

Table 1: Nonlinear Algorithm

**Algorithm 1** Nonlinear Algorithm for RWOC**Input:** $\mathbf{y} : n \times 1, X_1 : n \times p, X_2 : n \times q,$ learning rate  $lr$ , number of iterations  $k$ , number of IPOT iterations  $l$ ,number of IPOT inner iterations  $m$ , a parameter in IPOT  $\beta$ initialize:  $w_1^{(1)}, w_2^{(1)}$  $\mathbf{b} \leftarrow \frac{1}{n} \mathbf{1}_n$  $S^{(1)} \leftarrow \mathbf{1} \mathbf{1}^T$ **for**  $t = 1$  **to**  $k$  **do** $Y_1 \leftarrow \mathbf{y} - X_1 w_1^{(t)}$  $Y_2 \leftarrow X_2 w_2^{(t)}$  $C_{ij} \leftarrow (Y_1[i] - Y_2[j])^2$  $G_{ij} \leftarrow e^{-\frac{C_{ij}}{\beta}}$ **for**  $h = 1$  **to**  $l$  **do** $Q \leftarrow G \odot S^{(h)}$ **for**  $f = 1$  **to**  $m$  **do** $\mathbf{a} \leftarrow \frac{\mathbf{1}_n}{Q\mathbf{b}}, \mathbf{b} \leftarrow \frac{\mathbf{1}_n}{Q^T\mathbf{a}}$ **end for** $S^{(h+1)} \leftarrow \text{diag}(\mathbf{a})Q\text{diag}(\mathbf{b})$ **end for** $Loss \leftarrow \|Y_1 - SY_2\|_2^2$  $w_1^{(t+1)} \leftarrow w_1^{(t)} - lr \cdot \left. \frac{\partial Loss}{\partial w_1} \right|_{w_1=w_1^{(t)}}$  $w_2^{(t+1)} \leftarrow w_2^{(t)} - lr \cdot \left. \frac{\partial Loss}{\partial w_2} \right|_{w_2=w_2^{(t)}}$ **end for**