dilemma after dilemma, and the one thing which he always treats with implicit and absolute faith is the validity of dilemmatic reasoning, beyond which he can imagine nothing finer nor higher.

356.21–357.4 The movement. . . .] Peirce interprets here the transitions on the logical levels of Being, Essence, and Concept (ibid., §§79–83, 111).

The conception of opposing forces appears in Hegel's *Science of Logic* (vol. 1, bk. 2, §2, III.B) and *Phenomenology of Mind* (vol. 1, III.A). For another statement about Hegel's three stages of thought, see item 33 (230.8–23).

358.32–34 See p. 173n.2–4.

358.35–37 The. . . .] See note 355.35–356.1.

359.13–18 See a similar statement in item 33 (231.22–27).

359.19–21 Peirce wrote this first version of Chapter VI before he compiled the table of contents. He later decided to devote the chapter to a different topic, "Logical Algebra and Logic of Relatives," which is the title noted in the table of contents (which he soon replaced with "The Logical Algebra of Boole"). This early version of Chapter VI is included here on account of its internal interest and its having been dubbed an "Excursus" by Peirce himself.

359.22–360.40 The following passage is found in an earlier draft of this first version:

Mathematics and natural science had both gloriously begun their modern career, at the time when the clever Port Royal Logic,—*L'Art de Penser,*—of Arnault and Niccole was written. But the bearing of the new scientific movement on the art of thinking was not yet understood. Even the principles of Descartes were not deeply understood by the writers of that book. The preface tells how it came to be written. A young prince expressed a desire to study logic; whereupon one of the author's of the treatise undertook to write out in three days all the recommendations of good sense touching the right use of reason. We see in this that the impression of a cultivated scholar of that day was that the greater part of the traditional logic was rubbish; and that the great discoveries of his day had been made without it, and almost inspite of it. But it is narrated, that when he came to his task, he found it not by anymeans so easy as he had believed it to be; and, after all, the methods described in this treatise are nothing but the syllogism and the dilemma—a noticeable example of the difficulty of escaping from the trammels of old logical formulae, even after reason has begun to practice more efficient methods.

361.1–2 See note 359.19–21.

361.31–32 The following passage is found after these two lines in an earlier version. It indicates that Peirce's first intention was not to make this particular form of algebra (using the streamer notation) the central part of the chapter, probably because he wanted to turn to a discussion of the logic of relatives.

I relegate the development of this form of algebra to an appendix; although I recommend the examination of it, to the serious student, as an instructive lesson (of a negative kind) in mathematical method. Although this algebra is *logically* more perfect than any other, in that it has no unnecessary symbols and that it overtly exhibits every logical relation; yet it is found to be *mathematically* very bad, because its rules of transformation are not such as we can naturally remember and apply at a glance, and because its too explicit modes of statement are intricate on that very account.

361.34–362.6 This. . . .] Despite Peirce's disclaimers, his "blank" is really a second logical symbol in addition to the "streamer." In effect, Peirce quickly admits this by employing in place of the blank a small circle. The role of the blank, or the small circle, is the same that "the false" played earlier. What we have in the blank, then, is simply one guise in which "the pseudoproposition" appears.

362.1 An earlier draft shows that, at this point, Peirce thought of simplifying his notation system for the sake of the typist. The proposed new system is also the one used at the end of this item (371.26–32).

To facilitate the setting of the type, I will separate the streamer from the cross, and print the four propositions just given thus:

$$\bar{a} + b$$
$$\bar{a} + \bar{b} + c$$
$$\overline{\bar{a} + b} + c$$
$$\bar{a} +$$

The fundamental rules of this algebra are four, as follows:

I. If $\bar{a} + c$ and $\bar{b} + c$, then $\overline{\overline{\bar{a} +} + b} + c$.

II. If $\overline{\overline{\bar{a} +} + b} + c$, then $\bar{a} + c$ and $\bar{b} + c$.

III. If $\overline{\bar{a} + \overline{\bar{b} + c} +} +$, then $\overline{\bar{a} + b} + c$

IV. If $\overline{\bar{a} +} +$, then a.

But these rules are perplexing and not readily apprehended. To bring them into a familiar shape we may first introduce two abbreviations. Namely, instead of $\bar{a} +$, we may just as well write $\bar{a}$; and in place of $\overline{\bar{a} + \bar{b}} + +$ we will write simply *ab*. We next proceed to deduce from the above four rules, certain others more convenient in practice.

362.7 By . . . *copula*] See notes 341.5–6 and 335.10–20.

362.11–14 Let. . . .] Here the role of the pseudoproposition is played by the lowercase *n*. See note 361.34–362.6.

363.7 As the previous line is a version of *"modus ponens"* (or "detachment"), so is this line a *limited* version of the deduction theorem. See "Algebra of Logic," W4:163–69 and 173(2).

363.8–9 Any. . . .] This claim is a bit optimistic, unless implicit principles are at play. In the light of the preceding note, the most that Peirce's "two propositions" can guarantee is Positive Implication.

363.27–29 This is the Peircean version of "Semisubstitutivity of Implication." This principle also will underlie the permissions for erasure in evenly enclosed, and insertion in oddly enclosed, areas in Peirce's existential graphs, developed a decade later; see, for example, CP 4.394–594.

366.4–9 The. . . .] In Peirce's infinity symbol (∞) we have the germ of the blank spaces of his existential graphs. The symbol is used, in effect, to indicate "the true" or, equivalently, the denial of the pseudoproposition.

366n.1–5 Peirce frequently emphasized this point with respect to the not explicitly quantified conditional (or implication) statements of his algebra of logic. See, for example, the beginning of pt. III in item 30 (p. 177) and CP 3.448.

369.5 The following passage comes from a set of pages whose position in the manuscript cannot be identified with any certainty, but it seems appropriate to quote it here, after the ten rules of the algebra. Peirce's concluding reference to a calculus is parallel to 363.1–17.

By means of these rules, we can easily draw all qualitative conclusions concerning single individuals. For example, suppose we have given the premises $\overline{\overline{a}+b}+c$ and $\overline{a}+c$. The rule of substitution gives us $\overline{\overline{c}+b}+c$. Substitute the circle for *c* in this proposition and its denial is $\overline{\overline{\overline{O}+b}+O}+$ or $\overline{O}+b$, which is true. Consequently, from $\overline{\overline{c}+b}+c$ follows *c*, as the conclusion from the premises. Again, suppose we have the premises $\overline{\overline{\overline{a}+b}+m}+n$ and $\overline{\overline{\overline{b}+c}+l}+m$. By the rule of substitution, we can substitute $\overline{\overline{b}+c}+l$ for m, in the first premise. This gives $\overline{\overline{\overline{a}+b}+\overline{\overline{\overline{b}+c}+l}}+n$. But the formula of syllogism is $\overline{\overline{a}+b}+\overline{\overline{b}+c}+\overline{a}+c$. This gives $\overline{\overline{\overline{a}+c}+l}+\overline{\overline{a}+b}+\overline{\overline{b}+c}+l$. For suppose we have the premises $\overline{i}+\overline{j}+k$ and $\overline{k}+l$. The second shows that we can substitute *l* for *k* in the first, giving $\overline{i}+\overline{j}+l$. Hence, from $\overline{i}+\overline{j}+k$ follows $\overline{k}+\overline{l}+\overline{i}+\overline{j}+l$; and putting $i=\overline{a}+b$, $j=\overline{b}+c$, $k=\overline{a}+c$, we have the inference just drawn. We may therefore substitute $\overline{\overline{a}+c}+l$ for $\overline{\overline{a}+b}+\overline{\overline{b}+c}+l$ and thus obtain the conclusion $\overline{\overline{\overline{a}+c}+l}+n$.

Having thus converted our *language* into an *algebra*, we have now to go on to convert the *algebra* into a *calculus*. In other words we have to find a systematic and easy method of proceeding from any set of premises to their conclusion.

369 *[diagram]*] See Venn, *Symbolic Logic*, ch. 5, especially p. 106.

371.15–16 logical. . . .] The reference is to Jevons's *Pure Logic* and Mitchell's "On a New Algebra of Logic."

372.7 paper. . . .] The reference is to "On the Algebra of Logic," item 30.

373.7–13 Here Peirce goes to special pains to say that the notation $\overline{A}+B$ indicates what nowadays is called material implication.

374.31–36 Peirce seems to be making use of a finite version of the 1915 Löwenheim-Skolem theorem, namely that if F is satisfiable in every finite domain, then F is $\aleph_0$-satisfiable; that is, if F is n-satisfiable, then F is $(n+1)$-satisfiable. Peirce's proof is also similar to Löwenheim's: roughly, a product vanishes, that is, is κ-satisfiable, if its κ term (and so also each term $\lambda < \kappa$) vanishes.

376.4–5 This is a direct precursor of the rules of insertion and deletion in existential graphs (EG), although the rules in EG call for insertion in *oddly* enclosed and deletion in *evenly* enclosed areas (for example, CP 4.492, Rule 1). The roles of "even" and "odd" for "streamer" logic are opposite to the roles of these concepts in EG because whatever appears under an even number of streamers will appear in the graphs within an odd number of cuts, and whatever appears under an odd number of streamers will appear in the graphs within an even number of cuts. The outermost cut in EG does not have a correlate streamer.

378.10–16 This rule corresponds to the rules of iteration and deiteration in existential graphs (for example, CP 4.492, Rule 2).

379.5–9 The token . . . arbitrary.] For a parallel passage, see item 30 (162.16–20).

379.11–19 Indices. . . .] For a parallel passage, see item 30 (163.5–15).
380.2–6 Demonstrative. . . .] For a parallel passage, see item 30 (163.15–19).
381.14–16 The quotation, including the reference to Antoine Laurent Lavoisier, is a slightly emended version of a passage from the "Fixation of Belief" which continued, following "open," with "by manipulating real things instead of words and fancies" (W3:243–44).
382.4–5 Here Peirce makes his equal-sign stand for a meta-logical analogue of the "material equivalence" connective, unlike his usual practice of making it stand for logical equivalence. Nothing, however, in the document seems to hinge in any important way on this convention.
388–420 Item 57 is an early version of Peirce's contributions for the letter E for the *Century Dictionary,* many of them hastily prepared and some of them alphabetically out of order. The method—and basis for the *Century*—was to modify and expand definitions from the *Imperial Dictionary.* Peirce's definitions, only a small sample of his voluminous contributions to the *Century,* were written to be read against the corresponding definitions in the *Imperial.* There are occasional personal remarks about *Century* staff members or *Imperial* contributors.
388.4 See A] The reference is to Peirce's logic definition for "A," which has not been found in the manuscripts. In the *Century,* Peirce points out that "A," in logic, signifies the universal affirmative proposition, and he makes some historical remarks about the use of the vowels *A, E, I,* and *O* in traditional logic (and about the use of letters in mathematics).
388.5–6 Under 2 . . . :] The reference is to the *Imperial:*

> 2. The terraqueous globe which we inhabit. It is one of the planets, and the third in order from the sun, its orbit embracing those of Mercury and Venus, but being within the orbits of all the other planets. The earth is endowed with two principal motions: first, a motion round its axis, from west to east, in twenty-four hours; and secondly, a motion of revolution round the sun. It is the first of these motions which produces the phenomena of day and night, and the apparent diurnal revolution of the heavenly bodies. The time in which the earth's rotation is performed is measured by the interval which elapses between two transits of the same fixed star over the meridian of any place, and this interval is always precisely the same. It is called a sidereal day, and forms a perfectly uniform measure of time. The revolution of the earth about the sun is performed in an elliptic orbit, having the sun in one of the foci, and its mean distance from the sun, as calculated by Mr. Hind from Leverrier's determination of the solar parallax, is 91,328,600 miles. The time in which the earth performs a revolution in its orbit with respect to the fixed stars is 365 days, 6 hours, 9 minutes, 9.6 seconds. This is called the sidereal year. (See YEAR.) The plane which contains the earth's orbit is called the ecliptic. The earth's axis is inclined to this plane in an angle of 66°32′4″, whence the earth's equator is inclined to the ecliptic in an angle of 23°27′56″. This inclination, which is called the obliquity of the ecliptic, gives rise to the phenomena of the seasons. The figure of the earth is that of an oblate spheroid of revolution, the polar axis being to the equatorial diameter in the ratio of 301 to 302. The equatorial diameter is nearly 7925 English miles, the polar diameter about 7898 miles, and the mean diameter 7912 miles. Two-thirds of the earth's surface are covered with water; its mass compared with that of the sun is nearly as 1 to 355,000; its mean density is to that of water as $5\frac{1}{2}$ to 1.

388.20 For . . . :] The reference is to the *Imperial:*

Easter is the first Sunday after the full moon which happens upon or next after the 21st of March; and if the full moon happens upon a Sunday, Easter day is the Sunday after; but properly speaking, for the "full moon" in the above the "fourteenth day of the moon" should be substituted.

389.1 For . . . :] The reference is to the *Imperial:*

It consists of spirits of wine flavoured by a few drops of different essential oils blended so as to yield a fine fragrant scent.

389.4 Note. . . .] The phrase "on the ebb" does not appear in the *Imperial.*

389.5–8 ECCEITY] Peirce provides a fuller definition under "Haecceity," to be published in W6.

389.20 For . . . :] The reference is to the *Imperial:* "1. In *anc. astron.* a circle the centre of which did not coincide with that of the earth."

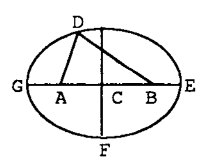
389.23–24 The reference is to the *Imperial:*

Thither his course he bends
Through the calm firmament; but up or down,
By centre or *eccentric,* hard to tell. *Milton.*

3. *[. . .]* (*b*) A circle described about the centre of an elliptical orbit, with half the major axis for radius.

389.25 For . . . :] The reference is to the *Imperial:*

2. In *astron.* the distance of the centre of a planet's orbit from the centre of the sun; that is, the distance between the centre of an ellipse and its focus. Thus in the ellipse DEFG, of which A and B are the foci, and C the centre, AC or BC is the *eccentricity.*



390.2 Ferguson] James Ferguson.

390.3 Add the words] The reference is to the *Imperial:*

Selecting; choosing; not original nor following any one model or leader, but choosing at will from the doctrines, works, &c., of others; specifically applied to certain philosophers of antiquity who did not attach themselves to any particular sect, but selected from the opinions and principles of each what they thought solid and good.

390.5 For . . . :] The reference is to the *Imperial:*

1. A great circle of the sphere supposed to be drawn through the middle of the zodiac, making an angle with the equinoctial of about 23°27′, which is the sun's greatest declination. The ecliptic is the apparent path of the sun, but as in reality it is the earth which moves, the ecliptic is the path or way among the fixed stars which the earth in its orbit appears to describe to an eye placed in the sun. The angle of inclination of the equator and ecliptic is called the obliquity of the ecliptic. It has been subject to a small irregular diminution since the time of the earliest observations on record. In 1839 it was 23°27′46″. Its mean diminution per century is about 48″.

390.11 The reference is to the *Imperial:*

> *Plane of the ecliptic,* an imaginary plane which passes through the ecliptic, and is indefinitely extended. In this plane the earth's orbit is situated.

390.12 For . . . :] The reference is to the *Imperial:*

> A straining; an exertion of strength or power, whether physical or mental; endeavour; strenuous exertion to accomplish an object; as, the army, by great *efforts,* scaled the walls; distinction in science is gained by continued *efforts* of the mind.
>
> There certainly is a kind of moral excellence implied in the renunciation of all *effort* after display. *Abp. Whately.*

390.17 Effort. . . .] The reference is to Thomas Sydenham who, in the Hippocratic tradition, treated the patient as a whole and regarded disease as "an effort of Nature, who strives with might and main to restore the health of the patient by the elimination of the morbific matter" (*Medical Observations,* p. 29).

390.23–32 *Dombey and Son,* ch. 1, p. 5. In the original, "beg" is italicized.

390.33 de Gasparis] Annibale de Gasparis (1819–1892), Italian astronomer.

390.36–391.3 Retain. . . .] The reference is to the *Imperial:*

> The *ego,* as the subject of thought and knowledge, is now commonly styled by philosophers the *subject,* and *subjective* is a familiar expression for what pertains to the mind or thinking power. In contrast and correlation to these, the terms *object* and *objective* are now in use to denote the *non-ego,* its affections and properties, and, in general, the really existent as opposed to the ideally known. *Reid.*

Fleming's *Vocabulary of Philosophy* cites *Works of Reid,* without mentioning William Hamilton, the editor, who supplemented the volume with interpretive essays, and is the author of the quoted passage; see *The Works of Thomas Reid,* Note B: "On Presentative and Representative Knowledge," p. 806.

391.2 Fleming] William Fleming (1791–1866) wrote a manual of moral philosophy, a book on political economy, and the *Vocabulary.*

391.6–7 Put . . . :] The reference is to the *Imperial:*

> 1. In *philos.* the opinion of one who thinks everything uncertain except his own existence; the doctrine which refers the elements of all knowledge to the phenomena of personal existence; subjective idealism. See IDEALISM.—2. A passionate love of self, leading one to refer all things to one's self, and to judge of everything by its relation to one's interests or importance; egotism; selfishness.

391.12 Aegina was discovered by Josef Stephan (1835–1893), an Austrian physicist.

391.13 Coggia] Jean Eugène Coggia (b. 1849), a French astronomer.

391.20 To . . . :] The reference is to the *Imperial:* "2. In *astron.* the passing of an inferior planet from the disc of the sun in a transit."

391.22 To . . . :] The reference is to the *Imperial:* "1. One of the cardinal numbers."

391.24–25 Add. . . .] The following is the definition from the *Imperial:*

"*Elaborative faculty,* in *metaph.* the intellectual power of discerning relations and of viewing objects by means of or in relations; the understanding of the German philosophers; the discursive faculty; thought." See Hamilton's *Lectures on Metaphysics,* p. 276.

391.26 Strike. . . .] The reference is to the *Imperial:* "The measure of the elastic force of any substance is called its *modulus of elasticity.*"

391.27 The reference is to the *Imperial:*

> *Elastic curve,* a curve formed by an elastic blade, fixed horizontally by one of its extremities in a vertical plane, and loaded at the other extremity. The loaded end by its gravity bends the blade into a curve.

391.29 See. . . .] The figure is not in the manuscript.

392.3 Under . . . :] The reference is to the *Imperial:* "1. The quality of being elastic; the inherent property in bodies by which they recover their former figure or state, after external pressure, tension, or distortion."

392.8 Young] Thomas Young (1773–1829) was an English physician, physicist, and Egyptologist.

392.17 Fresnel] Augustin Jean Fresnel.

392.21 For . . . :] The reference is to the *Imperial:* "1. The act of choosing; choice; the act of selecting one or more from others."

392.24–25 The reference is to the *Imperial:*

> 3. Power of choosing or selecting; choice; voluntary preference; free-will; liberty to choose or act; as, it is at his *election* to accept or refuse.
>
> Nor headlong carried by the stream of will,
> Nor by his own *election* led to ill. *Daniel.*

392.26 Add] The reference is to the *Imperial:*

> Of or pertaining to Elea (L. *Velia*), a town of Magna Græcia; specifically, an epithet given to a sect of philosophers that originated in Elea. The founder of the school was Xenophanes.

392.28 ELECTRA] A fallacy named for a character in Sophocles' play *Electra.* Another variety of the same sophism can be found in Aristotle, *De Sophisticis Elenchis* 179a26–35; Diogenes Laertius attributes "Electra" to Eubulides. See H. Aldrich, *Artis logicae rudimenta,* ed. by H. L. Mansel (London: Rivingtons, 1862).

392.33 Peters] Christian Henry Frederick Peters.

392.34–35 Strike. . . ."] The reference is to the *Imperial:* "Called also *Schultze's Chronograph.*"

393.10–16 See *Metaphysics* 4.3.1014a26–4.1015a13.

393.24–26 The following. . . .] The reference is to the periodic law of Dmitry Ivanovich Mendeleyev. By June 1869, Peirce had published a pre-Mendeleyev table of the elements (see "Pairing of the Elements," W2:282–84, and the relevant notes).

395.6–8 *The Tempest,* 5.1.316–18.

395.9–10 *Othello,* 3.3.464–65.

395.15–17 *The Merry Wives of Windsor,* 4.2.180–86.

395.18 *Twelfth Night,* 3.4.136–37.
395.19–20 There's. . . .] *Much Ado about Nothing,* 2.1.357–58.
395.34 Meanings . . . :] The reference is to the *Imperial:*

> 2. Arising from first principles; natural. *"Elemental repugnancy." Sir T. Browne.*— 3. Relating to elements or first principles; simple; elementary. *"Elemental* knowledge." *Burke.*

396.6–7 *Arte of Logicke,* bk. 6, ch. 2; in the original, "elench" and "syllogism" are capitalized, and the former is followed by "(sayeth hee)" and the latter ends with "e."
396.8–18 Grote, *Aristotle,* 2:376–77. In the original, "sophistical," "elenchus," "refutation," "syllogism" and "pseudo" are capitalized, and "conclusion" is followed by "contradictory to or refutative of some enunciated."
396.20 For . . . :] The reference is to the *Imperial:*

> 7. In *trigonometrical surv.* height; altitude; height above the surface of the earth; angular height, or angle of elevation. The angle of elevation of any object is the angle formed by two straight lines drawn from the observer's eye, the one to the top of the object and the other parallel to the horizon, both lines being in the same vertical plane.

396.25 For . . . :] The reference is to the *Imperial:* "Ten and one added; as, *eleven* men."
396.27–30 The first *Imperial* definition of "eleven" as noun was: "The sum of ten and one." Peirce's remark refers to his suggested changes to the definition of "eleven" in both its adjectival and nominal forms.
396.31–36 The definition given here is essentially Sylvester's. Etienne Bézout (1730–1783) in "Sur le degré des équations résultantes de l'évanouissement des inconnues" (1764), systematized the process of determining the signs of the terms of a determinant. He showed that for n-homogeneous linear equations in n-unknowns, the vanishing of the determinant of the coefficients (that is, the vanishing of the resultant) is the condition required for the existence of non-zero solutions.

Sylvester's so-called "dialytic" method of eliminating the variable is in fact equivalent to Bézout's. Elimination (of an unknown) from a set of simultaneous equations is equivalent to deriving another set of equations from the original set in such a way that the new set does not contain the unknown to be eliminated but which is still satisfied by any values of the remaining unknowns which satisfy the original equations.
397.32 Add] The only special definition under "ellipsoid" in the *Imperial* is for "ellipsoid of revolution."
397.36 Poinsot] Louis Poinsot.
398.4–5 Strike . . . Add] The reference is to the *Imperial:* "1. Pertaining to an ellipse; having the form of an ellipse; oblong, with rounded ends."
398.28–29 Elliptic . . . :] Legendre integrals (also called improper or incomplete elliptic integrals) of the first, second, and third kind were first presented by Adrien-Marie Legendre (1752–1833) in his *Traité des fonctions elliptiques* (Paris, 1825–26).

399.1–2 For . . . above] The reference is to the *Imperial:*

> The quality of being elliptical; deviation from the form of a circle or sphere; specifically, in reference to the figure of the earth, the difference between the equatorial and polar semi-diameters divided by the equatorial; as, the *ellipticity* of the earth is $\frac{1}{293.153}$.

399.5 Under 5 add] The reference is to the *Imperial:*

> 5. In *astron.* the angular distance of a planet from the sun, as it appears to the eye of a spectator on the earth; apparent departure of a planet from the sun in its orbit; as, the *elongation* of Venus or Mercury.

399.7 Palisa] Johann Palisa (1848–1925), German astronomer.
399.13–14 "On an Application of the New Atomic Theory," p. 82; Henri Victor Regnault investigated oxidation as substitution.
399.15 For . . . :] The reference is to the *Imperial:* "Issuing from another."

> "*Emanative* effects." *Glanville.* [Rare.]

399.18–22 See *Monitio logica,* bk. 1, ch. 17, p. 58. The original reads:

> It *sometimes happens that a Cause causes the Effect* by the Mediation of some Action which is said to be its Causality in causing. Such a Cause we call Active; sometimes by its own *Existence* without any Causality distinct from its Existence; and this by some is *call'd Emanative:* Which Word tho' feign'd with Repugnance to the Analogy of the *Latin* Tongue, yet is it to be used upon this Occasion till a more Convenient can be found out.

399.29–30 "On Affirmation and Negation—on Propositional Forms—on Breadth and Depth—on Syllogistic, and Syllogistic Notation," in *Discussions on Philosophy and Literature,* p. 622; in the original, "contrary" is italicized.
400.1 For . . . :] The reference is to the *Imperial:* "Brought into act; brought from possibility into real existence; open; evident."

> "The internal *elicit* act of the will." *South.*

400.3 The. . . .] The expressions "elicit" and "imperate" are commonly used to distinguish two ways in which an action can be voluntary. Aquinas makes this distinction in *Summa theologica,* 1st pt. of 2nd pt., qu. 6, art. 4:

> The act of the will is twofold: one is its immediate act, as it were, elicited by it *[elicitur],* namely *to will;* the other is an act of the will commanded by it *[imperatur],* and put into execution by means of some other power: *e.g., to walk* and *to speak,* which are commanded by the will to be executed by means of the power of locomotion.

400.4 *Whole Works,* vol. IX, bk. II, ch. III, rule VI; the original reads: "Thus to give alms is a proper and elicit act of charity."
400.5–6 The reference is to the entries "Elicit" in Fleming's *Vocabulary of Philosophy* and "Elicitus" in Chauvin's *Lexicon Philosophicum.*
400.7–8 *Interest Deposed,* p. 10. The original reads: "The Schooles dispute

whether in moralls the external Action superadds anything of good or evill to the internall elicit acts of the will."

400.9 Add . . . :] The reference is to the *Imperial:*

> The act of eliciting; the act of drawing out. *Bp. Bramhall.*

400.10–12 *A Defence,* p. 140; in the original, the words "elicitation," "drawing," "intend," and "mention" are followed by commas, and the first two are in quotation marks; "act" is followed by a semicolon and "schools" is capitalized.

400.13–15 The . . . :] The reference is to the *Imperial:*

> [L. *emotio,* from *emoveo, emotum-e,* out, up, and *moveo,* to move.] A moving of the mind or soul; any excitement of sensibility; a state of excited feeling of any kind; specifically, in *mental science,* one of the threefold divisions of the human mind, the other two being *volition* and *intellect.* There are three kinds of emotion: pleasure, pain, and an excitement that partakes of neither, as wonder or astonishment. Pleasurable emotions have a healthy physical effect, and those of pain an unhealthy one. Every strong feeling has a certain outward expression. Under violent emotion the whole muscles of the body may be affected, but in less extreme cases the expression is confined to the three centres of movement of the face—the mouth, eyes, and nose, the former being the most expressive. The voice is also instinctively affected.

400.24 See vol. 1 of his *Collected Works,* note 2, pp. 396–97. The passage reads:

> There is another word to which Madame de Staël, and other writers on the German philosophy, annex an idea peculiar to themselves; I mean the word *experimental* or *empirical.* This epithet is often used by them to distinguish what they call the philosophy of Sensations, from that of Plato and of Leibnitz. It is accordingly generally, if not always, employed by them in an unfavourable sense. In this country on the contrary, the experimental or inductive philosophy of the human mind denotes those speculations concerning mind, which, rejecting all hypothetical theories, rest solely on phenomena for which we have the evidence of consciousness. It is applied to the philosophy of Reid, and to all that is truly valuable in the metaphysical works of Descartes, Locke, Berkeley, and Hume.
>
> Nor are the words, *experimental* and *empirical,* by any means synonymous in our language. The latter word is now almost exclusively appropriated to the practice of Medicine; and when so understood always implies a rash and unphilosophical use of Experience. "The appellation Empiric," says the late Dr. John Gregory, "is generally applied to one who, from observing the effects of a remedy in one case of a disease, applies it to all the various cases of that distemper." The same remark may be extended to the word *Empirique* in the French language, which is very nearly synonymous with *Charlatan.* In consequence of this abuse of terms, the epithet *experimental,* as well as *empirical,* is seldom applied by foreign writers to the philosophy of Locke, without being intended to convey a censure.

400.27–29 See *Critique* A220 (B267).

400.30 Put. . . .] The reference is to the *Imperial:*

> 1. The act of attempting to equal or excel, in qualities or actions; rivalry; desire of superiority, attended with effort to attain it; ambition to equal or excel.

401.1 Under . . . :] The reference is to the *Imperial:* "1. To meet face to face; particularly, to meet suddenly or unexpectedly; as, I *encountered* him just as I was turning the corner."

401.9–10 *Monitio logica,* bk. 1, ch. 18, p. 68. The original reads:

> *Ax.* 3. The End of *which,* is that which the Efficient desires.
> *Ax.* 4. *For* which, for whose sake, or for which, the Efficient desires such an End.

401.20 Strike . . . :] The reference is to the *Imperial:*

> That branch of science which investigates the laws regulating physical or mechanical forces, as opposed to vital. The whole range of physical phenomena thus forms the subject of its consideration.

401.24 *Hermes,* bk. 1, ch. 9, p. 173; in the original, "verbs" and "energies" are capitalized, and "verbs" and "called" are followed by commas.
401.25–27 *Lectures on Metaphysics,* p. 124; in the original, "act," "operation," and "energy" are italicized.
401.31 See ACT] The reference is to Peirce's *Century Dictionary* definition of "act," which has not been found in the manuscripts. The first two definitions under "act" in the *Century* (two of several contributed by Peirce) are the following:

> 1. An exertion of energy or force, physical or mental; anything that is done or performed; a doing or deed; an operation or performance. 2. A state of real existence, as opposed to a possibility, power, or being in germ merely; actuality; actualization; entelechy.

402.1–2 *On the Study of Character,* p. 202; in the original, commas follow the words "temperament" and "energize."
402.6–7 Peirce took this quotation from the third definition of ENERGY in the *Imperial.*
402.17 Young] See note 392.8.
402.18 Thomson] William Thomson is generally known as Lord Kelvin.
402.18 Rankine] William John Macquorn Rankine.
402.32 The. . . .] Peirce refers to the *Imperial's* examples of engine-turning, which illustrated ornamental designs of the kind etched on the backs of expensive watches and engraved on bank-notes. Although the reproductions in the *Imperial* are of poor quality, they were retained in the *Century.*
403.7–10 Caird, *Hegel,* p. 15. The first occurrence of the word "enlightenment" is capitalized and italicized in the original.
403.11 To . . . :] The reference is to the *Imperial:* "1. The number nine."
403.13 For . . . :] The reference is to the *Imperial:* "Languor of mind arising from lack of occupation; want of interest in present scenes and surrounding objects; listlessness; weariness; tedium; lassitude."
403.18 Retain. . . .] The reference is to the *Imperial:*

> The only fault of it is insipidity; which is apt now and then to give a sort of *ennui,* which makes one form certain little wishes that signify nothing. *Gray.*

404.3–4 See *Metaphysics* 9.8.1050a.
404.13–16 The soul. . . .] See *De Anima* 2.2.414a.
404.18–20 Cudworth. . . .] Cudworth discusses the concept of plastic nature in ch. 3 of his *True Intellectual System,* where he repeatedly com-

pares it to Aristotle's nature (*physis*); the reference to entelechy has not been found. The concept of vital principle was introduced by Nehemiah Grew (*Cosmologia sacra;* London, 1701), not by Cudworth.

404.19–20 Leibniz. . . .] See *La Monadologie,* §18.

405.3–5 The. . . .] The reference is to Duns Scotus's doctrine of transcendentals.

405.10–11 Not located.

405.12–13 Not located.

405.14 For . . . :] The reference is to the *Imperial:* "Dissipation of energy; loss of usefulness."

405.15–17 See Clausius, "Ueber . . . Wärmetheorie," p. 390.

405.18–20 See Tait, *Sketch of Thermodynamics,* ch. 3, especially p. 125.

405.21 For . . . :] The reference is to the *Imperial:*

> To count or tell, number by number; to reckon, as a number of things, each separately; to number; to count; to compute; hence, to mention in detail; to recount; to recapitulate; as, to *enumerate* the stars in a constellation. "*Enumerating* the services he had done." *Ludlow.*
>
> It would be useless to *enumerate* details. *Brande.*

405.25–26 1880 Census, p. xx.

405.27–29 Transpose . . . :] The reference is to the *Imperial:* "1. The act of enumerating; the act of counting or reckoning a number of things, each separately; computation.—2. An account of a number of things in which mention is made of every particular article."

405.30–31 1880 Census, p. xviii.

406.1 To. . . .] The reference is to the *Imperial:* "One who enumerates or numbers; specifically, in Britain, one who at the decennial census, takes the census of the inhabitants within a minor district."

406.2 For . . . :] The reference is to the *Imperial:*

> 3. That which is enunciated; announcement; statement; intelligence; information. Every intelligible *enunciation* must be either true or false. *A. Clarke.*
> 4. In *geom.* the words in which a proposition is expressed. If the enunciation respect a particular diagram it is called a *particular enunciation;* otherwise it is a *general enunciation.*

406.5–7 *Monitio logica,* bk. 1, ch. 27, p. 108. The original reads: "*Ax.* 2. An Enunciation is an Oration, Form of Speech or Declaration, in which somewhat true or false is pronounc'd of another."

406.22 See PROPOSITION] Peirce is probably making a forward reference to the definitions he expected to write (or perhaps had already written) for the *Century.* He in fact contributed a very long definition, with several illustrative quotations and special terms. His two leading definitions are the following:

> 3. A representation in thought or language of an act of the mind in thinking a quality or general sign, termed a *predicate,* to be applicable to something indicated, and termed a *subject.*
> 4. In *math.,* a statement in terms of either a truth to be demonstrated or an operation to be performed.

406.23 For . . . :] The reference is to the *Imperial:* "Pertaining to enunciation; declarative."

406.25 For . . . :] See definition 4 in note 406.31.

406.31 Add] The reference is to the *Imperial:*

> 1. A wrapper; an inclosing cover; an integument; as, the *envelope* of a letter or of the heart.—2. In *fort.* a work of earth in form of a parapet or of a small rampart with a parapet, raised to cover some weak part of the works.—3. In *bot.* one of the parts of fructification surrounding the stamens and pistils. The envelopes are formed of one or more whorls of abnormally developed leaves.—4. In *astron.* the dense nebulous covering of the nucleus or head of a comet, frequently rendering its edge indistinct. Called also a *Coma.*

407.7 Add . . . :] Apparently to be added to the following three examples of the first definition:

> Base *envy* withers at another's joy,
> And hates that excellence it cannot reach. *Thomson.*
>
> All the conspirators, save only he,
> Did that they did in *envy* of great Cæsar. *Shak.*
>
> Many suffered death merely in *envy* to their virtues and superior genius. *Swift.*

407.8–9 *Essay,* bk. 2, ch. 20, §13; in the original, "Envy" is in italicized capital letters, "desire" is followed by a comma.

407.18 Lilius] Aloysius Lilius, or Luigi Lilio Ghiraldi, astronomer and physician from Naples, and the author of the calendar system adopted by Pope Gregory XIII.

407.22–24 The *Imperial* article to which Peirce refers is the following:

> **Epact** (ē'pakt), *n.* [Gr. *epaktos,* brought in or on-*epi,* on, and *agō,* to lead.] In *chron.* the excess of the solar month above the lunar synodical month, and of the solar year above the lunar year of twelve synodical months. The epacts then are *annual* and *menstrual* or *monthly.* Suppose the new moon to be on the 1st of January; the month of January containing 31 days, and the lunar month only 29 days, 12 hours, 44 minutes, 3 seconds, the difference, 1 day, 11 hours, 15 minutes, 57 seconds, is the *menstrual epact.* The *annual epact* is nearly 11 days; the solar year being 365 days, and the lunar year 354.

407.25 Strike. . . .] The *Imperial* article to which Peirce refers is the following:

> **Epagoge** (e-pa-gō'jē), *n.* [Gr., a bringing on or to-*epi,* on, and *agō,* to lead.] In *rhet.* oratorical induction; a figure of speech which consists in demonstrating and providing universal propositions by particulars.

407.30 For . . . :] The reference is to the *Imperial:* "In *rhet.* of or pertaining to epagoge; inductive."

408.1 In . . . :] The reference is to the *Imperial:*

> 2. In *astron.* a collection of tables or data showing the daily positions of the planets or heavenly bodies in general; a publication exhibiting the places of the heavenly bodies throughout the year, and giving other information regarding them for the use of the astronomer and navigator; an astronomical almanac, such as the

Nautical Almanac and Astronomical Ephemeris, published by order of the British admiralty.

408.6 For . . . :] The reference is to the *Imperial:*

In *logic* and *rhet.* a syllogism having the truth of one or both of its premises confirmed by a proposition annexed (called a *prosyllogism*), so that an abridged compound argument is formed; as, all sin is dangerous; covetousness is sin (for it is a transgression of the law); therefore covetousness is dangerous. "For it is a transgression of the law" is a prosyllogism, confirming the proposition that "covetousness is sin."

408.7–8 See *Topics* 8.2.162a16–17.

408.10 Add] The reference is to the *Imperial:* "Pertaining to *Epictetus,* a Stoic philosopher in the time of the Roman emperor Domitian."

408.13 Under . . . :] The reference is to the *Imperial:*

1. Properly, a follower of Epicurus, who taught that pleasure and pain are the chief good and evil, that peace of mind, based on meditation, is the origin of all good; his ethical system has been popularly misrepresented as being characterized by gross sensualism.

408.15 Under . . . :] The reference is to the *Imperial:* "1. Pertaining to Epicurus; as the *Epicurean* philosophy or tenets."

408.17–18 Bentham and Mill] The reference is to their utilitarian doctrines.

408.20 Add] The reference is to the *Imperial:*

In the Ptolemaic system of astronomy, a little circle, whose centre moves round in the circumference of a greater circle; or a small circle, whose centre, being fixed in the deferent of a planet, is carried along with the deferent, and yet by its own peculiar motion carries the body of the planet fastened to it round its proper centre.

408.23 For . . . :] The reference is to the *Imperial:*

In *geom.* a curve generated by the movement of a curve upon the convex or concave side of another fixed curve; specifically, the curve generated by the movement of a curve upon the convex side of another curve, that generated by the movement of a curve upon the concave side of a fixed curve being called a *hypocycloid;* more specifically, a curve generated by any point in the plane of a movable circle which rolls on the outside of the circumference of a fixed circle. The curve that moves is the generating curve, the other being the base. The describing point is not necessarily in the circumference of the generating curve, but may be anywhere in a radius or its prolongation.

408.26 Roemer] Ole Römer (or Roemer), noted chiefly for calculating the speed of light.

408.28 For . . . :] The reference is to the *Imperial:* "Pertaining to the epicycloid, or having its properties."

409.3–5 (This. . . .)] The reference is to the *Imperial:*

In *geom.* the curve traced by a point in the plane of a circle which rolls on the convex side of a fixed circle. The curve thus generated belongs to the family of roulettes, and becomes an epicycloid when the generating point is in the circumference of the rolling circle. *Brande.*

409.6–7 Under . . . :] The reference is to the *Imperial:*

2. In *astron.* (*a*) the date at which a planet or other heavenly body has a given position. (*b*) An arbitrary fixed date, for which the elements used in computing the place of a planet or other heavenly body at any other date are given. *Goodrich.*

409.8 Strike . . . :] The reference is to the *Imperial:*

1. The same in magnitude or dimensions, value, qualities, degree, and the like; neither inferior nor superior, greater nor less, better nor worse; as, an *equal* quantity of land; a house of *equal* size; a person of *equal* bulk; *equal* angles; two commodities of *equal* value; men of *equal* rank; bodies of *equal* hardness or softness; two motions of *equal* velocity.

409.29–30 The reference is to the *Imperial:*

All men are created *equal. Jefferson*

Thou therefore also taste, that *equal* lot
May join us, *equal* joy, as *equal* love. *Milton.*

7. Being on the same terms; enjoying the same or similar benefits.

409.32–35 It is surprising that Peirce did not mention the formal property of transitivity, either in connection with the sign of equality or in the preceding definition for "equal." He might also have mentioned his usual concern that equality not be viewed as a simple conception (see "Algebra of Logic," W4:169n.3–170n.12).

409.32 Strike . . . :] The reference is to the *Imperial:*

4. In *math.* a comparison of two quantities which are in effect equal, though differently expressed or represented. It is usually denoted by two parallel lines, =; thus $3x + 4y = 20$; that is, $3x$ added to $4y$ are equal to 20.—*Ratio of equality,* the ratio of two equal quantities.

410.1 For . . . :] The reference is to the *Imperial:* "In the Ptolemaic system of astronomy, an imaginary circle used for determining the motions of the planets."

410.5–6 and 9 The reference is to the *Imperial:*

2. In *alg.* a proposition asserting the equality of two quantities, and expressed by the sign = between them; or an expression of the same quantity in two dissimilar terms, but of equal value; as, $3s. = 36d.$ or $x = b + m - r$. In the latter case x is equal to b added to m, with r subtracted, and the quantities on the right hand of the sign of equation are said to be the value of x on the left hand. An equation is termed simple, quadratic, cubic, or biquadratic, or of the first, second, third, or fourth degree, according as the index of the highest power of the unknown quantity is one, two, three, or four. And generally an equation is said to be of the 5th, 6th, nth, &c., degree, according as the highest power of the unknown quantity is of any of these dimensions.

410.18–19 Abelian equation] Named for Niels Henrik Abel (1802–1829) who introduced these equations in his *Mémoire sur une classe particulière d'équations résolubles algébriquement* (1829), written two months before his death.

410.34 Under 2] See note 410.5–6.

411.9 Equation of equinoxes] The reference is to the *Imperial:* "in *astron.* the difference between the mean and apparent places of the equinox."
411.18 The following are the definitions as they appear in the *Imperial:*

Equator n. 1. In *astron.* that imaginary great circle in the heavens, the plane of which is perpendicular to the axis of the earth. It is everywhere 90° distant from the celestial poles, which coincide with the extremities of the earth's axis, supposed to be produced to meet the heavens, and its axis is this produced axis. It divides the celestial sphere into the northern and southern hemispheres. During his apparent yearly course the sun is twice in the equator, at the beginning of spring and of autumn. Then the day and night are equal, whence the name *equinox.* —2. In *geog.* that great circle of our globe, every point of which is 90° from the poles, which are also its poles, its axis being also the axis of the earth. It is in the plane of the celestial equator. All places which are on it have invariably equal days and nights. Our earth is divided by it into the northern and southern hemispheres. From this circle is reckoned the latitude of places both north and south.—*Magnetic equator,* a line which pretty nearly coincides with the geographical equator, and at every point of which the vertical component of the earth's magnetic attraction is zero; that is to say, a dipping needle carried along it remains horizontal. It is hence called the *aclinic line.*

Equatorial n. An astronomical instrument, contrived for the purpose of directing a telescope upon any celestial object of which the right ascension and declination are known, and of keeping the object in view for any length of time, notwithstanding the diurnal motion. For these purposes a principal axis resting on firm supports is placed parallel to the axis of the earth's rotation, and consequently pointing to the poles of the heavens. On this polar axis there is placed, near one of its extremities, a graduated circle, the plane of which is perpendicular to the polar axis, and therefore parallel to the equator. This circle is called the *equatorial circle,* and measures by its arcs the hour angles, or differences of right ascension. The polar axis carries a second circle, called the *declination circle,* the plane of which is at right angles to that of the equatorial circle. This last circle has a telescope attached to it for making observations, and which moves along with it in the same plane. The name *equatorial,* or *equatorial instrument,* is sometimes given to any astronomical instrument which has its principal axis of rotation parallel to the axis of the earth.

The *Imperial* definition for "equatorial" as adjective is:

Pertaining to the equator; as, *equatorial* climates; the *equatorial* diameter of the earth is longer than the polar diameter.—*Equatorial telescope* or *instrument,* an equatorial (which see).

411.23 For . . . :] The reference is to the *Imperial:*

1. Being at an equal distance from some point or place.
The fixed stars are not all . . . *equidistant* from us. *Ray.*
2. In *geom.* a term of relation between two things which are everywhere at the same or at equal distances from each other.

411.28 Insert] The reference is to the *Imperial:* "To balance equally; to keep even with equal weight on each side; to keep in equipoise."
411.30 Strike . . . :] The complete definition of "equilibrium" in the *Imperial* runs as follows:

1. In *mech.* equipoise; equality of weight or force; a state of rest produced by the mutual counteraction of two or more forces, as the state of the two ends of a lever

or balance, when both are charged with equal weight, and they maintain an even or level position, parallel to the horizon. When two or more forces acting upon a body are so opposed to each other that the body remains at rest, although one of them would move it if acting alone, those forces are said to be *in equilibrium,* that is, equally balanced. See STATICS.—*Stable, unstable,* and *neutral* or *indifferent equilibrium.* When a body, being slightly moved out of any position in which it rests upon another body, always tends to return to its position, and, being left to itself, will roll back of its own accord into it, that position is said to be one of *stable equilibrium;* when the body will not thus return to its previous position, its position is said to be one of *unstable equilibrium;* and when a body, being moved more or less from its position of equilibrium, will rest in any of the positions in which it is placed, and is indifferent to any particular position, its equilibrium is said to be *neutral* or one of *indifference.* A perfect sphere, of uniform material, resting upon a horizontal plane, is in a state of *neutral equilibrium;* an oblate spheroid, with its axis of rotation vertical, is in *stable equilibrium,* while a prolate spheroid, with its axis vertical, is in *unstable equilibrium* on the same plane. A body suspended by its centre of gravity is in a state of *neutral* or *indifferent equilibrium.* If a body be suspended by any other point it will be in a state of stable equilibrium when its centre of gravity is perpendicularly below the point of suspension, but if the centre of gravity be above the point of suspension the equilibrium will be unstable.—2. A state of just poise; a position of due balance; as, to preserve the *equilibrium* of the body; take care you do not lose your *equilibrium.* 3. In the *fine arts,* (*a*) the just poise or balance of a figure or other object so that it may appear to stand firmly. (*b*) The due equipoise of objects, lights, shadows, &c.—4. Equal diffusion or distribution, as of temperature, which all bodies on the earth tend to produce, of the electric fluid in its natural undisturbed state, &c.—5. Equal balancing of the mind between motives or reasons; a state of indifference or of doubt, when the mind is suspended in indecision, between different motives or the different forces of evidence.—6. Equality of influence or effect; due or just relationship.

Health consists in the *equilibrium* between these two powers. *Arbuthnot.*

7. In *politics,* balance of power. See under BALANCE.—*In equilibrio,* in a state of equilibrium.

It is *in equilibrio*
If deities descend or no. *Prior.*

411.30–412.6 It is surprising that Peirce does not mention static and dynamic equilibrium, but the distinction appears (as stable and unstable equilibrium) in the passage he accepted from the *Imperial.*

412.7–8 See note 411.30.

412.11–12 *A Discourse of Conscience,* p. 5; the original reads: "the Ballance is turned, and where ever this happens, there is an end of the Doubt or *Æquilibrium*"

412.13 See note 411.30.

412.32 For . . . :] The reference is to the *Imperial:* "In *logic,* an equivalence between two or more propositions."

413.1–3 *Monitio logica,* bk. 1, ch. 32, p. 127. The original reads: "As to the Æquipollency or like Value of Enunciations in Sense, which differ in Words, which the Author says something of, I thought it of little Significancy to be mentioned here."

413.4–6 *Lectures on Logic,* App. V, p. 522; "equipollence" is capitalized in the original.

413.7–9 *The Romance of the Rose,* 7076–78.

413.10 For . . . :] The reference is to the *Imperial:* "In *logic,* having equivalent signification, force, or reach."

413.20 a word. . . .] Peirce gives the information after Prantl, *Geschichte der Logik,* 1:684.

413.33 The reference is to the *Imperial:*

> *Equivocal* generation is the production of plants without seed, or of insects or animals without parents in the natural way of coition between male and female. *Harris.*
>
> Unfinished things one knows not what to call,
> Their generation's so *equivocal. Pope.*

414.1–5 *Monitio logica,* bk. 1, ch. 17, p. 63. The original reads:

> The Effect is said to be contained in the Cause, either *formally* or *eminently.* . . . When *Eminently,* or the Cause by a *Nobler Sort* of Virtue produces the Effect, it is said to be *Æquivocal,* and *is better than its* Effect; as, when *Light* produces *Heat,* or the *Architect an House.*

414.9–10 See. . . .] See note 413.33.

414.13 Under . . . :] See note 413.33.

414.14–15 Also. . . .)] The reference is to the *Imperial:*

> This visible world is but a picture of the invisible, wherein, as in a portrait, things are not truly, but in *equivocal* shapes, and as they counterfeit some real substance in that invisible fabric. *Sir T. Browne.*

414.22 Under . . . :] The reference is to the *Imperial:*

> 1. An ambiguous term; a word susceptible of different significations.
> I loved you almost twenty years ago; I thought of you as well as I do now; better was beyond the power of conception; or, to avoid an *equivoque,* beyond the extent of my ideas. *Bolingbroke.*

414.24–27 *Arte of Logicke,* bk. 1, ch. 7; the original reads:

> Equivokes be such things as have one selfe name, and yet be divers in substance or definition, as a naturall dogge, and a certaine starre in the firmament are both called by one name in Latine *Canis,* yet they be nothing like in substance, kinde or nature.

414.28 For . . . :] The reference is to the *Imperial:* "1. The Horse's-head, a northern constellation consisting of ten stars."

414.32 Lacaille] Nicolas Louis de Lacaille.

415.1 Foerster and Lesser] Wilhelm Förster and O. L. Lesser, German astronomers who were observers at the Berlin Observatory in 1860.

415.6 For . . . :] The reference is to the *Imperial:* "In *physics,* the unit of work done by a force which, acting for one second upon a mass of one gramme (15.4 grains troy), produces a velocity of a centimetre (.3937 inch) per second."

415.10 The . . . text] See the preceding note.

415.14 Add] The reference is to the *Imperial:* "Therefore."

415.24 Perrotin] Joseph Perrotin (1845–1904), French astronomer.

415.25 Add] The reference is to the *Imperial:* "Pertaining to disputation or controversy; controversial; captious."

416.1 For . . . :] The complete *Imperial* definition for "error" runs as follows:

1. A wandering or deviation from the truth; a mistake in judgment by which men assent to or believe what is not true; a mistake as to matter of fact; a misapprehension.

In my mind he was guilty of no *error,* he was chargeable with no exaggeration, he was betrayed by his fancy into no metaphor, who once said, that all we see about us, King, Lords, and Commons, the whole machinery of the state, all the apparatus of the system, and its varied workings, end in simply bringing twelve good men into a box. *Brougham.*

2. A mistake made in writing, printing, or other performance; an inaccuracy; an oversight; falsity; as, a clerical *error;* an *error* in a declaration.—3.† A wandering; excursion; irregular course.

He (Æneas) through fatall *errour* long was led
Full many yeares. *Spenser.*

Driven by the winds and *errors* of the sea. *Dryden.*

4. A transgression of law or duty; a mistake in conduct; a fault; a sin; iniquity; transgression.

Who can understand his *errors?* cleanse thou me from secret faults. Ps. xix. 12.

If it were thine *error* or thy crime,
I care no longer. *Tennyson.*

5. In *law,* a mistake in the proceedings of a court of record either in fact or in law, entitling the unsuccessful party to have the case reviewed. Proceedings in error were abolished in civil cases by the Judicature Act of 1875, appeal being substituted; but they may still be taken in criminal cases, for which the court of review is the Queen's Bench. An appeal in error is made by means of an original writ, called a *writ of error.* —6. In *astron.* the difference between the places of any of the heavenly bodies as determined by calculation and by observation.—7. In *math.* the difference between the result of any operation and the true result.—*Error of a clock,* the difference between the time indicated by a clock and the time which the clock is intended to indicate, whether sidereal or mean time.

416.3, 7–8, and 18 See note 416.1.

416.15 Charles P. G. Scott was in charge of preparing the etymologies for the *Century.* William D. Whitney, the Editor-in-Chief, was the authority for comparative philology.

417.3 See definition 5 in note 416.1.

417.4–6 Originally. . . .] The classification of Aristotle's works into "esoteric" and "exoteric," which first occurred in Lucian, was recognized by Cicero (*De finibus bonum et malorum,* bk. 5, §5) who, however, used only the term "exoteric" and left the opposite class unnamed, referring to it as "alterum limatius" ("the other more elaborate").

417.9–15 *Aristotle,* 2:52; in the original, "exoteric doctrine" is enclosed in quotation marks and "esoteric" is italicized.

417.16 The examples Peirce wanted struck are the following:

The philosophy of the Pythagoreans, like that of the other sects, was divided into the exoteric and *esoteric;* the open, taught to all; and the secret, taught to a select number. *Warburton.*

> Enough if every age produce two or three critics of this *esoteric* class, with here and there a reader to understand them. *De Quincey.*
>
> On the testimony of a phrase in Aristotle, it is supposed that Plato, like Pythagoras, had exoteric and *esoteric* opinions; the former being, of course, those set forth in his Dialogues. *G. H. Lewes.*

417.33–418.7 *Essay,* bk. 3, ch. 3, §15. In the original, the final sentence is its own paragraph; the semicolon is a comma; the parenthetical phrase "as I do . . . genus" is in commas; the words "first," "essence(s)," "species," "idea(s)," "genus," "sort," "sortal," and "nominal" are italicized, as is the last occurrence of "real"; "anything" is two words; and in their first occurrence, "genus" and "sort" are followed by commas.

418.8 The reference is to the *Imperial:*

> Whatever makes a thing to be what it is, is properly called its *essence.* Self-consciousness, therefore, is the *essence* of the mind, because it is in virtue of self-consciousness that the mind is the mind-that a man is himself. *Ferrier.*

418.9–11 *Measure for Measure,* 2.2.117–20; Peirce omitted line 118 of the original. For other instances of Peirce's use of this quotation see CP 5.519 and 6.238–71.

418.14 Liebig's. . . .] The reference is to Justus Liebig's method of organic analysis of the juices of the flesh.

418.18 The quotation appears under ESSENCE, number 6, in the *Imperial.*

418.24 The quotation appears under ESSENCE, number 5, in the *Imperial.*

418.25 The quotation appears under ESSENCE, number 3, in the *Imperial.*

418.26 The reference is to the *Imperial:*

> Here be four of you, as differing as the four elements; and yet you are friends: as for Eupolis, because he is temperate and without passion, he may be the fifth *essence. Bacon.*

418.29 examples . . . Sidney] The reference is to the *Imperial:*

> I could have resign'd my very *essence. Sidney.*

418.31 Boyle] See his *New Experiments,* Exp. xxv, p. 99.

419.12 The quotation appears under ESSENTIAL, number 2, in the *Imperial.*

419.14 Add . . . :] There are no examples under ESSENTIALLY in the *Imperial.*

419.15–17 *Monitio logica,* bk. 1, ch. 15, p. 50. The original reads:

> *Ax.* 8. Causes likewise are said either to be *Essentially* or *Accidentally* subordinated.
>
> *Ax.* 9. Essentially are those of which one *depends upon another, when,* and *in as much as it causes.*

419.22 The definition in the *Imperial* is: "The star γ of the constellation Draco, interesting as being the star by the observation of which Bradley was led to the discovery of the aberration of the fixed stars."

419.25 Add . . . :] The reference is to the *Imperial:* "5. Unchangeable; existing at all times without change; as, *eternal* truth."

419.26–27 Strangely . . . :] The complete definition in the *Imperial* is:

> 1. In *astron.* and *physics,* a hypothetical medium of extreme tenuity and elasticity supposed to be diffused throughout all space (as well as among the molecules of which solid bodies are composed), and to be the medium of the transmission of light and heat.
>
> There fields of light and liquid *ether* flow. *Dryden.*
>
> 2. In *chem.* a very light, volatile, and inflammable fluid, produced by the replacement of the hydrogen of organic acids by alcohol radicles. It is lighter than alcohol, of a strong sweet smell, susceptible of great expansion, and has a pungent taste. A mixture of vapour of ether with atmospheric air is extremely explosive. Its formula is $(C_2H_5)_2O$.

419.28–29 It was. . . .] See Aristotle, *De Mundo* 392a5–31.

419.30–31 *Poetical Works,* p. 382 (from "To one who has been long in city pent").

419.32 See note 419.26–27.

420.1 Add] The reference is to the *Imperial:* "Relating to manners or morals; treating of morality; containing precepts of morality; moral; as, *ethic* discourses or epistles."

420.4 Under . . . :] The reference is to the *Imperial:*

> 1. The science which treats of the nature and laws of the actions of intelligent beings, these actions being considered in relation to their moral qualities; the science which treats of the nature and grounds of moral obligation; the science of moral philosophy, while teaches men their duty and the reasons of it; the science of human duty.

421–23 Peirce's letter contains the first known design for the use of electrical circuitry for computing. Marquand had contributed a paper on his logic machine, "A Machine for Producing Syllogistic Variations," to *Studies in Logic,* the most permanent monument to Peirce's teaching at the Johns Hopkins. A thorough description of the several machines built by Marquand, and of the logic involved, is found in Martin Gardner's *Logic Machines and Diagrams* (New York: McGraw-Hill, 1958), pp. 106–113, which includes, on p. 109 an extensive quote from Peirce's "Logical Machines" (*American Journal of Psychology* 1 [1887]: 165–70). An early reference to Marquand's machine, and his plans to adapt it to the use of electricity, appears in Baldwin's *Dictionary of Philosophy and Psychology* under "Logical Machine." (See also Kenneth L. Ketner's "The Early History of Computer Design: Charles Sanders Peirce and Marquand's Logical Machines," *Princeton University Chronicle* 45 [1984]: 186–224).

421.17–18 You. . . .] Not until sometime in the late 1930s did the American mathematician Claude Elwood Shannon, employing ideas similar to Peirce's, show how to use electrical circuits to carry out arithmetic operations and show that the calculus used for defining these circuits is equivalent to Boolean algebra. Shannon may have been influenced by the Jevons-type logical machines built by the Ukrainian physical chemist Aleksandr

Nikolaevich Shchukarev (1864–1936) and his colleague Pavel Smitrievich Hrushchov (1849–1909), who were in turn influenced by the description of Jevons's machines given by the Russian-Polish logician and logic historian Ivan Vladimirovich Sleshinskii. Shchukarev demonstrated his logic machine in April 1914 at a lecture at the Polytechnical Museum in Moscow. The first effective electronic digital computer using Boolean algebra for solving large systems of simultaneous differential equations was built by John Vincent Atanasoff at Iowa State College (now Iowa State University) in 1940. See Alice R. Burks and Arthur W. Burks, *The First Electronic Computer: the Atanasoff Story* (Ann Arbor: University of Michigan Press, 1988).

421.18–22 My. . . .] Marquand's machine could handle with ease only syllogisms with only universal premisses.

422.12 your last paper] This is Marquand's "A New Logical Machine" (*Proceedings of the American Academy of Arts and Sciences* 21 [1886]: 303–307), which he read at the November 1885 meeting of the Academy.

Bibliography of Peirce's References*

°Abbot, Francis Ellingwood. *Organic Scientific Philosophy: Scientific Theism.* Boston: Little, Brown, & Co., 1885.

°———. "The Philosophy of Space and Time." *North American Review* 99 (1864): 64–116.

The American Ephemeris and Nautical Almanac. Washington: Bureau of Navigation, 1855–.

°Aristotle. *Aristoteles Graece.* Edited by Immanuel Bekker. 2 vols. Berlin: Georg Reimer, 1831.

Astronomisches Jahrbuch. Bonn and Berlin: Dummler, 1776–.

Aubrey, John. "The Life of Mr. Thomas Hobbes, of Malmesburie." Appendix 5 to *Lives of Eminent Men.* In vol. 2 of *Letters Written by Eminent Persons in the Seventeenth and Eighteenth Centuries and Lives of Eminent Men,* edited by John Walker. London: Longman, Hurst, Rees, Orme, & Brown, 1813.

°Bacon, Francis. *Advertisement Touching a Holy War.* In vol. 13 of *The Works of Francis Bacon,* collected and edited by James Spedding, Robert Leslie Ellis, and Douglas Denon Heath. 14 vols. London: Longman, Green, Longman, & Roberts, 1857–74.

°———. *Novum Organum; or, True Suggestions for the Interpretation of Nature.* Newly translated by Andrew Johnson. London: Bell & Daldy, 1859.

°———. *Opera omnia.* Francofurti ad Moenum, impensis J. B. Schonwetteri, 1665.

°Bacon, Roger. *Compendium studii philosophiae.* In vol. 1 of *Opera quaedam hactenus inedita,* edited by J. S. Brewer. London: Longman, Green, Longman, & Roberts, 1859.

Bain, Alexander. *On the Study of Character.* London: Parker, Son, and Bourn, West Strand, 1861.

Bentham, George. *Outline of a New System of Logic, with a Critical Examination of Dr. Whately's "Elements of Logic."* London: Hunt & Clarke, 1827.

*The degree symbol (°) indicates works Peirce is known to have had in his possession. In most cases, the edition listed here is the one known or assumed to have been used by Peirce.

°Blundeville, Thomas. *The Arte of Logicke.* London: W. Stansby, 1619.
°Boole, George. *An Investigation of the Laws of Thought, on which are founded the Mathematical Theories of Logic and Probabilities.* London: Walton & Maberly, 1854.
Bourne, H[enry] R[ichard] Fox. *The Life of John Locke.* 2 vols. New York: Harper & Brothers, 1876.
Boyle, Robert. *New Experiments Physico-Mechanical, Touching the spring of Air, and its Effects, Made, for the most part, in a New Pneumatical Engine.* London: printed by Miles Flesher for Richard Davis, 1682.
Bramhall, John. *A Defence of True Liberty from Antecedent and Extrinsecal Necessity.* In vol. 4 of *The Works of the Most Reverend Father in God, John Bramhall, with A Life of the Author and a Collection of his Letters.* Oxford: John Henry Parker, 1844.
°Browne, Thomas. *Religio Medici.* Facsimile of the first edition published in 1642. With an introduction by W. A. Greenhill. London: E. Stock, 1883.
°———. *The Works of Sir Thomas Browne.* Edited by Simon Wilkin. London: Henry G. Bohn, 1852.
°[Burgersdicius]. *Monitio Logica or, An Abstract and Translation of Burgersdicius His Logick, By a Gentleman.* London: Ric. Cumberland, 1697.
Caird, Edward. *Hegel.* Edinburgh: Wm. Blackwood & Sons, 1883.
°Cayley, Arthur. "A Sixth Memoir upon Quantics." *Philosophical Transactions of the Royal Society of London* 149 (1860): 61–90.
[Census of 1880]. *A Compendium of the Tenth Census (June, 1880), Compiled Pursuant to an Act of Congress Approved August 7, 1882.* Washington, DC: Government Printing Office, 1883.
Chaucer, Geoffrey. *The Poetical Works of Geoffrey Chaucer.* 3 vols. Edited by Arthur Gilman. Boston: Houghton, Mifflin and Co., 1879.
°Chauvin, Etienne. *Lexicon Philosophicum.* Leovardiae: Franciscus Halma, 1713.
°Cicero, Marcus Tullius. *M. Tullii Ciceronis Opera Omnia.* Edited by K. F. A. Nobbe. 11 vols. Leipzig: Karl Tauchnitz, 1849–50.
Clausius, Rudolf. "Ueber verschiedene für die Anwendung bequeme Formen der Hauptgleichungen der mechanischen Wärmetheorie." *Annalen der Physik und Chemie* 125 (1865): 353–400.
°Clifford, William Kingdon. *The Common Sense of the Exact Sciences.* New York: D. Appleton & Co., 1885.
Connaissance des temps, ou des mouvements célestes, pour le méridien de Paris, à l'usage des astronomes et des navigateurs. Bureau de longitudes. Paris: L'Imprimerie de la République, 1797–.
°Cudworth, Ralph. *The True Intellectual System of the Universe: wherein All the Reason and Philosophy of Atheism is Confuted, and its Impossibility Demonstrated.* Translated by John Harrison. 3 vols. London: Thomas Tegg, 1845.
Daniel, Samuel. *The First Fowre Bookes of the Civile Wars between the Two Houses of Lancaster and Yorke.* London: printed by P. Short for Simon, 1595.
°De Morgan, Augustus. "On the Structure of the Syllogism, and on the Appli-

cation of the Theory of Probabilities to Questions of Argument and Authority [No. I]." *Transactions of the Cambridge Philosophical Society* 8 (1849): 379–408. [Presented 9 November 1846.]

°———. "On the Syllogism, No. III, and on Logic in General." *Transactions of the Cambridge Philosophical Society* 10 (1864): 173–230. [Presented 8 February 1858.]

°———. "On the Syllogism, No. IV, and on the Logic of Relations." *Transactions of the Cambridge Philosophical Society* 10 (1864): 331–58. [Presented 23 April 1860.]

°———. *Syllabus of a Proposed System of Logic.* London: Walton & Maberly, 1860.

°Descartes, René. *Oeuvres choisies.* New ed. Paris: Garnier Frères, 1865.

°Dickens, Charles. *Dombey and Son.* With illustrations by H. K. Browne. Philadelphia: Lea and Blanchard, 1848.

°Euclid. *Elementa.* [*The elements of Geometrie.* Translated by H. Billingsley.] Preface by John Dee. London: John Daye, 1570.

———. *The Elements of Euclid, Books I to VI, with Deductions, Appendices, and Historical Notes.* Edited by John Sturgeon Mackay. London and Edinburgh: W. & R. Chambers, 1884.

°Fechner, Gustav Theodor. *Elemente der Psychophysik.* 2 vols. Leipzig: Breitkopf & Härtel, 1860.

Ferrier, James F. *Institutes of Metaphysic/Theory of Knowing and Being.* London: William Blackwood and Sons, 1856.

°Fiske, John. *The Idea of God as Affected by Modern Knowledge.* Boston and New York: Houghton, Mifflin and Co., 1885.

———. *Outlines of Cosmic Philosophy, Based on the Doctrine of Evolution with Criticisms on the Positive Philosophy.* 2 vols. Boston: Houghton, Mifflin and Co., 1874.

°Fleming, William. *The Vocabulary of Philosophy, Mental, Moral, and Metaphysical; with Quotations and References; for the Use of Students.* London and Glasgow: Richard Griffin & Co., 1857.

°Fraser, Alexander Campbell. *Locke.* Philadelphia: J. B. Lippincott Co., 1890.

Gauss, Karl Friedrich. *Atlas des Erdmagnetismus nach den Elementen der Theorie entworfen.* Supplement zu den Resultaten aus den Beobachtungen des magnetischen Vereins unter Mitwirkung von C. W. B. Goldschmidt herausgegeben von Karl Friedrich Gauss und Wilhelm Weber. Leipzig, 1840.

°———. *Disquisitiones arithmeticae.* Leipzig: G. Fleischer, 1801.

———. *Intensitas vis magneticae terrestris ad mensuram absolutam revocata.* Göttingen: Dieterich, 1833.

———. *Theorematis fundamentalis in doctrina de residuis quadriticis demonstrationes et ampliationes novae.* Göttingen: H. Dieterich, 1818.

———. *Theoria motus corporum coelestium in sectionibus conicis solem ambientium.* Hamburg: F. Perthes & I. H. Besser, 1809.

°———. *Theory of the motion of the heavenly bodies moving about the sun* *in conic sections: a translation of Gauss's "Theoria motus."* With an appendix. Boston: Little, Brown and Co., 1857.

———. "Zur Bestimmung der Constanten des Bifilarmagnetometers." In vol. 5 of *Werke,* herausgegeben von der Königlichen Gesellschaft der Wissenschaften. Göttingen: Dieterichsche Universitäts-Druckerei, 1867.

Gellius, Aulus. *Noctium Atticarum.* 2 vols. Leipzig: B. G. Teubner, 1871.

Gilbert, Grove Karl. "Finley's Tornado Predictions." *American Meteorological Journal* 1 (1884): 166–72.

°Grote, George. *Aristotle.* 2nd ed., edited by Alexander Bain and G. Croom Robertson. 2 vols. London: John Murray, 1880.

°Hamilton, William. *Discussions on Philosophy and Literature, Education and University Reform.* New York: Harper & Brothers, 1856.

°———. *Lectures on Metaphysics and Logic.* 2 vols. Edited by H. L. Mansel and J. Veitch. Boston: Gould & Lincoln, 1860.

Harris, James. *Hermes or A Philosophical Inquiry Concerning Universal Grammar.* 7th ed. London: J. Collingwood, 1825.

°———. *The Works of James Harris, esq., with an account of his life and character,* by his son, the Earl of Malmesbury. Oxford: T. Tegg, 1841.

°Hegel, Georg Wilhelm Friedrich. *Encyclopädie der philosophischen Wissenschaften im Grundrisse.* 2nd ed. Heidelberg: August Osswald, 1827.

°———. *The Logic of Hegel.* Translated from *The Encyclopaedia of the Philosophical Sciences* by William Wallace. Oxford: Clarendon Press, 1874.

°———. *Werke: Vollständige Ausgabe durch einen Verein des Verewigten.* Edited by Philipp Marheineke et al. 18 vols. Berlin: Duncker & Humblot, 1832–40; 2nd ed., 1840–44. [Peirce owned vols. 1, 7, 16, 17, and 18 of the 1st ed., all others of the 2nd.]

°———. *Wissenschaft der Logik.* Edited by Leopold von Henning. Vols. 3–5 of *Werke.*

Helmholtz, H[ermann Ludwig Ferdinand von]. *Handbuch der physiologischen Optik.* [Vol. 9 of *Allgemeine Encyclopädie der Physik.* Edited by Gustav Karsten.] Leipzig: Leopold Voss, 1867.

°Hobbes, Thomas. *The English Works of Thomas Hobbes.* Edited by Sir William Molesworth. 11 vols. London: John Bohn, 1839–45. [Peirce owned the first volume.]

°———. *Leviathan, sive de Materia, Forma, & Potestate Civitatis Ecclesiasticae & Civilis.* In vol. 2 of *Opera Philosophica Omnia.*

°———. *Libros tres de Cive.* In vol. 2 of *Opera Philosophica Omnia.*

°———. *Opera philosophica quae latine scripsit omnia.* 8 parts in 2 vols. Amstelodami: J. Blaev, 1668.

———. *Thomas Hobbes Angli Malmesburiensis Philosophi vita.* London: William Crooke, 1681.

The Imperial Dictionary. See Ogilvie, John.

Jevons, William Stanley. *Elementary Lessons in Logic: Deductive and Inductive. With Copious Questions and Examples, and a Vocabulary of Logical Terms.* London: Macmillan & Co., 1870.

°———. *Pure Logic or the Logic of Quality apart from Quantity: with Remarks on Boole's System and on the Relation of Logic and Mathematics.* London: Edward Stanford, 1864.

°———. *Studies in Deductive Logic. A Manual for Students.* London: Macmillan & Co., 1880.

°Kant, Immanuel. *Critique of Pure Reason.* In Commemoration of the Centenary of its First Publication. Translated into English by F. Max Müller with an Historical Introduction by Ludwig Noiré. 2 vols. London: Macmillan & Co., 1881.

°———. *Kant's Introduction to Logic, and his Essay on the Mistaken Subtility of the Four Figures.* Translated by Thomas Kingsmill Abbott, with a few notes by Coleridge. London: Longmans, Green, & Co., 1885.

°———. *Kritik der reinen Vernunft.* Part 2 of *Sämmtliche Werke.*

°———. *Kritik der Urtheilskraft.* Part 2 of *Sämmtliche Werke.* (First English translation, by J. H. Bernard, not published until 1892; London and New York: Macmillan and Co.)

°———. *Immanuel Kant's sämmtliche Werke.* Edited by Karl Rosenkranz and Friedrich Wilhelm Schubert, 12 parts in 14 vols. Leipzig: Leopold Voss, 1838–42.

Keats, John. *The Poetical Works of John Keats.* Boston: Little, Brown and Co., 1845.

°Keynes, John Neville. *Studies and Exercises in Formal Logic, Including a Generalisation of Logical Processes in Their Application to Complex Inferences.* London: Macmillan & Co., 1884.

King, Lord [Peter]. *The Life of John Locke, with Extracts from His Correspondence, Journals, and Common-place Books.* New ed. 2 vols. London: Henry Colburn & Richard Bentley, 1830.

Lagrange, Joseph Louis. *Essai d'une nouvelle méthode pour déterminer les maxima et les minima des formules intégrales indéfinies.* Turin, 1762. [Offprint from *Mélanges de philosophie et de mathématique de la Société Royale de Turin,* 1760–61.]

Le Clerc, Jean. *Bibliothèque choisie, pour servir de suite à la Bibliothèque universelle.* 28 vols. Amsterdam: H. Schelte, 1703–18.

Legge, James. *The Life and Works of Mencius.* Philadelphia: J. B. Lippincott & Co., 1875.

°Leibniz, Gottfried Wilhelm. *La Monadologie.* In *Opera philosophica quae exstant latina, gallica, germanica omnia,* edited by Johannes Eduard Erdmann. Berlin: G. Eichler, 1840.

Lessing, Gotthold Ephraim. *Laokoon: oder, Über die Grenzen der Mahlerey und Poesie.* Berlin: C. F. Voss, 1766.

———. *Nathan der Weise. Ein dramatisches Gedicht, in fünf Aufzügen.* Berlin: C. F. Voss, 1779.

°Locke, John. *An Essay Concerning Human Understanding; with Thoughts on the Conduct of the Understanding.* 3 vols. London: C. Bathurst, 1795.

———. *A Letter Concerning Toleration.* 2nd ed. London: A. Churchill, 1690. [First published in Latin as *Epistola de tolerantia,* Goudae, 1689.]

°Macaulay, Thomas Babington. *Critical and Historical Essays, Contributed to The Edinburgh Review.* London: Longman, Brown, Green, & Longmans, 1850.

Machiavelli, Niccolò. *Discourses on the First Decade of Titus Livius.* Translated by Ninian Hill Thomson. London: K. Paul, Trench & Co., 1883.

———. *La Mandragola, commedia. La Clizia, commedia. Commedia senza titolo.* In vol. 6 of *Opere di Niccolò Machiavelli.* Florence: Gaetano Cambiagi, 1783.

°———. *The Prince.* Translated by N[inian] H[ill] T[homson]. London: K. Paul, Trench & Co., 1882.

[Mencius]. *The Chinese Classics.* Translated by James Legge. New York: Hurd and Houghton, 1870.

°Mill, John Stuart. *A System of Logic, Ratiocinative and Inductive: Being a Connected View of the Principles of Evidence, and the Methods of Scientific Investigation.* 6th ed. 2 vols. London: Longmans, Green, & Co., 1865.

°Milton, John. *The Poetical Works of John Milton. With Notes, and a Life of the Author.* New edition. 2 vols. Boston: Charles C. Little & James Brown, 1845.

°Mitchell, Oscar Howard. "On a New Algebra of Logic." In *Studies in Logic. By Members of the Johns Hopkins University,* 72–106. Boston: Little, Brown, & Co., 1883. [Edited by Peirce.]

°Montaigne, Michel Eyquem de. *Works of Michael de Montaigne: Comprising His Essays, Journey into Italy, and Letters, with Notes from All the Commentators, Biographical and Bibliographical Notices, etc.* By W. Hazlitt. New and carefully rev. ed., edited by O. W. Wight. 4 vols. New York: Hurd & Houghton, 1864.

The Nautical Almanac and Astronomical Ephemeris. London: H. M. Stationery Office, 1767–.

°Ogilvie, John. *The Imperial Dictionary of the English Language.* New edition, edited by Charles Annandale. 4 vols. London, New York: The Century Co., 1883.

°Perrin, Raymond S. *The Religion of Philosophy or the Unification of Knowledge: A Comparison of the Chief Philosophical and Religious Systems of the World—Made with a View to Reducing the Categories of Thought, or the Most General Terms of Existence, to a Single Principle, thereby Establishing a True Conception of God.* New York: G. P. Putnam's Sons, 1885.

Petrus Hispanus. *Summulae logicales.* Venice, 1597.

°Phoenix, John. *Phoenixiana; or, Sketches and Burlesques.* 8th ed. New York: D. Appleton & Co., 1856.

°Plato. *Phaedrus.* In vol. 1 of *The Works.*

°———. *The Republic.* In vol. 2 of *The Works.*

°———. *The Works of Plato. A New Literal Version, Chiefly from the Text of Stallbaum.* Translated by Cary, Davis, and Burges. 6 vols. London: Henry G. Bohn, 1851–55.

°Pope, Alexander. *The Poetical Works of Alexander Pope, with a Life by Rev. Alexander Dyce.* 3 vols. Boston: Little, Brown & Co., 1853.

°*The Port-Royal Logic, by Antoine Arnauld and Pierre Nicole.* 2nd ed., translated by Thomas Spencer Baynes. Edinburgh: Sutherland & Knox, 1851.

°Prantl, Carl. *Geschichte der Logik im Abendlande.* 4 vols. in 3. Leipzig: S. Hirzel, 1855–70.

°Ramus, Peter. *Dialecticae libri duo.* Francofurti, apud Andream Wechelum, 1579.

°Reid, Thomas. *The Works of Thomas Reid: Now Fully Collected, with Selections from His Unpublished Letters.* (8th ed. 2 vols. Edinburgh: MacLachlan & Stewart; London: Longman, Green, Longman, Roberts, & Green, 1880.) 5th ed., edited by William Hamilton. Edinburgh: MacLachlan & Stewart, 1858.

°Robertson, George Croom. *Hobbes.* Edinburgh and London: William Blackwood & Sons, 1886.

°Royce, Josiah. *The Religious Aspect of Philosophy. A Critique of the Bases of Conduct and of Faith.* Boston and New York: Houghton Mifflin Co., 1885.

Schelling, Friedrich. *Vorlesungen über die Methode des academischen Studium.* Tübingen: J. G. Cotta, 1803.

°Schröder, Ernst. "Exposition of a Logical Principle, as disclosed by the Algebra of Logic, but overlooked by the Ancient Logicians." In *Report of the Fifty-Third Meeting of the British Association for the Advancement of Science,* 412. London: John Murray, 1884.

°Shakespeare, William. *The Works of William Shakespeare.* 12 vols. Edited by Richard Grant White. Boston: Little, Brown & Co., 1857–65.

Sharp, John. *A Discourse of Conscience. The Second Part. Concerning a Doubting Conscience.* London: Printed for Walter Kettilby, at the Bishops Head, 1685.

South, Robert. *Interest Deposed, and Truth Restored.* 2nd ed. Oxford: W. Hall for George West, 1668.

Spencer, Herbert. *First Principles.* 1st ed. London: Williams and Norgate, 1862.

°Spenser, Edmund. *The Poetical Works of Edmund Spenser.* 5 vols. Boston: Little & Brown, 1839.

———. *The Principles of Biology.* 2 vols. New York: D. Appleton and Co., 1864–67.

Stewart, Dugald. *Dissertation: Exhibiting the Progress of Metaphysical, Ethical, and Political Philosophy.* Vol. 1 of *The Collected Works.* 2nd ed. Edited by Sir William Hamilton. Edinburgh: Thomas Constable and Co., 1854.

Sydenham, Thomas. *Medical Observations Concerning the History and Cure of Acute Diseases.* In vol. 1 of *The Works of Thomas Sydenham. Translated from the Latin Edition of Dr. Greenhill with a Life of the Author by R. G. Latham.* London: Sydenham Society, 1848.

°Sylvester, James Joseph. "On an Application of the New Atomic Theory to the Graphical Representation of the Invariants and Covariants of Binary Quantics." *American Journal of Mathematics* 1 (1878): 64–82.

Tait, P[eter] G[uthrie]. *Sketch of Thermodynamics.* 2nd ed. Edinburgh: David Douglas, 1877.

Taylor, Jeremy. *The Whole Works of the Right Rev. Jeremy Taylor, D.D.* 10 vols. London: Longman, Brown, Green, and Longmans, 1851.

°Tennyson, Alfred. *Poems.* A new ed. 2 vols. Boston: Ticknor & Fields, 1848.

Thomas Aquinas. *Summa theologica.* 17th ed. Edited by Nicolai [et al.]. Paris: Bloud and Barral, [1856].

°*The Thousand and One Nights, Commonly Called, in England, The Arabian Nights' Entertainments.* A new translation from the Arabic with notes, by Edward William Lane. Illustrated by W. Harvey. 3 vols. London: Charles Knight & Co., 1840–41.

Vasari, Giorgio. "Michelagnolo Buonarroti." In vol. 5 of *Lives of the Most Eminent Painters, Sculptors, and Architects.* Translated by Mrs. Jonathan Foster. London: George Bell & Sons, 1871.

°Venn, John. *Symbolic Logic.* London: Macmillan & Co., 1881.

Waldo, Leonard. *Multiplication and Division Table.* New York: J. Wiley & Sons, 1880.

°William of Ockham. *Summa logicae.* Paris: Johannes Higman, 1488.

Chronological List 1884–1886

Three kinds of materials are included in this list which (save the twenty-five manuscripts at the beginning) covers the middle of 1884 through the end of 1886:

1. All of Peirce's known publications, identified by P followed by a number. For these numbers and for further bibliographical information, see *A Comprehensive Bibliography of the Published Works of Charles Sanders Peirce,* 2nd ed. rev., ed. Kenneth Laine Ketner (Bowling Green, OH: Philosophy Documentation Center, 1986), the letterpress companion volume to the 161-microfiche edition of Peirce's published works.

2. All of Peirce's known manuscripts, typescripts, and annotated offprints, identified by MS followed by a number. These numbers reflect the Peirce Edition Project rearrangement and chronological ordering of the Peirce Papers, the originals of which are in the Houghton Library of Harvard University, and of papers found in other collections. Parentheses after the MS number give either the name or location of those collections, or they identify the Harvard manuscript number. For the latter, see Richard S. Robin, *Annotated Catalogue of the Papers of Charles S. Peirce* (Amherst: University of Massachusetts Press, 1967) and "The Peirce Papers: A Supplementary Catalogue," *Transactions of the Charles S. Peirce Society* 7 (1971): 37–57.

3. Those letters and letter drafts that are included in the edition, identified by L followed by a (Harvard) number. Parentheses give the location of letters not contained in the Peirce Papers.

Not included here are those items in *A Comprehensive Bibliography* that merely mention Peirce's Coast Survey duties and observations (in the annual *Report of the Superintendent of the United States Coast Survey*) or give mere titles or descriptive notes of papers which were presented at professional meetings or on other occasions, but for which there are no manuscripts.

The twenty-five manuscripts at the beginning of the following list (MSS 466–490) belong to Peirce's "Study of Great Men," begun at the Johns Hopkins University in the fall of 1883 (but not yet completed when he left the university less than a year later). As it came to preliminary fruition only in the late fall of 1884, it was decided to include the "Study" in the present volume: and, as the later manuscripts are closely connected with the earlier ones, to include the latter in the chronological list of the present volume (rather than that of volume 4).

Manuscripts and a few rarely republished items that have appeared in earlier editions are identified in brackets at the end of the given entry. CP refers to the *Collected Papers;* HPPLS to *Historical Perspectives on Peirce's Logic of Science: A History of Science,* ed. Carolyn Eisele (Berlin: Mouton, 1985); N to *Charles Sanders Peirce: Contributions to THE NATION,* 4 parts, ed. Kenneth L. Ketner and James E. Cook (Lubbock: Texas Tech Press, 1975–88); and NEM to *The New Elements of Mathematics,* 4 vols., ed. Carolyn Eisele (The Hague: Mouton, 1976).

Dates of publication or composition appear to the right; those in italics are Peirce's own. Descriptive or supplied titles are given in italic brackets, and journal titles are abbreviated. Letters and numbers in boldface indicate that the item is published in the present volume.

MS 466 (1120) — fall 1883–fall 1884
[Early List of Great Men].

MS 467 (1120) — fall 1883–fall 1884
[Preliminary List of Great Men].

MS 468 (1120) — fall 1883–fall 1884
[Additional List of Great Men].

MS 469 (1120) — fall 1883–fall 1884
Materials for a List of 300 Great Men.

MS 470 (1120). Item 2 — fall 1883–fall 1884
Materials for an Impressionist List of Great Men.

MS 471 (1120). Item 3 — fall 1883–fall 1884
My list of great Men. [Cf. MSS 466–468 and 472–474.]

MS 472 (1120) — fall 1883–fall 1884
[List of Great Men, A].

MS 473 (1120) — fall 1883–fall 1884
[List of Great Men, B].

MS 474 (1120) — fall 1883–fall 1884
[List of Great Men, C].

MS 475 (1120). Item 4 — fall 1883–fall 1884
[Men of Feeling, Action, Thought].

MS 476 (1120) — fall 1883–fall 1884
[Notes on Montaigne]. [Cf. MS 488.]

MS 477 (1120, L 482) — fall 1883–fall 1884
[Notes on Great Men].

MS 478 (1120). Item 5 — fall 1883–fall 1884
[Notes on Archimedes, Abel, Lagrange, and Gauss].

MS 479 (1120). Item 6 — fall 1883–fall 1884
[Notes on Leonidas].

MS 480 (1120). Item 7 — fall 1883–fall 1884
[Notes on Mencius].

MS 481 (1120). Item 8 — fall 1883–fall 1884
[Notes on Michelangelo].

MS 482 (1573, 1278). Item 9 — fall 1883–fall 1884
/Notes on Ockham and Machiavelli/.
MS 483 (1120, 1278, 1573). Item 10 — fall 1883–fall 1884
/Notes on Pythagoras/.
MS 484 (1120). Item 11 — fall 1883–fall 1884
/Notes on Rabelais/.
MS 485 (1120). Item 12 — fall 1883–fall 1884
Questions on Great Men /First Questionnaire/.
MS 486 (1120) — fall 1883–fall 1884
/Questionnaire Responses for Short List of Great Men/.
MS 487 (1120) — fall 1883–fall 1884
/Assorted Lists of Rankings of Great Men/. [Cf. MSS 525 and 526.]
MS 488 (1120). Item 16 — fall 1883–fall 1884
/Questionnaire Responses for Montaigne, Palissy, Machiavelli, and Lessing/. [Cf. MS 476.]
MS 489 (1120). Item 17 — fall 1883–fall 1884
/Questionnaire Responses for Short List of 48 Great Men/.
MS 490 (1120). Item 18 — fall 1883–fall 1884
/Questionnaire Responses for Short List of 24 Great Men/.

P 290. Item 1 — 1884
"Determinations of Gravity at Allegheny, Ebensburgh, and York, Pa., in 1879 and 1880." *Coast Survey Report 1883,* 473–87.
MS 506 (527, 1573). Item 20 — summer 1884
On the Algebra of Logic: Part II.
MS 507 (747, 547, 278). Item 21 — summer 1884
/Fragment on the Algebra of Logic/.
MS 508 (527). Item 22 — summer 1884
On the Algebra of Logic (Second Paper).
MS 509 (547) — summer–fall 1884
/On the Algebra of Logic/.
MS 510 (911, 1369) — summer–fall 1884
/On Three Kinds of Signs/. [Cf. section I of P 296.]
MS 511 (230) — summer–fall 1884
/Geometric Calculations/.
MS 512 (1120) — fall 1884
Questions on Great Men. [Cf. MSS 516 and 518.]
MS 513 (1100) — fall 1884
On Small Differences of Sensation. By C. S. Peirce and J. Jastrow. [Typescript of P 303. See MS 565.]
L 197 (National Archives). Item 23 — *1 October 1884*
Letter, Peirce to J. E. Hilgard.
MS 514 (National Archives) — *4 October 1884*
/Peirce's 1883–84 Coast Survey Report/. [See P 311.]
P 303. Item 24 — 17 October 1884
"On Small Differences of Sensation," by C. S. Peirce and J. Jastrow. *Memoirs of the National Academy of Sciences* 3 (1885): 75–

83. [CP 7.21–35. See also the abstract in the *Johns Hopkins University Circulars* (4 [Jan. 1886]:46) and the notice in *Mind* (11[1886]:128), as well as MSS 513 and 565.]

MS 515 (S 102) October–November 1884
The Old Stone Mill at Newport. [See P 293.]

MS 516 (1120). Item 13 *8 November 1884*
Questions on Great Men *[*Revised Questionnaire*]*. [Cf. MSS 512 and 518.]

P 292. Item 25 14 November 1884
"The Numerical Measure of the Success of Predictions." *Science* 4, 453–54. [NEM 3:682–83; HPPLS 580–81. For a German version, see O 304.]

MS 517 (National Archives) *14–21 November 1884*
Comparison of Thermometers.

MS 518 (1120) November–December 1884
*[*Questions on Great Men: Toward the Printed Questionnaire*]*. [Cf. MSS 512 and 516.]

MS 519 (Max H. Fisch Papers) November–December 1884
*[*Questions on Great Men: Printed Questionnaire*]*. [See pages 61 and 62 in this volume.]

MS 520 (1119). Item 14 November–December 1884
*[*Questions on Great Men: Final Questionnaire, with Instructions*]*.

MS 521 (1119) November–December 1884
*[*Questionnaire Responses for Great Men and Women*]*.

MS 522 (1119). Item 15 November 1884–1890
*[*Questionnaire Responses for Michelangelo, Hobbes, and Locke*]*.

P 293. Item 26 5 December 1884
"The 'Old Stone Mill' at Newport." *Science* 4, 512–14. [Cf. MS 515.]

P 279a *15 December 1884*
"The Sugar Problem." *New York Evening Post,* 16 December 1884, p. 2. [Same as P 279b.]

P 279b. Item 27 18 December 1884
"The Reciprocity Treaty with Spain." *Nation* 39, 521. [N 1:65–66.]

MS 523 (National Archives) *22 December 1884–18 February 1885*
*[*Smithsonian Pendulum Observations and Computations*]*.

MS 524 (National Archives) *27 December 1884–12 February 1885*
Sheet Readings: Smithsonian.

MS 525 (1573, 1120) 1884
*[*Great Men: Classifications and Rankings*]*. [Cf. MSS 487 and 526.]

MS 526 (1120). Item 19 1884
*[*Great Men: Classifications and Rankings*]*. [Cf. MSS 487 and 525.]

MS 527 (839) 1884
*[*Notes on Algebra and Logic*]*.

MS 528 (1580, 278, 747) 1884
[Mathematical Definitions].
MS 529 (747) 1884
[From page 5 of Schopenhauer's The World as Will and Idea].
MS 530 (1599) 1884
[Annotated Copies of G. Cantor's Articles in Acta Mathematica 2:4 (1883)].
MS 531 (1095, 839) 1884–85
[Gravity Determinations with Simple and Reversible Pendulums].
MS 532 (1095) 1884–85
Notes on the process of determining gravity by the reversible pendulum.
MS 533 (1095, 1094) 1884–85
Effect of Flexure of Piece carrying Knife.
MS 534 (1095, 278, 1574, 1062, 1120) 1884–85
[On the Flexure of Pendulums].
MS 535 (1095, 839, 1120, 278) 1884–85
[On the Use of the Noddy]. [Cf. P 315.]
MS 536 (1095) 1884–85
[On the Effect of the Flexure of Pendulums]. [Cf. P 316.]
MS 537 (1122) 1884–85
[Advertisement for Lectures on Great Men].
P 306a. Item 28 1 January 1885
"The Spanish Treaty Once More." *Nation* 40, 12. [N 1:67.]
P 306b. 3 January 1885
"The Spanish Treaty Once More." *New York Evening Post,* p. 4. [Same as P 306a.]
P 339. Item 29 24 January 1885
["Testimony on the Organization of the Coast Survey"]. *Miscellaneous Documents of the Senate of the United States* 82 (1886): 370–78. [HPPLS 629–38, excluding last two and one-half pages.]
P 296. Item 30 1885
"On the Algebra of Logic: A Contribution to the Philosophy of Notation." *American Journal of Mathematics* 7, 180–202. [CP 3.359–403. Cf. MSS 506–510 and 538.]
MS 538 (567). Item 31 spring 1885
Notes on the Algebra of Logic. [CP 3.403A–M; all but the last page. Cf. P 296.]
MS 539 (519). Item 32 *20–25 May 1885*
Studies in Logical Algebra.
MS 540 (1369) summer 1885
Our American Plato: a review of Royce's "Religious Aspect of Philosophy."
MS 541 (1369). Item 33 summer 1885
An American Plato: Review of Royce's *Religious Aspect of Philosophy.* [CP 8.39–54, without the first paragraph.]

MS 542 (1369) summer 1885
An American Plato. [Typescript of MS 541.]
MS 543 (National Archives) July–August 1885
*[*Comparisons of Standards*]*.
MS 544 (National Archives) July–December 1885
*[*Gravitation Journal*]*.
MS 545 (895, S 56). Item 34 summer–fall 1885
*[*Notes on the Categories*]*. [CP 1.353; part of second paragraph of Segment 2.]
MS 546 (901). Item 35 summer–fall 1885
One, Two, Three: Fundamental Categories of Thought and of Nature. [CP 1.369–372; without the last seven paragraphs, and four sentences omitted in first paragraph.]
MS 547 (901). Item 36 summer–fall 1885
*[*Measurement Scales and the Absolute*]*.
MS 548 (910). Item 37 summer–fall 1885
Types of Third Degenerate in the Second Degree.
P 300 10 August 1885
"The Coast Survey Investigation." *New York Evening Post,* 14 August 1885, p. 3. [Peirce's letter to the editor also published in *Science* 6 (21 August 1885): 158; i.e. P 317.]
MS 549 (National Archives) 14 August 1885
*[*Peirce's 1884–85 Coast Survey Report*]*. [See P 331.]
P 307. Item 38 3 September 1885
["Clifford's *The Common Sense of the Exact Sciences*"*]*. *Nation* 41, 203. [N 1:68–69.]
MS 550 (1095) October 1885
Corrections to Appendix 15 [P 315]. Report for 1884.
MS 551a (1600) October 1885
*[*Corrected Offprint of "On the Use of the Noddy" (P 315)*]*.
MS 551b (Max H. Fisch Papers) October 1885
*[*Corrected and Annotated Offprint of "On the Use of the Noddy" (P 315)*]*.
MS 552 (1095) October 1885
Corrections to Appendix 16 [P 316]. Report for 1884.
MS 553a (1600) October 1885
*[*Corrected and Annotated Offprint of "Note on the Effect" (P 316)*]*.
MS 553b (Max H. Fisch Papers) October 1885
*[*Corrected and Annotated Offprint of "Note on the Effect" (P 316)*]*.
MS 554 (1370) October–November 1885
*[*Review of Perrin's *The Religion of Philosophy*]*. [Cf. P 308.]
P 308. Item 39 19 November 1885
["Perrin's *The Religion of Philosophy*"*]*. *Nation* 41, 431. [N 1:69–70. Cf. MS 554.]

MS 555 (1368). Item 40 fall–winter 1885
[Kant's Introduction to Logic]. [CP 1.35; first paragraph only.]
MS 556 (1367). Item 41 fall–winter 1885
[Fiske's The Idea of God].
MS 557 (1095) 30 November 1885–18 January 1886
Ithaca Journal Record.
P 311 1885
"Determinations of Gravity by Pendulum Experiments, and Comparisons of Standards in Europe and in the United States." *Coast Survey Report 1884,* 40. [Cf. MS 514.]
P 312 1885
["On the Comparison of the Meter to the Yard"]. Coast Survey Report 1884, 81. [HPPLS 600–601.]
P 314 1885
["Thermometer Corrections"]. Coast Survey Report 1884, 442–43. [Letter, Peirce to Superintendent Hilgard, 29 April 1884.]
P 315. Item 42 1885
"On the Use of the Noddy for Measuring the Amplitude of Swaying in a Pendulum Support." *Coast Survey Report 1884,* 475–82. [Cf. MSS 535, 550, 551a, and 551b.]
P 316. Item 43 1885
"Note on the Effect of the Flexure of a Pendulum upon its Period of Oscillation." *Coast Survey Report 1884,* 483–85. [Cf. MSS 536, 552, 553a, and 553b.]
MS 558 (527) 1885
[Annotated Offprint of "On the Algebra of Logic" (P 296)].
MS 559 (1095) 1885
Calculation for Gautier Pendulums.
MS 560 (1095) 1885
[Calculations for Time Reductions]. [Cf. P 334.]
MS 561 (S 103) 1885
[Prayer of the Church Militant: Timed Writing Exercise].
L 1 (Abbot Papers). Item 44 *31 December 1885* and *11 January 1886*
Two Letters, Peirce to F. E. Abbot.
MS 562 (1066, 1067) 1885–86
[Tables of Excesses and Residuals]. [Cf. P 385.]
MS 563 (1088) 1885–86
On Gravity as an Index of the movements of the Earth's crust.
MS 564 (1089, 1095) 1885–86
Determinations of the Lengths of Decimetre Scales Nos. 3, 4, and 5.
MS 565 (1100) 1885–86
[Corrected Offprint of "On Small Differences of Sensation" (P 303)]. [See MS 513.]
MS 566 (1595, 1575, 1120) 1885–86
[Lexical Notes on Universities].

MS 567 (S 25). Item 45 — *5 January 1886*
Fundamental Properties of Number.
MS 568 (National Archives) — *January–July 1886*
*[*Gravitation Journal*]*.
P 326. Item 46 — 11 February 1886
"Dr. F. E. Abbot's Philosophy." *Nation* 42, 135–36. [N 1:71–74.]
MS 569 (1059) — *23 April 1886*
*[*Asteroid/Planetoid Data*]*.
MS 570 (National Archives) — *4–27 June 1886*
*[*Hoboken Time Observations*]*.
MS 571 (1016) — June 1886–June 1887
Color Studies.
MS 572 (897). Item 47 — summer 1886
One, Two, Three: Kantian Categories.
MS 573 (905). Item 48 — summer–fall 1886
One, Two, Three.
MS 574 (904, 736) — summer–fall 1886
*[*Conceptions of First, Second, Third*]*. [CP 1.337, one paragraph only.]
MS 575 (906, 736). Item 49 — summer–fall 1886
One, Two, Three: An Evolutionist Speculation.
MS 576 (906) — summer–fall 1886
One, Two, Three: An Evolutionist Speculation.
MS 577 (736, 904) — summer–fall 1886
*[*Degrees of Secondness and Thirdness*]*.
MS 578 (906, 909). Item 50 — summer–fall 1886
*[*First, Second, Third*]*.
MS 579 (National Archives) — *9 September 1886*
*[*Peirce's 1885–86 Coast Survey Report*]*. [Cf. P 356.]
P 331 — 1886
"Gravity Determinations and Experimental Researches at Washington, D.C., and in Virginia." *Coast Survey Report 1885,* 37–38. [Cf. MS 549.]
P 334. Item 51 — 1886
"Note on a Device for Abbreviating Time Reductions." *Coast Survey Report 1885,* 503–508. [Cf. MS 560.]
P 335. Item 52 — 1886
"On the Influence of a Noddy on the Period of a Pendulum." *Coast Survey Report 1885,* 509–510. [See MS 587.]
P 336. Item 53 — 1886
"On the Effect of Unequal Temperature upon a Reversible Pendulum." *Coast Survey Report 1885,* 511–12. [See MS 588.]
MS 580 (1247) — fall–winter 1886
The Beauties of Ebratum.
MS 581 (736) — fall–winter 1886
*[*The Three Elements of Reality*]*.

MS 582 (736, 737). Item 54 fall–winter 1886
Qualitative Logic. [CP 7.451–462; chs. 1 and 2 only. NEM 4:101–15; preface, ch. 3 (without variants), and ch. 6 (version entitled The Logical Algebra of Boole). Cf. MS 593.]

MS 583 (1599) December 1886
[Annotated Offprint of A. B. Kempe's "Memoir on the Theory of Mathematical Form" (1886)].

MS 584 (532). Item 55 1886
The Logic of Relatives: qualitative and quantitative.

MS 585 (537). Item 56 1886
An Elementary Account of the Logic of Relatives.

MS 586 (1167). Item 57 1886
[Words in E for the *Century Dictionary*].

MS 587 (1600) 1886
[Corrected Offprint of "On the Influence of a Noddy" (P 335)].

MS 588 (1600) 1886
[Corrected Offprint of "On the Effect of Unequal Temperature" (P 336)].

L 269 (Marquand Papers). Item 58 *30 December 1886*
Letter, Peirce to A. Marquand.

MS 589 (1573, 278, 1059, 1574, 1152, 1177) 1886–87
[Dictionary Definitions].

MS 590 (1078) 1886–87
[Notes on International Weights and Measures].

MS 591 (1085) 1886–87
[South American Weights and Measures].

MS 592 (New York Public Library) 1886–87
A classified list of modern pounds, perhaps a quarter of these in recent use.

MS 593 (839, 963, 1317) 1886–90
[Qualitative Logic]. [Typescript of MS 582.]

Essay on Editorial Method

Like the first four volumes of *Writings of Charles S. Peirce,* Volume 5: 1884–1886 (W5) presents a wide range of topics and forms. Peirce's topics range from biography to political commentary, geodesy to applied physics, psychology to cosmology, and logic to philosophy. The results of these studies are expressed as statistical analysis, technical report, book review, political editorial, logical argumentation, and mathematical proof. The editing challenge provided by this range of materials is magnified by the fact that thirty-nine of the fifty-eight items in this volume are published in toto for the first time.

Consequently, the techniques of editing the two general kinds of materials, published and unpublished, vary according to the nature of the work and the circumstances of its creation. The previously published items included in the present volume reveal fairly diverse textual histories—sixteen items appeared during Peirce's lifetime in six different publications, each with its own house style. Later, Peirce made alterations in offprints for five of these items. The remaining forty-two items were not published during his lifetime, but the textual history is no less diverse: Peirce wrote thirty-two of them exclusively by hand; the copy-text for nine more was prepared wholly on a typewriter, and all but two of these (items 41 and 45) contain alterations in pen; and copy-text for one (item 49) is split between manuscript and typescript.

Though many manuscript and typescript pages have had to be reassembled, most of them are in good physical condition. But some chipped, folded, or torn edges with cellophane tape do occur. There are also brittle, singed, or brown leaves, soiled pages, water smudge damage, ink smears, spots, or blots, and carbon copy smudging.

Special Editing Issues

Nearly all the unpublished items in W5 pose editing problems based on different aspects of Peirce's working methods and the ordering system under which the manuscript pages have survived. As Peirce left them, these items are working papers to some degree, and they remain unpolished; eight of them are fragmentary or incomplete (excluding the anomalous instance of fragmentation in item 17, discussed in its headnote). One item, "Qualitative Logic" (item 54), includes what may be called discrete versions of the text. Although there are what at first appear to be different "drafts" of large sections of four chapters in item 54, closer analysis suggests otherwise. Besides occasionally generating genuine "drafts" or revised leaves for sections of text throughout the project, Peirce often was inclined to start again and thus to create a parallel chapter that may repeat the title, or have the same or similar opening sentence, but then diverges from the direction of the first to make a different, complete version of the material. Such discrete versions—more common in the first four volumes of the present edition—are only found in item 54 in this volume, and they continue to illustrate Peirce's practice of composing more than one discrete version of a paper or chapter, and are sufficiently well developed to warrant inclusion in W5.

Editing item 54 involved the reordering of a complex, nearly book-length portion of the manuscript as it exists in the Peirce Papers at Harvard. Two other large-scale projects in this volume are scattered through an even broader section of the Harvard collection, and items selected from these materials required a leaf-by-leaf examination of the general folder chronology established by Robin. Items 2–19 represent selections from Peirce's study of great men that had to be recovered and then internally reordered to establish a sense of the chronology and an understanding of the statistical approaches that Peirce tried in successive stages of his study. The last major project in this volume is represented by eight interrelated items. Items 34–37 and 47–50 record two foundational stages, roughly a year apart, of Peirce's unpublished book on the categorial foundation of science. In the latter case as well, distinct items in the series are scattered through the Harvard folders. Often, the recovery process does not yield complete documents. Some of these items consist of self-contained fragments which are complete enough to stand alone, but which must be reordered to recover Peirce's final intention as much as possible. The portion of this essay devoted to copy-text decisions identifies the rationale used in editing these problematic groupings.

As noted earlier, the majority of Peirce's manuscripts in W5 are "working copies." Most of them reflect his editing or altering with excisions, additions, interpolations, transpositions, false starts, unfinished revisions, and memos to himself. Though he continues to work toward clarity of meaning and felicity of style, occasionally he still omits letters from words. At other times, and especially in more hastily inscribed items, he repeats a character or syllable. In the relatively complete but unpublished items, editorial emending has been somewhat less necessary. Peirce clearly remains conscious of the value of details in spelling and punctuation, as well as of the importance of accurate choices for words from several possibilities. Relatively little punctuation, such as a missing dash, parenthesis, bracket, or quotation mark, has required emendation. Peirce has generally ripened his work and thus forestalled unwanted pruning.

Transcription

The first step in editing and preparing manuscripts for publication is the transcription process. To assure readers that they have before them what Peirce actually wrote, certain transcription guidelines based on modern editorial theory and on the available working materials are followed.

The Peirce Edition Project owns a microfilm copy of the Charles S. Peirce Papers deposited in the Houghton Library of Harvard University, as well as two sets of photocopies that are more complete than the microfilm. We also own photocopies of all known letters and manuscripts in other libraries and archives. (However, uncatalogued material in other depositories remains to be examined and acquired.) A physical description of each manuscript is included with our photocopies. The legibility of our photocopies is generally good, partly because Peirce used either black or dark blue ink as his basic writing medium. When he used graphite pencil or different colors of ink and pencil for revisions and annotations, the legibility of our copies lessens markedly; two or three of his carbon-copy typescripts are very nearly illegible, even in the original in the Houghton Library. After the initial transcriptions are made from the photocopies, in which process the microfilm is often consulted, they are team-proofread twice by two different teams of editors against the photocopies, and passages that are difficult to read are marked. One of the editors then verifies the text by rereading the transcripts aloud to a second person against the originals in the Houghton Library (or another depository), paying particular attention to the "questionable" and "difficult to read" passages. At that time such features as holes, ink blots, or colored pen and pencil contained in the originals but not distinguishable on the copies are noted. The transcripts are then revised to coincide with the original manuscripts.

During the process of transcription, Peirce's revisions, misspellings, misplaced or omitted punctuation, and the like are typed or described as they appear on the manuscript page. All inscriptions, unless otherwise noted, are by Peirce. As we move from initial transcription to the copy-text editing copy, material that Peirce crossed out is omitted, as is accompanying punctuation that he failed to delete; his careted-in revisions are inserted, passages he marked for transposition are transposed, and his instructions for moving large blocks of material are followed. For the most part, Peirce clearly marked such revisions and instructions. When he did not, the following guidelines for resolving difficulties are used.

If the context permits, Peirce's uncertain revisions and unmarked marginal notations that represent revision are inserted into the text. An accompanying textual note indicates the original placement of the material on the manuscript page. When the revision or marginal annotation cannot be incorporated into the text without creating an incoherent reading, it is reproduced in the Textual Notes, again with an explanation of its physical placement within the manuscript. What the reader will not find reproduced in the text are Peirce's own pre-copy-text deletions, his page numbers and doodles, or annotations by other persons. Peirce's instructions are followed but not reproduced.

Selection of Copy-Text

The items in this volume published by Peirce during his lifetime pose no problems regarding copy-text, though the rationale for the choice of the later rather than earlier publication in the instance of "The Reciprocity Treaty with Spain" is discussed in the textual headnote to item 27. Determining copy-text for Peirce's unpublished manuscripts has been relatively unproblematic, but three specific groups of items, as well as item 54, require detailed analysis.

ITEMS 2–19. *[*STUDY OF GREAT MEN*]*

The Great Men manuscripts all stem from a study that Peirce began during his final year at the Johns Hopkins. But how does this open-ended statistical study of comparative biography fit into the chronology of the present edition? His fall 1883 course on the psychology of great men provided the subject matter, and the manuscripts show that he and his students continued to work informally on the study through the spring, summer, and fall of 1884 and even later; some of the documents—for example, the Locke questionnaire in item 15—cannot have been completed before 1890. Although the earliest of the manuscripts in the Great Men series undoubtedly coincide with unrelated documents published in W4, all the items selected from the series are grouped in the beginning of W5 to give a sense of the focus and development of the entire Great Men project.

Peirce's project went on intermittently for a number of years, and the surviving portions are scattered throughout several folders in the Harvard collection. Most of the material is contained in Harvard MSS 1120a and 1120b, but some is found in MSS 1119, 1122, 1122s, 1278, and 1573. Few of these papers contain dates; a number were reworked periodically, especially at the turn of the century when several published studies on great men prompted Peirce to return to his project. The result was a revised list of great men and a survey of "The Century's Great Men of Science" (P 760; 1901), scheduled for a subsequent volume of our edition. But the great majority of leaves were composed during the early days of the project, and can be grouped into sequences that, even if not precisely dated, bring out the basic character and progression of the Great Men study.

Thirty-one distinct documents have been recovered and reconstructed from the Harvard folders. The earliest of these are twelve impressionist lists used to define the data base of great men. Some are drafts, and others fragments of larger lists, indicating that Peirce and his students invested much time and care in this preliminary stage. Three of the narrowed lists (items 2–4) representing different organizational approaches are published here. (A possible final listing, leaves 102–115 of MS 1120a, was heavily annotated when Peirce revisited the material at the turn of the century. This list no longer retains the character of the early compositions, and must be considered a document of the later series of great men materials.)

Nine of the recovered documents are biographical notes and sketches of great men. Although the dating is uncertain—the paper suggests that some items were written during the early phase—all nine are grouped together. Seven texts from this grouping are published here as items 5–11.

There are six versions or fragments of questionnaires. Again, one or more of these may have been used near the beginning of the project but, in the absence of textual evidence, they are grouped as a distinct phase of the study. Three questionnaires reveal the evolution of this phase, and they are published here as items 12–14. The final questionnaire (item 14) appears in the Harvard folders with a number of filled-in questionnaires on great men. Only a few are complete, and these are published as item 15. Four other sets of answers and study results have been recovered from other portions of the Harvard folders, and these round out the series as items 16–19. Textual headnotes for the great men selections account for discarded pages in these documents, as well as for some of the documents not published here.

Throughout the series, the documents present the physical characteristics of statistical research: the great names remain at the center of the process, but charts, listings, and evaluative categories replace the normal narrative structure of biographical evaluation. This hybrid form of manuscript—halfway between scientific data base and comparative biography—presents special editing problems in a number of items.

Item 2 consists of a roughly alphabetical progression of names that Peirce sorted into categories of relative importance. The category headings vary slightly from page to page, but the basic template remains throughout, permitting emendation of the variant headings without altering Peirce's data or intent. For a number of names, Peirce subsequently added (prefixed) punctuation marks and other prefix and suffix symbols which, taken together, represent a weighting system. Although the specific meaning of many of these symbols is unclear today, they are generally retained as part of the developing data base. In item 3, Peirce used an alphabetical listing to assign a numerical weighting system to some names. These numbers are also found in items 6 and 17, clearly representing an evolution of the weighting symbols in item 2, and are retained here and throughout the remaining items of the series.

But item 3 and other pieces also carry markings by Peirce that are clearly peripheral to content. Here, for instance, he has checked off nearly every name and marked a few with a question mark. The end result is the cancellation of some names and the insertion of others; these revisions are considered to be ordinary alterations, but the directive checks and question marks are considered extra-textual and are silently removed. The silent removal of these and any other extra-textual marks (such as dividing lines and rules) is noted in the relevant textual headnotes.

In item 4, the rough inscription of the manuscript and Peirce's heavy alteration of the data require reformatting of the copy-text to restore clarity and coherence. The item consists of three general groupings of occupational categories, one grouping to a page; each page contains specific occupational headings, and each heading tops a column list of great men from that specific occupation. The expanding lists of names eventually overran column headings and page margins. As a consequence, preservation of the sequence of the headings is secondary to establishing a clear reading text of the chart. The categories of *Actor* (35[c4].23), *Other* (35[c4].27), *Lawyer* (36[c4].12), *Other* (37[c4].14), and *Historian* (38[c3].5) have been shifted under adjacent columns to restore the margin boundaries and provide adequate column spacing. In the final chart, titled "Men of Thought," the names of philosophers, mathematicians, physicists, and religionists overflow into columns of other categories. Peirce's directional lines establish placement within the three-tiered chart, but the increased length has forced the second and third tiers onto a separate page.

Items 5 through 11 are fairly straightforward narrative notes on a number of great men, and they reveal the way Peirce and his students abstracted biographical information for use in the evaluative questionnaires that mark the next phase of the project. Item 12 (MS 485) comprises the earliest surviving questionnaire. It is a rough working draft of 69 evaluative questions grouped under eleven categories. The categories follow a biographical progression, from ancestry through the stages of life and onto an assessment of impact and genius, with a final category on character and self-esteem. Although the questions are arranged in this order, they retain non-sequential numbers based on an earlier unsorted list or "bank" of questions (a single-page fragment of such a list survives as the last leaf of MS 485). Item 13, a revised version of the questionnaire dated 8 November 1884, lacks numbers for any of the categories or questions. It contains 83 questions and recombines them into nine categories; the category on character is the largest with 32 questions, including many on genius and personality.

Peirce had the final version of his questionnaire printed, and it is reproduced on pages 61–62. It retains the nine categories but abbreviates the questions into phrases or single words, possibly to provide maximum space for annotation. The two-page format accommodates 116 questions; the second page is almost exclusively devoted to questions of character, now totalling 49. Item 14 is a copy of the form with generic comments by Peirce to clarify some of the abbreviated questions. The questionnaire itself is a frame for Peirce's comments; his words, and not the form, represent copy-text. Consequently, only entries bearing Peirce's commentary are reproduced. The printed words or phrases for these entries appear in boldface; the full questionnaire on pages 61–62 allows complete reconstruction of the manuscript.

Item 15 includes individual questionnaire responses for Michelangelo, Hobbes, and Locke. Again, only questionnaire entries followed by Peirce's responses are reproduced, with boldface text for the form's printed words and phrases at the beginning of each entry. Marginal and verso comments by Peirce are not considered copy-text, and appear in textual notes.

Evidence for Peirce's further development or use of the final questionnaire has not survived, but a different line of research using the earliest questionnaire (item 12) led to several comparative assessments of great men. Item 16 includes handwritten responses for Montaigne, Palissy, Machiavelli, and Lessing. Peirce followed the shorter list of questions as arranged in item 12 (MS 485), using the numbers alone to stand for the questions. The composition is sketchy and rough, with many insertions and overruns. Marginal notations that are not direct responses to questions are reproduced in textual notes.

Items 17 and 18 (MSS 489 and 490) chart answers to the item 12 questions for a number of the great men identified in the earliest items of the series. Item 17 consists of twelve copies of a two-page printed form containing a vertical list of 48 great men. Two copies are blank or unannotated, but each of the others includes specific responses to two or three of the MS 485 questions identified by number (and a version of the question) at the top of the form. Template matter such as category headings (omitted by Peirce) and the wording of the questions written at the top of each form (sometimes fragmentary) have been taken from MS 485; this standardization of the layout is recorded in the Emendations. As this working manuscript (MS 489) has no compelling internal order, the pages have been arranged to follow the topical order of MS 485. Lines and other marks used to divide or align the responses are identified in the headnote and silently removed.

Whereas questions answered in item 17 correspond to a 22-question section from the second half of item 12, item 18 responds to a 20-question section from the first half of item 12, but only covers 24 of the 48 great men charted in item 17. Peirce used a hand-written four-column field with six names to a column to chart each set of responses, thus repeating the form two or three times per page. The manuscript has been reformatted into the single chart structure of item 17, and template headings are again introduced by emendation from item 12. Calculations and a partial chart of death ages are recorded in textual notes.

Item 19 consists of a classified ranking of 39 great men in four columns; a classified ranking of the same group (excluding Morphy, but adding Constantine, King David, and Humboldt) in one column; and a charted ranking of the second list (excluding Grotius, but adding Comte). The rough and irregular layout of the four columns in the first list has been aligned, and dividing lines have been silently removed.

ITEM 24. "ON SMALL DIFFERENCES OF SENSATION"

Peirce wrote "On Small Differences of Sensation" with Joseph Jastrow, one of his students at the Johns Hopkins, of whom he said in a letter to President Gilman on 7 March 1884: "I have guided and aided Mr. Jastrow, one of my pupils, in an experimental enquiry of a logico-psychological nature, which has certainly been advantageous to him and which will be an addition to science of some consequence." Jastrow later noted that he and Peirce "participated equally as subject and observer" (*A History of Psychology in Autobiography,* ed. Carl Murchison [Worcester, MA: Clark University Press, 1930], 1:136) and, in an obituary of Peirce, he said: "It was

Mr. Peirce who introduced me to the possibility of an experimental study of a psychological problem. He provided the problem, the instruments which I set up in my room, the method, and the mode of reaching the results; these were printed over our joint names" (*Nation* 98 [14 May 1914]: 571). Peirce and Jastrow presented the article on 17 October 1884 at the meeting of the National Academy of Sciences in Newport, RI, under the title "On Minimum Differences of Sensation." MS 513, a sixteen-page green-ribbon typescript headed with the present title, contains unfinished charts with data points added later by an unknown hand in pencil and ink. The finished article (P 303) appeared in *Memoirs of the National Academy of Sciences* (1884), and there is a corrected offprint (MS 565) in the Harvard Papers.

The many charts in MS 513 were not expanded (or in some cases even completed) with the same view toward publication as the text, and may be regarded as pre-copy-text notes toward the completed article. The finishing and rearrangement of the data and charts occurred only in P 303, which reflects the true focus and development of the article; consequently, P 303 is accepted as copy-text. Peirce's revisions found in the offprint are accepted as emendations to the copy-text. But for some reason Peirce failed to correct the misplacement of two charts from the main body of the texts that, perhaps due to the confusing arrangement of the charts in MS 513, were erroneously appended to a footnote in P 303. As this placement makes no sense in the context of the footnote, the charts have been placed (according to the sense of the reading in MS 513) at 127.5 in the present edition.

ITEMS 34–37 AND 47–50

Items 34–37 are a related series of manuscripts written in 1885 that represent Peirce's return (his first in several years) to the matter of categories. Item 34, the fragmentary "Notes on the Categories," is presented in four segments, the first lacking a complete beginning and end, and the following three reassembled from a number of leaves torn from a tablet. Peirce did not paginate the text, and the number of missing pages is unknown. The surviving segments were arranged in a topical sequence based on an analysis of the internal evidence.

Items 47–50 represent a second phase of activity focused on the description of the categories and, taken together, they form the beginning of a book-length project on evolutionary cosmology eventually titled "A Guess at the Riddle." Item 49, "One, Two, Three: An Evolutionist Speculation," is comprised of two reassembled typescript leaves and six autograph pages on loose-leaf paper. The typescript leaves (leaves 2 and 2_1 of Harvard MS 906) incorporate Peirce's holograph corrections from an earlier typed version (leaves 3–4 of Harvard MS 906); both typescripts break at the same point, leaving Peirce's handwritten leaves (leaves 216–224 of Harvard MS 736) of the most complete manuscript form as copy-text for the rest of the document. A major variant fragment of the manuscript section is reproduced in the Textual Notes.

ITEM 54. QUALITATIVE LOGIC

Item 54 is nested in a consecutive section of the Harvard folders, but the complex manuscript (MS 582) contains a number of parallel sections, restarts, and false starts that require significant reordering of the copy-text. The internal evidence contained in Peirce's own layers of revision, together with the external evidence of a subsequent typescript fragment (MS 593), supplies the basis for critical editing.

The Harvard sequence of the manuscript presents an obscured view of Peirce's recursive and intense pattern of re-writing. Reordering has been accomplished by analyzing the choices of variant sections within the manuscript, and by consulting the typescript, which includes the preface, the first three chapters, and part of the fourth (as well as many transcription errors and revisions not by Peirce); the typescript reveals that Peirce reordered and discarded a number of leaves from the first half of the work, and even brought leaves forward from later portions of the manuscript, all without rewriting the text. Moreover, the typescript is useful in indicating the reordering of the manuscript leaves without presenting a new and discrete version of the first four chapters, thus allowing the reordered manuscript to stand as copy-text for the entire work.

"Qualitative Logic" was a book-length project with ten chapters (according to Peirce's table of contents), but only six chapters and a preface survive. The preface (leaves 3–13) consists of a single consecutive version, but the rest of the manuscript requires careful editorial attention. Chapters I and II grew out of a common draft sequence of Chapter I (leaves 27–34). Peirce subsequently drafted a longer Chapter I (leaves 14–26) and a new start on Chapter II as a distinct title (leaves 42–44); his final extant version of Chapter II is in leaves 35–41. Only the final version of the first two chapters is included in the present edition.

Chapter III, "The Modus Ponens," presents a complex textual history requiring large-scale reordering. Peirce's initial draft of the chapter runs from leaf 56 through 64 (leaf 64 restarts leaf 61); a later draft (leaves 45–55) develops a fuller discussion of the modus ponens, and is clearly Peirce's choice for the first portion of the chapter—the subsequent typescript begins with this second version. But the typescript goes on to include the final five leaves of the first version (60, 64, 61–63), as well as seven leaves from the last portion of the manuscript (leaves 280–281, 205–208, 201, 279). Leaves 280–281 and 205–208 form a coherent fragment that describes important distinctions of terms involving the modus ponens; but leaves 60–64 involve applications of these distinctions, and form a coherent fragment only when placed with the leaves (201 and 279) following the discussion of terms contained in the first fragment. This rearrangement, which completes the process Peirce initiated in the typescript, requires the deletion of two surviving leaves from the first version (60 and 64). Consequently, the copy-text leaves are rearranged in the typescript sequence, and leaves 61–63 are further moved to form two coherent and significant fragments that conclude the chapter.

Four attempts at Chapter IV appear in a continuous portion of the manuscript (leaves 65–145). An incomplete fragment, titled "The Traditional Syllogistic" (leaves 122–134), is not published here. A revised version of the same title (leaves 113–121) is complete and, due to the opening transition from Chapter III, precedes Chapter IV proper in this edition. Chapter IV itself is based on leaves 65–112, which contain sequential page numbers (not by Peirce) from 1 through 48. Two major insertions appear in the present edition between leaves 68 and 69. The new paragraph opening leaf 69 was cancelled by Peirce, indicating a possible insertion or restart. Two such passages are identifiable within the larger body of the manuscript. The MS 593 typescript inserts a reordered transcription of leaves 135–145 (less 135–136 and 144–145), originally a fourth version of Chapter IV. Although the typescript ends here, Peirce's cancelled passage on leaf 69 picks up as a restart on leaf 270. Leaves 270–278 form a coherent conclusion to the discussion of complex syllogism introduced in the first insertion. The fact that the paper of these leaves matches that of the earlier rather than the intervening section suggests that a now-lost sequential connection once actually existed. The reordering and insertion of these manuscript leaves into the main body of Chapter IV reflects the revision begun by Peirce.

The typescript does not include the final two chapters of MS 582, and the remaining copy-text was assembled on internal evidence alone. Both versions of Chapter V, "Dilemmatic Reasoning" and "The Dilemma," are complete and, in the absence of evidence to suggest that one discrete version rather than the other constitutes Peirce's final intention, both are published here. His original conclusion for "Dilemmatic Reasoning" (leaf 158 and leaf 2 of Harvard MS 736s) has been replaced with a fuller conclusion (leaves 202–204) identifiable by a restart of leaf 158 on leaf 202.

The "excursus" to Chapter VI is presented in the surviving manuscript sequence (leaves 159–164). Chapter VI proper consists of leaves 167–188, but nearly a hundred leaves (189–281) follow. Peirce used twenty leaves to revise or amplify Chapters III–V as recorded above; 28 leaves actually belong to the separate work "One, Two, Three" (item 49); the remaining leaves contain fragmentary material on logical algebra, but only nine are relevant to Chapter VI. Leaf 197 is a restart of leaf 173, and it branches into a set of alternate rules of logic recorded on leaves 189–196. This mature variant fragment parallels the development of Chapter VI and appears at the conclusion of the main text.

The discrete versions of Chapters III–VI accepted for item 54 are the only parallel texts published in W5. As they cannot be collated, each stands as a separate section (as would any other single-text item). They merit publication because they are carefully written, fully developed, and well argued, and each represents a distinct departure in content from parallel items.

General Editing Policies

As mentioned earlier, the wide-ranging nature of Peirce's work and the complexities of his composing process mandate carefully developed editorial policies. With published items, the editing focuses on the effort to locate and eliminate editorial or printing corruptions and thereby to restore Peirce's original intentions. With autograph manuscript items, the focus is on reproducing Peirce's written text as accurately as possible and emending it on the basis of the evidence contained within the items themselves. At times, of course, Peirce will misspeak himself or produce a passage that is confusing or vague. In such cases, there is emendation only in public documents and when his intent is ascertainable. When such certainty is lacking, his eccentricities and anomalies are allowed to stand as he wrote them and, when necessary, explained in a note. The following is a discussion of the guidelines used in emending the texts of Peirce's private and public documents.

Peirce's writings are presented in a clear reading text. His published words are emended when there are errors or inconsistencies, or when the evidence suggests that previous editorial decisions have overturned Peirce's intentions. The unpublished writings require a different editorial approach. They are distinguished as either private or public documents and are then edited accordingly. Private documents include letters, drafts of letters, journals and notebooks, and a few manuscripts. None was ever intended for publication, nor did Peirce intend them for public presentation or consumption. Their content or ideas were for personal use only, as exemplified by items 23, 32, 44, and 58 in the present volume. All other documents are considered public. Besides the universal italic printing of all variables, private documents are emended only in the following four instances: spelling errors are corrected, periods are added at the ends of sentences except before the beginning of a new paragraph, apostrophes are inserted in the plural forms of variables (but not in the possessive forms of proper names or other substantive words), and Peirce's incomplete revisions are completed.*

Public documents include those that conform to genres normally intended for publication, whether or not they are so prepared, and whether they are personal working copies, early drafts, work sheets, or outlines; lectures written, in outline or fair-copy form, for presentation in lecture courses or at public conferences; and all manuscripts or typescripts that contain topics and ideas that are related to those Peirce published and that he would have been willing to submit for public comment and criticism. Such public documents are emended as follows. All spelling and grammatical errors are corrected, incomplete revisions completed, and all variables emended with italics. In addition, all missing punctuation is inserted, and inconsistencies within a document are corrected and noted. Slips of the pen and typographical errors are corrected. The Great Men items, however, have been emended minimally; for although they are public documents, they are prepared (here and there) as though they were private. Some of them have also been reformatted, a procedure that is discussed in the relevant headnotes.

The variant spellings "premiss" and "premise" are allowed to stand ac-

*The only exception occurs in item 32, where a comma is necessary for clarity of meaning (219.19).

cording to Peirce's usage. Other inconsistencies within a public document are corrected. Occurrences of any form of the word "meter" are emended to Peirce's preference for the spelling as "metre." Acceptable nineteenth-century spelling forms in all languages, as well as acceptable variant spellings of proper names, are retained. Our standard reference is the *Oxford English Dictionary.* However, Peirce's preferred spelling of "coëfficient" with the dieresis, which is not cited in the OED, occurs several times in the manuscripts included in W5; all occurrences without the dieresis are emended. In all manuscripts in this period, Peirce inscribed "st," "nd," "rd," and "th" in the superscript position following the appropriate number (for convenience's sake, they are on the line in typewritten pieces). In published pieces, the ordinals are superscripted to conform to Peirce's style; "2^{d}" and "3^{d}" are emended to "2^{nd}" and "3^{rd}." (When abbreviated ordinals are typed on the line, these mechanical exceptions attributable to the typewriter have been changed to superscript ordinals.) Peirce also regularly used the nineteenth-century calligraphic convention of double underlining superscript portions of abbreviations such as "M^{r}" or "1^{st}." In this edition, such conventions remain above the line without underlining and without Peirce's inconsistent use of the period. Decimal points are lowered from mid-line to on-line; multiplication dots are raised to mid-line from the on-line position. Every ambiguous 3-em dash for an omitted word or phrase is placed mid-line. Ambiguously placed terminal punctuation is given with punctuation outside the single terminal quotation mark and inside the double quotation mark. As Peirce's placement of quotation marks (single and double) when used in combination with a comma, colon, or semicolon is not always precise enough to make his intention clear, such combinations are interpreted by present standard punctuation practices. Superscripts and subscripts modifying the same character are stacked vertically rather than diagonally. These procedures are applied to conflicting publishers' styling also. A.M. and P.M., and B.C. and A.D. (with or without periods), are printed, according to modern practice, in small capitals.

As a mathematician and logician, Peirce knew that lowercase variables are to be represented in italic form; he insisted on that in publications, but he rarely underlined all lowercase variables in manuscripts. He was considerably less insistent on the practice for capital variables. When Peirce italicized some variables but left the rest to be changed from roman (or left variables undifferentiated from other text in wholly italic typescripts), they are italicized and noted as emendations. If Peirce was inconsistent in his underlining of variables, emendation to italic is noted as economically as possible. Exceptions regarding these variables, or any other cited readings, are recorded when the rest are cited as emended in a document: so that the original state of each variable can be reconstructed as it was in the copy-text.

Every effort has been made to verify all of Peirce's quotations in editions he is known to have owned or had available, and to identify and verify quotations that are not marked as such in the manuscripts. A quotation is allowed to stand as he gives it, even if it differs from the original. Information on his quotations and their sources may be found in the Notes. Peirce's book and chapter titles and other bibliographical citations are emended to conform to the style of the present edition. When, in referring to his own writings (contained in the present volume), Peirce gives specific page numbers, these are replaced with the appropriate page numbers in the present volume; original page and volume numbers are retained when the publication or manuscript appears elsewhere in the edition. Text that is not by Peirce (as at the end of items 27 and 28) is indicated by vertical marginal lines. They are not used in item 29 which, being a kind of dialogue, does not require such a means of identification.

To reduce the variety of different publishing house styles and to make understandable Peirce's obscure references, title notations are printed to conform to the modern practice of italicizing book and periodical titles and placing chapter and article titles in quotation marks. Lengthy published quotations are restyled to conform to our way of printing extracts.

Manuscript material requires further editorial intervention from time to time, in most cases because of Peirce's revisions. In the process of revising, he sometimes created grammatical errors or at other times failed to complete his intended revision by crossing out a necessary word or phrase but not going back to replace it. In such instances, his original word or phrase is reinstated, along with a citation of each occurrence (in the Emendations). When he added introductory clauses to already inscribed sentences but failed to lowercase the first word of the original sentence, his misplaced capitalization is emended. Abbreviations are left as Peirce gave them, with periods added where needed. Periods are also added at the end of sentences where Peirce's pen skipped or he inadvertently omitted them.

Peirce meticulously collected offprints of his published papers and, as he often corrected and annotated them, they provide an important source of emendation of the text. Further sources for emendation of published items include Peirce's lists of corrections and published errata lists (as in items 42 and 43), in which he notes oversights or printer's errors.

All emendations described thus far are listed, and the sources for each are cited, in the Emendations list for each item.

In one of Peirce's previously published scientific papers (item 1), and also in one manuscript (item 2), there are lengthy tables with repeated column headings on successive pages. These headings are moved to accommodate different page breaks. Similarly, Peirce and his printers used ditto marks for repeated data within columns in these tables, with full text printed for the first entry on each page. Appropriate text is substituted for ditto marks, and these are used in place of text when page breaks in this edition differ from the original printing. In order to avoid ambiguity between discrete formulas and run-on formulas (those more than one line in length), we do the following: single-line discrete formulas are individually centered on the page, and run-on formulas are broken at operational signs or after quantifiers, aligned at the left, and centered on the longest line.

Titles of Peirce's previously published items are printed in italics, and unpublished manuscript titles in roman type. Untitled published reviews are given titles, which are italicized and set within italic brackets; titles of the reviewed works themselves in such titles are in roman. Descriptive titles of published letters are in italics, of unpublished letters in roman. All titles and headings appear without periods. Such phrases as "By Charles Peirce" or "C. S. Peirce," which appear at the beginning or end of a published paper or unpublished manuscript, are deleted and noted in the Textual Apparatus. The symbols or page-by-page numbering systems that Peirce or his publishers used for footnotes are replaced by a single series of arabic numerals for each paper. In place of varying numbers of ellipsis points to mark omissions in quoted material, standard form is followed. Various printer's conventions for first lines of text or paragraph opening have been dropped, and all items except those that are letters or lists begin with the usual paragraph indention.

Alterations in the Manuscripts

Beginning with volume 5, the textual apparatus for unpublished items contains a new section entitled Alterations, which lists Peirce's own corrections and revisions made during composition or upon review. The purpose of the list is to enable readers to retrace the main steps in the development of the author's thought by providing brief and clear descriptions of those alterations the editors deem relevant for such a reconstruction. The published Alterations list is derived from the Complete List of Alterations in the Manuscripts, which is compiled early on in the editing work and which includes every revision, however minute, that Peirce made. As many alterations are too insignificant to be worth reporting, and as publishing a complete list would unnecessarily increase the size of our books, we publish only selected lists of alterations, which include all revisions and corrections whose report significantly furthers a close understanding of the evolution of Peirce's thought and its expression. The matter of selection is of course a difficult one. In order to reduce the subjectivity and to strengthen the consistency of the selection as much as possible, the different types of alterations are given precise technical definitions and specific selection rules.*

The fundamental principle underlying the selection is that only those alterations that manifest a "critically significant" change of intention in the process of composition should appear in the published Alterations list. To decide whether a correction is critically significant, the following maxim, inspired by Peirce's own pragmatic maxim, is applied: consider whether the change of intention possibly manifested in the alteration produces an effect which might conceivably modify a reader's perception or understanding of the altered passage; if there is any such conceivable effect, then the alteration is critically significant.

There are eight categories of alterations. (1) Simple insertions: they mostly consist of interlineations, sometimes accompanied by lines or carets

*What follows is a brief summary of André De Tienne's "Selecting Alterations from the Complete Alterations List for Inclusion in the Apparatus," a 29-page paper with 8 pages of charts.

that indicate their precise location; they may also appear in margins or on the line itself. (2) Simple deletions, i.e. excisions of words or parts of words made by blotting out, crossing out, erasing, etc., either before inscribing the next word or at a later time while revising. (3) Complex insertions, or insertions that replace deleted material; they can themselves contain internal deletions and insertions, sometimes on many levels, but they are not themselves fully deleted. (4) Superimpositions of letters or words (characters written over one another): they may be defined as the contraction of a complex insertion. (5) Simple transpositions, i.e. some textual unit is moved from one place to another within a bigger textual unit; a subcase of this is inversion, when two consecutive letters or words exchange their position. (6) Complex transpositions, or the combination of a simple transposition with some other kind of alteration, which latter is needed to finalize the transposition itself. (7) Typeface and character-style alterations, made to increase or decrease the emphasis given to a word or set of words. (8) Canceled alterations, which divide into interrupted alterations (alterations that were abandoned before their completion) and complete cancellations (alterations that were abandoned after they were fully carried out).

As these eight categories of alterations resort to only two basic operations, those of removing and of adding, the selection rules that are applied to them rest on a double consideration: whether the textual part that is removed (through deletion or transposition; absent from the critical text) is informative or not, and whether the textual part that is added (through insertion or transposition; present in the critical text) is dispensable or not. A deletion is informative if it carries a critically significant piece of information that is not found in the edition text. An addition is dispensable if it is part and parcel of the author's revised intention and if it is not absolutely necessary for the integrity of the sentence in which it is found. It is indispensable if it belongs to the author's first intention and is necessary for the sentence to function correctly. Alterations lists published in our volumes include only those revisions and corrections that are, at least in some of their parts, critically significant and informative, or critically significant and dispensable, or both at the same time. Thus, there may be manuscripts for which we do not publish an Alterations list, even though they do contain alterations (as in items 13, 14, and 41).

A Note on Peirce's Typewriter

Some observations regarding Peirce's use of the typewriter and his views on unsigned articles are relevant to his generating of texts. As noted in W4, Peirce himself had typewritten in 1881 his note entitled the "Width of Mr. Rutherford's Rulings," and in 1883 a typist for the Coast Survey had typed a copy of the "Report of a Conference on Gravity Determinations" for the compositor. On 14 August 1885, Peirce wrote Coast Survey Superintendent Thorn from Ann Arbor to request a Hammond typewriter to replace his survey party's machine, which had sustained bed-frame damage in trans-port. Use of such a damaged typewriter would likely have contributed to the irregular spacing of Peirce's original typescript of item 41, which dates from fall-winter 1885. It was not until 2 April 1887 that Thorn authorized "the purchase of a Hammond Typewriter for use in preparing memoirs" and, in fact, "decided to supply . . . [him] at once with the latest improved Hammond Typewriter . . . with two sets of type." But as early as summer 1886 Peirce had prepared item 47 on a machine that had roman and italic type. The second typewritten leaf of item 49, which dates from summer-fall 1886, also has italic words in a predominantly roman text. Item 56, also from 1886, is in roman type except for the italic title. All these documents have irregularities in alignment and spacing, probably attributable to machine damage and/or to Peirce's imperfect command of the typewriter. The remaining three typescripts (items 45, 50, and 57), exclusive of the Great Men typescripts (items 12 and 13), also have spacing irregularities due to a faulty typewriter and/or to the lack of skill of the typist, though this is so to a lesser degree in item 45, which has no words run together. Words that are typewritten without space between them or that are inappropriately broken by spacing are not listed as emendations. Irregular spacing of punctuation and characters typed between lines that should be on the line are corrected silently. Characters that are typewritten in the same space are considered consecutive and not emended. Italic typescripts are represented in roman type, and any problem of representation of italic text (such as undifferentiated variables in an italic typescript) is addressed in the headnote to the item or in textual notes. When Peirce intended an arabic numeral one, but there was only an italic font roman numeral on the typewriter, it is represented with an arabic one.

Symbols

Within the Text

Italic brackets enclose titles and other text supplied by the editors (including parts of words in damaged documents that have been reconstructed).

Italic brackets enclosing three ellipsis points indicate one or more lost manuscript pages.

Italic brackets enclosing a blank indicate that an incomplete discussion occurs before the end of the manuscript page.

Sets of double slashes mark the beginning and end of Peirce's undecided alternate readings; the single slash divides the original from the alternative inscription.

Within the Apparatus

All page, column, and line numbers refer to the present edition (running heads, diagrams without text, and drawings do not count as lines). Parenthetical numbers following the line number indicate the specific occurrence of the keyed reading in that line; when the keyed reading refers to all occurrences, no parenthetical numbers are used. Footnotes are counted separately from the text and are indicated by "n" following the page number. In items 2–4, 16–19, and 23, lines are counted individually column by column.

An asterisk (*) preceding page and line numbers indicates that the emendation or alteration is discussed in a textual note.

¶ is the sign of a paragraph indention.

A vertical stroke (|) indicates a line-end break and is used to clarify an emendation or an alteration description; when, in the Alterations, it indicates a page break, it is noted as such.

The word *formula,* used in italic brackets in the lemma, indicates a reading of an otherwise lengthy or intricate citation in which (in emendations) a mark of punctuation or some other careted variant occurs, or in which (in alterations) there are but a few simple changes.

IN EMENDATIONS ONLY:

Readings to the left of the roman bracket (the lemmas) are from the present (critically edited) edition, those to the right from the copy-text or other collated texts. The bracket is followed by an abbreviation or number that identifies the source of the emendation; a semicolon precedes the rejected readings. The source for all readings, other than the editors (E), is given in the headnote for each item.

The abbreviation *et seq.* after a page and line number indicates a "class-emendation": all readings subsequent to the cited reading are identically emended (and exceptions are listed after the word *Except*). If the same emendation occurs more than once, but in scattered places in a given item, the additional instances are listed after the word *Also*.

The abbreviations *ital.* and *rom.* indicate that the reading to the left of the bracket was originally printed either in italic type (underlined in manuscripts) or in roman type (not underlined in manuscripts or in italic typescripts).

The phrase *not present* signifies the lack of corresponding text in the copy-text or other collated source texts.

The word *reinstate* applies to manuscript material only and indicates that the reading to the left of the bracket was deleted by Peirce in the original, but that the editors have reinstated it.

The wavy dash (~) stands for the same word or expression to the left of the bracket.

The caret (‸) signals the absence of punctuation, operational signs (e.g., +, −, or the mid-line multiplication dot), or mathematical and scientific symbols (e.g., degrees, minutes, seconds).

IN ALTERATIONS ONLY:

Readings to the left of the first roman bracket represent the result of the author's revisions described to the right. In most cases, they coincide with the present edition. But in a few cases, when the result of the alteration had to be emended, an imploded diamond (◊) preceding page and line numbers indicates that the lemma represents the alteration prior to its emendation by the editors; the imploded diamond invites the reader to examine the corresponding lemma in the emendations for the exact reading in the edition text.

A small black diamond (◆) within an alteration description indicates that the immediately following words are affected by the description enclosed within the next set of square brackets; double and triple black diamonds indicate subsidiary alteration descriptions (and the number of consecutive black diamonds indicates the number of closing brackets needed to complete the total description). When there is no ambiguity as to the words affected by the description within brackets, no black diamond is used.

All words to the right of the first roman bracket are in roman when taken from the copy-text, in italic when part of the alteration description. Words that are underlined in the copy-text are given in roman and underlined.

Identical alterations are listed after the word *Also* at the end of their first keyed description, but only if they immediately follow each other. Otherwise, their description is repeated.

The following non-punctuated abbreviations are adopted to describe the type of each alteration and its location on the copy-text page: *ab* for *above, add* for *added* (i.e., inscribed either on the line without being squeezed in, or very close to the line without interlining, or in the margin), *aft* for *after, bef* for *before, bel* for *below, del* for *deleted by hand, del-t* for *deleted by typing over, ins* for *inserted* (i.e., squeezed in on the line), *intl* for *interlined* (i.e., set between two lines, or above or below a line), *intl-c* for *interlined with caret or a careted line, inv* for *inverted, ov* for *over* (i.e., inscribed over the letters of the original writing without interlining), *transp* for *transposed, transp-c* for *transposed with a caret or a careted line, w* for *with (w line* or *w arrow).* The conjunction *and* is used to concatenate parts of an alteration description that have the same subject. All other words used in the description are self-explanatory.

Textual Apparatus

The Textual Apparatus provides (together with the Essay on Editorial Method) a nearly complete record of what has been done in the editing process, and it presents the necessary evidence for the editorial decisions that have been made in this critical edition. It consists of fifty-eight sections, corresponding to the number of items published in the present volume, and each section contains up to five separate subdivisions. Each of the fifty-eight items begins with its identifying number in the volume and its (running-head) short title. It is followed by an untitled headnote, and by Textual Notes, Emendations, Line-End Hyphenation, and, if the item is a manuscript, Alterations. The last four are printed in reduced type, the last three in double columns.

The headnote describes the genesis of the text and the occasion for which it was written. It designates the copy-text and collated variant documents (and their sigla, in square brackets) and their pre-copy-text forms. It provides source information for unpublished manuscript items not deposited in the Harvard Peirce Papers, and information on their dating, and it gives a physical description of the manuscript (including number of leaves, paper size, watermarks, medium of inscription, and so on); it also describes fragmentary pages and other portions of the manuscript not included in the edition.

The Textual Notes discuss and explain in detail readings adopted in the edition that represent complex or interesting textual cruxes. They represent either an emendation or a retention of the copy-text reading and specify why, in certain problematical or anomalous instances, the copy-text has or has not been emended. They explain problematic restorations of missing words or portions of words, describe the manuscript page placement of revisions in the margins or on preceding or following versos that have been incorporated into the text or, when Peirce's final intention in those revisions is unclear, reproduce those revisions and annotations that cannot be incorporated. They also provide fuller verbal description of certain complex alterations that could not be represented adequately in the Alterations.

Emendations provide a record of all changes, in both substantives and accidentals, made in the copy-text to produce the critical text. They record the change or correction of single letters or of words, of mathematical and scientific formulas, and of such accidentals as spelling and punctuation. The matter of emendations and their presentation is discussed in detail in the Essay on Editorial Method and in Symbols.

Line-End Hyphenation is a list of those compounds or possible compounds that are hyphenated at the ends of lines in the copy-text. They are resolved according to known Peirce usage and, consequently, are printed in this edition either as hyphenated words or as single unhyphenated words. (The list of compounds hyphenated at the ends of lines in the present volume appears as a separate section before the Index.)

Alterations provide a selected list of changes made by the author in the course of writing the manuscript or upon reviewing it. Such changes include deletions, simple and complex insertions, simple and complex transpositions, superimpositions of letters or words, typeface and character-style alterations, and cancellations. Only those alterations that are critically significant and either dispensable or informative are recorded in this list. (The matter of alterations, including their selection and presentation, is discussed in detail in the Essay on Editorial Method and in Symbols.)

Two further lists are prepared in the editing process—a full Historical Collations list and a complete List of Alterations in the Manuscripts, which yields the published selected Alterations lists—but neither is published in the edition. (They are available to interested persons for the cost of photocopying.)

1. *Determinations of Gravity, 1884*

Copy-text is P 290, the publication in the 1883 *Coast Survey Report.* The 1882 *Coast Survey Report* lists Peirce's "Results for Force of Gravity" as Appendix 23 in its "Contents of Appendices" with this parenthetical note: "(Omitted—see page 557.)." It is further explained that "Owing to the already bulky proportions of this volume, Appendix 23 */*here entitled "Experimental Researches on the Force of Gravity,"*]* has been transferred to, and will appear in, the Annual Report of the Superintendent for the year 1883." The 1883 Appendix was reprinted (Washington: Government Printing Office, 1884) with the cover title *Methods and Results: Determinations of Gravity at Stations in Pennsylvania 1879–1880,* and with "(APPENDIX No. 19, COAST AND GEODETIC SURVEY REPORT FOR 1883.)" above the title. Pages 1–15 of the reprint are textually identical with pages 473–87 of the *Report* Appendix 19, with the following exceptions: reset page numbers; the omission of "+24.3" (5.11) and the μ (22.12[2]); tables of data on pages 16–58 which are not included in the *Report* Appendix; typographical differences in the title's fonts and lineation; the spacing of letters in "seconds" (7.13); and broken type in two commas. The tables at 2.26–3.44 in the original publication have been moved to this more appropriate place after 2.25. Fragments of an early form of this material are found in MS 448.

In about December 1880 Peirce drafted for Superintendent C. P. Patterson an annual report whose first three pages (one half the report) gave a summary of the Pennsylvania work, and, on 5 January 1881, he sent him preliminary results of the work at York. Exactly one year later, he sent J. E.

Hilgard, Patterson's successor, "a memorandum of the plan of my work for the remainder of the fiscal year." It included this statement: "Reports on force of gravity at Cambridge, Baltimore, Washington and the Alleghany series of stations to be sent in." Peirce notified Hilgard on 9 April 1882 that because of "city surveys of doubtful accuracy" the elevation of the pendulum station at the Allegheny observatory "above the railway stations is not very well determined" and also "that the country in the neighborhood should be roughly sketched by an experienced topographer. I recommend that if possible this work be done before next winter."

However, on 21 April, Richard D. Cutts, Assistant in Charge of the Survey Office, wrote Peirce that the printing of the *Report* of 1881 should "proceed immediately" and, on 29 June, he wrote again to ask if Peirce's paper on "Determinations of Force of Gravity at Points in Pennsylvania" would "certainly be forthcoming and, if so, that . . . [it] be in hand by the 15th of July at the very latest." In a letter of 6 July, Hilgard urged Peirce to "devote . . . attention" to the material he was to provide for the 1881 *Report*, and warned him that "If the manuscript is not sent in by July 20th, it will be impossible for the papers to appear in this Report."

Peirce was unable to revise the Allegheny results prior to the Survey's deadline for the 1881 *Report.* By 31 October 1882, Peirce wrote to Hilgard regarding his party's "operations . . . during . . . October":

> The computation of the Allegheny work has been revised. This was done because
> 1st An error was detected for one of the partial swings in the correction for atmospheric effect.
> 2nd Because a slightly erroneous value of the correction to the thermometer has been used.
> 3rd Because there was a difficulty about the rate of the chronometer on one of the days*[.]*
> 4th Because Professor Langley having determined the pressure correction of the clock, that ought to be taken account of in the discussion of the rates.

On the same date he also submitted to Hilgard a "scheme of work for my party during the current *[fiscal]* year." Under *"Office Work"* Peirce included "Preparation of reports on Allegheny, Ebensburgh, York, Baltimore, Cambridge." Finally, on 30 April 1883 he informed Hilgard from New York that on that day he would send his

> report upon Gravity at Allegheny, Ebensburgh, York. I was unable to get any more written & what I have done is somewhat hastily executed & stands in need of my revision. I send it in a box with other papers which would be useful to [my assistant] Preston if he revises the proofs. . . . The paper I send on to you ought not to go to the printers without being copied as if lost I have no other copy.

On 3 May 1883, Hilgard acknowledged receipt of Peirce's "package containing" the paper regarding which Peirce observed, in a 2 June letter to Hilgard: "If the package did not contain the complete (though hastily written) memoir on the Allegheny . . . work, some part of it has been lost." Apparently, according to a 27 June 1883 letter of Cutts to Charles Schott, the Assistant in Charge of the Computing Division at the Coast Survey Office, Cutts sent Peirce's then-designated Appendix 23 "report on the

'Force of Gravity'" to Schott for Henry Farquhar, a Coast Survey employee, as "the best person to make the compilation," as suggested by the Superintendent. Farquhar had worked under Peirce's supervision on the Allegheny observations. By 7 July, Schott submitted to Cutts "Farquhar's report on his examination of Assist. Peirce's M.S. on pendulum results" at Pennsylvania stations. It had been referred to Schott regarding the advisability of immediate publication of Peirce's manuscript in the *Coast Survey Report* for 1882, but Schott advised postponement of publication until Peirce could revise and complete his manuscript. In a letter dated 1 October, Hilgard instructed Peirce to revise his Allegheny report "and correct the proof sheets when it is printed." However, apparently sometime even before 1 October 1883, Farquhar's examination of the report had been sent to Peirce, for on that day Peirce wrote Hilgard denying Farquhar's contention that he had tampered with the database. On 3 October, Peirce telegraphed Hilgard: "Dangerous Illness of Wife retains me today *[;]* please send my manuscript." On the same day he also wrote Hilgard in further detail regarding Farquhar's adjustments to the database and his recommendation that some sections of Peirce's manuscript be omitted:

> I desire to see my manuscript. If Mr. Farquhar has really made all the alterations in it of which he speaks, it is the greatest outrage I ever knew to be perpetrated upon an author and I shall call upon you to protect me against such a wrong.

Peirce also took exception to Farquhar's claim to credit for determining the site longitude, and to his contention that Peirce provided little on-site supervision. The dispute took months to settle; eventually, Schott recommended publication, but offered to cut sections in his 13 May 1884 letter to the Superintendent:

> In accordance with your direction I have carefully read over the MS. report by Assist. Peirce on his results of pendulum work during 1879 & 1880.
>
> I find the subject matter presented in a systematic way, part discussion and results, part record, and should the paper be too bulky the following pp. of record may be omitted viz[.]: pp.14 to 39 incl.
> 60 to 75
> 89 to 121
> They are really not necessary to the understanding of the results, though in my opinion I should rather see them published for the reason that the time determinations and the rates adopted are of prime importance for an estimation of the accuracy of the results.

Schott also recommended a title change, for the following reasons:

> The results at the three stations are comparable only in the shape of absolute measures of the length of the second's pendulum (reduced to equator and the sea level) and the observations still appear in part experimental.... The observations are not differential.... the title Determination of the length of the second's pendulum at three stations in Penn. would define the contents closer than the present title.

At least parts of these suggestions were taken, and apparently at the last minute, for 15.17–19 and 22.13–15 and 17–18 seem to be interpolations.

Emendations

1.10 *et seq.* metres] E; meters *All plural and singular occurrences; also* millimeters *and* centimeters
1.19 *in vacuo*] E; *rom.*
1n.1–2 *American Ephemeris*] E; *rom.*
2.4 *et seq.* centre] E; center
2.15 C] E; C. *Also* 15.8, 11; 22.13
2.29 radius] E; radins
3.17 7^{th}] E; 7^{h}
3.17 5^{th}] E; 5^{h}
3.35(1) products.] E; ~‸
4.3 T_d] E; T_{d} *Also* 4.4, 6.14
4.3 T_u] E; T_{u} *Also* 4.4, 6.14
4.21 timepieces] E; time-pieces
4.35 Do.] E; *not present*
6.5 *Coast Survey Report*] E; *rom. Also* 6.11–12, 6n.1; 7.19–20, 29 (C. S. Report); 10.5; 23.16
6.10–11 consequence] E; consequeuce
6.27 1.0064470] E; 1^{s}.0064470
6n.1 "Measurements] E; ‸*Measurements*
6n.1 of Gravity] E; *not present*
6n.1 at . . . Stations,"] E; *at . . . Stations,*‸
7.13 seconds] E; ~′ *Also* 7.31 (Seconds′), 22.34, 23.7
8.11 Street] E; street *Also* 8.14, 16.14
8.21 Farquhar,] E; ~‸
9.14 Centre] E; Center *Also* 9.16
9.29 4] E; 7
10.8 *Report*] E; *rom.*
10.18(1) $^{\mu}$] E; *not present Also* 10.19, 21.29
11.19 where] E; ¶Where
11.31 Knife] E; ~, *Also* 11.35, 12.4
12.39 spider-lines] E; ~‸~
13.26 Ebensburgh] E; Ebensburg
14.4 cm] E; ~.
14.30 μ.] E; *not present*
14.37 -standard] E; ‸~
14.38 μ.] E; μ
15.24 *s.*] E; *s*
16.12 Alleghenies] E; Alleghanies
16.16 Alley] E; alley
16.26 viz.] E; ~‸ *Also* 18.21
16.26 2^{nd}] E; 2d
16.28 3^{rd}] E; 3d
17n.8(2,3) Do.] E; do‸ *Also* 17n.9, 18.14
17n.12 minute,] E; ~‸
18.6 *C.S.R.*] E; *rom. Also* 18.8, 10, 12, 16, 17
18.34 7–8.] E; ~‸
20.26 eye-and-ear] E; ~‸~‸~
22.16 $1^{m}_{\cdot}$0002884.] E; 1^{m}.0002884‸

Line-End Hyphenation

8.32 foot-rests
11.15 knife-edges

2. *Impressionist List, 1883–84*

Copy-text is MS 470, eight pages inscribed in blue ink on each recto and the first and last verso of six leaves of lined white paper measuring 8 × 10 in., with a "Massasoit Company" watermark. The leaves are halves of formerly single sheets torn from a sewn notebook; some of the leaves still form a single sheet. The first leaf is water-damaged and the ink has run; there are also blots, splattered ink, smudges, some additional water damage, and pen trials throughout. Otherwise the manuscript is carefully inscribed and moderately altered. "Leonidas|*Twin* (Cléombrotus)" appears upside down in the bottom right corner of the last page. Headings from the first page have been retained by the editors throughout, while the rest, which vary in "Provisionally Rejected" and "Provisionally Excluded" and in underlining and capitalization, have been removed; so have a horizontal line below the title and the numbers 31, 203, and 142 inscribed on the last page at the foot of each of the first three columns, respectively. Though these numbers apparently comprise the sums of the number of names in the three columns, only the first column sum is correct. The x's, points, question and exclamation marks, and equal signs (all of which were added or inserted by Peirce after the full list of names was completed) constitute a valuation system used by Peirce and his students to give a preliminary sense of relative greatness. The exact meanings are uncertain, but rankings in subsequent documents indicate that the exclamation point was a sign of certain greatness, while x's and question marks probably indicated names of lesser importance. An earlier attempt at this list appears in MS 469, entitled "Materials for a List of 300 Great Men."

Textual Notes

26(c2).17 J. van Artevelde] The question mark to the left of this entry, which Peirce underlined, is inscribed in a circle; so are the marks at "Fabricius" (27[c2].38). The editors have removed the circles and underlining.

27(c2).37 81 Euripides] Though a left-handed diagonal (deletion) line has been drawn through the number, it is retained to continue the sequence of enumeration.

30(c2).11 207 Rubens] This was followed, at the bottom of the leaf, by "Rembrandt"; but Peirce deleted it and then inserted it in the appropriate place as "202 Rembrandt."

30(c2).41–42 Peirce had originally inscribed "Wallace" between "Wagner" and "Watt," but then crossed it out and inscribed "Bruce" to the right of it. "Bruce" has been deleted because it appears above as entry 37 (26[c4].15).

31(c3).7 "Winckelmann" is inserted to the right between "Wilson" and "Wolsey."

Emendations

26(c1).4 *First*] E; *The* ~
*26(c2).17 J. van Artevelde] E; *J‸ von Artevelde*
26(c2).18 Ph.] E; ~‸
26(c2).32 Bismarck] E; Bismark
26(c3).17 Giordano] E; Jordano
27(c2).30 Geo.] E; ~‸
*27(c2).37 81] *reinstate*
27(c2).42 Frederick] E; Fréderick
27(c2).47 D.] E; D
27(c3).23 Diez] E; Dietz
27(c3).35 Gluck] E; Glück
27(c4).9 Fénelon] E; Fénélon
28(c1).5 Kepler] E; Keppler
28(c2).12 Wm.] E; ~‸ *Also* 28(c3).13, 22; 29(c2).46, (c4).16
28(c2).15 al-Rashid] E; ~‸~
28(c2).31 A.] E; A
28(c2).33 math.] E; ~‸
28(c2).42 Laplace] E; La Place
28(c2).52 Livingstone] E; Livingston
28(c3).7 Händel] E; Händell
28(c3).17–19 Howard (philanthropist)] E; ~,‸~‸
28(c4).9 Gen^1.] E; ~‸
29(c2).10 Stuart] E; Stewart *Also* 29(c2).18
29(c2).12 de'] E; da‸
29(c2).27 Munkácsy] E; Munkacsy
29(c2).35 Palissy] E; Pallisy
29(c2).49 Priestley] E; Priestly
29(c3).8 Mendelssohn] E; Mendelsohn
29(c3).13 Merchiston] E; Mercheston
29(c3).15 Niccola] E; Niccolà
29(c3).22 Père] E; Pere
29(c4).14 Palladio] E; Paladio
30(c2).14 Savonarola] E; Savanarola
30(c2).16 Sir] E; ~.
30(c2).26 Geo.] E; ~:
30(c2).37 Turgeneff] E; Turgenef
*30(c2).41 Wagner] E; ~|Bruce
30(c3).8 del Sarto] E; del Sarte
30(c3).13 Thad.] E; ~:
30(c3).14 Ph.] E; ~:
30(c3).22 Thucydides] E; Thucidides
30(c3).30 Vieta] E; Vietà
30(c3).33 Walther] E; Walter
*31(c3).7 Winckelmann] E; Winkelmann

Alterations

27(c3).33 Vega] *bef del* x
27(c4).12 !!Gambetta x] *aft del* 101
28(c2).48 140] *bef del* ??
◊29(c4).14 Paladio] P *ov* A; d *ov* gg

30(c2).4 Rachel] a *ov* i

*30(c2).41 Wagner] *ab del* Wallace [*bef add* Bruce]

30(c3).13 Stevens] v *ov* ph

30(c3).27 Gustavus] *aft del* 249

30(c3).28 Vauban] *aft del* 2

31(c2).5 Zenobia] e *ov* w

3. *My list of great Men, 1883–84*

Copy-text is MS 471, four pages carefully inscribed in black ink on four leaves of lined white paper measuring 8 × $10\frac{1}{16}$ in., with a "Massasoit" watermark. Peirce moderately altered the manuscript, which has some ink blots on the last leaf. In the top margins of all four leaves, Peirce inscribed several names for possible inclusion in his list; those he did not include are recorded in the textual note below. Nearly all the names in the list have a left-handed check before them. A number of names are preceded by a question mark or by a check and a question mark; Peirce deleted these names and replaced them with new ones. The editors have (silently) removed the checks and question marks, as well as a horizontal line below the title. MSS 466–468 and 472–474 are additional attempts at Lists of Great Men.

Textual Notes

32.1 In the top margin above the title and the top margins of the remaining three leaves, Peirce inserted 35 names (and struck out 20) for evaluation and possible inclusion; only three of the names are not included in the final list: "Chopin," "Claude Lorraine," and "Gen^{1} Gordon."

34(c3).31 "Knox" was originally followed by "Bopp," but Peirce struck out the latter when he moved the name into the place for "Corneille." He then added "Moltke," apparently unaware that he had already inserted that name after "Mozart."

Emendations

32(c1).13 Artevelde] E; Arteveldt

32(c2).11 Bouddha] E; Boudha

32(c2).19 Charles] E; Charls

32(c3).8 Bismarck] E; Bismark

33(c1).9 Geo.] E; ∼‸

33(c1).10 Q.] E; Q

33(c1).20 Alex.] E; ∼‸

33(c1).31 Loyola] E; Loyala

33(c1).32 Leonidas] E; Leonids

33(c2).13 Wm.] E; ∼‸

33(c2).29 Johannes] E; Joannes

33(c2).36 Molière] E; Moliere

33(c3).3 Gluck] E; Glück

33(c3).9 W. R.] E; W R

33(c3).11 al-Rashid] E; er‸Rasheed

33(c3).33 Mendelssohn] E; Mendelsohn

34(c1).18 Savonarola] E; Savanarola

34(c1).20 W.] E; W

34(c1).29 Vesalius] E; Vasalius

34(c2).3 Philip] E; Phillip

34(c2).33 Skeptic] E; skeptic

34(c2).37 Diez] E; Dietz

34(c3).25 Thucydides] E; Thucidides

*34(c3).31 Knox] E; ∼|Moltke

Alterations

32(c2).23 Chrysostom] *intl ab del* Cortez

32(c2).26 Apollonius of Perga] *intl bel del* Colbert

32(c3).22 Bopp] *aft del* Corneille

33(c1).6 Dickens] *intl ab del* Sir Fr Drake

33(c1).29 Hume] *intl ab del* Livingston

33(c1).33 Ptolemy] *intl ab del* Lucretius

33(c1).36 Michelangelo] *intl ab del* Mahomet

33(c2).7 Fermat] *intl ab del* Farragut

°33(c2).13 Wm Herschel] *intl bel del* Gambetta
33(c2).29 Johannes Müller] *aft del* Marlow
33(c3).31 Moltke] *aft del* Jan
33(c3).32 Morphy] *aft del* Fielding
34(c2).2 Phidias] *intl ab del* Pindar

4. *Men of Feeling, Action, Thought, 1883–84*

Copy-text is MS 475, three leaves of medium-weight, lined white paper measuring $7\frac{7}{8} \times 9\frac{7}{8}$ in., with a "Carew Co." watermark; the last leaf is torn in the upper left corner. These working lists, very roughly inscribed on each recto in pencil and revised, sometimes in black ink, with numerous insertions, direction lines, and other alterations, have been reformatted in the present edition for clarity and coherence. Horizontal lines below 35.3 and 37.1 have been removed.

Textual Notes

36(c4).11 *Lawyers*] This heading originally appeared between *Statesmen* and *Personalities*, but was eventually squeezed out of the template by names under the adjacent columns. For reasons of formatting, it has been moved to the end of the final column, to follow the two headings (*Philanthropist* and *Explorers*) that conclude the page.

37(c4).14 Peirce inscribed this column in the upper right margin of the leaf, because there was no space left elsewhere. The overflow of names in several categories has forced a page break after the top tier of columns in the present edition to restore adequate spacing and margins. The *Other* column is part of the top tier and, for reasons of formatting, has been moved to the end of the fourth column.

38(c3).5 *Historian*] Peirce inscribed this column in the only clear space remaining between the second and third tiers of headings on the manuscript page. For reasons of formatting, the column has been moved to the end of the last column.

Emendations

35(c1).10 Mendelssohn] E; Mendelsohn
35(c1).20 Turgeneff] E; Turgenef
35(c3).8 Burns] E; Buns
35(c4).6 R. W.] E; R W
35(c4).17 J. J.] E; J. J
35(c4).28 Savonarola] E; Savanarola
36.1 Men of Action] E; *not present*
36(c1).13 K.] E; K
36(c1).14 Q.] E; Q
36(c1).16 al-Rashid] E; ~‸Rasheed
36(c1).27 Philip] E; Phillip
36(c2).4 van] E; von
36(c2).8 Charles] E; Charls
36(c3).5 Bismarck] E; Bismark
36(c4).2 *Personalities*] E; Person
36(c4).3 Jeanne] E; Jeann
37(c1).2 *Philos.*] E; *rom.*
37(c1).8 Bp.] E; ~‸
37(c1).14 Ed. v.] E; ~‸ ~‸
37(c1).23 Ockham] E; Ôckhân
37(c1).27 Spinoza] E; Spinoze
37(c2).2 *Math.*] E; ~‸
37(c3).12 Wm.] E; ~‸ *Also* 37(c4).13
37(c3).14 A. von] E; ~‸ ~.
37(c3).16 Kepler] E; Keppler
37(c4).2–3 *Moralist & Reformer*] E; *rom.*
37(c4).8 *Linguist*] E; *rom.*
37(c4).11 Diez] E; Dietz
38(c1).4 St.] E; ~‸ *Also* 38(c1).20
38(c1).12 Th.] E; ~‸
38(c2).20 Vesalius] E; Vasalius
38(c3).4 Dr. Livingstone] E; ~‸ Livingston
38(c3).5 *Historian*] E; *rom.*
38(c3).7 Thucydides] E; Thucidides

Alterations

35(c1).4 *Musician*] M *ov* P *and ov* A
35(c2).14 Michelangelo] Miche *ov* Mil
35(c2).23 Titian] *ab del* Turenne [*ab del* Toussaint]
35(c4).9 Herodotus] *ab del* Lessing
35(c4).10 Lavater] *bel* Other *and ab del* Paul Morphy *and transp w line ab del* Lessing
36(c2).2 *Generals*] G *ov* St
36(c3).7 Colbert] *aft del* Be
37(c1).20 Lucretius] *ab del* others
37(c1).21 J. Mill] *ab del* Lav
37(c1).24 Plato] *ab del* Pythagoras
◊37(c1).27 Spinoze] *ab del* Thales [*ins w line*]
37(c2).6 Cauchy] *intl ab del* Copernicus
37(c2).15 Jacobi] *ab del* Keppler
37(c4).4 M. Aurelius] *ab del* Ho

5. *Archimedes, Abel, Lagrange, and Gauss, 1883–84*

Copy-text is MS 478, two pages on two rectos of a folded sheet of medium-heavy weight, lined white paper with a watermark of "Carew Co. Improved" above a shield, each leaf measuring 8 × 10 in. The bottom right corner has been torn from the first leaf, and some ink blots appear on the second; moreover, there is an incomplete list of great men on the verso of the first leaf. Although the manuscript seems hastily inscribed, it has only a few alterations and, as detailed in the textual note below, the last six lines are written in the margins of the second page. The irregular punctuation of the manuscript has been retained, except where coherence required emendation. The editors have removed a horizontal line following the Archimedes and Abel entries.

Textual Note

40.30–36 Peirce inscribed the first two lines diagonally in the lower left margin; the next four lines vertically in the upper portion of the left margin; and the last line horizontally in the top margin.

Emendations

39.5 Hiero] E; Hero
39.8 minister)] E; ~‸
39.14 professors,] E; ~‸
39.16 university] E; univesity
39.24 Professor] E; Professo
40.4 Göttingen] E; Gottingen *Also* 40.17, 23
40.4 Kästner] E; Köstner
40.7 *Disquisitiones Arithmeticae*] E; *rom.*
40.10 Lagrange's] E; Langrange's
40.18 *Theoria Motus*] E; *rom.*
40.32–33 secondary.] E; ~‸

Alterations

39.18 Berlin] *intl ab del* Paris
40.15 strong] *intl*
40.34 could] co *ov* w

6. *Leonidas, 1883–84*

Copy-text is MS 479, one-half page inscribed in dark blue ink on a leaf of medium-heavy, stiff, and lined white paper measuring $7\frac{13}{16} \times 9\frac{7}{8}$ in., with a watermark of "Carew Co. Improved" above a shield. A portion of the top right margin is torn away. There are only a few alterations. Some calculations and doodles appear on the lower half of the recto, and a list of great men (with numerical values) on the verso.

Textual Notes

41.4 The date is inscribed (perhaps as an afterthought) in the left margin.
41.11 Following this line in the manuscript, Peirce's calculations of relative greatness appear among doodles and three repetitions of the number 1000 (perhaps a note that 1000 men—300 Spartans and 700 Thespians—died with Leonidas). Peirce averaged four subjective rankings of value in each calculation, indicating that three of his students were working with him at the time. Two columns of numbers are averaged, yielding 2.8 and 5.4, and suggesting that different aspects of greatness were weighed before a ranking closer to the higher number (and thus lesser value) for Leonidas was decided upon.

Emendation

41.8 citizens,] E; ~‸

Alterations

41.3 Leonidas] *bef del* 17th
41.3 17th] *intl-c*
41.8 except] *bef del* two bodies. One was
41.8 citizens] *bef del* .
41.8 700] *aft del* The other
41.8 the] th *ov* so

7. *Mencius, 1883–84*

Copy-text is MS 480, five pages on five leaves of medium-heavy, lined white paper measuring $7\frac{13}{16} \times 9\frac{7}{8}$ in. and bearing a watermark of "Carew Co. Improved" above a shield. Peirce inscribed the manuscript in dark blue ink, and numbered the five pages in the upper right corner, the numbers preceded by "Mencius." There are few alterations in this manuscript, which is more carefully inscribed than some of the other notes for Peirce's "Study of Great Men."

Emendations

42.4 i.e.] E; i.e
42.5 The] E; *no* ¶~
42.5 Lû,] E; ~.
42.7 belonged] E; *not present*
42.8 Born] E; *no* ¶~
42.12–13 disciples.] E; ~‸
42.15 practice.] E; ~‸
42.25 saying,] E; ~‸
42.26(2) as] E; at
43.3 office.] E; ~‸
43.3 heaven.] E; ~‸
43.26 goodness,"] E; ~,‸
43.26 "is] E; ‸~
43.28 damming] E; damning
43.30 them] E; then
43.33 Wisdom.] E; ~‸

Alterations

42.6 700] *aft del* 711-694 BC
42.19 impracticability] *third* i *ov* l
42.20 years] *bef del* and
42.23 was] *ov* is
42.27 in] *aft del* of the
42.28 second] s *ov* n
43.24 Some] *aft del* Mencius held
43.38 I] *aft del* There
44.7 2.6] *aft del* 3 *and* 6 *ov* 75

8. *Michelangelo, 1883–84*

Copy-text is MS 481, five unnumbered pages on five leaves of thick, stiff, somewhat chipped, wove, laid, and unlined white paper measuring $8\frac{1}{8} \times 10\frac{1}{16}$ in. Peirce inscribed the manuscript on each recto in pencil, made some alterations on the first page, and revised the next two more extensively. But emendations have been made only when necessary, and the editors have removed horizontal lines (appearing in the manuscript) below lines 45.6, 12.

Textual Notes

45.25 Much . . . sarcasm?] This clause has been added, in two lines extending into the left margin, before what was already an indented paragraph.
45.28–29 To the left of this passage Peirce drew a curved line; its purpose is unclear.
46.35 Peirce neglected to lowercase "Small" after having added "Rather."

Emendations

45.3 Buonarroti-Simoni] E; ~‸~
45.9 Florence] E; florence
45.17 Apr] E; *no* ¶~
45.17 Ghirlandaio] E; Ghirandajo *Also* 45.22
45.17 years.] E; ~‸
45.18 year,] E; ~‸
45.19 Soon] E; *no* ¶~
45.22 1489] E; *no* ¶~
45.23 Magnificent.] E; ~‸
*45.25 Much] E; *no* ¶~
45.25 Torrigiano.] E; Torriggiano‸
45.29 mason.] E; ~‸
46.1 Giorgio] E; Georgio
46.2 country."] E; ~‸‸
46.6 Lorenzo's] E; Lorezo's
46.11 de'] E; di‸
46.13 Cardiere] E; Cardieri
46.13 Lorenzo.] E; ~‸
46.15 Savonarola] E; Savanarola
46.17 learning.] E; ~‸
46.24 Sarcastic.] E; ~‸
46.25 disease.] E; ~‸
46.26 Generosity.] E; ~‸
46.27 people.] E; ~‸
46.27 unfortunate.] E; ~‸
46.29 fury.] E; ~‸
46.30 calculations.] E; ~‸
46.31 height.] E; ~‸
46.31 shoulders. Slender] E; ~‸ slender
46.34 head.] E; ~‸
46.34 temples.] E; ~‸
*46.35 small] E; Small
46.38 remarkable] E; remakable

Alterations

45.3 Buonarroti] *ab del* or Buonarotti
45.4 1475] 5 *ov* 4
45.6 5] *aft del* 3 youn
45.7 31] 1 *ov* 5
45.9 among the] *intl-c*
45.15 that] *intl ab del* but
45.17 Apr 1] *intl-c*
45.19 Soon after] *intl ab del* 1485
45.22 to] *aft del* for
*45.25 Much given to sarcasm?] *add*
45.29–30 Drawing . . . precocity.] *add*
46.4–6 Impossible . . . activity] *ins*

9. *Ockham and Machiavelli, 1883–84*

Copy-text is MS 482, consisting of two and one-half pages on three leaves of medium-weight, white unlined paper measuring $6\frac{3}{4} \times 8\frac{1}{4}$ in. The manuscript is inscribed in dark blue ink, has several ink blots and smudges, many pen trials, and a moderate number of alterations. Two versos of this manuscript are actually rectos in the next manuscript. "Hail, holy number . . . " (MS 483, 51.27–32) is on the verso of the first page (47.3–22), "Honor thy father & mother . . . " (MS 483, 51.7–16) on the verso of the last page (48.14–20). The irregular punctuation of the manuscript has been largely retained, but the editors have removed horizontal lines (appearing in the manuscript) below 47.15 and 48.20.

Textual Note

48.15 to restore] Peirce accidentally canceled "to," the last word in the third line of the third page, with pen trials.

Emendations

47.5 Stowe] E; Stow
47.8 *inceptor.*] E; ~‸
47.14 Bavaria.] E; ~‸
47.18 offices.] E; ~‸
48.3 Cesare] E; Cesar
48.4 1506.] E; ~‸
48.11 Medici] E; Medeci
48.14 Too] E; *no* ¶~
48.15 *The Prince*] E; the Prince
*48.15(1) to] E; *reinstate*
48.19 widow] E; widdow

Alterations

47.17 One] O *ov* An
47.18 66 of them] *intl ab del* Many

10. *Pythagoras, 1883–84*

Copy-text is MS 483, nine unnumbered pages on nine leaves of somewhat chipped, medium-weight, unlined white paper measuring $6\frac{3}{4} \times 8\frac{1}{4}$ in. and inscribed in dark blue ink. A moderate number of alterations occur in these notes, and there is a smudged ink blot and a pen trial on the fourth leaf. An unrelated list of several great men is scribbled in the lower half of the last page. The irregular punctuation of the manuscript has been largely retained. An unfinished pre-copy-text version survives on an additional leaf inscribed on both recto and verso. Regarding material on the verso of the seventh and ninth leaves, see the headnote to item 9.

Textual Note

50.15 3 male 4 female] In the manuscript, these two phrases are on two lines following a brace; they have been set onto one line, and the brace has been changed to a comma.

Emendations

49.9 (poet)] E; (~‸
49.23 II).] E; ~.)
50.5 sick-bed] E; ~‸~
*50.15 children,] E; ~{
50.16 scholars,] E; ~‸
50.21 murdered. Cretona] E; murderd. Crotona
50.21–22 BC (aged 60)] E; *BC* ‸~ ~‸
50.22 Sybaris.] E; ~‸
50.30 muscle,] E; ~‸
50.31 noted.] E; ~‸
50.32 sentiments,] E; ~‸
50.38 obey,] E; ~‸
51.2 feast.] E; ~‸
51.7 heroes.] E; ~‸
51.10 remember] E; remembe
51.12 fault] E; faut
51.14 Courage,] E; ~‸
51.17 catechism] E; catachism
51.18 τετρακτύς.] E; ~‸
51.20 words] E; word
51.21 things.] E; ~‸
51.21 Music.] E; ~‸
51.21 powerful? γνώμη] E; ~‸ ~)

Alterations

°49.9 (poet] *intl*
49.11 passion] *aft del* taste
49.13 Study] S *ov* s
49.14 (20 years)] *intl*
49.23 ordeal] *intl ab del* trials
50.19 aristocrats] *intl ab del* persons
°50.22 aged 60] *intl*
50.23 fortune.] *bef del* He
50.24 σύστημα] *intl*
50.30 organ] *bef del* teste
50.30–31 Physiognomy carefully noted] *intl-c*
50.31 biography, their] *intl-c*
50.32 their sentiments] *intl-c*
50.34 occasionally] *intl-c*
51.9 Do] *intl ab del* Exercise
51.15 showeth] e *ov* s *and* th *intl*
51.21 Music] *intl ab del* Harmon
51.22 quart] *intl ab del* full manger

11. *Rabelais, 1883–84*

Copy-text is MS 484, four pages on two leaves of unlined white paper torn from a tablet and measuring $5\frac{1}{16} \times 7\frac{15}{16}$ in.; it is inscribed in dark blue ink on both recto and verso of the two leaves, and there are several pen trials and ink stains. Peirce made relatively few alterations in this manuscript; its irregular punctuation has been largely retained.

Textual Note

52.4 In the manuscript, "Son" is on the same line as "*Rabelais*" (52.3).

Emendations

*52.4 Son] E; *no* ¶ ~
52.4 neither.] E; ~‸
52.6 Touraine] E; Tourraine
52.10(1) Angers] E; Anger
52.11 acquaintance] E; aquaintance
52.13 wrong,] E; ~‸
52.14 right.)] E; ~.‸
52.15 Learned] E; learned
52.16 Confiscated] E; Confiscatd
52.23–24 physician,] E; ~‸
52.24 astronomer,] E; ~‸
52.26 Lyon] E; Lyons
52.28 Employed] E; *no* ¶ ~
53.4 du Bellay] E; de ~
53.5 Learned] E; Learded
53.19 1553] E; ~.

Alterations

52.9 convent of] *intl-c*
52.11 boys] *aft del* young [*intl ab del* pe]
52.29 demonstrations] d *ov* fe

12. *First Questionnaire, 1883–84*

Copy-text is MS 485, an italic, black-ribbon typescript consisting of two pages on leaves of medium-weight, white paper with frayed and torn edges, measuring $7\frac{7}{8} \times 10\frac{3}{4}$ in. with "The American Linen Paper" watermark. There are many handwritten additions in blue ink, and two in pencil (55.2, 5), and the manuscript is written entirely by hand from 56.12 onward. The punctuation in these autograph entries is left as is (except at 56.22, which is emended to conform with other headings) to indicate the unfinished state of the manuscript. The spacing of the typewritten punctuation is irregular. The use of parentheses has been regularized (that is, opening and closing parentheses for one-digit numbers, closing parenthesis for others). A fragmentary, numerically sequential list (31 to 62, with 56 missing) survives on a partly typewritten, partly autograph additional leaf.

Textual Notes

55.2 60bis] Peirce added this number in pencil, and numbers 19, 41, and 24 in ink (55.11, 30, 32).

55.8 Peirce inscribed this line in ink below the "18)" entry and drew a direction line for its insertion above the "47)" entry.

55.34 Peirce inscribed entry "7" in ink in the right portion of the page and drew a direction line for its insertion above entry "66)." The entries at 55.35–36 were also added in ink; the latter is in the top margin with a direction line pointing below the inserted entry "66)."

56.8–10 Entries "64)," "68)," and "69)" were inscribed by hand in the right portion of the page and inserted below entry "63)" with a direction line. Entry "68)" was inscribed below "69)" and transposed above it with a line.

Emendations

54.6 Parents'] E; Parent's
54.9 53)] E; ~‸ *Closing parenthesis also added in* 55.11; 56.10, 12, 13, 25–28
54.9 directions?] E; ~‸
*55.2 60bis)] E; 60 Bis‸
55.5 62bis] E; (~
55.5 Complexion?] E; ~‸
55.6 65] E; (~
55.8 appear?] E; ~‸
55.15 age?] E; ~‸
55.21 particularly] E; particuliarly *Also* 55.23
55.27 ready-made] E; ~‸~
*55.34 (7)] E; 7
55.34 imitators?] E; ~‸
55.35 66] E; (~
55.35 youth?] E; ~‸
55.36 67] E; (~
56.8 cessation?] E; ~.
*56.9 68] E; (~
56.9 time] E; *not present*
56.9 preferably work?] E; preferadly ~.
56.10 69)] E; 69‸
56.10 stimulants?] E; ~‸
56.22 peculiarities.] E; ~‸
56.23 (6)] E; 6
56.24 (6bis)] E; ‸~‸
56.27 Self-estimate] E; ~‸~

Alterations

◊54.9 53 . . . directions] *add*
◊*55.2 60bis] *add*
◊55.5 62bis) Complexion?] *intl ab del* 65) Tempament
◊55.6 (65) . . . take?] *add*
*55.8 25) . . . appear] *ins w arrow*
*55.11 19] *add*
55.14 VII] *add*
55.15 at what age] *add*
55.18 VIII] *add*
*55.30 41)] *add*
*55.32 24)] *add*
◊*55.34 7 . . . imitators] *add w line*
◊*55.35 (66) . . . youth] *ins*
◊*55.36 (67) . . . ?] *ins w line*
56.1 IX] *add*
56.7 63) . . . ?] *add*
◊*56.8 64) . . . cessation.] *add w line*
◊*56.9 (68) . . . work?] *add w line*
◊*56.10 69 . . . stimulants] *add w line*
56.11 X] *add*

13. *Revised Questionnaire, 1883–84*

Copy-text is MS 516, an italic, purple-ribbon typescript consisting of two and one-quarter pages on three leaves of thin, white paper with frayed, folded, and torn edges, measuring $8\frac{7}{16} \times 10\frac{7}{8}$ in. with "The American Linen Paper" watermark. Two handwritten alterations, and the page numbers and date ("1884 Nov. 8.") on the second and third pages, are in blue ink. The spacing of punctuation has been regularized but, otherwise, punctuation has been left as is to indicate the unfinished state of the manuscript. MSS 512 and 518 contain additional attempts at Questions on Great Men.

Emendations

57.13 brothers and sisters] E; ditto
58.1–2 distinguished] E; dist|tin-guished
59.16 marvellous] E; marv=llous
59.31 weaknesses] E; weeknesses

14. *Final Questionnaire, 1883–84*

Copy-text is MS 520, a printed form consisting of two folded leaves of heavy, blue-gray paper measuring $7\frac{15}{16} \times 13\frac{1}{16}$ in., with a "Linen 1876" watermark below an image of a crane. The edges are frayed and chipped (especially at the top of the first leaf), the folds are splitting, and cellophane tape holds together the upper right corner of the first leaf. Only entries followed by Peirce's handwritten annotations or commentary, inscribed in black ink, are reproduced. Printed words and phrases from the form appear in boldface in the present edition. MS 518 contains an earlier printed version of the questionnaire. The complete printed form is reproduced (without annotations) on pages 61–62.

Emendation

63.24 small] E; Small

15. *Michelangelo, Hobbes, and Locke, 1883–84*

Copy-text is MS 522, six pages on five leaves of the same blue-gray folded sheets as the printed form in MS 520. Some of the top edges are frayed. The Michelangelo and Hobbes questionnaires are inscribed in blue ink, with a few alterations in black ink, and the Locke questionnaire is inscribed in black ink, with a few ink smears as well as an unexplained "66" in the lower left corner of the page. Commentary inscribed in the margins and on the versos of three leaves is reproduced in the Notes. The evidence of "Hereditaments" instead of "Hereditary traits," as well as the addition of the "Concentration" entry in black ink, indicates that the Michelangelo form was printed before the others. Some of the biographical references used in the Hobbes and Locke questionnaires (with dates as late as 1886 and 1890) reveal that Peirce's work on the Great Men project continued well beyond the period of the present volume. Only entries followed by Peirce's handwritten responses are reproduced. The editors have removed braces inscribed before stacked responses. The forms' printed entries are in boldface in the present edition. A number of sketchily answered printed forms for Alexander the Great, John Calvin, Queen Elizabeth, and several others are in MS 521.

Textual Notes

65.5–31 On the second leaf verso of the Michelangelo questionnaire, Peirce made detailed notes to prepare or possibly to augment the abbreviated entries of the

"FAMILY," "CHILDHOOD," and "YOUTH" categories. They are recorded below, in linear sequence, with sources (where known) supplied in italic brackets. "Mother fell from horse & was dragged along the ground; yet it did her no harm. Born 1475 March 6. 2 AM." "Grandmother Alessandra (Mona Lessandra) di Brunaccio Brunnacci. 66 years old when Michelagnolo was born. d. 1494 Therefore at 85. 'So well known as Madonna Alessandra'" "Apprenticed to Ghirlandaio without premium and even to receive salary from the first. Ghirlandajo soon exclaimed 'He knows more than I do' and became jealous of him. He neglected drawing & took to sculpture without a master. Found by Lorenzo the magnificent in his garden, making head of fawn. Critic*[i]*sms, corrections. Taken into Lorenzo's house boarded clothed & supplied with money. *[Ghirlandaio's attributed quotation appears in Vasari's* The Lives of the Painters, *Vol. 5, p. 230, an edition Peirce owned]* ¶Had pastboard helmet into which a candle could be stuck. Used very poor quality candles. ¶Expression, agreeable, animated, decided." "Did not do much after 1550"

69.1–2 Became . . . Doctor.] This is inscribed vertically in the left margin, extending from "YOUTH" to "Height," and almost certainly was intended to go with the "Bent" entry.

69.24–70.6 Four marginal notations appear to the right of the second column of entries in the "CHARACTER" section of the Locke questionnaire. These notes do not correspond to specific categories, instead, appear to be ancillary comments. They are given here in linear sequence, with expanded citations (when known) in italic brackets. "Always preaching the innocence of pleasure"; "Insisting of direct *perception* of truth-p. 115" *[Locke,* Essay Concerning Human Understanding, *bk. 2, ch. 32, sec. 16; Vol. 2, p. 115 of the edition Peirce owned]*; "'Innate is a word Mr. Locke poorly plays on,' says Shaftesbury p. 117." *[Fraser,* Locke *(1890); the reference is to the 3rd Earl of Shaftesbury (1671–1713), Locke's pupil]*; "'Ideas are nothing but actual perceptions in the mind.'" *[Fraser, p. 124]*

69.24–70.6 On the versos of both leaves of the Locke questionnaire, Peirce made several rough biographical notes based largely on material in Locke biographies by Fox Bourne (1876) and Fraser (1890). These notes augment the abbreviated entries in the "CHARACTER" category which dominates the second page of the form. The notes are not keyed to specific questionnaire entries, and follow here in linear sequence (without preserving spacing, indentation, and line breaks); references to Peirce's sources are supplied (when known) where Peirce fails to do so. "The 5 great pleasures: Health, Reputation, Knowledge, Luxury of doing good to others, hope of heaven." *[Fraser p. 25]* "I have been as miserable as possible at the loss I endured in leaving you. p. 58" *[Fraser]* "A man of turbulent spirit, clamorous and discontented." "Says Anthony à Wood. That is, spirit of individualism. *[Fraser, p. 18; Anthony Wood was Locke's fellow-student at Christ College, Oxford]* "'Has on several occasions behaved himself very factiously & undutifully to the Government' Fraser, 74"; "Taste for books of travel. Fond of work"; "Sagacious, shrewd, humorous 'I know no such rack in the world to draw out men's thoughts as a well-managed bottle." *[Fraser, p. 81]* "Secretive, perhaps timid. Never could fully forgive Limborch for revealing his authorship of 1st letter on toleration" *[Philippus van Limborch, to whom Locke dedicated his* Epistola de Tolerantia*]*; "Declined embassy to Brandenburg which he ardently desired to accept, for fear of health." "'I believe there is not in the world such a master of taciturnity and passion.' Fraser p. 61." "Extremely well-regulated in every respect. Strict personal economy."

"It is an idle and useless thing to make it our business to study what have been other men's sentiments in things where reason is the only judge." "Contempt for 'enthusiasm'" *[Bourne, Vol. I, p. 237]*; "Lord Ashley formed him; but he was 34 when he met Ashley." *[Locke met Lord Ashley, Earl of Shaftesbury, in 1666 while a student at Oxford.]* "'A man,' said Sydenham, before Locke had distinguished himself, 'whom, in the acuteness of his judgment, and in the simplicity, that is, in the excellence of his manners, I confidently declare to have amongst the men of our own time few equals and no superior.'" *[Bourne, Vol. I, p. 219]* "Often a man of business, always a man of the world, without much undisturbed leisure."

Emendations

65.10 Alessandra] E; All
66.7 Torregiano] E; Torreggiano
66.16 legible] E; Legible
67.1 great] E; Great
67.5 Savonarola] E; Savanarola
67.19 *Medea*] E; *rom.*
67.36 palsy] E; palsey
67.38 *Leviathan*] E; *rom.*
68.5 says] E; Says
68.8 Irritabile] E; Iirritabile
68.12 Vol. I] E; ~‸ I.
68.13 *Opera Latina*] E; *rom.*
68.13 Ed.] E; ~‸
68.26–27 only, Thomas,] E; ~‸ ~‸
68.26 Locke] E; Lock
69.1 him.] E; ~‸
69.7 pointed] E; pointd
69.8 Eyebrows] E; Eveybrows
69.13 *Essay*] E; *rom.*
70.3 *Éloge historique*] E; *rom.*
70.4 *Life*] E; *rom.*

Alterations

65.12 He,] ***intl ab del*** older?
67.20 before] ***intl ab del*** at
67.28 shaped] ***intl ab del*** form
68.17 Grandson] ***intl ab del*** Son
68.19 good] ***aft del*** Probably
°68.26–28 Lock never married] ***intl ab del*** 2 years before he was mar
68.36 Church] ***aft del*** medicine? [? ***del by editors***]

16. *Montaigne, Palissy, Machiavelli, and Lessing, 1883–84*

Copy-text is MS 488, five pages of text on four different leaves. The Montaigne Responses are inscribed in blue ink on the recto of a leaf (of a folded sheet) that measures $7\frac{15}{16} \times 10$ in. and is medium-weight, lined white paper with a "Carew Co. Improved" watermark; additional comments, reproduced in the Notes, appear in the top margin of the leaf and elsewhere, and *"Ockham"* is inscribed on the verso of the otherwise blank second leaf. The Palissy Responses are inscribed in black ink on the first verso of a folded sheet of white stationery from the "Mount Vernon Hotel, West Monument Street, Baltimore, 188__," measuring 5×8 in., with a "Congress" embossing in the bottom left corner; it has two alterations. The Machiavelli Responses are fairly carefully inscribed in pencil on both sides of a leaf of laid white paper (lined on one side only), measuring $4\frac{15}{16} \times 7\frac{13}{16}$ in. The Lessing Responses are inscribed in pencil on a leaf of lined white paper, measuring 5 $\times$ 8 in.; it has a number of alterations and one blue ink blot. To indicate the incomplete state of the manuscript, the editors have (except for a few regularizations and emendations) retained the irregular punctuation of the manuscript; a horizontal line dividing "Lessing" from the following entry has been removed. MS 476 contains one page of related notes on Montaigne.

Textual Notes

71(c1).11–15 In the margin to the left of "53" to "9," Peirce inscribed, descending vertically, and referring, among others, to Montaigne's father, grandfather, and great grandfather, "fr gr ggr story of Uncle."
71(c1).17–18 Father . . . died.] This later addition is written to the right of *"Childhood"* with two separation lines.
71(c2).14–15 Says . . . 30] Peirce added this sentence below entry 25 and to the right of entries 47, 46, 19.
72(c2).1–4 Excessively . . . music] This passage is inscribed to the left of entries 49,

37, and 38, with a direction line to 37. "Unusually" was apparently added later, but Peirce neglected to lowercase "Delicate."

72(c2).15–24 Peirce had no space at the bottom of the page to inscribe entry 42 and thus added it vertically to the right of entries 17 through 2. It has been placed where Peirce had inscribed "42" in the manuscript.

Emendations

71(c1).7 Noble] E; *ital.*
71(c1).8 8] E; 6
71(c1).10 Eleven months] E; *ital.*
71(c1).12 doctors.] E; ~‸
71(c1).14 *Br. & sisters*] E; *rom.*
71(c1).19 *Childhood*] E; *rom.*
71(c1).23 *Youth*] E; *rom.*
71(c1).29 Delicate.] E; ~‸
71(c2).7 face.] E; ~‸
71(c2).11 62bis] E; ~. *Also* 73(c1).43, 74(c1).35 (60bis)
71(c2).14 cruelty.] E; ~‸
71(c2).17 At] E; *not present*
71(c2).21 *Progeny*] E; *rom.*
72(c1).11 *Work*] E; *rom.*
72(c1).15 Scepticism.] E; ~‸ *Also* 72(c2).11
72(c1).20 *Genius*] E; *rom.*
72(c2).1 V.] E; V
72(c2).1 indeed.] E; ~‸
72(c2).3 delicate . . . smell.] E; Delicate . . . ~‸
72(c2).5 good] E; Good
72(c2).11 Extreme vanity.] E; extreme Vanity‸
72(c2).17–18 *Essays*] E; *rom. Also* 72(c2).20
72(c2).19 intelligence.] E; ~‸
72.26 B. Palissy] E; *B‸ Pallisy*
73(c1).44 *Prince*] E; *rom. Also* 73(c2).32
73(c1).44 aet.] E; ~‸
73(c2).28 dau.] E; ~‸
74(c1).10 great] E; Great *Also* 75(c1).4
74(c1).14 painter.] E; ~‸
74(c1).20 9.] E; 9. one dau
74(c1).21 Broth.] E; ~‸
74(c1).40 *Laocoon*] E; *rom.*
74(c1).40 *Nathan*] E; *rom.*
75(c2).2 Dilatory.] E; ~‸
75(c2).3 6bis] E; 6

Alterations

71.5 Montaigne] *bef del* {what
71(c1).6 *Parents*] *aft del* 1)
71(c1).12–13 Hereditary . . . bladder] *intl*
71(c1).15 older] *intl-c*
*71(c1).17–18 Father . . . died.] *ins*
71(c1).29 Delicate] *ins w line*
71(c1).29–30 Good . . . bad.] *intl ab del* Never very robust
71(c2).7 Full face] *intl*
71(c2).8 Squat] *intl ab del* Well proportiond
*71(c2).14–15 Says . . . 30] *ins*
71(c2).22 33] *aft del* 32
72(c1).15 What . . . know?] *ins w line*
72(c1).16 Pleasure . . . aim] *ins*
72(c1).25 or less.] *intl*
*72(c2).2 Unusually] *add*
72(c2).6 Very bad indeed] *intl ab del* V Poor?
73(c1).13 Tolerably] T *ov* N
73(c1).33 Good] *aft del* Moderate
73(c1).34 Probably] Pro *ov* Not
73(c2).9 Great] *aft del* Vast
74(c1).14 Ggr.] g *ov* r
74(c1).20 9.] *bef del* None
74(c1).34 active] *ins*
74(c2).12 19] *bef del* Did'nt live
74(c2).24 No] *aft del* Hard
74(c2).37 comedy] *intl ab del* drama

17. *48 Great Men, 1883–84*

Copy-text is MS 489, twenty-four pages on twenty-four leaves of white paper measuring $5\frac{1}{2} \times 8\frac{1}{2}$ in., with a "Saint James" watermark, which represent twelve two-page printed forms. "QUESTION" is printed in the top left, and the names (in large and small capitals) follow below. Peirce's inscriptions are in blue ink. The subject headings (which are omitted from MS 489), the wording of the questions (which is sometimes fragmentary), and the order of the questions have been taken from MS 485 (item 12). The subject headings are represented in boldface type in the present edition, but the following are omitted: dashes before some of the answers to question 36; braces when a stacked answer fits on one line; short horizontal lines (and a long line below "Chasles" at 86[c1].13) or x's in the left margin between some of the listed names up to 81(c1).6; marks below "Constantine" and "Rienzi" (except at 83[c1].14) to divide the list of names into fourths; vertical lines (on two pages) dividing the answers into columns; and horizontal lines that align the answers on a number of pages. The editors have punctuated abbreviations, but irregular punctuation has generally been left as is to indicate the unfinished state of the manuscript. Two leaves with questions 65 and 45 (but no responses) and another leaf with neither number nor question (but with several numerical inscriptions and with what seems to be a blotting of Peirce's signature) also survive.

Textual Notes

77(c1).4 Julian] Peirce corrected the misprinted "e" to "a" here and at 84(c1).26 and 86(c1).26. The editors have corrected all others.

77(c1).8 Machiavelli] The printed name had 2 c's, but Peirce deleted the first here and at 85(c1).4. The editors have corrected all others.

82(c1).22 Peirce inscribed calculations in the right margin for Helmholtz, Morphy, Emerson, Virgil, Grotius, and Moltke. Three subjective rankings of value are averaged together in each calculation, indicating that Peirce and at least two of his students were actively engaged in this stage of the great men project.

92.3–18 "Men of feeling probably often intermittent" is inscribed vertically in the right margin.

Emendations

76(c1).27*et seq.* Helmholtz] E; Helmholz
76(c2).16 Boundless] E; Bundless
76(c3).25 Tyndall] E; Tyndal *Also* 78(c3).19
76(c3).26 Dr.] E; ~‸
77(c2).2 amiable] E; aimiable
77(c2).14 Amiable] E; Aimiable
77(c3).28 Maecenas] E; Mecaenas
78.3 imitators?] E; ~‸
80.2 youth?] E; ~‸
80(c2).10 Caracas. Picturesque] E; Carracas. Picturesqu
80(c3).8 decidedly] E; decidly
82(c2).9 Statesman] E; Statesmen
82(c2).18 Math.] E; ~‸ *Also* 83(c2).3, 13
82(c2).21 *Thought*] E; *Though*
82(c2).23 math. & physiol.] E; ~‸ & ~‸
83(c2).3 *Thought*] E; *rom.*
83(c2).9 player] E; playe
83(c2).13 Philos.] E; ~‸
83(c2).18 phil.] E; ~‸
84(c2).7 Conserved.] E; ~‸
85(c3).2 independent] E; independant
85(c3).6 desirable] E; desireable
88.2 cessation?] E; ~.
88.3 time] E; *not present*
88.3 preferably work?] E; preferadly ~.
88.4 stimulants?] E; ~‸
92(c2).7 Contin.] E; ~‸ *Also* 92(c2).8, 10, 15, 16; 93(c2).2, 7, 11, 13, 14
92(c2).9 Intermitt.] E; ~‸
92(c2).13 health] E; healt
92(c2).14 Cont.] E; ~‸
92(c3).9 self-willed] E; Self-~
92(c3).15 strong] E; Strong *Also* 92(c3).19; 93(c3).15, 18
92(c3).19 V.] E; V *Also* 94(c4).13; 95(c4).2, 16
93(c3).6 self-controlled] E; ~‸~

Alterations

77(c2).6 taciturn] *intl ab del* silent
77(c2).9 Wonderful] *intl ab del* Great
77(c3).7 No] N *ov* ?
77(c3).19 Probably] *intl ab del* Doubtful
78(c3).20 No] *aft del* Hard
80(c2).13 Near] *intl ab* Florence
80(c2).26 Lovely] *aft del* No
81(c2).2 Plain] *aft del* Ugly
82(c2).10 General & statesman] *intl ab del* Ruler
83(c2).4 *Feeling*] *intl ab del* Though
83(c2).6 *Feeling*] *intl ab del* Thought
83(c2).12 *Feeling*] F *ov* T *Also* 83(c2).16
83(c2).18 *Feeling*] *intl ab del* Thought
83(c2).18 Writer] Wr *ov* Ph
83(c2).19 Religionist] Re *ov* Phi
83(c2).20 *Feeling*] F *ov* T
84(c3).6 not] *intl-c*
84(c3).6 seem] s *ov* h
84(c3).11–12 Building . . . implied.] *intl bel del* Holy nation
85(c2).11 following] *intl ab del* turn
85(c3).4–5 merely . . . life-] *intl ab* while desire
86(c3).9 No] N *ov* ? *Also* 86(c3).15
88(c3).13 Early] E *ov* N
89(c2).3 Yes] *aft del* No
89(c2).10 Think] *aft del* No
89(c3).13 Mornings] M *ov* ?
90(c3).10 ?] *aft del* Probably
91(c2).13 Voluntarily] *aft del* Ye
93(c3).5 Nothing] N *ov* ?
◊93(c3).6 Very self controlled] *aft del* Self *and intl bel del* Probably not very strong
93(c3).13 Vacillating] *aft del* •Singular, Eccentric [*ab del* Weakness more striking than strength]
93(c3).15 V. strong] *intl ab del* Great
94(c4).16 Great] *intl ab del* Very good in
95(c2).6 Large] *aft del* Prob
95(c4).17 Prob.] P *ov* G

18. *24 Great Men, 1883–84*

Copy-text is MS 490, nine pages on eight leaves of lined white paper (separated halves of formerly folded sheets) measuring 8 × 10 in. and inscribed in blue ink on the eight rectos and the third verso. The manuscript is dog-eared, the second leaf is torn, and a pen trial and doodle appear on the sixth leaf. The first leaf has a "Massasoit" watermark; the other seven a "Carew Co. Improved." On most of the pages, the list of twenty-four Great Men is arranged in four columns with six names each. MS 485 provides the sequence and text for the questions, which are identified only by number (except for 62 which also has "Other Pecul."), and for the omitted subject headings (except for "Effect of Age," which Peirce included). Abbreviated names in the manuscript are spelled out. The document is reformatted to accord with the edition text of MS 489, and subject headings are printed in boldface type. Question "55" at the top of leaf two has no responses and has thus been omitted. Punctuation for abbreviations has been supplied, but the irregular punctuation of the rest of the manuscript is left as is to indicate its incomplete state. A related page, entitled "Supplementary answers to Questions," appears on the verso of the third page in MS 526 (item 19).

Textual Notes

97(c2).8, (c3).10 Peirce inscribed the responses for 62 and 62bis in the phrase "Gilbert *tall*, cheerful complexion" in the top margin of the third leaf's recto with an incomplete direction line to "Gil" in the list of names. We have inserted *"tall"* in the appropriate place for question 62 and "cheerful complexion" for the response to question 62bis.

97(c2).9–10 olive . . . Meagre.] A separation line between name and response, followed by a crude brace, has been removed.

98(c1).7 In the tabulated answers to question 47 ("First great work at what age?"), Peirce wrote "42" after "Barnev" and then deleted the number.

98(c3).5 *Basis of a division,* inscribed above the responses to question 47, is omitted.

99(c3).7,8,15,16 In question 18, Attila, Barneveldt, Hampden, and Julian are preceded by "x"; it is not clear why.

99(c3).28 Below the four columns of numbers in question 18, Peirce inscribed two sets of incomplete numbers (intended it seems, to indicate the mean age), then "Calculate expectation" followed by a table of twelve names and sets of numbers (whose significance is not clear), and then "Die 6 years too soon."

100(c4).5 In the spaces left blank in question 13, there are horizontal lines in the manuscript which, however, stand for names rather than answers.

101(c2).28 Below the responses to question 4, Peirce inscribed "XIX 5," "XVIII 3," "XVII 3," and "XVI 7," indicating the total number of men in each of the four centuries.

101(c3).28 Below the responses to question 3, Peirce inscribed a table of names of Great Men according to nationality (with the following headings: "Jew," "Greek," "Latin," "Italian," "French," "Ame|Engli.," "Germ & Dutch," "Slav," and "Negro") and the appropriate names thereunder.

102(c2).8 Preceding question 17, at the top of the manuscript page, Peirce has inscribed "(22) Already answered"—which has been omitted.

102(c3).31–32 The affirmative answers (to question 16) for Wellington and Grotius are each followed by an "x" (whose meaning is not clear); the editors have omitted both.

103(c2).15 Peirce added "An" but neglected to complete the alteration; similar oversights account for the emendations of "Gr. honored" (also 103[c3].11) and "Immensely admired" (103[c3].19–20).

Emendations

96(c2).17 prob.] E; ~‸ *Also* 96(c2).20
96(c3).12 strong] E; stron
97.3 Complexion?] E; ~‸
97(c2).6 Impressive countenance] E; impresive Countence
97(c2).9 compl.] E; ~‸
97(c3).9 Châtain] E; Chatain
98.2 appear?] E; ~‸
98(c3).11 *Laocoon*] E; *rom.*
98(c4).13 *Nathan*] E; *rom.*
100.2 age?] E; ~‸
100(c3).19 s.] E; s
100(c3).19 dau.] E; ~‸ *Also* 100(c3).20, 29
100(c3).29 sons] E; son
100(c4).14 Ordinary] E; Ordinay
101(c2).5 B.C.] E; B C *Also* 101(c2).11
101(c2).21 B.C.] E; *not present*
101(c3).9 *Ital.*] E; ~‸ *Also* 101(c3).18
101(c3).12 *Eng.*] E; ~‸ *Also* 101(c3).13, 27
101(c3).15 Eng.] E; ~‸
101(c3).16 Ital.] E; ~‸
101(c3).20 English] E; Englis
101(c3).23 *Engl.*] E; ~‸
101(c3).24 *Am.*] E; ~‸
102.3 particularly] E; particuliarly
102(c3).14 Rather] E; Rathr *Also* 102(c3).26
103.2 ready-made] E; ~‸~
103(c2).15 opening] E; Opening
103(c2).21 Found] E; ~,
103(c2).23 at] E; a
103(c2).28 interest,] E; ~‸
103(c3).11 Gr. honored] E; ~‸ Honored
103(c3).12 greatly] E; grealy
103(c3).18 Consid.] E; ~‸
103(c3).19–20 Immensely admired] E; Immensly Admired
103(c3).22 fully] E; Fully
103(c3).24 Greatly] E; Grealy

Alterations

96(c2).13 v. poor] *intl ab del* fair
96(c2).15 v. poor] *intl ab del* Invalid
96(c2).20 prob. fair] *intl ab del* moderate
96(c4).10 remark] *intl ab del* yes
96(c4).13 good] *intl ab del* rema
*98.7 Barneveldt] 42 *del in Qu. 47*
98(c3).18 21] *aft del* ?
98(c4).18 40 46] *bef del* 56
98(c4).19 40] *intl bel del* 39
100(c2).8 Yes] *ov* —
100(c2).19 Yes] *ov* —
100(c2).26 Yes] *ov* —
100(c3).10 1] *aft del* 0
101(c3).6 *Slav*] *intl ab del* Hun
102(c2).13 Yes] *intl ab del* No
102(c4).30 Yes] *ov* No
*103(c2).15 An] *ins*
103(c2).18 unexplored] *intl ab del* new
103(c2).18 science] *bef del* ready
103(c3).7 Not fully app.] *intl ab del* Good de
103(c3).10 Not app.] *intl ab del* Scarce
103(c3).17 Greatly] *ov* app.
103(c3).25 A good] *intl ab del* consi
103(c3).28 Unduly] *intl bel del* perhaps [*intl bel del* Fully]

19. *Classifications and Rankings, 1883–84*

Copy-text is MS 526, consisting of four pages on four leaves of white paper. The first three leaves, separated halves of formerly folded sheets, are chipped and dog-eared, lined, and inscribed in blue ink, have a "Carew Co. Improved" watermark, and measure $7\frac{7}{8} \times 10\frac{1}{32}$ in. The verso of the third leaf, entitled "Supplementary answers to Questions," actually belongs with MS 490. The fourth leaf is unlined, measures $5 \times 8\frac{3}{16}$ in., and is inscribed in black ink. List 1 is on the first page, List 2 on the next two pages, and List 3 on the fourth page. Horizontal, vertical, and diagonal dividing lines have been removed from the first list, and the four columns (somewhat irregular in the manuscript) have been made uniform. MS 487 consists of related lists of rankings, and MS 525 of related lists of classifications and rankings.

Textual Notes

104(c2).8 Peirce inserted all the occurrences of "Bent" and "Precocious" after inscription of the body of List 1. At 104(c2).16 and 104(c3).7, 14, 22 he did not have space to add them below the name and thus inscribed them above. In all instances, the editors have moved them below the name as Peirce intended.
104(c3).15 Lagrange 3.6] The editors have not changed the values for Lagrange ("3.6" in List 1, "3.5" in Lists 2 and 3); the value was probably somewhere in between the two.
104(c4).12 Peirce inscribed "4.1" before rather than after "Swedenborg" due to lack of space at the right edge of the page.
104(c4).26 Below the final column, Peirce has written "40," the total of the number of names in this list before he canceled one "Morphy," which he had inscribed twice; the editors have deleted the number.
105.18 Peirce inscribed the "Humboldt" entry to the right of "Young" without a direction line for its insertion below "Young."
105.32 The calculated value of "Rabelais" apparently fell between "4.5" and "4.6," and thus, in the second list, Peirce inscribed it between the two. Eventually he decided that "4.5" is the more appropriate value.
105.34 "4.7," which falls between "4.6" and "4.8" is the only blank entry in List 2 and has been omitted.
106(c2).14 "Macchiavelli" has been emended to conform with the spelling at 104(c4).9, 105.35.

Emendations

104(c2).12 2.5] E; 2‸5
104(c3).7 *(Bent)*] E; ‸~‸ *Also* 104(c2).8, (c3).14 (Bent), 22, (c4).18
104(c3).8 4.5] E; 4‸5
104(c3).15 Lagrange] E; Langrange
104(c4).5 *Writer &*] E; *rom. Also* 104(c4).10
104(c4).7 3.5] E; 3‸5
104(c4).8 *(Precocious)*] E; ‸Precocius‸ *Also* 104(c2).16 *(Precios),* (c3).11 Precocious, (c1).13, (c3).19, (c4).13 (‸~‸)
*104(c4).12 Swedenborg 4.1] E; 4.1 Swedenborg
104(c4).24 Palissy] E; Pallisy *Also* 105.14
105.3 Personality] E; *Pernality*
105.17 Young —] E; ~‸
*105.18 3.1 Humboldt] E; Humboldt 3.1
105.19 Turenne—General] E; ~‸ ~—
105.25 Barneveldt] E; Barnevelt
105.29 Statesman] E; Statement
105.30 Statesman] E; Stateman *Also* 105.35
*105.32 4.5] E; 4.5|4.6
*105.34 4.8] E; 4.7|4.8
105.39 Talma —] E; ~‸
105.41 Demagogue] E; Demagoge
*106(c2).14 Macchiavelli] E; Machivelli
106(c2).18 Talma,] E; ~‸

Alterations

104(c1).14 *Statesman*] *intl bel del* Ruler
104(c1).17 Hampden] *intl bel del* Grotius
104(c2).17 Augustus] *bel intl del* Preco
104(c4).10–11 Father of a Religion] *intl ab del* Religious Leader
104(c4).26 Rienzi] *ab del* Morphy
105.2 Poet] *aft del* Gree
105.14 3.0 . . . Personality] *intl*
105.16 3.0 . . . Biologist] *intl*
105.16 Biologist] *bef del* Personality
105.21 3.5 . . . Mathematician] *intl*
105.23 3.6 . . . Writer] *intl* [Critic *aft del* Dram]
105.25 Statesman] *aft del* Polit
105.29 4.1] 1 *ov* 2
105.38 5.2] 5 *ov* S
◊105.41 Demagoge] *intl ab del* Statesman Patrio

20. *Algebra of Logic: Part II, 1884*

Copy-text is MS 506, three and one-half pages on four somewhat chipped and dog-eared leaves of unlined, slightly heavy and stiff white paper measuring $7\frac{7}{8} \times 10\frac{5}{8}$ in., with an "American Linen Paper" watermark. The manuscript is carefully inscribed in black ink but has a moderate number of alterations. "On the" in the title appears on a torn and folded-down portion of the first leaf. The editors have added a colon to the main title and have deleted "By C. S. Peirce" and a dividing line, which follow after the subtitle. There is also a pre-copy-text version of the first page, with the same title, subtitle, and author line.

Emendations

107.9–10 "On . . . Logic"] E; ‸*On . . . Logic*‸
107.24 *et seq.* *X*] E; *rom. All capital and lowercase variables rom. except x* 107.26

Alterations

107.13 modifying] *intl ab del* changing
107.14 to] *bef del* ass
107.16 of] *bef del* mode

107.18 alternative] *intl ab del* simpler [*intl ab del* greater]
107.19 an aggregate.] *intl ab del* a part.
107.27 *A*] *bef del* with something dro
108.8 makes use of] *intl ab del* contains
108.16–17 negation . . . means of] *intl ab del* nothing can be done with
108.17–18 alone. Indeed . . . unbalanced,] *intl ab del* only without that of negation.
108.18 unbalanced,] , *ins bef del* operation

21. *Fragment on the Algebra of Logic, 1884*

Copy-text is MS 507, a fragmentary manuscript consisting of four chipped leaves of medium-weight, lined white paper. The first two, separated halves of one formerly folded sheet, measure $7\frac{13}{16} \times 9\frac{29}{32}$ in.; the last two $7\frac{13}{16} \times 9\frac{7}{8}$ in. All four have a "Carew Co. Improved" watermark and are inscribed on each recto in blue ink. There are a number of alterations in the manuscript. Two additional leaves of related fragmentary material also survive.

Emendations

109.4 schema] E; shema
109.10 *et seq.* *i*] E; *rom. All lowercase variables rom. except i* 109.12(2), 14(2)
109.13 $\Sigma_i a$,] E; $\Sigma_\mathrm{i}\mathrm{a}_\wedge$
109.13 *a*.] E; a
109.15 *j*'s] E; $\mathrm{j}_\wedge$s
110.12–13 term), . . . mark,] E; $\sim)_\wedge \ldots \sim_\wedge$

Alterations

109.16 To] *intl-c*
109.16 each] e *ov* E
109.16 some *i*] *intl-c*
109.16 *l*.] . *ins bef del* to some
109.19 some *j*.] *bef del* The signs of addition and multiplication are taken in their usual logical significations.
110.13 individual] *intl ab del* mark
110.13 positive] *aft del* affirm
110.26 ¶The] T *ov* t *and aft del* In place of
110.26 may be written] *intl ab del* we may write
110.28 formula] a *ov* ae
110.30 may . . . form] *ab del* $\Sigma_\mathrm{i}\Pi_\mathrm{j} = (\Pi$

22. *Algebra of Logic (Second Paper), 1884*

Copy-text is MS 508, a fragmentary, black-ribbon, italic typescript (with lowercase and capital variables not underlined or otherwise differentiated from the remaining text, and thus emended), consisting of eight pages on eight leaves that vary from light to slightly heavy and stiff, chipped white paper. The first two leaves measure $8\frac{7}{16} \times 10\frac{1}{2}$ in., with a "Holyoke Linen" watermark; the remaining six $8\frac{7}{16} \times 10\frac{7}{8}$ in., with "The American Linen Paper" watermark. There are a number of alterations; some are typed, others handwritten both in blue ink and in black ink. Five additional typed fragmentary leaves of pre-copy-text material also survive (one with extensive handwritten black ink notations on the verso).

Textual Notes

111.4 Peirce abandoned the opening of this typescript (¶"I begin by briefly restating the elementaryprinciples oflogical algebra, in the light of conceptionsgd") after the second line, which is followed by nearly three lines of random characters typed without breaks. The editors have deleted the five lines.

113.1–114.10 Peirce generally made the claw by typing hyphen and colon and then drawing connecting lines by hand. In this passage he neglected to draw the connecting lines.

113.13–15 As Peirce used the times sign throughout MS 508, the editors have regularized these three instances where multiplication dots are used.

115.diagram The seventh horizontal row mistakenly had two additional dots to the right, which have been removed.

115.29 Peirce made Greek capital pi and sigma by typing II (capital i's) and E and then drawing the cross bar for pi and tracing sigma over E in ink. He neglected to do the ink-drawing in this instance.

Emendations

*111.4 I] E; *no* ¶I
111.6 New] E; new
111.7 signifies or] E; sigifies of
111.10 mind,] E; ~.
111.11 is] E; si
111.13 universe] E; universre
111.14 described] E; desctibed
111.19 Or] E; On
111.26 pointed] E; ponted
111.27 connecting] E; conneting
111.27 system] E; sustem
112.5 non-existence] E; nonexistence
112.8 non-griffins] E; nongriffins
112.11 simple] E; sipmle
112.21 *et seq.* *a*] E; *All capital and lowercase variables undifferentiated from italic text*
112.29–30 *et seq.* v . . . f] E; *not bold*
112.31 v;] E; v.
113.1 formulae] E; fomulae
113.16 definitions] E; definition
114.28 —<, ×,] E; —< ×
115.2 De Morgan's] E; De Motgan's
115.12 one-to-one] E; ~‸~‸~
*115.29 Πs and Σs] E; IIs ~ Ês

Alterations

All alterations are by hand, except those on pp. 112–113.

111.24 point out to] *intl ab del* show
112.28(2) true] *bef del* and
112.36 each] ea *ov* th
113.32(2,3) *b*] *ov* c
114.11 antecedent] *intl ab del* consequent
114.12 (having f for its consequent)] *intl-c* and for [f *ov* a]
114.14(1) ,] *ins*
114.14 *x*,] *ins*
114.23(2) to] *intl-c*
114.24 refers] *bef del* to
114.29(2) ,] *ins*
114.29 in virtue] *aft del* we have
114.32 four] *intl ab del* three
114.34(3) Σ] *ov* (*and aft ins* (

23. *Peirce to Hilgard, 1884*

The copy-text of this letter, which is in a bound volume in the National Archives (Item 7, Entry 22, Record Group 23) and has been added to L 197, consists of twelve lined white leaves with a "Whiting Paper Co." watermark, inscribed on each recto in black ink (now faded) and an additional leaf, a printed map torn from a copy of the 1882 *Coast Survey Report* (facing p. 269); Peirce has marked the relevant stations with bold dots in black ink. The twelve pages are numbered in parentheses in the center of the top margin in red ink by someone other than Peirce. "Give here full address to which reply should be sent" is printed flush left (with a pointing hand) in italics at the top of the first page, followed by a line of dots on which Peirce inscribed "Office of Weights & Measures." "U.S. Coast and Geodetic Survey," is printed in the center of the next line, and that is followed, flush right, by two more lines of dots (the second with a comma and "188 .") on which Peirce inscribed "Baltimore Md," and "October 1, [188]4." respectively. Leaf 6 ends with "19," which Peirce mistakenly repeats at the beginning of the next leaf. As this is a private document, neither the "19" nor the repeated "45" (on leaf 8) has been corrected, nor have the omitted "90" (at the bottom of leaf 10) and "108" (on leaf 11) been supplied. There are many alterations and some ink blots in the letter. Five related leaves, dated 30 June 1885 and entitled "List of Stations of Provisional Scheme for Gravitation Survey," also survive in the National Archives.

Textual Notes

117.17 I] Peirce inscribed "I" with an apostrophe as he started to write "I'm"; but he thought better of the colloquial contraction, tried to wipe off the apostrophe, which is very faint in the manuscript, and then added "am."

117.29–30 Peirce revised this sentence but neglected to delete the article after "and."

118(c1).29 x 20.] The cross is no longer visible because of the hole made by the binding cord in the National Archives volume; but there was a station in Montreal, and there is a cross in the 30 June 1885 "List of Stations."

Emendations

116.25 Bermudas] E; Bermuda's
116.25 Guadeloupe] E; Guadalupe *Also* 119(c2).17
117.18 Mississippi] E; Missisipi
117.28 Townsend] E; Townshend
*117.29 and] E; ~ a
118(c1).2 *Easternmost*] E; *Eastermost*
118(c1).33 Crawford's or Willey's] E; Crawfurds ~ Wylie's
118(c2).9 Apalachicola] E; Appalachicola
118(c2).19–20 St. Philip] E; Phillip
118(c2).26 Vicksburg] E; Vickburg
118(c2).33 Milwaukee] E; Milwaukie
119(c1).8 Matamoros] E; Metamoras
119(c1).30 de Fuca] E; Fernandez
119(c2).10 Reikiavik] E; Reikavik
119(c2).12 Shoals] E; shoals
119(c2).16 Galapagos] E; Gallapagos

Alterations

116.11 gravity] *aft del* pendul
116.12 drawing] *aft del* fixing
116.18 by] *ov* in
117.4 geological] *aft del* geograp
117.8 lofty] *aft del* large
117.11–12 and churches.] *intl-c*
117.25 to] *intl ab del* in
117.28 at] *bef del intl-c* an
117.29 stations] *second* s *ins*
117.30 work] *bef del* be
119(c1).29 Port] P *ov* F
119(c2).3 *Alaska*] *ov del* Oceanic

24. *Small Differences of Sensation, 1884*

Copy-text is P 303, the publication in *Memoirs of the National Academy of Sciences.* It is emended by MS 565, Peirce's corrected offprint [565], inscribed on the cover to "Prof. John W. Langley | with the regards | of C S Peirce." (There is also an unannotated offprint among the Joseph Jastrow Papers at Duke University.) MS 513, a pre-copy-text typescript of the article, contains unfinished charts with data points added later by an unknown hand; although not reliable in its details, it is useful in determining the true order of the two charts beginning at 127.5.

Peirce and Jastrow first presented the article at the meeting of the National Academy of Sciences in Newport, RI, under the title "On Minimum Differences of Sensation," which is cited in the *Report of the National Academy of Sciences for the year 1884* (Washington: Government Printing Office, 1885), p. 12. *Memoirs of the National Academy of Sciences* (1884) includes P 303, the only published form of the article, which appears under the present title with the notation "READ OCTOBER 17, 1884 | By C. S. Peirce and J. J. Jastrow". An abstract of the article was published in the *Johns Hopkins University Circulars* (4 [January 1886]: 46), and an anonymous notice in *Mind* (11 [January 1886]: 128).

Textual Notes

126.18 below.] "First group" is not present in MS 513, and it is deleted in MS 565.
127.5 The tables for the second through fourth groups, which appear at the end of note 4 in P 303, have been moved here, because their printed placement creates a nonsensical reading for both note and text. In the absence of subsequent clarifying revisions by Peirce, the charts are returned to their pre-copy-text location (in MS 513).
127n.3 Mr.] The period is visible only in the offprint in MS 565, not in other printings.
133.5 This information and the data at 133.17, 22, and 33 appear in the left-hand column of MS 513, in which the tables are set up vertically rather than horizontally.

Emendations

122n.1 *Elemente der Psychophysik*] E; *rom.*
123.4 phenomenon,] 565; ∼‸
123.33 predictable] E; predictible
123n.3 *θt*] E; θt *Also* 123n.7
124.5 which.] 565; ∼?
124.12(2) is] E; *not present*
124.21 the sensations] 565; them
124.26–27 of. . .results] 565; *not present*
124.26–27 observer's] E; observers
124n.9 times,] E; ∼‸
125.23 change,] E; ∼‸
*126.18 below.] 565; ∼.|*First group.*
126.20 First Group.] 565; *not present*
126.20 Second Group.] 565; *not present*
126.30 Third Group.] 565; *not present*
126.30 Fourth Group.] 565; *not present*
*127n.3 Mr.] 565; ∼‸
127n.36 594] E; 504
129.31 heaviest,] E; ∼‸
129.35 lightest,] E; ∼‸
130.15 diminished,] E; ∼‸
131.13 17] 565; 28
131n.1 December] E; ∼,
131n.1 January] E; ∼,
131n.2 perfected,] E; ∼‸
131n.4 operator's] E; operator
132.32 &c.] E; etc.
133.38 16.0 ± 0.3] 565; 16
134n.17 strengthened] 565; strengtheued

Line-End Hyphenation

129.20 left-handed

25. *Success of Predictions, 1884*

Copy-text is P 292, the publication in *Science,* which may have grown out of a course described in the *Johns Hopkins University Circulars* of June 1884 (3, 119): "Mr. C. S. Peirce gave a course . . . in Probabilities, twice weekly during the second half-year." Peirce's article appears in the section of "Letters to the Editor" and is signed "C. S. Peirce" at the end. According to an abstract in the *Bulletin of the Philosophical Society of Washington,* this article was critically discussed and facetiously commented on by M. H. Doolittle at the 3 December 1884 meeting of the Society ("The Verification of Predictions," 7, 122–27).

Emendations

136.19–20 *Meteorological Journal*] E; *meteorological journal*

138.5 *p*] E; *p* ·

26. *The Old Stone Mill, 1884*

Copy-text is P 293, the publication in *Science.* An anonymous notice of Peirce's measuring of the Stone Mill at Newport, where he attended the National Academy of Sciences meeting on 14–17 October 1884, appeared in the *New York Evening Post* (12 December 1884, p. 4) and was reprinted the next day (p. 1). An anonymous response to Peirce's article appeared in the *Nation* (39 [18 December 1884]:522, under "Notes"), and a letter to the editor in the *Post* eleven days later (supplement section, p. 1). "C. S. Peirce" appears at the end of the published article. There is a three-page, handwritten, pre-copy-text fragment in MS 515, which corresponds to 139.3–140.4; its leaves are numbered (rather curiously, because they read consecutively) 1, 8, and 5.

Emendations

139.8 anyone] 515; any one

143.2 archeological] E; archeologi-|ical

27. *Reciprocity Treaty with Spain, 1884*

Copy-text is P 279b, the publication in the *Nation,* which is emended by P 279a, the publication in the *New York Evening Post* (16 December 1884, p. 2, col. 3); in the latter, the title is "The Sugar Problem" [279a]. Both are probably from the same standing type, for the *Nation* was published after 1881 by the Evening Post Publishing Company as the weekly summary magazine of the *Post.* Collation indicates that the only variations between P 279b and P 279a are the titles (neither of which is Peirce's), the salutations to the *Evening Post* instead of to the *Nation,* the absence of brackets before and after the editor's comment, and the absence of a final editorial signature in the *Post.* The missing hyphen in P 279b is apparently the result of dropped type, for there is space for it on the line. As early as 1 December 1884 and thereafter throughout the month, the *Evening Post* carried notices regarding the Spanish Treaty. The 11 December issue of the *Nation* had a discussion of the treaty as its first item, which had appeared in the *Evening Post* three days earlier in somewhat different form. However, Peirce's letter was informed by a knowledge of the reports and discussion in the *Evening Post* regarding the treaty. Vertical marginal lines indicate non-Peirce text.

Emendation

144.7 non-Spanish] 279a; ~-Span|ish

Line-End Hyphenation

145.25 sugar-growing

28. *Spanish Treaty Once More, 1885*

Copy-text is P 306a, the publication in the *Nation,* which also appeared two days later in the *New York Evening Post* (p. 4, col. 1), probably from the same standing type (P 306b). Collation indicates that the only variations between P 306a and P 306b are the use of different fonts in the title, the salutation to the *Evening Post* instead of to the *Nation,* the absence of comma and 1884 after the date, and the final editorial signature by the editor of the *Evening Post* rather than the *Nation.* Vertical marginal lines indicate non-Peirce text.

Emendation

148.14–15 One thirty-second] E; ~ -~-~ *Also* 148.18

Line-End Hyphenation

147.7 non-Spanish
148.6 non-Spanish

29. *Testimony on the Coast Survey, 1885*

Copy-text is P 339, whose full title is "Testimony before the Joint Commission to Consider the Present Organization of the Signal Service, Geological Survey, Coast and Geodetic Survey, and the Hydrographic Office of the Navy Department, with a View to Secure Greater Efficiency and Economy of Administration of the Public Service in Said Bureaus, Authorized by the Sundry Civil Act Approved July 7, 1884, and Continued by the Sundry Civil Act Approved March 3, 1885." The *Miscellaneous Documents* were published by the Government Printing Office in Washington, D.C.

Emendations

149.11 Office] E; office *Also* 149.14, 16; 150.9, 38; 151.25, 27, 31; 154.28; 155.2; 157.8; 158.6, 32; 159.2, 34
149.11 Weights and Measures] E; weights ~ measures *Also* 149.15, 16; 150.9; 151.25, 32; 158.6
150.22 *et seq.* metre] E; meter *All forms*
153.6 Gallatin] E; Gallatia
155n.2 ounce.] E; ~‸
156.21 troy] E; Troy
157.7 *pourparlers*] E; *pour parlers*
160.23 Bureau] E; bureau *Also* 160.29

30. *On the Algebra of Logic, 1885*

Copy-text is P 296, the publication in the *American Journal of Mathematics* (in the third line "By C. S. Peirce" follows the subtitle). MS 558 contains an offprint with one (surviving) annotation by Peirce; all other annotations and instructions are by the editors of the *Collected Papers* who cut, pasted, and annotated the offprint as printer's copy for their edition. According to the *Report of the National Academy of Sciences* (1884, p.13), Peirce presented "On the Algebra of Logic" at the 14–17 October 1884 meeting of the Academy in Newport, RI, and it was discussed by Messrs. Powell and Peirce. A number of items in the present volume (and one in the preceding volume) as well as several other manuscripts are related to this article: MS 509, seven discontinuous leaves on logical algebra; MS 510, a three-leaf fragment of an early draft of part of section I; items 20, 21, and 22, which are continuations of Peirce's 1880 "On the Algebra of Logic" (W4:163–209) or restatements of the principles of logical algebra in light of O. H. Mitchell's paper; and item 31.

Textual Note

173n.6 this error] In MS 558, Peirce inscribed in the margin, "But it was not an error!!! See my original demonstration in marginal note." The marginal note has not been located.

Emendations

163n.1 *of the*] E; *not present*
164.12 pronouns] E; pronoun
164n.1 *Logic. By Members*] E; ~, by members
164n.1 *of . . . University*] E; *rom.*
164n.1 Little,] E; ~&
164n.2 & Co.] E; *not present*
167.27 *[formula]*] E; ~.
167.30 *[formula].*] E; ~‸ *Also* 181.32, 187.7, 189.12
168.23 2[nd]] E; 2d *Also* 174.11, 181.21, 182.18, 184.3
172.16 formulae] E; formulæ *Also* 173.13, 173n.8, 15; 187.2, 8; 188.7
172.27 thing's] E; things
173.6 $\bar{x}$.] E; $\bar{x}$
174.12 3[rd]] E; 3d *Also* 183.3, 184.4
175.16 true,] E; ~‸
176.18 $(m \prec i)$,] E; ~‸
177.6 $\overline{\bar{x}+y}$] E; $\overline{\bar{\bar{x}}+\bar{y}}$
177.7 $\bar{x}+y$] E; $x+y$
177.21 *Barbara*] E; *rom.*
178.1 character,] E; ~‸
179n.17 $\left(\frac{7}{12}\right)$] E; $\left(\frac{7'}{12}\right)$
182.13 *et seq.* Πs . . . Σs] E; Π's . . . Σ's
182.28 $\Pi_j\Sigma_i$] E; $\Pi_i\Sigma_j$
183.12 indispensible] E; indispensable
185.25 q_{kj}] E; q_{jk}
186.22 excludes,] E; ~‸
186.26 includes,] E; ~‸
187.8 gives] E; give

Line-End Hyphenation

178.8 well-known

31. *Notes on the Algebra of Logic, 1885*

Copy-text is MS 538, twenty-eight pages of text on leaves of very heavy, stiff, and unlined white paper measuring $7\frac{1}{16} \times 9\frac{1}{16}$ in. Separated halves of formerly single sheets, the leaves have a watermark of a crown above a shield and "A. P. & S. Antique Parchment Note Paper," and they are inscribed in blue ink, with numerous alterations in black ink. (There are also black ink and pencil alterations by Paul Weiss, one of the editors of the *Collected Papers.*) At the top of the first page, Peirce inscribed but then deleted "(For the American Journal of Mathematics)"; he expanded the title by the added word "Notes"; between it and the "By C. S. Peirce" byline, he inserted "(To follow Am. J. Math. VII. 202.)"; below the byline, he deleted "(Continued from Vol vii. p. 202)"; between it and the body of the text, Weiss inscribed "Note." A small portion of the bottom of the first leaf has been torn off. The copy-text is emended by an amanuensis copy [538Am] (which does not include 196.3–197.20), consisting of thirteen damaged and brittle thin graph paper leaves that are separated halves of formerly single sheets inscribed in black ink and measuring $9\frac{1}{8} \times 11\frac{13}{16}$ in. Enclosed at the beginning of the amanuensis copy is a page (on the verso of Harvard University, Department of Philosophy, stationery) inscribed by Weiss with notations for the *Collected Papers.* Collation indicates that the amanuensis copy was transcribed from MS 538 and that any differences between it and the copy-text (except for the 191.1 emendation) are due to scribal errors. The presence of "the" before "mathematics" (198.17) suggests that the amanuensis copy was made before Peirce deleted "the" in MS 538 or that the copyist inadvertently transcribed the deletion. Twenty-two leaves of discarded earlier versions of different parts of MS 538 also survive.

Textual Notes

194.drawing The drawing has been excised from the manuscript (probably by the editors of the *Collected Papers,* who apparently used it as printer's copy), and "cut being made" is inscribed in an unknown hand in pencil beside the excision. The amanuensis copy of the drawing, used in the present text, is somewhat different from the one in CP 3.403E.

195.18–21 Only these four lines are inscribed on the tenth manuscript leaf. The rest is blank except for two pencil check marks below the text.

198.6 The lower half of the manuscript leaf after this point is blank, except for a pencil check mark.

198.20–22 Peirce probably inscribed this sentence as an indented footnote, but then deleted the asterisk after "proposition" and before "As"; he did not, however, delete the brackets, which are not quite large enough to include "many exceptions." Paul Weiss has drawn parentheses over the brackets. As suggested by the amanuensis copy, the sentence is run on as a bracketed interpolation.

201.14 $\bar{1}_{eh}$] One of the editors of the *Collected Papers* has deleted the incorrect "c" and inscribed an "e" above in pencil.

Emendations

191.1 on] 538Am; On
191.4 183] E; 197
191.6 *et seq.* a_{ijk}] E; *rom. All capital and lowercase variables rom. except* n 194.2, 3, 9, 18; 201.19; i 194.7, 11; 197.19; 200.5; 201.8; u 194.8, 10; 203.2; u^2 194.9; m 195.9, 18; 202.17; l 195.20; 197.20; q, r 198.7; b 198.29; 199.8; a 198.29, 35; 199.8; 200.24; 201.8; 202.17; k 197.19; 198.34; 200.9, 10; d 199.8; 201.8; c 199.8; 200.16, 201.8; 202.17; j 197.20; 200.5; x 200.10, 11, 15; 201.1; y 200.11, 15; 201.2; f 200.24; 201.8; g 201.8; w 203.2
191.13 1_{ii}] E; 1
191.15 f] E; *not bold Also* 191.17
192.22 distribution-formula] E; ~ ‸~ *Also* 192.31
192.26 association-formula] E; ~‸~
193.7 in the middle . . . 182] E; at the bottom . . . 196
193.13 · etc.] E; ‸~‸ *Also* 193.15
193.14 $a_2a_3\cdot$] E; $\mathrm{a_2a_3}$‸
193.16 etc.] E; ~‸ *Also* 193.21, 23–26; 196.14, 16, 17
*194.drawing *[drawing]*] 538Am; *not present*
195.6 utility] E; ultility
196.2 $+\,ax\bar{b}$] E; *not present*
196.3 183] E; 198
197.1 an] E; and
198.1 $\bar{1}_{jy}$] E; 1_{jy}

198.24 186] E; 200
199.12 187] E; 200
199.25 $\Pi_\lambda \Sigma_k$] E; $\Pi_\lambda \Sigma_q$
*201.14 $\bar{1}_{eh}$] E; 1_{ch}
201.17 *x*'s] E; $\sim_\wedge\sim$
201.18 *y*'s] E; $\sim_\wedge\sim$

Line-End Hyphenation

191.19 distribution-formulae

Alterations

191.3 *On*] *aft del* Notes
191.8 a] *ov* 1
191.19 formulae] *bef del* *
191.19 This] *aft del* foot-note.*
191.20 third] *intl ab del* fourth
192.15–16 *any . . . true*] *add underline*
193.32 theorem] *intl ab del* proposition
193.32 of] *intl ab del* for
194.8 as in] *intl-c*
194.18 of] *ov* is
194.19 all] *bef del* can
194.20 by] *ov* for
194.23 If the proposition] *bel del* If there are
195.22 artifice] *intl ab del* evolution
195.24 taking] *bef del* the whole or
195.24(1) the] th *ov* a
195.25(2) the] t *ov* a
195.26 this] is *ov* e
196.11 Confining] *aft del* Considering
196.11 attention] *bef del* at fir
196.19 The first index] *bel del* In the case of the
196.26(1) Π_i] Π *ov* Σ
197.1 a] *intl ab del* a
197.1 Π] *bef del* s
197.1 may] m *ov* c
197.3 under what circumstances] *intl-c ab del* how
197.7 prevent] *bef del* the
197.6 *[formula]*]*bel del* $\Sigma_i \prec \Sigma_i\Sigma_j \mid \Sigma_i a_i \prec \Sigma_i\Sigma_j a_i a_j$
197.16 1_{ij}] l *ov* a
197.26(2) is] *bef del* only a
197.29 value of the] *intl-c*
197.30 every quantity has] *intl ab del* there is
198.4 of association and] *intl-c*
198.9–10 [Compare . . . natures.]] *intl aft intl del* Angelic natures
198.12 subsisting] *bef del* only
198.17 Of] *intl ab del* For
198.19–20 every proposition] every[thing *del and bel intl* proposition]
198.34 in the factors] *intl-c*
199.14 the square of this] *intl-c ab del* this
199.16 Identifying] *aft del* We *and* I *ov* i *and* ing *add*
199.20 the] *bef del* equation
199.21 Only] *aft del* We ♦get [*intl ab del* have] then
200.5 it] *bef del* does
200.11 a] *intl-c*
200.19 Σ_b] Σ *ov* Π
200.19 Σ_e] Σ *ov* Π
200.19 r_{kde}] e *ov* b
200.19 y_e] e *ov* b
202.7 twice into] *intl ab del* by
202.10(2) $\bar{r}_{\gamma ab}$] $\bar{r}$ *ov* u
202.11 $r_{\alpha ab}$] αab *ov* abα
202.14 We have universally] *bel del* The simple omiss
202.19 Σ_a] *aft del* Σ_δ
202.19 1_{cb}] *ov* r
203.2 last] *bef del* form

32. *Studies in Logical Algebra, 1885*

Copy-text is MS 539, a dark brown leather notebook with a loose binding containing forty-seven leaves of medium weight, gilt-edged, and unlined white paper measuring $6\frac{9}{16} \times 7\frac{31}{32}$ in. Peirce inscribed the dated notes of this private document in blue ink on the recto of each leaf, which he numbered from 5 to 93 (odd numbers only) in the top right corner (95 to 99 are numbered in pencil by another hand); some of the rectos are only partially inscribed. Many of the verso pages contain commentary, notes, or formulae related to the text on facing recto pages, and all of these are recorded in textual notes below. The principal notations on the versos facing pages 73, 75, and 79 and 81 appear to be actual additions to the copy-text and have therefore been incorporated (even though Peirce did not indicate where they should be placed). Textual notes indicate the rationale for placement in this edition. On page 5, Peirce has written "C. S. Peirce 37 Grammercy Park|New York," which is followed by a horizontal line and the title, and that in turn is followed by "1885 May 20." The editors have moved the date to the head of the first journal entry in the notebook, and have removed horizontal lines below 204.7, 204.19, 213.29, 215.7.

A summary of the close relationship between items 30, 31, and 32 is included in the item 30 headnote. The unrevised state of the text of item 32 reveals dead ends, loose ends, and cryptic references to the earlier items. Many of Peirce's comments within the text (as well as his verso remarks) are notational and even rhetorical, indicative of the experimental nature of his effort to explore and extend quantification theory.

Textual Notes

204.4–6 Peirce wrote the following commentary on the facing verso: "See proviso p 21 *[*206.22–24*]*. ¶Note also slight difficulty on p 23 *[*207.1–2*]*. ¶This principle implies the unlimited character of the universe of quality. Can be applied to objects when their universe is unlimited."

204.23–205.8 Peirce wrote the following on the facing verso: "Try to do this with the identical formula," which is followed on successive lines by the canceled fragment "$\Pi_k \bar{q}_{ki} \bar{q}_{kj} + q_{ki}$" and by "$\Pi_l (q_{li} q_{lj} + q_{li} \bar{q}_{lj} + \bar{q}_{li} \bar{q}_{lj})$ ¶This does not do." Opposite the fifth step (205.6–8) of the same example, he wrote "$q_{li} \bar{q}_{mi} \bar{q}_{mj} q_{lj} + \bar{q}_{li} q_{mi} q_{lj} \bar{q}_{mj} + \bar{q}_{li} q_{mi} \bar{q}_{lj} q_{mj}$" before canceling the middle term.

205.16–19 At this point on the facing verso, Peirce defines the extent to which q_{ki} and $\bar{q}_{kj}$ may be eliminated: "They always will be eliminable so long as we do not uselessly introduce q_{ki} and $\bar{q}_{kj}$."

205.29–31 Peirce worked a variant expression of the premiss on the facing verso:

This does not seem correctly expressed
$\Pi_i \Sigma_a \Pi_b \Sigma_c$
$x_a(\bar{r}_{iab} + \bar{y}_b + r_{icb} \bar{1}_{ac})$
The form given is the negative of
$\Sigma_i \Pi_a \Pi_b \Pi_c \Pi_d \Sigma_e$
$y_e(r_{ide} + \bar{x}_d)(\bar{r}_{iac} + \bar{r}_{ibc} + \bar{x}_a + \bar{x}_b + \bar{y}_c + 1_{ab})$
But perhaps the two forms are identical,

"This hardly follows," which appears at the bottom of the same verso, probably refers to the facing text on the recto: "There must then be some undiscovered method of elimination" (206.6).

207.20 On the facing verso, Peirce has noted that "Though this proposition is necessary, it is pretty hard to justify it by my rule" (which reference may be to the rule opening the *Studies* at 204.4–6 or to the general rule developed in item 30 beginning at 182.11).

208.28 Peirce's verbal statement of this proposition appears on the facing verso: "In words. Either nothing at all is x or else there is a relation that every y bears to some x to which nothing else bears the same relation."

209.8–9 Peirce's facing verso comment opposite his rule of elimination anticipates his own challenge to the eliminatory process that begins the next recto in the manuscript (209.11–19): "But is this permissible?"

209.22 Also] The word is preceded, flush left, by "N" (probably the beginning of the word "Now"), which Peirce forgot to delete.

210.6–10 Peirce has written the following on the facing verso: "But there seem insuperable difficulties to this, at least as a thing to be put into practice."

210.25 $\Sigma_1 1_{11}$] Peirce defined "1" as a dyadic identity relation in the logic section of item 30: "We may adopt a special token of second intention, say 1, to express identity, and may write 1_{ij}." In using this relation for item 32, Peirce wrote "$\Sigma_1 1_{11}$" (185.5–9). The editors have supplied the second relate and recorded the change as an emendation.

211.3 Peirce wrote "No; not quite" on the facing verso, opposite the tentative reduction of the proposition at 211.2.

211.4–8 On the verso facing the proof of the falseness or contingency of the proposition at 211.3, Peirce developed a second example of contingency:

Take the proposition
$\Sigma_\gamma \Pi_\beta \Sigma_4 \Pi_5 \Sigma_6 \Pi_7 \Sigma_8 \Pi_9$
$y_4(\bar{r}_{\beta 45} + \bar{z}_5 + r_{\beta 65}\bar{1}_{46}) + \bar{x}_7 + r_{\gamma 78} z_8(\bar{r}_{\gamma 98} + 1_{79})$
Put $r_\beta = 1$ and drop tokens having γ as index
$\Sigma_4 \Pi_7 \Sigma_8 (y_4 + \bar{x}_7 + z_8)$
which is again non-identical

The last line of the proposition originally read "$\Sigma_4(y_4\bar{z}_4 + \bar{x}_7 + z_8)$."

211.9–10 To prove. . . .] Peirce's facing verso commentary reads: "But it dont follow because a proposition is contingent that its contradictory is necessary, does it?"

211.17–18 Peirce developed a diagram of these substitutions on the facing verso:

$\Sigma_\alpha \Pi_1 \Sigma_2 \Pi_3$
$\Sigma_\beta \Pi_4 \Sigma_5 \Pi_6$
$\Pi_\gamma \Sigma_7 \Pi_8 \Sigma_9$

211.19 Peirce's first stage of the reduction has a long horizontal line running below the formula, with three vertical marks at the beginning, middle, and end, pointing respectively to $r_{\alpha 12}$, $r_{\beta 25'}$ and $\bar{r}_{\gamma 15}$.

212.21–27 On the facing verso, Peirce provided the following notes on raising the power of the Boolian:

The first power instead of yielding
$\Pi_1 \Sigma_2 (\bar{x}_1 + y_2)$ gives only
$\Pi_1 \Pi_4 \Sigma_2 \Sigma_5 (\bar{x}_1 + x_5 + \bar{y}_4 + y_2)$
The second power
$\bar{x}_1 + \bar{x}_{1'} + 1_{13} 1_{1'3'} + y_2 y_{2'} r_{\alpha 12} \bar{r}_{\alpha 32} r_{\alpha 1'2'} \bar{r}_{\alpha 3'2'}$
$+ \bar{y}_4 + \bar{y}_{4'} + 1_{46} 1_{4'6'} + x_5 x_{5'} \bar{r}_{\beta 45} \bar{r}_{\beta 65} r_{\beta 4'5'} \bar{r}_{\beta 6'5'}$
$+$ etc
And so any power will yield an identical proposition.
Ordinary reasoning can hardly show that the proposition holds even for ultrainfinites

214.8–10 On the facing verso opposite the 3rd step formula, Peirce noted: "Corrollary | $a_{ijk}\bar{a}_{iyz} \prec \bar{1}_{jy} + \bar{1}_{kz}$." Opposite the 4th and 5th steps, Peirce noted: "Development | Combined these"

215.12–22 "See De Morgan|Postpone this till have read De Morgan" is diagonally inscribed to the left of this passage; just above that inscription is a diagram (beneath another canceled diagram) that attempts to conceptualize the passage in graphic terms.

215.28–33 At an angle across the verso opposite the entire sum formulation, Peirce wrote: "Required to abbrieviate or simplify the formulae."

216.4–6 Peirce has drawn two lines connecting Σ_1 to Σ_5 and Π_2 to Π_4; they cross between "the" and "*x*'s."

216.9–11 This short version of the premiss introduced at 216.4 appears on the facing verso. Peirce's reference to the subsidiary token *m* at 218.13 indicates his intention to incorporate the passage into the body of the work, although he did not specifically mark it for insertion. The editors have placed the passage where it best fits the context of the recto manuscript page.

216.12–17 Peirce wrote the following vertically down the facing verso page: "But what about the fourth power? ¶You have to proceed at once to the sixth, n'est ce pas?"

216.14 Whereas the dot at $x_{51} \cdot x_{52}$ and the first two dots at 216.16 are multiplication dots, the dots in the subscript numbers (215.32 onward) separate discrete numbers that would otherwise be unclear.

217.1–5 This passage appears at the top of the verso facing Peirce's initial sequence of operations (216.18–31,) with the premiss identified at 216.19. Although it is not marked for insertion, the context suggests that it is a continuation of the summary Peirce began at the bottom of the facing recto; a second sequence of operations with the same premiss opens the next recto (beginning 217.6). Below the verso passage is a general statement of the paradigm applied to the operation: "$(a + b)(c + d) \prec a + c + bd$"; it appears to be notational and is not considered a part of the copy-text.

217.12 The emendation here and at 219.2 results from the fact that, whereas Peirce deleted a phrase and a portion of a formula, he neglected to delete the preceding comma and plus sign, respectively.

217.14 On the facing verso, Peirce wrote the following formula (which appears to be a representation of a stage in the identifications described at 217.15–17):

$$\{\bar{y}_2 x_1 + x_1(\bar{r}_{\alpha 12} + r_{\alpha 32}\bar{1}_{13}\}(\bar{x}_1\bar{r}_{\alpha 12} + x_1 r_{\alpha 12})(\bar{x}_3\bar{r}_{\alpha 32} + x_3 r_{\alpha 32})$$
$$\bar{y}_2 + x_1 r_{\alpha 32}\bar{1}_{13}x_3$$

217.30–218.9 This alternate procedure is developed on versos 78 and 80, with continuation instructions bridging both pages. Peirce left no further instructions, but his specific reference to the procedure at 219.32 indicates his intention to incorporate the passage into the body of the work. Context indicates that the procedure provides an alternative to the "need of introducing again the last result" noted at the end of recto 79; it has been inserted before the summation that opens recto 81.

218.diagram Design of the printed page gives exaggerated prominence to this diagram, which in the manuscript is an interlinear notation just inside the left margin of the page; it was probably drawn to help Peirce visualize possible identifications for the restatement at 218.14–18.

219.1–5 Peirce wrote the following notation on the facing verso:

Note the formulae
$\Pi_a \Sigma_b(\bar{x}_a + x_b)$ From the square of 1
$\Pi_a \Sigma_b \bar{x}_a x_b = 0$
$\Pi_a \Sigma_b(x_a + \bar{x}_b)$
etc.

219.10–20 Peirce's development of the premiss led him to make the following verso comments: "But this development is not used" (facing 219.10–12); "Peculiar process | Very important" (facing 219.14); and "Note this proceeding" (facing 219.17–18).

219.21 Peirce has inscribed a direction line between this formula and the following clause, and a line below the formula connecting $\bar{r}_{\beta 32}$ and the second $r_{\beta 32}$.

219.30 of] Although the word in the manuscript looks like "off," the second f is merely a retracing of the first.

219.30–32 Opposite this reference to the method of solution for the problem at 218.21–219.5, Peirce noted on the facing verso: "Method of manipulating Boolian."

The comment is punctuated by two hand icons pointing to the recto reference. A third oversize hand icon to the left of the comment points to the top verso page and, by extension, to the earlier pages of the notebook, where the method may be found.

220.3–11 The symbols used in the following verso comments (and in the comments facing 220.12–18)—dagger, cup, and negative sign—belong to Peirce's notation for the logic of relatives and are defined in W4:338–41. Peirce made the following verso comments on the *Numbers* section: "Abbrieviated" (facing 220.4–5); "$\breve{e}_\infty$" and "$\Pi_A \breve{e}\infty$" (facing 220.6); "$e\infty + 1$" and "$\Pi'_A \;\, e\infty_A$" and "$\Pi_A \Pi_B (1_{AB} + (e\infty)_A + (e\infty)_B$" (facing 220.7); "$(\breve{\bar{e}} \dagger \bar{e}) + 1$" and "$\Pi_B \Pi'_A \;\, (\breve{\bar{e}} \dagger \bar{e})_{AB}$" and "$\Pi_A \Pi_B \{ 1_{AB} + (\breve{\bar{e}} \dagger \bar{e})_{AB} \}$" (facing 220.8); "$(\bar{e} \dagger \breve{\bar{e}}) + 1$" (facing 220.9); "$\infty \bar{e}$" (facing 220.10); and "$(\bar{q} \dagger \breve{e})\infty + 1, (qe\breve{\bar{q}}) + q$" and "$\Pi_\alpha \Sigma_B \Pi_E (\bar{q} \dagger \breve{e})_{AB} + (qe\breve{\bar{q}})_{\alpha\alpha} + q_{\alpha E}$" (facing 220.11).

220.12–18 Peirce made the following notation on the facing verso:

$$(\bar{q} + \bar{c}) \dagger \Sigma_i \;\, [\{r_i, (\bar{r}_i \dagger 1)\} \infty], [\{\bar{r}_i, (\bar{r}_i \dagger 1)\{ \infty]$$

$$(c \dagger \bar{e} \dagger \breve{\bar{c}}) + \bar{1}$$

Above the first line are the canceled false starts "$\bar{q} + \bar{c}$" and "$[\bar{q} \dagger \Sigma_i \;\, \{r_i, (r_i \dagger 1)\} \infty] + \bar{c}$"; to the left of the second line is the canceled false start "$\bar{c}$."

220.18 Below the formula, Peirce drew lines that connect $_\alpha$ to $_\alpha$, $_A$ to $_A$, and $_B$ to $_B$.

220.21 On the facing verso, Peirce wrote "$\Pi_\beta \infty \{x[(\bar{r}_\beta \dagger \bar{y}) + \bar{1}r_\beta]\}$," which is followed by a canceled "$(\bar{1}r_\beta)_{12}$."

220.25–26 To the right of these lines, Peirce has inscribed two rows of dots, five and four respectively; a vertical line is drawn between the two first dots.

Emendations

204.4 Every] E; *no* ¶ ~
204.4 *et seq.* q_α] E; *rom. All capital and lowercase variables rom. except m* 204.13, 215.7; *l* 206.14; *a* 207.5; 213.14, 16(2), 18(2); *c* 207.5; *q* 207.5; *x* 208.7, 209.27(2), 215.26; *b* 213.18; *n* 215.6; *i* 215.24; *y* 215.25; *z* 215.25(1)
204.4 as] E; *not present*
205.17 wherever] E; where-|ever
205.25 q_{ki})] E; q_{ki} ‸
206.21 204] E; 7
206.25 above] E; on p. 17
207.8 on p 206] E; at the bottom of p 21
207.10 desirable] E; desireable
207.11 on p. 206] E; at the bottom of p. 17
207.20 $q_{ja}q_{jc}$] E; $q_{ja} + q_{jc}$
207.29–30 non-ultrainfinite] E; ~ ‸ ~ *Also* 209.27, 31
208.10 206] E; 17
208.14 important] E; importantant
208.15 204] E; 7
208.27 y's] E; ys *Also* 215.26, 216.5
208.33 1's] E; 1s
209.9 $\prec$ *[formula]*] E; ‸ ~
209.15 pp. 183–84] E; p 198
*209.22 Also] E; N ~
209.24 it] E; its
209.30 q's] E; ys
210.8(2) α] E; $\underline{\alpha}$
210.25 $\Sigma_1 1_{11}$]E; $\Sigma_1 1_1$
211.6 = *[formula]*] E; ‸ ~
211.10 210] E; 45
212.2 *[formula]*)] E; ~ }
212.14 Boolian] E; ~|$r_{\gamma 78} r_{\gamma 7'8'} r_{\gamma 7''8''}$
214.14 1_{ii}] E; 1_i
214.20 Σ^m] E; Σ^n
214.22 *[4 dittos]*] E; *[last ditto missing] Also* 215.9 (*[first ditto missing]*)
215.26 x's] E; x‸s *Also* 216.5
216.24 *[formula]*}] E; ~ ‸ *Also* 217.25
217.3 (] E; {
217.5 (*[formula]*)] E; { ~ ‸
*217.12 true.] E; ~,.
218.8 ($\bar{r}_{\beta 12}$] E; { ~
218.13 216] E; 72
218.22 216] E; 73
219.2 1_{67}] E; ~ + $\bar{}$
219.12 $(r_{\gamma jh}\bar{1}_{gj})$] E; $(r_{\gamma jh}\bar{1}_{gj}$ ‸
219.19 3,] E; 3
219.20 $[\Sigma_a \Sigma_b]$] E; { ~
219.28 219] E; 87
219.29 218–19] E; 85

219.30 217] E; 79
220.1 219] E; 87
220.2 pp 217–18] E; p 78
220.15 object,] E; $\sim_{\wedge}$
220.28 $\bar{1}_{ad}$.] E; $\bar{1}_{ad\wedge}$

Alterations

204.11 premise] e *ov* op
205.9 and] *aft del* again
205.10 *[formula] bel del* $\Pi_l \Pi_k \; (q_{li} + \bar{q}_{lj})\,(q_{ki} + \bar{q}_{kj})$
205.16 proposition] *intl ab del* equate
205.18 but Π_k] *intl ab del* & may
205.24 *[formula]*] *bel del* $\Pi_l \Pi_k \; (q_{li} q_{kj}) + (\bar{q}_{li} \bar{q}_{ki})\,(q$
205.24 Σ] *ov* Π
205.27 y] *intl ab del* B
205.27 x] *ov* A *Also* 205.28
205.28 y] *ov* B
205.29 Π_i] i *ov* a
205.31 r_{iac}] c *ov* b
205.31 x_d] *intl-c*
206.2 Π_f] *bef del* $r_a(\bar{x}_a + r_{iab} y_b)$
206.5 *[formula]*] *bel del* $\Sigma_i \Pi_a \bar{x}_a$
206.14 This] *aft del* Now
206.31 q_k] *intl-c*
206.33 That definition runs that] *ab del* $\Pi_a \Sigma_i \Pi_i$ [*ab del* $\bar{x}_a$ +]
206.34 Π_f] *intl-c*
207.1 cannot] not *ins*
207.13 Fermatian] *intl ab del* mathe
207.16 Logic] *aft del* Theory
207.18 $x_b x_c$] *intl-c*
207.20 $\Pi_a \ldots \Pi_d$] *bel del* ◆$\Pi_a \Pi_b \Sigma_i$◆◆$\Pi_c \Pi_j$ [*intl-c and aft del* ◆◆◆$\Pi_a \Sigma_i \Sigma_b$ [*bel del* $\Sigma_i \Pi$]] [*del by editors*] {$\bar{x}_a$ [x *ov* a] + $r_{iab}(q_{ja} q_{jc} + \bar{q}_{ja} q_{jc} + \bar{r}_{icb}$ ◆ $(q_{kb} q_{kd} + \bar{q}_{kb} q_{kd} + \bar{r}$ [*intl-c and del by editors*]
207.22(1) 1] *ov* r
207.25 $\Pi_j \ldots \Pi_n$] *bel del* $x_e y_f$ [*bel del* $\Pi_j \Sigma_e \Sigma_f$]
207.26 r_{jeh}] h *ov* f
207.33(2) Σ] *ov* Π
208.19–20 or . . . circumstances] *intl-c*
208.21 among] *bef del* ◆its relates. [*bel intl del* the kinds] thin
208.22 incompatible] i *ov* c
208.23 explicit] *intl-c*
208.27 at least] *intl-c*
208.28(1) Σ] *ov* Π
209.4(1) x] *ov* y
209.5 $\bar{y}_2$] *bef del* $\bar{y}_5$
209.6 Boolian] *aft del* factor
209.6(2) 2] *ov* 5
209.6–7 and putting . . . 3] *intl-c*
209.8 $r_{\alpha 46}$] 6 *ov* 3
209.14 $\Pi_1 \Sigma_2 \Pi_3$] *ab del* $1_{12} x_2$
210.15 Π_7] *aft del* Σ_γ
210.19 Σ_4] *aft del* $\Pi_2 \Sigma_3$
210.19 Π_7] *aft del* $\Pi_5 \Sigma_6$
210.23 Σ_α] α *ov* i
211.5 $\bar{y}$] *ov* 1
211.6 $\Sigma_1 \ldots z_8$)] *bel del* $= \Sigma_1 x_1 \bar{y}_1 + \bar{x}_7$
211.11 $\Sigma_7 \Pi_8 \Sigma_9$] *transp-c from aft* Π_6
211.16 Σ_β] *transp-c from aft* Π_3
211.16 Σ_α] *transp-c from aft* Σ_7
211.17(2) and] *aft del* $r_\gamma \equiv$
211.19 $\bar{r}_{\gamma 15}$] *bef del* $+ \bar{z}_5$
211.20 z_5] *aft del* r_β
211.20 $r_{\gamma 93}$] 3 *ov* 5
211.28 $x_9 \bar{z}_0$] z *ov* x
211.30 Π_7] *aft del* Σ_9
211.30 $\bar{z}_0$] *aft del* x_9
212.2 Π_a] *aft del* Π_e
212.2 Π_e] *intl-c*
212.5 Σ_8] Σ *ov* Π
212.11 1_{79}] 7 *ov* 9
212.12 This must give] *ab del* $\Pi_e \Pi_a \Pi_b \Sigma_c \Sigma_d \Pi_f \Pi_g \Pi_f$ [*ab del* $\Pi_a \Pi_b \Pi_c \Pi$]
212.14 To get . . . Boolian] *ab del* $\bar{x}_7 + \bar{x}_{7'} + \bar{x}_{7''} + r_{\gamma 78} r_{\gamma' 7' 8'} r_{\gamma'' 7'' 8''}$ z_8◆$z_{8'}$[z *ov* 8]$\underline{z}_{8''}$ ◆$(\bar{r}_{\gamma 98}$ [*bel* $\bar{1}_{89}$] + ◆$\bar{r}_{\gamma' 9' 8'}$ [*bel* $\bar{1}_{77'}$] + $\bar{r}_{\gamma'' 9'' 8''}$ + $1_{79} 1_{7'9'} 1_{7''9''}$
212.14 cube] *intl ab del* multiply
212.16 $\bar{r}_{\gamma' 9' 8'}$] *intl*
213.5 and the] *aft del* but
213.8 l_{11} l_{12} etc] *bef del* l_{21} l_{22} etc
213.25 x_{bc}] b *ov* a
213.25(1) l] *ov* x
214.1(1) j] *ov* e
214.2(2) Σ_i] *aft del* Π_i Π_k
214.4 Raising] *bel del* Multiplicati
214.12 $a_i \bar{a}_i \prec 0_i$] *bel del* $a_i \prec 0_i$
214.21 except perhaps] *intl ab del* but
215.1 something] *bef del* or other
215.3 *[formula]*] *bel del* $\Sigma_j^2 \Sigma_i \prec \Sigma_i \Sigma_j^2$
215.5 there are] *intl ab del* everything
215.11 m women] *intl ab del* there are

215.11 each] *intl-c*
215.15 by] *aft del* only once
215.26(1) j] *ov* d
215.26 or] *intl ab del* and
215.29 Π_z] z *ov* α
215.31 y_6] 6 *ov* 4
215.31 $\bar{r}_{j67}$] 67 *ov* 25
215.31 q_{z7}] 7 *ov* 5
216.12 The conclusion] *bel del* The first only
216.14 $\Pi_{41}\Pi_{42}\Sigma_{51}\Sigma_{52}$] *aft del* ◆$\bar{\mathrm{y}}_4 +$ [*bel del* $\Pi_4\Pi_{40}\Pi_{41}$]
216.25 My difficulty] *bel del* $\mathrm{P}_{\mathrm{ij}}\ \bar{\mathrm{P}}_{\mathrm{kl}} \prec \bar{1}_{\mathrm{ik}} + \bar{\mathrm{P}}_{\mathrm{jl}}$
216.28 $\bar{r}_{\alpha.11.41}$] 4 *ov* 2
216.28(1,2) $+1$] 1 *bel del* $^-$
217.11 x_1] 1 *ov* 3
*217.12 true] *bef*, [*del by editors and bef del* because any power will contain a term exclusively in r's]
217.23 Now we have] *bel del* ◆$\{\bar{\mathrm{r}}_{\mathrm{i46}} + \bar{\mathrm{r}}_{\mathrm{i47}} + \bar{\mathrm{r}}_{\mathrm{i56}} + \bar{\mathrm{r}}_{\mathrm{i57}} + 1_{45} + 1_{67}\}$ [*bel del* ◆◆$\Sigma_{\mathrm{i}}\Pi_4\Pi_5\Pi_6\Pi_7$ [*bel del* Now we have]]
217.23 Now we have] *ab del* ◆$\Pi_4\Pi_5$◆◆Σ_{i}[*transp-c from aft* Π_7] $\Pi_6\Pi_7\mathrm{r}_{\mathrm{i}}$[*ab del* $\Pi_4\Pi_5\Pi_6\Pi_7\Sigma_{\mathrm{i}}\mathrm{r}_{\mathrm{i45}}\mathrm{r}_{\mathrm{i67}}$ $(\bar{\mathrm{r}}_{\mathrm{i47}}$◆◆$\bar{\mathrm{r}}_{\mathrm{i65}}$ [*intl-c*] $+ 1_{57}$ ◆◆$+$ [*ov*)] $1_{46})$]
218.3 2] *ov* 1
218.3 1] *ov* 2
218.3 Hence] *bef del* $\Pi\alpha\Pi$
218.4 1_{24}] $^-$ *del ab* 1 *and* 2 *ov* 1
218.5 The conclusion.] *ab del* $\Pi_1\Sigma_\alpha\Pi_2\Sigma_3\Pi_4\ \{\mathrm{r}_{\alpha 42}\ (\bar{\mathrm{r}}_{\alpha 32} + 1_{43}) + 1_{14}\}$ | $\Pi_1\Pi_2\Sigma_\alpha\Pi_3\Pi_4$ | ¶ On the whole it is absurd to say that 1 has the relation | $(\bar{\mathrm{r}}_{\alpha 12} + \mathrm{r}_{\alpha 32}\bar{1}_{13})$ | to 2, unless there are just two individuals and no more.
218.8 Σ_β] β *ov* α
219.5 6.7] *ov* 3.5
219.11 $\Sigma_a \ldots \Sigma_j$] *ab del* $\mathrm{x}_1\bar{\mathrm{y}}_2 + \mathrm{p}_{\mathrm{x}} + \mathrm{q}_{\mathrm{x}} + (\bar{\mathrm{p}}_\alpha + \mathrm{x}_{\mathrm{c}}(\bar{\mathrm{y}}_\alpha + \mathrm{r}_{\alpha\mathrm{cd}})$
219.11 Σ_e] *intl-c*
219.11 Σ_f] *intl-c*
219.11 Π_β] *intl-c*
219.11 Σ_i] *intl-c*
219.11 Π_γ] *intl-c and* Π *ov* Σ
219.11 Σ_j] *aft del* Σ_{i}
219.12(1,3) $\bar{p}$] p *ov* q
219.12 $r_{\gamma jh}$] *aft del* Σ_{j}
219.13 $+ r_{\beta 12}$] *aft del* $\bar{\mathrm{r}}_{\beta 12}$
219.16(1,2,5) $_a$] *ov* 1
219.16 $_b$] *ov* 2
219.16 $_\beta$] *ov* α
219.16 $_{32}$] 3 *ov* 1
219.17 *[formula]*] *bel del* Identify
219.17 Σ_1] Σ *ov* Π
219.31 integrant] nt *ov* 1
219.31 its] *intl ab del* can
220.6 $\Pi_A\Sigma_B e_{AB}$] *ab del* $\Pi_{\mathrm{A}}\Pi_{\mathrm{B}}\Pi_{\mathrm{C}}(\bar{\mathrm{e}}_{\mathrm{CA}}\bar{\mathrm{e}}_{\mathrm{CB}} + 1_{\mathrm{AB}})$
220.11(2) e] *ov* q
220.13 that] *bef del* every number is in a relation
220.14–15 and . . . an object] *intl*
220.15 numbers] nu *ov* obj
220.17 $\Pi_\alpha \ldots \Pi_c$] *bel del* $\bar{\mathrm{c}}_{\alpha\mathrm{A}}$ [*bel del* $\Pi_\alpha\Pi_{\mathrm{A}}\Sigma_{\mathrm{i}}\Pi$ [*bel del* $\Pi_\alpha\Pi_{\mathrm{A}}\mathrm{c}_{\alpha\mathrm{A}}$]]
220.17 Σ_b] Σ *ov* Π
220.20 r_{aba}] ba *intl ab del* ab
220.20 1_{ad}] 1 *bel del* $^-$ *and* ad *intl bel del* ab
220.21 x] *ov* y
220.22 The antecedent] *bel del* The first pas
220.28 the] e *bef del* se

33. *An American Plato, 1885*

Copy-text is MS 541, eighty pages (numbered 1–23, 23bis, 24–79) inscribed in dark and light blue ink on eighty leaves of medium-heavy, laid, and unlined white paper measuring $7\frac{7}{8} \times 5\frac{1}{16}$ in. Remnants of red adhesive on the left edge of each leaf suggest that they were torn from a tablet. Although the manuscript is, on the whole, carefully inscribed, there are many alterations, all in light blue ink. Note 2 is on a smaller leaf, attached with a straight pin and cellophane tape to the bottom of page 23 of the manuscript. "By C. S. Peirce" appears below the title, and "C. S. Peirce|4 Kirkland Place|Cambridge" on the verso of the damaged last leaf. MS 540 contains fifteen numbered pages of an earlier version of the review, as well as twenty-eight discarded leaves of still earlier attempts. MS 542 is a carbon copy typescript of MS 541, and it is altered with typewritten carbon copy and autograph carbon copy impressions, blue-black ink, and pencil; none of the inscriptions is in Peirce's hand. Almost certainly prepared for, and altered by someone at, the *Popular Science Monthly* (and later written on by the editors of the *Collected Papers*), the typescript has no textual authority. In any case, the review was returned to Peirce, for on 28 October 1885 he wrote to William James from Madison, WI, regarding E. L. and W. J. Youmans, who edited the *Popular Science Monthly:* "I wrote Youmans,—at his particular request,—a notice of Royce's book. I was a long time over the book & I wrote I thought something really very good, for me; but Youmans wouldn't print it, i.e., he made such a wry mouth that I relieved him of it."

Textual Notes

230.29 that] Peirce had first inscribed "them"; he changed "m" to "t" but neglected to alter "e" to "a."

233.40 *[S]*pace] The top-left corner of the last sheet of the manuscript has been torn off and with it the first letter of the first word. Below the word, "space" is pencilled in in the hand of Arthur Burks.

Emendations

221.2–3 *Religious . . . Philosophy*] E; *rom.*
221.21 Dr.] E; *no* ¶ ∼.
222.14 make,] E; ∼‸
222.29 *Republic*] E; *rom.*
223.1 judge," . . . "who] E; ∼,‸ . . . ‸∼
223.33 philosophers] E; philo-|phers
224n.1 Mitchell] E; *ital.*
224n.1–2 *Studies in Logic. By Members . . . University*] E; Logical Studies by members of the Johns Hopkins university
224n.2 Peirce] E; *ital.*
224n.2 *The . . . Mathematics*] E; *rom.*
224n.2 vol.] E; ∼‸
225.13 than] E; that
225.27 *A*:*B* . . . *A*:*A*] E; *rom.*
225.33 Whately] E; Whateley
225n.3 self-consciousness."] E; ∼.‸
226.2 lightning] E; lightening
226.5–6 "I . . . know."] E; 'I . . . ∼.‸
226.10 Is] E; In
226.33 innumerable] E; in-|merable
227.35 out of] E; of out
228.1–2 indistinguishable] E; indis-tis-|tinguishable
228.3 that] E; ∼|that
228.38 reality] E; real
229.18 The] E; the
229.32 In] E; It
230.28 *Popular . . . Monthly*] E; *rom.*
*230.29 that] E; thet
231.9 identify] E; identity
231.18 sentiment] E; ∼,
231.33 is] E; in
232.24 for] E; For
232.37 behold," . . . "made] E; ∼,‸ . . . ‸∼
233.8 a-hunting] E; ∼‸∼
233.24 I] E; I.
233.26 other] E; Other
233.27 premises,] E; ∼‸
233.39 II . . . undertakes] E; II., . . . ∼,
234.7 'as . . . lute.'"] E; "∼ . . . ∼.‸"

Line-End Hyphenation

225.20 nerve-cells
230.17 heat-lightning
232.29 self-destructive

Alterations

221.5 I] *intl ab del* we
221.6 my] *intl ab del* our
221.7 me] *intl ab del* us
221.11 and feel] *intl-c*
221.11–12 hold to it.] *intl ab del* are converted.

221.16 system] *intl ab del* method
221.18 contains] *intl ab del* possesses
221.19 the study of] *intl-c*
221.24 in passages] *intl-c*
221.27 hidden] *intl-c*
222.3 glance] *bef del* for a moment
222.5 in] *bef del* its being
222.7 they] *bef del* will
222.8 admitting] *bef del* Dr. R
222.8 borrowed] *intl ab del* drawn
222.9 people] *bef del* will
222.14 people] *bef del* will at once
222.14 at once,] *intl-c*
222.15 namely,] *intl-c*
222.16 will as an] *intl ab del* volitional
222.18(1) that] *bef del* the
222.21 the settlement . . . in] *intl-c*
222.36 any] *ins* ny
222.37 were] *intl ab del* if
222.37 to be] *intl ab del* were
222.39 at once] *intl-c*
222.39–40 the very . . . been] *intl ab del* what you were
222.40 after] *intl ab del* for
223.4 Being] B *ov* be
223.4 —no matter who—] *intl-c*
223.8 ultimately] *intl-c*
223.8–9 in point of fact] *intl-c*
223.9 I] *bef del* dont
223.9–10 understand] *intl ab del* see
223.16 this] *intl ab del* that
223.18 that] *bef del* he
223.21 is] *bef del* that
223.22 an] *intl ab del* the
223.27 how] *bef del* the subject
223.30 in doing] *intl ab del* to do
223.31 been] *bef del* in the mind
223.37 that] at *ov* e
223.37 logical machine] *intl ab del* system of formal logic
223.39 to] *bef del* the
224.1 the present writer] *intl ab del* C. S. Peirce
224.4 treatises] *intl ab del* works
224.4 the science] *intl ab del* Logic *and* the [e *ov* is]
224.6–7 to be so] *intl-c*
224.7 others] *intl ab del* rest
224.7 Grassmann] *aft del* and only
224.7 alone] *intl-c*
224.7 pursuing] *intl ab del* have pursued
224.8 one] *intl ab del* only
224.19 are] *bef del* absolute
224.21 mesmerizer,] *aft del* hypnotizer,
224.34 my] *intl ab del* our [*intl ab del* my]
224.35 I] *intl ab del* we
224.35 may] a *ov* u
225.3(1) so] *intl-c*
225.6 I] *intl ab del* it
225.7 should] sh *ov* w
225.8 distinguished] *bef del* and are bound together into a real continuum
225.9 we] *bef del* forget
225.9(2) that] *bef del* there is
225.26–27 to the . . . *A*:*A*.] *intl-c*
225.29 Outward] O *ov* o
225.30 nutrition,] *bef del* (synthesis).
225.33–34 drawing] dr *ov* wh
225.34 leaving] *intl ab del* losing
225.35 of] *bef del* forms.
226.16 possible.] *intl ab del* probable
226.35 as probably erroneous] *intl-c*
226.36 all] *intl-c*
226.36 beliefs] *intl ab del* matters
226.36 beyond] *bef del* all doubt for prob
226.40 concerning one] *intl ab del* about a certain
227.1 or another] *intl-c*
227.3(1) been] *intl ab del* got
227.5 confidence] *intl ab del* certainty
227.6 that] *intl ab del* which
227.7(1) of] *bef del* bei
227.8 be] *intl ab del* were
227.9 questions] *bef del* were to
227.14 the] e *ov* is
227.14–15 of survival . . . and nearer] *intl ab del* becomes indefinitely near
227.15 zero] *intl ab del* certainty
227.20 problem] *intl ab del* question
227.21 asked] *intl-c*
227.23 greater than] *intl ab del* as great as
227.25 otherwise] *bef del* not.
227.27 case,] *bef del* every question (or
227.27 infinitesimal] *intl ab del* finite
227.31 admission] *intl ab del* proposition
227.38 be] *aft del* by
227.40 for the sake of argument] *intl-c*
228.8 that] at *ov* ere
228.9 timbers] *intl ab del* beams
228.13–14 if . . . matter,] *intl ab del* if this is never admitted

228.16 (to . . . expression)] *intl-c* [a *ov* the *and* popular [r *bef del* ly]]
228.18 get] *ov* be
228.19 of] *bef del* rea
228.21 ;] *ov* .
228.23 ordinary life] *intl ab del* practice
228.26–27 centre of the] *intl-c*
228.28 ;] *ov* .
228.28 shall] *intl-c*
228.31–32 or not] *intl-c*
228.32 are] *intl ab del* really will
228.32 actually] *bef del* get
228.32 God,] *bef intl-c del* or not
228.33 a universal] *intl ab del* general
228.37 about reality] *intl-c*
228.39 theory] the *ov* vie
228.40 Himself] H *ov* h
228.40 it] *intl ab del* and
228.40–229.1 thus . . . him] *intl ab del* therefore be
229.2 intend to] intend *intl ab del* mean [*bef* to *and* mean to *intl ab del* shall]
229.3 putting into print.] *intl ab del* presenting.
229.4–5 a tendency . . . universe that] *intl* [so *intl ab del* a ne]
229.6 upon innumerable atoms] *intl bel del* in the universe
229.6–7 has . . . result.] *intl ab del* tends to the accomplishment of ends.
229.7 the ends . . . about] *intl ab del* these ends
229.8 intelligence] *second* i *ov* e
229.15 for that] *intl ab del* which
229.19(2) to] *bef del* indicate
229.19 point . . . to] *intl-c*
229.21 aimless] *intl ab del* unwise
229.22 pursuit.] *bef del* It is
229.24 as if] *intl ab del* like
229.25 A] *bef del* genuine
229.27 a man] *intl ab del* one
229.29 with him] *intl-c*
229.30 receive . . . development] *intl ab del* become developed.
229.31(1) to] *bef del* fancy
229.35 with] *intl ab del* in
229.35 strongest] *bef del* way
229.40 renewed] *intl ab del* new
230.2(1) they] *bef del* have
230.2 find] fi *ov* n
230.4 not] *bef del* for
230.8 questions.] *intl ab del* doubts.
230.12 It . . . see] *intl-c ab del* I don't see
230.24 Dr. Royce] *aft del* ¶One of the difficulties of the Hegelians is to know into which of their pigeon-holes to file away the scientific thinkers. They say the scientists are in the naive stage; and yet ◆scientists [*intl ab del* they] are the most critical and sceptical of all men. A German [Ge *ov* ge] philosopher [*bef del* will] of the true blood will never comprehend how a scientific man can doubt the axioms of geometry. I myself, ◆to take the readiest example, [*intl-c*] am one of the [*bef del* scien] minds formed under the influence of natural science: [: *ov* ;] I positively disbelieve the exact conformity of nature to any laws whatever. One would think, then, that I ought to have reached the second stage of thought. But no,—we are still naïve because we do not doubt just for the fun of doubting but only when we have positive reason to do so; ◆though this [*intl ab del* which] comes to saying that our doubt is genuine and not put on.
230.24 in] *bef del* mo
230.26 then] *intl-c*
230.27 back] *intl-c*
230.29 how] *bef del* it
230.30 when . . . ethics.] *intl ab del* in an example.
230.36 will] *bef del* be
230.39 very much] very *intl ab del* over [*bef* much *and* over much *intl-c*]
231.3(1) the] e *ov* is
231.8–10 my . . . world.] *intl ab del* God, or desire to see ◆a certain [*intl-c*] order and fitness prevail and love of my fellow being, which is a sort of esprit de corps, an elevated and enlarged selfishness.
231.13 Sinai),] *bef del* other
231.14 idealist] *intl ab del* realist
231.15 sentiment,–] *intl ab del* feeling,
231.15 Two] *aft del* These
231.16 such] *intl-c ab del* an
231.18 while] *intl-c ab del* an
231.19 is only] *intl ab del* rests on
231.19 has] h *ov* w

231.27 is . . . restricted.] *intl ab del* appears to me to be almost childish.
231.28 appears] *intl ab del* certainly seems
231.30 yet] *bef del* he is so little m
231.35 state] *second* t *ov* g
232.13–14 The . . . out.] *add*
232.20 realism] *intl ab del* idealism
232.21 this] *intl-c*
232.22 adduces] *intl ab del* brings [*bef del* forward]
232.22 testimony of] *intl-c ab del* pr
232.24 But] *ins*
232.24 fancy I] *intl-c*
232.24–25 in cold weather] *intl-c*
232.25 willing to take] *intl ab del* taking *Also* 232.26
232.26 Scepticism,] *aft del* Then *and* S *ov* s *and* , *add*
232.26 then,] *intl-c*
232.26 shares] *intl ab del* shares [*intl ab del* reconciles]
232.27 thus] *intl ab del* then
232.28 namely,] *add*
232.28 reconcile] *bef del* all
232.28 So] *add aft del* Thus,
232.32 acquired] *intl ab del* reached
232.33 this] t *ov* ?
232.33 at last] *intl ab del* attaine
233.4 that] *intl-c*
233.7 admitted] *bef del* that
233.9 the axiom] *intl-c*
233.9 ,] *ov* .
233.11 Here] *intl ab del* Now
233.18 at] *intl ab del* with
233.26 But] *add*
°233.26 Other] *bef del* parts *and* Ot *ov* Par
233.27 the evil eye.] *intl ab del* envy
233.37–38 may . . . that] *intl-c aft del* will think *and bef intl del* on the whole
233.38 charity] *bef del* better.
233.38 is not . . . all.] *add* [much lower *intl-c bef del* low]
233.40 God.] *bef del* In reference

34. *Notes on the Categories, 1885*

Copy-text is MS 545, forty-seven pages inscribed in blue ink on the rectos of forty-seven leaves of heavy, laid, unlined white paper measuring $7\frac{7}{8} \times 5\frac{1}{8}$ in. Remnants of red adhesive on the left-hand side indicate that they were torn from a tablet. Some leaves are smeared, stained, or chipped, and there are many alterations and revisions. The lower-left corner of the fifth leaf is torn away, so that Peirce's inscription of "It is" (235.29) only partially remains (though it is interlined in lead pencil by the editors of the *Collected Papers,* who have made other lead, red, and blue pencil inscriptions in the manuscript). Three additional pre-copy-text leaves, one of which seems to be an earlier attempt at note 1, also survive.

Textual Notes

237.2 Here follow, in the bottom half of the manuscript page, the first five lines of a paragraph completed on the following page. Peirce crossed out the first five lines, but not the remainder on the next page. (The paragraph is reproduced in the Alterations.)
237.5 //badly/ill//.] Both words are followed by a period; only one has been retained.
237.26 reasons] Peirce neglected to lowercase the first letter in "reasons" when he added the introductory adverb.
237.37 right] Peirce's original word was "write"; someone other than Peirce deleted it and interlined the word "right" above it. (This correction is not recorded in the Alterations list.)
238.34 unlike] Peirce mistakenly interlined (with a caret) "unlike," although the same word is already there, clearly written.
240.11 signs] Peirce neglected to lowercase the first letter in "signs" when he added the introductory adverb.

240.33 Here follow, at the bottom of the manuscript page, the first two lines of a sentence completed on the following page. Peirce crossed out the first two lines, but neglected to do so with the next page. (The sentence is reproduced in the Alterations.)

Emendations

235.12 so-called] E; so called
235.20 comparatively] E; compartatively
235.24(2) the] E; The
236.17 *à*] E; *a*
236n.9 Chance.] E; ~‸
236n.16 Kant's] E; Kants
236n.16 negation,] E; ~‸
*237.5 badly] E; ~.
237.18 is] E; *not present*
*237.26 reasons] E; Reasons
*237.37 right] E; write
238.1 three] E; a ~
*238.34 unlike] E; ~ unlike
238n.1 "On . . . Categories."] E; ‸~ . . . ~‸
238n.1 *Proceedings . . . Sciences,*] E; *rom.*
238n.2 1867] E; 1865
239.18 resultant;] E; ~‸
239.21 end.] E; ~‸
239.35 exemplified] E; explified
240.4 Cogito.] E; ~‸
240.9 relation,] E; ~‸
*240.11 signs] E; Signs
241.5 *et seq.* *P*] E; *rom. All capital and lowercase variables rom.*
241.11 it] E; in
241.18 *Statistical.*] E; ~‸
241.18 *S*'s are *P*'s] E; Ss ~ Ps
241.19 *Particular.*] E; ~‸
241.22 propositions] E; proposition

Alterations

235.6 implied] *aft del* implicitly involved in all reasoning, they must be in
235.6 reasoning] *bef del* (for reasoning supposes [*bef del* logic) a] logic, though it may not distinctly recognize all the details of [*bef del* it)] the science)
235.6 the conceptions they employ] *intl-c ab del* they
235.10 is] *intl ab del* seems
235.10 hard] *aft del* to ♦me [*intl ab del* be]
235.11(1) he] *intl ab del* Kant
235.11–12 he made . . . Judgment"] *intl ab del* of the distinctions of judgments
235.13 still,] *bef del* I cannot must say that
235.13 Kant's] *intl ab del* the
235.14 If the] *bef del* list of
235.15 his] *intl ab del* Kant's
235.16 thought,] *bef del* I think it must be admitted that the conceptions in these distinctions would have been shown to have an apriori character
235.20 at least] *intl-c*
◊235.20 compartatively] *bef del* at least
235.20 at all events] *intl ab del* certainly conceptions
235.21–22 whatever . . . philosophy.] *intl bel del* although it might not be easy to say what the precise degree of their importance might be
◊235.23–24 A . . . that] *add bef* The categories . . . in
235.26 used] *bef del* with a
235.28 substance] *intl ab del* subject
236.1 "Mercy] *intl ab del* Omniscience
236.2 So] *bef del* the conception of
236.4 thus] *intl ab del* putting
236.10 was] w *ov* is
236.11 his] *intl ab del* the
236.14 utterly] *intl ab del* totally
236n.7 a given] given *intl ab del* certain [*aft* a *and* a certain *intl ab del* some]
236n.7 states] st *ov* as
236n.8 latter] *intl ab del* former
236n.9 *Law*] L *ov* l
236n.9 former] f *ov* l
236n.9 Chance] C *ov* c
236n.9 The] *bef del* third disti
236n.11 .] *ov* ;
236n.11 law.] *bef del* and therefore of evolution.
236.21 distinct] *bef del* series
236.21 will be . . . present.] *intl ab del* must be postponed to another occasion.
236.22 thought] *intl ab del* appre-

hended
236.23 just] *intl-c*
236.25 may] *intl ab del* might
236.25 is] *intl ab del* was
237.1 perhaps] *intl-c*
*237.2 philosophy.] *bef del* ¶Many minds fail to remark the security against error afforded by Kant's method of searching [se *ov* in] for the categories. We might fling ourselves into the ♦war of the [*intl ab del* storm [*intl ab del* ocean]] elements [*intl ab del* of thought] as Hegel does and endeavor [call ♦up [*intl ab del* out] the categories in their order by the direct necromancy of thought. First, Second, Third, is a series of conceptions that one might have hit upon by such a method. But in such a proceeding we have no assurance against ♦the [*intl-c*] mistakes and misconceptions, which are so abundant in the Hegelian dream. [*from* call up *manuscript page is del by editors*]
237.3 assurance] *aft del* secur
237.16 and be . . . rules;] *intl-c ab del* ;
237.18 will] *intl ab del* is
237.18 be] *intl-c*
237.19 ever] *intl-c*
237.19 brought] *intl ab del* carried
237.20 satisfy] *intl ab del* assure
237.21 the] *bef del* rules are accurate
237.24 Now . . . that] *bel del* ♦¶Now I undertake to show that there are [*bel del* ¶The purpose of the present contribution]
237.24 Now] *intl ab del* But
237.24–25 are really] *intl ab del* truly
237.25 so that they are] *intl ab del* ♦and making [*intl ab del* forming]
237.25 fundamental] *bef del* characters of
237.26 thought.] . *ov* , *and bef del* and ♦I [*bel intl del* on]
*◇237.26 Furthermore, Reasons . . . holding] *intl ab del* following up the trail we shall ♦find [*bef intl-c del* reason to hold]
237.27 the] *intl-c*
237.28 mind,] *bef del* and
237.28 again] *intl-c*
237.29 finally] *intl-c*
237.29 elementary] a *ov* s
237.32 the new] *intl ab del* the [*ov* my]
237.33 efforts. Yet] efforts. *intl ab del* labor. [*bef del* I can, however, so aid him ♦that [*bel intl* Yet]]
237.33 that] *intl-c*
237.34 read] *bef del* carefully
237.34 this] is *ov* e
237.34 paper] *aft del* whole
237.34 will . . . understand] will *bef del* ♦have no difficulty [*bel intl del* understand] ♦in maki [*bel intl* have come to understand] be able to conceive
237.35 outset,] *bef del* it may be useful
238.1 I] *bef del* calle
238.1 three] *bef del* -fold
238.2 place,] *bef del* we may
238.5 *Red*] R *ov* r
238.6 Sound] S *ov* s
238.15–16 can neither] can[not *del bel intl* neither]
238.16 nor suppose] *intl-c*
238.16 Taller] T *ov* t
238.19(2) other] *intl ab del* ma
238.24 and] *bef del* in f
238.24 many] *bef del* con-
238.25 definitely] *bef del* conceived
238.31 three.] *bef del* Imagine something definite which presents no likeness nor relation to
238.32 the] *bef del* possibility
238.36 and pristine] *intl-c*
238.36 Think] *bef del* how
239.2 defined.] *bef del* This will give a hint of the category of First.
239.3 word] *bef del* freedom
239.5 it] t *ov* s
239.7 it] *bef del* gives The
239.8 that] *bef del* some straining is necessary to ♦attain [*aft del* ser]
239.8 conception] *bef del* ;
239.8–9 is . . . straining;] *intl ab* whereas . . . difficulty
239.9–10 immediately] *intl ab del* much
239.12–13 A . . . upon] *intl ab del* Relativity, not of thought merely but of existence,
239.13 something else, something] *intl-c*
239.14 determined] ed *ab del* ation
239.14 in real relation to something,] *intl-c* [relation[ship *del bel intl* to something]]
239.15 effort,] *bef del* power,

239.14 in real relation to something,] *intl-c* [relation[ship *del bel intl* to something]]
239.15 effort,] *bef del* power,
239.15 an end] *intl-c*
239.15 end] *bef del* events ♦actually [*intl-c*] coming to pass
239.15 experiences] *intl ab del* ideas *and bel intl del* phenomena &
239.16 This] *bef del* concep
239.18 (and analysis)] *intl-c*
239.18 resultant] *intl-c*
239.20 Third] T *ov* t
239.20–21 the beginning and the end] *intl-c*
239.26 relationship. It] . *ov, and* I *ov* i
239.34 genera] *intl ab del* species [*intl ab del* classes]
239.38 "Si cogito, sum."] *intl ab del* If I think, I am
240.3–5 Si . . . Ergo, sum.] *to right of del* If I think, I am|I think|Therefore, I am.
240.6 *Cogito*] *intl ab del* I think,
240.8 Sum] *intl ab del* If I think, I am,
240.8 is the fact . . . the] *intl bel del* sets two pr sep a relation between two facts; it involves a
240.9–10 Si cogito . . . medium or] *intl ab del* I am, syllogizes or combines the other two and gives the resultant, it is the
240.10 words or] *intl-c*
240.11 single] *bef del* propositions
240.11 facts,] *bef del* as I think, I am
240.14 or premises] *intl-c*
240.16 three] *intl-c*
240.23(2) real] *bef del* states of
240.26 the] *ov* a *and bef del* certain
240.29 perhaps] *intl-c*
*240.33 variable.] *bef del* The two distances equated; then, may really be equal within the |limits of their ambiguity or inherent uncertainty, or they may be inequal beyond those limits, or the ♦bands [*intl ab del* limits] of uncertainty of the two quantities may be each partly covered and partly uncovered by the other. [*from* limits of *manuscript page is del by editors*]
240.33 margin] *intl ab del* limits
240.34 includes] s *ins bef del* those
241.1 predicament] *intl ab del* state
241.4 are] *bef del* probable
241.9 that] *bef del* no reasoning from finite probabilities can ever produce probability unity or zero, no no reasoning from these can produce a fin
241.10–11 reasoning;] *bef del* but
241.21 the] *bef del* most
241.22 of] *bef del* judgements
241.23 or . . . hazard] *intl-c*
241.24(1) stated] *intl ab del* certain
241.25 result brought about] result *bef del* ♦forced upon us [*bef intl* brought about] by ♦observation [*bel intl del* the conditions] or the limited experience necessitated
241.25 a] *bef del* fact

35. *One, Two, Three, 1885*

Copy-text is MS 546, thirty-nine pages inscribed in blue ink (and numbered in the upper right corner on thirty-nine leaves of medium-weight, laid, and unlined white paper measuring $7\frac{15}{16} \times 5\frac{1}{8}$ in. The first and last leaves are chipped and dog-eared, and the word "Three" in the title (still extant when the manuscripts were microfilmed nearly thirty years ago) has disappeared with a small piece of paper that has broken off. Leaves 30 and 31 have stains and ink blots. Although the manuscript is on the whole carefully inscribed, there are a number of alterations, as well as several lead and red pencil inscriptions by one of the editors of the *Collected Papers.* "By C. S. Peirce" (below the title) and a horizontal line beneath the by-line have been removed in the present edition. Two additional pre-copy-text leaves, one numbered "8" and the other unnumbered (though an earlier version of page 16), also survive. It is possible that MS 546 is the beginning of, and actually belongs with, the 15 pages of MS 547.

Textual Note

244.35 roadway-connection] Peirce actually inscribed an equal sign to indicate the intended hyphen (even though the two words are not broken at the end of a line).

Emendations

242.19 Elijah] E; Elijahah
243.6 which] E; who
243.9 dilemmatic] E; dilemattic
243.33 *et seq.* *A*] E; *rom. All capital variables rom.*
243.34 separately;] E; ~,
244.5 *B* constitutes] E; B, constitute
244.5 degenerate] E; degerate
244.7 said,] E; ~‸
244.29 nowhere] E; no where
244.34 network] E; net work
244.37 fact).] E; ~.)
246.17 passion,] E; ~‸
246.17 exercise] E; excercise
246.29 1^{st},] E; 1^{st}‸
246.36 as] E; are
247.6 physiological] E; physological
247.15 nervous system] E; ~-~
247.21–22 usually consider] E; consider usually
247.23 property,] E; ~‸
247.31 at] E; an

Line-End Hyphenation

242.6 three-fold
247.25 nerve-cell

Alterations

242.6 logical] *bef del* science
242.6 analytics] t *ov* s
242.9 the] *bef del* question
242.26 by itself] *intl-c*
243.6 not] *bef del* lead them
243.7 The] *aft del* But *and* T *ov* t
243.11 on] *bef del* precise
243.15 Names] N *ov* n
243.18 all] *intl-c*
243.18 is the] *intl ab del* consists of
243.19 subject] *aft del* objec
243.21 particular] *intl-c*
243.22 describing it;] *intl ab del* asserting and
243.22 name or] *intl-c*
243.37 If we] *intl ab del* We might
244.2 *A*'s] A *ov* or
244.3–4 To borrow this expression] *intl ab del* In the same sense
244.4 we may say that] *intl-c*
◆244.5 constitute] *aft del intl-c* may
244.7 divisible into] *intl ab del* either
244.8 character,] *intl ab del* rela
244.13 fact] *intl ab del* character
244.15 take] *intl ab del* use
244.15 include] *intl ab del* mean
244.22 plural] pl *ov* du
244.22 facts] *intl ab del* characters
244.34 *road about a*] *intl-c*
244.37 (relation)] *intl-c*
244.38 coexistence] *intl ab del* congeries
244.39 degenerate form of] *intl-c*
244.40 plural] *intl ab del* triple
245.1 facts] *bef del* none
245.1 the] th *ov* is
245.4 *signified,*] *bef del* idea
245.4 *produced*] *intl ab del* created
245.8 which] *bef del* is brou
245.10 logical] *intl-c*
245.11 general] g *ov* n
245.14–15 really determined by] *intl ab del* an index of
245.18 in its premises] *intl-c*
245.26 do] *bef del* into
245.27 connected] *bef del* by
245.31 faculties] *intl ab del* powers [*bef del* or modes]
246.3 variation] *intl ab del* variety
246.4 and anticipatory] *intl-c*
246.7–8 be brought] *intl ab del* come
246.13 itself] *aft del intl-c* in
◆246.17–18 every excercise of will] *intl-c*
246.21 whether there be not an] *intl ab del* what there
246.21 element] *bef del* there is
246.22 nor] *bef del* volition
246.23 and-inference] *intl-c*
246.24 activity] *intl ab del* volition

246.26 getting hit,] , *ov* .
246.26 of meeting] *aft del* We are conscious
246.29 true] *intl ab del* real
246.32 Consciousness] *aft del* Active or dual *and* C *ov* c
246.33 resistance,] *bef del* externality
246.34 sense of] *intl-c*
246.36 elementary] *intl-c*
246.40 viewed] *intl-c*
246.40–247.1 whenever] *bef del* the
247.16 quite] *intl ab del* very
247.21 quest] *aft del* search
247.21 usually] *intl-c*
247.24 .] *ov* , *and bef del* and [*bel intl del* probably of] reproduction
247.33 will] *bef del* of
247.35 life] *intl ab del* consciousness

36. *Measurement Scales and the Absolute, 1885*

Copy-text is MS 547, fifteen pages inscribed in blue ink (and numbered, in the upper right corner, 81 to 95) on the rectos of fifteen leaves of heavy, laid, and unlined white paper measuring $7\frac{7}{8} \times 5\frac{1}{16}$ in. with brownish edges. Remnants of red adhesive on the left-hand side indicate that they were torn from a tablet. There are two ink smears on page 88, as well as a number of deletions and other alterations in this otherwise carefully written manuscript. It is possible (though impossible to prove) that MS 547 actually belongs with the 39 pages of MS 546; for there are similar discussions, in similar contexts, both in "A Guess at the Riddle" (1887–88) and in "The Architecture of Theories" (1891).

Emendations

248.12 be] E; *not present*
248.15 and] E; any
248.22 *et seq.* *axy*] E; *rom. All lower-case variables rom. except* 248.23; 249.7, 8
248.23 *d*] E; *d*,
249.11 coincident).] E; ∼.)
249.20 points).] E; ∼.)
250.3 is] E; if
250.7 E] E; O
251.1 tangent] E; circle
251.12 been] E; *not present*

Alterations

248.4 Colors] *aft del* Various different scale
248.6 such] *bef del* measu
248.8 single] *intl ab del* distinct
248.11 to] *bef del* just on
248.14 algebraical] *aft del* mathematical
248.15 the first] *aft del* one
248.24 on] *aft del* assigned *and intl ab del* by
248.26 grades] *aft del* two
249.4 such] *intl ab del* so
249.10 this] i *ov* e *and* s *bef del* e
249.18 is] *aft del* no
249.19 two] *aft del* the
250.3 obtained] *aft del* affor
250.7–8 point of] *intl-c*
250.8 its] *intl ab del* the
250.9 shifted] *aft del* moved
250.10 through] *aft del* betwe
250.11 exception] *first* e *ov* o
250.26 different] *intl-c*
250.26 point] t *ov* ts
250.33 capable of] *intl-c*
250n.2 would be a] *intl ab del* is a
250n.2–3 conception] *aft del* and true
251.1 any] *intl-c*
251.1 point] t *bef del* s
251.1 whatever] *intl-c*
251.2 at] *bef del* a
251.2(1) different] *bef del* distance from
251.3 rather] *intl-c*
251.8 there] *aft del* prog
251.10 growth] *intl ab del* advance
251.12 since] si *ov* in
251.12 that] a *ov* e
251.14 absolute] *intl-c*

37. *Types of Third, 1885*

Copy-text is MS 548, one and one-half pages inscribed in light blue ink on two leaves of heavy, stiff, laid white paper, lined on one side, measuring $7\frac{7}{8} \times 9\frac{7}{8}$ in., with a "Whiting Paper Co" watermark. The first leaf is water-stained and has a rusty paper clip mark in the top margin, and both leaves are torn, chipped, discolored, dog-eared, and brown-edged. The manuscript is carefully inscribed and has few alterations. Its exact date of composition is difficult to determine; the earliest possible date is summer-fall 1885 but, given the close connection of its content with that of the 1886 "One, Two, Three" essays, it may have been written as late as the fall of 1886. The editors have removed a horizontal line below the title.

Textual Notes

252.15 Peirce left extra space between "A" and "Philadelphia" and drew three diagonal lines and a fourth imaginary line consisting of three x's, presumably representing the three cities, but the arrangement of the lines and x's is the reverse of the cities' geographical positions. The drawing, whose meaning is unclear, has been omitted by the editors.

252.16–17 Approaches B] Although inscribed in the left margin before "Centaur," the phrase clearly refers to both lines; consequently, it has been moved after the brace that has been added by the editors.

252.20 *A. . .C.*] Peirce mistakenly interlined this sentence between "*X* . . . *Z*" and "resemble one another."

252.23 There is a crude brace, whose meaning is not clear, before Detaille; it has been removed by the editors.

252.25 F] There is a hint of a prime that Peirce began to inscribe after the letter; the editors have left it out.

253.1–3 Peirce had originally written these lines in the following order: 253.1, 252.26, 253.2. He corrected his mistake by a semicircular line in the left margin connecting G with G′.

Emendations

252.15 A] E; A.
252.18 *et seq.* *X*] E; *rom. All capital variables rom.*
252.22 C] E; C.
252.23 D] E; D.
252.23 Detaille] E; Detaillé
253.6 character] E; charcter

Alteration

252.16 mixture] *intl ab del* junc

38. *Clifford's Common Sense, 1885*

Copy-text is P 307, the publication in the *Nation.* Though Haskell's *Index to the Nation* does not list Peirce as the author of this unsigned review, Max Fisch has attributed it to Peirce on the basis of internal evidence, especially the reference to F. E. Abbot's concept of space.

Emendation

256.1 *n*] E; *rom. Also* 256.6

39. *Perrin's Religion of Philosophy, 1885*

Copy-text is P 308, the publication in the *Nation,* which is unassigned in Haskell's *Index.* Peirce is no doubt the author of this unsigned review, for three pre-copy-text pages (two consecutive, one fragmentary) survive in MS 554.

Emendations

257.3 *Philosophy or*] E; ~; or,

257.3(2)–7 *The . . . God.*] E; *rom.*

40. *Kant's Logic, 1885*

Copy-text is MS 555, consisting of two and one-quarter pages inscribed in dark blue ink on the rectos of three leaves of medium-weight, lined white paper measuring $7\frac{7}{8} \times 9\frac{15}{16}$ in., with a "Massasoit" watermark. All three leaves are chipped, and the last is torn. This somewhat hastily inscribed, incomplete manuscript is moderately altered. One of the editors of the *Collected Papers* has inscribed the title "Merits and Defects of Kant" in blue pencil in the top margin of the first page and made other inscriptions in lead pencil on all three pages. An additional discarded leaf also survives.

Textual Notes

258.10 time] In the manuscript, Peirce's omitted "time" is interlined in pencil, with a caret, by one of the editors of the *Collected Papers.*

258.15 Kant] In the manuscript, Peirce's omitted "Kant" is interlined in pencil, with a caret, by one of the editors of *Collected Papers.*

258.17 was] Peirce inadvertently deleted the word; one of the editors of the *Collected Papers* inscribed it in pencil in the left margin.

258.23 the latter] In the manuscript, "process" is interlined both in ink with a caret and in pencil below the ink interlineation. Both seem to have been inscribed by the editors of the *Collected Papers* who, however, did not incorporate it into their version of the manuscript (CP 1.35).

Emendations

258.3–4 *Kant's . . . Figures*] E; *rom.*
258.6 Green,] E; ~‸
258.8 *Critic . . . Reason*] E; *rom.*
*258.10 time] E; *not present*
*258.15 Kant] E; *not present*
*258.17 was] E; *reinstate*
258.21 *Critic*] E; *rom.*
259.1 system] E; sytem
259.5 *Figures*] E; ~,

Alterations

258.7–8 gives . . . to] *intl ab del* calls
258.8 and] *aft del* logic
258.13 is] *aft del* wa
258.13–14 played a part in] *intl ab del* been made use of
259.5 consequences] *aft del* im

41. *Fiske's Idea of God, 1885*

Copy-text is MS 556, a one-page purple-ribbon typescript of medium-weight, laid, and unlined white paper measuring $7\frac{15}{16} \times 9\frac{15}{16}$ in. There are no autograph alterations, only a few smears caused by the ribbon. The left-margin alignment and the spacing of several words and punctuation marks are irregular.

Textual Note

260.10 The emendations here and at 260.18, 21, 23 are the result of the (right-margin) limitations of Peirce's typewriter; as it apparently had no right-margin release and as the three words thus could not be completed, they were completely retyped on the next line (but the initially typed characters were not deleted).

Emendations

260.3 *The . . . Affected by Modern Knowledge*] E; *no* ¶The . . . affected by modern knowledge
260.6 was] E; wes
260.8 *Idea of God,*] E; "Idea of God,"
*260.10 all] E; al ~
260.18 ever] E; eve ~
260.21(1) the] E; th ~ *Also* 260.23(1)
260.21–22 *Outlines . . . Philosophy*] E; "Outlines . . . Philosophy"
260.26 evolutionists] E; evolotionists
260.27 variations,] E; ~‸
260.28 numbers] E; nombers
261.3 evolutionism.] E; ~‸
261.6 freedom] E; freedon
261.6 deflect] E; diflect
261.7 necessitated] E; neceisitated

42. *On the Use of the Noddy, 1885*

Copy-text is P 315, the publication in the 1884 *Coast Survey Report* (which has "By C. S. Peirce, Assistant" below the title). It is emended by MS 550, Peirce's autograph list of "Corrections to Appendix 15. Report for 1884," consisting of four numbered pages of errata and commentary [550]; by MS 551a, an offprint carefully corrected in Peirce's hand [551a]; and by MS 551b, an informally corrected, revised, and annotated offprint (from the Max H. Fisch Collection) with a printed errata list for appendix 15 tipped in and corrected by Peirce [551b]. Corrections and comments in the offprints are in black ink unless otherwise indicated in the Textual Notes. Most of the authorial corrections appear in all three post-copy-text documents, but slight variations require the inclusion of each as a distinct source of emendation. Some fragmentary pre-copy-text pages and a few other related leaves are in MS 535.

Peirce wrote to J. E. Hilgard, Superintendent of the Coast and Geodetic Survey, on 2 September 1884: "I should like to read a paper on the Noddy to the American Association. . . . I write to Professor Rees to say that I will read a paper in December but that I cannot have the thing complete in so short a time. The first part can appear by itself"; and on 11 March 1885: "I have busily occupied myself with the two papers for the Report which I shall mail next week. I have materially improved the theory of the Noddy . . . I have also made the flexure problem more general." On 29 October 1885, he wrote to Hilgard's successor, F. M. Thorn: "The printing [of Appendix 15] . . . is marred by various bad errors, and I send a list of corrections which I ask to have printed on a slip and inserted in each copy of the papers and the Report"; and on 12 November 1885, he wrote to B. A. Colonna, Assistant in Charge of the Coast Survey Office:

I return herewith the copy for slips of Errata.

I have no time to discuss the question of who was at fault for each error; nor have I the means of distributing the blame between the Printing Office and the C. S. Office. All our printing is badly done in every respect, but especially as to corrections of algebraical matter. The general causes are in my opinion the following:

1. Mathematical printing is a special art for which the Public Printer is not equipped.
2. He takes very little pains to make up for his bad equipment by extra precautions.
3. The universal experience is that authors should read their own proofs of mathematical analysis.
4. There is no one in the office who really understands proof-reading of such matter.
5. Various regulations, mainly of the printing office, but also of our office, interfere with the proper execution of the work.

A correct copy of the mathematical analysis cannot be made. The author cannot do it, soon after writing, because he has to consult his own or others' papers with various notation and occasionally will commit a slip. Nobody else than the author can do it. The only way is to have the author read the proofs, unless an accomplished mathematician is at hand who is also an accomplished proof-reader, who can & will carefully read the paper. . . .

Again, the limits of integration are in a dozen cases either omitted or printed wrong. Mr. Farquhar [an employee of the Survey] thinks the omissions excused by saying "perhaps there but illegible." Printing must be legible. That is what it is for. If not legible, the proof-reader must have it made so. . . .

I make these remarks, because I seem called upon to do so. I think the remedy for our bad printing lies, first, in a special appropriation to supply deficiencies in the Printing Office, to get better type, a regular mathematical compositor, etc., second, in sending the author proofs until he is satisfied and in making the Printing Office red tape yield to that requirement. I have had lots of mathematical papers printed different places in this country and abroad. They have all been printed to my satisfaction except those done by the Public Printer in Washington, none of which have satisfied me.

MS 551a appears to be printer's copy prepared by Peirce for resetting the text, which however was not reset; collation and typeface comparison reveal that 551a, 551b, and all other examined copies of P 315 were printed from the same type. There is no clear indication of priority among Peirce's three surviving attempts at correction. All corrections in MSS 550, 551a, and 551b are emended into the copy-text; Peirce's occasional failure to mark the correction on all three documents is recorded in the Emendations. Peirce also made full-scale revisions in MS 551b which are accepted as sources of emendation, but extra-textual annotations are represented in the Textual Notes.

The printed errata tipped into MS 551b represents only a partial list of Peirce's corrections, but his holograph correction of new errors introduced by the printer of the errata sheet are useful in confirming his intentions in 551b proper. A corrected version of the errata sheet has been tipped into P 315, but it exhibits no authority and contains no new variants.

None of the subscript variables is printed in italic in either printed errata sheet. This discrepancy indicates yet another misunderstanding between author and printer-Peirce underlined none of the variables in MSS 550, 551a, and 551b, because he assumed that the Coast Survey Office staff would know that variables should be printed in italic. As the Textual Notes reveal, Peirce's frustration with the printer erupts in all three sources of emendation. MS 551b, which apparently received the greatest attention (in that Peirce both corrected and revised the text), includes a page-by-page tally of misprints.

Textual Notes

263.39 T] In MS 550, Peirce indicated that all upsilons "should be upright," and he so marked them in MS 551a.
264.1 In the 12 November 1885 letter to B. A. Colonna, Peirce noted that "where s is printed for S, Mr Farquhar thinks it a good excuse to say 'The *s* is a capital in small type.' People are not supposed to read with a microscope. The letter should not only be an S but look like one." In MS 550 Peirce noted after the correction, *"Very important!"*; regarding the occurrences at 266.4 and 266.6 not corrected in MS 551a, he wrote, "Perhaps these *are* uppercase. The previous ones are *not.*" Only MS 551b and the errata correct the occurrences at 264.6(1) and 266.8; and only the errata and MSS 550 and 551b correct 264.8(2). The instance of omission at 264.1(2) is corrected by MSS 550, 551a, and 551b; at 264.8(1) by MSS 550 and 551b.
264.1 In MS 551b, Peirce has added "$+ x_0$" to the right of the first equation.
264.2 $\theta\cdot$] In his letter to Colonna, Peirce stated: "Mr. Farquhar's remarks on my corrections show some of his faults as a proof-reader. Thus he does not think the office at fault for printing $D_t ds$ twice for $D_t\theta \cdot ds$, because the θ is a little blotted in my copy. But D_t ds means nothing, and a mathematical paper cannot be properly corrected by a person who reads it unintelligently." In MS 550 he wrote, *"This looks like culpable negligence on the part of the C. S. Office proof-reader."* The multiplication dot is not present in MS 551b.
264.2 In MS 551b, Peirce has added "$+D_t x_0$" to the right of the equation.
264.6–7 In MS 551b, Peirce has added the following to the right of the equation: "$+\int_0^s \cos\theta D_t\theta\, ds \cdot D_t x_0 + h\cos\theta_s D_t\theta_s D_t x_0 + \frac{1}{2}(D_t \cdot x_0)^2$."
264.12 ∂] Peirce noted in MS 550: "The sign of *variation* used four times in place of the sign of partial differentiation. The more perplexing because the calculus of variations might have been used at this point!"
264.12 In MS 551b, Peirce has changed "∂T" to "∂(T − E)."
265.2 η] Peirce noted "bad type" for the eta in MS 551a; in MS 551b he traced the eta by hand, because the printed one was either too faint or not present.
265.11 There is no period after the formula in MSS 551a and 551b, and the period in MS 550 and the errata is ambiguous, as it ends the sentence and also immediately follows the formula.
265.13 In MS 551b, Peirce added, in light blue ink, "$D_t^2 x_o +$" before the equations. He did the same at 267.18.
265.14 In MS 551b, Peirce added, in light blue ink, "$hD_t^2 x_o +$" before the equation.
266.17–22 In MS 550, Peirce indicated that these equations were "Not properly aligned"; he also noted their misalignment in MS 551a.
266.25 and 26 In MS 551b, Peirce added, in light blue ink, "$+D_t^2 x_o$" after the two equations.
267.6 In MS 551b, Peirce added, in light blue ink, "$+D_t^2 x_o$" above the first comma.
267.12 Let] In MS 550, Peirce indicated that there should be "No break" here — that is, no new paragraph.
267.13 x_p] Peirce seems to have read the miniscule superscript sigma as s, which he inscribed in MS 550. The printed errata curiously note "for χ^s read χ_p." Peirce corrected the chi's to x's in the MS 551b errata, but did not notice the superscript s.
267.14 x_p] Peirce noted in MS 550 that this printing error was *"Very bad!"* The original errata sheet also printed the x's as chi's. Peirce corrected the latter and the omitted D in the formula in the errata in MS 551b.

267.21–30 In MS 551b, Peirce wrote, in light blue ink, "np cos ψ – sin ψ + nq sin ψ" in the left margin and "Den essentially negative" in the right margin; from the latter phrase, he drew a direction line to the end of each of the two equations.
267.26–30 In MS 550 Peirce wrote, "Numerator of second member badly spaced." The numerator has been aligned with the first member.
268.11–12 In MS 551b, Peirce added an alternate form of the second half of the equation, which extends into the right margin

$$= \frac{\frac{s}{h}\frac{\cot\psi}{\psi} - 1}{h + \frac{\gamma^2}{h} + \frac{s^2}{h}\left(\frac{2\cot\psi}{\psi} + \frac{\tan\frac{1}{2}\psi}{\psi}\right)\frac{\tan\frac{1}{2}\psi}{\psi} + S - \frac{s^2}{h}\frac{\cot\psi}{\psi}}$$

268.14–15 In MS 551b, "$h_t^2\sigma^2$" is inscribed in blue ink in the left margin between the two lines.
268.15 In MS 551b, S is inscribed in the left margin with a forked pointer to the sigma and psi in the term "$\sigma p\psi$" in the numerator to indicate that the expression might have been written as S or Sp.
269.8 In MS 551b, Peirce has added "= 0" to the right of the equation.
270.15–16 In MSS 550, 551a, and 551b, Peirce closed the large space that appears in P 315 between $\frac{g}{h}$ and p' and between sinh and ψ (where p' and ψ are printed next to their respective commas).
270.24–33 In MS 551b, "$\psi - 2$ (coth ψ – cosec $\psi) = \frac{1}{12}\psi^3$ $(1 - \frac{1}{10}\psi^2$ etc)" is diagonally inscribed in the margin to the left of these equations.
271.2–3 In MS 551b, "$\frac{1}{12}\psi^4 + \frac{1}{180}\psi^6 +$ etc" is diagonally inscribed in the left margin with a forked direction line to "(cosh ψ – 1) + ψ sinh ψ."
271.7 In MS 551b, the following is inscribed in the lower left margin below the equation:

In an actual case
$p' = 8.494$
$(h^2 + \gamma^2 - \sigma^2)\sinh\psi + \sigma p'\psi = 48.07$
$\gamma^2\sigma[\psi\sinh\psi - 2(\cosh\psi - 1)] = 1.0125$

273.10 air.] In MS 551b, Peirce has added the following text in the left margin with a direction line and caret for insertion before the period: "and still more to the communication of motion to the support. The theory of this is simple. The support simply takes a motion such as the noddy tends to give it."
273.23 In MS 551b, "$\frac{\rho_1^2 + \gamma^2}{\rho^1}\vartheta$" is inscribed in the space to the left of the equation; "$(h + \rho)\vartheta + \sigma\chi$" is crossed out, but a direction line is drawn to the following inscription below the final text on the page:

This is right but
$$= \frac{\gamma^2 + \rho_1^2}{-\rho_1}\vartheta$$

Emendations

262.16 *et seq.* centimetre] E; centimeter *All forms; also* millimeter
262.23 These] 550, 551a, 551b; They
262.25 *et seq.* centre] E; center
263.11 on] 550, 551a, 551b; or
*264.1(1) *et seq.* $\int_0^S$] 550, 551a, 551b; $\int_0^S$ *All S's in upper limit of integration are small, except not present* 264.1(2), 8(1)

*264.2 $D_t\theta\cdot$] 550, 551a, 551b; D_t
264.3 $D_t\theta\cdot$] 550, 551a, 551b; $D_t\cdot$
264.3 *[formula].*] E; $\sim_\wedge$ *Also* 264.8, 12, 16, 18, 25; 265.2, 6, 14, 16, 22; 266.9(2), 14; 267.11, 18; 268.2, 12, 16; 269.8, 20; 270.2, 10; 271.7, 12, 14, 23; 272.3, 7, 11, 14, 17; 273.1, 25(3)
264.6(1) $\int_0^S$] 551a, 551b; $\int^S$
264.8 $\frac{\epsilon}{M}$] 551a, 551b; $\frac{E}{M}$
*264.12 ə] 550, 551a, 551b; δ *Also* 264.16
264.13 The] 551a; ¶~ *Also* This 264.19; Let 265.1; Then 265.3; Differentiating 265.15; This 270.3
264.13 *et seq.* coëfficients] E; coefficients *Both plural and singular forms*
*265.11 $D_s\theta.$]550, 551a, 551b; $D_{ss}\theta_\wedge$
265.20 θ_3] E; $\theta_3,$
266.2 $2\pi\sigma$] 551b; $\frac{\sigma}{2\pi}$
266.8 θ_s] E; θ_s
266.26 h +] 550, 551a, 551b; h ∓
*267.12 Let] 550; ¶~
*267.13 x_ρ] 550, 551a, 551b; x^s
*267.14 x_ρ] 550, 551a, 551b; $x\rho$
267.18 $\sigma D_t^2\chi$] 551b; $\rho D_t^2\chi$
267.18 $\frac{\pi^2}{T^2}$] 551b; $\frac{\delta 2}{T2}$
268.2 $[(h^2 \ldots \sigma p\psi]]$ 551b; $-_\wedge($ $\sim \ldots \sim_\wedge$
268.6 $\frac{\pi^2}{gT^2}$] 551b; $\frac{\delta^2}{gT^2}$
268.9 $\frac{\pi^2}{gT_1^2}$] 551b; $\frac{\delta^2}{gT_1^2}$
268.11 $\frac{\pi^2}{gT_2^2}$] 551b; $\frac{\delta^2}{gT_2^2}$
269.21 Gudermannian] E; gudermannian
270.7(1) θ_s] E; θ_ς
270.7 $p' \ldots q'$] 551b; $p^1 \ldots q^1$ *Also* 270.9, 10, 15
270.19 D_t^2] E; D_t^2
270.29 np'] 551b; $n\rho'$
271.2–3 $+\psi \ldots + \sigma p'\psi$] 551b; $-\psi \ldots - \sigma p'\psi + 2h\sigma$ (cosh $\psi - 1$)
271.5 $\frac{+\sigma p'\psi}{+\psi \sinh\psi}$] 551b; $\frac{-\sigma p'\psi + 2h\sigma(\cosh\psi - 1)}{-4\sinh\psi}$
271.6–7 $\frac{p'}{\ldots + \sigma p'\psi} = \frac{\ldots}{\ldots + S + S\frac{\sigma}{h}\coth\psi}$] 551b; $\frac{p'}{\ldots \sigma p'\psi + 2h\sigma(\cosh\psi - 1)} = \frac{\ldots}{\ldots - S - s\frac{\sigma}{h}\coth\psi + 2\sigma\tanh\frac{1}{2}\psi}$
271.12 $D_t\theta \ldots D_t\vartheta$] 550, 551a, 551b; $D\,\theta \ldots D\,\vartheta$
271.14(2) $D_t\xi$] 550, 551a; $D\,\xi$
272.6 $[h + \rho)\vartheta + \sigma\chi]]$ 551b; $[(h + \varsigma)\vartheta + \sigma\chi_\wedge$
272.19 harmonic] 551b; sort of
272.21 nearly . . . of] 551b; not much affected by
272.21 (T − T'),] 551b; $\sim_\wedge$
272.21–22 and . . . reached] 551b; until it would reach
273.5 again,] E; $\sim_\wedge$
273.12 $\int_0^S$] E; $\int_0^S$
273.21 ϵ] E; e *Also* 273.23
273.21 $\frac{t}{2T'}$] E; $\frac{t}{2T'}$
273.28 and] 550, 551a; —
274.4 affords] 550, 551a, 551b; is

Line-End Hyphenation

266.2 wave-length

43. *Flexure of a Pendulum, 1885*

Copy-text is P 316, the publication in the 1884 *Coast Survey Report* (which has "By C. S. Peirce, Assistant" below the title). It is emended by MS 552, Peirce's autograph list of "Corrections to Appendix 15. Report for 1884" on one page numbered "5" [552] that follows the first four pages of the Appendix 15 "Corrections"; by MS 553a, a corrected, revised, and annotated offprint [553a]; and by MS 553b, another corrected, revised, and annotated offprint (from the Max H. Fisch Collection) [553b]. All holograph markings in the offprints are in black ink unless otherwise indicated in the Textual Notes. The majority of authorial changes appear in all three post-copy-text documents, but slight variations (especially in the revised passages) require the inclusion of each as a distinct source of emendation. Some fragmentary pre-copy-text pages and a few other related leaves are in MS 536.

As early as 11 March 1885, Peirce had written to J. E. Hilgard that he was working on his paper on "the flexure problem" and would have it ready in a week. On 29 October 1885, he notified F. M. Thorn that the printing of Appendix 16 was, like that of Appendix 15, "marred by various bad errors" and that he enclosed a list of corrections (for more on which see the headnote to item 42). There is no clear indication of priority among Peirce's three surviving attempts at correction. All the corrections and revisions indicated in MSS 552, 553a, and 553b are emended into the copy-text; Peirce's occasional failure to mark the correction or revision on all three documents is recorded in the Emendations. Extra-textual annotations are represented in the Textual Notes.

A published list of "Corrections," printed below those for Appendix 15 on a half-page tipped into P 315, is incomplete and contains pagination errors. It also contains new corrections which cannot be traced to any authorial source. These variations are not accepted as emendations, but are recorded in the Textual Notes.

Textual Notes

275.12 γ_0] The "bad type," a subscript zero that looks more like a "c," which Peirce noted in MSS 550 and 553b, has been corrected.

276.15–18 ψ] The published corrections (to pages 483 and 484 rather than the printed "Pages 484 and 485") read: "The formulae do not agree with the definition of ψ. They would be right if ω were everywhere substituted for ψ, or if ψ were defined so as to be synonymous with ω." In MSS 552 and 553b, Peirce has changed the printed omegas to psi's.

277.8 In MS 553a, Peirce added, after the second equation, "$= \mathrm{mm_0}\left(\mathrm{h}_0^2 + \gamma^2\right)\left(\mathrm{h}^2 + \gamma^2\right) + \mathrm{m}^2\mathrm{r}^2\gamma^2$." In the second equation in the next line, he inscribed diagonally below the equal sign another equal sign, as though to continue with an alternate formula.

277.11 In MS 553a, Peirce added "$+\frac{\mathrm{N}}{\mathrm{M}}$" and then continued, beginning in the left margin, with the following text: "for L is small compared with M and N that whether ϵ be large or small."

277.13–14 In MS 553b, Peirce changed the three instances of "*mh*" to "$m_0 h_0$"; he crossed out the parenthetical expression in the third equation, and wrote above it, in light blue ink, "$\mathrm{g(mh + m_0h_0}\ \psi^{-1})$."

277.16–19 In MS 553b, Peirce has rewritten, after the final text on the last page, the parts of the formulas following the equation signs. They are inscribed in light blue ink and read as follows:

$$\mathrm{gmh}\left(\mathrm{m_0}\left(\mathrm{h}_0^2 + \gamma_0^2\right) + \mathrm{mr}^2 + \mathrm{m_0hh_0mhr} + \frac{\gamma^2}{\mathrm{h}}\left(\mathrm{m_0h_0} + \mathrm{mr}\right)\right.$$

$$\mathrm{gmh}\left(\mathrm{m_0}\left(\mathrm{h}_0^2 + \gamma_0^2\right) + \mathrm{mr}^2 + \mathrm{m}\left(\mathrm{h}^2 + \gamma^2\right) + 2\mathrm{mrh}\right.$$

277.16 *gmh*] In MS 553b, Peirce has deleted *"mh"* from *"gmh."*
277.16, 20 In MS 553b, Peirce has crossed out "$(1 + \psi^{-1})$" only in the first of the two lines; but in both instances, he wrote "$(mh + m_0 h_0 \psi^{-1})$" above the parenthetical expression.
277.17 *mh*] In MS 553b, Peirce has added subscript zeros to the two letters.
277.18 *gmh*] In MS 553b, Peirce has added subscript zeros to the last two letters.
277.18–19, 21 In MS 553a, Peirce has rewritten these two formulas (and added a third) in the lower margin of the page:

$$\psi M = gmh\mathfrak{M}\mathfrak{H}\mathfrak{L} + ghm\ \{\mathfrak{M}\mathfrak{H}\mathfrak{L} - mh(r + h) - y^2 + m_0\frac{h_0}{h}(h^2 + \gamma^2) + mr\frac{\gamma^2}{h}\}$$

$$\psi N = g^2mh\mathfrak{M}\mathfrak{H} + g^2mh\ (\mathfrak{M}\mathfrak{H} - mh)\psi$$

$$\psi L = mh$$

277.20 In MS 553b, Peirce first added subscript zeros to the last two letters in *"g^2mh,"* but then deleted the zeros as well as the two letters; he added subscript zeros both to *"mh"* and *"m^2h^2."*
277.21(1) *g^2mh*] In MS 553b, Peirce has added subscript zeros to the last two letters.
277.25–27, 278.6–7 In MS 552 and the printed errata, Peirce indicated that the letters M, H, and L should be printed in "Old English type"; in MS 553b, he marked the 21 occurrences as "Old English"; in MS 553a, he rewrote the three letters in the first three lines only.
278.16 Following the printed text in MS 553a, Peirce has added the following: The way I seem to have set out was very simple $\delta\theta = -\omega\theta$

$$\theta - \theta_0 = -\omega\theta \qquad \theta = \frac{1}{1+\omega}\theta_0 = (1-\omega)\theta_0 \qquad \delta\theta = -\omega(1-\omega)\theta_0$$

$$U = \tfrac{1}{2}gm_0h_0\theta_0^2 + \tfrac{1}{2}gmr\theta_0^2 + \tfrac{1}{2}gmh(1-2\omega)\theta_0^2 + \tfrac{1}{2}gmh\omega\theta_0^2$$
$$= \tfrac{1}{2}gm_0h_0\theta_0^2 + \tfrac{1}{2}gm(r+h)\theta_0^2 - \tfrac{1}{2}gmh\omega\theta_0^2$$

$$E = E' - mrh\left(D_t\theta_0\right)^2\omega - m\left(h^2\gamma^2\right)\left(D_t\theta_0\right)^2\omega.$$

Emendations

275.10 *et seq.* centre] E; center *All forms*
275.16 coëfficient] E; coefficient
275.18 *[formula].*] E; $\sim_\wedge$ *Also* 276.-11, 16, 18(2), 28, 31; 277.14, 23; 278.7
275n.10 may] 553a; must
*276.15–18 ψ] 552, 553b; ω
277.2 *gmr*] 552, 553a, 553b; *gm L Also* 277.6
277.9 $-E^2$] 553a, 553b; $-E$
277.11 $\frac{M}{L}$.] 552, 553b; $\sim_\wedge$ ¶If we measure the elasticity by holding the second piece in a rigidly horizontal position and observing the angular sagging at the joint due to the weight of the first piece, we have for the value of the angle

$$\psi = \frac{g\ m_0\ h_0}{\epsilon}$$

277.12 the . . . ψ] 552, 553b; *not present*
277.19 ψ] 553a, 553b; *not present*
*277.25 $\mathfrak{M}$] 552, 553a, 553b; M
*277.26 $\mathfrak{M}\mathfrak{H}$] 552, 553a, 553b; MH *Also* 278.6, 7
*277.27 $\mathfrak{M}\mathfrak{H}\mathfrak{L}$] 552, 553a, 553b; MHL *Also* 278.6, 7

44. *Peirce to Abbot, 1885–86*

The copy-text, L 1, consists of four folded sheets of unlined white paper measuring $9\frac{13}{16} \times 7\frac{7}{8}$ in., with a "Cranes Distaff" watermark, and is deposited in the Abbot Papers in the Harvard University Archives. The first letter is inscribed on two pages, the second on eight, and both are only moderately altered. One ink blot occurs in the first, two in the second. The editors have removed the underlining from the signature in the second letter. Published with permission of the Harvard University Archives.

Emendations

279.12 *group.*] E; ∼‸
280.36 any] E; an
282.5 aristotelis.] E; ∼‸

Line-End Hyphenation

280.30 philosophy-|as-it-is
281.6 things-|in-themselves

Alterations

279.15 while you] *intl ab del* but
279.17–18 of which it is] *intl ab del* that possess it
280.4 speak] *intl ab del* say
281.1 to] *intl ab del* of
281.35 *do not*] *intl ab del* would
281.36 I . . . matter.] *intl*

45. *Fundamental Properties of Number, 1886*

Copy-text is MS 567, a one-page purple-ribbon typescript on one leaf of medium-weight, very stiff, laid, white paper measuring $9\frac{31}{32} \times 7\frac{31}{32}$ in., with a "Guggenheimer and Weil" watermark. The typescript is in very poor condition, dust- and water-stained, dog-eared, and torn, and the typing is sometimes faint. The spacing of its punctuation is irregular; there are no alterations.

Textual Notes

283.6 The emendations here and at 283.14 and 20 are the result of the limitations of Peirce's typewriter; without a right-margin release, Peirce (or his typist) was unable to type the period and two hyphens, respectively.
284.8 *[*wa*]*nts] Due to dust staining of the lightly typewritten word the "nt" is almost illegible; only the "s" is clearly visible.

Emendations

283.3 Jan.] E; ∼‸
*283.6 characters.] E; ∼‸
283.10 possessing] E; possess|sing
283.12 1,] E; 1
283.14 essential] E; essen|tial *Also* 283.20
283.27 positive] E; positiye
284.5 one.] E; ∼‸
284.9 true.] E; ∼‸

46. *Dr. F.E. Abbot's Philosophy*

Copy-text is P 326, the publication in the *Nation.* Though Haskell's *Index to the Nation* does not list Peirce as the author of this unsigned review, Abbot himself does in his diary. The printing of the review in the *Nation* is sometimes faint, and the "i" in "chain" (286.22) and the dot in "is" (286.23) are dropped.

Emendations

285.8 *Scientific Theism*] E; 'Scientific Theism'
286.22 chain] E; cha n
287.30 141] E; 139

Line-End Hyphenation

288.27 vortex-motions

47. *One, Two, Three: Kantian Categories, 1886*

Copy-text is MS 572, two and one-half pages of a purple-ribbon typescript on three soiled leaves of medium-weight, white paper measuring $9\frac{15}{16} \times 8$ in., with a "Guggenheimer and Weil" watermark. The left rough edge of the leaves suggests that they were torn from a tablet. The first leaf has turned brown, the second is torn, and some corners of all three are dog-eared. There are several alterations: some typewritten, others handwritten in blue ink. The title and the immediately following "By C. S. Peirce" (which the editors have removed) are in italic type; the rest is in roman. The alignment in the left margin, as well as the spacing of some punctuation and of several words, is irregular. The verso of the first leaf, containing only the mis-typed title "FATEGORIES," represents a false start.

Like items 34–37 of the year before, items 47–50 are the result of Peirce's renewed interest in the categories which, during the next two or three years, will culminate in "A Guess at the Riddle," and they are to be viewed as preliminary attempts, more or less elaborate, at expounding the core of the theory. Item 47 links Peirce's undertaking to the philosophical tradition and introduces evidence for the irreducibility of the element of chance, and item 48 gives expression (in a pre-Socratic tone) to the riddle itself: "What is the world made of?" Items 49 and 50 try each to give a systematic presentation of the three categories and their degrees of degeneracy, the second being the more finished; they were both written in the summer-fall of 1886, as the following excerpt from a 20 August 1886 letter to Edward S. Holden, Peirce's friend in Madison, intimates:

> You remember that I told you something of a sort of evolutionist speculation of mine. This has grown much since in two directions. In the first place, I have found how to make mathematical deductions from it respecting nature, and have quite a list of these, which agree remarkably with some of the most striking characters of the laws of physics; so that it is no longer a mere speculation but a great working hypothesis of science, destined to play a great part in the future. In the second place, the skeleton of my ideas has filled itself out on the philosophical side, so that my book will be a real manual of philosophy, leaving no question untouched. It seems to me that it will be very important from this point of view, and will wheel the philosophers and Hegelians into harmony with scientific thought.

(It is worth noting that Peirce's letter, like item 50, is typed entirely in italic, and that the same typewriter was used for both.)

Textual Notes

293.17 probable] Peirce (or his typist) realized that the word would not fit in the space remaining in the line and thus abandoned it after having typed "p"; he neglected to delete the letter.

293.37(2) is] Peirce typed "wil," left three spaces, and then decided on "is"; he neglected to delete "wil."

Emendations

292.6 axioms] E; aqioms
292.9 triangle] E; triangles
292.10(1) the] E; he
292.16 is] E; are
292.28 positive] E; positiye
293.7 that] E; *not present*
293.14 of] E; sf
*293.17 probable] E; p ~
293.37 rational] E; ratinal
*293.37(2) is] E; wil ~
293.38(1) account] E; a count
293.39 attempt] E; atempt
293.40 and] E; the
294.1 of] E; if

Alterations

292.20 constant] *intl ab del* quantity
292.25 number] *intl ab del* number or value
292.27(1) nearer] er [r *ov* to] *add and bef del* that value which it
293.15 though] *intl ab del* but
293.26 the universe is] *intl ab del* we are
293.27 complete] *aft del* pure
293.29 Third] T *ov* t

48. *One, Two, Three, 1886*

Copy-text is MS 573, thirteen pages inscribed (and revised) in black and blue ink on eleven leaves of medium-weight, gilt-edged and unlined white paper measuring $6\frac{9}{16} \times 8$ in. The leaves have come unbound from a brown leather-covered notebook with a very brittle binding, and there are ink blots and some staining throughout. Peirce worked on this manuscript at two different times (in 1886), as is revealed by his use of two different inks (black and blue); the portions written in blue are later than the others (and are indicated in the Textual Notes). Copious commentary, inscribed on the versos of leaves four and five, is reproduced in the Textual Notes. The editors have removed horizontal lines below the title, "Contents," and "In Psychology" (294.3, 294.5, 298.10).

Textual Notes

294.3 Above the title, Peirce later inscribed, in blue ink, "Trichotomic. (The art of hair-cutting.)|Trischistic (or Trischismatic)."
294.8 Peirce added this line in blue ink.
294.17 Between "XI" and "XII," Peirce left space for two or three more lines of text.
294.20–21 Peirce later inscribed these sentences in blue ink, on an otherwise blank page following the two pages of "Contents."
294.23–295.3 The facing verso page in the manuscript contains the following inscriptions:

The moment the first is asserted, it is in some measure denied. The pure first precedes all assertion all differentiation. There is no synthetical unity in it, no consistency; it is just the manifold and bizarre of first impressions
Courage?
Beauty je ne sais quoi
Emotion, heart & soul, excitation, animation
Question [Answer = 2^{nd} Explanation = 3^{rd}]
A note of sound = 1^{st} Caprice [But choice = 3^{rd}]

Melody = 3^{rd}
First come first served
In nubibus, Baseless fabric of a vision. Idea Prototype
Monad. Sheer, pure, innocent, virgin. Not yet.
Individualism. Dawn. Initiative, first step. But second thoughts = 2^{nd}
Scattering Variety Manifold Prime of life
Strange, wondrous, bizarre.
Feeling, consciousness [But sense = 2^{nd} Will = 2^{nd} Intelligence = 3^{rd}.]
Life = 1st
Pleasure & Pain & all subjective Objective = 2^{nd} Thrill = 1^{st} Touch = 2^{nd}

295.4 Preceding the paragraph, which begins a new page, Peirce has inscribed, in the top left corner, "Couple|Gain|Property Wealth."

295.4–6 Facing this sentence on the verso page are the following inscriptions:

Duplicity Lie Negation. All exaggerated language & swearing, "supreme," "utter" "matchless" false root & branch
Positive = First. Superlative = Second. Comparative = Third.
Duel. Hit. Deed (But thought 1^{st} Word conduct 3^{rd}) Fall of the curtain. Fate (But chance = 1^{st})
Reflective Second. Identity. Hyperion to a satyr. Likeness. Plays second fiddler
Ebb. Second thoughts. Dine with Duke Humphrey Echo. Interior. Motive, allurement
[But excuse = 3^{rd}
Difficulty Duty
Flesh & blood.

295.5 gets hit] Above this phrase, Peirce has interlined "compulsion"; it is not clear how he intended to incorporate the word.

295.6 reality] There is a direction line between this word and "stubborn fact," which is in the space between the two paragraphs; it is not clear how Peirce intended to incorporate the phrase.

295.6 upshot.] In the blank space at the end of this paragraph, Peirce has inscribed "Here and now"; immediately preceding the next paragraph, he has inscribed, in the left margin, "Hard & fast|(Fluidity = 3^{rd})."

295.9 Exchange] Peirce interlined "compensation" above and "commerce" below this word.

295.9–18 Facing this list of words on the verso page are the following inscriptions:

Con Trans- (prep.) = 3^{rd} Ob = 2^{nd} [*Obstacle, Objective, occurrence*
Habit, training Substitute, deputy Utility Conduct (But Deed = 2^{nd})
Aid (but hindrance = 2^{nd}) Compact, contract Consummation Conditions
Giving (taking = 2^{nd}) Sociability Love Punishment

295.11 mixture] A short line is inscribed between "mixture" and "Whole," which appear as the first words in two successive lines.

295.31–298.14 IIa . . . Comparison.] This portion of the manuscript is inscribed in blue ink.

295.33 What is] Peirce had first written "was is," but then capitalized the first letter to correct the word; he thought better of it and interlined "What," but neglected to delete "Was."

295.36 the variety] Peirce had first written "the richness," but when he deleted the second word, he neglected to delete "the" at the end of the preceding line and the comma after "richness."

296.10 That . . . problem.] Peirce interlined this sentence but, having added "That," he neglected to delete the comma and to lowercase "Was."

296.15 This . . . Anaximenes] Peirce interlined this new beginning of the paragraph above the earlier one whose first and second lines ended and began with "hypo-" and "thesis," respectively; he neglected to delete "thesis."

296.41 distinct] Although it is not clear whether Peirce wrote "distinct" or "distant," the editors have opted for the more appropriate "distinct."

298.5–8 No. . . .] The difficulty of reconstructing Peirce's text, which he altered at least twice, leaves some uncertainty as to his exact meaning. Two alternate nearly equally plausible renderings are (1) "No, though the theory may not go deep enough, the inference was logical, for the main conception of a primal homogeneous material may be crude, but granted that, the primal matter must itself be living" and (2) "No, though the theory may not go deep enough, for the main conception of a primal homogeneous material may be crude, but granted that, the inference was logical that the primal matter must itself be living."

Emendations

294.7 logic.] E; ~‸
294.8 metaphysics.] E; ~‸
294.9 psychology.] E; ~‸
294.10 physiology.] E; ~‸
294.21 3^{rd}.] E; 3^{rd}‸
294.28 synthetizing] E; sythetizing
295.8 Process.] E; ~‸
295.8 time.] E; ~‸
295.10 third).] E; ~)‸
295.19 change.] E; ~‸
295.28 Hypothesis.] E; ~‸
*295.33 is] E; Was ~
295.38 Tracing] E; tracing
*296.10 That was] E; ~, Was
296.16 traced.] E; ~‸
296.28 Now . . . what] E; No . . .What
296.41 distinct] E; distint
298.6–8 that— . . . that—] E; ~‸ . . . ~,
298.9 III.] E; ~‸

Alterations

*294.8 IIa. In metaphysics] *add*
294.26 of] *bef del* One, Two, Three,
294.26 First . . . Third.] *intl aft del* or [*intl aft del* Three,]
294.27 the First] *aft del* One, or
294.27 at all] *intl-c*
294.28 synthetizing] izing *intl ab del* ised
294.28 Whole] W *ov* w
295.1 first,] *bef del* and
295.2 taken.] *intl ab del* one
295.4 last or term or end] *intl-c*
295.5–6 effect, . . . upshot.] *intl ab del* occurrence, otherness, dependence, result.
295.6 upshot.] *bef ins del* Con
295.8 Process] *intl*
295.8 Flow of time] *intl*
295.10 representative] ive *ov* ion
295.11 & legislation] *intl-c*
295.12 & active force] *intl-c*
295.17 2] *ov* 1
295.17 Place] *intl ab del* End
295.35 First] F *ov* f
295.35 Inquiry] *aft del* Science
*295.36 the variety] *aft del* the richness, [the *and* , *del by editors*]
295.36(2) the] th *ov* a
◇295.38 tracing] *aft del* To [*bel intl del* The idea] *and* ing *ov* e
295.38 of] *ins*
296.1 Induction] I *ov* i
296.1 Induction,] *bef del* could not was what those ◆early men [*bel intl del* inexperienced minds] could not possibly understand see ◆either [*first* e *ov* th] the need
296.2 those] *aft del* was a proceeding
296.2 possibly] *intl-c*
296.4 each step] *intl ab del* it
296.5 in an endless regress.] *intl-c*
296.7 their] ir *ins*
296.7 account] *intl ab del* history
296.7 explain] *intl ab del* account for
296.8 start from] *intl ab del* begin with
296.8(2) from] *intl ab del* with
◇296.10 That . . . problem.] *intl* [their conception of *intl bel del* what they had to do]
296.11 Now] *add*
296.11 every] e *ov* E
296.11 in its inception] *intl-c*
*296.14–15 seemed . . . Anaximenes]

intl ab del But Anaximenes improved this hypo-|
296.15–16 his line . . . traced] *intl with following four alterations*
296.15 his] *ov* by
296.15–16 reflection] *intl bel del* thought
296.16 easily] *bef del* to *and* i *ov partial* y
296.16 traced] *intl ab del* divine
296.17(2) had] *bef del* been sensed be
296.17 sprung] *intl ab del* determined *and* r *ov* a
296.18–19 is it . . . variety?] *intl ab del* can variety be determined by sameness?
296.19 fecund] *intl-c*
296.21 must] *bef del* then
296.21 have] *intl ab del* be
296.21 had] *add*
296.22–23 One word . . . soul.] *intl*
296.24 *arche*] *intl ab del* first
296.24 have been animated] *intl ab del* be lively
296.25 So] *intl ab del* Thus,
296.26 with] *bef del* the bringing out the
296.26 initial] *intl ab del* first
296.27 primary] *intl ab del* fundamental
°296.28 Now let us ask ourselves] *intl-c*
296.28 that object would] *intl ab del* is it *and* object *intl-c and* would *bef del* that
296.30 should abide as] *intl-c*
296.31 three] *intl ab del* mechan [*intl ab del* laws]
296.33–34 impregnable] *intl bel del* •force of [*bel intl del* irresistible]
296.34 strength] *intl ab del* force
296.34 position] *aft del* position [*bel intl del* logica]
296.35 the axiom which Spencer is blind to] *intl-c* [which *ov* that]
296.35 dead] *intl bel del* blind
296.36 mechanical] *aft del* and dead
296.36 causation] *intl ab del* determ
296.37 To] *aft del* For that the
296.37 effect] *intl ab del* bring
296.38 that] *bef del* about
296.38 homogeneous] homo *intl ab del* prin
296.38 must] *bef intl-c del* either
296.38 else] *intl-c*
296.39 can] *ov* will [*intl ab del* can]
296.39–40 Freedom . . . material.] *intl ab del* The material must itself have spontaneous freedom. *and* Freedom *aft del* Spont *and* be of *intl ab del* reside in
296.40 peculiarity in the] *intl bel del* •law or cause [*bel intl del* heterogeneity]
296.40–41 forces acting . . . it] forces . . . distinct *intl ab* apart [*del*] from •it [*ins bef del* the matter]
296.41 would] *intl bel del* will
296.41 such] *aft del* then
298.2–5 This is the clear . . . gainsay it] *transp from bel* living.
298.3 philosophical] *intl ab del* scientific
°298.5–8 though the theory may not go deep enough, the inference was logical that for the main conception of a primal homogeneous material may be crude, but granted that,] *intl-c* [for *intl ab del* and *and* main *intl ab del* whole *and* primal *intl-c*]
298.15–16 Plato's . . . Souls.] *add*

49. *One, Two, Three: An Evolutionist Speculation, 1886*

Copy-text is MS 575, consisting of (1) two leaves of purple-ribbon typescripts on medium-weight paper measuring $8 \times 10\frac{1}{32}$ in., with a "Guggenheimer and Weil" watermark, and (2) six leaves of medium-weight paper, ruled on both sides, measuring $7\frac{1}{16} \times 8\frac{15}{32}$ in., with a "Southworth's 1882 Linen Bank" watermark (with three perforated holes in the left margin for insertion in a loose-leaf binder) and inscribed in black ink. The second leaf is very faintly typewritten, and both leaves have turned brown, are torn and chipped, and their corners are folded. The typescript is roman except for the italic "*ob*" (300.1), "*Fate* and *chance*" (300.13), "*duel*" (300.14), and "*duplicity*" (300.15), and the spacing of typed punctuation is irregular. Handwritten alterations on the first page are in black ink. The handwritten pages are moderately altered throughout, but the last is extensively revised. They provide copy-text for 300.36 onward, beginning with "roads." (The leaf that serves as copy-text for 300.36–38 also includes a variant of the text at 300.27–36, which is given in the Textual Notes.) Seven additional leaves, rejected earlier variants, also survive. There is related holograph material in MSS 574 and 577, and two pages of related typescript material (the second with Peirce's inscribed alterations) are in MS 576. For more on the split copy-text of item 49, see the Essay on Editorial Method.

Textual Notes

300.7 This emendation and the one at 300.25 are the result of the limitations of Peirce's typewriter; without a right-margin release, Peirce (or his typist) was unable to type either a hyphen or an additional letter.

300.15 negation] This emendation, like those at 300.7 and 25, is the result of the limitations of Peirce's typewriter, and he failed to add the missing "n" by hand.

300.28–33 In the handwritten variant, "relations . . . logicians" reads "by the old logicians relations of reason"; the words "or peculiarity" are not present; the "second" (300.32) is "last"; and "is third," reads "is a third, for."

301.6 involves] Peirce's deleted "s" is reinstated because the second clause is not part of the subject of the verb.

Emendations

299.10 things] E; thiogs
299.34 Second] E; second
299.36 hit],] E; ~,]
299.38 constitutes] E; constitutas
300.4 first.] E; ~‸
*300.7 completely] E; comple| ~
300.12 over-weathered] E; ~‸~
*300.15 negation] E; negatio
300.15 *duplicity*].] E; ~.]
300.16 implies] E; imlies
300.25 is] E; i| ~
300.31 By] E; *no* ¶ ~
*301.6 involves] E; *reinstate* s
301.12 neighbor's] E; neighbors
301.26 aggregate] E; ~,

Alterations

299.2 knack] *intl ab del* habit
299.6–7 , & so be . . . chapters] *add*
299.12 First] *bef del* is that which
299.16 refer] *aft del* imply
299.25 talks inaccurately] *inv*
299.26 in] *aft del* but
299.29 present,] *intl-c*
299.30 or remote] *intl-c*
301.3 likewise] *intl ab del* also
301.12 contains] *aft del* is third
301.17–18 a character] character *intl ab del* fact [*aft* a *and* a fact *intl ab del* charac]
301.18 extremes] *aft del* latter
301.19 are comparisons,] *intl ab del* is a compact
301.23 alone] *intl-c*
301.26 different] *intl ab del* every
301.26–27 We . . . thirds.] *intl ab del* Such is an action which leads to an unforeseen result.
301.30 second] *intl ab del* thir
301.33 Washington] *intl ab del* Baltimore
301.35 is] *aft del* lies
301.38 fifths,] *bef intl-c del* and other plurals
301.38 are not] *intl ab del* present
302.4 philosophy,] *bef del* ♦and are the [*bel intl del* being]
302.5 particularly in that of] *intl ab del* obtrusively prominent in
302.6 his] *intl ab del* that of Hegel
302.6 noting] *intl ab del* recognition of
302.7 chiefly] *intl ab del* only
302.7 an] *aft del* what seems to me to be
302.8 force] *intl ab del* second
302.8 playing] *intl ab del* making
302.9 This] *aft del* I think that it is

this fact, mainly, which explains the very ◆unhegelian [*intl ab del* different] use which I am going to make of these conceptions in the chapters which follow.
302.10 in what follows] *intl-c*

50. *First, Second, Third, 1886*

Copy-text is MS 578, a purple-ribbon italic typescript of four pages on four leaves of light-weight, white paper measuring $9\frac{7}{16} \times 12\frac{15}{32}$ in. All leaves have browned edges; the first two are torn and chipped. There are numerous handwritten revisions on the first leaf (in blue ink), some on the second, and typewritten alterations in the entire manuscript. On the first leaf, a cross and a vertical line are inscribed in red pencil in the left margin (at 302.13–24); there is another cross in the left margin near the bottom of the first leaf (before 304.6) and in the top left corner of the second leaf (before 304.14). The spacing of punctuation and several words is irregular, and many characters that should have been typed on the line are above it. Some emendations were necessary when spaces for characters in the typescript were left blank. Capital variables are not underlined or otherwise differentiated from the remaining italic text in the typescript (as is the *ob* at 305.4) and, consequently, are emended. A fragmentary italic typewritten leaf with a cross in red pencil in the upper left corner also survives. There is related holograph material in MSS 574 and 577, and two pages of related typescript material (the second with Peirce's inscribed alterations) are in MS 576.

Textual Notes

302.26 discovery] Instead of a semicolon, something resembling a section sign occurs in the typescript.
303.9 right] This emendation, the one at 307.1, and the first at 308.4 stem from the (right-margin) limitation of Peirce's typewriter.
303.25–26 Peirce neglected to delete "than" at the end of this sentence.
305.22–23 conception.] The final part of the "n" and the period are missing because Peirce ran out of room when he interlined the word in ink at the right edge of the leaf.
306.4–5 means . . . third] Except for the last two letters of the last word, this passage is typed diagonally at the bottom of the second leaf of the manuscript. When Peirce (or his typist) realized that the leaf had begun slipping from the typewriter, he typed the words anew on the first line of the following leaf, but neglected to delete the last line of the previous leaf.

Emendations

302.17 happily] E; happiln
302.17 something] E; ssmething
*302.26 discovery;] E; ~ʃ
302.26–27 unphilosophical] E; unphilossphical
*303.9 right] E; righ| ~
303.14 opinion.] E; ~‸
303.25 fiction] E; Fiction
*303.26 to triads] E; triads than
303.28 significant] E; significat
303.31 so] E; ss
304.1 Third] E; Therd
304.7 of . . . think] E; od . . . thiok
304.8 philosophy] E; plilossphy
304.8 unities] E; uneties
304.11 initiative,] E; ~‸
304.15 notion] E; notilon
304.31 involved] E; in ~
304.37 is] E; iis *Also* 306.23, 307.31
305.4 *ob*] E; *rom.*
305.14 return] E; seturn
305.15 over-weathered] E; ~‸~
305.20 duplicity).] E; ~.)
*305.22–23 conception.] E; ~‸

305.33 realizable] E; realizeable
305.36 *et seq.* *A* and *B*] E; A ~ B *All capital variables undifferentiated from italic text*
306.1 thing] E; theng
306.2 or] E; of
306.3 any] E; anwy
*306.4 means,] E; means, any process, any thing that brings anything aboot is a thi ~,
306.5 third] E; hird
306.7(1) only] E; inly
306.10 stop,] E; sto,
306.11 new] E; neu
306.11 higher] E; higer
306.12 show] E; shon
306.18 new] E; nen *Also* 306.20
306.23 violated] E; voilated
306.23(1) the] E; tihe
306.23 would] E; woold
306.30 regarded] E; ~ as
306.32 Now] E; Non
306.33 ever] E; ewer
306.35 any] E; anoy
306.36 univalent] E; uniyalent
306.38 capable] E; capabe
306.38 which] E; wrich
306.39 any] E; aoy
307.1 secondness] E; secondnes
307.4 unite] E; uuite
307.10 obvious] E; obgious
307.10 of] E; Of
307.17 with] E; nith
307.18 rather] E; rathee
307.18 finality.] E; ~..
307.19(1) in] E; iin
307.22 calling] E; callinm
307.22 second] E; iecond
307.24 Latin language] E; Latio lauguage
307.28(2) began] E; begao *Also* 307.29
307.28 mouse,] E; ~‸
307.29 sensuously] E; sensuoosly
307.30 now] E; non
307.31 grades,] E; ~‸
307.33 secondness;] E; ~.;
307.33 degenerate] E; degeneratene
307.34 (to] E; Zto
307.34 there is] E; thre ii
307.39 in] E; iu
308.2 Now] E; Nsw
308.4 second] E; secon
308.4 one] E; ine
308.6 identity,] E; ideutity, consist
308.8 upon] E; opon
308.9(1) man] E; main
308.9 himself] E; himielf
308.12 indistinguishable] E; indiitingoishable

Alterations

Except in 305.7 all alterations are by hand.

303.6 are] *intl ab del* is certainly
303.11 practical] *transp-c from aft* life *and* l *bef del* ly
303.13 strong] *intl ab del* some
303.14 In regard . . . opinion] *intl ab del* I say nothing in disparagement of these facts. The philosophy of two comes i within my scheme. Perhaps somebody will find out sometime what seven and other numbers may mean. Meantime
303.14–15 only . . . develope] *intl ab del* want to show how much ther ii in
303.23 time?] *bef del* These questions puzzles we shall try to guess at answers for.
303.24–25 Several religions . . . false.] *transp-c from aft* Hesperides *and* Several *intl ab del* Many *and* several *intl bel del* many
303.25 For] *intl ab del* But
◊303.25 fiction] F *ov* f
303.25 even] *intl ab del* is far
◊303.26 than truth] *ins bef del* ♦given to [*intl ab del* remarkable for its *and bef intl* is inclined]
303.26 triads] *bef* than ♦is [*intl-c and del*] truth [*del*]
303.26 The] *intl ab del* There are
303.28 etc. point . . . fact.] *intl-c*
303.31 no] *intl-c*
303.32 nor] *intl-c*
303.32 analysis] *intl ab del* thinking, for example
303.32 much] *intl ab del* hardly an
303.32 the] *intl ab del* their
303.33 of them] *intl*
303.34 is . . . draw] *intl ab del* would be a power thrown away here. These *and* is *bef del* of no use for his purpose
303.34 ideas] *bef del* are

303.36 useless] *intl ab del* of no use
303.36 express] *intl ab del* convey
303.39 however] *intl w line*
303.39–40 which shall point] *intl ab del* but pointing
304.1 First,] *aft del* ¶The ideas of
304.1 beginning] *aft del* ideas of
304.8–9 and so] *intl-c ab del* thus
304.9 imply] y *bef del* ing
304.11 initiative] *intl-c*
304.17 Let] *intl ab del* The instant that
304.17 be] *intl ab del* is
304.17 already] *intl-c*
304.30 whatever] *aft del* that
304.38 Compulsion] *intl ab del* Force
305.7 acknowledge] *intl ab del* conceive
*305.22–23 conception] *intl ab del* idea
305.27 Yet] *intl ab del* But
305.27 notion] *intl ab del* idea

51. *Device for Abbreviating Time Reductions, 1886*

Copy-text is P 334, the publication in the 1885 *Coast Survey Report.* In his annual report to F. M. Thorn dated 14 August 1885, Peirce noted that "the principal achievement of the year was the invention and execution of a new method of observing the pendulum, by which the observations can receive their final reductions in the field . . . I also presented a paper on the computation of time observations which I consider valuable." On 4 November 1885, he sent Thorn several papers and indicated that he was "busily occupied" with several others. (MS 560 consists of two pages of calculations of time reductions related to P 334.) Bound in with this letter (in the National Archives) is a note by Edward Goodfellow, in charge of editorial administration at the Coast Survey Office, stating: "Original paper of Mr. Peirce—*On the Reduction of Observations for Time*|With Mr. Schott's criticism given to him for review." Peirce wrote again to Thorn the following April (in a letter stamped as received 8 April 1886) and said: "A paper by me on Reduction of Time Observations was lately returned to me from your office to be revised for publication in the 1885 Report; and it was accompanied by a valuable memorandum of objections by Asst Schott. I herein reply to these objections and request this reply should be referred to Asst Schott for his opinion." Schott's criticism of Peirce's paper is extant among the Peirce manuscripts at Harvard, but the original paper returned to Peirce has not survived; nor does the revised version or another copy that may have served as printer's copy for P 334.

Emendations

309.7 field-reduction] E; $\sim_{\wedge}\sim$ *Also* 316.35
309.9 *et seq.* coëfficients] E; coefficients
310.17 $\cos^2 \delta$] E; $\cos 2\, \delta$ *Also* 310.18
311.25 *least-square*] E; $\sim_{\wedge}\sim$
312.12 *Clamp E*] E; *rom. Also* 313.1
312.14 *Clamp W*] E; *rom. Also* 313.1
312.30 H.] E; H
314.6 v^2] E; ν^2
314.17 sin] E; Sin
314.19 $0^s.99$] E; $^s.99$
314.21 *[formula]*.] E; $\sim_{\wedge}$
315.24 t] E; τ
315.35 H. Draconis] E; H Draco
316.29 a'] E; a^1 *Also* 316.34
316.32 *[formula]*.] E; $\sim_{\wedge}$
316.34 .001.] E; $.\sim_{\wedge}$

52. *On the Influence of a Noddy, 1886*

Copy-text is P 335, the publication in the 1885 *Coast Survey Report.* It is emended by MS 587, Peirce's annotated offprint [587]. Peirce's paper was enclosed with his 4 November 1885 letter to F. M. Thorn (see the headnote to item 51), and it probably served as printer's copy; but it has not survived.

Textual Note

318.17 $\frac{h'}{h}$] Peirce neglected to underline the variables in this handwritten insertion.

Emendations

317.9 simple] 587; single
317.12 *et seq.* centre] E; center
317.16 τ',] E; τ'
317.17 T,] E; T
317.21 *[formula]*.] E; ~‸ *Also* 318.1, 6, 8, 14
318.8 $\frac{\mu\mu'}{ll'}$] 587; ~)
318.16 $\frac{1}{100}$] E; ~,
*318.17 $\frac{h'}{h}=$] 587; *not present*

Line-End Hyphenation

317.5 pendulum-support

53. *On the Effect of Unequal Temperature, 1886*

Copy-text is P 336, the publication in Appendix 17 of the 1885 *Coast Survey Report.* It is emended by MS 588, Peirce's annotated offprint [588]. Peirce sent the fair copy of his paper (and also the item 52 paper) with his 4 November 1885 letter to F. M. Thorn; it probably served as printer's copy for P 336, but it does not survive. The editors have removed "Appendix No. 17" and "By C. S. Peirce, Assistant," which appear, respectively, above and below the title.

Textual Note

319.13–14 In the offprint, Peirce inscribed "(the mean temperature being given)" in the right margin, and "part" in the left margin, to be inserted after "temperature" and "this," respectively. However, he went on to cancel both insertions.

Emendations

319.14 *et seq.* centre] E; center *All occurrences except* 320.28
319.23 *[formula]*.] E; ~‸ *Also* 320.20, 26; 321.1, 7, 17(5)
320.12 coëfficient] E; coefficient
320.20 $\frac{\gamma_2}{hl}$] E; ~.
321.16 10500] 588; 105000

54. *Qualitative Logic, 1886*

Copy-text is MS 582, one hundred seventy-four discontinuously numbered pages on as many leaves of white paper, lined on both sides, measuring $7 \times 8\frac{7}{16}$ in. The first one hundred forty-three (except for the contents table) are heavy and stiff with a "Westlock" watermark and two perforated holes in the left margin for insertion in a loose-leaf binder. The table of contents and leaf 142 onward in the reordered copy-text formed from the manuscript (beginning at 361.1 in the present edition) are medium weight and have a "Southworth's 1882 Linen Bank" watermark and three holes in the left margin. Some of the leaves are chipped, slightly torn, water-stained, soiled, cracked, and have ink blots or smears, and one leaf has cellophane tape on the verso. Peirce carefully inscribed and moderately altered this manuscript in black ink. Some of the pages are numbered in ink by Peirce, others are numbered (not by Peirce) in lead and blue pencil. Red pencil arrows are inscribed in the left margin at 326.9, 328.16–17, and 329.20. MS 593, a typescript of the preface and first four chapters with autograph alterations not by Peirce, also survives. It is valuable in determining the sequence of the copy-text leaves, but otherwise unreliable because of its many transcription errors. Eighty-four additional rejected leaves also survive in MS 582.

Textual Notes

331.26 Peirce's rewriting of the first version of Chapter III ends here. The following incomplete paragraph is inscribed by Peirce in what seems to be a different pen on the same leaf: "Every addition or improvement to our knowledge, of whatsoever kind, comes from an exercize of our powers of perception. In necessary inference, my observation *[ ]*." The bottom third of the leaf is blank. Peirce starts over with a new version of this fragment on the next leaf and continues on with new material to 332.22.

333.2–4 An . . . reasoning.] In the margin to the left of this sentence, Peirce wrote "Postpone to Modus Tollens."

334.6 About two-thirds of the leaf is blank after this line.

335.5–6 takes . . . inference] Only this phrase is inscribed on the manuscript leaf, the rest is blank.

335.10 One] Peirce began this paragraph with a cursive capital *I*, not a roman numeral one.

338.29 About two-thirds of the leaf is blank after this line.

341.7 $\overline{S}\!\rceil\!\!+$] Peirce inscribed the streamers usually with a curve connecting the vertical and top horizontal lines. For typesetting reasons, they are represented as straight right angles.

343.25 About half of the leaf is blank after this line.

347.24–26 *Barbara* . . . syllogism.] Peirce inscribed this sentence at the bottom of the leaf after 347.27–28 and indicated with a direction line that it be inserted.

349.10–14 In . . . *A*.] Peirce inserted this passage at the bottom of the leaf after 349.18.

349.18 About two thirds of the leaf is blank after this line.

352.30 About half of the leaf is blank after this line.

359.14 greater] Peirce interlined "of a" with a caret but neglected to supply the necessary modifier "greater."

363.4 use.] The manuscript leaf has another sentence, canceled by Peirce, which is incomplete or originally continued on a now lost leaf. The first page of another version leading to the alternate set of rules has the following sentence at the end of the leaf: " $\sim$. To convert our logical symbol$\rceil\!\!+$ into an algebra, we require seven fundamental rules, which I proceed to set forth." Other substantive readings in this leaf, which begins at 362.32, are: "expresses" for "signifies" (362.34), "or rather" for "and, indeed" (362.35), "still" is not present (363.1), and "body" for "code" (363.2).

Emendations

323.3 I] E; I.
323.4 II] E; ∼.
323.7 IVbis] E; ∼.
323.9 VI] E; ∼.
323.10 VII] E; ∼.
323.12 Inference] E; ∼.?
323.13 Continuity] E; ∼.
324.19 are] E; *not present*
325.4 *Méthode*] E; *Methode*
325.12(3) of] E; *not present*
325.26 added] E; add
330.1 Chapter] E; *ital.*
330.2 The] E; *ital.*
330.3 *et seq.* *P*] E; *rom. All capital and lowercase variables rom. except* *C* 348.1; *s* 348.4, 18; *m* 348.20; 362.13, 21; 363.28, 29, 31; 364.5, 11; *k* 348.27; *p* 349.11; *a* 361.29, 31, 33, 36; 362.2, 3, 11, 21; 363.6(2), 7(1), 17; 366.2(1, 2); 368.15; 369.3, 8(1), 11, 14; 370.6, 22, 28(3); 371.22, 26(1); *b* 361.29; 362.12, 21; 363.6(2), 7(1); 366.9, 11, 12; 368.14, 15; 369.3, 8(1), 11; 370.6, 22; 371.6, 7, 22, 26(1); *c* 362.12, 21; 368.12, 13; 370.4, 29; *l* 362.12, 21; *n* 362.14, 21; 363.27(2), 29; 364.5, 11, 15; *d* 368.9; 370.4
330.10 In] E; At
330.22 hereafter).] E; ∼.)
331.6 balanced] E; bananced
331.19 or] E; *ital.*
332.9(1) the] E; *reinstate*
332.30 reader:] E; ∼‸
333.23 to] E; *not present*
334.6 it).] E; ∼.)
334.34 a] E; an
334.36(2) is] E; it
*335.10 One] E; I
336.2 example:] E; ∼‸
336.25 *animal*] E; *ani|mal*
338.8 *S*'s] E; Ss *Also* 353.1
338.8 *P*'s] E; Ps *Also* 353.2
338.26 following:] E; ∼‸ *Also* 339.19
338.30 In] E; *no* ¶∼
339.3 *M;*] E; M
339.11 *P*).] E; P)‸
339.15 *syllogism,*] E; ∼‸
340.8 *syllogism.*] E; ∼‸
341.9 chose,] E; ∼‸
341.24 follows,] E; ∼.
342.10 *identity*] E; ∼,
344.5 is] E; ∼,
346.13 non-existence] E; ∼‸∼
347.8(1) *P.*] E; P *Also* 348.36(2)
347.15 Architecture] E; Architech-ture
347.32(1) *M.*] E; M
347.33 *Baroko.*] E; ∼‸
348.24 *P;*] E; P
348.29 *M,*] E; M
349.9 some, *Felapton . . . Ferison,*] E; ∼‸ Felapton . . . ∼‸
349.19 *Baralipton*] E; *Baralpton*
349.20 *Dimaris*] E; *rom.*
349.25 *Fesapo.*] E; *rom.*
349.25 *Fresison*] E; *rom.*
349.36 Baroko: Darapti.] E; ∼. ∼:
351.9 ἔξω] E; ἐξω
351.9 *fallaciae*] E; *rom.*
351.30 gnaws] E; knaws *Also* 351.31
352.14 achievement] E; achievment
352.40 *P*'s,] E; P's,"
353.16 not-*P*] E; ∼‸P *Also* 353.18, 358.10
353.17(2) not-*S*] E; ∼‸S *Also* 358.10
353.20 I] E; which I
353.29 *inessential*] E; *inessen*-|tial
353.34 *copartients*] E; *rom.*
354.6 *coinherent*).] E; ∼.)
354.30 syllogisms:] E; ∼‸
354.32(2) *Z*'s] E; Zs *Also* 354.33(2)
354.32(2) *Y*'s] E; Ys
354.33(2) *X*'s] E; Xs
354.34 *Barbara*] E; *rom.*
354.36 *Darii*] E; *rom.*
354.39 *Darapti*] E; *rom.*
355.11 some] E; a ∼
355n.1 Keynes,] E; *Keynes:*
355n.1 *Formal Logic*] E; *rom.*
355n.1 Chap. VIII,] E; ∼‸ ∼.
356.15–16 earth," . . . "neither] E; ∼,‸ . . . ‸∼
356.35 *B*'s . . . *A*'s] E; Bs . . . As
356.40 and] E; and ∼
358.12 "If . . . true."] E; ‸if . . . ∼.‸
358.25 ever] E; never
*359.14 greater] E; *not present*
359.19 Chapter] E; *ital.*
359.30 species, is,-] E; ∼, — ∼‸
360.31 consequents).] E; ∼.)
360.37–38 Arnauld and Nicole] E; Arnault ∼ Niccole
362.13 Bible] E; bible
362.14 men's] E; mens'
362.22 *b*] E; a
363.9 to] E; *not present*
363.27 1] E; I

363.30 rule] E; *not present*
364.16 odd] E; add
364.33 2] E; II
366.7 *a*] E; $\overline{a\,+}$
367.17 is] E; *not present*
369.7 I.] E; *I*
370.5 *c*] E; c,
370.17 *Rule*] E; *rom. Also* 370.24, 371.9
371.14 abbreviations] E; abbrieviations *Also* 371.24 (abbrieviation)
371.28 *[formula].*] E; ∼‸
371.32 *dilemma.*] E; ∼‸

Line-End Hyphenation

326.19 hack-drivers

Alterations

323.10 Inferences] *intl ab del* Syllogisms
323.15 to guide] *intl ab del* for
323.17 while] *intl ab del* though
323.26(2) of] *intl ab del* with
324.2 distinct] *intl ab del* correct
324.12 so much the better; I] *intl ab del* I shall be better pleased, and
324.13–14 his investigation.] *intl ab del* such an examination
324.19 only] *intl ab del* no
324.21 i.e.] *intl ab del* or
324.22 The] *aft del* Men never reasoned rightly in those days;
324.23 regulated] *aft del* barbarism
324.30 when] *aft del* before very much it had done much except
324.32 fairly] *intl-c*
324.33 early] *intl-c*
324.37 foundation] *aft del* means of
324.38 the Epicureans] *aft del* such
325.1 in the 16th Century] *intl-c*
325.3 imported] *intl ab del* added
325.9 dignity] *aft del* ob
325.14 contained] *aft del* have invo
325.15 lessons] *aft del* highly important
325.16 and contributed even more to the progress of civilization] *intl-c*
325.19 conclusions] *intl ab del* inferences
325.20 carried] *aft del* involved
325.23 few] *intl ab del* small
325.27 Coördinates] *aft del* invention of geometrical
325.27 of] *intl ab del* by
325.28 great] *intl ab del* the
325.29 forming] *intl ab del* and having
325.30 not to say the principal] *intl-c*
325.33–34 It . . . outweighs] It . . . logic *intl ab del* which ♦far [*bef intl-c del* it] outweighs in ♦importance [*bel intl del* by far] ♦the whole art of [*bel intl* for it decidedly outweighs]
325.36 The] *aft del* In *and* T *ov* t
326.3 trust] *intl ab del* hope
326.5 existing] *intl ab del* which has existed
326.5 since] *intl ab del* from
326.5 birth] *intl ab del* dawn
326.5 is] *aft del* may
326.5 process] *intl ab del* way
326.22 mental] *intl-c*
326.28 a special kind of] *intl-c*
326.29 latter] *intl ab del* former
326.34 exercized] *intl ab del* had
327.7 practice] *aft del* habit
327.10–11 of all] *intl ab del* kind of
327.14 as good an] *intl ab del* the best
327.19 capacity for] *intl ab del* power of
327.19 Primitive] *aft del* But *and* P *ov* p
327.19–20 however] *intl-c*
327.25–26 principally] *intl ab del* mos
327.39 involuntary] *aft del* unconscious
328.2 nevertheless] *intl-c*
328.8 good] *intl ab del* strong
328.11 yet] *intl ab del* though [*intl ab del* but]
328.16 the] *intl ab del* a
328.32 that there is] *intl ab del* of
329.9 carry] *aft del* lead
329.10 that] at *ov* us
329.33 as a Cartesian] *intl-c*
330.10 case] *intl ab del* rate
330.23 hypothetical] *aft del* first pr
331.2 now] *intl-c*
331.5 physically] *intl-c*
331.10 facts] *intl ab del* events
331.29(1) my] *intl ab del* our *and aft del* what we
331.38 be] *aft del* consist

331.39 by] *bef del* all the points of
331.39 an] n *ins*
331.39 imaginary] *intl-c*
332.9 then] *bef del* the line enclosing
332.10 is imagined to lie] *intl ab del* lies
332.24 apprehend] *intl ab del* have
333.10 in] *bef del* every [*bel intl del* each [*bel intl del* any]]
333.10 each] *aft del* ¶1 st. If ◆the [*bef del* pr] conclusion is necessarily true, whatever the premises may be;
333.12 It follows that] *intl ab del* Accordingly,
333.12 logically] *intl-c*
333.15 Take] *aft del* He may think that *and* e *ov* ing
333.20 thought] *intl ab del* said
333.21(2) is] *aft del* of it
333.22 allow] *intl ab del* make
333.27 consequent] *intl ab del* antecedent
333.28 antecedent] *aft del* consequent
333.31 For] *intl-c*
333.31 to] t *ov* T
333.34 making it the] *intl ab del* saying that it is a
333.34–35 of a hypothetical proposition] *intl ab del* to any antecedent
334.4 (If *P* then *X*)] *intl-c*
334.8 ¶To say] *aft del* could not reject the proposition. It is true that such a state of knowledge is itself impossible, in a certain sense; but it is more or less a very useful conception.
334.11(1) I] *intl ab del* we
334.12 vast number] *intl ab del* universe
334.20 possible] *intl-c*
334.21 It] *aft del* It is also a valid inference, because this really is so
334.25 we were] *intl ab del* nothing
334.27 retained] *aft del* had
334.28(2) a] *aft del* also *and bef del* true
334.28 something] *intl ab del* what is
334.29 a] *aft del* further
334.30 principle] *intl ab del* proposition
334.34 instance,] *bef del* the
334.34 command to be] *intl ab del* order is
334n.1 is] *bef del* one w
334n.1 in] *bef del* though
335.1(1) so] *ov* this
335.2 mode of] *intl ab del* reasoning
335.3 is] *intl ab del* was
335.4 form] *aft del* mode of
335.21 This] is *ov* e
335.25 of] o *ov* b
335.25–26 This is essentially the same as the relation of antecedent to consequent.] *intl-c*
335.26 it] *ins aft del* this
335.28 A] *aft del* The
335.28 judgment] *intl ab del* belief
335.28 is] s *ov* n
335.29 following] *aft del* subseque
335.30 arises from] *intl ab del* depends on
335.31 to say that *C* is] *intl ab del* of being
336.6 general] *intl-c*
336.9–10 is contained among] *intl ab del* lies within
336.20 syllogism] *intl ab del* proposition
336.23 general] *intl-c*
336.25 *salamander*] *intl ab del* griffin
336.27 In] *aft del* In logic, it is to be understood that when propositions
336.28 categorical] *intl-c*
336.32 premises] *aft del* premise or
337.1 follows] *intl ab del* may be inferred
337.7 every] *aft del* an [*bel intl del* any]
337.30 for] *aft del* of the thing
338.8 are] *aft del* lie
338.15–16 separately] *intl ab del* distinct
339.4 logically] *intl ab del* necessa
339.4 If S then P.] If *aft del* "*and* P. *bef del* "
339.7 would] *intl-c*
339.7 follow] w *bef del s*
339.10 would follow] would *intl-c bef* follow [w *bef del s*] *Also* 339.12, 13, 17, 18, 26, 28; 340.6, 7
339.12 from the premise] *intl ab del* ◆we can [*intl ab del* it follows from]
339.12–13 we can conclude] *intl-c ab del* to
339.29 In] *intl ab del* As
340.16 ordinary] *intl-c Also* 340.20
340.21 ordinary man] ordinary *intl-c ab* man [*intl ab del* prophet]
340.21 fish] *intl ab del* whale
340.24 ordinary] *intl-c*
340.24 fish] *intl ab del* whale

340.26 prophet is an ordinary] *intl ab del* man swallowed
341.9 putting] *intl-c*
341.14 distinguish] *intl ab del* indiss
341.17 but] *bef del* as a moral consequence
341.18–19 or could fail to know what would kill him] *intl ab del* or could fail to know that physical consequence
341.21 denote] d *ov* b
341.23 a] *aft del* the mental hypothes suppositions
341.26 while] *intl ab del* and
341.28 It is] *aft del* ¶We have seen that | $\overline{\mathrm{A} \dashv}\, (\overline{\mathrm{B} \dashv}\, \mathrm{C})$ | and $\overline{\mathrm{B} \dashv}\, (\overline{\mathrm{A} \dashv}\, \mathrm{C})$ | ◆mean [*aft del* are true and fa] the same thing; that is, they are by logicall necessity either both true or both false. ◆It is [*bef del* therefore] unnecessary
341.33 a] *intl ab del* the
341.33 consequent] t *ov* ce
341.33 from] *aft del* of
342.2 whole] *intl-c*
342.8 spoken] *intl-c*
342.8 without] *bef del* at
342.14 from] *aft del* $\overline{\mathrm{A} \dashv}\, \overline{\mathrm{B} \dashv}\, \mathrm{C} = \overline{\mathrm{B} \dashv}\, \overline{\mathrm{C} \dashv}\, \mathrm{A}$;
342.15 *B.*] *bef del* that is, we can always transpose antecedents.
342.25 *ponens.*] . *ov* , *and bef del* as a valid inference.
343.24 syllogism.] *bef del* and *and* . *ov* ,
343.26 ¶It] *aft del* ¶Still, more complicated forms could easily be given, were we not hampered by the imperfection of ordinary language, which does not permit them to be clearly set forth. But such complications would, after all, introduce no new principles of inference.
343.28 indeed indispensible] *intl-c*
343.29 or habit] *intl-c*
343.30 by] *aft del* both
343.32 find] *aft del* be
344.2 applying] *aft del* putting rules to any applicat
344.5 The common] *intl ab del* As the usual
344.5 that] *intl-c ab del* ,
344.17 thought or] *intl-c*
344.28 thus] *intl-c*
344.29 discovered] *bef del* in this by syllogism
344.30 from] *intl ab del* of
344.30 cannot] *bef del* even in the simplest cases,
344.33 and] *bef del* the progress
344.37 intellect] *intl ab del* understanding
345.5 of] *bef del* ph
345.13 that] *bef del* renders
345.15 their] *bef del* stude
345.25 who] *bef del* use
345.31 involve] *intl ab del* introduce
346.14–15 in the case of] *intl ab del* with
346.15 Affirmative] *intl ab del* Negative
346.17 his] *intl ab del* the
346.19 is in his own mind] *intl ab del* he means
346.23 that] th *ov* or
346.30 one] *intl ab del* one *and intl bel del* the major premise
346.34 one] *intl ab del* each
347.15 and other chaster forms of Gothic] *intl-c*
347.18 three] *intl-c*
347.19 each of] *intl-c and* each [e *ov* a]
347.23 are] *aft del* have
*347.24–26 *Barbara* . . . syllogism.] *add*
347.24 my] *aft del* the
347.24 a case of] *intl ab* the
347.38 particular] *intl-c*
348.5 inference called] *intl-c*
349.9 from *all* to] *intl ab del* of subalter
*349.10–14 In . . . Some *B* is *A.*] *add*
349.17 a] *intl ab del* the
349.30 *Fapesmo*] *intl ab del* Dabitis
349.33 list] *intl ab del* names
349.33 moods] *bef del* are all summed up in the
350.8 thing] *intl ab del* object
350.8 would] w *ov* c
350.17 poet] *intl ab del* man
350.18 William] *intl ab del* Swan of Aron
350.18 poet] *intl ab del* man
350.20–21 with . . . affirmatives] *intl-c*
350.24 true] *bef del* with
350.27 will] *bef del* be a dilem
350.27 the] *intl ab del* a
350.31 false] *intl ab del* true
350.33 true] *intl ab del* false
350.35 a] *bef del* kin
351.4 been satisfied with] *intl ab del* handed down

351.8 ἐλέγχοι παρὰ τὴν λέξιν,] *intl-c*
◊351.9 οἱ ἔξω τῆς λέξεως,] *intl-c*
351.27 when] *bef del* it is assumed to be the same thing
351.29 these.] *intl ab del* the following
351.36(1) friar] *intl ab del* monk
352.3 ¶3. *Ignoratio*] *aft del* ¶3. Fallacia consequentis or non sequitur
352.25 deduced] *intl ab del* obtai
352.37–38 more ably] *intl ab del* better
352.39 so that] *intl ab del* and
353.2 some *P*'s.] *intl ab del* all the
353.18 Something . . . *P.*] *intl ab del* Not everything is either S or P.
353.19 to] *intl ab del* upon
353.19 which] *bef del* I ha
353.30 he calls each] *intl ab del* each class is
353.34 *copartients;*] ; *ov* .
353.34 each] *aft del* He calls
354.15–16 are properly] *intl ab del* may be
354.28 any] y *bef del* th
354.35 can] *intl-c*
354.35 substitute] e *bef del* s
355.11 some] *intl ab del* good many
355.34 imported] *intl ab del* introduced
356.5 disparagement] *intl ab del* disparaging remarks
356.5 so enlightened a] *intl-c ab del* a modern
356.28 the] *aft bel* but
356.37 magnitude] *intl ab del* quantity
357.16 Now] *bef del* a
357.16 syllogisms] *last* s *ins and bef del* may
358.11 infer] *bef del* If not P, then not not S
358.17 *P*] *bel del* ¯
358.25 No] *intl ab del* The
358.25 for] *intl ab del* of
358.25 been] *bef del* very accurately settled
358.36 drove] *intl ab del* displaced
358.36 the] *bef del* schola
359.4 has] *intl ab del* is
359.11 seems] *bef del* superficially
359.11 denied;] ; *ov* . *and bef del* We shall examine this a little further on; ◆and [*aft del* meantime] shall find that notwithstanding this appearance,
359.32 are] *bef del* usually
359.37 from] *bef del* the
360.1 of] *bef del* the
360.3 as we regard the matter,] *intl ab del* in our point of view
360.10 an embryologist,] *intl-c*
360.25 are] *bef del* involve
360.29 a] *bef del* class-
360.29 denotes] s *ov* d
360.29 *names*] s *ov* d
360.29 the] *bef del* indiv
360.30 of] *intl ab del* to
361.13 In the case of] *intl ab del* For
361.14 our] *intl ab del* the
361.17 body] *intl ab del* system
361.17 being] *intl ab del* may be
361.17 expressed,] *bef del* and [*bef intl-c del* then]
361.18 may be] *intl-c*
361.20 body] *intl ab del* system
361.22–23 besides . . . examined.] *intl-c*
362.3 antecedent] *intl ab del* conclusion
362.3 consequent] *intl ab del* premise
362.15(2) would] *intl ab del* does
*363.4 use.] *bef del* The rules for our algebra will be ◆sufficient [*bef intl-c del* to prove ◆◆anything [*bel intl del* every] (I do not say satisfactory)], if and only if, they enable us to prove the two propositions, that
363.8–9 enable us] *intl-c*
363.13 with] *bef del* all
363.24 laws] *intl ab del* rules
364.1 rule holds] *intl ab del* proposition
364.32 that] *bef del* corre
366n.1 our] *aft del* this
366n.4 non-] *intl-c*
367.17 and] *bef del* two
368.1 true,] *bef del* and
368.30 For] *aft del* For put C = O. Then $\overline{\mathrm{C}\,+}\,\mathrm{L}$ becomes true
368.30 replaced] *intl ab del* put
369.1 *C* is] *intl ab del* all three are
369.3 Any] *aft del* Given
369.14 *a*] *bef del* fact, x
369.17 *b*] *intl ab del* another fact y
369.19 *c*] *intl ab del* z
369.19 oval] *intl ab del* area
369.20 *d*] *intl ab del* w
369.21 $\overline{b\,+}\,c$] *intl ab del* $\overline{\mathrm{y}\,+}\,\mathrm{z}$
369.21(2) *b*] *intl ab del* y
369.21(2) *c*] *intl ab del* z
369.22 $\overline{a\,+}\,\overline{b\,+}\,c$] *intl ab del* $\overline{\mathrm{x}\,+}\,\overline{\mathrm{y}\,+}\,\mathrm{z}$

369.23 $\overline{b \dashv}\, c$] *intl ab del* $\overline{y \dashv}\, z$
369.23 *a*] *intl ab del* x
370.1 *a*] *ov* x *Also* 370.3
370.1 *b*] *ov* y *Also* 370.3
370.1 *c*] *ov* z *Also* 370.3, 4, 5
370.1 *d*] *ov* w *Also* 370.3, 4, 5
370.4 substituting] *bef del* a for x, b for y, and
370.15 *c*] *ov* b
370.21 of *a* and *b*.] *intl ab del* representing
370.21–22 Consequently] Con- | *intl ab del* the *and* sequently *aft del* area
370.25(1,2) *c*] *ov* b
370.26 proof of] *intl-c*
370.27 combination,] *bef del* with its proof shows that $\overline{a \dashv \overline{a \dashv b \dashv} \dashv}$
370.28 expresses] *intl ab del* rep
370.28 and] *bef del* b
371.2 *b*] *ov* a
371.2 *c*] *ov* b
371.3 serves] *intl ab del* proves
371.4 fact] *bef del* negated
371.5 It] *intl ab del* This
371.6 *b*] *ov* a *Also* 371.7, 8
371.25 take the following forms] *intl ab del* become

55. *The Logic of Relatives, 1886*

Copy-text is MS 584, fifteen pages inscribed in black ink on fifteen leaves of slightly heavy, stiff, lined white paper measuring $7\frac{15}{16} \times 9\frac{7}{8}$ in., with a "Massasoit Company" watermark. Several leaves are soiled, cracked, or torn. Although the "l" in "qualitative" (372.2) is crossed, the manuscript is on the whole carefully inscribed. The editors have added a colon after the main title because the subtitle appears on a separate line, and have removed a dividing line that follows, as well as "By C. S. Peirce." In the upper right corner of the first leaf, "1885" is inscribed in pencil by one of the editors of the *Collected Papers*. Seventeen pre-copy-text leaves also survive.

Textual Notes

372.6 In revising this sentence, Peirce neglected to delete the last word in the canceled phrase "make use of."
377.23 Peirce inscribed a peculiar squiggle that might have been a comma after "$\overline{W_3 \dashv}$"; the editors have deleted it.

Emendations

372.6 employ] E; ~ of
372.7 Vol.] E; ~‸
372.7–8 *American Journal of Mathematics*] E; *rom.*
372.13 reasoning] E; reasonining
372.13 *et seq.* *A*] E; *rom. All capital and lowercase variables rom. except n* 374.14(2), 15, 32, 37, 39; 375.15, 19(2); 376.22(1); 377.16, 17
372.18 non-existent] E; nonexistent
373.16(3) is,] E; ~‸ *Also* 373.30(2), 32(1), 33(2)
374.26 constituent] E; constuent
375.9 true,] E; ~‸
376.6 $\overline{D \dashv}$] E; D
377.2 *W*'s . . . *T*'s] E; Ws . . . Ts
377.16 etc.] E; ~‸
377.21 $\overline{W_3 \dashv}$] E; $\overline{W \dashv}$,
*377.23 $\overline{W_3 \dashv}$] E; $\overline{W \dashv}$,
378.3 Next] E; *no* ¶ ~
378.8 *Z*] E; Z.
378.11 even] E; ever
378.14 odd] E; add

Alterations

*372.6 employ] *intl ab del* make use
372.7 notation] *bef del* I [*ov* u]
372.23 possible] *intl-c*
372.24 may] *bef del* not
373.22 that is] *intl ab del* or
373.25 yield] *intl ab del* form

373.26 the] *bef del* men [*bel intl del* analytical merit]
373.31 true] *intl ab del* false
373.31(2) false] *intl ab del* true

374.1 true.] *bef del* $\overline{\overline{A+}\,\overline{B+}\,\overline{C+}\,C+}$ F will mean that either $\overline{A+}\,\overline{B+}\,\overline{C+}\,C$ is false or F true, that is, that $\overline{A+}\,\overline{B+}\,\overline{C+}\,C$ is false, that is, that A
374.2(1) false] fa *ov* tr
374.9 lines] *aft del* long
374.21 every] *intl ab del* all
374.32 or lower] *intl-c*
374.33 then] *intl ab del* second
374.37 $(n + 1)^{\text{th}}$] *intl-c and* + *ov*$^{\text{th}}$
375.9 if] *intl-c*
375.9 true] *intl-c*
375.11 Now] *bef del* all
375.27 every] *intl ab del* any
376.12 add] *bef del* or take an a
376.12 under] *intl ab del* covered by
376.13 expression,] *bef del* which cannot be rendered in
376.14 even] *intl ab del* single
376.15 thus] *intl ab del* so
376.15 far] *bef intl del* ♦met w [*ab del* occurre]
376.16 erased] *bef del* in an expression
376.17 a similar] *intl ab del* the same
376.25 part] *intl ab del* component [*intl ab del* part of]
376.33 *[formula]*] *aft del* $\overline{A_1 +}$ A
377.1(1) erasures] *intl ab del* omissions
377.1 results] *bef del* are omiss
377.2 not] *intl ab del* but
377.9(2) true] *intl ab del* false
377.9 false] *intl ab del* true
377.14 let] *intl ab del* suppose
377.19 and let *Y* be] *intl ab del* and that it
377.26 only] *intl-c*
378.11 direct] *intl-c*
378.14 can] *bef del* add as
378.16 as a] *bef intl-c del* direct

56. *Elementary Account of the Logic of Relatives, 1886*

Copy-text is MS 585, an incomplete black-carbon-copy italic typescript of four pages on four leaves of medium light-weight, brittle white paper measuring $9\frac{7}{16} \times 14\frac{15}{16}$ in. The first and last leaves are singed, and all four are stained and turning brown. Several corners are folded down, and the folding in half of the leaves has caused tears in all but leaf two. In the lower left corner of the last leaf, there are holes made by a pin. Lowercase and capital variables are not underlined or otherwise differentiated from the remaining italic text and, consequently, are emended. The typing is almost illegible in a number of places (386.38–39 in particular) because of the smudging of the carbon copy, the soiled paper, and the deterioration of the leaves. The alignment of the left margin and the spacing of punctuation are irregular, and some characters are mistakenly typewritten above rather than on the line. The incomplete "elementary exposition of the" is typed above the title, and "By C. S. Peirce" is typewritten on the same line as the title. On the verso of leaf four is a variant of 385.25–27, which differs only in punctuation and format. Peirce made alterations in black ink at 379.5 and 386.36–37, and there are several carbon-copy alterations, both autograph and typewritten. Two italic typewritten pre-copy-text leaves and their identical carbon copies also survive.

Textual Notes

379.10 A line of dots occurs in the space on the last line of the paragraph, a line of hyphens and random characters is interlined, and there are hyphens below the latter part of the interlineation.
380.17 not] The (right-margin) limitation of the typewriter forced Peirce (or his

typist) to retype the whole word at the beginning of the next line. Similar incomplete words appear at 380.27; 382.28; 383.13; 385.28; 386.2; 386.20; 387.14, 17.

380.37 a . . . resembles] Peirce's typewritten interlineation of "a picture" is above undeleted "they"; the final "s" in "resembles" was added afterward, so that there is no space between it and the next word.

381.2 important.] The following characters, irregularly spaced, occur in the space on the last line of the paragraph: "g^g o p q^q o $^{p\,g}$ g^g."

381.16 open] The paraphrased quotation, which actually begins with "of his alembics . . . ," breaks off in the middle of the line. It continues, "with one's eyes open, by manipulating real things instead of words and fancies." See "The Fixation of Belief" in W3:244.

382.35 T] The space for the "T" is blank in the carbon copy, and the "T's" are very faint at 382.37.

383.1 $] Peirce used the dollar symbol to denote infinity because his typewriter lacked the latter symbol.

386.17 Four "8's" occur above "danger," nine above "coward" and the "n" in "nor" (386.18), three above the last three letters in "ruining" (386.18), two above "do" (386.23), and three above "not" (386.24[1,2]).

386.17 chancellor] Due to the (right-margin) limitation of Peirce's typewriter, the final "r" is not present. A similar instance ("d" in "would") occurs at 386.32(2).

Emendations

379.12 relation] E; relations
379.18 says] E; say
379.25 far] E; for
379.29 pronoun] E; pronouns
*380.17 not] E; no| ∼
380.19 its] E; t ∼
380.22 diagrams,] E; ∼.
380.24 icon: q^q.] E; ∼.∼‸
380.24 *et seq.* q^q] E; *rom. All capital and lowercase variables undifferentiated from italic text*
380.27 regard] E; regar| ∼
380.37 a picture] E; they ∼ ∼
381.9–10 construction] E; construc|-tion
381.13 When,] E; Whe ,
*381.16 open] E; o
381.31 whatever] E; whate er
382.15 rightly] E; rightl
382.28 and] E; a| ∼
*382.35 T] E; *not present*
382.38 and] E; ∼ =
383.13 either] E; eithe| ∼
383.21 3^{rd},] E; 3rd.
383.28 4^{th},] E; 4th.
384.25 protoplasmic] E; protop asmic
384.36 $\overline{m}$] E; m
385.1 $\overline{a}$] E; $\bar{\mathrm{o}}$
385.5 protoplasmic] E; protoplas|mic
385.7 Next] E; ∼ m$\bar{8}$
385.11 perish,] E; ∼‸
385.13 $a\bar{c} + (\bar{a}$] E; $\mathrm{o}\bar{\mathrm{c}} + (\bar{\mathrm{c}}$
385.20 the] E; th
385.28 board] E; boar| ∼
385.28(1) or] E; ∼ or
385.28 shareholder;] E; ∼.; *Also* 384.31 (share|holder)
385.34 one] E; on
386.2 once] E; onc| ∼
386.11 have] E; ha e
386.12 assistance] E; asssistance
386.14 reduce] E; reduc
*386.17 chancellor] E; chancello
386.20 campaign] E; campaig| ∼
386.20 I] E; l
386.28 the] E; t e
386.32(2) would] E; woul
386.34 alienate] E; aliena|ate
386.35 vassalage.] E; ∼,.
386.35–36 abbreviations] E; abbrieviations
386.38 $\overline{qy}$] E; $\bar{\mathrm{q}} + \bar{\mathrm{y}}$
387.2 premises] E; ∼.
387.11 $(t + zv)\ (\bar{v} + \bar{y} + w\bar{z})$] E; $(\mathrm{t} + \mathrm{zu})\ (\bar{\mathrm{v}} + \bar{\mathrm{y}} + \mathrm{x}\bar{\mathrm{z}})$
387.12 $(\bar{u} + y)$] E; $(\mathrm{v}^{-} + \mathrm{y})$ (
387.14 shall] E; s| ∼
387.17 stands] E; stan| ∼

Line-End Hyphenation

379.27 influencing-and-being-influenced-by-the-|external-world *Also* 380.14 (influencing-and-being-|influenced-by-the-external-world)

57. *Words for the Century Dictionary, 1886*

Copy-text is MS 586, an incomplete italic typescript consisting of twenty-nine pages (numbered 1 to 30, with page 3 missing) on twenty-nine leaves of medium-weight, laid, white paper measuring $7\frac{7}{16} \times 10\frac{7}{8}$ in., with "The American Linen Paper" watermark. The first leaf is a purple-ribbon typescript, three leaves are purple-carbon-copy typescripts, and the remainder are black carbon copies. Peirce's alterations are in blue-black and black-brown ink or are carbon impressed. Other alterations were ribbon-typewritten after the carbon copy had been prepared. The manuscript is bound together with a faded pinkish-purple ribbon, the left corners of the leaves are folded down (and the last leaf is torn), and there are stains, ink blots, and carbon and ink smears. The spacing of punctuation and of several words is irregular. Lowercase and capital variables are not underlined or otherwise differentiated from the italic text and, consequently, are emended. Peirce inscribed a line after all the listed elements except gold, thulium, decipium, columbium, and philippium. The *Century Dictionary* entries for some definitions and comments on pages 6, 12, and 23 (392.21, 26; 399.27, 400.1; 411.23, 24), are not in alphabetical order, but due to the rough state of the typescript the editors have left them as they appear there. Three pre-copy-text leaves and a purple carbon-copy page entitled "Genuine Imitation|Cologne Water," which is not related to the "Eau De Cologne" entry, also survive.

Textual Notes

388.7–19 In the right margin of the typescript, Peirce wrote by hand, vertically: "He had halved axes. I restore and make other corrections *[.]* But little remains of this."

390.11 ecliptic, *[. . .]*] Page 3 of the typescript is missing. Peirce's revisions for ECLIPTIC continue onto a partially visible (but unreadable) typed line along the bottom edge of page 2 (and presumably onto the missing page 3). Page 4 begins with this closing fragment of Peirce's (discussion of "efficient cause" within the) definition of EFFICIENT: "versal and particular. 8th, into proximate and remote. Medical men follow Galen in dividing causes of disease into predisposing, exciting and determining."

390.12 In the right margin, Peirce inscribed "Replaced in galley."

390.34–35 In the right margin, Peirce inscribed "Restored in galley"—as he did (with lowercase "restored") at 405.15–20.

393.6 Peirce interlined "In math." above "One," but neglected to lowercase the "O."

397.21 *too small*] In this italic typescript Peirce used the old European convention of spaced letters to indicate italics. His intention has been followed in the edition text for emphasis and for bibliographical references (408.2–5).

399.11 *x*′ . . . *z*′] Because primes were not available on the typewriter, they are represented by apostrophes in this typescript.

400.24 EMPIRICAL] Peirce neglected to delete an "E" typed in the double-space between this entry and the one preceding it.

401.28 2.] Peirce neglected to delete a "2." typed in the double-space above.

403.8 age;] The semicolon in this typescript was made by typing a comma in the same space as a colon. In this instance, the comma follows the colon.

403.22 Here and at 411.15, 414.28, and 418.1, the last line at the bottom of the page is retyped at the top of the next page; Peirce neglected to delete the last line in all four instances.

404.28 In the right margin, Peirce inscribed "Corrected in galley!!"; at 404.35–36, the same phrase appears with only one exclamation mark.

408.15–16 sensualistic] This emendation (and those at 413.2 and 414.9) is due to the fact that the typist decided to begin the word anew on the next line but neglected to delete the previously typed character or characters.

409.6 Strike] Typed with the three letters "ike" in one space at the end of the line, the word is retyped at the beginning of the next line; Peirce neglected to delete the first form.

417.33 whereby] Typed with the letters "her" in one space at the end of the line, the word is retyped at the beginning of the next line; Peirce neglected to delete the first form.

419.3 and] Typed with the letters "nd" in one space at the end of the line, the word is retyped at the beginning of the next line; Peirce neglected to delete the first form.

Emendations

388.12 acquired] E; aquired

388.12 metres] E; meters

388.13 191st] E; 191-st

388.14 45°] E; 45″

388.17 days, 6 hours, 9 minutes,] E; ~‸ 6 ~‸ 9 ~‸

388.20 EASTER. . . . read:] E; ~‸ . . . ~‸

389.17 angle] E; angla

389.24 modern] E; modrn

389.31 eccentric] E; ecc|centric

390.9 23° 27′] E; ~‸ ~‸

390.10 is] E; ia

390.28 Countess] E; countess

390.30 "Able] E; *no* ¶"~

390.31 "To] E; *no* ¶"~

390.31 Peps] E; Pepps

390.34 Any] E; any

391.3 *Works*] E; *rom.*

391.10 XVIIIth] E; XVIIIthr

391.17 Egotistical] E; Egotiatical

391.24 ELABORATIVE.] E; *not present*

391.27 elastic] E; elstic

392.1 resistance] E; resistence

392.3–5 ELASTICITY] E; ELASTICTY *All capital and lowercase forms*

392.11 Coëfficient] E; Coefficient

392.17 strain] E; train

392.35 Called.] E; ~‸

*393.6 one] E; One

393.26 Mendelejeff] E; Mendeljef

394.21 Philippium] E; Phillipium

394.27 10.] E; ~‸

395.3 etc.; fire,] E; ~, ~,

395.3 lightning] E; lightening

395.16 of] E; ~ fortune of

395.19 "There's] E; ""'~

395.22 points, lines,] E; ~‸ ~‸

395.23–24 *Elements*] E; "Elements"

395.24 orderly] E; ordetly

395.28 viz.] E; ~‸

395.35 principle] E; princzple

396.3 indemonstrable] E; indemonatrable

396.7 Blundeville] E; Blundvile *Also* 414.27 (Blundevile)

396.13 to] E; of ~

396.21 generally] E; genrally *Also* 416.16

396.28 order to that] E; ordet ~ rhat

396.29 involved] E; involvdd

396.30 conception] E; conceptuon

396.31 coëfficients] E; coefficients *Also* 401.18–19, 410.16

397.1 equations] E; e5uations

397.9 ELLIPSIS.] E; ~‸

397.16 H.] E; H

398.2 whose] E; whse

398.16 means] E; maens

398.22 that] E; th t

398.25 a] E; A

398.30 *et seq.* k^2] E; k² *All lowercase variables undifferentiated from italic text*

399.1 $\frac{1}{293}$] E; 1–293

399.3(1) the] E; ~the

400.12 Bramhall] E; Bramhal

400.20 2.] E; 2, +

400.20–21 exercise] E; excercise

*400.24 EMPIRICAL] E; E|~

400.25 2] E; II

400.30 first.] E; ~‸

401.3 other's] E; others

401.16 Objective] E; Objecti've

401.20 out] E; our
*401.28 2.] E; 2.|2.
401.28 opposed] E; opposes
401.32 power,] E; ~.,
402.18 signification] E; signifcation
402.19 distinguishing] E; distinuishing
402.21 Greek] E; Oreek
403.3 *[formula].*] E; ~‸
*403.8 age;] E; ~:,
403.21 and] E; ~ and
403.21 Latin] E; latin
*403.22 example] E; example "Innocence is ens" means that innocence is actually ~
403.30 non-existence] E; nonexistence
404.1–2 character).] E; ~.)
404.6 actuality).] E; ~.)
404.9 worked.] E; ~,
404.11 (kinesis).] E; ((~)‸
404.22 which] E; awhich
404.26 first] E; ~,
404.27 major (minor)] E; ~, (~,)
404.35 L. entitative] E; Lentitative
405.4 distinguished] E; ditinguished
405.10 gav'st] E; gavst
405.15 1] E; *no* ¶1
405.26 family] E; fimily
405.33 successively] E; sucessively *Also* 406.29
406.8 Simple] E; *no* ¶ ~
406.13 pure)] E; ~‸
406.21 respect] E; respecct
406.22 nature,] E; ~‸
406.30 disappear] E; dissappear *Also* 416.28
406.32 surface] E; ssurface
407.18 calendar.] E; ~,
407.22 (I] E; ‸I
408.2 chief] E; cfief
408.3 *temps*] E; t e m s
408.3–4 1766), . . . 1776),] E; ~)‸ . . . ~)‸
408.8 doubt.] E; ~‸
*408.15–16 sensualistic] E; s ~
408.17 those] E; whose
408.30 forms] E; foms
*409.6 Strike] E; Strike ~
409.26 superposed.] E; ~‸
409.29 (The] E; ‸~
409.31 Equal] E; *no* ¶~
410.15(2) equation] E; aquation
411.15 in] E; in passing between the brain and periphery, partly ~
412.5 weight] E; wright
412.8 text.)] E; ~.‸
412.8 7] E; 7.
412.20 earth's] E; earths
412.26 22^{nd}] E; 22d
413.2 says] E; ~ s
413.10 EQUIPOLLENT] E; EQUUPOLLENT
413.15 EQUISEGMENTAL] E; EQUUSEGMENTAL
413.28(2) Equivocal] E; Eqivocal
413.28(2) generation] E; genration
413.34 Equivocal] E; *no* ¶~
414.9 See] E; Se ~
414.14 Also,] E; ~—
414.18 EQUIVOCATION] E; EQUUVOCATION
414.27 substance, kind,] E; ~‸ ~‸
414.28 EQUULEUS] E; EQUULEUS. For 1 put: | ~
415.12 One] E; *no* ¶~
415.17 Watt's] E; Watts
415.20–21 Achernar] E; Archarnar
415.27 questions] E; quastions
415.29 star] E; *not present*
416.16 wrong-doing] E; ~‸~
416.29–30 circumstances] E; citcumstances
417.7 Pythagoras.] E; ~,
417.17 when] E; hwen
417.19 L. essentia] E; Lessentia
417.20 a] E; A
*417.33 whereby] E; wher ~
418.1 agree] E; agree to certain abstract ideas, to which we have annexed ~
418.13 it] E; its
418.19 or] E; ot
418.27 discourse.] E; ~,.
*419.3 and] E; and ~
419.18 head.] E; ~‸
419.23 genuine.] E; ~‸
419.23–24 polymorphism] E; plymorphism
419.30 angel's] E; angels
420.1 agreement] E; agreeement
420.7 of] E; osf

Line-End Hyphenation

400.14 hog-latin

Alterations

All alterations are by hand, except a few typed deletions *(del-t)*

389.6–7 constitutes] *bef del* the difference between an individual and a general essence
390.7 apparent] *intl ab del* relative
391.13 discovered] *bef del* by Dorelly
392.27 alone is] *inv*
393.34 (197)] *add*
393.38 88] *add*
394.5 114] *add*
394.11 90] *add*
394.12 (141)] *add*
394.14 (207)] *add*
394.14 234] *add*
394.19 (145)] *add*
394.20 183] *add*
394.28 239] *add*
394.38 199] *add*
394.38 195] *add*
395.30 Hence] *bef del-t* the
397.12–13 At its extremity erect] *intl ab del* To that draw
401.13 aimed at as] *intl ab del* referred
401.13 a means] *intl-c*
402.33 muzzle] *intl ab del* snout
403.27 characters] *bef del-t* depend
404.16 whenever] *intl ab del* because
410.3 which] *bef del-t* a planet was supposed
410.24 form] *intl ab del* sort
410.37 centre] *intl ab del* orbit
410.39 sun.] *bef del* In the case of the sun, it is is the difference between the mean and true anomaly.
411.31 A] *intl ab del* That
415.3 as a figure] *intl-c*
415.8 twice] *intl-c*
416.22 sum] *aft del-t* errors of the
416.29 taken] *aft del-t* the constant error
417.20 In Greek,] *intl-c*
418.12 Any] *intl ab del* A
418.34 difference] *aft del-t* definition
420.2 veracity] *intl-c*

58. *Peirce to Marquand, 1886*

Copy-text is L 269, a four-page letter now in the Marquand Papers in Princeton University Library. It is inscribed in blue-black ink on a folded sheet of unruled, heavy white paper with a watermark of "Hand Made Royal Irish Linen|Marcus Ward & Co" surmounted by a crown. Each page measures $8\frac{1}{4} \times 5\frac{1}{4}$ in., and the letter is folded for mailing. There is some smudging, and there are very few alterations. Published with permission of Princeton University Library.

Textual Note

422.12–13 The postscript is inscribed vertically in the left margin of the first page of the letter.

Emendation

421.23 *et seq.* *abcdef*] E; *rom. All lowercase variables rom.*

Alteration

421.7 it] *bef del* highly

Line-End Hyphenation in the Edition Text

The following lists those compound words hyphenated at the ends of lines in the critical text of the present edition that, in being quoted or transcribed from the text, must retain their hyphens. All other possible compounds hyphenated at the ends of lines should be transcribed as single words.

20.31	one-fourth
52.14	Fontenay-le-Comte
114.1	non-relative
158.17	machine-shops
168.13	class-names
192.21	association-formula
207.29	non-ultrainfinite
230.11	*arrière-pensée*
255.39	non-euclidean
280.30	philosophy-as-it-is
302.13	bread-winning
309.7	least-square
354.38	major-particular

Index