$$y_i = x_i^T \beta + \xi_2 \tau^{-\frac{1}{2}} \sqrt{\tilde{v}_i}\, z_i$$

$$\underset{1\times1}{y_i} \quad \underset{p\times1}{x_i} \quad \underset{1\times p}{\beta}$$

$$\tilde{v} \mid \tau \sim \prod_{i=1}^{n} \tau \exp(-\tau \tilde{v}_i) \qquad \tilde{v} = (\tilde{v}_1, \cdots, \tilde{v}_n)$$

$$z \sim \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} z_i^2\right) \qquad z = (z_1, \cdots, z_n)$$

$$\beta_k \mid s_k \sim \frac{1}{\sqrt{2\pi s_k}} \exp\left(-\frac{\beta_k^2}{2 s_k}\right), \quad N(0, s_k), \quad k=1,\cdots,p$$

$$s_k \mid \eta^2 \sim \frac{\eta^2}{2} \exp\left(-\frac{\eta^2}{2} s_k\right)$$

$$\tau \sim \text{gamma}(a, b), \quad \eta^2 \sim \text{gamma}(c, d)$$

$$\tau, \eta^2 \sim \tau^{a-1} \exp(-b\tau) \, (\eta^2)^{c-1} \exp(-d\eta^2)$$

spike and slab:

$$\beta_k \mid s_k \sim (1-\pi_0) N(0, s_k) + \pi_0 \delta_0(\beta_k), \quad k=1,\cdots,p$$

$$f(y \mid -) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi \tau^{-1} \xi_2^2 \tilde{v}_i}} \exp\left\{-\frac{(y_i - x_i^T\beta - \xi_1 \tilde{v}_i)^2}{2\tau^{-1}\xi_2^2 \tilde{v}_i}\right\}, \quad \xi_1 = 0$$

$$\bullet [\beta_k \mid -] \propto \exp\left\{-\frac{1}{2}\sum_{i=1}^{n} \frac{(y_i - x_i^T\beta)^2}{\tau^{-1}\xi_2^2 \tilde{v}_i}\right\} \times$$

$$\left\{(1-\pi_0)\frac{1}{\sqrt{2\pi s_k}} \exp\left(-\frac{\beta_k^2}{2 s_k}\right) I(\beta_k \neq 0) + \pi_0 \delta_0(\beta_k)\right\}$$

$$\propto \exp\left\{-\sum_{i=1}^{n} \frac{1}{2\tau^{-1}\xi_2^2 \tilde{v}_i} \overbrace{\left(y_i - x_{ik}\beta_k - \sum_{j=1, j\neq k}^{p} x_{ij}\beta_j\right)^2}^{(x_{ik}\beta_k - \tilde{y}_{ik})^2}\right\} \times$$

$$\left\{(1-\pi_0)\frac{1}{\sqrt{2\pi s_k}} \exp\left(-\frac{\beta_k^2}{2 s_k}\right) I(\beta_k \neq 0) + \pi_0 \delta_0(\beta_k)\right\}$$

let $\tilde{y}_{ik} = y_i - \sum_{j=1, j\neq k}^{p} x_{ij}\beta_j$

$$\propto \exp\left\{-\sum_{i=1}^{n}\frac{1}{2\tau^{-1}\xi_2^2\tilde{V}_r}\left(x_{ik}^2\beta_k^2 - 2x_{ik}\beta_k\cdot\tilde{y}_{ik} + \tilde{y}_{ik}^2\right)\right\}\times$$

$$\left\{(1-\pi_0)\frac{1}{\sqrt{2\pi\delta_k}}\exp\left(-\frac{\beta_k^2}{2\delta_k}\right)I(\beta_k\neq 0) + \pi_0\,\delta_0(\beta_k)\right\}$$

$$\propto (1-\pi_0)\frac{1}{\sqrt{2\pi\delta_k}}\exp\left\{-\frac{1}{2}\left[\left(\sum_{i=1}^{n}\frac{\tau x_{ik}^2}{\xi_2^2\tilde{V}_r}+\frac{1}{\delta_k}\right)\beta_k^2 - 2\sum_{i=1}^{n}\frac{\tau\tilde{y}_{ik}x_{ik}}{\xi_2^2\tilde{V}_r}\beta_k\right.\right.$$

$$\left.\left.+\sum_{i=1}^{n}\frac{\tau}{\xi_2^2\tilde{V}_r}\cdot\tilde{y}_{ik}^2\right]\right\}I(\beta_k\neq 0) +$$

$$\pi_0\exp\left\{-\sum_{i=1}^{n}\frac{\tau}{2\xi_2^2\tilde{V}_r}\tilde{y}_{ik}^2\right\}\delta_0(\beta_k)$$

$$\propto (1-\pi_0)\frac{1}{\sqrt{2\pi\delta_k}}\exp\left\{-\frac{1}{2}\left[\left(\sum_{i=1}^{n}\frac{\tau x_{ik}^2}{\xi_2^2\tilde{V}_r}+\frac{1}{\delta_k}\right)\beta_k^2 - 2\sum_{i=1}^{n}\frac{\tau\tilde{y}_{ik}x_{ik}}{\xi_2^2\tilde{V}_r}\beta_k\right]\right\}$$

$$\cdot\cancel{\exp\left\{-\sum_{i=1}^{n}\frac{\tau}{2\xi_2^2\tilde{V}_r}\cdot\tilde{y}_{ik}^2\right\}}\cdot I(\beta_k\neq 0) + \pi_0\,\delta_0(\beta_k)\cdot\cancel{\exp\left\{-\sum_{i=1}^{n}\frac{\tau\tilde{y}_{ik}^2}{2\xi_2^2\tilde{V}_r}\right\}}$$

$$\propto (1-\pi_0)\frac{1}{\sqrt{2\pi\delta_k}}\exp\left\{-\frac{1}{2}\left[\left(\sum_{i=1}^{n}\frac{\tau x_{ik}^2}{\xi_2^2\tilde{V}_r}+\frac{1}{\delta_k}\right)\beta_k^2 - 2\sum_{i=1}^{n}\frac{\tau\tilde{y}_{ik}x_{ik}}{\xi_2^2\tilde{V}_r}\beta_k\right]\right\}$$

$$+\ \pi_0\,\delta_0(\beta_k)$$

$$\sigma_k^2 = \left(\sum_{i=1}^{n}\frac{\tau x_{ik}^2}{\xi_2^2\tilde{V}_r}+\frac{1}{\delta_k}\right)^{-1},\quad \tilde{\mu}_k = \frac{\sum_{i=1}^{n}\frac{\tau\tilde{y}_{ik}x_{ik}}{\xi_2^2\tilde{V}_r}}{\sum_{i=1}^{n}\frac{\tau x_{ik}^2}{\xi_2^2\tilde{V}_r}+\frac{1}{\delta_k}}$$

$$\propto (1-\pi_0)\,\delta_k^{-\frac{1}{2}}\underbrace{\frac{1}{\sqrt{2\pi\sigma_k^2}}\exp\left(-\frac{1}{2\sigma_k^2}(\beta_k-\tilde{\mu}_k)^2\right)}_{N(\tilde{\mu}_k\cdot\sigma_k^2)}\cdot(\sigma_k^2)^{\frac{1}{2}}\cdot$$

$$\exp\left(\frac{1}{2}\left(\sum_{i=1}^{n}\frac{\tau\tilde{y}_{ik}x_{ik}}{\xi_2^2\tilde{V}_r}\right)^2\cdot\sigma_k^2\right)\longrightarrow\left(\exp\left(+\frac{1}{2\sigma_k^2}\cdot\tilde{\mu}_k^2\right)\right)$$

$$+\ \pi_0\,\delta_0(\beta_k)$$

$$\propto (1-\pi_0)\, s_k^{-\frac{1}{2}} \cdot (\sigma_k^2)^{\frac{1}{2}} \cdot \exp\left(-\frac{1}{2}\left(\sum_{i=1}^{n} \frac{\tau\, \tilde{y}_{ik}\, x_{ik}}{\xi_2^2\, \tilde{v}_i}\right)^2 \cdot \sigma_k^2\right) \cdot N(\tilde{\mu}_k, \sigma_k^2)$$

$$+ \pi_0\, \delta_0(\beta_k)$$

$$l_k = P(\beta_k = 0 \mid \text{rest})$$

$$= \frac{\pi_0}{\pi_0 + (1-\pi_0)\, s_k^{-\frac{1}{2}} (\sigma_k^2)^{\frac{1}{2}} \exp\left(-\frac{1}{2}\left(\sum_{i=1}^{n} \frac{\tau\, \tilde{y}_{ik}\, x_{ik}}{\xi_2^2\, \tilde{v}_i}\right)^2 \cdot \sigma_k^2\right)}$$

$$\tilde{y}_{ik} = y_i - \sum_{j=1,\, j\neq k}^{p} x_{ij}\, \beta_j$$

$$\sim (1-l_k)\, N(\tilde{\mu}_k, \sigma_k^2) + l_k\, \delta_0(\beta_k)$$

- $[s_k \mid -]$

If $\beta_k \neq 0$, $\quad [s_k \mid -] \propto \frac{1}{\sqrt{2\pi s_k}} \exp\left(-\frac{\beta_k^2}{2 s_k}\right) \cdot \exp\left(-\frac{\eta^2}{2} s_k\right)$

$$\propto \frac{1}{\sqrt{\cancel{2\pi}\, s_k}} \exp\left\{-\frac{1}{2}\left[\eta^2 s_k + \beta_k^2 \cdot s_k^{-1}\right]\right\}$$

$$\sim \text{inv. Gaussian}\left(\mu = \sqrt{\frac{\eta^2}{\beta_k^2}},\ \lambda = \eta^2\right),$$

If $\beta_k = 0$, $\quad [s_k \mid -] \propto \exp\left(-\frac{\eta^2}{2} s_k\right) \quad \sim \exp\left(\frac{\eta^2}{2}\right)$

- $[\tilde{v}_i \mid -] \propto \frac{1}{\sqrt{\tilde{v}_i}} \exp\left(-\frac{(y_i - x_i^T \beta)^2}{2\tau^{-1} \xi_2^2\, \tilde{v}_i}\right) \exp(-\tau \tilde{v}_i)$

$$\propto \frac{1}{\sqrt{\tilde{v}_i}} \exp\left\{-\frac{1}{2}\left[2\tau \tilde{v}_i + \frac{\tau (y_i - x_i^T\beta)^2}{\xi_2^2}\, \tilde{v}_i^{-1}\right]\right\}$$

$$\sim \text{inv. Gaussian}\left(\mu = \sqrt{\frac{2\tau \cdot \xi_2^2}{\tau (y_i - x_i^T \beta)^2}},\ \lambda = 2\tau\right)$$

• $[\tau \mid -] \propto \tau^{\frac{n}{2}} \exp\left\{-\frac{1}{2}\sum_{i=1}^{n} \frac{(y_i - x_i^T\beta)^2}{\tau^{-1}\xi_2^2 \tilde{v}_r}\right\} \tau^{n} \exp\left(-\tau \sum_{i=1}^{n} \tilde{v}_r\right) \cdot$

$\tau^{a-1} \exp(-b\tau)$

$\propto \tau^{a+\frac{3n}{2}-1} \exp\left\{-\tau\left[\sum_{i=1}^{n}\left(\frac{(y_i - x_i^T\beta)^2}{2\xi_2^2 \tilde{v}_r} + \tilde{v}_r\right) + b\right]\right\}$

$\sim \text{Gamma}\left(a + \frac{3}{2}n,\ \sum_{i=1}^{n}\left(\frac{(y_i - x_i^T\beta)^2}{2\xi_2^2 \tilde{v}_r} + \tilde{v}_r\right) + b\right)$

• $[\eta^2 \mid -] \propto \prod_{k=1}^{p} \frac{\eta^2}{2} \exp\left(-\frac{\eta^2}{2} s_k\right) \cdot (\eta^2)^{c-1} \exp(-d\eta^2)$

$\propto (\eta^2)^{p+c-1} \exp\left\{-\eta^2\left(\sum_{k=1}^{p} \frac{s_k}{2} + d\right)\right\}$

• $[\pi_0 \mid -] \propto \prod_{k=1}^{p}\left[(1-\pi_0)\frac{1}{\sqrt{2\pi s_k}} \exp\left(-\frac{\beta_k^2}{2 s_k}\right) I(\beta_k \neq 0) + \pi_0 \delta_0(\beta_k)\right]$

$\times\ \pi_0^{c-1} (1-\pi_0)^{d-1}$

if $\beta_k = 0$, $[\pi_0 \mid -] \propto \pi_0^{c-1}(1-\pi_0)^{d-1}\ \pi_0^{p} = \pi_0^{p+c-1}(1-\pi_0)^{d-1}$

$\sim \text{Beta}(p+c, d)$

if $\beta_k \neq 0$, $[\pi_0 \mid -] \propto (1-\pi_0)^{p}\ \pi_0^{c-1}(1-\pi_0)^{d-1}$

$= \pi_0^{c-1}(1-\pi_0)^{p+d-1} \quad \sim \text{Beta}(c, p+d)$

let $Z_k = \begin{cases} 1, & \text{if } \beta_k \neq 0 \\ 0, & \text{if } \beta_k = 0 \end{cases}$

$[\pi_0 \mid -] \sim \text{Beta}\left(p + c - \sum_{k=1}^{p} Z_k,\ d + \sum_{k=1}^{p} Z_k\right)$