PROBLEM SET 6

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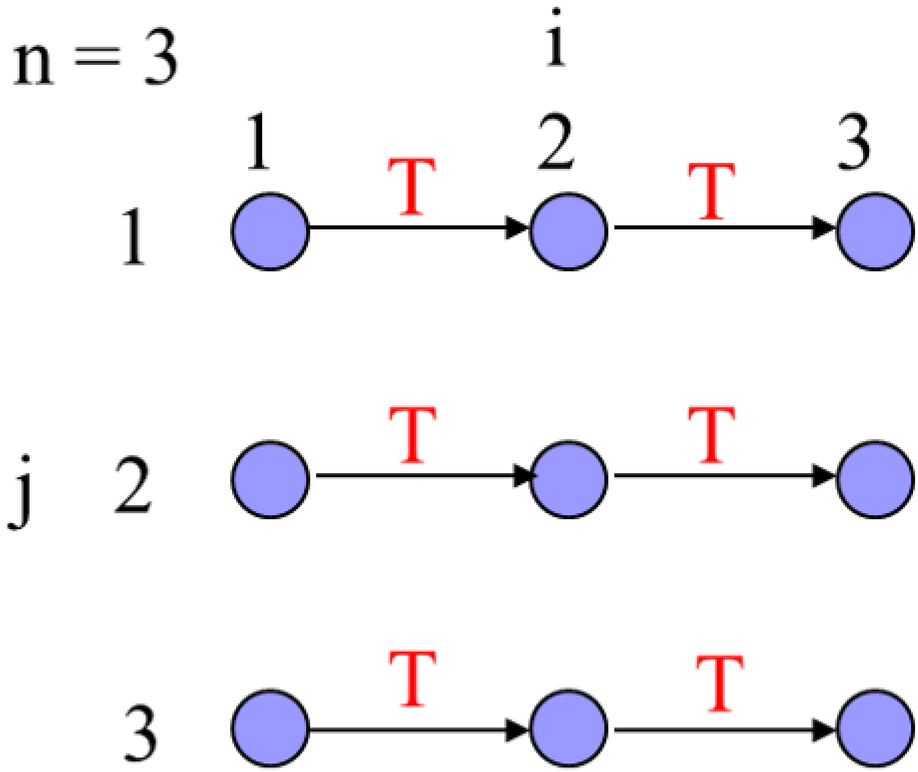
Problem 1

Dependences:

- $S_1[i,j] \to T\ S_1[i+1,j]$
- $S_1[i,j] \to A\ S_2[i,j]$

LGD:



Parallel algorithm:

- No dependences between different j, which is the way to parallelize.

```
#pragma omp parallel for private(i) schedule(static)
for (j=1; j<=n; j++)
    for (i=1; i<=n; i++) {
        S1: a[i][j] = a[i-1][j] + b[i][j];
        S2: b[i][j] = c[i][j]
    }
```

Problem 2

Dependences:

- $S_1[i] \rightarrow T\ S_1[i+1]$
- However, it can be formulated like: $a[i] = 0 + 1 + 2 + \ldots + i = (1+i) * i/2$

```
a[0] = 0;
#pragma omp parallel for schedule(static)
for (i=1; i<=n; i++)
    a[i] = (1+i)*i/2;
```

Problem 3

Dependences:

- $S_1[i] \rightarrow T\ S_1[i+1]$
- $S_2[i] \rightarrow T\ S_2[i+1]$
- $S_2[i] \rightarrow T\ S_1[i+1]$

It's suitable to deploy the reduction diagram:

```
double sum = 0.0;
double sign;
#pragma omp parallel for reduction(+:sum) private(sign) schedule(static)
for (i = 0; i < n; i++) {
    sign = 1;
    if (i%2 != 0){
        sign = -1;
    }
    sum += sign/(2*i+1);
}
pi = 4.0*sum;
```

Problem 4

Refer from this [notes]() for the whole proof:

> ```
> In 2 steps:
>
> THEOREM 2.7 : [Knuth's 0/1 Principle]
> ============  If a sorting algorithm that performs only element
> comparisons and exchanges sorts all sequences of zeroes and ones then
> it sorts all sequences of arbitrary numbers.
>
> proving         SORTS ALL 0/1 s  ====> SORTS ANY SEQUENCE
>
> is = to proving   CAN NOT SORT A SEQUENCE ====> CANNOT SORT ALL 0/1s
>
> PROOF:
>
> ------ Let f be a monotonic function: x <= y  ======> f(x) <= f(y)
> ```

```
Obviosuly, if a compare/exchange algorithm

 transforms      (x1,x2,....,xn)  into (y1, y2, ...,yn)
it also transforms  (f(x1),f(x2),...,f(xn)) into (f(y1),f(y2),...,f(yn))

Suppose that it sorts x to obtain a y sequence where  y(i) > y(i+1) UNSORTED
Define f as : f(x)=0 for x < y(i)  and f(x)=1 for x >= y(i)
Then the algorithm transforms the 0/1 sequence (f(x1),f(x2),...,f(xn))
into the 0/1 sequence
      (f(y1),f(y2),...f(yi),f(y[i+1])...,f(yn)) which is NOT SORTED.

         0  ....    0  1      0      1 .. 1
```

-

```
Comments by Mark Allen on the conditions for a sorting
algorithm to be called a "comparison-based sorting algorithm":
-----------------------------------------------------------------
As far as the 0/1 proof, I think the conditions on a "comparison
based sorting algorithm" would be the following (These conditions
should be sufficient, and although I don't prove it I'm quite
confident I can write a counter-example program if you try to
weaken them much):

1.) The only two operations which can be used to modify the
    data in the array are
    a.) compare and exchange, and
    b.) swap (unconditionally).
2.) For the logical expression that direct the flow of control
    in our program, we can use any kind of logical expression,
    as long as this expression does not vary with the contents
    of the array.

    "Contents" meaning the value of A[0], A[1], .., and A[n-1].
    So the number of elements in the array, n, is fair game to
    use in our logical expressions.

The goal of these conditions would be that given an array and a
non-decreasing function f, the algorithm performs all the exact
same swaps, on elements of the array x1,x2,..,xn as it does on the
array f(x1),f(x2),..,f(xn), or if it does not perform some swap on
say f(xi) and f(xj) where it did on xi and xj, it is because
f(xi)==f(xj).

Another desirable result of these conditions is that (since the
flow of control is unrelated to the contents of the array) two
arrays of equal size get sorted in the same ammount of time.

The first condition is obvious.  It is what we have been saying the
whole time.  The only two operations which can be used to modify
the data in the array are
  1.) compare and exchange

  2.) swap (unconditionally)
```

By the way, the compare and exchange operation may test for <, <=, >, >=, or == between the two array elements in making its decision of whether or not to exchange those two elements.

The second condition is less obvious and I've never heard it stated when people discuss comparison based sorting algorithms, but it is just as important. We must restrict what kinds of logical expressions can be used to alter the flow of control. [Eg. the logical expressions in "if" and "while" statements.]

The condition is that we can make any kind of logical expression we want as long as neither expression varies with the contents of the array.

Note: this condition implies in particular that we may NOT use array elements in comparisons that are related to the flow of control of the program (even if _both_ elements are array elements). The ONLY time such comparisons happen is inside a "compare exchange" command.

To see the problem if we allow comparisons between elements of the array to be used in an if statement, here is an example of such a program that contradicts the 0/1 principal:

```
Make a couple loops that go through comparing all possible pairs
of array elements, and keeps track of whether any of them have
equal values.
  If (we find _any_ two elements that are equal OR
      the array has only one or two elements) then
    we say "ah, I like this array, I'll sort it correctly."
    next we sort using some legitimate sorting algorithm
  else
    we say "nope, I think I'll just output garbage this time."
  endif
End of algorithm.
```

Note that the first condition of this "if" statement will always evaluate true when we do a sequence of 0's and 1's. So it will always sort these correctly, but it will not do all arbitrary sequences.

The problem is that the condition in the "if" statement varies with the contents of the array.

I just wanted to point this out because I consider it quite dangerous to oversimplify what it means for an algorithm to be comparison based.

I don't mean to overcomplicate it either. If we wanted to do that we could write a grammar that generates all legitimate comparison based algorithms :) . It wouldn't be too informative though, I don't think.

A straightforward OpenMP example:

```
for(phase = 0; phase < N; phase++) {
    if(phase & 1){
        #pragma omp parallel for private(i) shared(arr)
        for(i = 2; i < N; i+=2)
            if(arr[i-1] > arr[i])
                swap(&arr[i-1],&arr[i]);
    } else {
        #pragma omp parallel for private(i) shared(arr)
        for(i = 1; i < N; i+=2)
            if(arr[i-1] > arr[i])
                swap(&arr[i-1],&arr[i]);
    }
}
```