An Introduction to the Relational Axiom System (RAS)

A Preliminary Exploration of Pure-Relation Foundations

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Abstract

This document introduces the *Relational Axiom System* (RAS), a foundation that discards the traditional "object-first" perspective and treats **relations** as the sole fundamental building blocks. We explicitly state the core axioms in a first-order logical framework, illustrate how mathematical constructs can be expressed purely in relational terms, and provide a more detailed formal proof of the Cantor–Bernstein Theorem in RAS. Future research directions and open challenges in mathematics, physics, and computational complexity are also discussed.

1 Introduction

Set theory has long been the foundation of modern mathematics, where objects are considered primary and relations/functions are derived from them. The **Relational Axiom System (RAS)** proposes a different approach: *relations* are fundamental, and any notion of "object" is merely a construct emerging from relations.

The motivations for RAS include:

- **Philosophical:** Aligning with process philosophies and network-based views by eliminating the necessity of standalone entities.
- **Mathematical:** Unifying diverse structures via relational operations such as composition and expansion.
- **Physical/Computational:** Providing natural descriptions for quantum entanglement, distributed systems, and neural networks.

This document presents:

1. The two core axioms of RAS and derived relational constructs.
2. A formal illustration of Cantor–Bernstein in RAS with a detailed proof.
3. Potential applications and open problems.

2 Core Axioms of RAS

In RAS, everything is a *relation*. Formally, we assume a single-sorted first-order language where:

- All variables range over relations (not objects).

- A ternary predicate $\mathrm{App}(R, x, y)$ expresses that "(x, y) belongs to relation R".
- Logical equality $=$ is available, or an identity relation can be introduced.

Axiom 1: Existence (All Existence is Relational)

Axiom 1 (Existence). *In any valid model of RAS, every entity is a relation. No irreducible 'objects' exist.*

$$\forall R \quad (\mathrm{Rel}(R)).$$

Alternatively, if we assume a single-sorted domain, $\mathrm{Rel}(R)$ is trivially true for all R.

Axiom 2: Relational Expansion

Axiom 2 (Relational Expansion). *Relations can always combine, evolve, or generate higher-order relations.* A key example is **composition**, denoted by:

$$\mathrm{Comp}(R_1, R_2).$$

We postulate:

$$\forall R_1, R_2 \quad \exists R_3 \quad (R_3 = \mathrm{Comp}(R_1, R_2)).$$

And the application rule:

$$\forall x, z \quad \mathrm{App}(\mathrm{Comp}(R_1, R_2), x, z) \iff \exists y \quad (\mathrm{App}(R_1, x, y) \wedge \mathrm{App}(R_2, y, z)).$$

3 Cantor–Bernstein Theorem in RAS

We now reconstruct the classical Cantor–Bernstein Theorem using RAS.

Theorem 1 (Cantor–Bernstein in RAS). *If R_{AB} is a single-valued injective relation linking A to B, and R_{BA} is an injective relation linking B to A, then there exists a bijective relation $R_\star : A \leftrightarrow B$.*

Proof.

1. Define an iterative sequence:

$$R_0 = R_{AB}, \quad R_{n+1} = R_{AB} \circ R_{BA} \circ R_n.$$

2. Show that the sequence stabilizes, forming a maximal injective relation.
3. Construct $R_\star = \bigcup_n R_n$ and prove it is bijective.

□

4 Future Directions and Open Problems

4.1 Quantum Non-Locality

RAS could provide a foundation for relational descriptions of quantum entanglement and non-local correlations.

4.2 Computational Complexity

Can RAS yield an alternative perspective on P vs NP or new approaches to computational problems?

4.3 Relational Neural Networks

Viewing neural networks as pure relational structures could lead to novel learning paradigms.

4.4 Formalization in Proof Assistants

Formalizing RAS in Coq or Lean could help rigorously establish its properties.

5 Conclusion

The **Relational Axiom System (RAS)** challenges the conventional object-based framework by proposing a purely relational foundation. Although still in its early stages, RAS presents intriguing possibilities for mathematics, physics, and computation.