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Image Denoising with Dictionaries

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Contents

Abstract

In image processing, removing such noises from an image problem is a crucial stage. Noises in an image mostly occur during the image's storage, transmission process, and acquisition process. Image denoising is consider as a huge issue found in various problems with image processing and computer vision. There are various existing methods to denoise image. This research interested in denoising image with dictionaries. The proposed algorithm denoises the image, while using the K-SVD algorithm to train the dictionary at the same time. The K-SVD algorithm is an iterative technique that alternates between sparse coding and updating dictionary. The essential aspect of a good image denoising model is that, while preserving the details in the image, it should completely remove noise as much as possible. This project proposed to implement alternative matching pursuit Fast Iterative Shrinkage Threshold Algorithm (FISTA) to replace the Orthogonal Matching Pursuit (OMP). The project started by implementing the K-SVD algorithm with artificial signals to develop the knowledge of the K-SVD, then expanded to introduce the unseen data signal by varying noise level to train the dictionary. Subsequent of the artificial signals, the K-SVD algorithm then use image dataset to imitate the real-world application to denoise image problem. The project seeks to find the image reconstruction capability by measuring the PSNR value between two images.

1 Introduction

In Linear Algebra, a basis of an n-dimensional space is a set of n linearly independent vectors in that space. Every vector in that space can be expressed as a unique way, linear combination of elements in that basis. The dictionary can be viewed as an over-complete basis, so that each vector in the same space can be expressed approximately as linear combinations of elements in this dictionary. Among all linear combinations above, we focus on one having minimum number of non-zero coefficients (sparse solutions). Dictionary Learning is the method whose objective is to find a good over-complete basis in relation to minimise the approximation error and finding sparsest solution given a set of vectors.

Images often contain noise, that may occur during the acquisition or transmission process. In many applications, removing such noise is a great benefit, and this may explain the vast interest in this problem and its solution. Image denoising, as one of the simplest image restoration and inverse problems, has been widely studied and analysed. The key behind such type of problems, is how to find good image model, in order to estimate the clean image from the corrupted image. Since there is no true image models, various tools and approaches have been applied to approximate what images are. The principle of image denoising is to suppress the noise, while preserving as many image structures and details as possible [1].

1.1 Aims

K-SVD algorithm has been extensively used in many applications such as medical image sensing, military, remote sensing and many more. Therefore, the group has decided to apply K-SVD algorithm into the image denoising problem. The experiment can be divided into two stages, which in the first stage, the group aims to develop foundation knowledge in K-SVD algorithm. While in the second stage, the group aims to experiment and improve the K-SVD algorithm by replacing the sparse coding Orthogonal Matching Pursuit (OMP) with an alternative sparse coding method which is Fast Iterative Shrinkage Threshold Algorithm (FISTA). The goals of this thesis are grouped in the following:

1) To evaluate the computational time between two different algorithms, K-SVD and modified version of K-SVD.
2) To assess the reconstruction effectiveness of the dictionary coefficient between two different algorithms, K-SVD and modified version of K-SVD.

1.1 Motivations

The application for image processing can be found in two main areas, one is improving graphical information for human interpretation and the other is processing image data for autonomous perception of the machine. In many fields, such as biology, astronomy, military, medical, satellite imaging etc. successful applications of image processing concepts can be found today. For instance, image denoising application can be used in medical imaging for the MRI scan machine to denoise the measured image to produce a clear image of brain tumour scan. In all these fields, the problem of recovering the original images from an incomplete and noisy image is faced by researchers. An image is often corrupted by noise in its acquisition and transmission process. Image denoising algorithm is used to improve estimation of an image corrupted by noise. The main objective of image denoising is to remove the noise as well as retaining as many details as possible in the image without adding artefacts [1].

2 Literature review

The denoising problem starts with an ideal image which contaminated by randomly distributed Gaussian noise. Noise, edge, and texture can be categorised as high frequency components, it is difficult to differentiate them in the process of denoising and some details could inevitably be lost by the denoise image [2]. Noise reduction is designed to reduce the noise in measured images while minimising the loss of image characteristic features. One of the classic ways to denoise an image is by applying filtering, spatial filters have been applied to image denoising problem which result in two types of filters: linear and non-linear [2]. Applying the linear filter to an image problem, successfully denoise the image however the image textures are not preserved. Linear filters were originally used to remove noise in spatial domain, but they do not preserve image textures. Mean filtering has been used to reduce Gaussian noise, but it results in over-smooth image. While Wiener filtering has been used to overcome the disadvantages of over-smooth, but it can easily blur sharp edges. In order to apply a Wiener filter, one needs to estimate the covariance matrix of the signal [2]. As for a non-linear filter such as median filtering, the amount of noise can be suppressed without any detection. Usually, spatial filters managed to remove noise to a sufficient level but then it may cause an image blurring as the result [2].

In recent years, the sparse representations technique has become a trend as it is widely used in image restoration application. It can be categorised as a dictionary learning based method in which it performs denoising process by learning a large group of patches from an image dataset such that each patch in the image problem can be represented as a linear combination of only few patches from the redundant dictionary [1]. Dictionary learning can be formulated as an optimisation problem in several ways. In basic operations such denoising image, a model of dictionary is helpful for the structuring the data [3].

In relation to the denoising issue and sparse representation, a research by Aharon, M, Elad, M & Bruckstein [4] concluded that the reconstruction of the noisy image can be done with the dictionary learning algorithms. Applying the K-SVD algorithm, finding the sparse representation vector, and updating the corresponding dictionary able to approximate the true image with minimum level of error. The process of updating the columns of the dictionary is performed in conjunction with an updated sparse representation that results in rapid convergence [4]. Finding the sparse representation in sparse coding stage could be done with any pursuit algorithms. One of the simplest algorithms is the Orthogonal Matching Pursuit (OMP) algorithms. The OMP is very basic, involving computation of internal products between the columns of the signal and dictionary [1]. Other than that, a well-known pursuit algorithm is the basis pursuit. It suggests a convexification of the problems by replacing ℓ_0-norm with an ℓ_1-norm [4]. For such of these two pursuit algorithms, it is stated that the K-SVD algorithm is flexible and able to work with any pursuit algorithms. This leads to a steady-state result with infinitely many iterations in which the denoise image can only produce an approximate image rather than the true image itself. The K-SVD algorithms developed in [4] applied the OMP as the sparse coding that essentially evaluates the sparse representation, $\boldsymbol{\alpha}$ and updating the dictionary. The algorithm sweeps through the columns and always use the most updated representation as they emerge from preceding dictionary update process. A synthetic experiment is being conducted using the K-SVD algorithm and other two algorithms which method of optimal direction (MOD) and *maximum a posteriori* (MAP)-based algorithm, there are some parameters needed to be assigned such as the size of matrix $\boldsymbol{D}$ (20 x 50), sample signals $\boldsymbol{Y}$ (200 with dimension of 20), sparsity level is 3 and it was set to 80 times of iteration. The outcome shows that the computed dictionary of three algorithms was compared with their initial generating dictionary and the K-SVD algorithm performed better those the other two algorithms.

The work in [4] experimenting the K-SVD algorithm in synthetic way, supporting the principle of K-SVD algorithm, work by Elad, M & Aharon, M 2006 [5] implemented actual image dataset to solve the image denoising problem. The approach taken is by developing a sparse and redundant representation over the trained dictionary. Having sparse and redundant representation introduced a shift invariance property which is desired in solving inverse linear problem. The work proposed to train the dictionary from the corrupted image using patches. This means that the K-SVD algorithm will be iterated between patch-by-patch sparse coding and updating the dictionary with the updated sparse representation [5]. For such large samples, X of size $\sqrt{N} \times \sqrt{N}$ $(N \gg n)$, the image patches will be introduced by $\sqrt{n} \times \sqrt{n}$ pixels as column vector $x \in R^n$. The reason of using small patches is considered as 1) when training dictionary takes place only small dictionaries can be composed, moreover, 2) a small dictionary indicates a locality of the algorithms outcome in which it simplifies the overall image treatment. However, using small dictionary when treating large image, may introduced artifacts on the block boundaries. Thus, to prevent the emerging artifacts, the work proposed on overlapping the patches as well as averaging the results. The experiment being conducted, using three different images which are called Peppers, House and Barbara. The denoising performance of the datasets being assessed with different noise level, σ $(1 \leq \sigma \leq 100)$, in which the larger the σ value, the noisier it becomes and the lower the PSNR result.

Throughout the previous discussions, we discussed about how K-SVD algorithm able to denoise an image problem. As mentioned in [4] the K-SVD algorithm was versatile and can work with any pursuit algorithm. This thesis proposed to replace the sparse coding OMP with an alternative sparse coding Fast Iterative Shrinkage Threshold Algorithm (FISTA). This algorithm is significant due to its versatility in solving various problems and its rapid convergence capability. FISTA was a fast version of Iterative Shrinkage Threshold Algorithm (ISTA). FISTA technique focus on computing the next iteration based on two or more previously computed iteration, rather than just using the previous one [6]. Using the FISTA as alternative matching pursuit algorithm introduce the Lagrange multipliers, $\boldsymbol{\lambda}$. This parameter was needed to balance between the sparsity and the approximation of the data problem. This study meant to explore the capability of FISTA to reconstruct a data signal problem and analysed the difference in output SNR error produced between the OMP and FISTA. The work will implement FISTA by integrating with the K-SVD as the dictionary learning algorithm.

3 Technical Background

3.1 Dictionary Learning Algorithm

Assume x and n are uncorrelated, for x is an ideal image that is measured in the presence of an additive zero-mean white Gaussian noise, n with standard deviation σ. While the measured image is y, thus the model can be written as:

$$y = x + n \qquad 3.1$$

The image denoising algorithm is designed to remove the noise from y, bringing the result as close to the original image, x as possible [5].

Several algorithms have been created for the task of dictionary learning. The K-SVD algorithm is one of the well-established algorithms. The objective of the K-SVD algorithm is to find a dictionary D and sparse representation alpha that minimises the following representation error, given a set of signals $X = [x_1, \dots, x_n]$:

$$\left(\widehat{\boldsymbol{D}}, \widehat{\boldsymbol{\alpha}}\right) = \boldsymbol{arg\,min}_{\boldsymbol{D},\boldsymbol{\alpha}}\ \boldsymbol{\lambda}\|\boldsymbol{\alpha}\|_{\mathbf{0}} + \frac{\mathbf{1}}{\mathbf{2}}\|\boldsymbol{D\alpha} - \boldsymbol{X}\|_{\mathbf{2}}^{\mathbf{2}} \qquad 3.2$$

The K-SVD algorithm is an iterative technique that alternates between sparse coding and updating dictionary. The algorithm involves the ℓ_0-norm and ℓ_2-norm, where it simply a regularisation in way to avoid overfitting problem. It reduces the parameters $(\widehat{\boldsymbol{D}}, \widehat{\boldsymbol{\alpha}})$ and shrink the equation in **3.2**. First the dictionary D with ℓ_2 normalised columns is initialised. Then the main iteration is composed of following stages:

- Sparse coding: In this step, D is fixed and the following optimisation problem is solved to compute the representation vector α for each example x_i:

$$i = 1, \dots, n \quad \boldsymbol{arg\, min}_{\alpha} \frac{\boldsymbol{1}}{\boldsymbol{2}} \|\boldsymbol{D\alpha} - \boldsymbol{X}\|_2^2 \; s.t \; \boldsymbol{\lambda}\|\boldsymbol{\alpha}\|_{\boldsymbol{0}} \tag{3.3}$$

  Approximate solution of the above problem can be obtained by using sparse coding algorithm such as Orthogonal Matching Pursuit (OMP).

- Dictionary update: This is where the K-SVD performed atom-by-atom in an efficient way. Following of the result of achieving sparse representation vector, the term to be minimised can be written as:

$$\{\widehat{\boldsymbol{D}}, \widehat{\boldsymbol{\alpha}}_{\boldsymbol{k}}, \widehat{\boldsymbol{X}}\} = \boldsymbol{arg\, min}_{\widehat{D}, \alpha_k, X} \frac{\boldsymbol{\mu}}{\boldsymbol{2}} \|\boldsymbol{X} - \boldsymbol{Y}\|_2^2 + \sum_{\boldsymbol{k}} (\boldsymbol{\lambda_k} \|\boldsymbol{\alpha_k}\|_{\boldsymbol{0}} + \frac{\boldsymbol{1}}{\boldsymbol{2}} \|\boldsymbol{D\alpha_k} - \boldsymbol{R_k X}\|_2^2) \tag{3.4}$$

3.2 K-SVD Algorithm

This section provides an introduction on how K-SVD algorithm is being used in image denoising. K-SVD is an iterative denoising method which learns from the entire range of pixels of noisy images. Each of the iteration comprises applying the orthogonal matching pursuit (OMP) to estimate coefficients for each patch and updating the dictionary using singular value decomposition for one column at a time [5]. The fundamental idea is that every signal from patches can be represented as a linear combination of a few columns from the redundant dictionary $\boldsymbol{D}$ [1].

In our early project, we replicate the experiment from M. Aharon, M. Elad and A. Bruckstein [4] which the works experimenting the ability of K-SVD to denoise an image. Starting from patches to the global objective function, we introduce $\boldsymbol{x}$ as a small image patch of size $\sqrt{\boldsymbol{n}} \times \sqrt{\boldsymbol{n}}$ pixels as a column vector of length $\boldsymbol{n}$. The sparse representation model can be formally described as $\boldsymbol{x} = \boldsymbol{D\alpha}$, where $\boldsymbol{\alpha} \in \boldsymbol{R}^{\boldsymbol{n}\times\boldsymbol{m}}$ is a sparse vector with $\|\alpha\|_0$ as non-zeros value. Assume $\boldsymbol{y}$, as the noisy version of $\boldsymbol{x}$, which tainted by an additive zero mean gaussian noise with standard deviation $\boldsymbol{\sigma}$ and $\boldsymbol{C}$ as the noise gain. The standard deviation can be varied to a random number such as 25,50 and 75. The sparse representation is obtained by solving:

$$\widehat{\boldsymbol{\alpha}} = \boldsymbol{arg\,min}_{\boldsymbol{\alpha}} \|\boldsymbol{\alpha}\|_0 \ \ \boldsymbol{s.t} \ \|\boldsymbol{D\alpha} - \boldsymbol{y}\|_2^2 \leq (\boldsymbol{C\sigma})^2 \tag{3.5}$$

This aim to recover the sparse representation vector of $\boldsymbol{x}$. The above optimization can be written also in a Lagrangian form:

$$\widehat{\boldsymbol{\alpha}} = \boldsymbol{arg\,min}_{\boldsymbol{\alpha}} \boldsymbol{\lambda}\|\boldsymbol{\alpha}\|_0 + \frac{\boldsymbol{1}}{\boldsymbol{2}}\|\boldsymbol{D\alpha} - \boldsymbol{y}\|_2^2 \tag{3.6}$$

Such that the constraint of $(\boldsymbol{C\sigma})^2$ becomes part of the penalty. The two problems can become equivalent with a proper choice of $\boldsymbol{\lambda}$ as the Lagrange multiplier.

Moving on to an arbitrary image size for $\boldsymbol{X}$ problem of size $\sqrt{\boldsymbol{N}} \times \sqrt{\boldsymbol{N}}$**,** its noisy version of $\boldsymbol{Y}$ and considering their patch extractions in every patch of $\boldsymbol{X}$**.** This leads to a problem for denoising:

$$\boldsymbol{arg\ min}_{\alpha_k, X}\ \frac{\mu}{2}\|X - Y\|_2^2 + \sum_k (\lambda_k \|\alpha\|_0 + \frac{1}{2}\|D\alpha_k - R_k X\|_2^2) \qquad 3.7$$

The first term describes the log-likelihood that demands a close relationship between the noisy image $\boldsymbol{Y}$ and the denoised image $\boldsymbol{X}$. The penalty term would have read as $\|\boldsymbol{X} - \boldsymbol{Y}\|_2^2 \leq (\boldsymbol{C\sigma})^2$**,** which reflects the direct relationship between $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ [1]. A selection of $\boldsymbol{\mu}$ must be in relationship with the constraint $(\boldsymbol{C\sigma})^2$. While the second term holds for the image prior to ensures that the constructed image, $\boldsymbol{X}$, in every patch $\boldsymbol{x_k} = \boldsymbol{R_k X}$ of size $\sqrt{\boldsymbol{n}} \times \sqrt{\boldsymbol{n}}$ has a sparse representation with bounded error. The parameter $\boldsymbol{R_k}$ stands for matrix $\boldsymbol{R_k} \in \boldsymbol{R}^{\boldsymbol{n \times N}}$, which put the patches together.

The discussion has been based on the assumption that dictionary $\boldsymbol{D}$ is known. Thus, what if the dictionary $\boldsymbol{D}$ is unknown? In the experiment for the unknown dictionary, the dictionary $\boldsymbol{D}$ is learned using the additive noise patches from the image $\boldsymbol{Y}$. The problem can be defined as:

$$\{\widehat{D}, \widehat{\alpha}_k, \widehat{X}\} = \boldsymbol{arg\ min}_{\widehat{D}, \alpha_k, X}\ \frac{\mu}{2}\|X - Y\|_2^2 + \sum_k (\lambda_k \|\alpha_k\|_0 + \frac{1}{2}\|D\alpha_k - R_k X\|_2^2) \qquad 3.8$$

Instead of solving all problem at once, we start with initialising the dictionary $\boldsymbol{D}$ as the overcomplete dictionary and set $\boldsymbol{X} = \boldsymbol{Y}$ to seek the optimal $\widehat{\boldsymbol{\alpha}}_{\boldsymbol{k}}$. Then iterate between OMP over all the patches and update the dictionary, $\boldsymbol{D}$ using the K-SVD method. Given those representations, we now update the dictionary $\boldsymbol{D}$. In doing so, the *Equation* **3.8** can be separated into two tasks [2]:

$$\widehat{\alpha}_k = \boldsymbol{arg\ min}_{\alpha_k}\ \lambda_k \|\alpha_k\|_0 + \frac{1}{2}\|D\alpha - x_k\|_2^2 \qquad 3.9$$

Given all the $\hat{\boldsymbol{\alpha}}_{\boldsymbol{k}}$ representations, and the updated dictionary, we now fix those parameters and seek to find $\boldsymbol{X}$. Referring to the expression in equation (1), the $\boldsymbol{X}$ problem can be solved in form:

$$\hat{X} = arg\ min_X \frac{\mu}{2} \|X - Y\|_2^2 + \sum_k \frac{1}{2} \|D\alpha_k - R_k X\|_2^2 \quad 3.10$$

After numbers of iterations, the dictionary admits a content that adapted to the measured image, and the representations now ready for the averaging stage which to produce the output image [1].

3.3 FISTA Algorithm

FISTA (Fast Iterative Shrinkage-Thresholding Algorithm) is an iterative algorithm for solving composite problems based on the use of proximal operators. FISTA is the new ISTA with an improved complexity result of $O(1/k^2)$. FISTA can be defined as:

$$\beta_k = S_{\frac{1}{L}}\left(w_k - \frac{1}{L}\left((X^T X + \mu I)w_k - X^T y\right)\right)$$

Where $S_\gamma(z) = \mathrm{sign}(z)(|z| - \gamma)$ and L is an estimation of the Lipschitz constant value. For this Lipschitz constant,L problem in the ℓ_1 optimization problem, it depends on the maximum eigenvalue of the $X^T X$, where X is the dictionary matrix [6]. Each iteration involves matrix-vector multiplication involving X and X^T followed by a shrinkage/soft-threshold step. The Lipschitz constant might be difficult to compute as the problem data goes larger. For more details refer to the original paper [6].

4 Method of experiments

Using the MATLAB environment, the first stage of the project, an experiment being developed by replicating the work in [3] to develop the understanding of the K-SVD algorithm. The experiment implementing artificial signal data to train the dictionary and ability to recover the target sparsity from the output coefficient. Setting the parameters dictionary size, $D = 20 \times 50$ each column was normalized to a unit ℓ_1-norm. Later, artificial data signals, $Y = 200$ was generated and added some random white gaussian noise, $snr = 30\text{dB}$. Using the OMP as the matching pursuit method, the targeted sparsity value was set to $S = 5$ and the maximum iterations were set to 120 times to train the dictionary. The experiment sought to find the relationship of RMSE value for each iteration. The Root Mean Square Error (RMSE) means the error representation between the generated signal and the denoise signal. The lower the RMSE value the better representation of the data.

Then, the experiment was expanded to introduce an unseen data signal. Using the unseen data signal, the trained dictionary from the generated data signal was used to solve the unseen signal in which we sought to find the capability of the trained dictionary to reconstruct the problem and returned the average value of the problem similar to the sparsity value, $S = 5$. This experiment was conducted in three cases where the noise level of generated and unseen data signal was varied:

Data	Case 1	Case 2	Case 3
Generated	No noise	Low noise	High noise
Unseen	No noise	No noise	Low noise

Table 4-1 Three cases of noise level

On the second part of the experiment using the same setting of parameters but we only change the matching pursuit method with FISTA algorithm instead of OMP. The FISTA algorithm MATLAB code from H. Kasai was used as a reference with a bit of modification to fit the K-SVD algorithm [7]. It is stated that any matching pursuit algorithm can be used with K-SVD to train the dictionary. However, introducing the FISTA as matching pursuit algorithm also introduce the new parameter lambda, λ as the Lagrange Multiplier. This parameter $\lambda > 0$ provides a tradeoff between fidelity to the measurements and noise sensitivity [6]. Thus, by varying the lambda values such $\lambda = 0.1, 0.3, 0.5 \text{ and } 0.9$, the experiment sought to find an optimum value which balance the sparsity and dictionary reconstruction coefficient. Other than that, the optimum lambda value should return an average value similar to the targeted sparsity value.

The experiment later, implement K-SVD algorithm with OMP to train dictionary from image dataset of brain tumor MRI and the image is introduce with low level of Gaussian noises using the MATLAB code developed by M. Elad [8]. The experiment sought to find the difference of Peak-Signal-Noise-Ratio (PSNR) between the noisy image and the denoise image as well as the reconstruction structure of those images.

5 Result

5.1 K-SVD with OMP in Artificial Signal Experiment

The first stage of the project, team had developed an experiment by replicating the work in [3] to develop understanding of the K-SVD algorithm. Using the method mentioned in Section 4,the experiment sought to find the returned average value of the output coefficient. Ideally, the average value should return a equal value the sparsity $S = 5$. Table 5-1 shows that the K-SVD with OMP matching pursuit able to reconstruct the generated and unseen data using the trained dictionary. This result was supported by validating the average value of the sparse coefficient for both cases. The average value for both cases produced equal value as the sparsity. Thus, it can be said that the K-SVD with OMP algorithm capable to reconstruct the problems.

Generated Data		*Unseen Data*	
Average	Standard deviation	Average	Standard deviation
5	0	5	0

Table 5-1 Comparison initial data vs unseen data for targeted sparsity and average

Other than that, the experiment sought to find the relationship of RMSE for each iteration. The Root Mean Square Error (RMSE) here means the error representation between original and denoise signals. The lower the RMSE value the better error representation of the data. Based on the Figure 5-1, it shows that the RMSE values of the generated data signals decreased with number of iterations. Later, the RMSE value started to reach a steady state condition at 120 of iteration. This shows that at certain point, the K-SVD algorithm had reached the error threshold and the error became constant for further iterations.

Table 5-2 shows the training time taken to train the dictionary was very fast about 7 seconds. While the time taken to solve the unseen data using the trained dictionary also fast. This is due to OMP have an advantage of its simplicity and fast application.

	Generated Data	*Unseen Data*
Training Time (sec)	7.44	6.32

Table 5-2 Training time taken to solve the generated and unseen data.

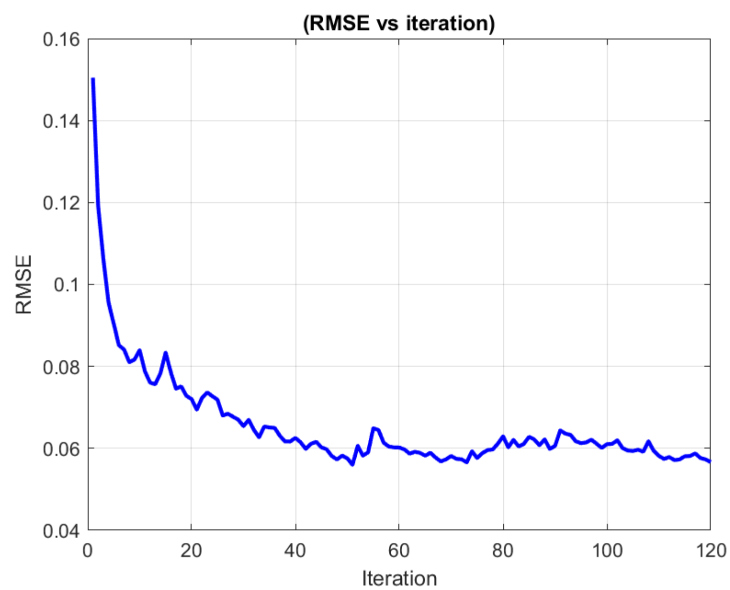


Figure 5-1 RMSE vs iteration using K-SVD with OMP

On the second part, the experiment was conducted in three cases of noise level. First case, there were no noise applied for both generated and unseen data. While, for the second case low noise value was only applied to the unseen data and lastly, a higher noise level was applied to the unseen data. This experiment was performed to analyse the SNR output error for the three cases of different noise level. Based on Table 5-3, the output SNR error for generated data in three cases were high which indicate the signal strength is stronger in relation to the noise level. However, the output SNR error for the unseen data were extremely low, which almost 12 dB difference in relative to the generated data values. This means that the trained dictionary was unable to reconstruct the unseen data signal effectively.

Noise level	*Output SNR Error (dB)*	
	Generated Data	Unseen Data
No/No	19.41	7.79
No/Low	19.21	7.81
No/High	18.79	7.73

Table 5-3 Output SNR Error for different noise level

Using the K-SVD as the dictionary learning algorithm, it able to train the dictionary from the generated data in an iterative technique that alternate between sparse coding of the generated data using the current dictionary and the dictionary atom was updated in the process to better fit the data. Then, the dictionary columns were updated in conjunction with the sparse representation coefficient associated with them. This resulting to a faster convergence.

5.2 K-SVD with FISTA in Artificial Signal Experiment

The previous experiment was done by implementing the K-SVD with OMP as the matching pursuit algorithm. Now, OMP algorithm was replaced with alternative matching pursuit which was FISTA algorithm. Using the similar setting parameters from the previous experiment and setting the Lagrange multiplier, $\lambda = 0.1, 0.3, 0.5 \text{ and } 0.9$. The experiment sought to find an optimum value of lambda which balance between the sparsity and good approximation of data. Finding the optimum lambda value required a trial-and-error process. Based on the Table 5-4, the optimum lambda value was 0.3, which the generated data produced the average value = 5.6 almost equal to sparsity = 5. Although, the average value of generated data almost equal to the sparsity value, the average value for the unseen data was almost twice than the generated data value, average = 9.64. This means that FISTA algorithm was unable to reconstruct the problem effectively due to the average value of the unseen data was almost twice larger than the generated data value. Increasing the lambda value more than result in over-penalized between sparsity and good approximation of the data which was bad.

λ	GENERATED DATA		UNSEEN DATA	
	Average	Standard deviation	Average	Standard deviation
0.1	12.16	2.36	16.8	2.12
0.3	5.60	2.32	9.64	3.48
0.5	2.84	1.60	6.32	3.20
0.9	1.29	0.78	2.36	1.93

Table 5-4 Average value for K-SVD with FISTA

Later, the experiment expanded to analyses the output SNR error for both generated and unseen data. This experiment was performed to analyse the SNR output error for the three cases of different noise level. Based on Table 5-5, the output SNR error for generated and unseen data decreased with larger lambda value. Taking the optimum lambda value = 0.3, the difference of output SNR error between generated and unseen data was about 8 dB. This can be said that FISTA algorithm incapable to reconstruct the problem coefficient effectively.

λ	*Output SNR Error (dB)*	
	Generated Data	Unseen Data
0.1	25.97	16.95
0.3	16.11	7.99
0.5	12.45	5.03
0.9	7.75	1.91

Table 5-5 Output SNR error for K-SVD with FISTA for different lambda

Table 5-6 shows the training time taken for FISTA to train and solve the generated data took much longer time which almost 100 seconds for different lambda parameter. While it took up to 200 seconds time even longer to solve the unseen data. This probably due to the computational stress that needed to compute the eigen value of $X^T X$.

	Training Time	
λ	Generated Data	Unseen Data
0.1	125.3	229.8
0.3	100.5	195.9
0.5	102.2	202.2
0.9	100.7	201.1

Table 5-6 Training time taken to solve generated and unseen data for FISTA

Figure 5-2 shows the RMSE vs iteration for different lambda values. We had discussed that the larger the lambda value results in over-penalised of the sparsity. Thus, at lambda = 0.5 and 0.9 it almost flattens the RMSE value for each iteration. While at lambda = 0.3, it produces much similar result to the OMP methods.

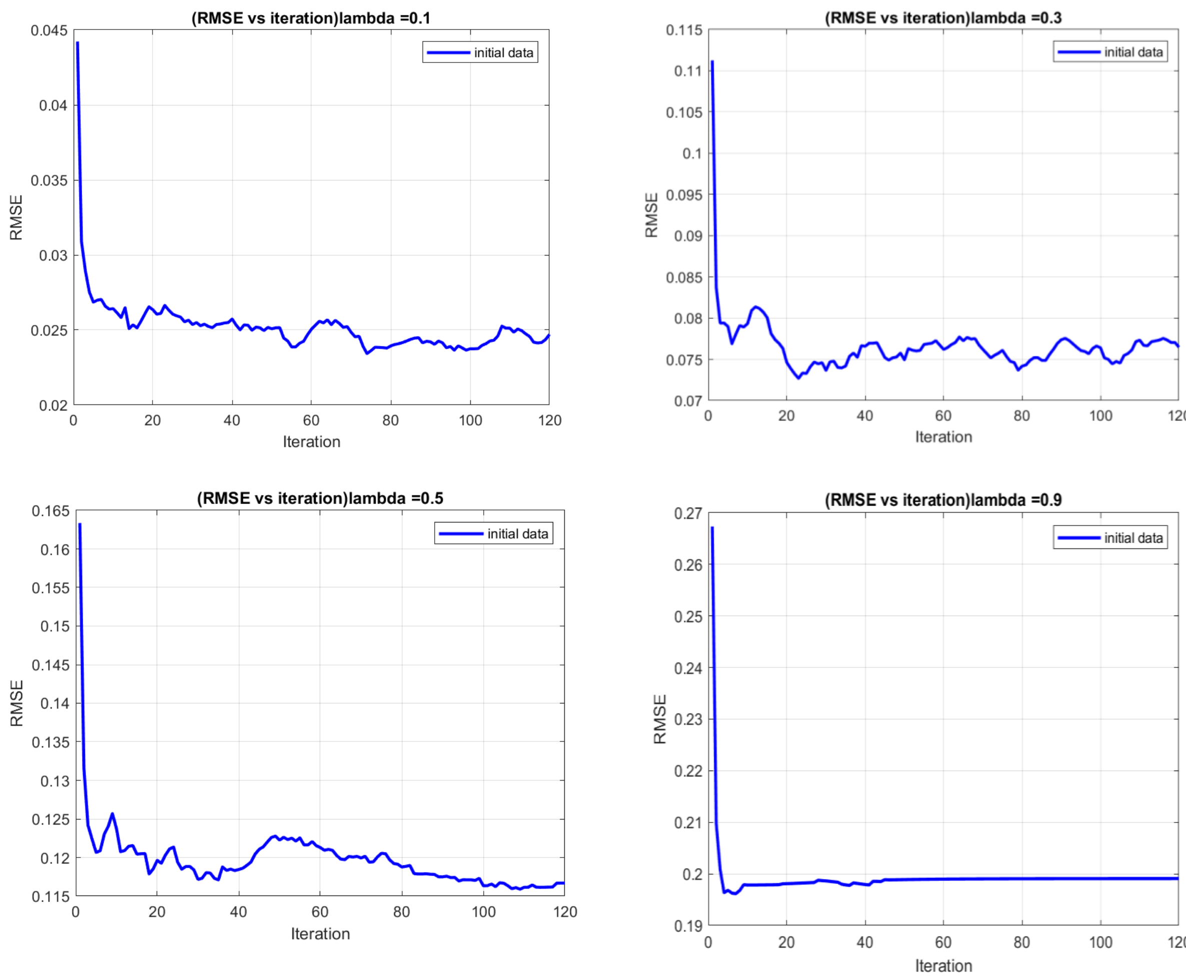


Figure 5-2 RMSE vs iteration for lambda = 0.1, 0.3, 0.5 and 0.9

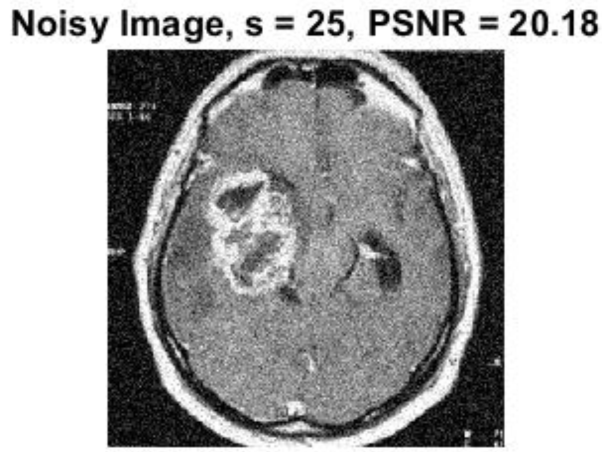
5.3 K-SVD with Image Dataset Experiment

Following of the K-SVD experiment with artificial signals, another experiment was developed using the MRI brain image as in Figure 5-3 (a) Original brain image. (b) Noisy brain image. (c) Denoise brain image with size 300×300 pixels as the image datasets as the image problem. The work proposed on creating the patches from the corrupted image. Following the K-SVD algorithm, we can assume a fixed dictionary and image patches, and seek to find the sparse representations. This can be done by the sparse coding method that deploys OMP. After finding the sparse representations, the dictionary was updated using the SVD operations and this process alternate up to 80 iterations. In order to quantify the denoising results from the experiment, we use the peak-signal to noise-ratio (PSNR) to measure the capability of reconstruction image between the noisy image and denoise image [3]. We compute the PSNR values in dB by calculating the RMSE between the original with noisy and denoise images. The higher the PSNR, the better the reconstructed image quality. Figure 5-3 (a) Original brain image. (b) Noisy brain image. (c) Denoise brain image successfully being denoise with about 7 dB of improvement compared to the (b) Noisy brain image. Thus, this concluded that the K-SVD algorithm using the OMP as matching pursuit able to denoise image problem through several iterations.

The dataset used for the project was about the MRI brain scan which was obtained from the open-source platform GitHub [8]. A brain tumour occurs when abnormal cells form within the brain. There were two main types of tumours: which cancerous and benign. The properties of the image that the project interested was due to the same black background colour and white colour for the brain image. The brain image created a same pattern around the edge of the brains which made the solving the approximation of the data using the trained dictionary much easier rather than having a random pattern of images.

Figure 5-3 (a) shows that the original image of the brain was clear from any noise in which later in Figure 5-3 (b) it was added with low noise level resulting into a corrupted structure of the image. Performing the K-SVD algorithm to train the dictionary based on the noisy image Figure 5-3 (b). It able to reconstruct and denoise the noisy image again as in Figure 5-3 (c). However, there were some structures of the image being over smoothen but able to preserve the details of the image such as edges of the image.

(a)



(b)

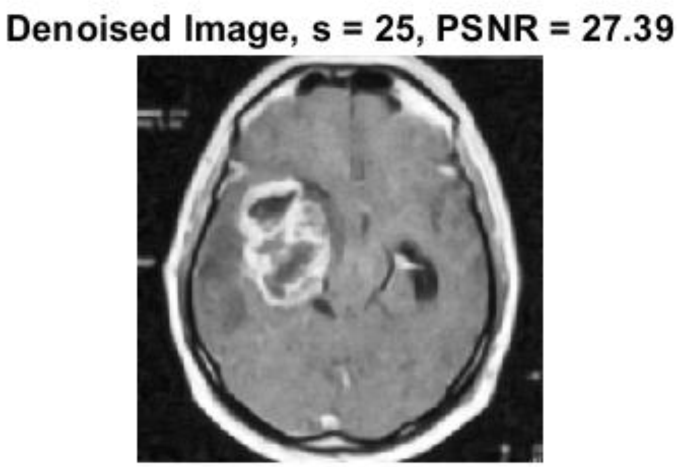


Figure 5-3 (a) Original brain image. (b) Noisy brain image. (c) Denoise brain image

6 Discussion

This discussion compared the results between K-SVD with OMP and K-SVD with FISTA. Based on the experiment K-SVD with OMP, the average value for both generated and unseen data produced same value as the sparsity. While using the FISTA as the matching pursuit the average value of the generated data almost equal to targeted sparsity value. However, the average value of the unseen data was twice bigger than the generated data. This means that the OMP able to approximately represent the data signal better compared to the FISTA. Nevertheless, for both methods using the K-SVD algorithm as dictionary learning techniques able to train the dictionary from the generated data signal successfully. In general, it is expected, the larger (denser) the dictionary, the better the resulting matching pursuit decomposition at a higher computational cost.

On top of that, the output SNR error for both methods produced almost similar results, where the output SNR error for the generated data produced higher SNR value while the unseen data produced lower SNR error. The difference between the output SNR error for both methods was huge which almost more than 8 dB made the unseen data SNR value much worst. The reason was due to the dictionary being trained with only one sample of data signal instead of multiple data signal which made the dictionary was not well trained to solve an unseen data signal. Ideally, the dictionary would be trained with larger dataset i.e., 500 data samples, 2000 data samples. Other than that, implementing FISTA as matching pursuit algorithm took longer computational time to solve the problem compared using the OMP. The residual vector in the OMP is always orthogonal to the columns that have been selected. As a result, no columns will be selected twice, and the number of selected columns will be increased as the iterations progress. The FISTA took longer time for the residual error to converge to zero as well as using the calculation of $X^T X$ which made the dictionary matrix larger to train the dictionary. Thus, conclude the modified version of K-SVD using the FISTA was unable to perform very well compared to the OMP.

Future work would require training the dictionary with larger dataset to well-trained the dictionary in order to solve the unseen problem effectively. This work would require the larger dictionary size to fit the large dataset thus longer computational time to train the dictionary. Further work would require experimenting with the varies of lambda, λ for a larger dictionary as the parameter needed to be modified with increasing of data samples and dictionary sizes. Finding an optimal lambda, λ parameter is difficult when the optimization problem is hard. Other than that, would be implementing the FISTA algorithm to solve a noisy image problem and comparing it with the K-SVD using the OMP.

7 Conclusion

As a conclusion, the K-SVD algorithm manage to train the dictionary and update the dictionary from the given problem dataset. While the matching pursuit algorithm OMP and FISTA able to recover the signal in iterative way, by making a local optimal selection at each iteration in the hopes of finding the global optimum solution at the end of the algorithm. However, the OMP is much faster to converge the residual to zero compared to the FISTA algorithm. While FISTA required an optimum lambda parameter to regularize between the sparsity and good approximation of data. OMP correctly find the less non-zero value positions compared to FISTA and able to estimate the non-zero coefficient resulting the average value to equal the desired sparsity value. The dictionary learning and its greedy algorithms have been extensively studied and were used in wide range of applications. The usefulness of dictionary learning is becoming increasingly essential las the demand for cheaper, faster, and more efficient devices grows.

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