Painting With Continuum Arms

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Abstract

In this paper, we use a continuum robot to paint some simple geometric objects. We will paint a cube for this paper, but the method we use can be generalized to painting objects with all kinds of shapes. To achieve our goal, we will divide this task into three steps. The first step is initialization, which will enable us to find a set of points that will cover the object to be painted. Second, we need to find a minimum number of painting points that will cover the whole cube. Third, we will find an optimal painting path which minimizes the distance of the robot's movement. After the initialization, we will transform our problem to a set cover problem and the third step can be addressed by a solving a traveling salesman problem. Both problems are NP-hard. It is hard to find the best solution if n is large. So we use approximation algorithms to find a satisfactory solution for our painting problem.

1 Introduction

In this paper, we will discuss how to find a good route for the continuum robot to paint a cube. The continuum robot we are dealing with is a three-part painting robot. It is called continuum robot because it is like an arm and more flexible and continuous than a regular painting machine. We assume that the sprayer will generate a sphere-shape space. If certain surface area on the cube lies within that sphere, we consider that area painted. Therefore, each painting position will cover certain part of the cube surface. We need to find a minimum set of points in a three-dimension space which will cover the whole surface of the cube. We also need to find the minimum traveling distance of the sprayer to traverse all the positions. You can refer to Figure 2, 3 and 4 to get an idea of this problem.

Several major topics are involved in our problem. One topic is covering subsets with patches, which is to divide the surface of a object into tiny units and analyze how to use patches to cover these tiny units[4]. In this way, we can often transform a irregular shapes to regular shapes, thus transforming a continuous math problem to a discrete one.

Another major topic involved is the Polynomial Approximation Scheme[2][3]. We will apply approximation algorithm for both the set cover problem and traveling salesman problem, both of which are np-hard problem. To get the optimal solution within acceptable time limit is just impossible when the problem set is large.

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Figure 1

1.1 Motivation and related work

This problem is similar to Minimum Covering with Travel Cost [1]. Finding the minimum points to cover the cube is closely related to Set Cover Problem [2]. Find a minimum spraying route is a Traveling Salesman Problem[3]. What makes this problem different from the classic TSP is that this problem is a TSP in three dimension. The solving of this problem is similar, though.

1.2 Results and techniques

By dividing the cube surface and the space, we transformed the problem to set cover problem and traveling salesman problem. Then we use approximation algorithms to find a satisfactory solution to the problems.

2 Preliminaries

The solution to this problem will be divided into three steps. The initialization, the Set Cover problem and the Traveling Salesman Problem. In the initialization step, we will divide the surface of the cube into grid with equal square cells. Each cell on the cube surface is represented by the coordinate of its center, which will be used to decide if it is covered by a painting position. We assume that the sprayer will generate a sphere. Each painting position will cover the cells whose center coordinates are within a distance of R from the painting position.

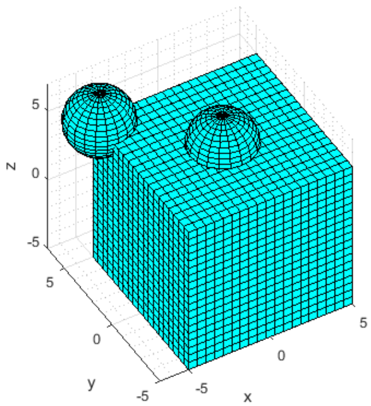

Figure 2

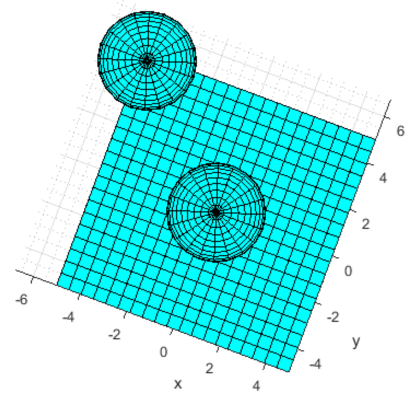

Figure 3

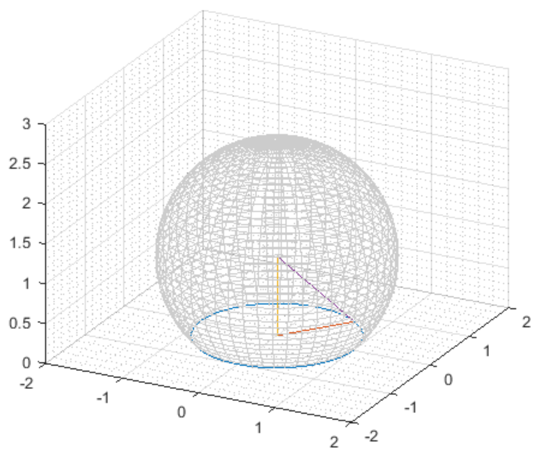

Figure 4

3 Section 1

The following is a detailed explanation of the three steps:

3.1 **Initialization**

The surface of the cube into grid. We wrap the cube with a bigger cube which is divided up into tiny cubes. We let the distance of the bigger cube to the object be R so that we don't need to consider the space outside the bigger cube. The volume covered by the sprayer is simplified as a sphere with a diameter R. Then we iterate through each center point of all the cubes and calculate its distance to the grid cell center and note down all the points that have a distance less than R(R is given) to the cell center point. We call this set of points covered by B. Figure 5 shows a intersection circle of the spraying sphere with the surface of the cube. All the squares lying within the circle are considered to be covered by the spraying sphere. To make things easier, we place the cube in such a way that the surface planes of the cube are parallel to the x, y and z plane.

Now I will demonstrate how to do the calculation for the tiny cubes over the top side of the cube. Given the diameter of the sphere R, the coordinate of the center of the sphere $(x\,y\,z)$ and the coordinate of the center of the intersection circle $(x\,y\,z')$,(Supposing that the intersection circle is parallel to the x-y plane) we can calculate the diameter of the intersection circle R'=$\sqrt{R^2-(z-z')^2}$. Let the side length of the cells be l, then point (a' b' c') is considered to be covered by $(x\,y\,z)$ if it satisfies the condition: $\sqrt{(x-x')^2+(y-y')^2} <= R' - \frac{\sqrt{2}l}{2}$.

It is easy to prove that for the tiny cubes that are not directly over one surface of the cube including those at the side formed by two surface planes and at the corner formed by three surface planes, we can apply the same rule to calculate the cover set of each tiny cube. See Figure 6. The only difference is that for tiny cubes at the side, the spraying sphere can be projected on to two surfaces so we need to calculate points from two planes. And for tiny cubes at the corner, the spraying sphere can be projected onto three surfaces so we need to calculates points from three planes.

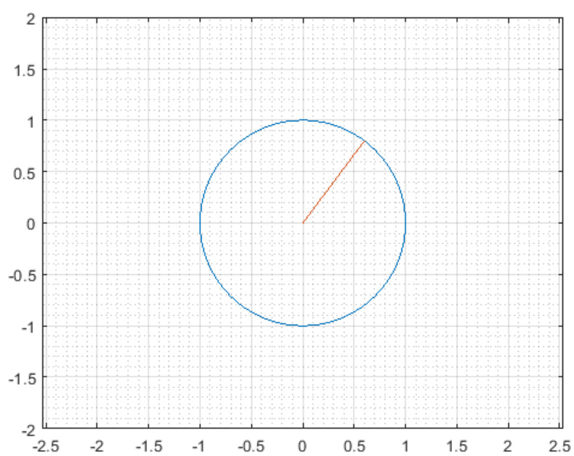

Figure 5

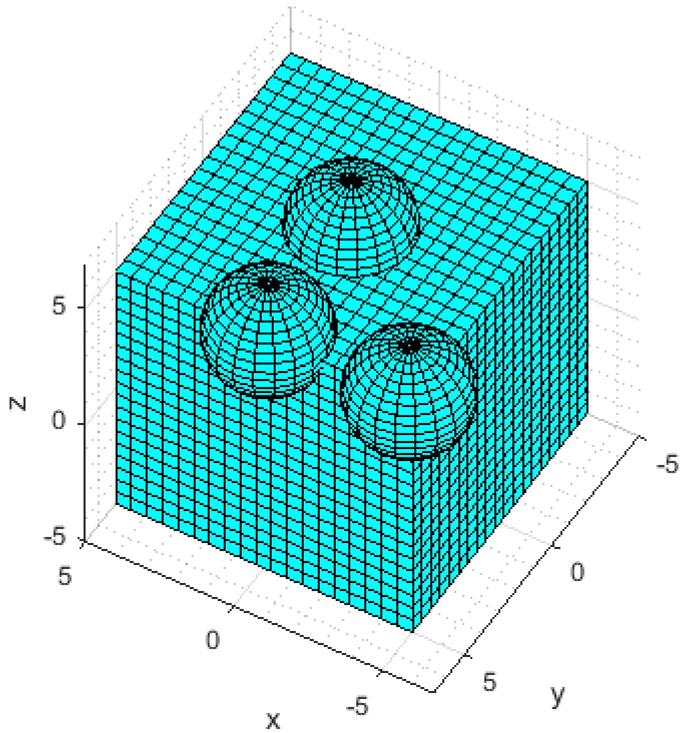

Figure 6

3.2 **Solve the Set Cover Problem**

After the initialization, we get a finite set A, which is the set of all the center points of the grid cells on the surface of the cube. We also have a finite set B, each of whose elements denotes a subset of A. Our goal is to find a minimum size subset $C \subseteq B$, such that every element of A belongs at least one subset of B:$A = \bigcup_{S \in B} S$

As this is a NP-hard problem, brute-force takes too much time. We will apply a greedy approximation algorithm to find a good but not necessarily the optimal solution.

The idea is to pick the element of B that covers the greatest number of elements not yet covered:

1. $T \Leftarrow A$
2. $C \Leftarrow \emptyset$
3. $While\ |T| \neq 0$
   do select an $S \in B$ that maximizes $|S \cap T|$
   $T \Leftarrow T - S$
   $C \Leftarrow C \cap S$
4. return C

3.3 **Solve the Traveling Salesman Problem**

After step 2. We have a set of points that represents all the points the sprayer needs to traverse. This is a traveling salesman problem in three dimension. If the number of points are small, we can use some brute-force with a time complexity of $O(n!)$ or dynamic programming algorithm with a time complexity of $O(n^2 * 2^n)$ to find the best solution. However, if the number of the points are large. Both of the methods are time-consuming to the degree of unbearable. So we have to apply some approximation algorithms to handle the task. The method we use is also approximation method with polynomial time complexity. We first construct a minimum spanning tree with prim's algorithm since it is a dense graph. Then we do a deep first search to traverse all the vertexes.

1. Find a Minimum Spanning Tree of our Graph and let T be the Minimum Spanning Tree
2. Let W be the walks obtained from performming a preorder traverse of T
3. Let C be the Hamiltonian Circle obtained from W by taking shortcuts
4. Return C

4 Simulation Results

We can simulate the whole process with java code. We need four input parameters for our calculation: The length of the cube $CubeLength$, the length of the cell on surface of the $CubeCellLength$, the range/radis of the sprayer $SphereRadius$ and the length of the cells dividing up the ourter box surrounding the cube $BoxCellLength$. For $CubeLength = 10$, $CellLength = 1$, $SphereRadius = 2$ and $BoxLength = 1$, we will get 99 painting points that will do the job. The overall moving distance is 3240.68. Figure 7 shows the routes for this setting. For $CubeLength = 20$, $CellLength = 2$, $SphereRadius = 3$ and $BoxCellLength = 2$, we will get 198 painting points. The overall moving distance is 3240.68. Figure 8 shows the routes for this setting. The calculation is fast and the time it take is almost ignorable.

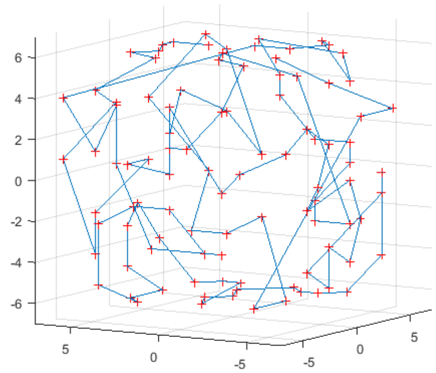

Figure 7

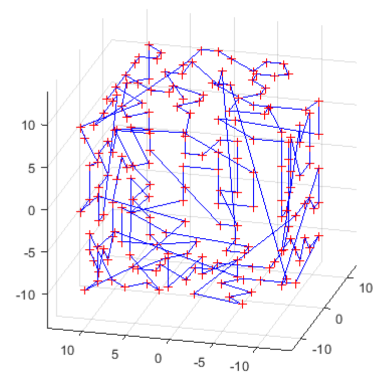

Figure 8

5 Concluding Remarks

The above three-step approach is just one way of dealing with this painting problem. We modeled the problem to two NP-hard problem and solved them with approximation method. There may be other approach to get it done. Also, this method can be improved in some way. For example, in the set cover step, we will end up choosing painting positions very close to the surface of the cube, which means a lot of painting positions obtained from initialization step is not needed. We can reduce a lot of unnecessary calculations by limiting the size of the bigger space in the first step.

References

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