BL-SS

$$Y = \underset{n\times q_1}{C}\,\underset{q_1\times 1}{b} + \underset{n\times q_2}{E}\cdot\underset{q_2\times 1}{\alpha} + \underset{n\times 1}{X}\underset{1\times 1}{\beta} + \underset{n\times q_2}{W}\underset{q_2\times 1}{\eta} + \underset{n\times 1}{\varepsilon}$$

$C = (C_1, \cdots, C_{q_1})$, $b = (b_1, \cdots, b_{q_1})^T$

$E = (E_1, \cdots, E_{q_2})$, $\alpha = (\alpha_1, \cdots, \alpha_{q_2})^T$

$X = G_j$, $\beta_{1\times 1}$, $W = (XE_1, \cdots, XE_{q_2})$, $\eta = (\eta_1, \cdots, \eta_{q_2})^T$

priors: $\beta, \eta_i, \pi_c, \tau_c^2, \lambda_c^2, \pi_e, \tau_{ei}^2, \lambda_e^2, \alpha, b, \sigma^2$

$y \mid - \ \propto (\sigma^2)^{-\frac{n}{2}} \exp\left(-\frac{1}{2\sigma^2}(y-\mu)^T(y-\mu)\right)$, $\mu = E(y)$

$\beta \mid \pi_c, \tau_c^2, \sigma^2 \sim (1-\pi_c) N(0, \sigma^2\tau_c^2) + \pi_c \delta_0(\beta)$,

$\pi_c \sim \text{Beta}(r_c, u_c)$, $\tau_c^2 \sim \text{gamma}(1, \frac{\lambda_c^2}{2})$

$\lambda_c^2 \sim \text{gamma}(\overset{(a_1,\ b_1)}{a_c, b_c})$

$\eta_i \mid \pi_e, \tau_e^2, \sigma^2 \sim (1-\pi_e) N(0, \sigma^2\tau_{ei}^2) + \pi_e \delta_0(\eta_i)$, $i = 1, \cdots, q_2$

$\pi_e \sim \text{Beta}(r_e, u_e)$, $\tau_{ei}^2 \overset{iid}{\sim} \text{gamma}(1, \frac{\lambda_e^2}{2})$

$\lambda_e^2 \sim \text{gamma}(a_e, b_e)$ (a_2, b_2)

$\sigma^2 \sim \text{inv. gamma}(s, h)$ $\left((\sigma^2)^{-s-1} \exp(-\frac{h}{\sigma^2}) \right)$

$\alpha \sim N_{q_2}(0, \Sigma_{\alpha 0})$

$b \sim N_{q_1}(0, \Sigma_{b0})$

full: $\pi(\sim \mid Y)$

$$\propto (\sigma^2)^{-\frac{n}{2}} \exp\left\{-\frac{1}{2\sigma^2}(y-\mu)^T(y-\mu)\right\} \times \underbrace{\exp\left(-\frac{1}{2}b^T\Sigma_{b_0}^{-1}b\right)}_{b} \times$$

$$\underbrace{\exp\left(-\frac{1}{2}\alpha^T\Sigma_{\alpha_0}^{-1}\alpha\right)}_{\alpha} \times \underbrace{\left[(1-\pi_c)(2\pi_c\sigma^2\tau_c^2)^{-\frac{1}{2}}\exp\left(-\frac{\beta^2}{2\sigma^2\tau_c^2}\right)I(\beta\neq 0) + \pi_c\delta_0(\beta)\right]}_{\beta}$$

$$\times \underbrace{(\lambda_c^2)^{a_c-1}\exp(-b_c\lambda_c^2)}_{\lambda_c^2} \times \underbrace{\frac{\lambda_c^2}{2}\times\exp\left(-\frac{\lambda_c^2}{2}\tau_c^2\right)}_{\tau_c^2} \times \underbrace{(\pi_c)^{r_c-1}(1-\pi_c)^{u_c-1}}_{\pi_c} \times$$

$$\prod_{i=1}^{q_2}\underbrace{\left[(1-\pi_e)(2\pi_e\sigma^2\tau_{ei}^2)^{-\frac{1}{2}}\exp\left(-\frac{1}{2\sigma^2\tau_{ei}^2}\eta_i^2\right)I(\eta_i\neq 0)+\pi_e\delta_0(\eta_i)\right]}_{\eta_i}$$

$$\times \underbrace{(\lambda_e^2)^{a_e-1}\exp(-b_e\lambda_e^2)}_{\lambda_e^2} \times \prod_{i=1}^{q_2}\underbrace{\frac{\lambda_e^2}{2}\times\exp\left(-\frac{\lambda_e^2}{2}\tau_{ei}^2\right)}_{\tau_{ei}^2} \times \underbrace{(\pi_e)^{r_e-1}(1-\pi_e)^{u_e-1}}_{\pi_e}$$

$$\times (\sigma^2)^{-s-1}\exp\left(-\frac{h}{\sigma^2}\right)$$

- $b \mid - \ \propto \exp\left\{-\frac{1}{2\sigma^2}(y-\mu_{(-b)}-Cb)^T(y-\mu_{(-b)}-Cb)-\frac{1}{2}b^T\Sigma_{b_0}^{-1}b\right\}$

$\Sigma_b = \left(\Sigma_{b_0}^{-1}+\frac{1}{\sigma^2}C^TC\right)^{-1}$, $\quad \mu_b = \Sigma_b\cdot\left(\frac{1}{\sigma^2}(y-\mu_{(-b)})^T\cdot C\right)^T$

$\sim N(\mu_b, \Sigma_b)$. $\quad \mu_{(-b)} = E(y) - Cb \quad$ (same)

- $\alpha \mid - \ \propto \exp\left\{-\frac{1}{2\sigma^2}(y-\mu_{(-\alpha)}-E\alpha)^T(y-\mu_{(-\alpha)}-E\alpha)-\frac{1}{2}\alpha^T\Sigma_{\alpha_0}^{-1}\alpha\right\}$

$\Sigma_\alpha = \left(\Sigma_{\alpha_0}^{-1}+\frac{1}{\sigma^2}E^TE\right)^{-1}$, $\quad \mu_\alpha = \Sigma_\alpha\cdot\left(\frac{1}{\sigma^2}(y-\mu_{(-\alpha)})^T\cdot E\right)^T$

$\sim N(\mu_\alpha, \Sigma_\alpha)$. $\quad \mu_{(-\alpha)} = E(y) - E\alpha \quad$ (same)

- $\beta \mid - \ \sim (1-l_c)N(\mu_\beta, \sigma^2\Sigma_\beta) + l_c\delta_0(\beta)$

$\cdot\ \mu_\beta = \Sigma_\beta X^T(Y-\mu_{(-\beta)})$, $\quad \cdot\ \Sigma_\beta = \left(\underset{1\times n}{X^T}\underset{n\times 1}{X}+\frac{1}{\tau_c^2}\right)^{-1}$

$$l_c = P(\beta=0\mid -) = \frac{\pi_c}{\pi_c+(1-\pi_c)(\tau_c^2)^{-\frac{1}{2}}|\Sigma_\beta|^{\frac{1}{2}}\exp\left\{\frac{1}{2\sigma^2}\cdot\Sigma_\beta\underbrace{(Y-\mu_{(-\beta)})^TX\cdot X^T(Y-\mu_{(-\beta)})}_{\|X^T(Y-\mu_{(-\beta)})\|_2^2}\right\}}$$

$\mu_{(-\beta)} = E(y) - X\beta$.

- $\eta_i \mid - \sim (1-l_{ei})\, N(\mu_{\eta_i}, \sigma^2 \Sigma_{\eta_i}) + l_{ei}\, \delta_0(\eta_i)$, $i = 1, \cdots, q_2$

$W_i^T \cdot (Y - \mu(-\eta_i))$

$$\mu_{\eta_i} = \Sigma_{\eta_i} \big(\overset{1\times n}{Y - \mu(-\eta_i)}\big)^T \cdot \underset{n\times 1}{W_i}, \qquad \Sigma_{\eta_i} = \big(\underset{1\times n}{W_i^T}\, \underset{n\times 1}{W_i} + \frac{1}{\tau_{ei}^2}\big)^{-1}$$

$$\mu(-\eta_i) = E(y) - W_i \cdot \eta_i \qquad y - cb - E\alpha - X\beta - (W\eta + W_i \cdot \eta_i) \qquad W[,-i] \cdot \eta[-i]$$

$$l_{ei} = P(\eta_i = 0 \mid -) = \frac{\pi_e}{\pi_e + (1-\pi_e) \cdot (\tau_{ei}^2)^{-\frac{1}{2}} \, |\Sigma_{\eta_i}|^{\frac{1}{2}} \cdot \exp\left\{ \frac{1}{2\sigma^2} \cdot \Sigma_{\eta_i} \, \| W_i^T (Y - \mu(-\eta_i)) \|_2^2 \right\}}$$

- $\frac{1}{\tau_c^2} \mid - \sim \begin{cases} \text{inv. gamma}\,(1, \underbrace{\tfrac{\lambda_c^2}{2}}_{\text{scale}}), & 若\ \beta = 0 \\ \text{inv. gaussian}\,(\sqrt{\frac{\sigma^2}{\beta^2} \lambda_c^2}, \lambda_c^2), & 若\ \beta \neq 0 \end{cases}$

- $\frac{1}{\tau_{ei}^2} \mid - \sim \begin{cases} \text{inv. gamma}\,(1, \overbrace{\tfrac{\lambda_e^2}{2}}^{\text{scale}}), & 若\ \eta_i = 0 \\ \text{inv. gaussian}\,(\sqrt{\frac{\sigma^2}{\eta_i^2} \lambda_e^2}, \lambda_e^2), & 若\ \eta_i \neq 0 \end{cases}$

- $$\lambda_c^2 \mid - \propto (\lambda_c^2)^{a_c + 1 - 1} \exp\big(-(b_c + \tfrac{\tau_c^2}{2})\, \lambda_c^2\big)$$

  $$\sim \text{gamma}\,(a_c + 1, \tfrac{\tau_c^2}{2} + b_c)$$

- $$\lambda_e^2 \mid - \propto (\lambda_e^2)^{a_e - 1 + q_2} \exp\big(-(b_e + \sum_{i=1}^{q_2} \tfrac{\tau_{ei}^2}{2})\, \lambda_e^2\big)$$

  $$\sim \text{gamma}\,(a_e + q_2, \sum_{i=1}^{q_2} \frac{\tau_{ei}^2}{2} + b_e)$$

let $Z_1 = \begin{cases} 1, & 对\ \beta \neq 0 \\ 0, & 对\ \beta = 0. \end{cases}$ $\quad Z_{2i} = \begin{cases} 1, & 对\ \eta_i \neq 0 \\ 0, & 对\ \eta_i = 0 \end{cases}$ $\quad i = 1, \cdots, q_2$

$\pi_c | -$ 对 $\beta \neq 0$, $\pi_c | - \propto (1-\pi_c)^{1+u_c-1} (\pi_c)^{r_c-1} \sim \text{Beta}(r_c, u_c+1)$

对 $\beta = 0$. $\pi_c | - \propto \pi_c^{1+r_c-1} (1-\pi_c)^{u_c-1} \sim \text{Beta}(r_c+1, u_c)$

- $\pi_c | - \sim \text{Beta}(r_c + 1 - Z_1, u_c + Z_1)$

- $\pi_e | -$ 对 $\eta_i \neq 0$, $\pi_e | - \propto (1-\pi_e)^{q_2+u_e-1} \cdot \pi_e^{r_e-1}$

$$\sim \text{Beta}(r_e, u_e + q_2)$$

对 $\eta_i = 0$, $\pi_e | - \propto \pi_e^{q_2+r_e-1} (1-\pi_e)^{u_e-1} \sim \text{Beta}(r_e+q_2, u_e)$

$$\pi_e | - \sim \text{Beta}\left(r_e + q_2 - \sum_{i=1}^{q_2} Z_{2i},\ u_e + \sum_{i=1}^{q_2} Z_{2i}\right)$$

- $\sigma^2 | - \ \text{inv. gamma}\left(s + \frac{n + Z_1 + \sum_{i=1}^{q_2} Z_{2i}}{2},\right.$

$$\left. h + \frac{1}{2}\left[(Y-\mu)^T(Y-\mu) + \frac{\beta^2}{\tau_c^2} + \eta^T D_{\tau e}^{-1} \eta\right]\right)$$

$$D_{\tau e} = \begin{pmatrix} \tau_{e1}^2 & & \\ & \ddots & \\ & & \tau_{eq_2}^2 \end{pmatrix} \qquad \sum_{i=1}^{q_2} \frac{\eta_i^2}{\tau_{ei}^2}$$