Final Exam

Instructions

The final exam is due on November 28th by **noon**. This is a hard deadline, so no exceptions. You must push your local repository back to GitHub before the deadline. Your repository must contain:

- The Python code you used to complete the project;
- A notebook file named `main.ipynb` that generates all required plots;
- A `report.pdf` file with your answers to the project questions. The report must also contain an Appendix with the code used to solve the project;
- All plots in the report must be self-contained. Self-contained means that a reader who only sees your figure (image and caption, but not the surrounding text) can understand what you are plotting. This translates to all plots having axis titles, correct units on the axis, and a caption that summarizes what is plotted.

This project makes use of stock and **options** data. Refer to the Data page and the questions below for instructions on how to download the data and which files to download (requires Duke login). You must complete all exercises for both of your stocks using the data at the 5-minutes sampling frequency, unless stated otherwise.

You can obtain the repository for this project by clicking **on this link**.

Questions

Exercise 0

During the exam you will use data on your two stocks, data on the market index (SPY), and data on options. If one of your stocks already is SPY, pick a different one from the available stocks and clearly state what stock you are using. Fill the table below with the stocks you will use for solving this exam:

Stock	Ticker
1	
2	
3	SPY
Option File	

Exercise 1 - The Hull-White Model for Options [40 points]

The Black-Scholes model implies that the value for out-of-the-money put options using realized volatility could be well below the market value. A problem with this model is that it treats volatility as constant until the expiration date, while we know that is not the case. The Hull-White (HW) options model accounts for random volatility and is very easy to implement. Let V denote the random annualized volatility between now and expiration of the option. The Hull-White price of an option is the expected value of the BLS price using V averaged over the distribution of V.

A reasonable model for the variance is that V is log-normal. Suppose that:

$$\log V \overset{d}{\sim} \mathcal{N}\left(\log 40 - \frac{1.20^2}{2}, 1.20^2\right)$$

The term being subtracted in the distribution is to ensure that $\mathbb{E}[V] = 40$ (expected value of V, not of $\log V$).

The Hull-White model can be implemented via simulation as follows:

- Simulate the volatility:

$$\tilde{V}_i = \mathrm{e}^{\log 40 - \frac{1.20^2}{2} + 1.20\tilde{Z}_i} \text{ where } \tilde{Z}_i \overset{d}{\sim} \mathcal{N}(0,1)$$

- Convert the volatility to appropriate units:

$$\tilde{\sigma}_i \equiv \frac{\tilde{V}_i}{100}$$

- Repeat the two steps above for $i = 1, 2, \ldots, 1000$.

- Compute the Hull-White price via:

$$\text{Hull-White Put price} = \frac{1}{1000}\sum_{i=1}^{1000} \text{BLS put price}(S_0, K, r, T, \tilde{\sigma}_i, q)$$

A. [15 points]

Consider a 90-day put option with the following characteristics:

$$\begin{aligned}
&S_0 = 50 \text{ (current stock price in dollars)}\\
&K = \text{strike price in dollars, various values below}\\
&r = 0.013 \text{ (interest rate in decimals)}\\
&T = 90/365 \text{ (time to expiration in years)}\\
&\sigma = 0.40 \text{ (annualized volatility in decimals)}\\
&q = 0.025 \text{ (dividend yield in decimals)}
\end{aligned}$$

Compute the BLS implied price for this put option (strike prices below) and its price implied by the Hull-White model. Fill the table below:

	Put Price implied by	
K	Black-Scholes	Hull-White
42.5		
44		
48		
50		
52		
55		

B. [5 points]

Plot the prices implied by both models against the strike-to-underlying ratio. Are the Hull-White put prices higher than the Black-Scholes put prices? Why or why not? In your last project you saw that prices implied by the Black-Scholes model would be below the market prices for the options. Could the random volatility in the Hull-White model help explain the under-pricing by the Black-Scholes model?

C. [20 points]

Repeat the exercise above using the actual options data from one of the options data files chosen at random. Specify the file you are using. Also add the actual market price to the plots. In order to compute the Hull-White prices for the options in your data set, you will need to change the log-normal distribution of V:

$$\log V \overset{d}{\sim} \mathcal{N}\left(\log \mu - \frac{1.20^2}{2}, 1.20^2\right)$$

Use the realized variance (in the appropriate units) as the value of μ. Since your options data are at the end of a trading day, you should have all of the returns for the same day available, thus compute the realized variance for this given day. Also report the average price from the Black-Scholes and Hull-White model across all options in your file.

Interpret the results. How do the models compare to the market price? Does stochastic volatility help with pricing out-of-the-money (OTM) put options? Is the pricing of OTM puts by the Hull-White model better than the Black-Scholes model?

Exercise 2 - Realized Semibetas and the CAPM [60 points]

A. [10 points]

The CAPM is one most used models for valuing risky assets. The core of the model is that there is a **linear** relationship between an asset's excess return (return above the risk-free interest rate) and the excess return of the market portfolio. There is an extensive literature on wether this model can explain the returns of different assets.

The CAPM implies the relationship:

$$\mathbb{E}[r_{stock}] = r_{risk-free} + \beta(\mathbb{E}[r_{market}] - r_{risk-free})$$

An asset with a higher beta should have a higher expected return than another asset with a lower beta.

We have studied a measure of covariance between two assets: the realized beta. Estimate the realized beta day by day for both of your assets in relation to the market index.

Fill the table below:

	$R\beta_{Stock1,Market}$	$R\beta_{Stock2,Market}$
Average		
Minimum		
25% Percentile		
50% Percentile		
75% Percentile		
Maximum		

Plot the realized beta for the two stocks and interpret the results.

B. [15 points]

Another strand of the literature argues that one of the underlying assumptions of the CAPM is too restrictive, and the linear relationship between asset returns and market returns is too simplistic. This literature argues that investors are averse to risk (volatility) only when the risk leads to losses, but that investors are **not** averse to risk when the risk leads to gains. Therefore, the relevant measure of risk is not the total volatility, but only the volatility due to the negative returns.

We can split the usual measure of variance, the realized variance, into two components:

$$\begin{aligned} \mathrm{RV}_t &= \sum_{i=1}^{n} r_{t,i}^2 \\ &= \underbrace{\sum_{i=1}^{n} r_{t,i}^2 \mathbb{1}_{\{r_{t,i}>=0\}}}_{\mathrm{RV}_t^+} + \underbrace{\sum_{i=1}^{n} r_{t,i}^2 \mathbb{1}_{\{r_{t,i}<0\}}}_{\mathrm{RV}_t^-} \end{aligned}$$

Compute the realized variance for both of your stocks, and also compute the two components of the variance: upside realized semivariance (RV_t^+) and downside realized semivariance (RV_t^-).

Fill the table below:

	Stock 1			Stock 2		
	RV_t	RV_t^+	RV_t^-	RV_t	RV_t^+	RV_t^-
Average						
Minimum						
25% Percentile						
50% Percentile						
75% Percentile						
Maximum						

Plot the realized variance and the semivariances. Interpret the results.

What happened to the realized semivariances during the 2008 crisis? Are there any patterns in the semivariances?

C. [10 points]

If the main concern of investors is the downside risk, then investors will care about the comovement of a stock with the negative returns of the market index. To assess how a stock comoves with the market index, we can plot daily returns of a stock against daily returns of the market index. Create a scatter plot showing the returns of your stocks (x-axis) against the returns of the market index (y-axis). Use lines to divide the plot in 4 quadrants that split positive from negative returns.

How does your stock comove with market returns? Interpret the relationship in each of the quadrants.

D. [5 points]

If the main concern of investors is the downside risk, then the covariation of a stock's returns with **positive** market returns should not be priced. That is, stocks that comove with the market return when the market returns are positive will yield no risk premium. On the other hand, the covariation of a stock's returns with **negative** market returns should be priced.

Suppose a stock's returns comove **positively** with the market returns when the market returns are negative. That is, whenever there is a negative market return, the stock return will also be negative. Does this stock demand a high risk premium or a low risk premium? Why? In other words, would a risk averse agent demand a high expected return in order to hold this stock in his portfolio?

Suppose a stock's returns comove **negatively** with the market returns when the market returns are negative. That is, whenever there is a negative market return, the stock return will be positive. Does this stock demand a high risk premium or a low risk premium? Why?

Based on your solution of exercise C, would you say your stocks comove positively or negatively with the market when the market is performing poorly? Based on the discussion above, would you expect your stocks to have high or low risk premiums?

E. [20 points]

A recent paper in the literature extends the idea of semivariances to the realized beta. Specifically, the authors created signed measures of comovement between two assets based on high-frequency observations. Define the following signed returns:

$$
\begin{aligned}
r^+_{t,i,stock} &\equiv \max(r_{t,i,stock}, 0) \\
r^-_{t,i,stock} &\equiv \min(r_{t,i,stock}, 0)
\end{aligned}
$$

Then, the realized beta can be decomposed into four terms:

$$
\begin{aligned}
R\beta_t &\equiv \frac{\sum_{i=1}^n r_{t,i,Stock1} r_{t,i,Stock2}}{\sum_{i=1}^n r_{t,i,Stock1}^2} \\
&= \underbrace{\frac{\sum_{i=1}^n r_{t,i,Stock1}^{+} r_{t,i,Stock2}^{+}}{\sum_{i=1}^n r_{t,i,Stock1}^2}}_{R\beta_t^{+,+}} + \underbrace{\frac{\sum_{i=1}^n r_{t,i,Stock1}^{+} r_{t,i,Stock2}^{-}}{\sum_{i=1}^n r_{t,i,Stock1}^2}}_{R\beta_t^{+,-}} + \\
&+ \underbrace{\frac{\sum_{i=1}^n r_{t,i,Stock1}^{-} r_{t,i,Stock2}^{+}}{\sum_{i=1}^n r_{t,i,Stock1}^2}}_{R\beta_t^{-,+}} + \underbrace{\frac{\sum_{i=1}^n r_{t,i,Stock1}^{-} r_{t,i,Stock2}^{-}}{\sum_{i=1}^n r_{t,i,Stock1}^2}}_{R\beta_t^{-,-}}
\end{aligned}
$$

Notice we are assuming there are no jumps in the stock, so using log-returns is justified.

The betas defined above are called realized semibetas. Compute the realized semibetas between your stocks and the market index.

Fill the table below:

	Stock 1 and Market Index				Stock 2 and Market Index			
	$R\beta_t^{+,+}$	$R\beta_t^{+,-}$	$R\beta_t^{-,+}$	$R\beta_t^{-,-}$	$R\beta_t^{+,+}$	$R\beta_t^{+,-}$	$R\beta_t^{-,+}$	$R\beta_t^{-,-}$
Average								
Minimum								
25% Percentile								
50% Percentile								
75% Percentile								
Maximum								

Plot the realized semibetas and interpret the results. Are the semibetas stable over time? Are there any visible patterns? Interpret the semibetas in the context of investors that are only concerned about downside risk. What do the magnitudes of $R\beta_t^{-,+}$ and $R\beta_t^{-,-}$ suggest about the risk premium of your stocks?