

一、矩阵

Matrix: Rectangular array of numbers:

$$\begin{array}{c} \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix} \quad \begin{array}{c} 2 \rightarrow \\ 2 \rightarrow \end{array} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$\uparrow \quad \uparrow$
 4×2 matrix
 $\mathbb{R}^{4 \times 2}$

$\uparrow \quad \uparrow \quad \uparrow$
 2×3 matrix

Dimension of matrix: number of rows x number of columns

矩阵元素

Matrix Elements (entries of matrix)

$$A = \begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix}$$

A_{ij} = " i, j entry" in the i^{th} row, j^{th} column.

$$A_{11} = 1402$$

$$A_{12} = 191$$

$$A_{32} = 1437$$

$$A_{41} = 147$$

$$\cancel{A_{42}} = \text{undefined (error)}$$

向量

Vector: An $n \times 1$ matrix.

$$y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}$$

$n = 4$

← 4-dimensional vector.

~~$\mathbb{R}^{2 \times 2}$~~

\mathbb{R}^4

$y_i = i^{th}$ element

$$y_1 = 460$$

$$y_2 = 232$$

$$y_3 = 315$$

1-indexed vs 0-indexed:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

1-indexed

$$y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

0-indexed

矩阵向量相乘与数组运算

Matrix Addition

$$\begin{array}{c}
 \downarrow \quad \downarrow \\
 \rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 0.5 \\ 2 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0.5 \\ 4 & 10 \\ 3 & 2 \end{bmatrix} \\
 \text{3x2 matrix} \quad \text{3x2} \quad \text{3x2}
 \end{array}$$

$$\begin{array}{c}
 \rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 0.5 \\ 2 & 5 \end{bmatrix} = \text{error} \\
 \text{2x2}
 \end{array}$$

Scalar Multiplication

real number

$$3 \times \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 6 & 15 \\ 9 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} \times 3$$

$$\begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} / 4 = \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & \frac{3}{4} \end{bmatrix}$$

这个矩阵除以4的结果

两个矩阵相乘

$$\begin{bmatrix} 1 & 3 \\ 4 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 16 \\ 4 \\ 7 \end{bmatrix}$$

$1 \times 1 + 3 \times 5 = 16$
 $4 \times 1 + 0 \times 5 = 4$
 $2 \times 1 + 1 \times 5 = 7$

Details:

$$\begin{array}{c}
 \underline{A} \quad \times \quad \underline{x} \quad = \quad \underline{y} \\
 \begin{array}{c} \text{Diagram of } A \text{ (rows highlighted)} \\ \text{Diagram of } x \text{ (columns highlighted)} \end{array} \\
 \begin{array}{c} \boxed{m \times n} \text{ matrix} \\ (m \text{ rows, } n \text{ columns}) \end{array} \quad \begin{array}{c} \boxed{n \times 1} \text{ matrix} \\ (n\text{-dimensional} \\ \text{vector}) \end{array} \quad \begin{array}{c} \boxed{m} \text{-dimensional} \\ \text{vector} \end{array}
 \end{array}$$

→ To get y_i , multiply A 's i^{th} row with elements of vector x . 在这个例子中 我们先看一下矩阵的维度

Example

$$\begin{array}{c}
 \begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & 3 & 0 & 4 \\ -1 & -2 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ -7 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ -7 \end{bmatrix} \\
 \boxed{3 \times 4} \quad \quad \quad 4 \times 1 \quad \quad \quad 3 \times 1
 \end{array}$$

$$1 \times 1 + 2 \times 3 + 1 \times 2 + 5 \times 1 = 14$$

$$0 \times 1 + 3 \times 3 + 0 \times 2 + 4 \times 1 = 13$$

$$-1 \times 1 + (-2) \times 3 + 0 \times 2 + 0 \times 1 = -7$$

矩阵乘法方法:

Details:

$$\begin{array}{c}
 \underline{A} \quad \times \quad \underline{B} \quad = \quad \underline{C} \\
 \begin{bmatrix} \quad \quad \quad \end{bmatrix} \times \begin{bmatrix} \quad \quad \quad \end{bmatrix} = \begin{bmatrix} \quad \quad \quad \end{bmatrix} \\
 \begin{array}{c} \boxed{m \times n} \text{ matrix} \\ (m \text{ rows, } n \text{ columns}) \end{array} \quad \begin{array}{c} \boxed{n \times o} \text{ matrix} \\ (n \text{ rows, } o \text{ columns}) \end{array} \quad \begin{array}{c} m \times o \\ \text{matrix} \end{array}
 \end{array}$$

特殊的矩阵运算

1、 矩阵的逆

2、 转置矩阵