Project 1 - xijie zhou

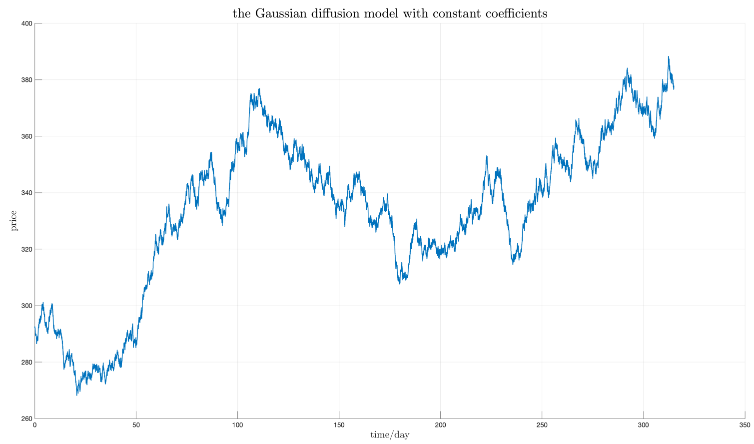
1 Exercise 1

1.1 A

n : steps per day(intervals in a day when we observe)
T : the total number of days when we observe
μ: average of returns per day
σ: volatility of returns per day

market index returns per year is μ*252 business days;
σ^2/252 business days is the volatility of market index returns in one year.

1.2 B



Interpret: This figure represents a Gaussian diffusion model with constant coefficients.The x-axis is the time in days and the y-axis is the asset price. We can see that the price fluctuations in the observation interval are very intense and generally W-shaped. At first, it experienced a sharp rise after the decline, and entered a period of volatility decline in about 110 days, and then began to rise again in about 200 days.

Code:

Listing 1: ex_1.m

```
n = 80;%steps per day
T = 1.25*252;%the whole days
```

```
mu = 0.03782/100;%local predictable risk premuim parameter
sigma = 0.011;%the risk of the outcome paramter
delta = 1/n;%discretization interval
x0 = log(292.58);

seed = 3005;
rng(seed,'twister');

% x = zeros(n*T+1,1);%create zero filled array
% z = randn(n*T+1,1);%create normal distribution
% x(1) = log(292.58);%define x(0)
%
% %simulate the process
% for i = 1:n*T
% x(i+1) = x(i) + mu*delta + sigma*sqrt(delta)*z(i+1);
% end
x = simDCC(x0,mu,delta,sigma,n,T);

%p = exp(x);%create price

t = 0:delta:T;%time axis

%plot
f = figure;
set(f,'units','normalized','outerposition',[0 0 1 1]);
plot(t,exp(x));
title('the Gaussian diffusion model with constant coefficients');
box off; grid on;
xlabel('time/day');
ylabel('price');
print(f,'-dpng','-r200','figures/1B');
close(f);
```

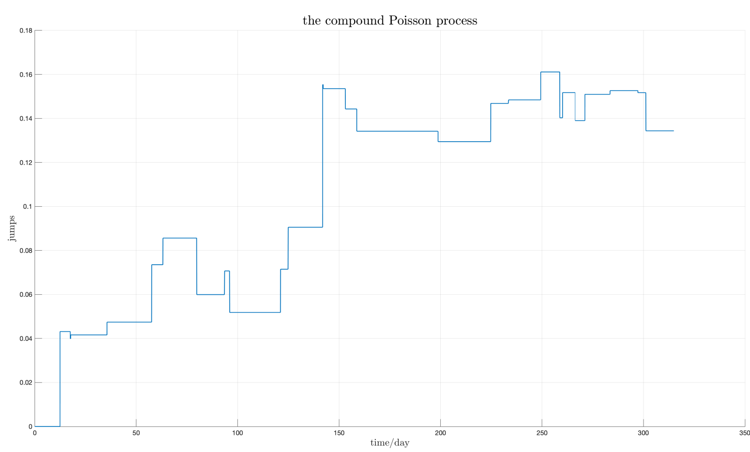
2 Exercise 2

2.1 A

λ: the intensity of jumps
σ_j: the volatility of jump sizes

σ_j is bigger than$\sigma/\sqrt{n}$ and it is the volatility of jumps which happen at δn, so there is an n in the denominator of σ_j

2.2 B



Interpret: This figure represents a compound Poisson process.The x-axis is the time in days and the y-axis is jump. We can see the jumps in this figure are irregular, because they arrive at random times, and the size of the jumps in this figure follow normal distribution.

Code:

Listing 2: ex_2.m

```
n = 80;%number of samples
T = 1.25*252;
sigma = 0.011;%lognormal sigma parameter
Lambda = 15/252;%Poisson lambda parameter
sigmaj = 18*sqrt(sigma*sigma/n);
Y = normrnd(0,sigmaj,1,n*T);%create normal random
    numbers
N = poissrnd(Lambda*T);%create random numbers from
    Poisson distribution
U = unifrnd(0,1,1,n*T);%create continuous uniform
    random numbers
delta = 1/n;%discretization interval

seed = 3005;
rng(seed,'twister');

% J = zeros(n*T+1,1);
% for i = 0:n*T
%     for k = 1:N
%      if U(k) <= i/(n*T)
%             J(i+1) = J(i+1)+Y(k);
%      else
%             J(i+1) = J(i+1);
%      end
%     end
% end
J = simcp(Y,U,N,n,T);

t = 0:delta:T;

f = figure;
set(f,'units','normalized','outerposition',[0 0 1 1]);
plot(t,J);
title('the compound Poisson process');
```

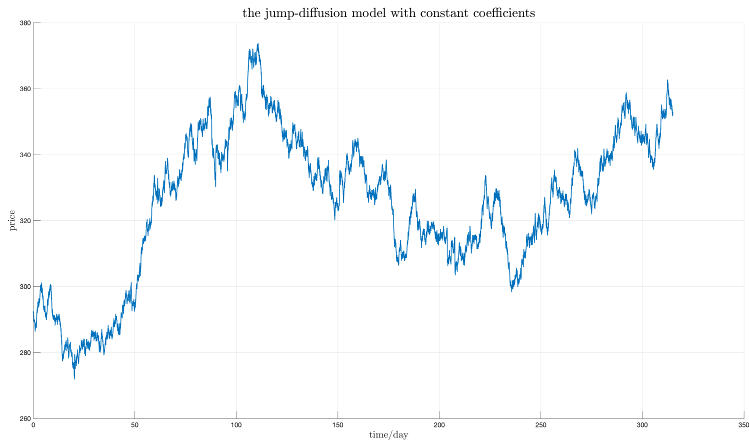
```
box off; grid on;
xlabel('time/day');
ylabel('jumps');
print(f,'-dpng','-r200','figures/2B');
close(f);
```

3 Exercise 3

3.1 A

1 and 4 are correct.Because a jump-diffusion model with constant coefficients is $dX_t = \mu * dt + \sqrt{c} * dW_t + dJ_t$, which is just the addition of the simulations from Exercise 1 and Exercise 2 ,so 1 is correct.As for 4, it simultaneously index both sides of an equation, which is exactly 1.So 1 and 4 are different appearances ,but same meaning.

3.2 B



Interpret: This figure represents a jump-diffusion model with constant coefficients. The x-axis is the time in days and the y-axis is the asset price.We can see this figure is kind of like the figure 1B, because it adds the jump part on the basis of figure 1B. Observe carefully, we can find they are different, like more or less volatile in some parts.

Code:

Listing 3: ex_3.m

```
n = 80;
T = 1.25*252;
mu = 0.03782/100;
sigma = 0.011;
```

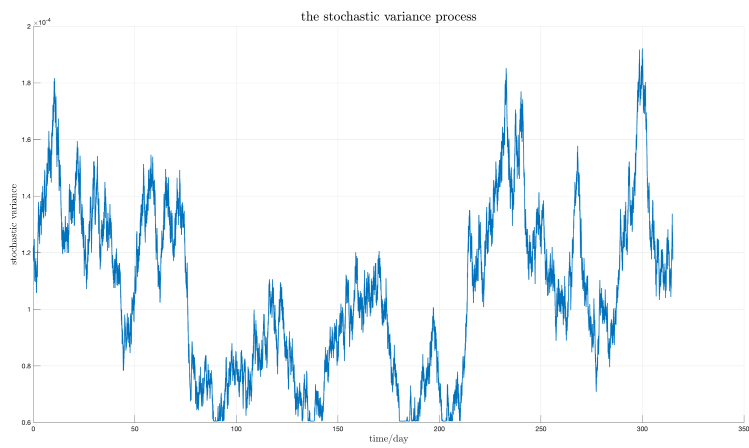
```
delta = 1/n;
Lambda = 15/252;
sigmaj = 18*sqrt(sigma*sigma/n);
Y = normrnd(0,sigmaj,1,n*T);
N = poissrnd(Lambda*T);
U = unifrnd(0,1,1,n*T);
z = randn(n*T+1,1);
x0 = log(292.58);

seed = 3005;
rng(seed,'twister');

% x = zeros(n*T+1,1);
% x(1) = log(292.58);
% for i = 1:n*T
% x(i+1) = x(i) + mu*delta + sigma*sqrt(delta)*z(i+1);
% end
x = simDCC(x0,mu,delta,sigma,n,T);

% J = zeros(n*T+1,1);
% for i = 0:n*T
%     for k = 1:N
%      if U(k) <= i/(n*T)
%              J(i+1) = J(i+1)+Y(k);
%      else
%              J(i+1) = J(i+1);
%      end
%     end
% end
J = simcp(Y,U,N,n,T);

p = exp(x+J);
t = 0:delta:T;

f = figure;
set(f,'units','normalized','outerposition',[0 0 1 1]);
plot(t,p)
title('the jump-diffusion model with constant coefficients');
box off; grid on;
xlabel('time/day');
ylabel('price');
print(f,'-dpng','-r200','figures/3B');
close(f);
```

4 Exercise 4

4.1 A

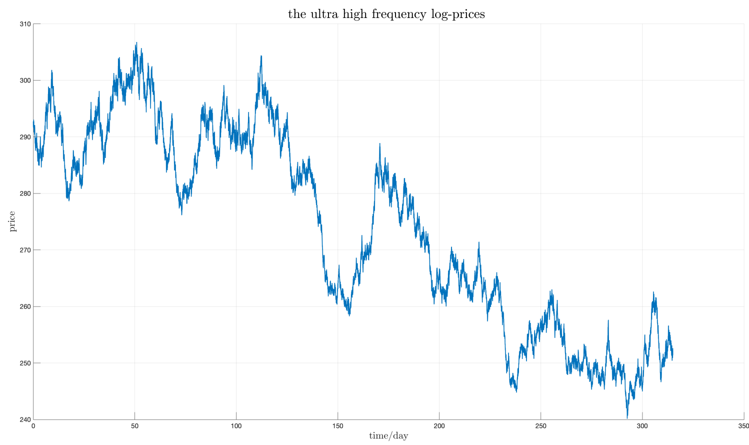
ρ: convergence rate
μ_c: mean of volatility
σ_c:volatility of volatility

4.2 B



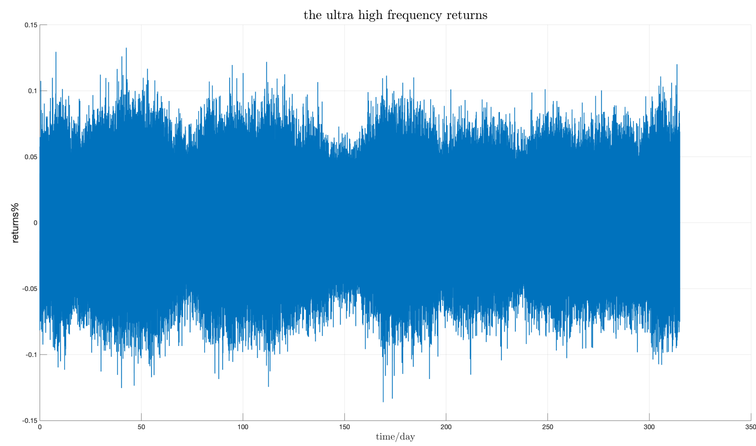
Interpret:This figure represents a stochastic variance process. The x-axis is the time in days and the y-axis is stochastic variance. We can see that the fluctuations in the observation interval are very intense and there is not an obvious trend in this figure.

4.3 C



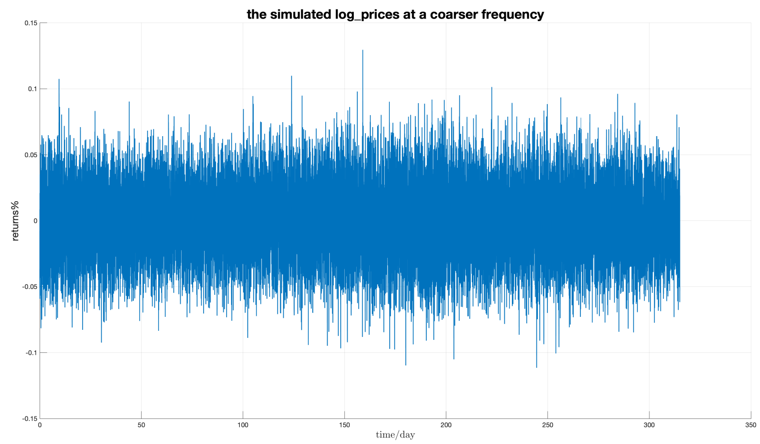
Interpret: This figure is about the ultra high frequency log-prices.The x-axis is the time in days and the y-axis is the asset price. We can see that the price fluctuations in the observation interval are very intense. The general trend of this figure is downward with sharp volatilities.

4.4 D



Interpret: This figure represents the ultra high frequency returns.The x-axis is the time in days and the y-axis is returns. We can see the return volatilities display a pattern which is the diffusions on the upper and lower sides of 0 on the y-axis are gathered after diffusion, and then repeat the cycle.

4.5 E



Interpret: This figure shows the returns we mentioned in the last question at a coarser frequency.The x-axis is the time in days and the y-axis is returns. In this figure, we can see the pattern is not as obvious as the former one.

Code:

Listing 4: ex_4.m

```
n = 80;
T = 1.25*252;
nE = 20*n;
rho = 0.03;
muc = 0.011^2;
sigmac = 0.001;
c0 = muc;
delta = 1/n;

seed = 3005;
```

```
rng(seed,'twister');

x = zeros(nE*T+1,1);
x0 = log(292.58);

deltaE = 1/nE;
z = randn(nE*T+1,1);

% c = zeros(nE*T+1,1);
% for j = 1:nE*T
%
%  c(j+1) = c(j) + rho*(muc-c(j))*deltaE+z(j+1)*sigmac
    *sqrt(c(j)*deltaE);
% end
%
% for j = 1:nE*T
%    if c(j) < muc/2
%       c(j) = muc/2;
%    end
% end
c = simsv(c0,rho,muc,deltaE,sigmac,nE,T);

t = 0:deltaE:T;

f = figure(1);
set(f,'units','normalized','outerposition',[0 0 1 1]);
plot(t,c);
title('the stochastic variance process');
box off; grid on;
xlabel('time/day');
ylabel('stochastic variance');
print(f,'-dpng','-r200','figures/4B');
close(f);


% question C
% for j = 1:nE*T
%     x(j+1) = x(j)+sqrt(c(j)*deltaE)*z(j+1);
% end
x = simlp(c0,x0,rho,muc,sigmac,deltaE,nE,T);

p = exp(x);

f = figure(2);
set(f,'units','normalized','outerposition',[0 0 1 1]);
plot(t,p);
```

```
title('the ultra high frequency log-prices ');
box off; grid on;
xlabel('time/day');
ylabel('price');
print(f,'-dpng','-r200','figures/4C');
close(f);

% question D
% deltax = zeros(nE*T+1,1);
% for j = 1:nE*T
%     deltax(j) = x(j+1)-x(j);
% end
deltax = simlr(x,nE,T);

f = figure(3);
set(f,'units','normalized','outerposition',[0 0 1 1]);
plot(t,100*deltax);
title('the ultra high frequency returns');
box off; grid on;
xlabel('time/day');
ylabel('returns%');
print(f,'-dpng','-r200','figures/4D');
close(f);

% question E
% xi = zeros(1+nE*T/n,1);
% xi(1) = x(1);
% for i = 1:n*T
%     xi(i+1) = x((i)*nE/n);
% end

% deltaxi = zeros(n*T+1,1);
% for j = 1:n*T
%     deltaxi(j) = x(j+1)-x(j);
% end
deltaxi = simlr(x,n,T);

t = 0:delta:T;

f = figure(4);
set(f,'units','normalized','outerposition',[0 0 1 1]);
plot(t,100*deltaxi);
title('the simulated log_prices at a coarser frequency
    ');
box off; grid on;
xlabel('time/day');
```

```
ylabel('returns%');
print(f,'-dpng','-r200','figures/4E');
close(f);
```