Supplementary Material for

The numerical solution for flapping wing hovering wingbeat dynamics

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A. Extended quasi-steady aerodynamic model

Here, we assume that the flapping wing fruit fly can keep hovering flight always, then the revised quasi-steady aerodynamic model can be adopted to estimate aerodynamic force of flapping wing thin plate passing through unsteady flow [6, 17, 18]. Considering the assumption and applicability of revised quasi-steady aerodynamic model realized by blade-element method, Whitney and Wood has broadly remarked its qualification in efficiently estimating 2D quasi-steady aerodynamic force under quasi-static assumption and disqualification in tackling 3D unsteady flow feature, such as wake capture [22]. Here, we shall derive aerodynamic forces and moments component analysis formulas for three types of aerodynamic mechanisms, namely, translational and rotational circulation mechanism and added mass effect. It's worth noting that the aerodynamic damping moment, which plays an important role in realizing smooth pitch rotation of wing planform [22], is introduced into the pitch-axis aerodynamic moment. And the aerodynamic damping moment results from the velocity gradient of each strip along chordwise differential elements due to pitch motion of wing planform [1-4, 22] is also included to complete the development of aerodynamic moments. Thus, the current aerodynamic model can be termed as an extended version of quasi-steady aerodynamic model.

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A.1 Aerodynamic forces

As seen in Fig. 1 from [12], the blade-element method assumes that aerodynamic force acting on the wing planform is the summation of force acting on each infinitesimal spanwise strip, and the local pressure drag acting on each strip is the summation of normal pressure drag acting on each chordwise differential elements [1-4, 17, 18, 22]. Thus, the aerodynamic forces in wing planform fixed frame, which are produced by translational circulation, rotational circulation, and added mass effect, can be given by

$$\boldsymbol{F}_{\mathrm{trans,y}}=\frac{1}{2}\rho R_{\mathrm{eff}}{}^{3}C_{\mathrm{aver}}\hat{F}_{\mathrm{trans}}C_{\mathrm{N}}\left(\alpha\right)\omega_{\mathrm{pal}}{}^{2} \tag{1}$$

$$\boldsymbol{F}_{\mathrm{rot},y}=\frac{1}{2}\rho R_{\mathrm{eff}}{}^{2}C_{\mathrm{aver}}{}^{2}\hat{F}_{\mathrm{rot}}C_{\mathrm{R}}\omega_{x}\omega_{\mathrm{pal}} \tag{2}$$

$$\boldsymbol{F}_{\mathrm{add,y}}=\frac{\pi}{4}\rho C_{\mathrm{aver}}{}^{2}R_{\mathrm{eff}}{}^{2}\hat{F}_{\mathrm{rot}}\left(\dot{\omega}_{z}+\omega_{x}\omega_{y}\right)+\frac{\pi}{4}\rho C_{\mathrm{aver}}{}^{3}R_{\mathrm{eff}}\hat{F}_{\mathrm{coeff,add,y}}\dot{\omega}_{x} \tag{3}$$

where ρ is the air density. The wing effective length (R_{eff}) is defined as the projected distance between the most proximal point (wing root) and the most distal point (wing tip) on the wing planform along the X_{s}-axis. The mean chord length (C_{aver}) is defined as the area of single wing divided by the wing effective length, $A_{\mathrm{w}}/R_{\mathrm{eff}}$. $\hat{F}_{\mathrm{trans}}$ and $\hat{F}_{\mathrm{rot}}$ can be termed as non-dimensional translational and rotational aerodynamic force, respectively [22]. And $\hat{F}_{\mathrm{coeff,add,y}}$ can be analogously termed as non-dimensional rotational added-mass aerodynamic coefficient. Their complete expression is listed in Table S1, where a series of non-dimensional parameters have been acquired, for example, non-dimensional x-root offset $\hat{x}_{\mathrm{r}}=x_{\mathrm{r}}/R_{\mathrm{eff}}$, non-dimensional radial distance $\hat{r}=r/R_{\mathrm{eff}}$, non-dimensional leading-edge profiles $\hat{z}_{\mathrm{le}}(\hat{r})=z_{\mathrm{le}}(r)/C_{\mathrm{aver}}$, non-dimensional trailing-edge profiles $\hat{z}_{\mathrm{tr}}(\hat{r})=z_{\mathrm{tr}}(r)/C_{\mathrm{aver}}$, non-dimensional chord distribution $\hat{C}(\hat{r})=C(r)/C_{\mathrm{aver}}$. And the aspect ratio ($AR$) is equal to $R_{\mathrm{eff}}/C_{\mathrm{aver}}$ for the single wing planform. C_{R} is the theoretical rotational coefficient with expression of $C_{\mathrm{R}}=\pi(0.75-\hat{x}_{0})$, and ω_{pal}, which is equal to $\dot{\phi}$, is defined as angular velocity of pitch axis line. Here, for translational aerodynamic force, its tangential component is neglected due to its little contribution in the case of wing planform incurred high angle of attack [5, 15, 16]. And the normal translational aerodynamic force coefficient ($C_{\mathrm{N}}(\alpha)$, α is angle of attack) is obtained from lift and drag coefficients using trigonometry. Within the community of flapping wing, the broadly adopted 2D quasi-static lift and drag coefficients in local flow for translational circulation mechanism are from the simple harmonic fitness relationships to the experimental data for dynamically scaled fruit fly wing model [6], like following:

$$\begin{cases} C_{\mathrm{L}}(\alpha) = 0.225 + 1.58\sin(2.13\alpha - 7.2) \\ C_{\mathrm{D}}(\alpha) = 1.92 - 1.55\cos(2.04\alpha - 9.82) \end{cases} \tag{4}$$

These formulas might be appropriately used to evaluate translational aerodynamic force for dynamically scaled wing planform with non-dimensional conformal feature. Thus, the normal translational aerodynamic force coefficient can be obtained by a simple transformation, $C_{\mathrm{N}}(\alpha) = \cos(\alpha)C_{\mathrm{L}}(\alpha) + \sin(\alpha)C_{\mathrm{D}}(\alpha)$.

Table S1 Non-dimensional aerodynamic parameters for different aerodynamic components

Aerodynamic Components	**Abbreviation**	**Calculation formulas**
Translational components	$\hat{F}_{\mathrm{trans}}$	$\hat{F}_{\mathrm{trans}} = \int_0^1 (\hat{r} + \hat{x}_{\mathrm{r}})^2 \hat{C}(\hat{r}) \mathrm{d}\hat{r}$
	$\hat{M}_{\mathrm{coeff,trans,z}}$	$\hat{M}_{\mathrm{coeff,trans,z}} = \int_0^1 (\hat{r} + \hat{x}_{\mathrm{r}})^3 \hat{C}(\hat{r}) \mathrm{d}\hat{r}$
	$M_{z,\mathrm{T,P}}$	$M_{z,\mathrm{T,P}} = \frac{1}{2}\rho R_{\mathrm{eff}}{}^4 C_{\mathrm{aver}} \hat{M}_{\mathrm{coeff,trans,z}}$
	$\hat{r}_{\mathrm{spw,cop,trans}}$	$\hat{r}_{\mathrm{spw,cop,trans}} = \frac{\hat{M}_{\mathrm{coeff,trans,z}}}{\hat{F}_{\mathrm{trans}}}$
	$M_{x,\mathrm{T,P}}$	$M_{x,\mathrm{T,P}} = \frac{1}{2}\rho R_{\mathrm{eff}}{}^3 C_{\mathrm{aver}}{}^2 \hat{F}_{\mathrm{trans}}$
Rotational components	$\hat{F}_{\mathrm{rot}}$	$\hat{F}_{\mathrm{rot}} = \int_0^1 (\hat{r} + \hat{x}_{\mathrm{r}}) \hat{C}(\hat{r})^2 \mathrm{d}\hat{r}$
	$\hat{M}_{\mathrm{coeff,rot,z}}$	$\hat{M}_{\mathrm{coeff,rot,z}} = \int_0^1 (\hat{r} + \hat{x}_{\mathrm{r}})^2 \hat{C}(\hat{r})^2 \mathrm{d}\hat{r}$
	$M_{z,\mathrm{R,P}}$	$M_{z,\mathrm{R,P}} = \frac{1}{2}\rho C_{\mathrm{aver}}{}^2 R_{\mathrm{eff}}{}^3 \hat{M}_{\mathrm{coeff,rot},z}$
	$\hat{r}_{\mathrm{spw,cop,rot}}$	$\hat{r}_{\mathrm{spw,cop,rot}} = \frac{\hat{M}_{\mathrm{coeff,rot,z}}}{\hat{F}_{\mathrm{rot}}}$
	$M_{x,\mathrm{R,P}}$	$M_{x,\mathrm{R,P}} = \frac{1}{2}\rho C_{\mathrm{aver}}{}^3 R_{\mathrm{eff}}{}^2 \hat{F}_{\mathrm{rot}}$
Aerodynamic damping	$\hat{M}_{\mathrm{coeff,rd},x}$	[a] $\hat{M}_{\mathrm{coeff,rd},x} = \int_0^1 \hat{z}_{\mathrm{rd}}(\hat{r}) \mathrm{d}\hat{r}$
	$M_{x,\mathrm{Rd,P}}$	$M_{x,\mathrm{Rd,P}} = \frac{1}{2}\rho C_{\mathrm{aver}}{}^4 R_{\mathrm{eff}} \hat{M}_{\mathrm{coeff,rd},x}$

components	$\hat{M}_{\text{coeff,rd,z}}$	$\hat{M}_{\text{coeff,rd,z}} = \int_0^1 \hat{C}(\hat{r})^3 (\hat{x}_{\text{r}} + \hat{r}) \mathrm{d}\hat{r}$
	$M_{z,\text{Rd,P}}$	$M_{z,\text{Rd,P}} = \frac{1}{6}\rho C_{\text{aver}}{}^3 R_{\text{eff}}{}^2 \hat{M}_{\text{coeff,rd,z}}$
Added-mass components	$\hat{F}_{\text{coeff,add,y}}$	[b] $\hat{F}_{\text{coeff,add,y}} = \int_0^1 \hat{C}(\hat{r})^2 \hat{z}_{\text{h}} \mathrm{d}\hat{r}$
	$\hat{M}_{\text{coeff,add,z,1}}$	$\hat{M}_{\text{coeff,add,z,1}} = \int_0^1 \hat{C}(\hat{r})^2 (\hat{r} + \hat{x}_{\text{r}})^2 \mathrm{d}\hat{r}$
	$\hat{I}_{xx,\text{am}}$	[b] $\hat{I}_{xx,\text{am}} = \int_0^1 \hat{C}(\hat{r})^2 \left(\hat{z}_{\text{h}}(\hat{r})^2 + \frac{1}{32}\hat{C}(\hat{r})^2 \right) \mathrm{d}\hat{r}$
	$\hat{I}_{xz,\text{am}}$	[b] $\hat{I}_{xz,\text{am}} = \int_0^1 (\hat{r} + \hat{x}_{\text{r}}) \hat{C}(\hat{r})^2 \hat{z}_{\text{h}}(\hat{r}) \mathrm{d}\hat{r}$
	$I_{xz,\text{am,P}}$	$I_{xz,\text{am,P}} = \frac{\pi}{4}\rho C_{\text{aver}}{}^3 R_{\text{eff}}{}^2 \hat{I}_{xz,\text{am}}$
	$I_{xx,\text{am,P}}$	$I_{xx,\text{am,P}} = \frac{\pi}{4}\rho C_{\text{aver}}{}^4 R_{\text{eff}} \hat{I}_{xx,\text{am}}$
	$M_{z,\text{R,P}}$	$M_{z,\text{R,P}} = \frac{\pi}{4}\rho C_{\text{aver}}{}^2 R_{\text{eff}}{}^3 \hat{M}_{\text{coeff,add,z,1}}$

[a] $\hat{z}_{\text{rd}}(\hat{r}) = \frac{1}{4}\left[\left|\hat{z}_{\text{le}}(\hat{r}) - \hat{\Delta}\right| \left(\hat{z}_{\text{le}}(\hat{r}) - \hat{\Delta}\right)^3 - \left|\hat{z}_{\text{le}}(\hat{r}) - \hat{\Delta} - \hat{C}(\hat{r})\right| \left(\hat{z}_{\text{le}}(\hat{r}) - \hat{\Delta} - \hat{C}(\hat{r})\right)^3 \right]$;

[b] $\hat{z}_{\text{h}}(\hat{r}) = \frac{1}{2}\hat{C}(\hat{r}) - \left(\hat{z}_{\text{le}}(\hat{r}) - \hat{\Delta}\right)$.

A.2 Aerodynamic moments

The aerodynamic moments in wing planform fixed frame, which are produced by the mechanisms of translational circulation and rotational circulation, aerodynamic damping and added mass effect**,** can be expressed as:

$$\boldsymbol{M}_{\text{trans},z} = M_{z,\text{T,P}} \operatorname{sign}(\alpha) C_{\text{N}}(\alpha) \dot{\phi}^2 \vec{e}_z \tag{5}$$

$$\boldsymbol{M}_{\text{trans},x} = M_{x,\text{T,P}} \operatorname{sign}(\alpha) \hat{Z}_{\text{cop,trans}}(\alpha) C_{\text{N}}(\alpha) \dot{\phi}^2 \vec{e}_x \tag{6}$$

$$\boldsymbol{M}_{\text{rot},z} = -M_{z,\text{R,P}} C_{\text{R}} \dot{\psi} \left|\dot{\phi}\right| \vec{e}_z \tag{7}$$

$$\boldsymbol{M}_{\text{rot,x}} = -M_{x,\text{R,P}} C_{\text{R}} \hat{Z}_{\text{cop,rot}}(\alpha) \dot{\psi} \left|\dot{\phi}\right| \vec{e}_x \tag{8}$$

$$\boldsymbol{M}_{\text{rd},x} = -M_{x,\text{Rd,P}} C_{\text{rd}} \dot{\psi} \left|\dot{\psi}\right| \vec{e}_x \tag{9}$$

$$\boldsymbol{M}_{\text{rd},z} = -M_{z,\text{Rd,P}} C_{\text{rd}} \dot{\psi} \left|\dot{\psi}\right| \vec{e}_z \tag{10}$$

$$\boldsymbol{M}_{\text{add},x} = \left(-I_{xz,\text{am,P}} \left(\ddot{\phi} \cos\psi \right) - I_{xx,\text{am,P}} \ddot{\psi} \right) \vec{e}_x \tag{11}$$

$$\boldsymbol{M}_{\text{add},z} = \left(-\frac{\pi}{2} M_{z,\text{R,P}} \left(\ddot{\phi} \cos\psi \right) - I_{xz,\text{am,P}} \ddot{\psi} \right) \vec{e}_z \,. \tag{12}$$

The complete derivation of (5-12) is detailed in the supplementary material and listed in Table S1 for concision. Here, $M_{z,\text{T,P}}$ and $M_{z,\text{R,P}}$ in (5) and (7), can be termed as translational and rotational aerodynamic moments, respectively. $M_{x,\text{Rd,P}}$ and $M_{z,\text{Rd,P}}$ in (9) and (10) can be termed as rotational damping coefficient. And $I_{xz,\text{am,P}}$, $I_{xx,\text{am,P}}$ and $M_{z,\text{R,P}}$ in (11) and (12) can be termed as added-mass moments coefficients, respectively. As to pitching moment component of wing planform arisen from the rotational circulation term, it is necessary to tackle the problem of chordwise acting point distribution of rotational normal aerodynamic force since it might play an improving or resisting role during the passivity of pitch reverse dynamic of wing planform [3]. Here, taking the difficulty of direct measurements of rotational moments into consideration, we assume that the COP's chordwise location distribution is the same as the one for translational normal aerodynamic force ($\hat{d}_{\text{cop}}(\alpha)$) in the light of translational circulation and rotational circulation stemming from the same mechanism of circulatory-and-attached-vortex force [19, 20].Thus the non-dimensional COP's chordwise location at a certain angle of attack (α) for particular strip, on which the translational and rotational aerodynamic force act normal, can be simplified as $\hat{Z}_{\text{cop,trans}}(\alpha)$ and $\hat{Z}_{\text{cop,rot}}(\alpha)$ in (6) and (8) after derivation like following expression:

$$\hat{Z}_{\text{cop,trans}}(\alpha) = \frac{\int_0^1 \hat{z}_{\text{cop}} \left(\hat{r}_{\text{spw,cop,trans}} \right) \left(\hat{r} + \hat{x}_{\text{r}} \right)^2 \hat{C}(\hat{r}) \text{d}\hat{r}}{\hat{F}_{\text{trans}}} \tag{13}$$

$$\hat{Z}_{\text{cop,rot}}(\alpha) = \frac{\int_0^1 \hat{z}_{\text{cop}} \left(\hat{r}_{\text{spw,cop,rot}} \right) \left(\hat{r} + \hat{x}_{\text{r}} \right) \hat{C}(\hat{r})^2 \text{d}\hat{r}}{\hat{F}_{\text{rot}}} \,, \tag{14}$$

where $\hat{r}_{\text{spw,cop,trans}}$ and $\hat{r}_{\text{spw,cop,rot}}$ are non-dimensional spanwise location of COP relative to z-axis of wing shoulder frame for translational and rotational aerodynamic force, respectively. For acting point of translational aerodynamic force, the non-dimensional distance of chordwise COP for spanwise COP strip relative to pitch axis line is given by

$$\hat{z}_{\text{cop}}\left(\hat{r}_{\text{spw,cop,trans}}\right)=\hat{z}_{\text{le}}\left(\hat{r}_{\text{spw,cop,trans}}\right)-\hat{C}\left(\hat{r}_{\text{spw,cop,trans}}\right)\hat{d}_{\text{cop}}\left(\alpha\right), \quad (15)$$

where $\hat{d}_{\text{cop}}(\alpha)$ is non-dimensional COP's chordwise position distribution about angle of attack relative to leading edge. Here, according to the scaling law, the non-dimensional fitting formula of COP for dynamically scaled fruit fly wing planform [7, 8, 22] is adopted with slight increase of 0.11 in the constant term due to the relative shift of pitch axis chosen here to trailing-edge, which is

$$\hat{d}_{\text{cop}}\left(\alpha\right)=\frac{0.82}{\pi}\left|\alpha\right|+0.16. \quad (16)$$

Similarly, for acting point of rotational aerodynamic force, the non-dimensional chordwise COP's location distribution for specific strip located in $\hat{r}_{\text{spw,cop,rot}}$ can be given by

$$\hat{z}_{\text{cop}}\left(\hat{r}_{\text{spw,cop,rot}}\right)=\hat{z}_{\text{le}}\left(\hat{r}_{\text{spw,cop,rot}}\right)-\hat{C}\left(\hat{r}_{\text{spw,cop,rot}}\right)\hat{d}_{\text{cop}}\left(\alpha\right). \quad (17)$$

For the translational and rotational circulation aerodynamic force components acting normal to each spanwise strip element, the temporal and spacial variable process of the COP's chordwise position distribution changing with local angle of attack has been diagramed in the supplemental video S1. Moreover, for the total translational and rotational circulation aerodynamic forces acting on the whole wing planform, the chordwise position distribution of COPs locked on the different spanwise specific strip are also plotted in blue and cyan solid circle, respectively.

In short, here, the **extended quasi-steady aerodynamic forces/moments model** is derived from previous revised quasi-steady aerodynamic model [6, 17, 18, 22], but it has two different points from the latter one. The first one is that it includes the contribution of aerodynamic damping moment along *z*-axis of wing shoulder frame, which is scarcely considered in previous research [3, 4, 22]. The second one is that the assumption of uniform distribution about non-dimensional COP's chordwise location for translational and rotational circulation aerodynamic mechanism is introduced to simplify the calculation of the rotational aerodynamic moment. Because of the difficulty of direct measurements of rotational moments and the lack of exploring possible COP's chordwise location for rotational circulation aerodynamic force [3, 4, 7, 8], the calculation of rotational aerodynamic moment is either neglected [22] or consciously executed by assumption that the COP's chordwise position distribution for aerodynamic force stemming from translational and rotational circulation is identical, and located at the geometric center of wing strip elements [3, 4].

B. Inertial forces and moments

According to at-scale experimental studies for flapping wing hovering flight aerodynamics [22], the isolation of inertial force arisen from the high angular acceleration at the COM plays a crucial role in measuring aerodynamic force especially during the neighbouring region of wing pitch reversal [17, 18]. Thus it is necessary to consider the contribution of inertial force to total instantaneous forces [9]. The inertial force in wing fixed frame can be given by

$$F_{\text{inert},y} = -m_{\text{wing}} \cdot {}^{rw}\dot{\upsilon}^{\text{w}}_{\text{com},y} = -m_{\text{wing}}\left(\ddot{\phi}\cos\psi x_{\text{com}} - (\ddot{\psi} - \frac{1}{2}\dot{\phi}^2 \sin 2\psi) z_{\text{com}}\right) \tag{18}$$

$$F_{\text{inert},z} = -m_{\text{wing}} \cdot {}^{rw}\dot{\upsilon}^{\text{w}}_{\text{com},z} = -m_{\text{wing}}\left(-\ddot{\phi}\sin\psi x_{\text{com}} - (\dot{\psi}^2 + \dot{\phi}^2 \sin^2\psi) z_{\text{com}}\right), \tag{19}$$

where m_{wing} is the mass of wing planform, ${}^{rw}\dot{\upsilon}^{\text{w}}_{\text{com},y}$ and ${}^{rw}\dot{\upsilon}^{\text{w}}_{\text{com},z}$ is linear acceleration of COM of wing planform. Then, the inertial moments in the wing fixed frame can be calculated through a cross-product operation

$${}^{rw}\boldsymbol{M}_{\text{inert}} = {}^{rw}\boldsymbol{p}^{\text{w}}_{\text{com}} \times {}^{rw}\boldsymbol{F}_{\text{inertial}} = \begin{bmatrix} x_{\text{com}} \\ 0 \\ z_{\text{com}} \end{bmatrix} \times \begin{bmatrix} 0 \\ {}^{rw}\boldsymbol{F}_{\text{inert},y} \\ {}^{rw}\boldsymbol{F}_{\text{inert},z} \end{bmatrix} = \begin{bmatrix} -z_{\text{com}}F_{\text{inert},y} \\ x_{\text{com}}F_{\text{inert},z} \\ -x_{\text{com}}F_{\text{inert},y} \end{bmatrix}, \tag{20}$$

And the inertial moments can be expressed in the form of components as following,

$${}^{rw}\boldsymbol{M}_{\text{inert}} = \begin{bmatrix} {}^{rw}\boldsymbol{M}_{\text{inert},x} & {}^{rw}\boldsymbol{M}_{\text{inert},y} & {}^{rw}\boldsymbol{M}_{\text{inert},z} \end{bmatrix}^T, \tag{21}$$

here, the components of ${}^{rw}\boldsymbol{M}_{\text{inert}}$ can be written as

$$\begin{cases} {}^{rw}\boldsymbol{M}_{\text{inert},x} = -z_{\text{com}}m_{\text{wing}}\left(\ddot{\phi}\cos\psi x_{\text{com}} - (\ddot{\psi} - \frac{1}{2}\dot{\phi}^2\sin 2\psi) z_{\text{com}}\right) \\ {}^{rw}\boldsymbol{M}_{\text{inert},y} = x_{\text{com}}m_{\text{wing}}\left(-\ddot{\phi}\sin\psi x_{\text{com}} - (\dot{\psi}^2 + \dot{\phi}^2\sin^2\psi) z_{\text{com}}\right) \\ {}^{rw}\boldsymbol{M}_{\text{inert},z} = -x_{\text{com}}m_{\text{wing}}\left(\ddot{\phi}\cos\psi x_{\text{com}} - (\ddot{\psi} - \frac{1}{2}\dot{\phi}^2\sin 2\psi) z_{\text{com}}\right) \end{cases}, \tag{22}$$

C. Total aerodynamic force acting normal to wing planform

According to the quasi-steady model for flapping wing hovering flight [17, 18], the total instantaneous aerodynamic force normal to wing planform can be represented as the summation of three force components

$$\boldsymbol{F}_{\text{aero},y} = \boldsymbol{F}_{\text{trans},y} + \boldsymbol{F}_{\text{rot},y} + \boldsymbol{F}_{\text{add},y}, \tag{23}$$

Obviously, the quasi-steady estimating model is unavailable to include some unsteady effects, such as the vortex starting effect during acceleration impulsively from rest [10], vortex shedding effect occurring at high angle of attack [21], wake capture due to wing planform intercepting its own wake during reciprocating oscillation [6], and induced flow effects depended on wing size and shape [14, 15, 20]. However, the current extended quasi-steady estimating model should not be discounted to evaluate flapping wing hovering aerodynamics as verified by following comparisons between estimating results including the inertial force and experimental results. The total instantaneous forces acting normal to wing planform is the sum of aerodynamic force and inertial force.

$$^{rw}\boldsymbol{F}_{\text{total},y} = \boldsymbol{F}_{\text{aero},y} + \boldsymbol{F}_{\text{inert},y} \, . \tag{24}$$

The instantaneous forces of single wing in right wing planform fixed frame are estimated with the wing morphological parametrization from [11, 12], flapping and pitch angles of approximate hovering fruit fly measured by Dr. Muijres [13] (Namely ϕ_{exp} and ψ_{exp} as seen in Fig. 3 in the text). The instantaneous aerodynamic forces include the instantaneous circulation force and added-mass force; both of them are acting normal to the wing planform but with different acting points (Fig. S1(a, b)). The instantaneous circulation force is the resultant of translational and rotational circulation components, both of them have different spanwise centers of pressure acting on specific strip. Here, in order to visualize the total effect of instantaneous concentrated force for three types of aerodynamic mechanisms but not to reflect the real physical essence of force distribution, the average chord length with non-dimensional location of pitch axis assumed at a specific value of 0.25, which is marked by the red dashed line during the dynamic process, is arbitrarily chosen as a specific spanwise strip to diagram the time and space shifting of centers of pressure along the chordwise direction of average chord length with variation of angle of attack for the circulation forces. The dynamic process of instantaneous forces for a complete stroke has been diagramed in the supplemental video S2. Here, two-dimensional diagrams of wing kinematics and instantaneous forces denote the magnitude (0.1 times) and orientation of normal force vectors, respectively. Black lines indicate the instantaneous position of the average wing chord at 15 temporally equidistant points. Small black circles mark the leading edge.

D. Torques in right wing root frame of reference

It is spontaneous to develop the total aerodynamic moments in the wing planform fixed frame in terms of the aerodynamic moments stemming from different aerodynamic mechanisms mentioned above. Hence, for single wing planform, the total aerodynamic moments along spanwise pitch axis and chordwise axis of wing planform fixed frame ($x_{rw}y_{rw}z_{rw}$) can be given by

$$\begin{cases} {}^{rw}\boldsymbol{M}_{\text{aero},x} = \boldsymbol{M}_{\text{trans},x} + \boldsymbol{M}_{\text{rot},x} + \boldsymbol{M}_{\text{rd},x} + \boldsymbol{M}_{\text{add},x} \\ {}^{rw}\boldsymbol{M}_{\text{aero},z} = \boldsymbol{M}_{\text{trans},z} + \boldsymbol{M}_{\text{rot},z} + \boldsymbol{M}_{\text{rd},z} + \boldsymbol{M}_{\text{add},z} \end{cases}, \tag{25}$$

After substituting the aerodynamic moment components from four types of mechanisms into (25), we have

$$ {}^{rw}\boldsymbol{M}_{\text{aero},x} = \begin{bmatrix} M_{x,\text{T,P}}\text{sign}(\alpha)\hat{Z}_{\text{cop,trans}}(\alpha)C_{\text{N}}(\alpha)\dot{\phi}^2 - M_{x,\text{R,P}}C_{\text{R}}\hat{Z}_{\text{cop,rot}}(\alpha)\dot{\psi}\left|\dot{\phi}\right| \\ -M_{x,\text{Rd,P}}C_{\text{rd}}\dot{\psi}\left|\dot{\psi}\right| - I_{xz,\text{am,P}}\left(\ddot{\phi}\cos\psi\right) - I_{xx,\text{am,P}}\ddot{\psi} \end{bmatrix}\vec{e}_x, \tag{26}$$

$$ {}^{rw}\boldsymbol{M}_{\text{aero},z} = \begin{bmatrix} M_{z,\text{T,P}}\text{sign}(\alpha)C_{\text{N}}(\alpha)\dot{\phi}^2 - M_{z,\text{R,P}}C_{\text{R}}\dot{\psi}\left|\dot{\phi}\right| \\ -M_{z,\text{Rd,P}}C_{\text{rd}}\dot{\psi}\left|\dot{\psi}\right| - \frac{\pi}{2}M_{z,\text{R,P}}\left(\ddot{\phi}\cos\psi\right) - I_{xz,\text{am,P}}\ddot{\psi} \end{bmatrix}\vec{e}_z. \tag{27}$$

Then the resultant moments including the inertia moments components along spanwise pitch axis and chordwise axis of wing planform fixed frame can be written as:

$$\begin{cases} {}^{rw}\boldsymbol{M}^{\text{pitch}}_{\text{total},x} = {}^{rw}\boldsymbol{M}_{\text{aero},x} + {}^{rw}\boldsymbol{M}_{\text{inert},x} \\ {}^{rw}\boldsymbol{M}_{\text{total},z} = {}^{rw}\boldsymbol{M}_{\text{aero},z} + {}^{rw}\boldsymbol{M}_{\text{inert},z} \end{cases}, \tag{28}$$

which constitute the vector of ${}^{rw}\boldsymbol{M}_{\text{total}} = \left[{}^{rw}\boldsymbol{M}^{\text{pitch}}_{\text{total},x} \quad {}^{rw}\boldsymbol{M}_{\text{inert},y} \quad {}^{rw}\boldsymbol{M}_{\text{total},z}\right]^T$. Further, the rotational matrix of ${}^{rr}_{rw}R$ is used to transform the total moment vectors back into the right wing root frame of reference ($x_{rr}y_{rr}z_{rr}$):

$$ {}^{rr}\boldsymbol{M} = {}^{rr}_{rw}\boldsymbol{R}\cdot{}^{rw}\boldsymbol{M}_{\text{total}} = \begin{bmatrix} \cos\phi\cdot{}^{rw}\boldsymbol{M}^{\text{pitch}}_{\text{total},x} - \sin\phi\cdot\sin\psi\cdot{}^{rw}\boldsymbol{M}_{\text{total},z} \\ \sin\phi\cdot{}^{rw}\boldsymbol{M}^{\text{pitch}}_{\text{total},x} + \cos\phi\cdot\sin\psi\cdot{}^{rw}\boldsymbol{M}_{\text{total},z} \\ \cos\psi\cdot{}^{rw}\boldsymbol{M}_{\text{total},z} \end{bmatrix}, \tag{29}$$

where, the z-axis component (namely, ${}^{rr}\boldsymbol{M}_z$), which is about the z_1-axis of the right wing root frame of reference ($x_{rr}y_{rr}z_{rr}$) (namely, the flapping axis of wing shoulder), can be denoted as ${}^{rr}\boldsymbol{M}^{\text{stroke}}_z$ with following analytic expression

$$ {}^{rr}\boldsymbol{M}^{\text{stroke}}_z = \cos(\psi)\cdot{}^{rw}\boldsymbol{M}_{\text{total},z} = \cos(\psi)\cdot\left(\boldsymbol{M}_{\text{trans},z} + \boldsymbol{M}_{\text{rot},z} + \boldsymbol{M}_{\text{rd},z} + \boldsymbol{M}_{\text{add},z} + {}^{rw}\boldsymbol{M}_{\text{inert},z}\right). \tag{30}$$

Reference

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