Adaptive Designs and Multiple Testing Procedures for Clinical Trials

October 2019

Hierarchical testing

Outline

- Mierarchical testing
- Semi-parametric procedures
- Closed testing
- 4 Graphical approaches
- Multiplicity in practice

Hierarchical testing

Hierarchical test procedures

- Suppose hypotheses can be ordered into a *pre-specified* hierarchy H_1, \ldots, H_K , before the data are observed
 - Clinical relevance
 - Dose concentration
 - Time sequence
- Hierarchical test procedures: tests the hypotheses in the pre-defined hierarchical order
 - Fixed sequence procedure
 - Fallback procedure

Hierarchical testing

 Each hypothesis is tested in the pre-specified sequence at level α until the first non-rejection

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- Rejection rule
 - ▶ if $p_1 \leq \alpha$, reject H_1 and continue; else stop
 - ▶ if $p_2 \le \alpha$ reject H_2 and continue; else stop
 - **.**..
 - if $p_k \leq \alpha$ reject H_k and continue; else stop

Fixed sequence procedure

- Each hypothesis is tested in the pre-specified sequence at level α until the first non-rejection
- Rejection rule
 - if $p_1 < \alpha$, reject H_1 and continue; else stop
 - if $p_2 < \alpha$ reject H_2 and continue; else stop

 - if $p_k \leq \alpha$ reject H_k and continue; else stop
- As soon as a hypothesis H_i cannot be rejected, $p_i > \alpha$, the procedure stops and all remaining hypotheses H_{i+1}, \ldots, H_K are not rejected

Fixed sequence procedure

Advantages

- Simple procedure
- Optimal (maximises power) if previous hypotheses rejected

Disadvantages

- Ordering of testing sequence is critically important, and may be based on subjective information
- Minimises power if a previous hypothesis is not rejected
- Once a hypothesis is not rejected, no further testing is allowed

Fallback procedure

 \bullet Again test each hypothesis in the pre-specified sequence, but split the α between hypotheses

Graphical approaches

- Assign α_i to hypothesis H_i , where $\sum_i^k \alpha_i = \alpha$
- H_1 is tested at level $\alpha_1' = \alpha_1$
- For $i \geq 2$, H_i is tested at level α'_i , where

$$\alpha_i' = \begin{cases} \alpha_i & \text{if } H_{i-1} \text{ is not rejected} \\ \alpha_i + \alpha_{i-1}' & \text{if } H_{i-1} \text{ is rejected} \end{cases}$$

- Test all hypotheses even if initial hypotheses are not rejected
- Fallback procedure is implemented in the multxpert R package

Example

- $\alpha = 0.025$
- p-values $p_1 = 0.03$, $p_2 = 0.004$, $p_3 = 0.01$
- For fallback procedure, suppose α split equally: $\alpha_1 = \alpha_2 = \alpha_3 = 0.025/3$

p-value	Fixed sequence	Fallback procedure
0.03	$\alpha = 0.025$	$\alpha_1' = 0.025/3$

Hierarchical testing

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0.004	Not tested	$\alpha_2'=0.025/3$

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0.01	Not tested	$\alpha_3' = 0.05/3$

Hierarchical testing

Semi-parametric procedures

Hierarchical testing

- Hypotheses H_1, \ldots, H_K
- ullet Aim to control the FWER at level α

Šidák procedure

- Hypotheses H_1, \ldots, H_K
- Aim to control the FWER at level α
- Šidák (also known as Dunn-Šidák) procedure uses the adjusted significance level

$$\alpha_{S} = 1 - (1 - \alpha)^{1/K}$$

- More powerful than Bonferroni for K > 1
- However, only guaranteed to control the FWER for independent or positively correlated test statistics
 - e.g. appropriate for many-to-one comparisons, where a positive correlation is induced by the control group

Hochberg

Hierarchical testing

- Ordered *p*-values $p_{(1)} < \cdots < p_{(K)}$ with corresponding hypotheses $H_{(1)}, \ldots, H_{(K)}$
- Uses same significance thresholds as the Holm procedure, but reversed (step-up rather than step-down)
- Rejection rule
 - If $p_{(K)} \leq \alpha$, reject $H_{(1)}, \ldots, H_{(K)}$ and stop; else continue
 - If $p_{(K-1)} \leq \alpha/2$, reject $H_{(1)}, \ldots, H_{(K-1)}$ and stop; else continue
 - **...**
 - If $p_{(1)} \leq \alpha/K$, reject $H_{(1)}$ and stop
- Find largest i such that $p_{(i)} \leq \alpha/(K-i+1)$ and reject all hypotheses $H_{(i)}, H_{(i+1)}, ..., H_{(1)}$

Hochberg

- More powerful than Bonferroni and Holm
- Again, only guaranteed to control FWER under certain correlation assumptions
 - e.g. When test statistics are independent or positively correlated

Hommel

- Rejection rule
 - Let i be the largest integer for which

$$p_{(K-j+i)} > \frac{i\alpha}{j}$$

for all
$$i = 1, \ldots, j$$

- If no such *i* exists, reject all hypotheses
- ▶ Otherwise, reject all $H_{(i)}$ with $p_{(i)} \leq \alpha/j$
- More powerful than Hochberg, but needs same distributional assumptions
- Hommel and Hochberg procedures are implemented in the multxpert R package

Closed testing

Closed test procedures

- General methodology to construct multiple testing procedures which strongly control the FWER
- Includes many well-known procedures as special cases

which strongly control the FWER

Hierarchical testing

General methodology to construct multiple testing procedures

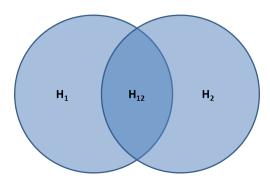
- Includes many well-known procedures as special cases
- Closed test procedures consider all *intersection hypotheses*

$$H_J = \bigcap_{i \in J} H_i, \qquad J \subseteq \{1, \dots, K\}$$

• Closure principle: An individual hypothesis H_i is rejected at familywise level α only if every intersection hypothesis H_I with $i \in J$ is rejected at local level α

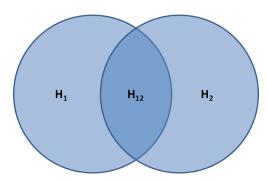
Hierarchical testing

Venn diagram for K = 2 hypotheses



Closure principle

Venn diagram for K = 2 hypotheses



- Test $H_{12} = H_1 \cap H_2$ using Bonferroni or Dunnett etc. at level α
- Test H_1 and H_2 using a level α test

Closure principle

K = 2 hypotheses

$$H_{12} = H_1 \cap H_2$$

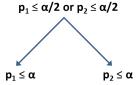
$$H_1 \qquad H_2$$

- Reject H_1 overall if H_{12} and H_1 and are rejected locally at level α
- ullet If K>2 , several intersection hypotheses have to be tested
- Different tests can be chosen for each (intersection) hypothesis

Closure principle Holm

Holm's procedure is the closure principle applied to Bonferroni:

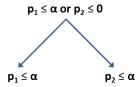
 H_{12} is rejected if either $p_1 \leq \alpha/2$ or $p_2 \leq \alpha/2$



Closure principle Fixed sequence procedure

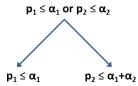
Bonferroni could also be applied with unequal splitting of the significance level into α_1 and α_2 , where $\alpha_1 + \alpha_2 = \alpha$

Setting $\alpha_1 = \alpha$ and $\alpha_2 = 0$ gives the *fixed sequence procedure*:



Closure principle Fallback procedure

In general, we recover the fallback procedure:



Hierarchical testing

$$k = 3$$
 hypotheses

$$H_1 \cap H_2 \cap H_3$$

$$H_1 \cap H_2$$
 $H_1 \cap H_3$ $H_2 \cap H_3$

$$H_1$$
 H_2 H_3

Closure principle

Fixed sequence procedure with a-priori fixed order $H_1 \rightarrow H_2 \rightarrow H_3$

$$H_{1} \cap H_{2} \cap H_{3}$$

$$p_{1} \leq \alpha$$

$$H_{1} \cap H_{2} \qquad H_{1} \cap H_{3} \qquad H_{2} \cap H_{3}$$

$$p_{1} \leq \alpha \qquad p_{1} \leq \alpha \qquad p_{2} \leq \alpha$$

$$H_{1} \qquad H_{2} \qquad H_{3}$$

$$p_{1} < \alpha \qquad p_{2} < \alpha \qquad p_{3} < \alpha$$

Holm

$$H_1 \cap H_2 \cap H_3$$

 $p_1 \le \alpha/3$ or $p_2 \le \alpha/3$ or $p_3 \le \alpha/3$

Graphical approaches

$$H_1 \cap H_2$$

 $p_1 \leq \alpha/2$ or $p_2 \leq \alpha/2$

$$H_1 \cap H_2$$
 $H_1 \cap H_3$
 $p_1 < \alpha/2 \text{ or } p_2 < \alpha/2$ $p_1 < \alpha/2 \text{ or } p_3 < \alpha/2$

$$H_2 \cap H_3$$

 $p_2 \le \alpha/2 \text{ or } p_3 \le \alpha/2$

$$H_1$$
 $p_1 < \alpha$

$$H_2$$
 $p_2 \le \alpha$

$$H_3$$
 $p_3 \leq \alpha$

Closed test procedures

Advantages

- Includes many well-known procedures as special cases
- Closed test procedures are more powerful than the procedures they are derived from

Graphical approaches

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• By construction, it is *coherent*: if null hypothesis H_I is rejected, all subsets $H_I \subseteq H_I$ are rejected as well

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- Any non-coherent multiple testing procedure can be replaced by a coherent one that is at least as powerful
- Any coherent multiple test controlling FWER is a closed test

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Disadvantages

- No natural point estimates or confidence intervals
- Can be a very large number of intersection hypotheses to test as k increases: worst-case is $2^K - 1$. However, shortcuts exist

Hierarchical testing

Motivating example: Diabetes trial

- Trial compares two doses D_1 or D_2 against placebo in diabetes patients for two endpoints
 - Primary endpoint: HbA1c
 - Secondary endpoint: Body weight
- There is a natural order: a primary endpoint is more important than a secondary endpoint
 - We test the primary null hypothesis first; only if this is rejected do we test the secondary hypothesis

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 - We test the primary null hypothesis first; only if this is rejected do we test the secondary hypothesis
- Both doses are equally important

Motivating example: Diabetes trial

- Objective: test all four hypotheses under strong FWER while reflecting clinical objectives
- Standard multiple testing procedures do not reflect the relative importance of the two endpoints or the underlying structure
- In general, need test procedures that can deal with complex trial objectives and multiple structured hypotheses: **Graphical approaches** (also known as chain or sequentially rejective procedures)







Graphical approaches

Secondary



Low dose



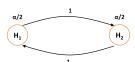
High dose

Graphical approaches

- **1** Hypotheses H_1, \ldots, H_K represented as nodes
- 2 Split of significance level α into $(\alpha_1,\ldots,\alpha_K)$
- \bullet " α propagation" through weighted, directed edges







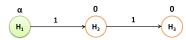
Graphical approaches

- Technical basis of the graphical approach: the graph defines a closed testing procedure with weighted Bonferroni tests for each intersection hypothesis
- Equivalent formulation of split of α using weights (w_1,\ldots,w_K) where $\sum_{i=1}^K w_i = 1$ and $\alpha_i = \alpha w_i$.
- α -propagation: If a hypothesis H_i can be rejected at level α_i (i.e. $p_i \leq \alpha_i$), propagate its level α_i to the remaining (not yet tested) hypotheses, according to a prefixed rule, and continue testing with the updated α levels.

Fixed sequence procedure

- Assume $H_1 \rightarrow H_2 \rightarrow H_3$
- Example fixed sequence procedure:



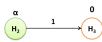


Examples

Fixed sequence procedure

- Assume $H_1 \rightarrow H_2 \rightarrow H_3$
- Example fixed sequence procedure:

H₂ rejected at level α



Fixed sequence procedure

- Assume $H_1 \rightarrow H_2 \rightarrow H_3$
- Example fixed sequence procedure:

 H_3 not rejected at level α (stop)



• *Bonferroni*: no α -propagation (no edges between nodes)

 $\alpha/2$



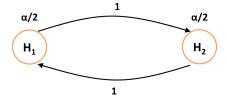
 $\alpha/2$



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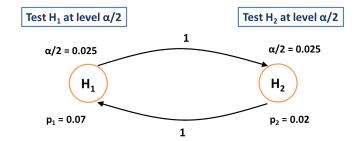
• Holm: includes α -propagation \rightarrow more powerful



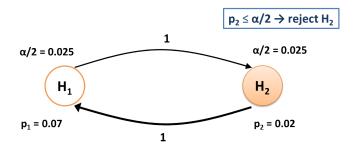
Examples

Hierarchical testing

Holm procedure with $\alpha = 0.05$

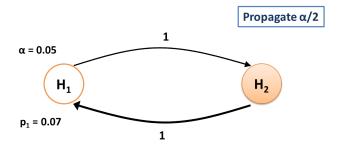


Holm procedure with $\alpha = 0.05$



Examples

Holm procedure with $\alpha = 0.05$



Holm procedure with $\alpha = 0.05$

Remove node for H₂

$$\alpha = 0.05$$

Holm procedure with $\alpha = 0.05$

Test H_1 at level α $p_1 > \alpha \Rightarrow$ do not reject H_1

 $\alpha = 0.05$

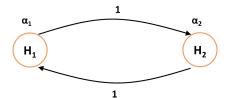


 $p_1 = 0.07$

Graphical approaches

Weighted Holm procedure

Use α_1, α_2 (where $\alpha_1 + \alpha_2 = \alpha$) instead of $\alpha_1 = \alpha_2 = \alpha/2$



Primary



 H_2

Graphical approaches

Secondary



 H_4

Low dose

High dose



Primary



 H_1

α/2



Graphical approaches

Secondary



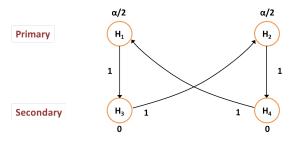
Low dose



High dose

High dose

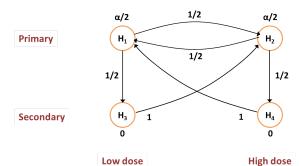
Diabetes example



Low dose

$$m{lpha} = egin{pmatrix} rac{lpha}{2} & rac{lpha}{2} & 0 & 0 \end{pmatrix} \qquad m{G} = egin{pmatrix} 0 & 0 & 1 & 0 \ 0 & 0 & 0 & 1 \ 0 & 1 & 0 & 0 \ 1 & 0 & 0 & 0 \end{pmatrix}$$

Hierarchical testing



$$m{lpha} = egin{pmatrix} rac{lpha}{2} & rac{lpha}{2} & 0 & 0 \end{pmatrix} \qquad m{G} = egin{pmatrix} 0 & 1/2 & 1/2 & 0 \ 1/2 & 0 & 0 & 1/2 \ 0 & 1 & 0 & 0 \ 1 & 0 & 0 & 0 \end{pmatrix}$$

Update algorithm

Hierarchical testing

Transition matrix $\mathbf{G} = (g_{ii})$, where g_{ii} is the fraction of α_i allocated to H_i if H_i is rejected.

Require $0 \le g_{ii} \le 1$, $g_{ii} = 0$ and $\sum_{k=1}^{K} g_{ik} = 1$ for $i, j = 1, \dots, K$.

Closed testing

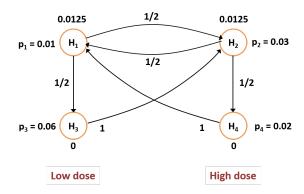
- Set $J = \{1, ..., K\}$
- Select a $j \in J$ such that $p_i \leq \alpha_i$ and reject H_i ; otherwise stop
- Update the graph:

$$egin{aligned} J &
ightarrow J \setminus \{j\} \ &lpha_\ell &
ightarrow egin{cases} lpha_\ell + lpha_j g_{j\ell} & ext{ for } \ell \in J \ 0 & ext{ otherwise} \ & g_{\ell k} &
ightarrow egin{cases} rac{g_{\ell k} + g_{\ell j} g_{jk}}{1 - g_{\ell j} g_{j\ell}} & ext{ for } \ell, k \in J, \ell
eq k, g_{\ell j} g_{j\ell} < 1 \ 0 & ext{ otherwise} \end{aligned}$$

3 If $|J| \ge 1$, go to Step 1; otherwise stop

(Bretz et al., 2009)

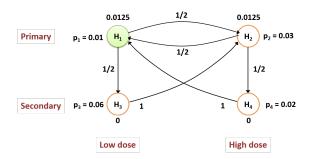
 $\alpha = 0.025$



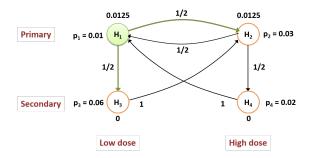
Graphical approaches

Diabetes example

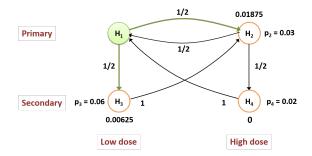
 $\alpha = 0.025$



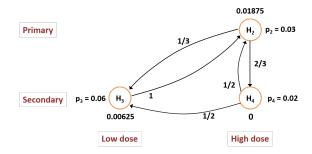
$$\alpha = 0.025$$



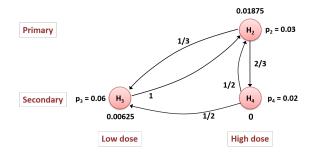
 $\alpha = 0.025$



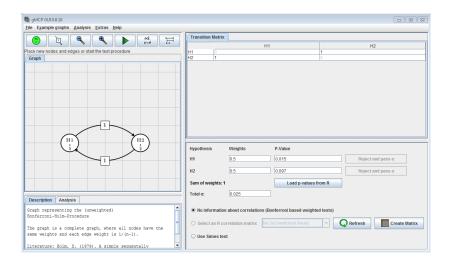
 $\alpha = 0.025$



$\alpha = 0.025$



gMCP package



Summary

- Graphical approach allows the following (Bretz et al. 2015):
 - Tailor advanced multiple test procedures to a structured families of hypotheses
 - Visualise complex decision strategies in an efficient and easily communicable way
 - Ensure strong FWER control
- Approach covers many common multiple test procedures as special cases
- Many possible extensions as well

Multiplicity in practice

A lot of advice available!

- Dmitrienko and D'Agostino (2013) Traditional multiplicity adjustment methods in clinical trials. Stat Med 32:5172-5218.
- Dmitrienko and D'Agostino (2018) Multiplicity Considerations in Clinical Trials. *NEJM* 378:2115-2122.
- Howard et al. (2018) Recommendations on multiple testing adjustment in multi-arm trials with a shared control group. SMMR 27(5):1513-1530.
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- Wang et al. (2011) Regulatory Perspectives on Multiplicity in Adaptive Design Clinical Trials throughout a Drug Development Program. J Biopharm Stat 21:846-59.
- Wason et al. (2014) Correcting for multiple-testing in multi-arm trials: is it necessary and is it done? Trials 15:364.

- Need to identify the adjustment method aligned with the structure of the clinical objectives, that provides highest statistical power.
- Use all available clinical and statistical information to arrive at the most appropriate and efficient method.
- Often need to do a simulation-based assessment of operating characteristics of procedures under trial-specific assumptions.
- Helpful to classify methods (see Dmitrienko and D'Agostino, 2013)
 - Logical restrictions
 - Distributional information

Choosing a multiple testing procedure Logical restrictions

- Group procedures into three classes:
 - Single-step that test all hypotheses simultaneously (e.g. Bonferroni)
 - Stepwise that rely on data-driven ordering (e.g. Holm and Hochberg)
 - Stepwise that rely on pre-specified ordering. (e.g. Fixed-sequence or Fallback)
- Single-step most basic. Stepwise more powerful; easier to tailor.

Choosing a multiple testing procedure Distributional information

- Again group into three classes:
 - Non-parametric procedures: without distributional assumptions (e.g. Bonferroni, Holm, Fixed-sequence, Fallback)
 - Semi-parametric methods: control under e.g. independence or positive correlation, but not full specification of a joint distribution (e.g. Hochberg and Hommel)
 - Parametric methods: control FWER only when joint distribution of test statistics fully specified (e.g. Dunnett)
- In general, power increases as more distributional assumptions are added

Regulatory view

- So far, we have been assuming that we have to adjust for multiplicity.
- Often we will need to, but not always.
- Latest (draft) regulatory guidance from the FDA and EMA:
 - EMA: Guideline on multiplicity issues in clinical trials (2016)
 - FDA: Multiple Endpoints in Clinical Trials (2017)

Software R

- multxpert package for common adjustments in single family (non-parametric, semi-parametric and parametric procedures) and multiple families (gatekeeping procedures).
 - http://multxpert.com/wiki/Software
 - "Multiple Testing Problems in Pharmaceutical Statistics" by Dmitrienko et al. (2009)
- gMCP for graph based multiple test procedures.
- multcomp package addresses multiplicity issues in general linear and non-linear models.
 - Adjustments for ANOVA, ANCOVA, regression models, and more.
 - "Multiple Comparisons Using R" by Bretz et al. (2016)

Graphical approaches

- Several commands support correcting for multiple comparisons.
- oneway (one-way ANOVA) commands
- Powerful test command for testing linear hypotheses from regress, stcox, logit, svy has an mtest option that supports Bonferroni, Holm, etc.

Summary

Hierarchical testing

• There is a lot of advice available on how to choose a multiple testing procedure.

Graphical approaches

- But no golden rule: have to tailor to your trial.
- Important as often required by regulatory authorities.
- Much software out there to do this in practice.

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