THE UNIVERSITY of ADELAIDE

IMAGE DENOISING WITH DICTIONARIES

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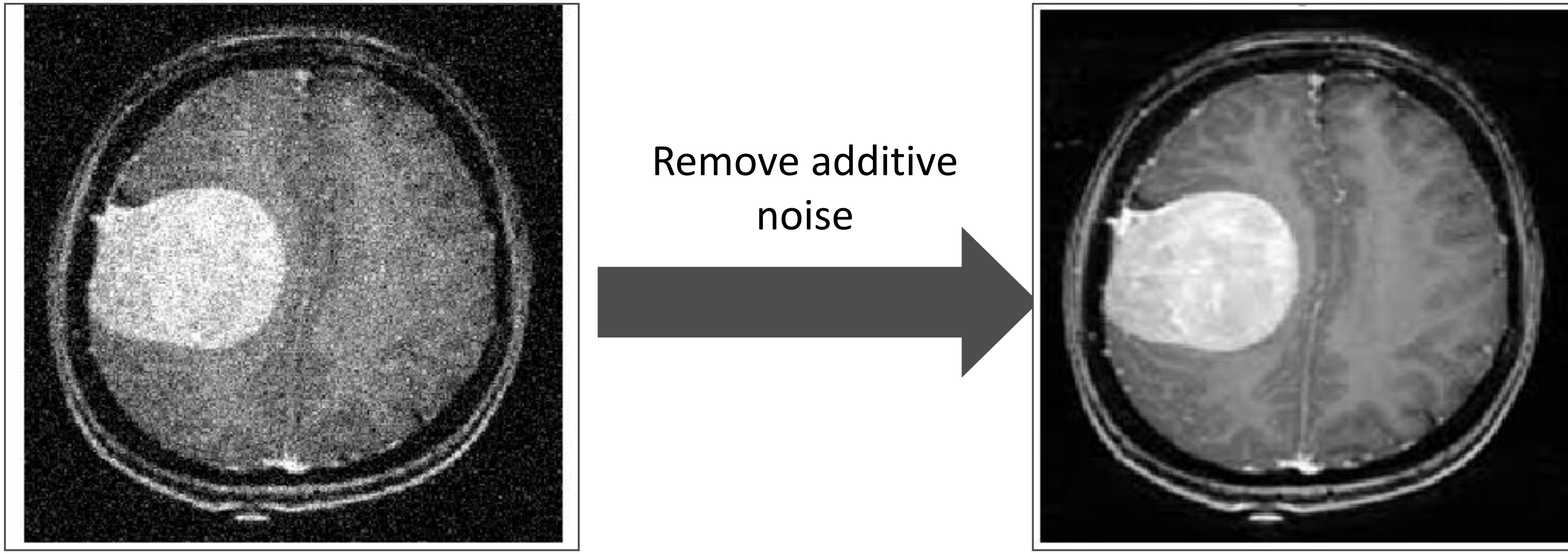
Outline

- Project Background
- K-SVD Dictionary Learning
- Orthogonal Matching Pursuit
- Artificial Experiment
- Individual Dictionary
- Universal Dictionary
- Future Work
- Conclusion

Project Background



$$\mathrm{y} = x + n$$

y : Measured image

x : True image

n : Gaussian noise

Dictionary Learning

- **Image denoising methods:**
  - **Spatial domain**
  - **Transform domain**
  - **Dictionary learning**

- **Dictionary learning aims at finding a dictionary in which some training data admits a sparse representation**

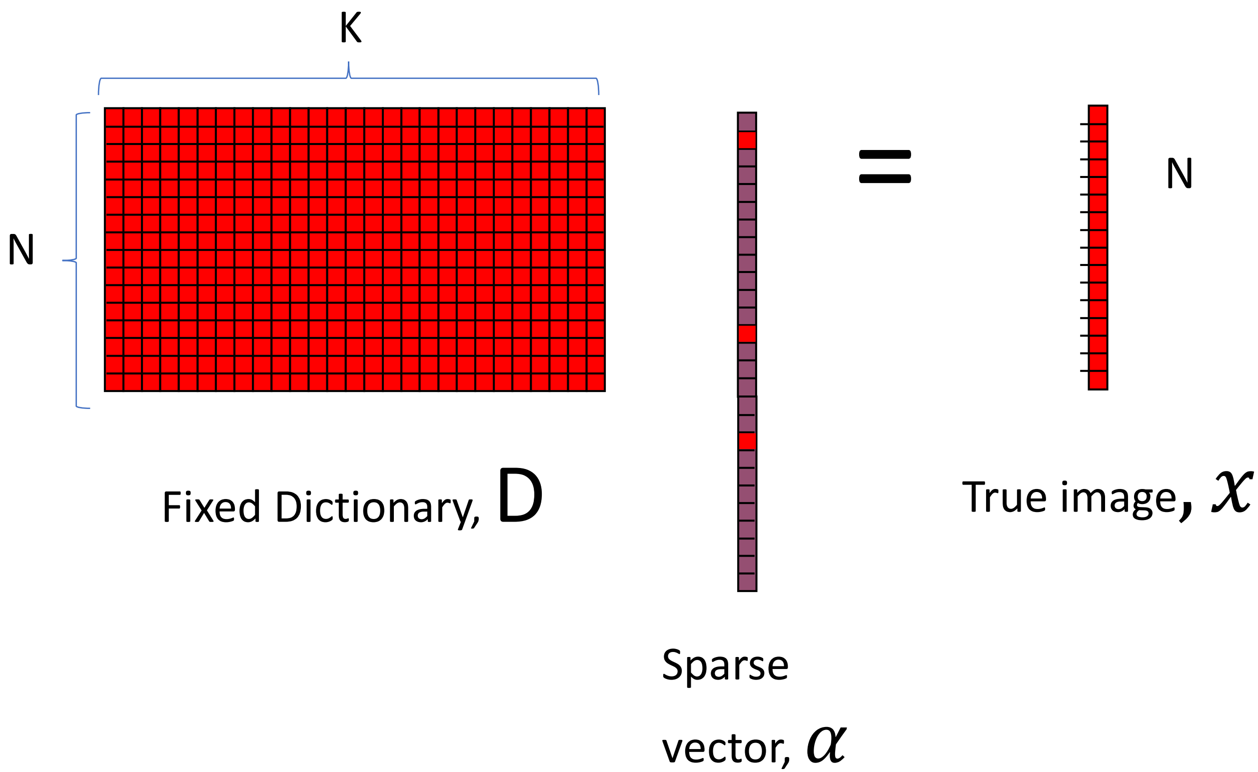
Dictionary Learning

- **Dictionary Learning Algorithms**
  - **K-SVD**
  - **Online Dictionary Learning**
  - **Stochastic Gradient Descent**
  - **Lagrange Dual Method**

Dictionary Learning

- We want to represent signal $\boldsymbol{x} \in \boldsymbol{R}^{\boldsymbol{n}}$
- A dictionary, $\boldsymbol{D} = [\boldsymbol{d}_{\mathbf{1}} \ldots \boldsymbol{d}_{\boldsymbol{k}}] \in \boldsymbol{R}^{\boldsymbol{n} \times \boldsymbol{k}}$
- Each $\boldsymbol{d}_{\boldsymbol{i}}$ is called an atom
- Goal: find a linear combination of
  a “ few ” atoms from $\boldsymbol{D}$ that is “ close ” to
  original signal $\boldsymbol{x}$

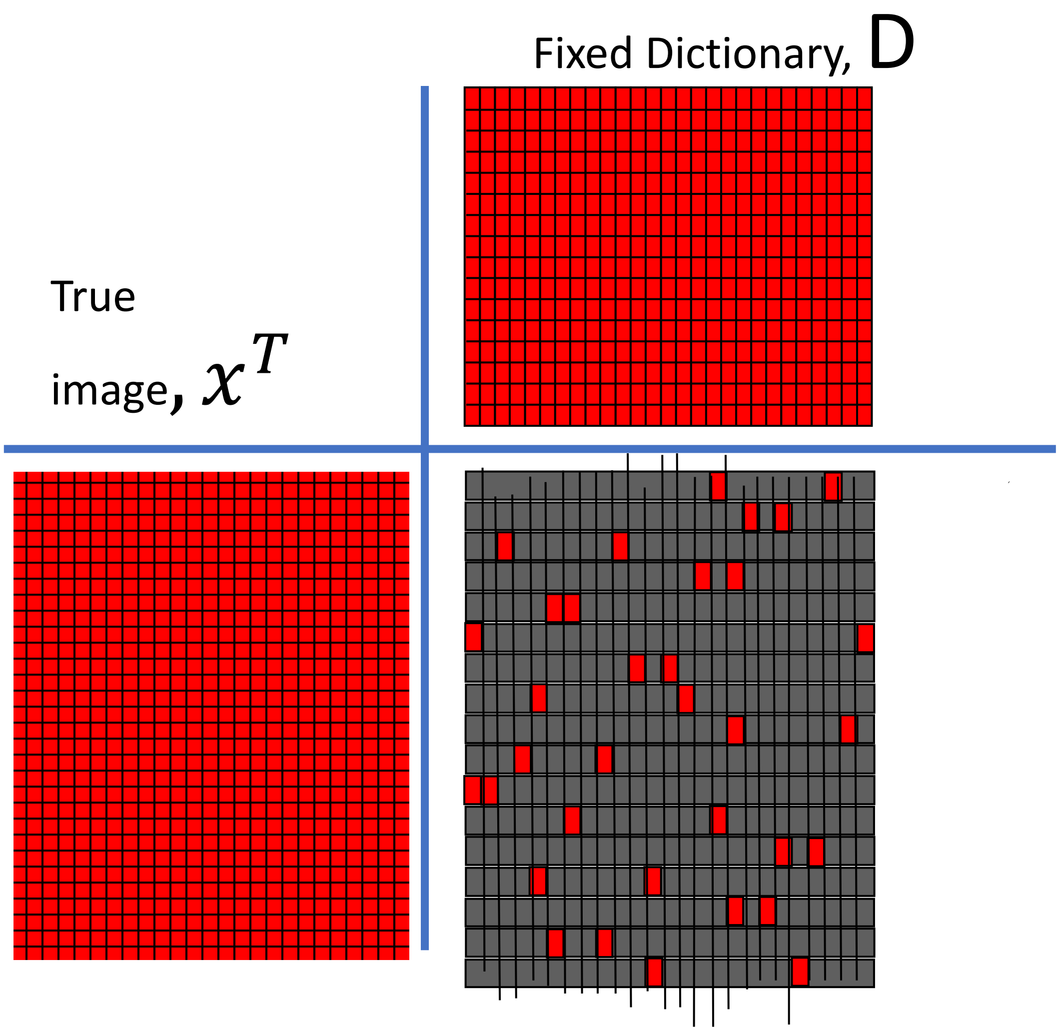
Dictionary and Sparse Representation

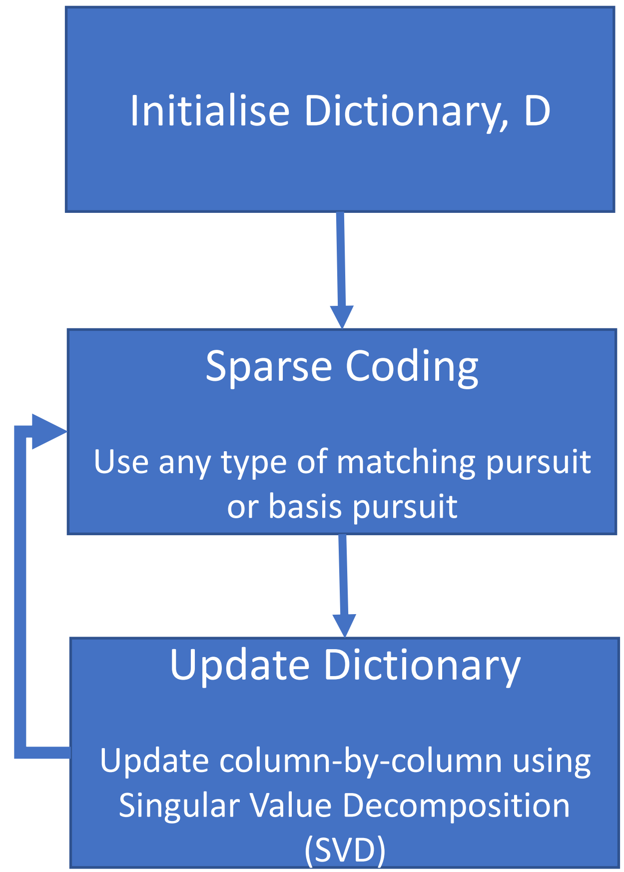


- The vector α is with few non-zeros
- Simple: Every signal is built as a linear combination of few atoms from the dictionary D

Source: Aharon, Elad & Bruckstein ('04)

K-SVD algorithm (General)

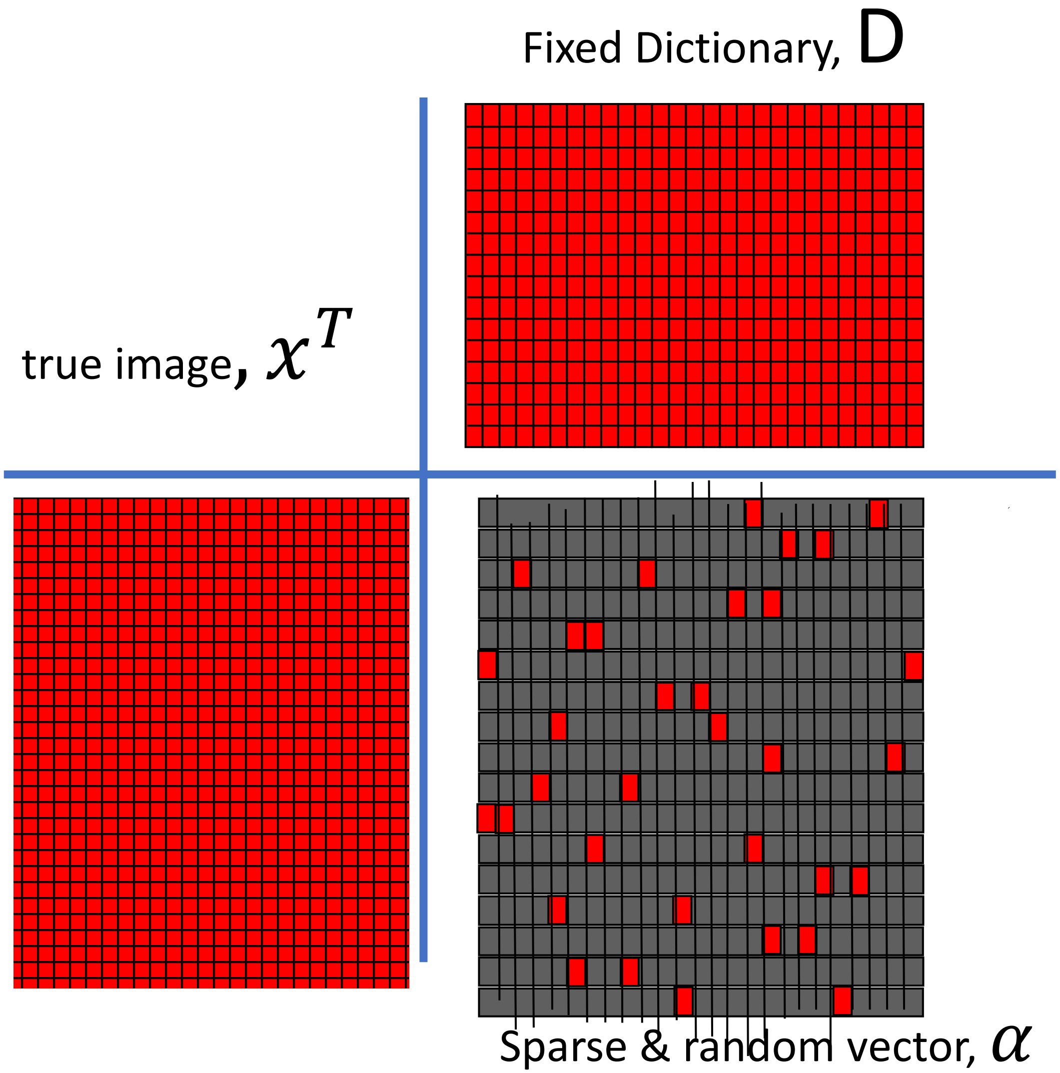




Source: Aharon, Elad & Bruckstein (‘04)

Sparse & random vector, α

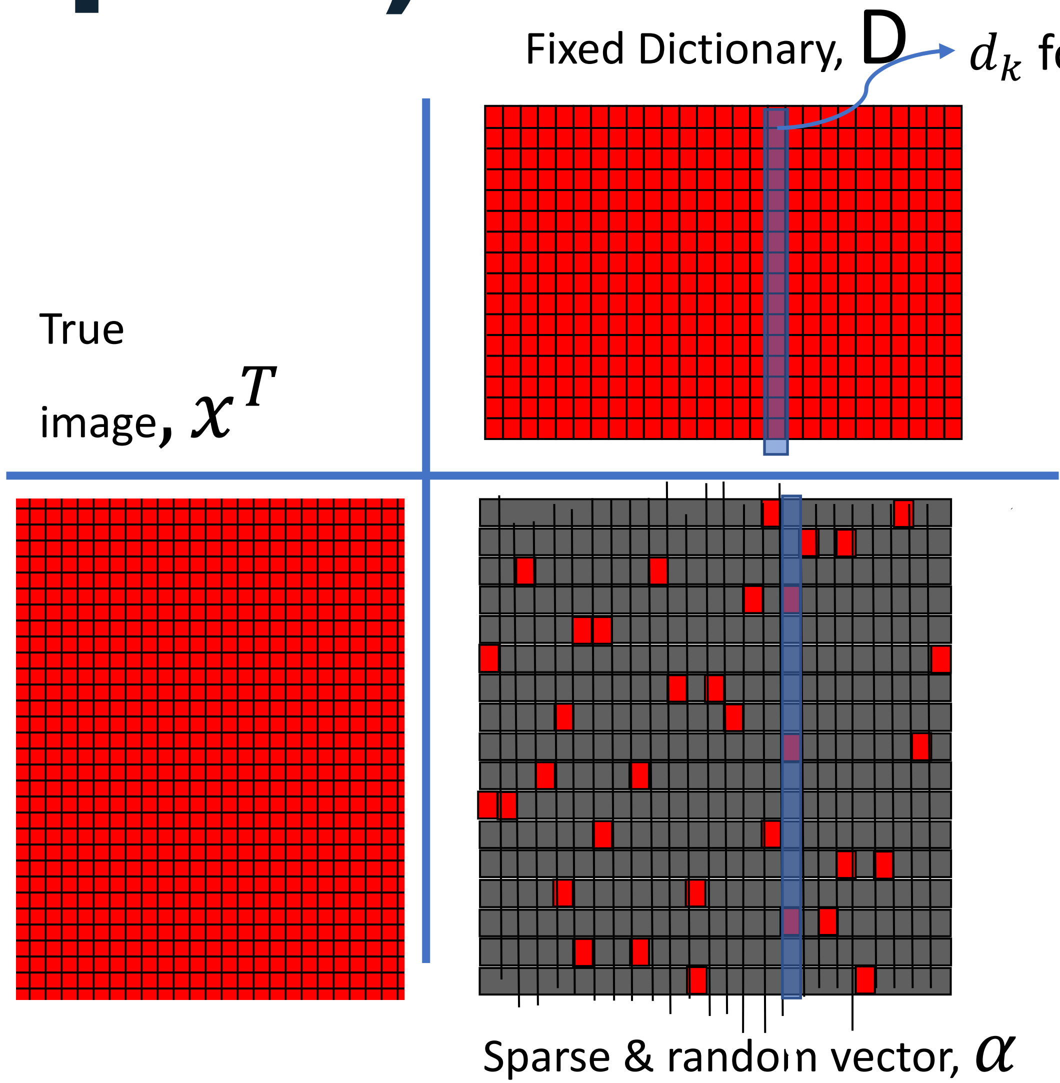
K-SVD algorithm (Sparse Coding)



$$\hat{\alpha} = \boldsymbol{arg\,min}\,\|\alpha\|_{\mathbf{0}}$$
$$\textbf{subject to}\ \|\boldsymbol{D}\alpha - x\|_{\mathbf{2}}^{\mathbf{2}}$$

Sparse coding by using any sparse coding algorithm

K-SVD algorithm (Dictionary Update)



Let us fix all α and D apart from the α_k column and seek both d_k and the α_k column to better fit the residual!

$$\boldsymbol{arg\,min}\ d_k \boldsymbol{\alpha}_k$$

subject to

$$\| d_k \boldsymbol{\alpha}_k{}^T - E_k \|_{\boldsymbol{F}}^{\boldsymbol{2}}$$

Residual, E_k

$$E_k = \sum \quad d_k \boldsymbol{\alpha}_k{}^T - x$$

Orthogonal Matching Pursuit (OMP)

1. Initialise residual, $r_0 = x - D\alpha = x$, $\quad \alpha_0 = 0$
2. Compute $E(k) = min\, z \|z \cdot d_k - r_{i-1}\|$

   for $1 \leq k \leq m$
3. Choose k_0 s.t. $\forall 1 \leq k \leq m, E(k_0) \leq E(k)$
4. Update S_i: $S_i = S_{i-1} \cup \{k_0\}$
5. Update coefficient: $\alpha_i = \min \alpha \|D\alpha - x\|_2^2$

   s.t. support $\{\alpha\}$ = S_i
6. Update residual: $r_i = x - D\alpha$

OMP Strategy: choose the next non-zero value such that reduce the "energy" in the residual as best as possible

Fast Iterative Shrinkage Threshold Algorithm (FISTA)

FISTA with constant stepsize

Input: $L = L(f)$ - A Lipschitz constant of ∇f.

Step 0. Take $\mathbf{y}_1 = \mathbf{x}_0 \in \mathbb{R}^n$, $t_1 = 1$.

Step k. $(k \geq 1)$ Compute

$$\mathbf{x}_k = p_L(\mathbf{y}_k), \tag{4.1}$$

$$t_{k+1} = \frac{1 + \sqrt{1 + 4t_k^2}}{2}, \tag{4.2}$$

$$\mathbf{y}_{k+1} = \mathbf{x}_k + \left(\frac{t_k - 1}{t_{k+1}}\right)(\mathbf{x}_k - \mathbf{x}_{k-1}). \tag{4.3}$$

Source : Beck A and Teboulle M, "A Fast Iterative Shrinkage-Thresholding Algorithm for Linear Inverse Problems," SIAM Journal on Imaging Sciences, vol. 2, no. 1, pp. 183-202, 2009.

- **Lipschitz constant in $\ell 1 - \text{regularisation}$ (LASSO) problem depends on the maximum eigenvalue of $D^T D$**
- **For large-scale problem this quantity is not always computable**

Artificial Experiment

- Analyse the performance of dictionary reconstruction
- Time taken for the algorithm to denoise the signal
- Output SNR error between the generated and unseen data

Artificial Experiment

- Apply alternative matching pursuit FISTA
- FISTA introduce new parameter λ, Lagrange Multiplier
- Optimum λ is needed to balance between sparsity and approximation of signal
- Dictionary size, D = 20×50 ,
- Artificial data size, X = 20×100, Sparsity, S = 5

Artificial Experiment

MATCHING PURSUIT	λ	AVERAGE SPARSITY OF COEFFICIENT		TRAINING TIME TO TRAIN DICTIONARY (SECONDS)		OUTPUT SNR ERROR OF THE SIGNAL (DB)	
		Generated Data	Unseen Data	Generated Data	Unseen Data	Generated Data	Unseen Data
OMP	-	5	5	7.44	6.32	19.41	7.79
FISTA	0.1	12.16	2.36	125.3	229.8	25.97	16.95
FISTA	0.3	5.60	9.64	100.5	195.9	16.11	7.99
FISTA	0.5	2.84	6.32	102.2	202.2	12.45	5.03
FISTA	0.9	1.29	2.36	100.7	201.1	7.75	1.91

- Optimum lamba for FISTA is $\lambda = 0.3$
- OMP performs better and faster than FISTA

Individual Dictionary

- 15 test images with the same size
- Cut the image into 8 by 8 small patches
- Added with Additive white Gaussian noise
- Use the same dictionary to denoise the image

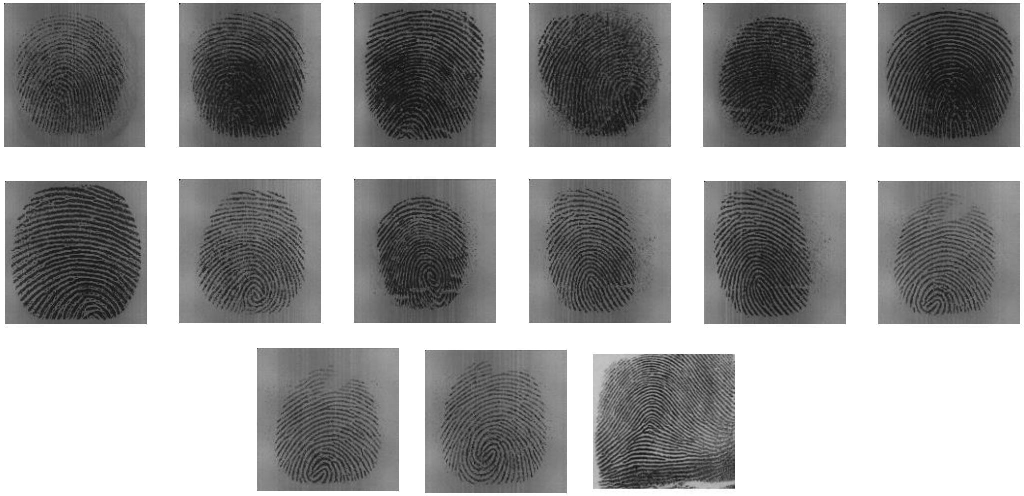
SNR = 14.9 dB

SNR = 22.6 dB

Universal Dictionary

- One trained dictionary from 15 test images using K-SVD algorithm
- Combine all the patches from various images
- Expand the 8 x 8 patches of the new noisy image using the trained dictionary, call OMP
- Size of dictionary used is 64 x 512 ($n = 64$, $k = 512$)
- Executed over 100 iterations

Universal Dictionary



15 test images to train the universal dictionary

Applied with universal dictionary

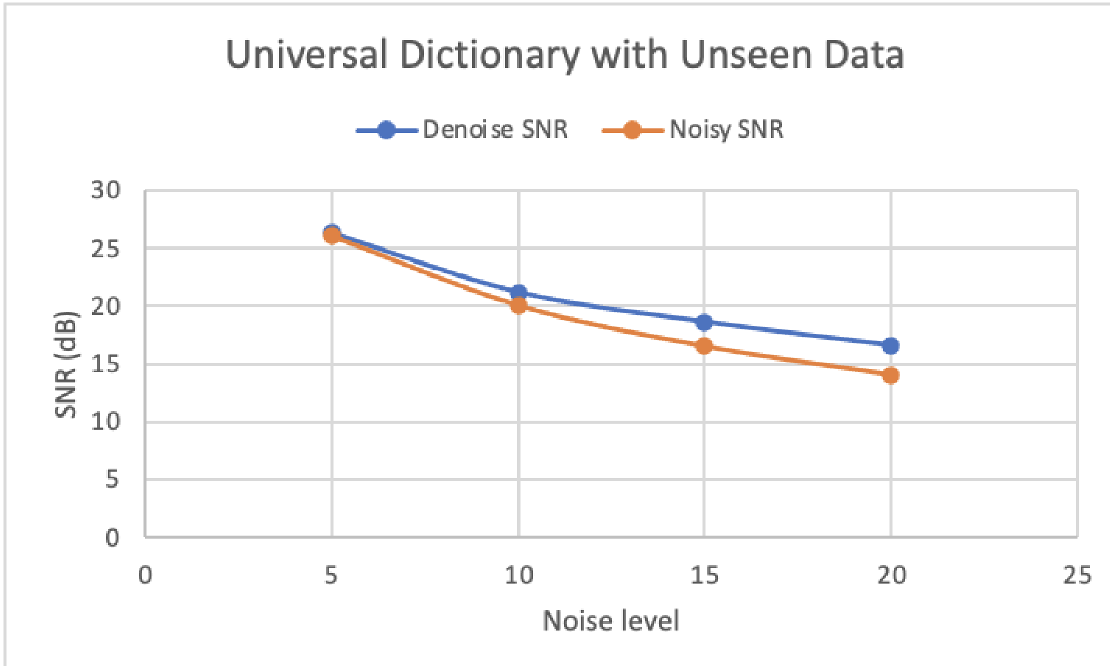
Unseen image (Noisy)

SNR = 14.06 dB

Unseen image (Denoised)

SNR = 16.6 dB

Result (Universal Dictionary)



- Improvement of approximately 2 dB
- All the images used are the same size

Comparison

- Individual dictionary has improvement of 6 dB
- Universal dictionary has improvement approximately 2 dB
- Universal dictionary is more practical in real world application

Conclusion

- K-SVD with OMP performs better than FISTA due to its simplicity and fast convergence to zero
- Universal dictionary is more powerful as it able denoise unseen data. The dictionary could be trained using larger dataset to make more useful for real world application

Future Work

- Trained dictionary with much more data to create denser dictionary
- Applied the image problem with FISTA
- Compare the universal dictionary with individual dictionary

Primary references

1. Elad, M & Aharon, M 2006, ‘Image Denoising Via Sparse and Redundant Representations Over Learned Dictionaries’, IEEE Transactions on Image Processing, vol. 15, no. 12, pp. 3736–3745.
2. Aharon, M, Elad, M & Bruckstein, A 2006, ‘K-SVD: An algorithm for designing overcomplete dictionaries for sparse representation’, IEEE Transactions on Signal Processing, vol. 54, no. 11, pp. 4311–4322.

Questions and Answers

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