Chapter 9: Morphological operations

形态学图像处理

Content Agenda

9.1: Preliminaries

9.2: Dilation and erosion

9.3 Opening and closing

9.4: The Hit-or-Miss transformation

9.5: Some basic morphological algorithms

9.6: Morphological operations on gray images

作业：**9.7,** 选作**9.19**

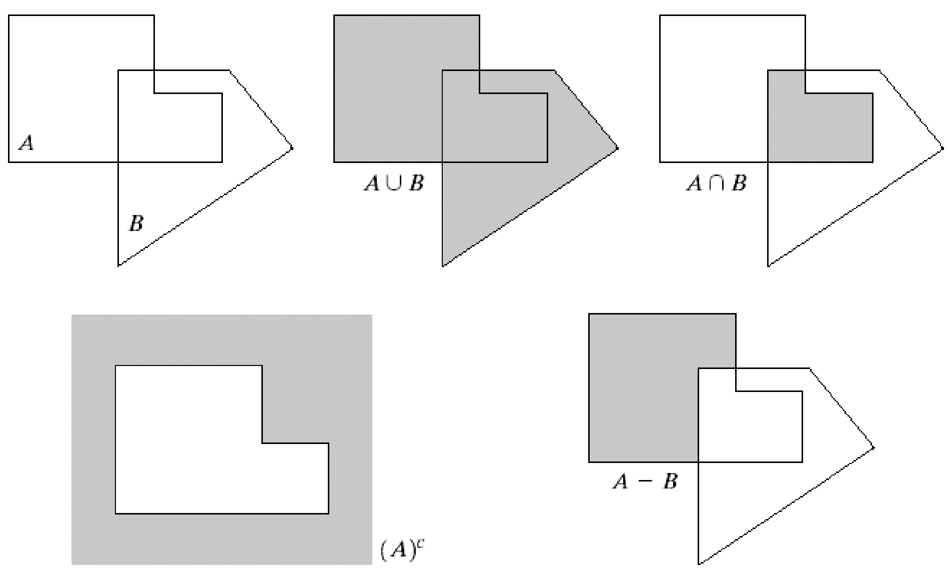
Image Processing —— Image Understanding ——Computer Vision

- **Low-level Processing: both input and output are images**
  Noise Reduction; Image Enhancement; Image Sharpening
- **Mid-level Processing: input images, output attributes of those images**
  Image Segmentation
  Image Indexing (Feature Extraction)
- **High-level Processing: related to computer vision**
  Image Analysis and Understanding

The materials in this chapter is a transition from a focus on purely image processing methods, whose input and output are both images, to processes in which the input are images, but the outputs are attributes extracted from those images, in the sense defined before. Tools such as morphology and related concept are a cornerstone of the mathematical foundation that is utilized for extracting “meaning” from an image. Other approaches are developed and applied in the remaining chapters of the book.

9.1: Preliminaries

- Set theory
  1. Union $A \bigcup B = \left\{ p \middle| \; or \;\; p \in B \right\}$
  2. Intersection $A \bigcap B = \{ p \mid p \in A \; and \;\; p \in B \}$
  3. Complement $A^c = \{ p \mid p \notin A \} = \Omega - A$
  4. Difference $A - B = \{ p \mid p \in A, p \notin B \} = A - (A \bigcap B)$

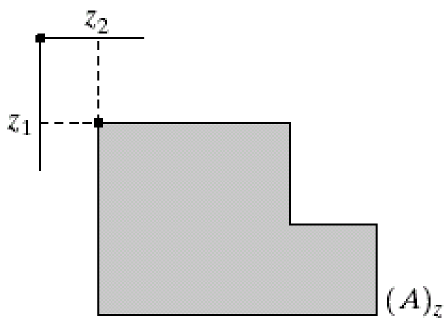


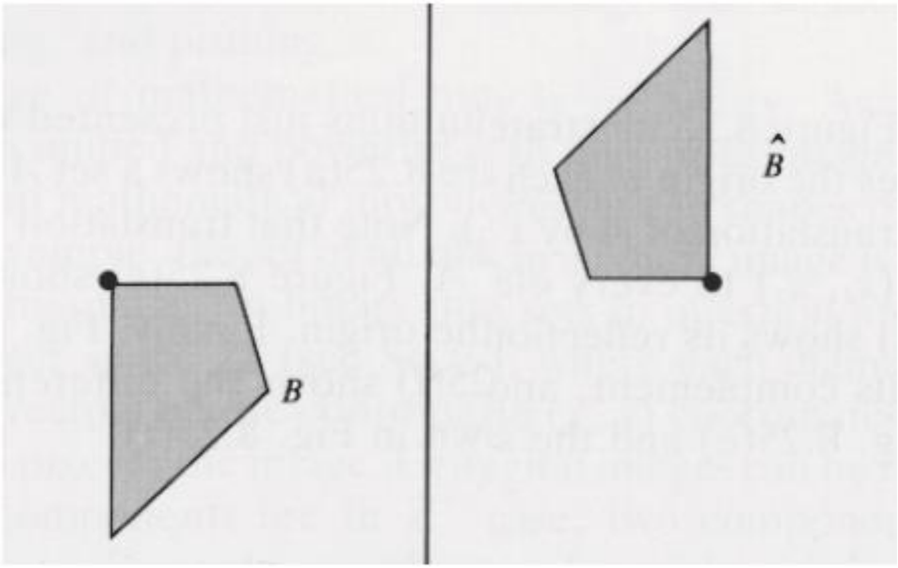
a b c
d e

FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

5. Special set operations for morphology

- Translation $(A)_z = \{p \mid p = a + z, a \in A\}$
- Reflection $\hat{B} = \{w \mid w = -b, b \in B\}$





Logic Operations Involving Binary Image

1. NOT
2. AND
3. OR
4. XOR
5. NOT_AND

Foreground=1,
background=0

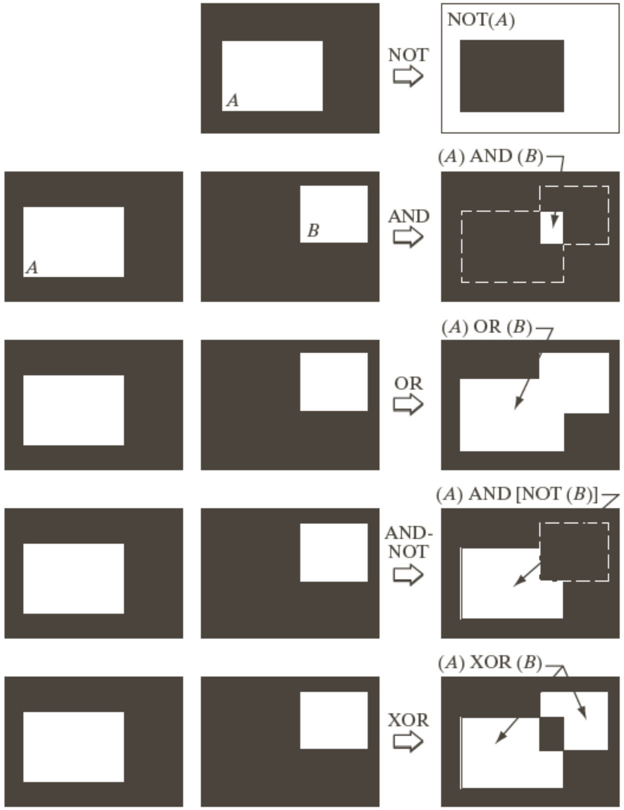


FIGURE 2.33 Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.

TABLE 9.1 The three basic logical operations.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

9.2 Dilation and Erosion

Terminologies and explanations: structure elements have to be rectangular arrays.

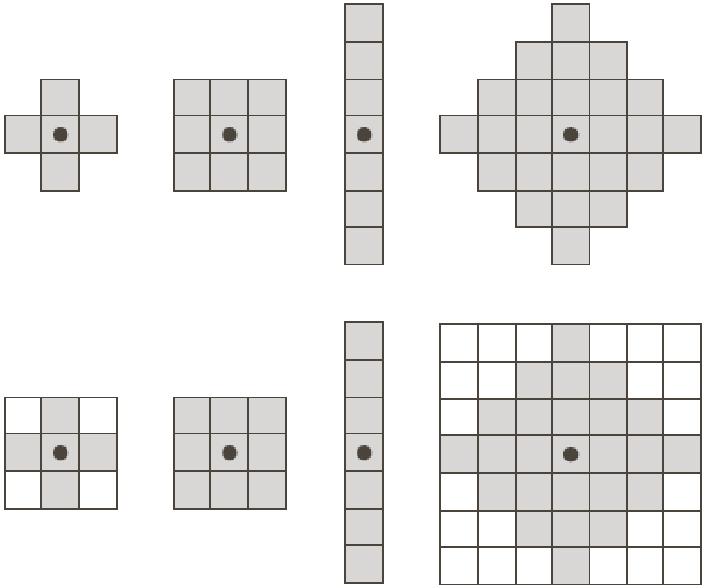
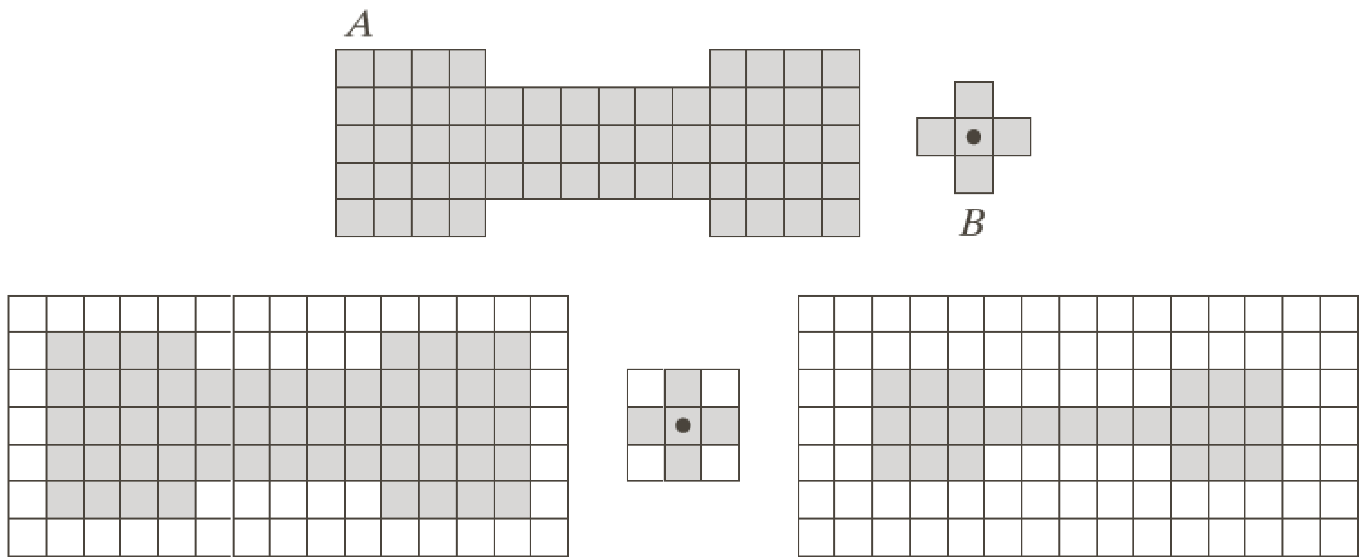


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

9.2 Dilation and Erosion

Terminologies and explanations: structure elements have to be rectangular arrays.



a b
c d e

FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

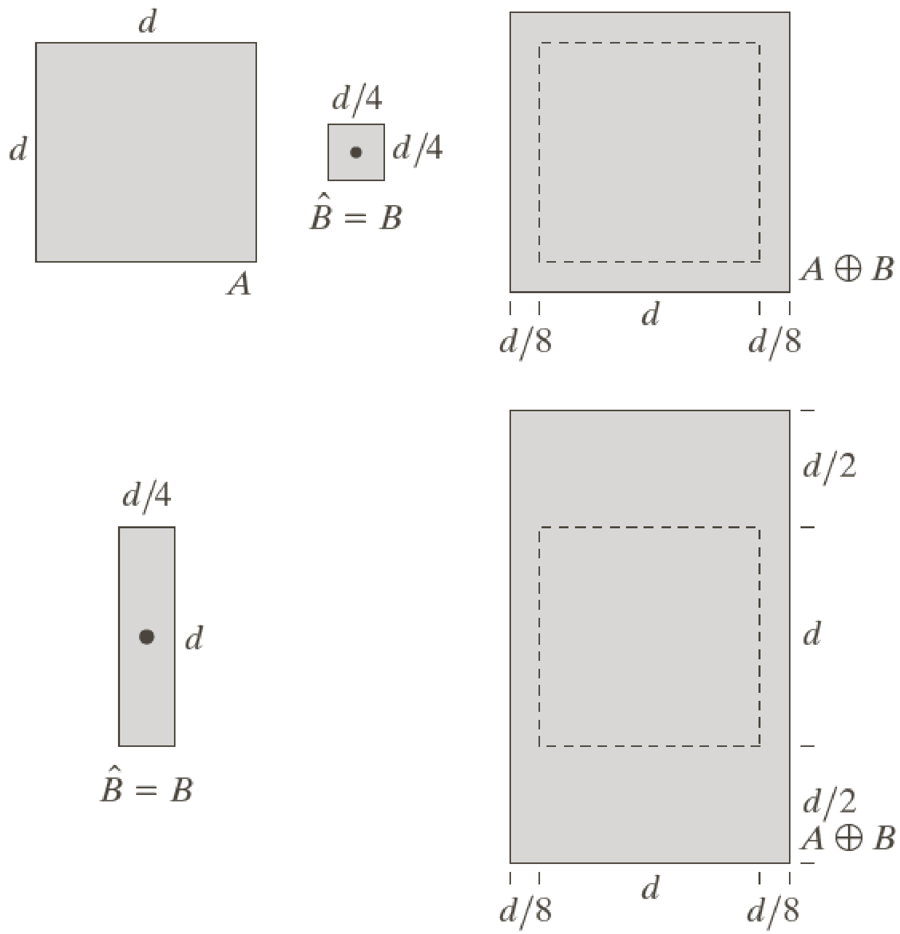
9.2 Dilation and Erosion

- Dilation of A by B: $A \oplus B$
  - 直观定义：给定图像*A*，任选一个结构元素（ structuring element ）*B*，用 $\hat{B}$ (或 *B*)的**中心点**贴着*A*的Boundary滑动，从而把*A*膨胀（或扩展）
  - With A and B as sets in Z^2, the *dilation* of A by B, is defined as

$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \varnothing \}$$

  - Equivalent definition:

$$A \oplus B = \left\{ z \mid [(\hat{B})_z \cap A] \subseteq A \right\}$$



a	b	c
d		e

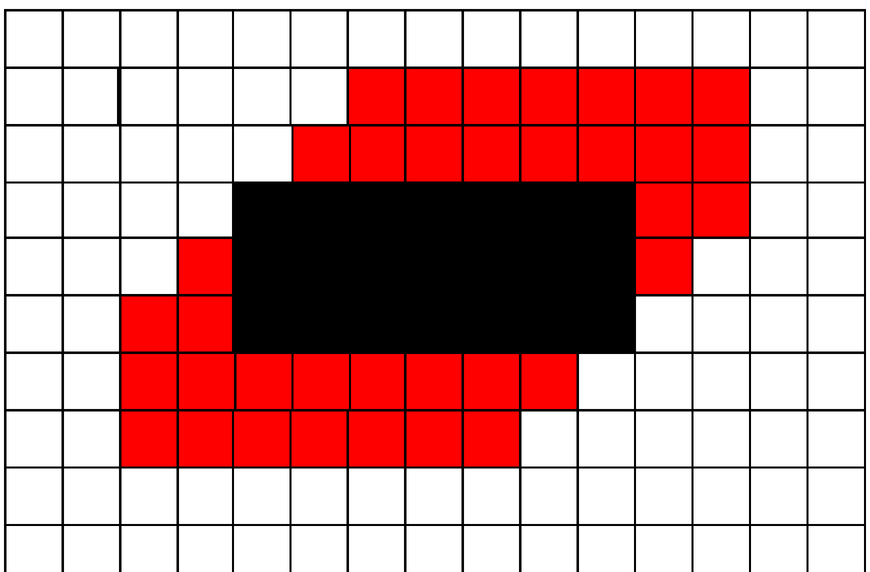
FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

Some examples of dilation

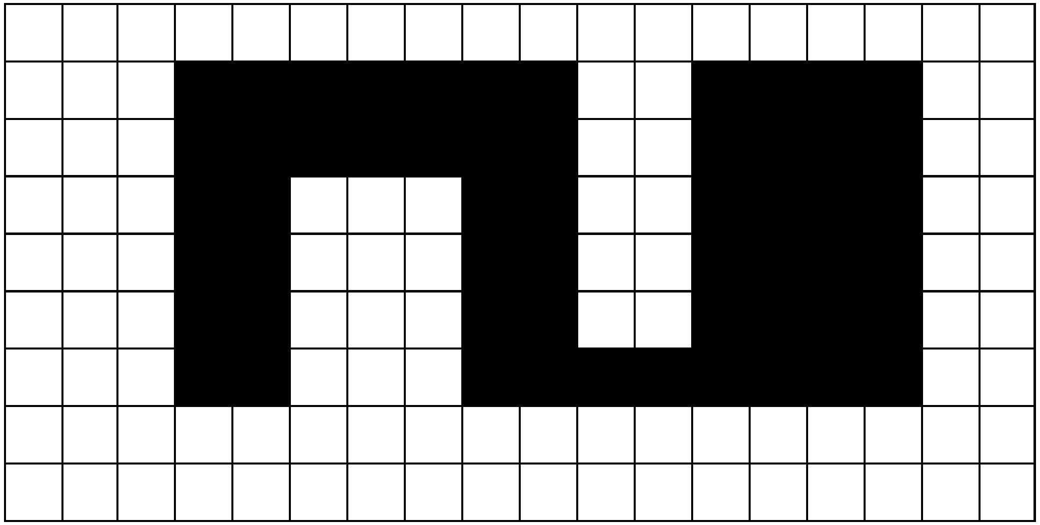
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

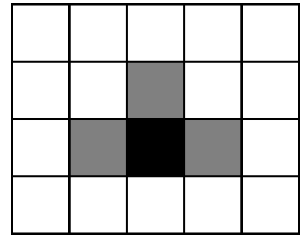
$$\begin{array}{ccccc} & & & & 1 \\ & & & 1 & \\ & & \boxed{1} & & \\ & 1 & & & \\ 1 & & & & \end{array}$$

左边A，右边B。 □内的1是B的中心元素。

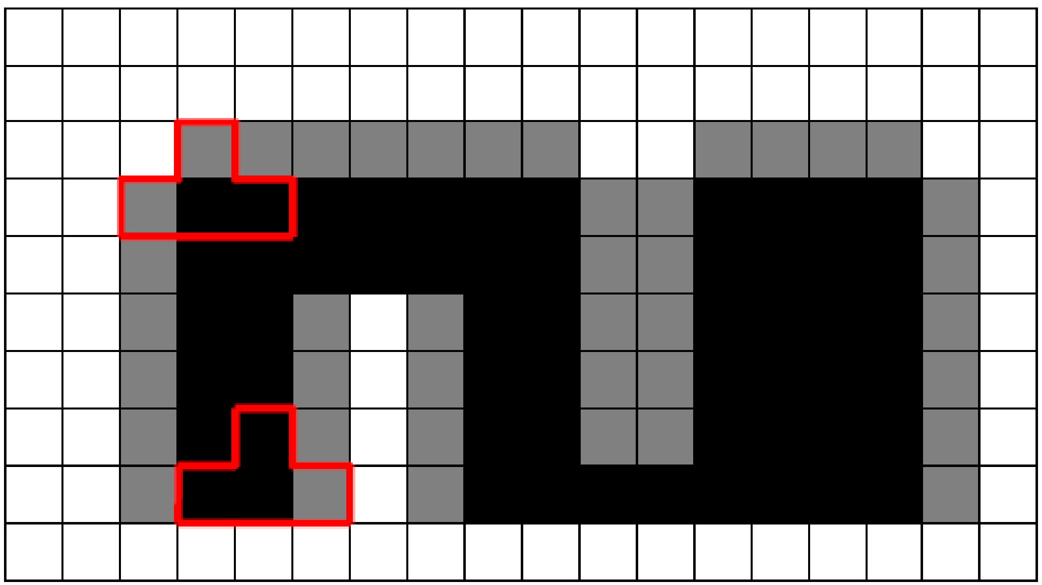


黑色是A，红色是用B膨胀出去的部分。



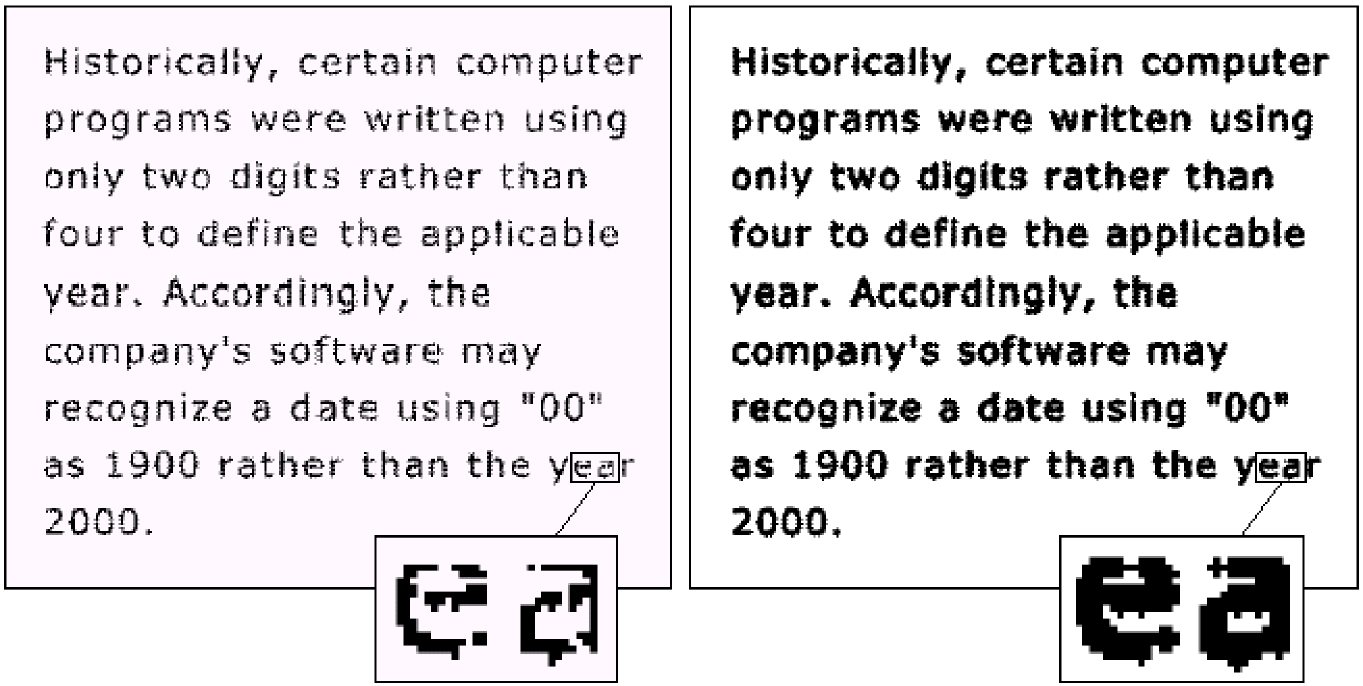


$\hat{B}$



$A \oplus B$

Bridging gaps



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Erosion A⊖B

For sets A and B in Z^2, the *erosion* of A by B, is defined as

$$A \ominus B = \{ z \mid (B)_z \subseteq A \}$$

任何一个点z而言，把结构元素B的中心放在这一点，如果B的所有元素都包含在A内，则该点z属于A⊖B

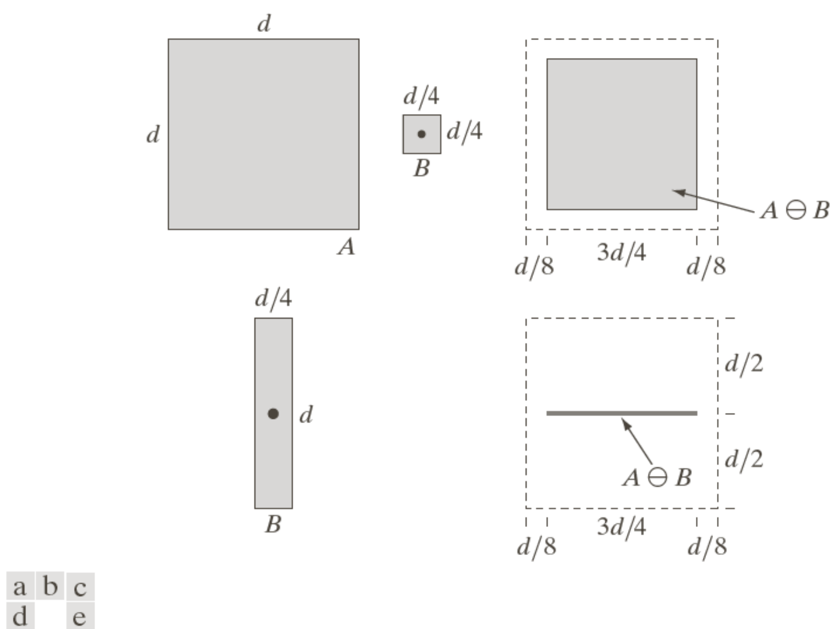


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

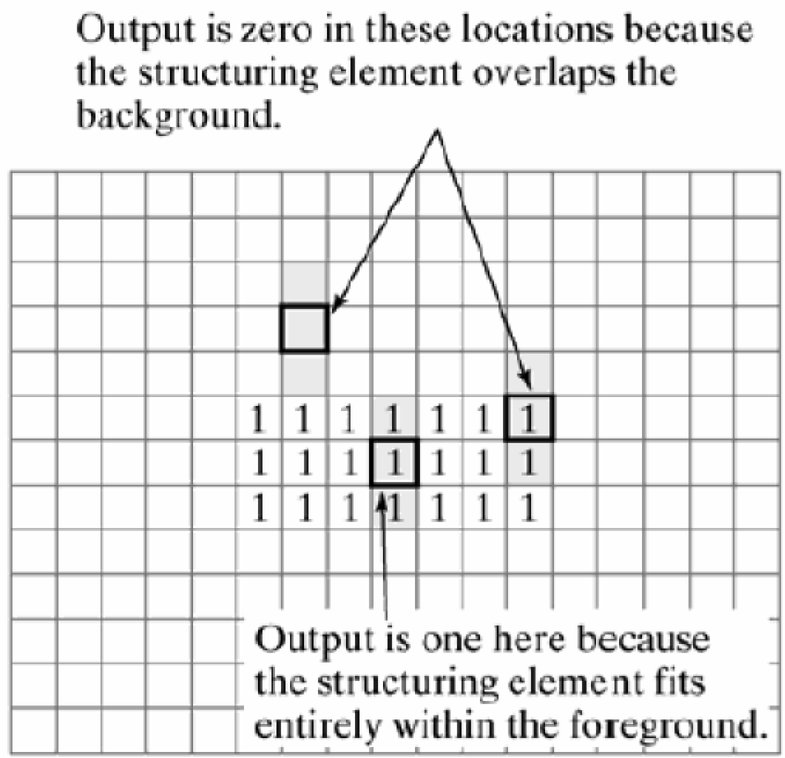
Question: if A=B?

1
$\boxed{1}$
1

B

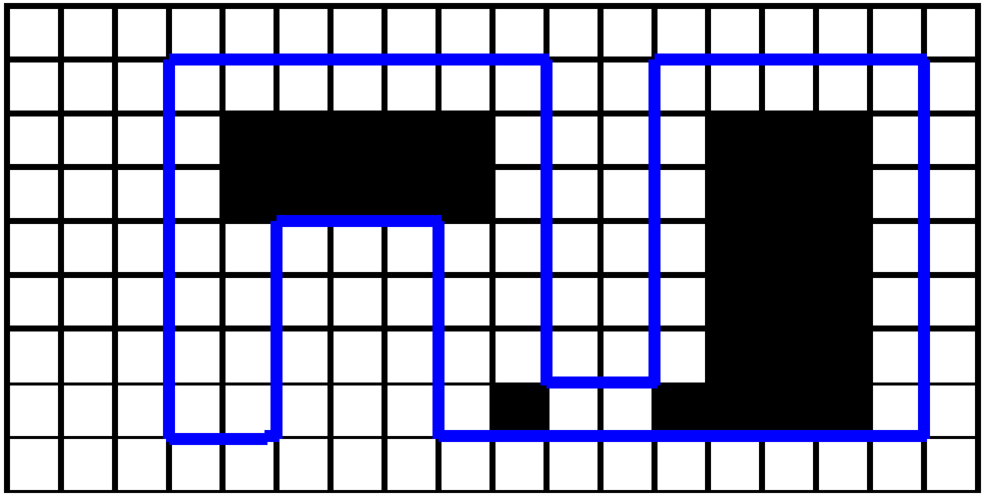
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

A

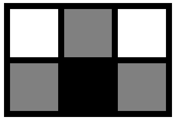


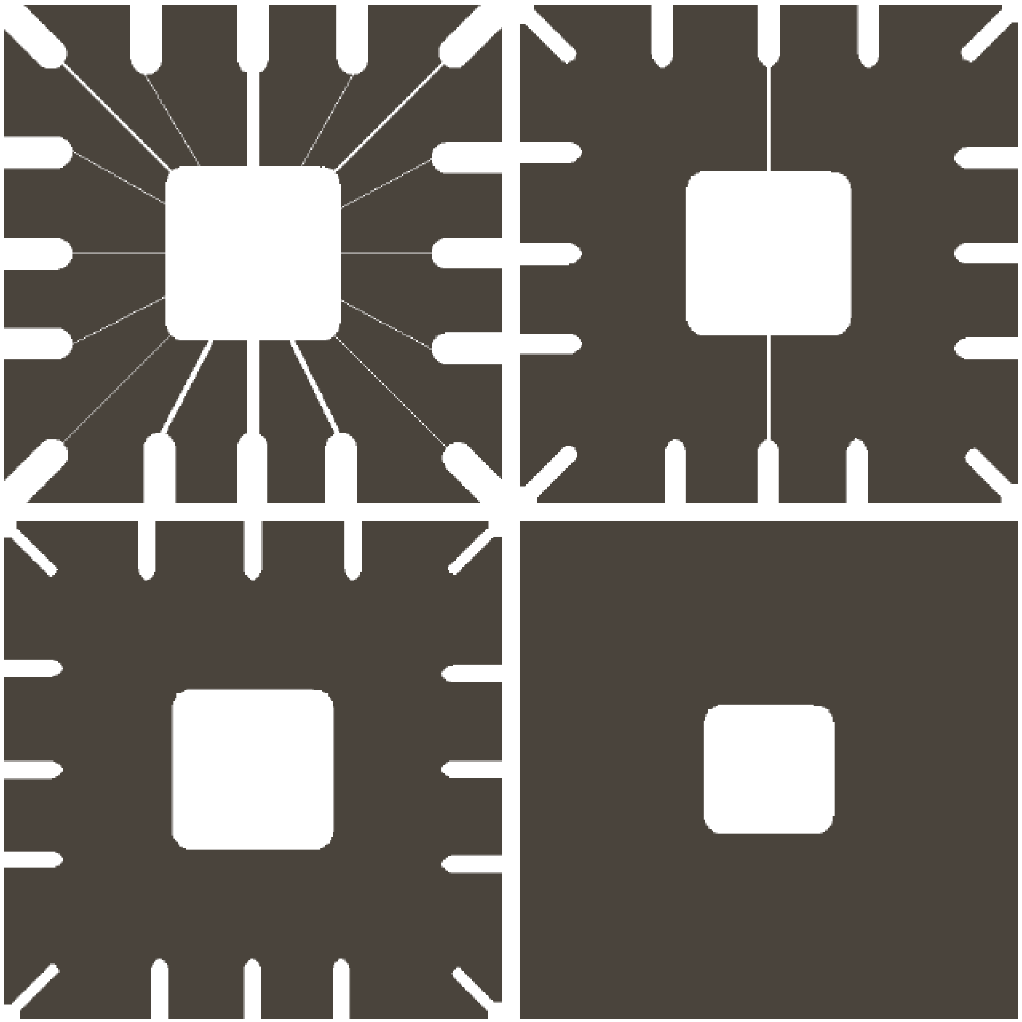
$A \ominus B$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0



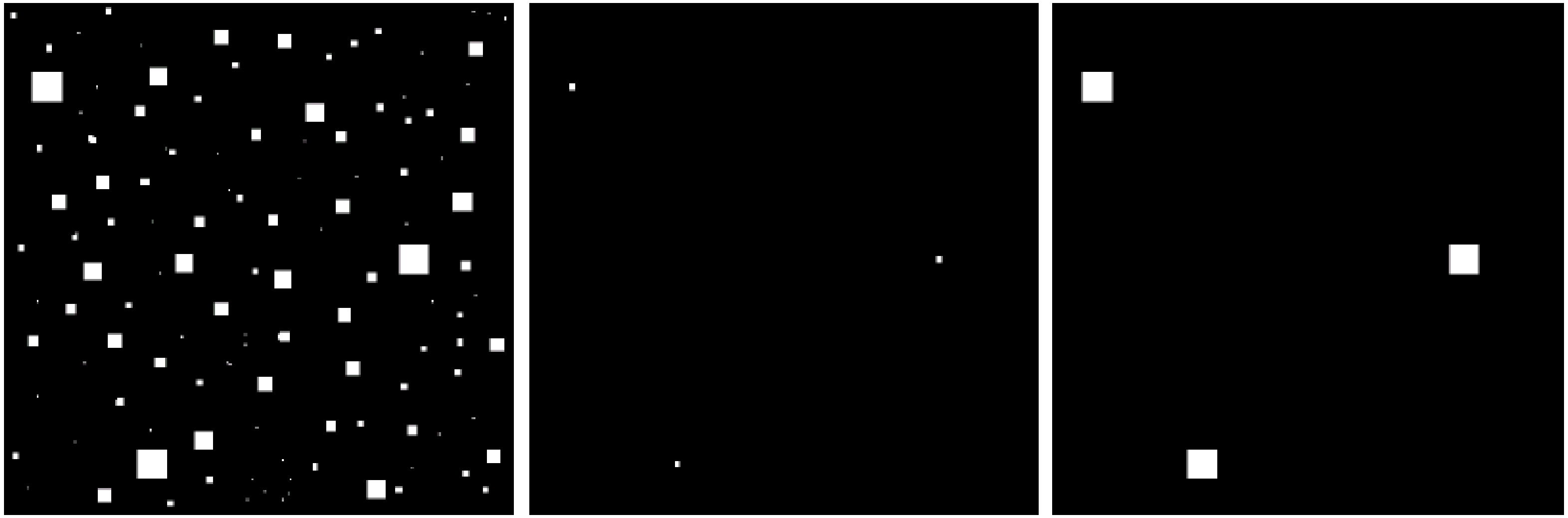
左圖中藍框內爲 A，B 如下圖所示





a b
c d

FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45, respectively. The elements of the SEs were all 1s.



a b c

Figure 9.7

(a) Binary image with squares of size 1, 3, 5, 7, 9 and 15

(b) The result of eroding image (a) with a structuring element of size 13X13.

(c) The result of dilating the 3 spot in image (b) by a structuring element of size 15X15.

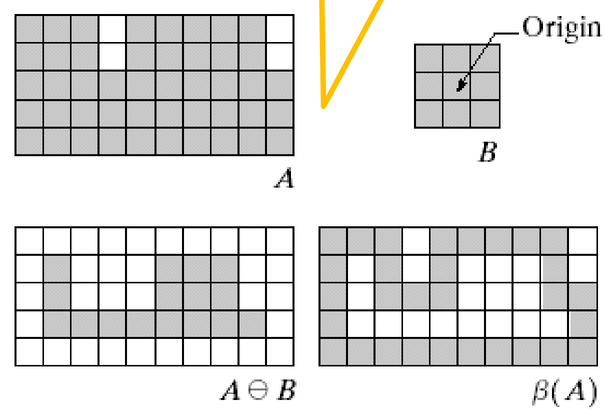
提取物体A的边界：

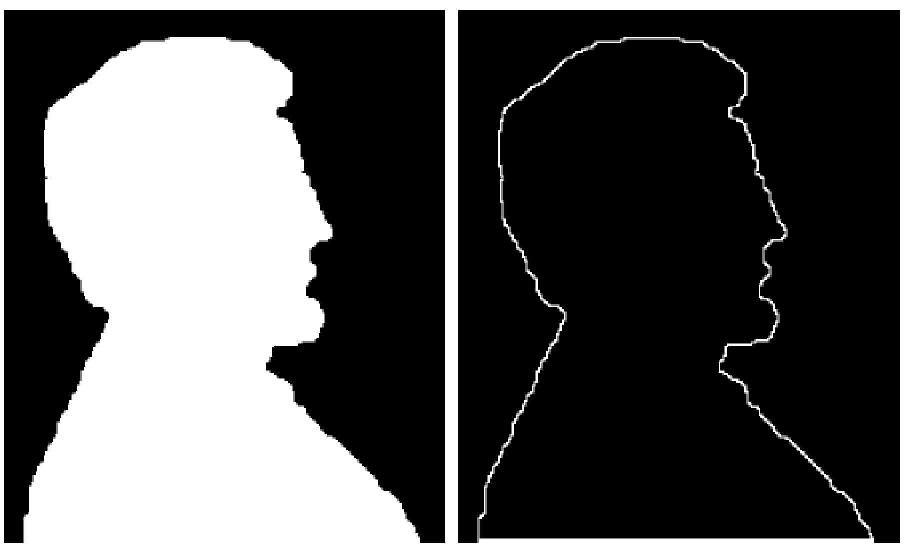
$A - A \ominus B$ (9.5-1)

白色表示0，灰（黑）色表示1

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.





a b

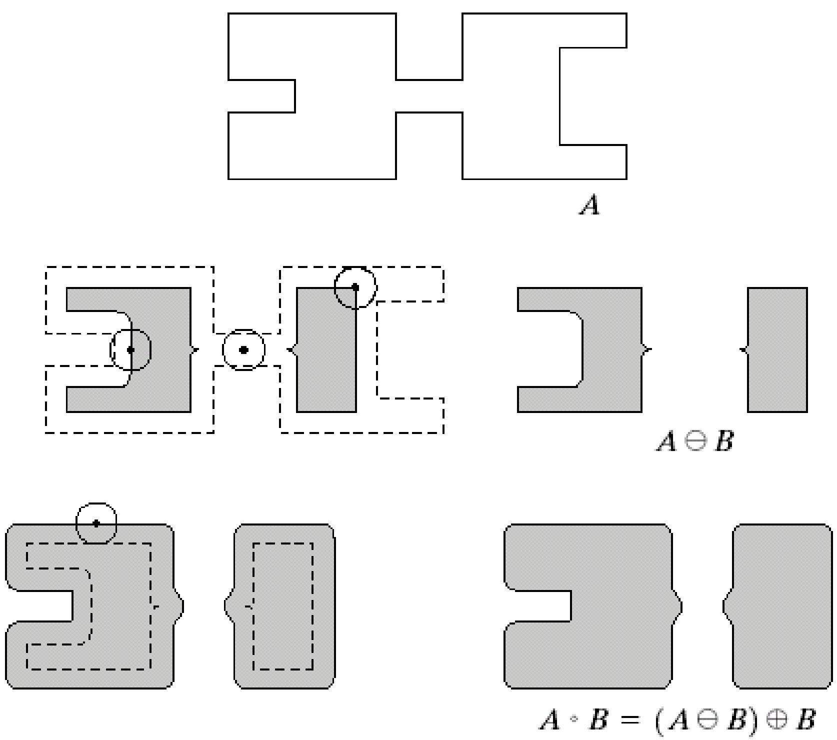
FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

9.3 Opening and Closing

- Opening (A∘B)

  the opening A by B is the erosion of A by B, followed by a dilation of the result by B.

$$A \circ B = (A \ominus B) \oplus B$$

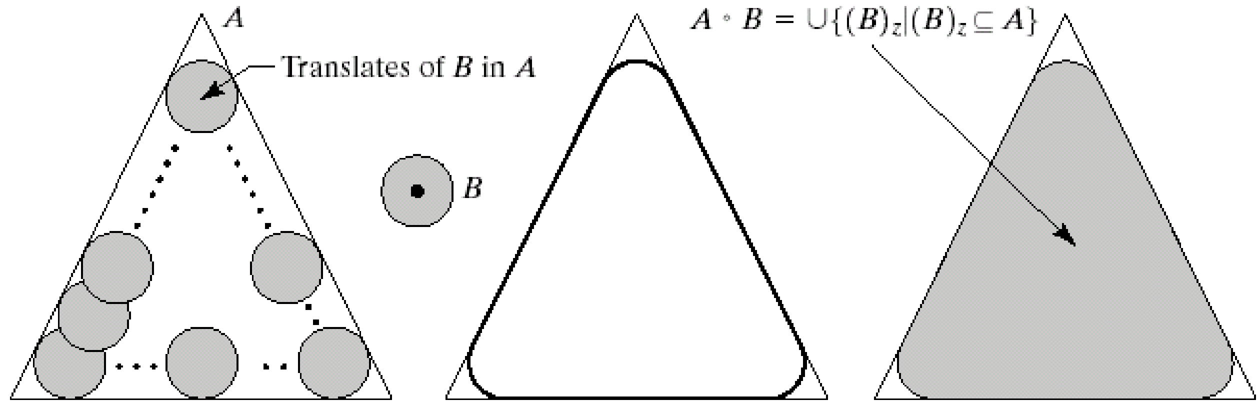


Opening separate objects

Opening generally smooths the contour of an object, break narrow isthmuses (峡) and eliminates thin protrusions (突起).

Since $A\circ B=\cup\{(B)_z \,|\, (B)_z \subseteq A\}$, $\rightarrow$ $A\circ B \subseteq A$.

Opening 的主要功能就是把外轮廓磨得较圆滑，但也会把 connected obiects 的细线切断开



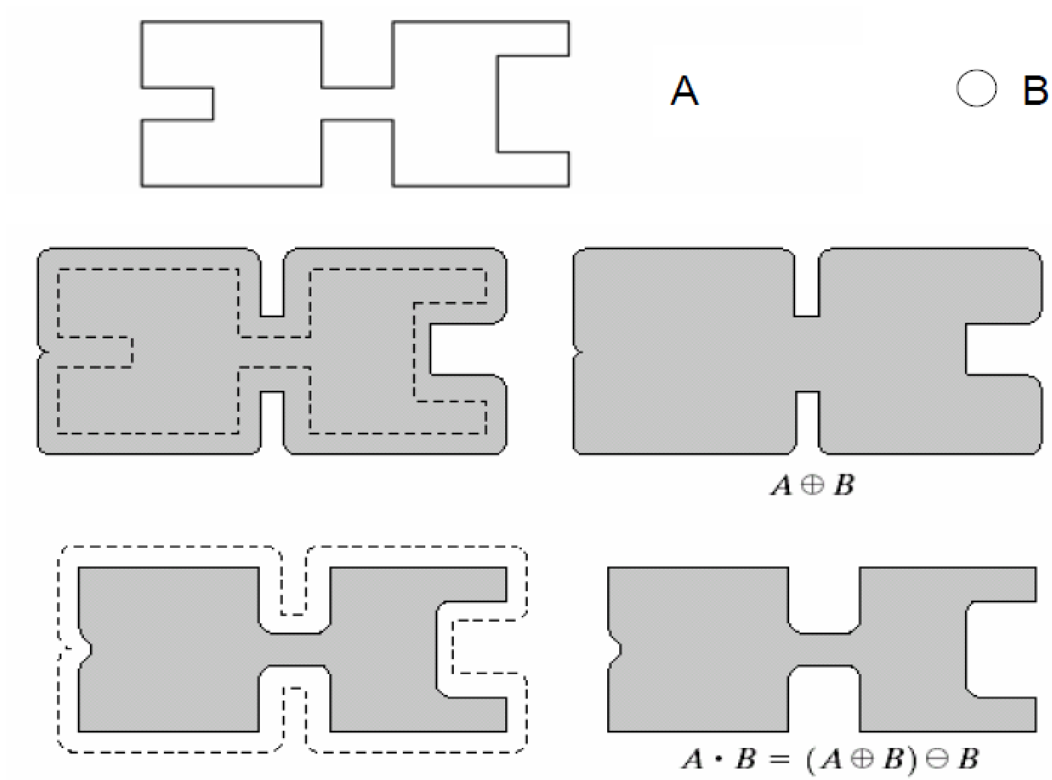
a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

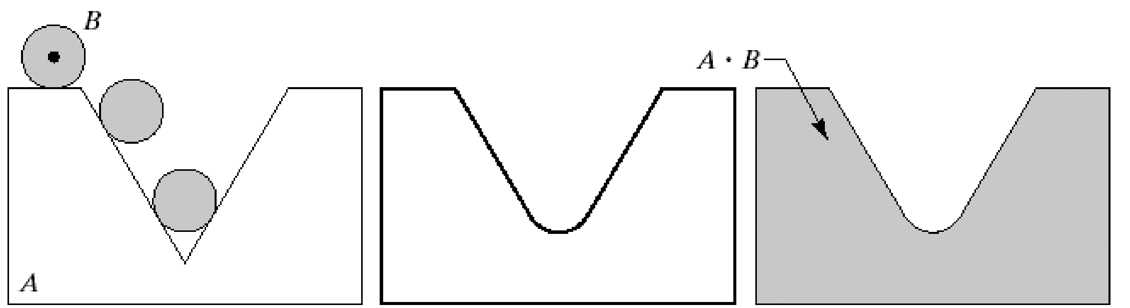
Closing (A·B)

The *closing* of set A by structuring element B is simply the dilation of A by B, followed by the erosion of the result by B

$$A \cdot B = (A \oplus B) \ominus B$$



Closing generally fuses narrow breaks and long thin gulfs, eliminates small hole, and fill gaps in the contour.



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

We can show that

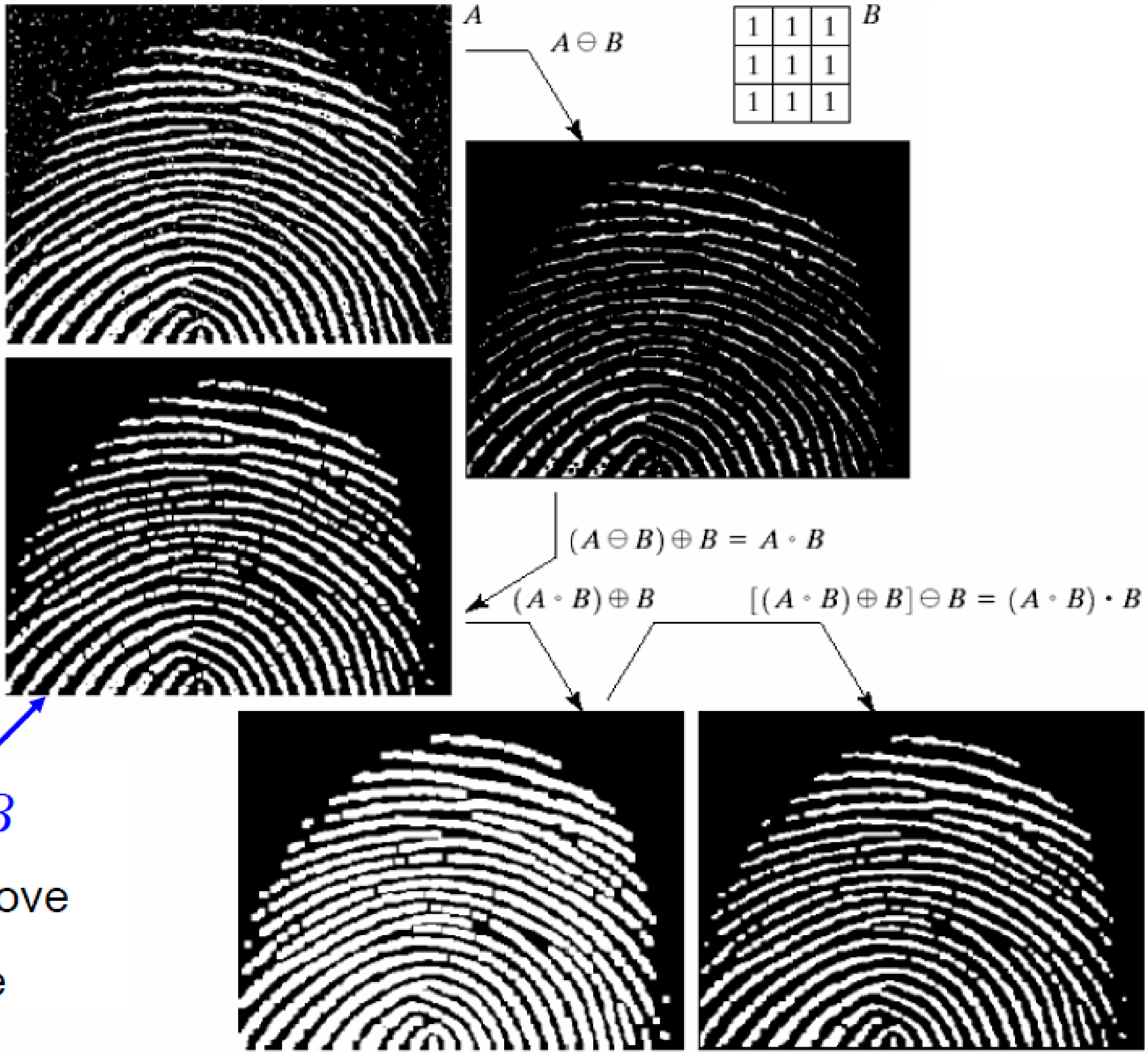
$$\because \quad A \cdot B \quad \Leftrightarrow (B)_a \cap A \neq \varnothing$$

$$\therefore \quad A \bullet B \supseteq A$$

Closing 的主要功能是：把内凹的角填起成圆弧

- Application Example of Opening and Closing

Opening and closing are frequently used to remove noises and to connect components fragmented by noises. In the following example, noises are first removed by opening, followed by a closing to re-connect fragmented lines.



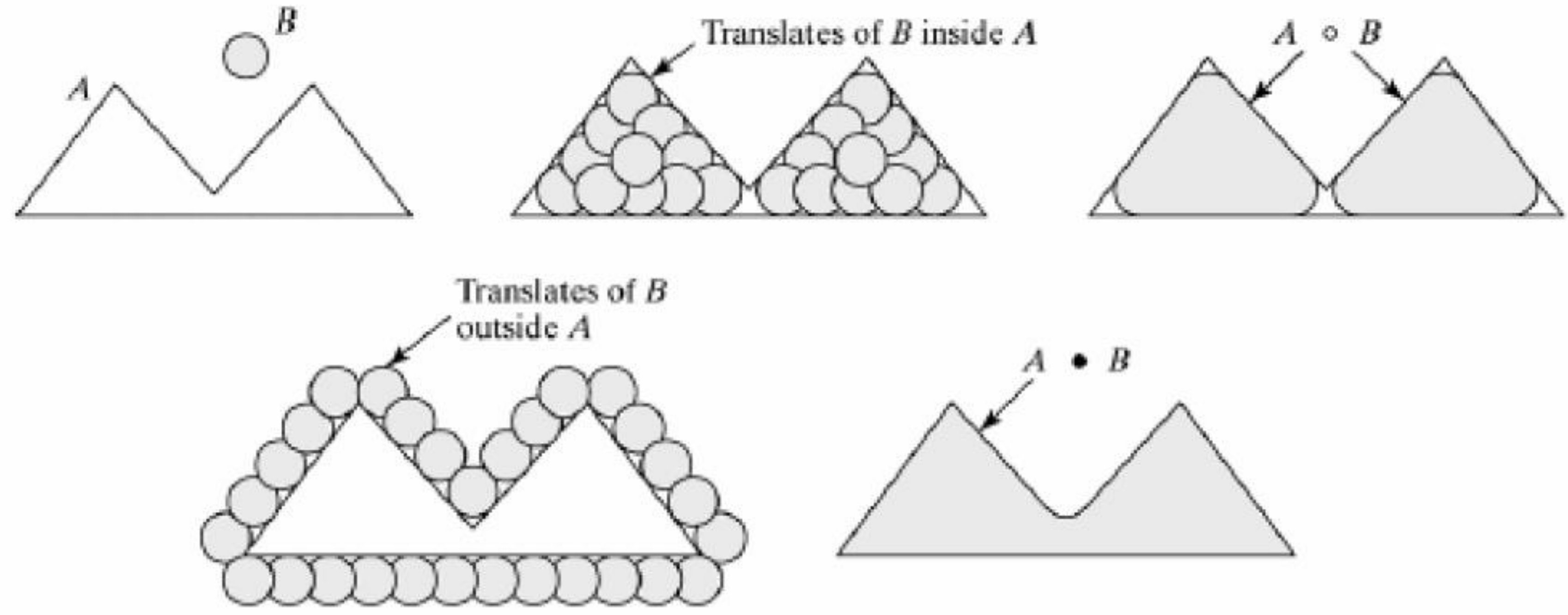
$A \circ B$

Remove noise

$(A \circ B) \bullet B$

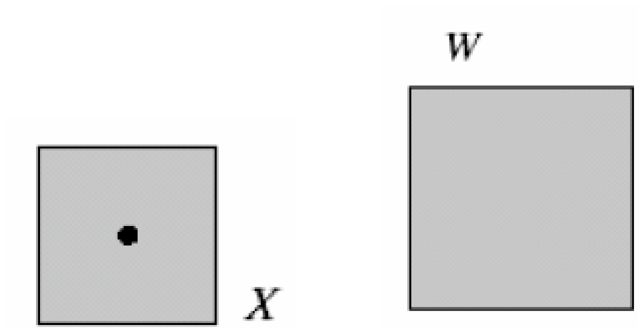
Connecting lines

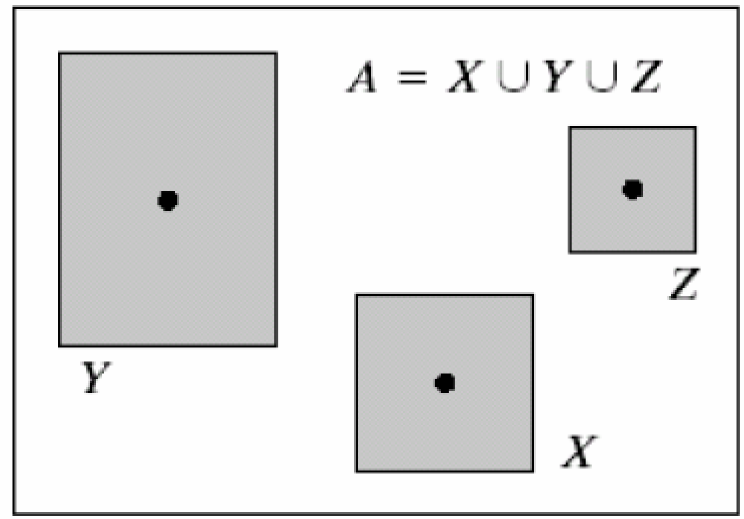
- Comparison of Opening and Closing



9.4 The Hit-or-Miss Transformation

Given X, to find X in the input image as shown on the right

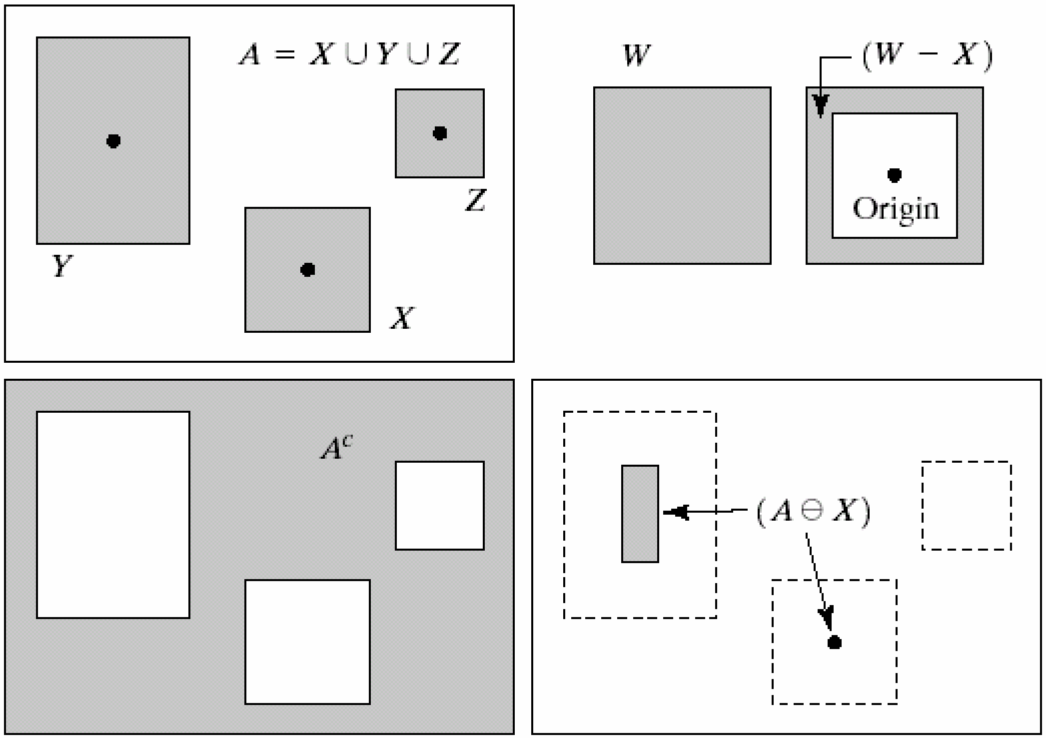


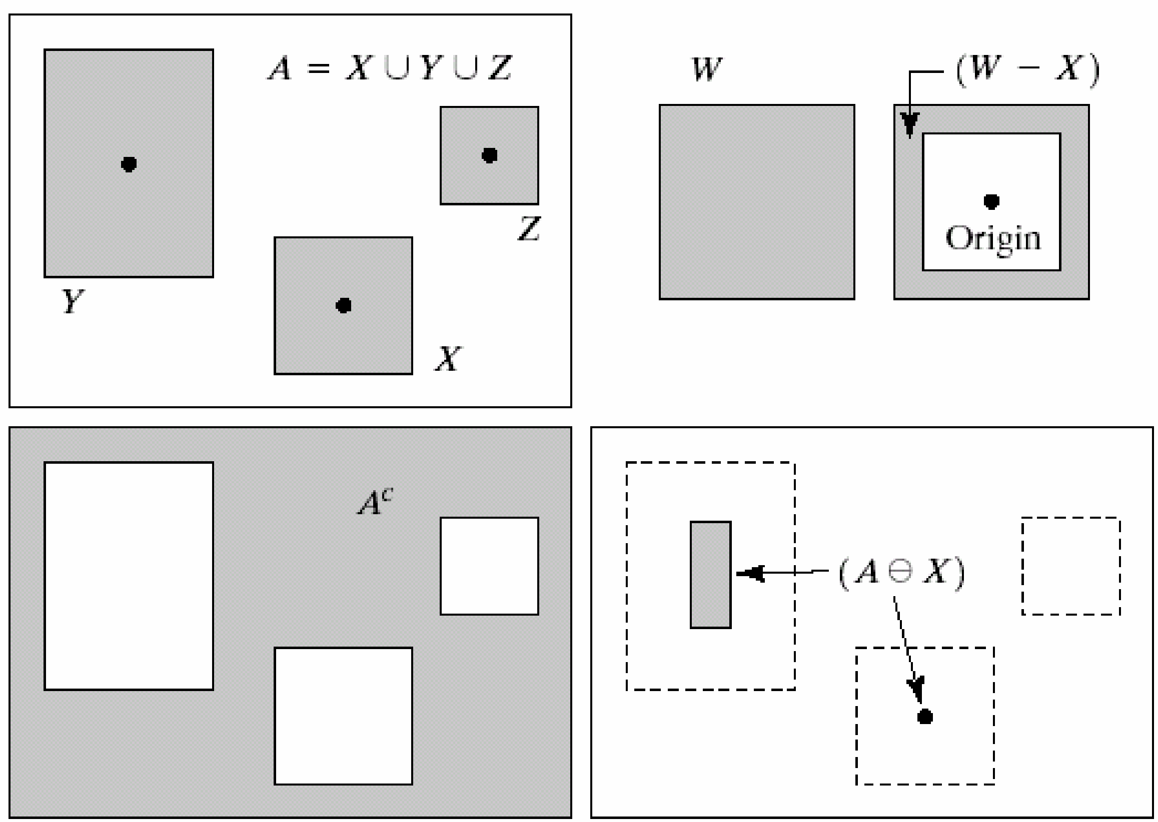


$$A \otimes X \triangleq (A \Theta X) \cap \left[A^c \Theta (W - X) \right] \quad \text{where } W \supset X$$

The Hit-or-miss transform is used to allocate (match) a given reference pattern in a test image. For instance to find the body X in an input image.

However, this approach requires that the X in the input image exactly matches the reference image X without any distortion, hence its direct practical applications are very limited. However, it does leads to some other interesting methods that might be useful in practical applications (such as convex hull).



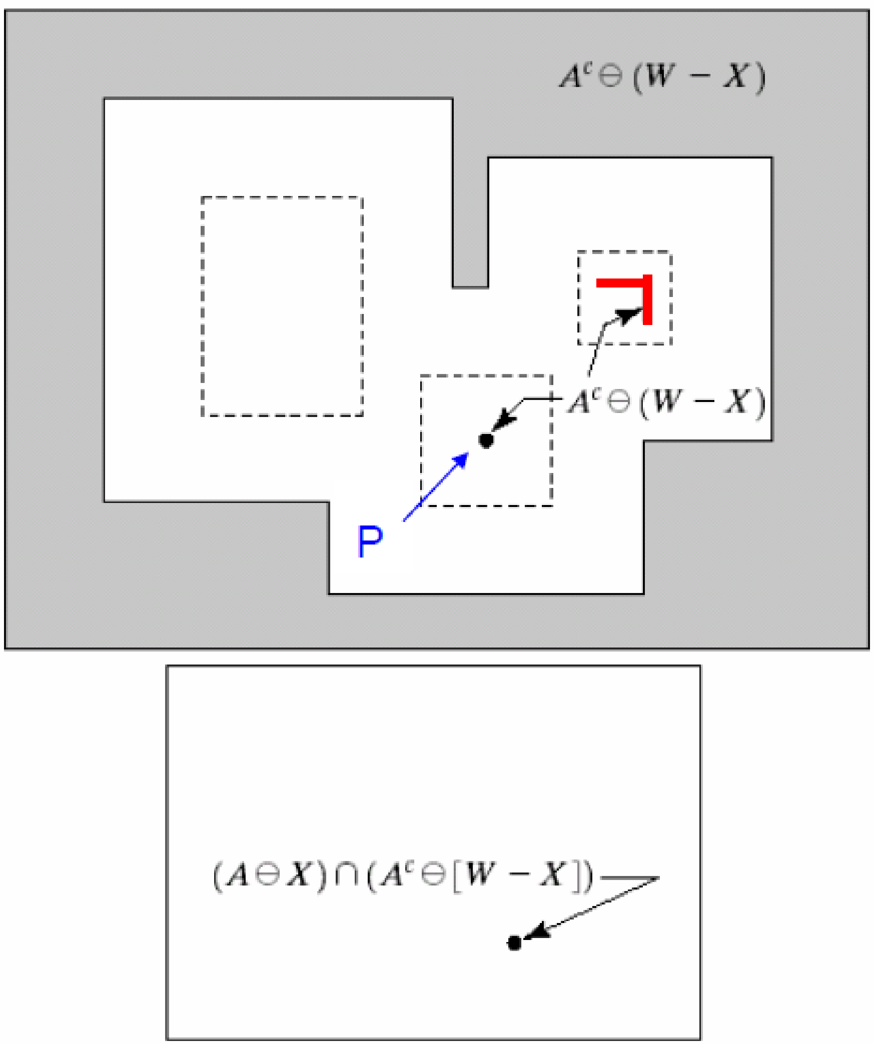


STEP 1: Design a window $W \supset X$ to obtain W-X

STEP 2: Find A^C

STEP 3: Erode A by X to get $A \ominus X$

Before the next step, please notice that, when center of (W-X) is at position P, $(W-X)_P \subseteq A^C$. Thus $P \in A^C \Theta (W-X)$



STEP 4: Erode A^c by W-X to get $A^c \ominus$(W-X)

※ we have shown

$P \in A^c \ominus (W-X)$

※ Similarly, the shape of Z can be completely covered by the shape of X, hence there exists a subset of Z, $Z_s \subset Z$ so that $(W-X)_{\hat{p}} \in A^c$ if $\hat{p} \in Z_s \subset Z$ i.e,

$$\hat{p} \in [A^c \Theta (W-X)]$$

STEP 5: to get $A \otimes X$

9.6 Morphological operations on gray images

9.6.1 Dilation $A \oplus B$ of gray images

Let A be the gray image dilated by the structuring element B, G be the resultant gray image and $b(x, y) \in B$, $g(s,t) \in G$, then

$$g(s,t) = (f \oplus b)(s,t) = \max\{ f(s-x, t-y) + b(x,y) \mid (s-x),(t-y) \in D_A; (x,y) \in D_B \}$$

One may follow three steps to obtain the intensity of $g(s, t)$ at a pixel coordinate (s, t)

(i) For each given (s, t), translate A around B (i.e., change x and y) such that $(s-x),(t-y) \in D_A$ and $(x,y) \in D_B$

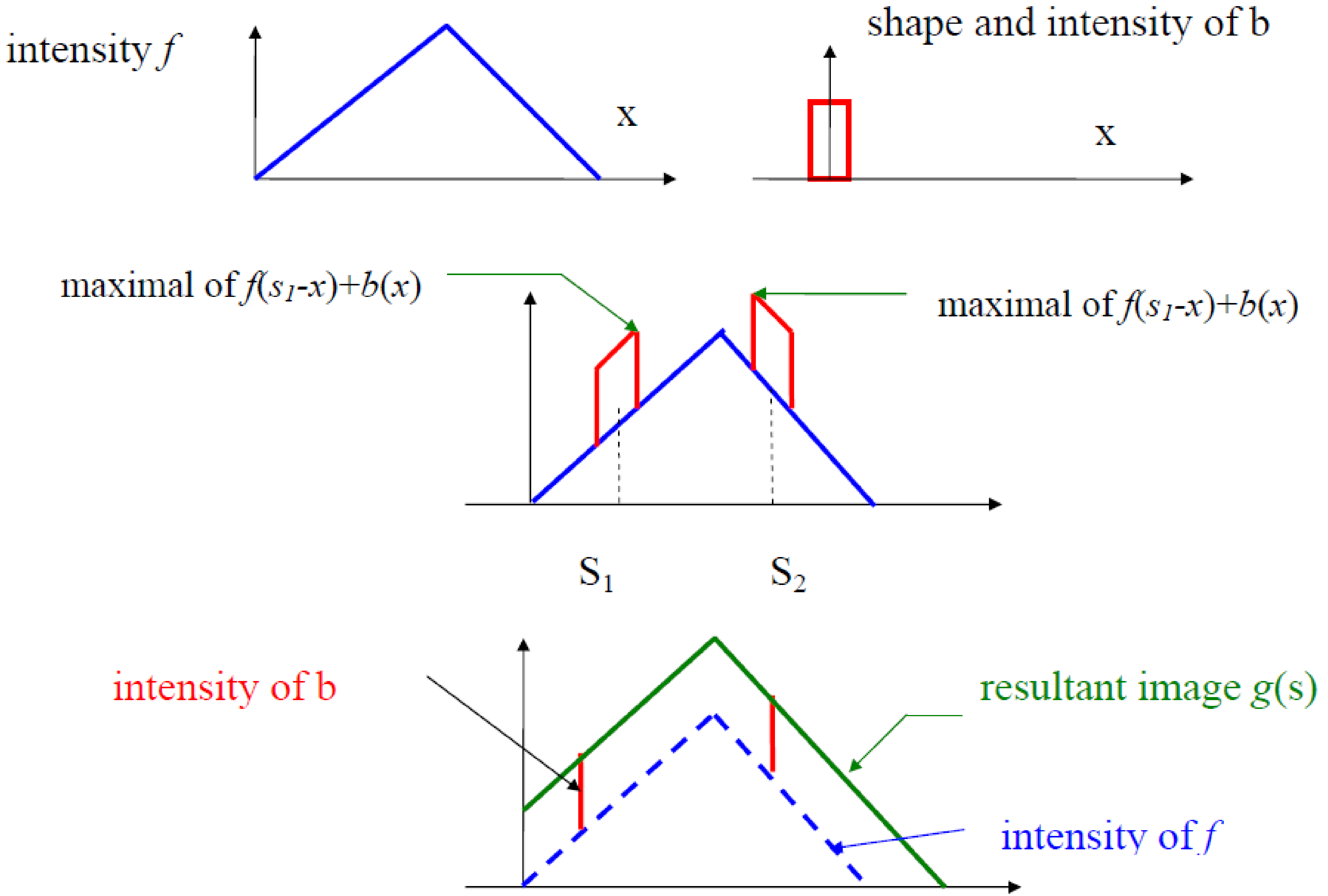
(ii) at each position (x, y), obtain $A(s - x, t - y) + b(x, y)$ with the given (s, t)

(iii) For the given (s, t), compare all intensity value $A(s - x, t - y) + b(x, y)$ under the condition that $(s-x),(t-y) \in D_A$ and $(x,y) \in D_B$, and choose the maximum value of $A(s - x, t - y) + b(x, y)$ as the value of $g(s, t)$.

Please notice that

- Resultant intensity is maximal of $A(s - x, t - y) + b(x, y)$, hence dilation generally increase the intensity of a gray image. （图像变亮）
- In the limiting case when the gray image is binary, the above definition reduces exactly to that defined for binary images.
- However, in the case of gray images, all pixels are changed (in terms of intensity) while binary operations changes only those pixels on the boundary.

The following example is a 1-D case



intensity f
x
shape and intensity of b
x
maximal of $f(s_1-x)+b(x)$
maximal of $f(s_1-x)+b(x)$
S_1
S_2
intensity of b
resultant image $g(s)$
intensity of f

9.6.2 Erosion $A \ominus B$ of gray images

Let A be the gray image eroded by B, G be the resultant gray image and $b(x, y) \in B$, $g(s,t) \in G$, then

$$g(s,t) = (f \Theta b)(s,t) = \min\{A(s+x, t+y) - b(x,y) \mid (s+x),(t+y) \in D_A; (x,y) \in D_B\}$$

One may follow three steps to obtain the intensity of $g(s, t)$ at a pixel coordinate (s, t)

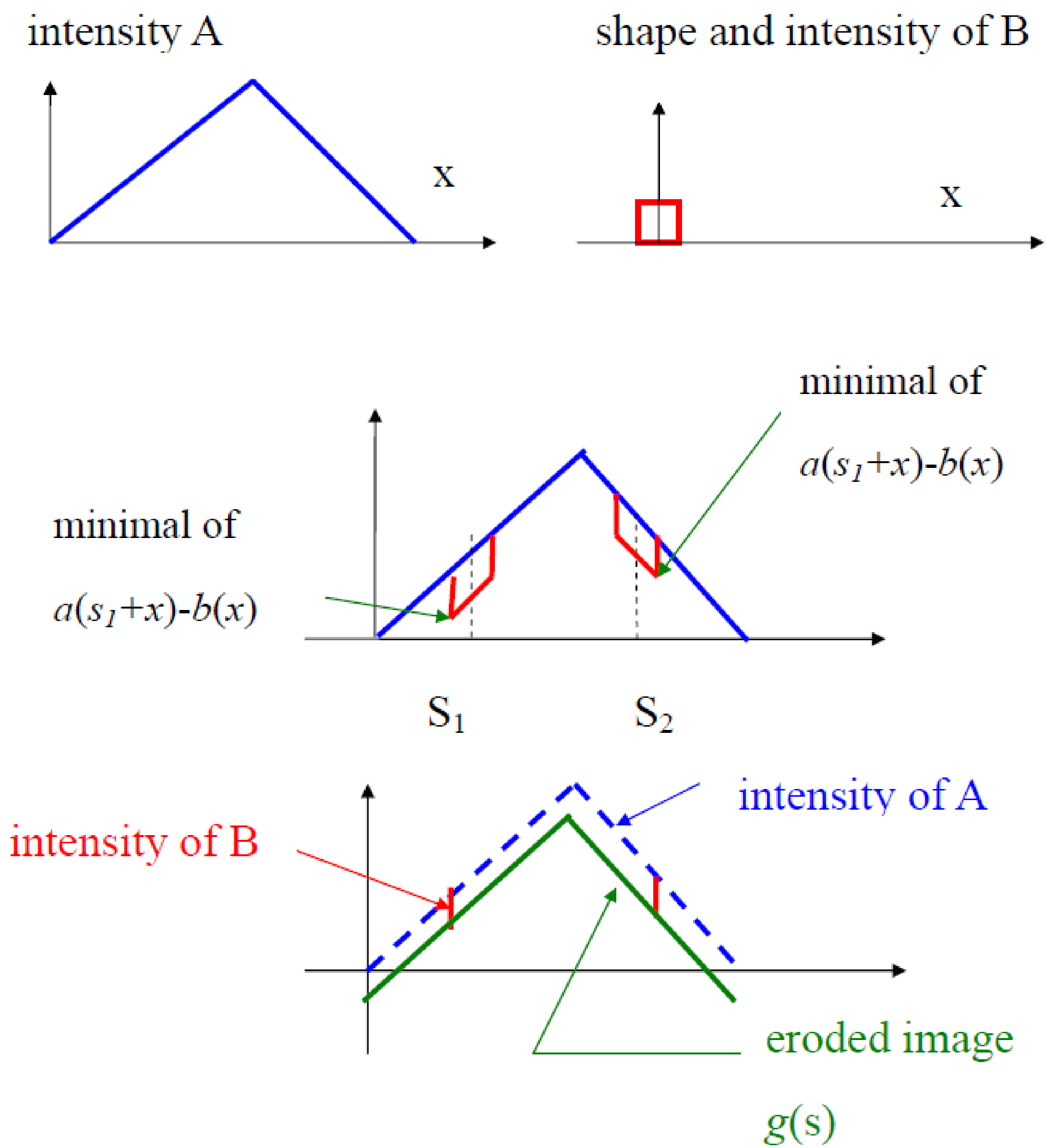
(i) For each given (s, t), translate A around B (i.e., change x and y) such that $(s+x),(t+y) \in D_A$ and $(x,y) \in D_B$

(ii) at each position (x, y), obtain $A(s + x, t + y) - b(x, y)$ with the given (s, t)

(iii) For the given (s, t), compare all intensity value $A(s + x, t + y) + b(x, y)$ under the condition that $(s+x),(t+y) \in D_A$ and $(x,y) \in D_B$, and choose the minimum value of $A(s + x, t + y) - b(x, y)$ as the value of $g(s, t)$.

Please notice that

- Resultant intensity is minimal of $A(s + x, t + y) - b(x, y)$, hence erosion generally decrease the intensity of a gray image.
- In the limiting case when the gray image is binary, the above definition reduces exactly to that defined for binary images.
- However, in the case of gray images, all pixels are changed (in terms of intensity) while binary operations changes only those pixels on the boundary.


intensity A
x
shape and intensity of B
x
minimal of
$a(s_1+x)-b(x)$
minimal of
$a(s_1+x)-b(x)$
S_1
S_2
intensity of A
intensity of B
eroded image
g(s)

Examples of dilation and erosion

Original image

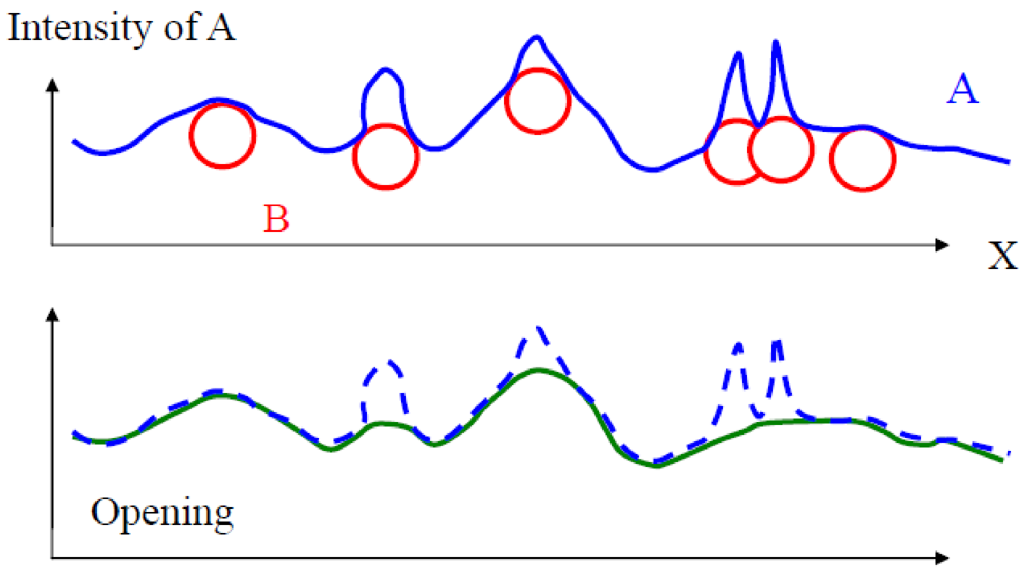
Dilation by a cubic

erosion

9.6.3 Opening and Closing

- Opening: erosion followed by dilation

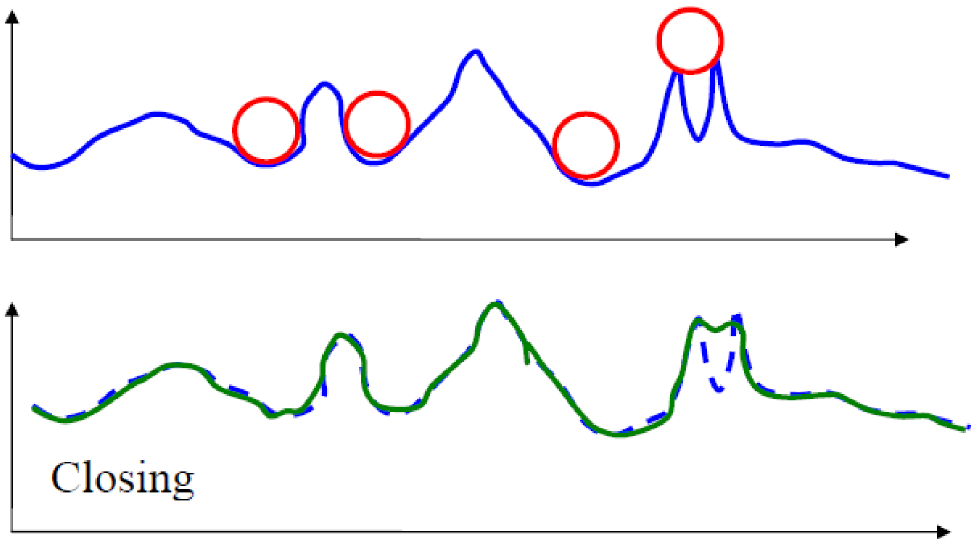
$$A \circ B = (A \ominus B) \oplus B$$



Remove small light details, while leaving the overall gray levels and larger right features

Closing: dilation followed by erosion

$$A \bullet B = (A \oplus B) \ominus B$$



Remove dark details from an image while leaving bright features relatively undisturbed

□ Application of gray-scale opening and closing

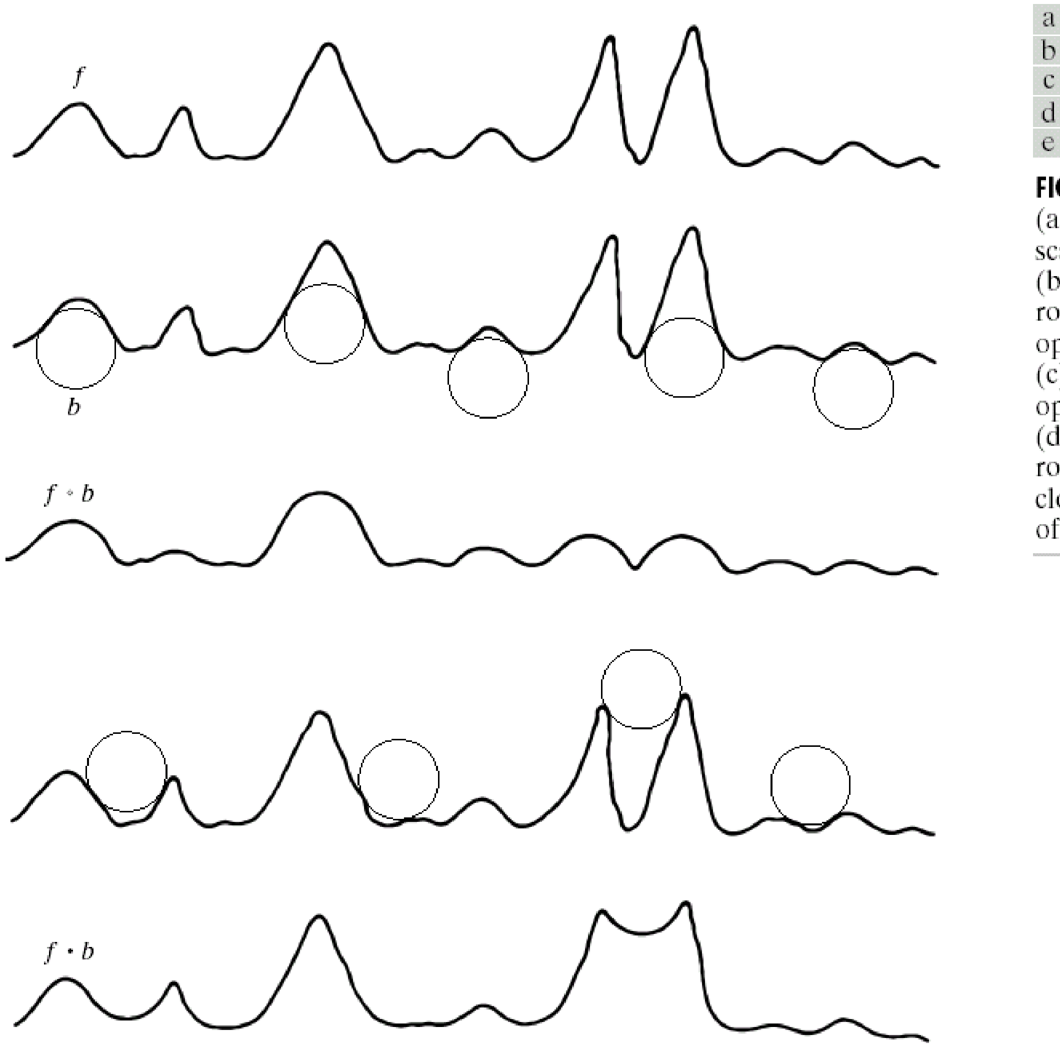
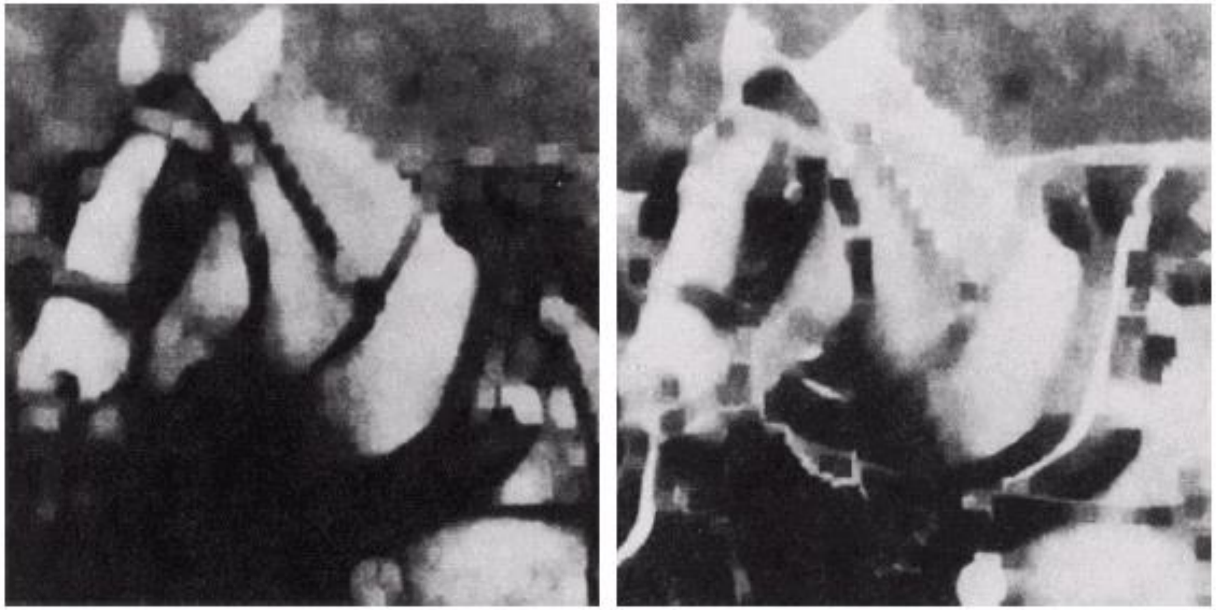


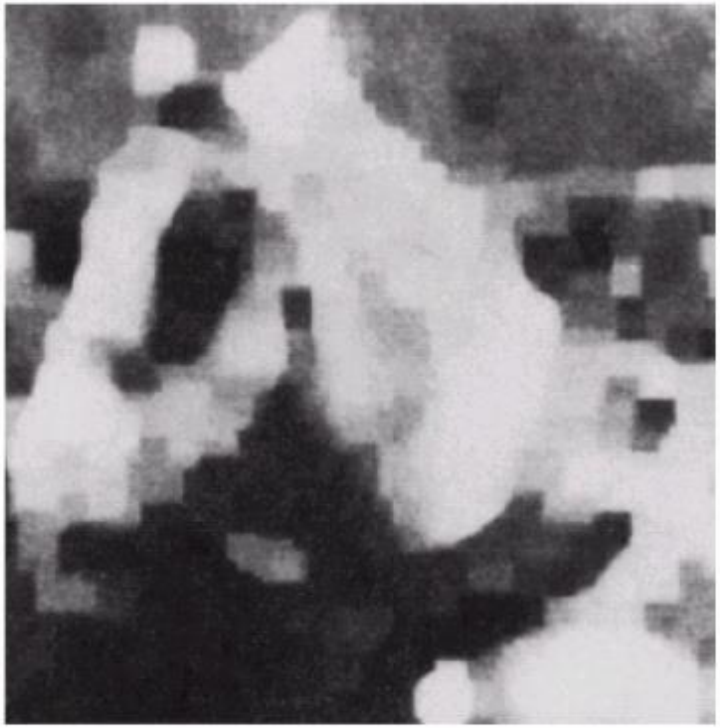
FIGURE 9.30 (a) A gray-scale scan line. (b) Positions of rolling ball for opening. (c) Result of opening. (d) Positions of rolling ball for closing. (e) Result of closing.



a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

- 9.6.4 Some Applications of Gray-Scale Morphology
  - Morphological smoothing = opening + closing



To remove all isolated bright spots (open) and all isolated dark spots (close)

- Morphological gradient = $(A \oplus B) - (A \ominus B)$

Morphological gradient can be used for edge detction

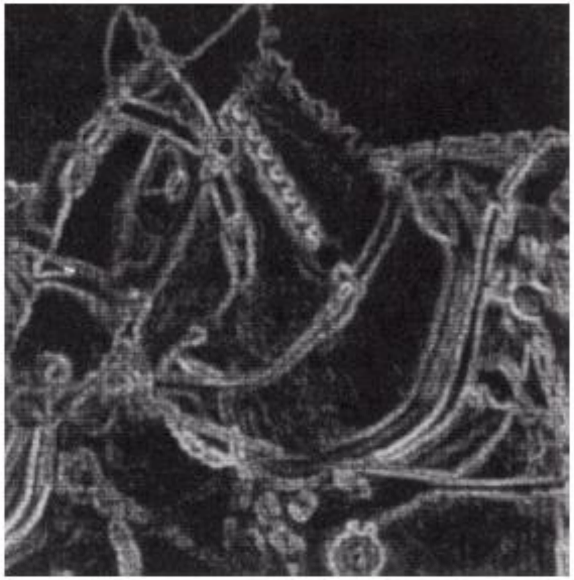


FIGURE 9.33 Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

- Top-hat transform = $f - (f \circ b)$

To enhance details in shadow while suppressing bright area

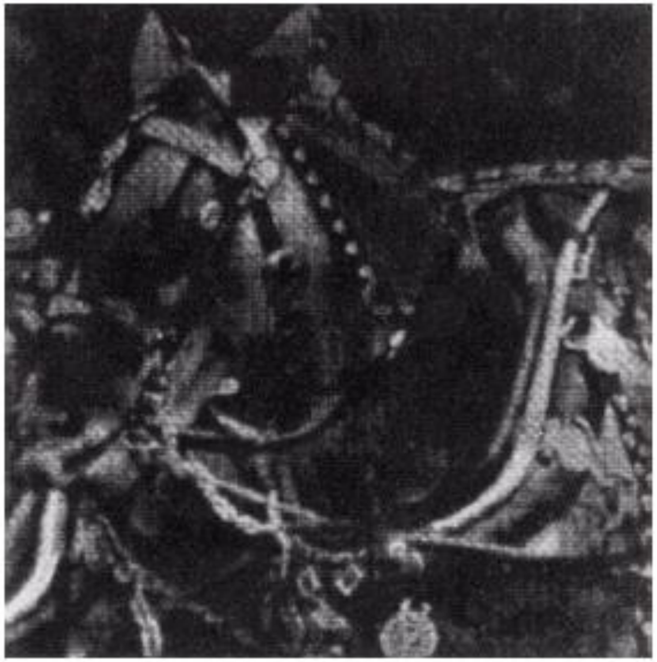
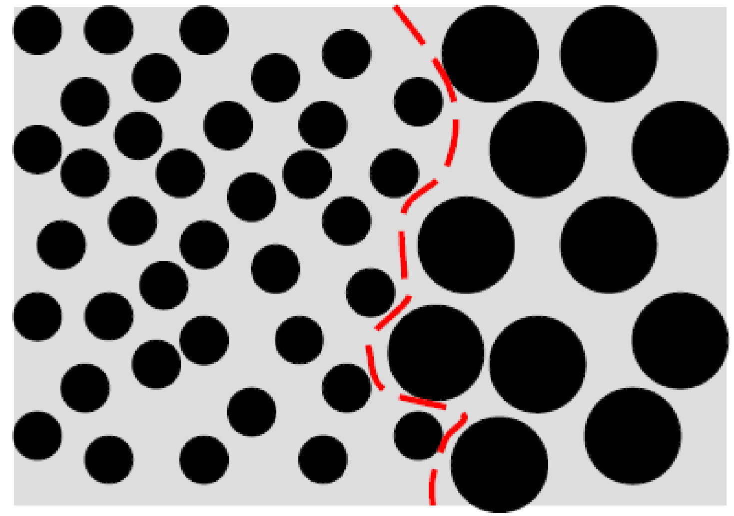


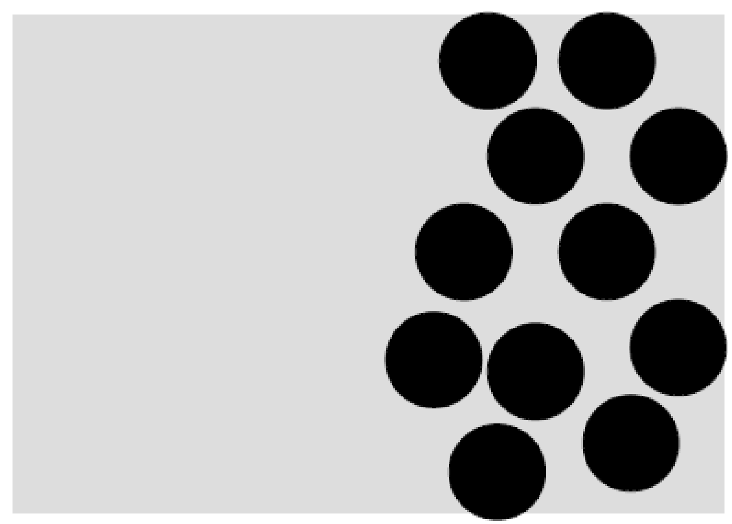
FIGURE 9.34 Result of performing a top-hat transformation on the image of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Textural segmentation

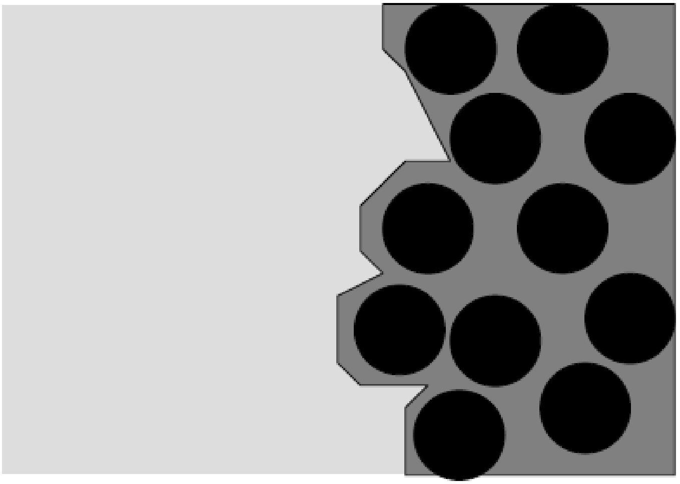
To find the boundary of large blobs and small blobs



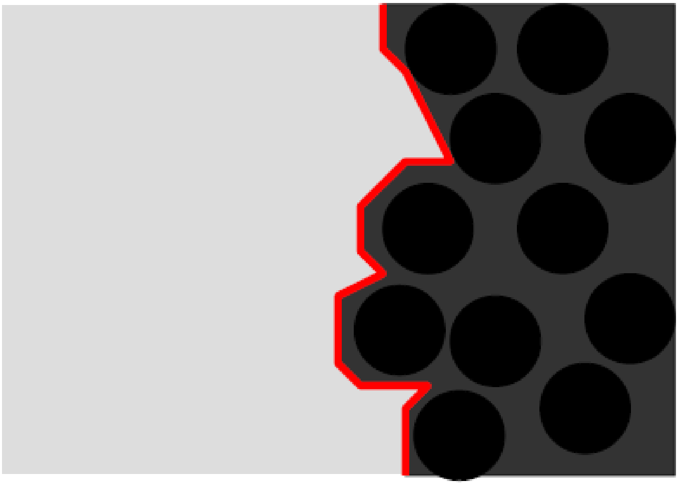
STEP 1: Use closing to remove small blobs



STEP 2: Use opening to reduce the intensity of the patches



STEP 3: Use a threshold to separate the two regions



The End

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