### Empirical Methods in Financial Econometrics: Project ${\bf 5}$

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# Exercise 1

## $\mathbf{A}$

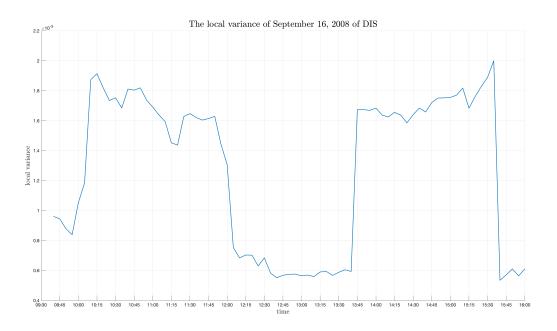


Figure 1: The local variance of September 16, 2008 of DIS

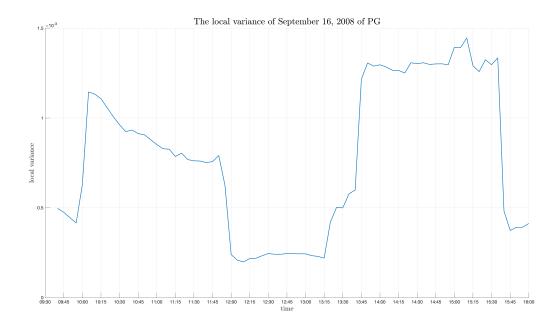


Figure 2: The local variance of September 16, 2008 of PG

Interpret: The local variance of DIS on September 16, 2008 shows a clear trend, which is like two mountain peaks. Specifically, the local variance rose sharply from 9:30 to 10:15 to  $1.9 \times 10^{-3}$ . Then, it fluctuated between 10:15 and 11:50, and dropped sharply to around  $0.7 \times 10^{-3}$  after 11:50, and continued stable to 13:40, sharply rising to around  $1.7 \times 10^{-3}$ . Next, it fluctuated from 13:45 to 15:35 until the highest point of  $2 \times 10^{-3}$ , and then drastically dropped to around  $0.5 \times 10^{-3}$  until the end.

The local variance of PG on September 16, 2008 also shows a clear trend, which is like two mountain peaks. Specifically, the local variance rose sharply from 9:30 to 10:05 to  $1.2 \times 10^{-3}$ . Then, it almost kept dropping between 10:05 and 11:50, and dropped sharply to around  $0.2 \times 10^{-3}$  after 11:50, and continued stable to 13:15, sharply rising to around  $1.3 \times 10^{-3}$ . Next, it fluctuated from 13:45 to 15:45 until the highest point of  $1.4 \times 10^{-3}$ , and then drastically dropped to around  $0.4 \times 10^{-3}$  until the end.

### $\mathbf{B}$

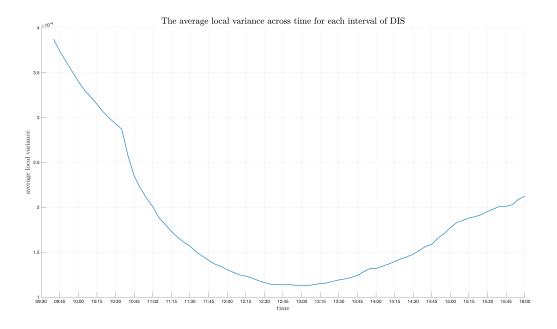


Figure 3: The average local variance across time for each interval of DIS

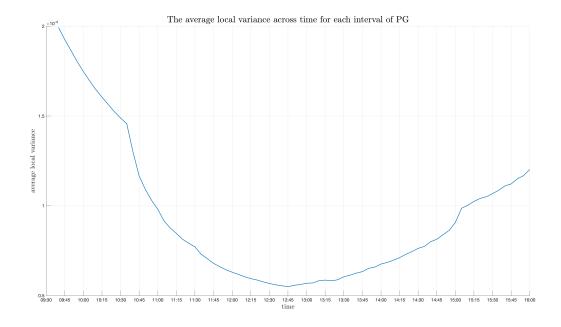


Figure 4: The average local variance across time for each interval of PG

Interpret: I can clearly find out the average local variance across time in the figures of DIS and PG are like "U" shape, which decremented from 9:30 to 12:50, incremented from 12:50 to 16:00, but didn't increment to the original point of 9:30, so it is higher on the left than on the right.

The plot of the time of day factor is also like a "U" shape. Because the time of the day factor is the weight of variance. It is easy to understand that the bigger the local variance is, the bigger the weight of variance, so they have similar shape.

## Exercise 2

### $\mathbf{A}$

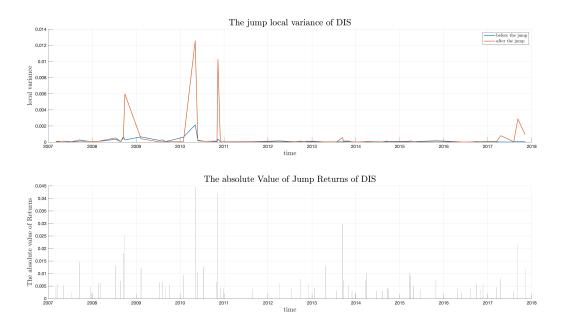


Figure 5: The jump local variance and the absolute value of jump returns of DIS

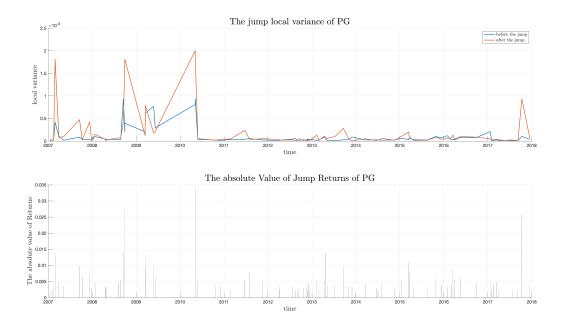


Figure 6: The jump local variance and the absolute value of jump returns of PG

Interpret: According to the two figures above, it can be seen that when there are jumps in returns, there is indication that the volatility will jump. For example, the plots of 2009, 2010, 2011 and 2017 can clearly show this. Another question can also be explained by these plots, which is there is relation between the moves in the volatility and the magnitude of the jump returns. The bigger the magnitude of the jump

returns are, the bigger the moves in the volatility.

# Exercise 3

## $\mathbf{A}$

The jumps per year of SPY:

| years | jumps |
|-------|-------|
| 2007  | 11    |
| 2008  | 5     |
| 2009  | 7     |
| 2010  | 8     |
| 2011  | 4     |
| 2012  | 17    |
| 2013  | 9     |
| 2014  | 14    |
| 2015  | 8     |
| 2016  | 18    |
| 2017  | 15    |
|       |       |

The average magnitude of the jumps per year of SPY:

| years | jumps  |
|-------|--------|
| 2007  | 0.0047 |
| 2008  | 0.0095 |
| 2009  | 0.0063 |
| 2010  | 0.0037 |
| 2011  | 0.0067 |
| 2012  | 0.0034 |
| 2013  | 0.0024 |
| 2014  | 0.0024 |
| 2015  | 0.0040 |
| 2016  | 0.0027 |
| 2017  | 0.0019 |
|       |        |

## $\mathbf{B}$

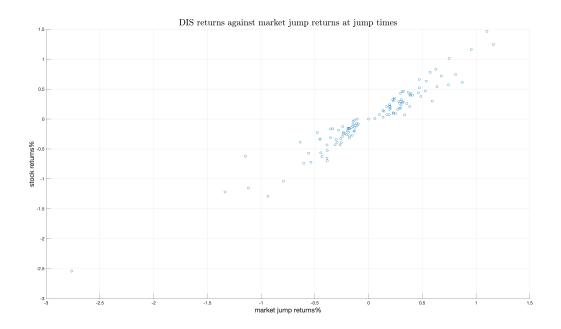


Figure 7: DIS returns against market jump returns at jump times

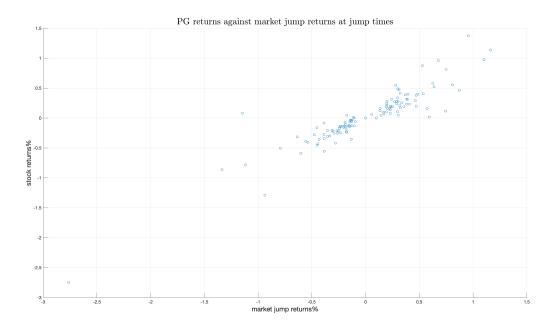


Figure 8: PG returns against market jump returns at jump times

Interpret: Based on the two figures above, I think a liner regression is plausible, because the scatters showed a tendency to gather near a line, and this line is increasing from left to right.

### $\mathbf{C}$

The OLS jump beta for DIS is 0.9942

The OLS jump beta for PG is 0.8269

Interpret: The OLS jump beta means that the jump returns of the market increases by one unit, and the jump returns of the specific stock will increase the jump beta units. Specifically, for DIS, the jump returns of the market increases by one unit, and the jump returns of it will increase 0.9942 units; for PG, the jump returns of it will increase 0.8269 units

## $\mathbf{D}$

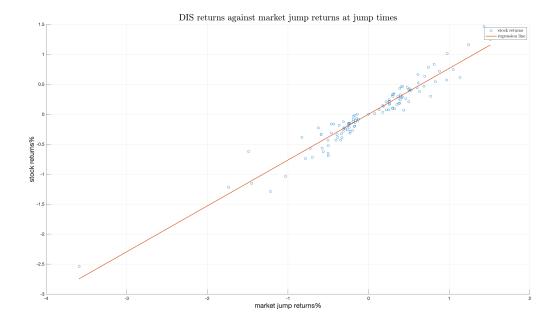


Figure 9: DIS returns against market jump returns at jump times with a regression line

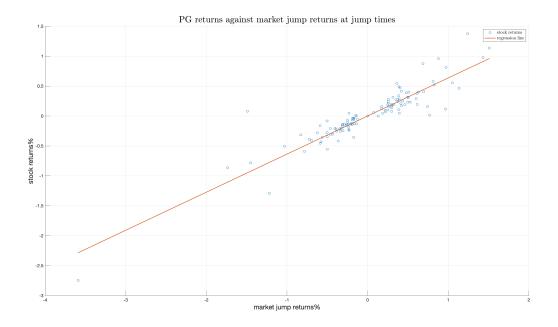


Figure 10: PG returns against market jump returns at jump times with a regression line

### $\mathbf{E}$

The value of 
$$\sqrt{\Delta_n \hat{V}_{\beta}}$$
 of DIS is 0.0512  
The value of  $\sqrt{\Delta_n \hat{V}_{\beta}}$  of PG is 0.0421

### $\mathbf{F}$

The confidence interval for the jump betas:

DIS:[0.8937;1.0946]

PG:[0.7443;0.9094]

## $\mathbf{G}$

I can hedge my position by short selling the market index futures (SPY futures). Specifically, when I have 200 million DIS, I need to short sell 198.8345 million SPY futures; when I have 200 million PG, I need to short sell 165.3727 million SPY futures.

Based on the confidence intervals, the range of values I need to short sell for DIS is [178.7461;218.9228] (million), for PG, is [148.8583;181.8870] (million) in order to hedge my position.

#### $\mathbf{H}$

```
For DIS:

\hat{\beta}_1 is 0.9586

\hat{\beta}_2 is 1.0710

\hat{V}_{\beta_1} is 0.4102

\hat{V}_{\beta_2} is 0.0567

For PG:

\hat{\beta}_1 is 0.7713

\hat{\beta}_2 is 0.9468

\hat{V}_{\beta_1} is 0.2849

\hat{V}_{\beta_2} is 0.0822
```

#### Ι

The confidence interval for the difference of the jump betas:

```
DIS:[-0.2651;0.0401]
PG:[-0.3108;-0.0401]
```

Interpret: The confidence interval of DIS contain the number 0, and the confidence interval of PG don't contain the number 0. The reason could be the jump beta of the period 1 and the period 2 of DIS could be the same, so there is 0 in the confidence interval, while the jump betas of the period 1 and the period 2 of PG can not be the same, so the underlying assumption which is the jump beta is constant is not always true.

## Code

The code is below:

Listing 1:  $ex_1.m$ 

```
% question A
[dates, prices] = load_stock('DIS.csv');
[dates_return,deltax] = log_returns(dates, prices);
[n,T] = size(deltax);

zoom_end_916 = find(dates_return==datenum(2008,9,16,16,00,0))/n;
deltax_zoom_916 = deltax(:,zoom_end_916);

kn = 11;
```

```
11
   c_hat_916 = local_variance(deltax_zoom_916,1,n,kn);
12
13
   f = figure(1);
  set(f,'units','normalized','outerposition',[0 0 1 1]);
14
   plot(dates_return(:,1),c_hat_916)
   datetick('x','HH:MM');
16
   title('The local variance of September 16, 2008 of DIS');
17
18 box off; grid on;
19 | xlabel('time');
   ylabel('local variance');
   print(f,'-dpng','-r200','figures/1A1');
21
22
   close(f);
23
24
   % question B
25
   c_hat = local_variance(deltax,T,n,kn);
   average_c_hat = sum(c_hat,2)/T;
26
27
   f = figure(2);
28
29
   set(f,'units','normalized','outerposition',[0 0 1 1]);
   plot(dates_return(:,1),average_c_hat)
  datetick('x','HH:MM');
32 | title('The average local variance across time for each interval of DIS');
   box off; grid on;
34 | xlabel('time');
   ylabel('average local variance');
   print(f,'-dpng','-r200','figures/1B1');
36
37
   close(f);
```

#### Listing 2: $ex_2.m$

```
1  % question A
2  [dates, prices] = load_stock('DIS.csv');
3  [dates_return,deltax] = log_returns(dates, prices);
4  [n,T] = size(deltax);
5  deltan = 1/n;
6
```

```
tau = tau_f(deltax);
8
  BV = bipower_var(deltax);
   alpha = 5;
9
   cutoff = alpha*deltan^0.49*sqrt(tau*BV);
11
12 rc = deltax;
13 rc(abs(deltax)>cutoff)=0;
14 rd = deltax;
15 rd(abs(deltax) <= cutoff) = 0;
16
17 | kn = 11;
18 | [c_hat_minus,c_hat_plus,dates_j,rd_j] = local_variance_j(dates_return,rc,
      rd,n,T,kn);
19
20 f = figure(5);
21 | subplot (2,1,1);
22 set(f, 'units', 'normalized', 'outerposition', [0 0 1 1]);
23 | plot(dates_j,c_hat_minus);
24 hold on
25 | plot(dates_j,c_hat_plus);
26 hold off
27 datetick('x','yyyy');
28 | legend('before the jump', 'after the jump')
   title('The jump local variance of DIS');
29
30 box off; grid on;
   xlabel('time');
31
   ylabel('local variance');
32
34 | subplot(2,1,2);
35 bar(dates_j,abs(rd_j));
   datetick('x','yyyy');
37 box off; grid on;
   xlabel('time')
38
   ylabel('The absolute value of Returns')
   title('The absolute Value of Jump Returns of DIS');
40
41
```

```
42 print(f,'-dpng','-r200','figures/2A1');
43 close(f);
```

Listing 3:  $ex_3.m$ 

```
% question A
1
   [dates_m, prices_m] = load_stock('SPY.csv');
3 | [dates_return_m,deltax_m] = log_returns(dates_m, prices_m);
   [n,T] = size(deltax_m);
4
   deltan = 1/n;
6
7
   tau_m = tau_f(deltax_m);
8
   BV_m = bipower_var(deltax_m);
   alpha = 5;
9
   cutoff_m = alpha*deltan^0.49*sqrt(tau_m*BV_m);
11
12
  rd_m = deltax_m;
13 rd_m(abs(deltax_m) <= cutoff_m) = 0;</pre>
14
  jumps = sum(rd_m~=0);
16 | y = zeros(12,1);
17
   [yyyy,mm,dd] = ymd_f('SPY.csv');
   for i = 1:11
18
19
       y(i+1) = sum(ismember(yyyy,2006+i));
20 | end
   jumps_y = zeros(11,1);
21
22
   for i = 1:11
       jumps_y(i) = sum(jumps(1,(sum(y(1:i))+1):sum(y(1:i+1))));
23
24
   end
25
   sum_jumps_y = sum(jumps_y);
26
27 | rd_m_y = sum(rd_m);
   mag_rd_y = zeros(11,1);
28
   for i = 1:11
29
30
       mag_rd_y(i) = sum(abs(rd_m_y(1,(sum(y(1:i))+1):sum(y(1:i+1)))));
   end
32 av_mag_rd_y = mag_rd_y./jumps_y;
```

```
33
34
   % question B
   [dates_1, prices_1] = load_stock('DIS.csv');
35
   [dates_return_1,deltax_1] = log_returns(dates_1, prices_1);
36
37
38
   j_m = (rd_m = 0);
39
   rd_j_1 = j_m .* deltax_1;
40
41 | f = figure(7);
42 | set(f, 'units', 'normalized', 'outerposition', [0 0 1 1]);
   scatter(rd_m(:)*100,rd_j_1(:)*100)
44 | title('DIS returns against market jump returns at jump times');
45 box off; grid on;
   xlabel('market jump returns%');
46
47
   ylabel('stock returns%');
   print(f,'-dpng','-r200','figures/3B1');
48
   close(f);
49
50
   [dates_2, prices_2] = load_stock('PG.csv');
51
52
   [dates_return_2,deltax_2] = log_returns(dates_2, prices_2);
53
   rd_j_2 = j_m .* deltax_2;
54
56 \mid f = figure(8);
   set(f,'units','normalized','outerposition',[0 0 1 1]);
57
   scatter(rd_m(:)*100,rd_j_2(:)*100)
58
   title('PG returns against market jump returns at jump times');
59
60 box off; grid on;
61 | xlabel('market jump returns%');
62 | ylabel('stock returns%');
   print(f,'-dpng','-r200','figures/3B2');
   close(f);
64
65
66
   % queation C
   betaj_1 = jump_beta(rd_m,deltax_1);
67
68
```

```
betaj_2 = jump_beta(rd_m,deltax_2);
   % question D
71
    x = rd_m(:) * 100 * 1.3;
72
73
74 \mid f = figure(9);
    set(f,'units','normalized','outerposition',[0 0 1 1]);
75
    scatter(x,rd_j_1(:)*100);
76
77 hold on
78 | plot(x,rd_m(:)*100 * betaj_1);
79 hold off
80 title('DIS returns against market jump returns at jump times')
81 box off; grid on;
82
    xlabel('market jump returns%')
83 | ylabel('stock returns%')
84 | legend('stock returns', 'regression line')
    print(f,'-dpng','-r200','figures/3D1');
85
    close(f);
86
87
88
    f = figure(10);
    set(f,'units','normalized','outerposition',[0 0 1 1]);
89
90 | scatter(x,rd_j_2(:)*100);
91 hold on
    plot(x,rd_m(:)*100 * betaj_2);
93 hold off
94 | title('PG returns against market jump returns at jump times')
95 box off; grid on;
96 | xlabel('market jump returns%')
97 | ylabel('stock returns%')
98 | legend('stock returns', 'regression line')
    print(f,'-dpng','-r200','figures/3D2');
100
    close(f);
102
    % question E
103 | V_beta_hat_1 = V_beta(deltax_1, betaj_1, deltax_m, rd_m, n, T);
104 | std_1 = sqrt(deltan*V_beta_hat_1);
```

```
106
                   V_beta_hat_2 = V_beta(deltax_2, betaj_2, deltax_m, rd_m, n, T);
107
                   std_2 = sqrt(deltan*V_beta_hat_2);
108
109
                  % question F
                   ci_betaj_1 = ci_f(0.05, betaj_1, std_1);
111
112
                  ci_betaj_2 = ci_f(0.05, betaj_2, std_2);
113
114 | % question G
115 | hedge_1 = 200 * betaj_1;
116 | hedge_2 = 200 * betaj_2;
117
                  hedge_1_r = 200 * ci_betaj_1;
118
                  hedge_2_r = 200 * ci_betaj_2;
119
120
                 % question H
121
                   zoom_start_11 = find(dates_return_1 == datenum(2007,1,3,16,0,0))/n;
122
                   zoom_end_11 = find(dates_return_1 == datenum(2011,12,30,16,0,0))/n;
123
                   rd_m_1 = rd_m(:,zoom_start_11:zoom_end_11);
124
                   deltax_1_1 = deltax_1(:,zoom_start_11:zoom_end_11);
125
                   deltax_m_1 = deltax_m(:,zoom_start_11:zoom_end_11);
126
127
                   zoom_start_12= find(dates_return_1==datenum(2012,1,3,16,0,0))/n;
128
                   zoom_end_12 = find(dates_return_1 == datenum(2017,12,29,16,0,0))/n;
129
                   rd_m_2 = rd_m(:,zoom_start_12:zoom_end_12);
130
                   deltax_1_2 = deltax_1(:,zoom_start_12:zoom_end_12);
                   deltax_m_2 = deltax_m(:,zoom_start_12:zoom_end_12);
132
                  betaj_1_1 = jump_beta(rd_m_1,deltax_1_1);
134
                   betaj_1_2 = jump_beta(rd_m_2,deltax_1_2);
                   V_{\text{beta}} = 
136
                   V_{\text{beta}} = 
138
                   zoom_start_21 = find(dates_return_2 == datenum(2007,1,3,16,0,0))/n;
                  zoom_end_21 = find(dates_return_2 == datenum(2011,12,30,16,0,0))/n;
                  rd_m_1 = rd_m(:,zoom_start_21:zoom_end_21);
```

```
141
    deltax_2_1 = deltax_2(:,zoom_start_21:zoom_end_21);
142
    deltax_m_1 = deltax_m(:,zoom_start_21:zoom_end_21);
143
144
    zoom_start_22 = find(dates_return_2 == datenum(2012,1,3,16,0,0))/n;
145
    zoom_end_22 = find(dates_return_2 == datenum(2017,12,29,16,0,0))/n;
    rd_m_2 = rd_m(:,zoom_start_22:zoom_end_22);
147
    deltax_2_2 = deltax_2(:,zoom_start_22:zoom_end_22);
148
    deltax_m_2 = deltax_m(:,zoom_start_22:zoom_end_22);
149
150
   betaj_2_1 = jump_beta(rd_m_1,deltax_2_1);
    betaj_2_2 = jump_beta(rd_m_2,deltax_2_2);
    V_beta_hat_2_1 = V_beta(deltax_2_1,betaj_2_1,deltax_m_1,rd_m_1,n,1260);
    V_beta_hat_2_2 = V_beta(deltax_2_2, betaj_2_2, deltax_m_2, rd_m_2, n, 1509);
153
154
   % question I
156
    std_11 = sqrt(deltan*(V_beta_hat_1_1 + V_beta_hat_1_2));
157
    ci_betaj_11 = ci_f(0.05, betaj_1_1-betaj_1_2, std_11);
158
159
    std_22 = sqrt(deltan*(V_beta_hat_2_1 + V_beta_hat_2_2));
   ci_betaj_22 = ci_f(0.05,betaj_2_1-betaj_2_2,std_22);
```