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2014 Mathematical Contest in Modeling (MCM) Summary Sheet

A Comprehensive Model to Measure the importance of nodes

Abstract

This task requires us to find the most influential researcher or paper in given academic circle. Firstly, we utilized some data extraction skills to gain the cooperation relationship between these researchers. We view researchers as nodes, and view their relationship as edge, in this way, we convert a net of cooperation to an undirected graph. For citation network, we use the similar way to build a directed graph. Next, we are supposed to find those relative important nodes in the complex network.

We read a quantity of mainstream algorithms which are used to measure the significance of node, and applied some of the most famous algorithm (such as PageRank, HITS, Centrality, etc.) to mentioned networks. However, there is some weaknesses.

Owing to the weaknesses of previous algorithm, we propose our model. In fact, we didn't abandon previous algorithms. Firstly we still utilize kinds of previous methods to calculate the importance of all nodes. Secondly, we find that some orders of different methods are similar, but some others are completely different. This phenomenon shows that different traditional methods really place emphasis on different aspect. What we are supposed to do is to retain those algorithms which differ greatly from each other rather than similar. Thus, we took advantage of PCA to select which algorithms we really need. Finally, applying these selected algorithms to networks, we can gain a comprehensive measure about nodes.

Next, we tested the sensitivity and effectiveness of our model. Firstly, we collected data about international trade between 24 countries in 1988.Similarly, we build the network about 24 countries' trade. Next, we applied our advanced model to rank the importance of countries. Then, compared our rank with GDP rank of 24 countries in 1988, we found consistence between our rank and GDP rank in some extent. So, our model is effective .Finally, we revise some data about international trade between 24 countries in 1988, and did the experiment again. We found our model is fairly sensitive to the changes of data.

Keyword: Pagerank, PCA, HITS, Centrality

Contents

Introduction

Clarification of the problem

Paul Erdős utilized his enormous co-authorships to construct a co-author network among mathematicians. Then, there comes several network-based evaluation tools such as Science Citation Index and H-factor that use co-author and citation data to determine impact factor researchers, publications and journals. It is getting significant to analyze network as it has great helpfulness for issue decisions and social situation reflections. In the ICM problem, we need to construct network models or algorithms scientifically to research the author influence based on co-author network and the paper influence based on citation network, and develop and complete our models or algorithms to implement it in other social network data.

Model design

For author influence measurement problem, we construct the co-author network with data from Erdős1. In order to achieve model easier, we only consider whether is there a co-authorship between two authors, not include the times they co-authored. In co-author network, we regard authors as nodes, co-authorship as edges. For co-authorship is an interrelationship, the co-author network is undirected. Some of the given data are useless for our models, therefore, first of all, we should take some skilled data extraction efforts to obtain the correct set of nodes.

Having built our co-author network, we analyze the network structure and node importance. We plan to obtain the quantified author influence by transforming it into corresponding node importance measurement. Naturally, we rise an awareness that the degree of a node indicates the sum of co-authors of an author, and generally, the author with more number of co-authors is more influential. However, it is too one-sided to measure author influence only by node degree, and there are still some other aspect related to influence measurement described as follow:

- Some authors that with not much co-authors are more influential if they have published important works.
- Generally, the author that have been co-authored with significant author is influential. The more number of significant authors they co-authored, the more powerful influence they have.
- If an author has co-authored with active authors (the one has much number of co-authors), his influence power could be strengthened.

We need to consider all the above factors and find out the index that can describe corresponding factor, take the comprehensive index as our measurement result.

For citation influence measurement problem, the main process of analyzing the problem is similar to the co-author one. With the given data, we abstract each paper as a node, and regard the paper citation relationship as edge. We should note that, different from the undirected graph of co-author network, the graph constructed on the basis of citation network is directed for the citation relationship is unidirectional. We define the direction of the edge as the direction from cited paper to citing paper. For the additional direction condition, on the basis of previous thinking, we also need to consider in-degree and out-degree of the node.

Through the establishment of the previous model, we continues to revise and improve our model to make it more universal, and implement it on a completely different set of network influence data to test the validity of the model.

Background of Network Science

Network science is an interdisciplinary academic field which studies complex networks such as telecommunication networks, computer networks, biological networks, cognitive and semantic networks, and social networks. The field draws on theories and methods including graph theory from mathematics, statistical mechanics from physics, data mining and information visualizationfrom computer science, inferential modeling from statistics, and social structure from sociology.

Model Preparation

Although network science is a “new” science, there’re still so many researchers who have been studied the network deeply. Let’s review some mainstream researches about this discipline.

●Eigenvector Centrality

Eigenvector centrality is a measure of the influence of a node in a network[1]. It assigns relative scores to all nodes in the network based on the concept that connections to high-scoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes.

Using the adjacency matrix to find eigenvector centrality. For a given graph $G := (V, E)$ with $|V|$ number of vertices let $A = (a_{v,t})$ be the adjacency matrix, i.e. $a_{v,t} = 1$ if vertex v is linked to vertex t, and $a_{v,t} = 0$ otherwise. The centrality score of vertex v can be defined as:

$$x_v = \frac{1}{\lambda} \sum_{t \in M(v)} x_t = \frac{1}{\lambda} \sum_{t \in G} x_t$$

where $M(v)$ is a set of the neighbors of v and λ is a constant.

●Weighted Degree

For one vertice, weighted degree is defined as the whole sum of edges’ weight (These edges must be linked to this vertice.) This index is used to measure the significance of vertices in the network.

●Graph Eccentricity

The eccentricity $\varepsilon(v)$ of a graph vertex v in a connected graph G is the maximum graph distance between v and any other vertex u of G. For adisconnected graph, all vertices are defined to have infinite eccentricity[2][3][4]

●Closeness Centrality

In connected graphs there is a natural distance metric between all pairs of nodes, defined by the length of their shortest paths. The farness of a node s is defined as the sum of its distances to all other nodes, and its closeness is defined as the inverse of the farness[5][6]. Thus, the more central a node is the lower its total distance to all other nodes. Closeness can be regarded as a measure of how long it will take to spread information from s to all other nodes sequentially[7].

An extension to networks with disconnected components has been proposed by Opsahl (2010)[8], and later studied by Boldi and Vigna (2013)[9]in general directed graphs:

$$C_H(x) = \sum_{y \neq x} \frac{1}{d(y,x)}$$

The formula above, with the convention $\frac{1}{\infty} = 0$, defines harmonic centrality.

●Betweenness Centrality

Betweenness is a centrality measure of a vertex within a graph (there is also edge betweenness, which is not discussed here). Betweenness centrality quantifies the number of times a node acts as a bridge along the shortest path between two other nodes[10].

The betweenness centrality of a node v is given by the expression:

$$g(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

where σ is the total number of shortest paths from node s to node t and $\sigma_{st}(v)$ is the number of those paths that pass through v.

●Modularity

Modularity is one measure of the structure of networks or graphs. It was designed to measure the strength of division of a network into modules[11] (also called groups, clusters or communities). Modularity is the fraction of the edges that fall within the given groups minus the expected such fraction if edges were distributed at random.

●Clustering Coefficient

In graph theory, a clustering coefficient is a measure of the degree to which nodes in a graph tend to cluster together.

The clustering coefficient for the whole network is given by Watts and Strogatz[12] as the average of the local clustering coefficients of all the vertices n:[13]

$$\bar{C} = \frac{1}{n}\sum_{i=1}^{n} C_i$$

●Weighted Clustering Coefficient

Barrat er al. were the first to propose a weighted version of the clustering coefficient[14]. Their definition reads as follows:

$$\widetilde{C_{i,B}} = \frac{1}{s_i(k_i - 1)} \sum_{j,k} \frac{w_{ij} + w_{ik}}{2} a_{ij} a_{jk} a_{ik},$$

where a_{ij} = 1 if there is an edge between i and j, and 0 otherwise.

●Local Clustering Coefficient

The local clustering coefficient of a vertex (node) in a graph quantifies how close its neighbors are to being a clique (complete graph).Duncan J. Watts and Steven Strogatz introduced the measure in 1998 to determine whether a graph is a small-world network[15].

Assumptions

- If two authors collaborate on a paper, the influence on both are equal.
- If a paper cites another one, it doesn't influence the cited one.

Our Model

Brief Introduction of Modeling

To solve these tasks, which are proposed by ICM, we have studied some mainstream measures or indexes that are used to evaluate the importance of nodes in a network. For instance, Closeness Centrality, Betweenness Centrality, and PageRank, etc.

Firstly, we build the co-author network and citation network separately according to data we have obtained. The specific process of build network is as follow.

To build the erdos1 network, we are supposed to know those researchers who cooperated with Erdös directly and how often they have cooperated before. We took advantage of some data extraction skills to extract useful data from raw data which is supplied by ICM, and acquired times of cooperation with Erdös from Google Scholar. Then we build the unweighted adjacency list according to cooperation relationship. Next, giving weight to edge, we think that if a researcher often cooperate with Erdös, he or she must have more influence in academic. So, if in adjacency list a_{ij} =1, indicates these two researchers have cooperated once. We set their edge as the product of their respective times of cooperation with Erdös. In this way, we can gained a weighted adjacency list.

Like that, we build the citation network in a similar way. But, it is worth noticing that this network is a directed graph.

Immediately following, we applied a series of traditional methods to assess the importance of nodes in different network.

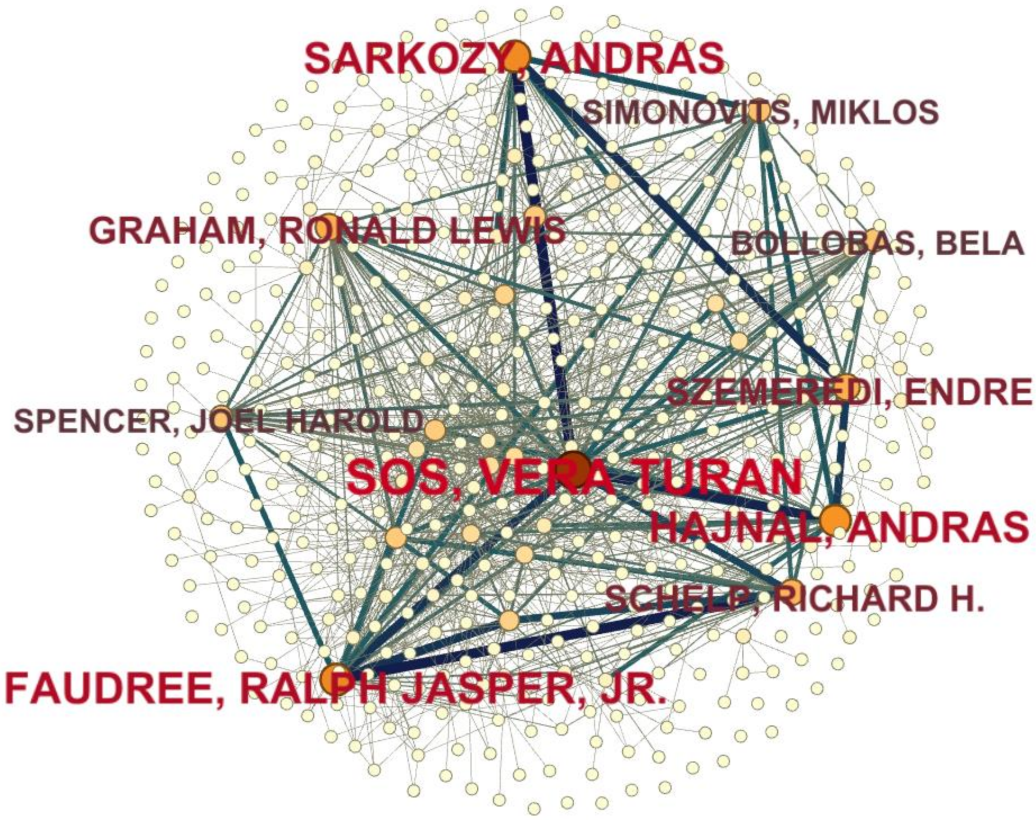
Compared the ranks of different methods, we summarized the strengths and weaknesses of different traditional methods.

To make up the weakness of traditional methods, we proposed our advanced model to evaluate the importance of nodes. Finally, we can determined which one is the most influential node in the network.

Presentation

Using Gephi (a software that professionally processes complex network), we can easily apply these traditional methods to co-author network and citation work. Finally, we acquired a set of feature data about network, besides, we also visualized data.

Co-author network



Figure[1]

Just as figure[1], this is an extremely complex network. Firstly, we analyze some properties of this network itself.

Table[1]

Index	value
average degree	6.411
Network Diameter	10

connected components	44
average cluster coefficient	0.343
average path length	3.823

According to table[1], we came to the conclusion that these researchers cooperate closely, meanwhile, this network has high information exchange rate, besides, there are so many academic circles in this co-author network.

Next, we chose 10 nodes from all 511 nodes as our study objects. Besides, for concision, we only adopted five traditional methods to assess the significance of 10 nodes. Utilizing Gehphi, we acquired the orders in accordance with five different methods.

Table[2]

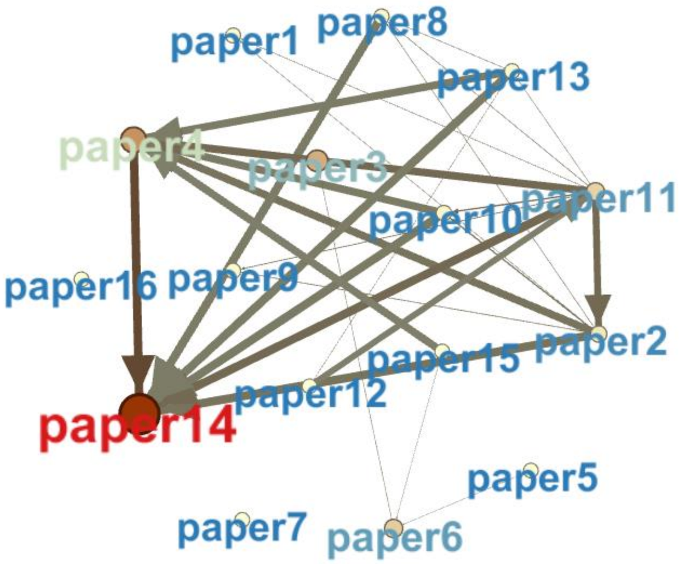
Id	Authority	Number of triangles	Closeness Centrality	Degre e	Betweenness Centrality
ALON, NOGA M.	1	1	10	1	7
GRAHAM, RONALD	2	3	11	2	8
RODL, VOJTECH	5	2	15	5	19
BOLLOBAS, BELA	4	4	12	4	10
HARARY, FRANK*	3	16	19	3	1
FUREDI, ZOLTAN	6	5	9	6	6
TUZA, ZSOLT	7	6	14	7	12
SOS, VERA TURAN	8	19	14	8	3
SPENCER, JOEL	9	8	17	9	17
PACH, JANOS	11	13	21	10	10

Notes that the numbers in the charts indicate the importance rank of 10 researchers in accordance with 5 different indexes, there are 511 nodes all together in the network.

According to the above data, we can see that for the same 10 nodes, different methods separately gave 5 different order. For instance, the order of Authority is similar to the order of Number of triangle and Degree. However, the order of Authority is completely different from the order of Closeness Centrality. This phenomenon shows that different traditional methods place emphasis on different aspect.

Citation network

Our data is obtained directly from PDF which ICM supplied. We select all 16 papers as our study object, label them in increasing order according to the primary order. In addition, to build the citation network, we acquired the citation relationship between these 16 papers from Google Scholar. Finally, we plotted the citation network by using Gephi.



Figure[2]

Similarly, we chose 10 nodes as study objects. For concision, we only adapted four traditional methods to assess the significance of 10 nodes. Utilizing Gehphi, we acquired the order of importance in accordance with four different methods.

Table[3]

Id	Authority	Closeness Centrality	Degree	Betweenness Centrality
paper14	1	11	4	6
paper11	10	6	1	1
paper4	2	1	3	4
paper10	9	5	7	8
paper2	11	7	2	5
paper8	3	1	6	7
paper13	5	4	5	3
paper12	14	10	10	10
paper9	8	14	13	13
paper1	6	12	11	11

Notes that the numbers in the charts indicate the importance rank of 10 papers in accordance with 4 different indexes, there are 16 nodes all together in the network.

The table[3] also shows that some ranks are similar, and some other ranks are totally different. So, this resemblant phenomenon shows that different traditional methods really place emphasis on different aspect.

As is above mentioned, different traditional methods have different emphasis, but every method has own weaknesses. The table [4] below is the strengths and weaknesses about some mainstream study about the significance of nodes.

Table[4]

Index	Strength	Weakness	time complexities
Degree	simple,intuitional	Only reflect partial feature of nodes	O(N)
Eigenvector	Consider the importance of node's neighbour	Simply Linearly overlay the topological features	O(N2)
Closeness	Make global influence to part of nodes via net	Unfit to random network and complex network	O(N3)
Betweenness	Consider the load information capacity of node	Unfit to complex network	O(N3)
Pagerank	Consider the global topological features of network	Neglect some actual factors	O(MI)
HITS	Low time complexities	The improper link of webpage can lead to mistakes of order	O(NI)

Although many researchers have proposed a large number of methods to measure the importance of nodes in the complex network, these methods yet can't assess the significance of nodes roundly. Our goal is to find a comprehensive evaluate model. Luckily, we find an advanced model based on principal components analysis.

Our Refined Model

In this Erdős Problem, we firstly apply this to a network model. And we take the relationship between to objects as a path (with or without a weight depending on the relationship between two nodes in the network).

As previously mentioned, there are an ocean of indices to compare the weight of a node in a network, and each of them stresses an aspect of the node. And in different conditions, these indices can some frustrate us with a weird rank. Because of this, we are here intending to come up with a relatively more method to deal with this situation, try our best to get rid of those short slabs.

However, considering all those indices, if we simply made a weighted sum out of them, we will inevitably driving onto another road which can probably give too much score in one aspect.

Out of all the viewpoint above, we decide to use the Principal Component Analysis, namely PCA, method.

Principal Component Analysis is a method which applies to condition that needs relatively fewer variables to explain most of the variations among the data.

This method assists us to transform the variables those has a high correlation to some independent or uncorrelated variables. Usually we prefer fewer variables but convey a good deal of information the original variables hold, and these are what we call Principal Component. Furthermore, we are then able to apply these Principal Component to assessing our sample.

This method is illustrated as follows:

●$x_1,\ldots, x_p$ is node

●p is the number of nodes.

●$c_1,\ldots,c_p$ is weight of each node.

●$s_1,\ldots,s_n$ is synthesis score of each n nodes

The PCA method aims to find a weight vector $c = (c_1,\ldots,c_p)^T$ to maximize

$$\mathrm{Var}(c_1X_1 + c_2X_2 + \cdots + c_pX_p) \qquad *$$

,in which $x = (x_1,..,x_p)^T$, and satisfies the condition

$$c_1^2 + c_2^2 + \cdots + c_p^2 = 1$$

And c is a PC, but this is not enough to hold most of the information the sample provides, so we need to find more PCs that not only maximize function *, but also convey a set of different information from each other(where in mathematic they are orthogonal) and with the help of some of them(that can explain 70%~80% of sample's information) we are able to achieve our goal. So we have this:

$$\begin{cases} Z_1 = c_{11}X_1 + c_{12}X_2 + \cdots + c_{1p}X_p \\ Z_2 = c_{21}X_1 + c_{22}X_2 + \cdots + c_{2p}X_p \\ \qquad\qquad\vdots \\ Z_p = c_{p1}X_1 + c_{p2}X_2 + \cdots + c_{pp}X_p \end{cases}$$

●Z_i is the i-th principal component.

We make use of Matlab to help us calculate the PCs, and the steps are:

1) Normalize our selected set of indices using

$$\widetilde{a_{ij}} = \frac{a_{ij}-\mu_j}{s_j}, i = 1,2,\cdots,n; j = 1,2,\cdots,m,$$

where $\mu_j = \frac{1}{n}\sum_{i=1}^{n} a_{ij}$, $s_j = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(a_{ij} - \mu_j)^2}, j = 1,2,\cdots,m.$

And

$$\widetilde{x_j} = \frac{x_j-\mu_j}{s_j}, j = 1,2,\cdots,m$$

is standardized indicator variables.

●m is the number of principal component analysis indicator variables.

●x_i ,i=1,⋯,m is the principal component analysis indicator variables.

●n is the number of evaluation object.

●a_{ij} is the j-th value of the i-th index in the evaluation object.

●μ_j is the j-th indicator sample mean.

●s_j is the j-th indicator sample standard deviation

2) Calculate the correlation matrix $\boldsymbol{R} = (r_{ij})_{m\times m}$

$$r_{ij} = \frac{\sum_{k=1}^{n} \widetilde{a_{ki}} \cdot \widetilde{a_{kj}}}{n-1}, i, j = 1,2,\cdots,m,$$

where $r_{ii} = 1$, $r_{ij} = r_{ji}$,

●r_{ij} is the correlation coefficient of the i and j-th indicators index

3) Calculate the eigenvalue and the eigenvector

4) Select our PCs, according to the accumulation contribution of sample's information:

$$b_j = \frac{\lambda_j}{\sum_{k=1}^{m} \lambda_k}, j = 1,2,\cdots,m,$$

and

$$\alpha_p = \frac{\sum_{k=1}^{p} \lambda_k}{\sum_{k=1}^{m} \lambda_k}$$

●b_j is information contribution rate of y_j

●α_p is cumulative contribution rate y_j

●y_j is the j-th principal component.

●λ_j is the j-th eigenvalues.

5) Calculate our synthesis score

$$Z = \sum_{j=1}^{p} b_j y_j$$

6) Analysis which indices are relatively important according to Pcs' eigenvector.

Solve problems with our refined method

For problem 2, we list the most significant person and their score in Erdös' Collaboration Network as in table[5]

Table[5]

Name	score
ALON, NOGA M.	6.506363
GRAHAM, RONALD LEWIS	5.267893
RODL, VOJTECH	5.039036
BOLLOBAS, BELA	5.038913
HARARY, FRANK*	4.777058
FUREDI, ZOLTAN	4.693739
TUZA, ZSOLT	4.418881
SOS, VERA TURAN	4.134165

SPENCER, JOEL HAROLD	3.670349
PACH, JANOS	3.64128

They are not only important in their local network but also make a big difference in the whole network. Let us take the third influential person RODL, VOJTECH as an example for our analysis:

And here are some of his other scores in those ‘unilateral measurement’ in table[6]:

Table[6]

Name	Authority	Number of triangles	Closeness Centrality	Degree	Betweenness Centrality
RODL, VOJTECH	5	2	15	5	19

Usually it’s hard to tell a person good or bad just from a specific aspect, like in this case, our RODL, VOJTECH seemed not to do very well in passing information in this network which was unveiled by the indices--Betweenness Centrality, in which he ranked 19th. However, RODL, VOJTECH played a quite important role in his local neighborhood as he had ranked 5th in HITS ranking as well as 2nd in the Number of Triangles which indicated he had a very concentrative neighborhood around him, and he was active being there with them. This always gives us suggestion that this is a man that matters a lot in the network. Above all, we can in some degree come up with the idea that RODL, VOJTECH did far more good than he ranked in Betweenness Centrality and this is supported well by our refined advanced method, in which he ranked 3rd.

In terms of problem 3, we will illustrate how our refined method works, and analyze the result to tell in order to getting a better reputation what should be done is the most effective choice, which will be proved in the consecutive part.

Step1) choose some basic measurement of the weighted graph:

Table[7]

indices	name
x1	Authority
x2	Betweenness Centrality
x3	Closeness Centrality
x4	Clustering Coefficient
x5	Degree
x6	Eccentricity
x7	Eigenvector Centrality
x8	In-Degree
x9	Local Clustering Coefficient
x10	Out-Degree
x11	PageRank
x12	Weighted Clustering Coefficient
x13	Weighted Degree
x14	Weighted In-Degree
x15	Weighted Out-Degree

Step2) we use MALAB to normalize each paper's different indices, and then we calculate the correlation matrix in this part to obtain the eigenvalue, contribution rate and the accumulated contribution rate.

Table[8]

lamda	contribution rate	accumulated contribution rate
6.046018	40.30679	40.30679
4.174173	27.82782	68.13461
2.671902	17.81268	85.94729
1.049496	6.996639	92.94393
0.5237	3.491332	96.43526
0.3048	2.032	98.46726
0.114069	0.760461	99.22772
0.055838	0.372253	99.59997
0.037475	0.249835	99.84981
0.020623	0.137484	99.98729
0.001869	0.012459	99.99975
3.75E-05	0.00025	100
2.65E-16	1.77E-15	100
1.37E-16	9.16E-16	100
5.05E-17	3.37E-16	100

Step3) we can tell from this table that the first five Principal Components have already reach an accumulative contribution rate of 96.43526%. And this settles the problem of PC choosing.

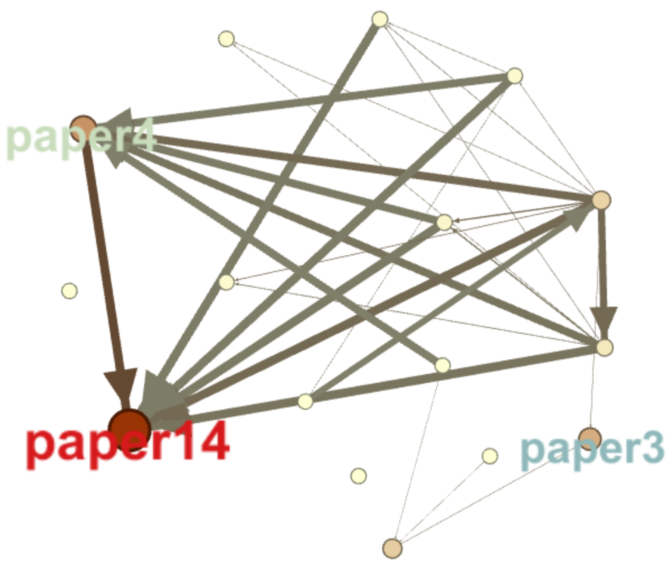
Step4) in this part, we are going to analysis the 5 PCs' eigenvector to give a suggestion on which part of ourselves or the outer worlds we should concentrate on in order to be the most efficient person in enhancing one's influence.

Table[9]

	x1	x2	x3	x4	x5	x6	x7
1	0.371039	0.054153	0.051908	0.219898	0.274617	-0.09138	0.346933
2	-0.08255	0.347689	0.395256	0.008113	0.337937	0.277069	-0.19267
3	-0.03004	-0.20867	0.005084	0.510423	-0.09888	0.034027	-0.12249
4	-0.01512	-0.41325	0.427628	-0.03539	-0.1479	0.72168	0.090197
5	-0.49429	-0.15198	-0.47998	0.075548	-0.00883	0.199615	0.23037

	x8	x9	x10	x11	x12	x13	x14	x15
1	0.372817	0.219898	0.036331	0.326199	0.224026	0.355947	0.353032	0.087342
2	-0.12203	0.008113	0.457606	-0.1543	0.051132	0.142062	-0.10046	0.454814
3	-0.05729	0.510423	-0.06778	-0.27394	0.502913	-0.16448	-0.20971	0.040688
4	0.064288	-0.03539	-0.20801	0.046858	-0.05556	0.08806	0.153578	-0.09364
5	-0.39368	0.075548	0.270181	0.202943	0.074303	0.24807	0.234578	0.080812

From the first line (the most effective PC) we can tell indices x_1 and x_8 have a direct and very strong effect in one's final score in the network. Take the 3rd ranked paper—paper 4(…) for example: it ranked 4th in the x_1 indices (HITS) and 5th in the x_8 indices (In Degree),and both of this two index contribute to it 3rd ranking. And apart from the more synthesis ranking this method offers to us, it meanwhile gives us a clue to increase the paper's reputation on putting effort into 'citation reputable papers'(of course in an academic way!), and this will help this paper to gain a its fame quite fast.



Figure[3]

To finish this part, we would also like to compare our refined method to some 'unilateral indices', such as PageRank. Picture n shows a PageRank score weighted graph (in which the higher the score the bigger the dot), From which we can tell the first three most important papers are paper 14, paper 4 and paper 3. However, from the graph, we can also make a somehow intuitive judgment that paper3 only with a small references (actually 2) doesn't deserve this high ranking. And our refined method indeed provides us a relative more 'correct answer' with paper3 ranking 15th in the whole graph.

Evaluation of our Method

Verification and sensitivity analysis

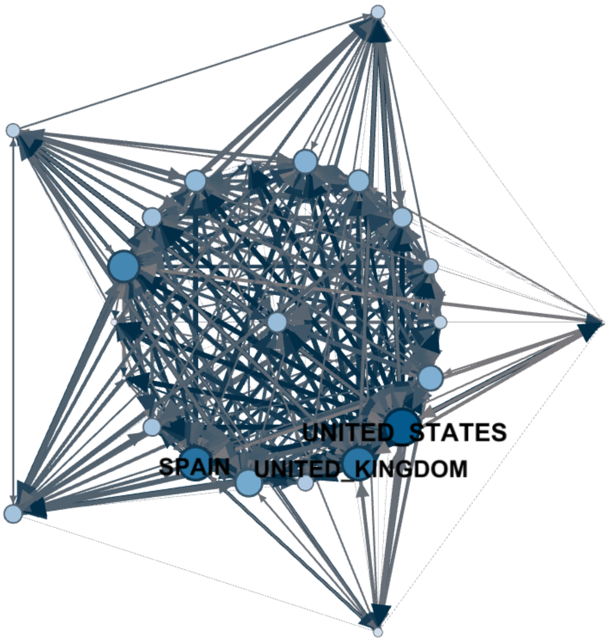
Here we give our approach to problem 4 and give our sensitivity analysis of our model.

We take 5 different kinds of trade cooperation among 24 different counties as our index to compare the importance of different countries in this network. And furthermore, we integrate these five cooperation as a whole (with or without a trade cooperation and the intensity of the cooperation) to apply our refined method here. And finally we compare our rank with the GDP rank between these 24 counties then to make an assessment, as well as offering a method to enhance a certain country's influence. Detailed analysis is as follows:

Step1) the country we choose :

YUGOSLAVIA，UNITED_STATES，UNITED_KINGDOM，THAILAND，SYRIA，SWITZERLAND，SPAIN，PAKISTAN，NEW_ZEALAND，MADAGASCAR，LIBERIA，JAPAN，ISRAEL，INDONESIA，HONDURAS，FINLAND，ETHIOPIA，EGYPT，ECUADOR，CZECHOSLOVAKIA，CHINA，BRAZIL，ARGENTINA，ALGERIA.

There are 24 countries here. The data were selected by Wasserman and Faust (1994)[16] from a list of 63 countries given by Smith and White (1988)[17]. After our analysis here is a trade-intensity graph between these countries, in which we labeled the most influential 3 countries: United States, United Kingdom, and Spain.



Figure[4]

Table[10]

Name	score
UNITED_STATES	1
SPAIN	2
JAPAN	3
UNITED_KINGDOM	4
SWITZERLAND	5

Step2) Apply our refined method, and the top 5 rank as in tablenx.

And here we offer the GDP rank in 5 countries according to these 24 countries:

Table[11]

Name	GDP

UNITED_STATES	1
JAPAN	2
UNITED_KINGDOM	3
SPAIN	6
SWITZERLAND	7

It is quite apparent that our score has quite an excellent result which suits the GDP rank very well.

Then we march our sensitivity analysis. Thailand ranks 15th among these 24 countries, and in our analysis just the same as previous did, we come up with that, among these various indices for Thailand, it is a pretty efficient way to enhance its trade cooperation with more countries to enhance Thailand's influence. And only with a slight increase, we saw a 2 rank rise to 13th position.

Merits

The light spot of our model is that we take various ranking indices into account, and analysis their correlation to successfully achieve a goal of overall assessment of a node in a certain network. This strength not only makes our model quite robust to be able to apply to various network models intending to measure the importance of a certain node, but also offers us a way to give advice on which part needs to be strengthened to obtain an effective improvement in the influence in the network.

Defects

However, what we should still carry on is that this model is relatively complex which is a little troublesome to access to the final refined overall rank.

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