$$y_i = x_i^T \beta + \xi_2 \tau^{-\frac{1}{2}} \sqrt{\tilde{v}_i}\, z_i$$

$1\times1 \quad p\times1 \quad 1\times p$

$$\tilde{v} \mid \tau \sim \prod_{i=1}^{n} \tau \exp(-\tau \tilde{v}_i) \qquad \tilde{v} = (\tilde{v}_1, \cdots, \tilde{v}_n)$$

$$z \sim \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} z_i^2\right) \qquad z = (z_1, \cdots, z_n)$$

$$\beta_k \mid s_k \sim \frac{1}{\sqrt{2\pi s_k}} \exp\left(-\frac{\beta_k^2}{2 s_k}\right), \quad N(0, s_k), \quad k=1,\cdots,p$$

$$s_k \mid \eta^2 \sim \frac{\eta^2}{2} \exp\left(-\frac{\eta^2}{2} s_k\right)$$

$$\tau \sim \text{gamma}(a, b), \quad \eta^2 \sim \text{gamma}(c, d)$$

$$\tau, \eta^2 \sim \tau^{a-1} \exp(-b\tau) \, (\eta^2)^{c-1} \exp(-d\eta^2)$$

spike and slab:

$$\beta_k \mid s_k \sim (1-\pi_0) N(0, s_k) + \pi_0 \delta_0(\beta_k), \quad k=1,\cdots,p$$

$$f(y \mid -) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi \tau^{-1} \xi_2^2 \tilde{v}_i}} \exp\left\{-\frac{(y_i - x_i^T\beta - \xi_1 \tilde{v}_i)^2}{2\tau^{-1}\xi_2^2 \tilde{v}_i}\right\}, \quad \xi_1 = 0$$

$$\bullet [\beta_k \mid -] \propto \exp\left\{-\frac{1}{2}\sum_{i=1}^{n} \frac{(y_i - x_i^T\beta)^2}{\tau^{-1}\xi_2^2 \tilde{v}_i}\right\} \times$$

$$\left\{(1-\pi_0)\frac{1}{\sqrt{2\pi s_k}} \exp\left(-\frac{\beta_k^2}{2 s_k}\right) I(\beta_k \neq 0) + \pi_0 \delta_0(\beta_k)\right\}$$

$$\propto \exp\left\{-\sum_{i=1}^{n} \frac{1}{2\tau^{-1}\xi_2^2 \tilde{v}_i} \overbrace{\left(y_i - x_{ik}\beta_k - \sum_{j=1,\, j\neq k}^{p} x_{ij}\beta_j\right)^2}^{(x_{ik}\beta_k - \tilde{y}_{ik})^2}\right\} \times$$

$$\left\{(1-\pi_0)\frac{1}{\sqrt{2\pi s_k}} \exp\left(-\frac{\beta_k^2}{2 s_k}\right) I(\beta_k \neq 0) + \pi_0 \delta_0(\beta_k)\right\}$$

let $\tilde{y}_{ik} = y_i - \sum_{j=1,\, j\neq k}^{p} x_{ij}\beta_j$

$$\propto \exp\left\{ -\sum_{i=1}^{n} \frac{1}{2\tau^{-1}\xi_2^2 \tilde{v}_i} \left(x_{ik}^2 \beta_k^2 - 2 x_{ik}\beta_k \cdot \tilde{y}_{ik} + \tilde{y}_{ik}^2 \right) \right\} \times$$

$$\left\{ (1-\pi_0) \frac{1}{\sqrt{2\pi s_k}} \exp\left(-\frac{\beta_k^2}{2 s_k} \right) I(\beta_k \neq 0) + \pi_0 \delta_0(\beta_k) \right\}$$

$$\propto (1-\pi_0) \frac{1}{\sqrt{2\pi s_k}} \exp\left\{ -\frac{1}{2}\left[\left(\sum_{i=1}^{n} \frac{\tau x_{ik}^2}{\xi_2^2 \tilde{v}_i} + \frac{1}{s_k} \right) \beta_k^2 - 2 \sum_{i=1}^{n} \frac{\tau \tilde{y}_{ik} x_{ik}}{\xi_2^2 \tilde{v}_i} \beta_k \right.\right.$$

$$\left.\left. + \sum_{i=1}^{n} \frac{\tau}{\xi_2^2 \tilde{v}_i} \cdot \tilde{y}_{ik}^2 \right] \right\} I(\beta_k \neq 0) +$$

$$\pi_0 \exp\left\{ -\sum_{i=1}^{n} \frac{\tau}{2\xi_2^2 \tilde{v}_i} \tilde{y}_{ik}^2 \right\} \delta_0(\beta_k)$$

$$\propto (1-\pi_0) \frac{1}{\sqrt{2\pi s_k}} \exp\left\{ -\frac{1}{2}\left[\left(\sum_{i=1}^{n} \frac{\tau x_{ik}^2}{\xi_2^2 \tilde{v}_i} + \frac{1}{s_k} \right) \beta_k^2 - 2 \sum_{i=1}^{n} \frac{\tau \tilde{y}_{ik} x_{ik}}{\xi_2^2 \tilde{v}_i} \beta_k \right] \right\}$$

$$\cdot \cancel{\exp\left\{ -\sum_{i=1}^{n} \frac{\tau}{2\xi_2^2 \tilde{v}_i} \cdot \tilde{y}_{ik}^2 \right\}} \cdot I(\beta_k \neq 0) + \pi_0 \delta_0(\beta_k) \cdot \cancel{\exp\left\{ -\sum_{i=1}^{n} \frac{\tau \tilde{y}_{ik}^2}{2\xi_2^2 \tilde{v}_i} \right\}}$$

$$\propto (1-\pi_0) \frac{1}{\sqrt{2\pi s_k}} \exp\left\{ -\frac{1}{2}\left[\left(\sum_{i=1}^{n} \frac{\tau x_{ik}^2}{\xi_2^2 \tilde{v}_i} + \frac{1}{s_k} \right) \beta_k^2 - 2 \sum_{i=1}^{n} \frac{\tau \tilde{y}_{ik} x_{ik}}{\xi_2^2 \tilde{v}_i} \beta_k \right] \right\}$$

$$+ \pi_0 \delta_0(\beta_k)$$

$$\sigma_k^2 = \left(\sum_{i=1}^{n} \frac{\tau x_{ik}^2}{\xi_2^2 \tilde{v}_i} + \frac{1}{s_k} \right)^{-1}, \quad \tilde{\mu}_k = \frac{\sum_{i=1}^{n} \frac{\tau \tilde{y}_{ik} x_{ik}}{\xi_2^2 \tilde{v}_i}}{\sum_{i=1}^{n} \frac{\tau x_{ik}^2}{\xi_2^2 \tilde{v}_i} + \frac{1}{s_k}}$$

$$\propto (1-\pi_0) s_k^{-\frac{1}{2}} \underbrace{\frac{1}{\sqrt{2\pi\sigma_k^2}} \exp\left(-\frac{1}{2\sigma_k^2} (\beta_k - \tilde{\mu}_k)^2 \right)}_{N(\tilde{\mu}_k \cdot \sigma_k^2)} \cdot (\sigma_k^2)^{\frac{1}{2}} \cdot$$

$$\exp\left(\frac{1}{2} \left(\sum_{i=1}^{n} \frac{\tau \tilde{y}_{ik} x_{ik}}{\xi_2^2 \tilde{v}_i} \right)^2 \cdot \sigma_k^2 \right) \longrightarrow \left(\exp\left(+ \frac{1}{2\sigma_k^2} \cdot \tilde{\mu}_k^2 \right) \right)$$

$$+ \pi_0 \delta_0(\beta_k)$$

$$\propto (1-\pi_0)\, S_K^{-\frac{1}{2}} \cdot (\sigma_K^2)^{\frac{1}{2}} \cdot \exp\left(+\frac{1}{2}\left(\sum_{i=1}^{n} \frac{\tau\, \tilde{y}_{ik}\, x_{ik}}{\xi_2^2\, \tilde{U}_i}\right)^2 \cdot \sigma_K^2\right) \cdot N(\tilde{\mu}_K, \sigma_K^2)$$

$$+ \pi_0\, \delta_0(\beta_K)$$

$$l_K = P(\beta_K = 0 \mid \text{rest})$$

$$= \frac{\pi_0}{\pi_0 + (1-\pi_0)\, S_K^{-\frac{1}{2}}\, (\sigma_K^2)^{\frac{1}{2}} \exp\left(-\frac{1}{2}\left(\sum_{i=1}^{n} \frac{\tau\, \tilde{y}_{ik}\, x_{ik}}{\xi_2^2\, \tilde{U}_i}\right)^2 \cdot \sigma_K^2\right)}$$

$$\tilde{y}_{ik} = y_i - \sum_{j=1,\, j\neq k}^{p} x_{ij}\, \beta_j$$

$$\sim (1-l_K)\, N(\tilde{\mu}_K, \sigma_K^2) + l_K\, \delta_0(\beta_K)$$

- $[S_K \mid -]$

If $\beta_K \neq 0$, $[S_K \mid -] \propto \frac{1}{\sqrt{2\pi S_K}} \exp\left(-\frac{\beta_K^2}{2 S_K}\right) \cdot \exp\left(-\frac{\eta^2}{2} S_K\right)$

$$\propto \frac{1}{\sqrt{\cancel{2\pi} S_K}} \exp\left\{-\frac{1}{2}\left[\eta^2 S_K + \beta_K^2 \cdot S_K^{-1}\right]\right\}$$

$$\sim \text{inv. Gaussian}\left(\mu = \sqrt{\frac{\eta^2}{\beta_K^2}},\ \lambda = \eta^2\right),$$

If $\beta_K = 0$, $[S_K \mid -] \propto \exp\left(-\frac{\eta^2}{2} S_K\right) \sim \exp\left(\frac{\eta^2}{2}\right)$

- $[\tilde{U}_i \mid -] \propto \frac{1}{\sqrt{\tilde{U}_i}} \exp\left(-\frac{(y_i - x_i^T \beta)^2}{2\tau^{-1} \xi_2^2\, \tilde{U}_i}\right) \exp(-\tau \tilde{U}_i)$

$$\propto \frac{1}{\sqrt{\tilde{U}_i}} \exp\left\{-\frac{1}{2}\left[2\tau\, \tilde{U}_i + \frac{\tau (y_i - x_i^T \beta)^2}{\xi_2^2}\, \tilde{U}_i^{-1}\right]\right\}$$

$$\sim \text{inv. Gaussian}\left(\mu = \sqrt{\frac{2\tau \cdot \xi_2^2}{\tau (y_i - x_i^T \beta)^2}},\ \lambda = 2\tau\right)$$

$$\bullet\ [\tau|-] \propto \tau^{\frac{n}{2}} \exp\left\{-\frac{1}{2}\sum_{i=1}^{n}\frac{(y_i - x_i^T\beta)^2}{\tau^{-1}\xi_2^2\tilde{v}_r}\right\} \tau^n \exp\left(-\tau\sum_{i=1}^{n}\tilde{v}_r\right)\cdot$$

$$\tau^{a-1}\exp(-b\tau)$$

$$\propto \tau^{a+\frac{3n}{2}-1}\exp\left\{-\tau\left[\sum_{i=1}^{n}\left(\frac{(y_i-x_i^T\beta)^2}{2\xi_2^2\tilde{v}_r}+\tilde{v}_r\right)+b\right]\right\}$$

$$\sim \text{Gamma}\left(a+\frac{3}{2}n,\ \sum_{i=1}^{n}\left(\frac{(y_i-x_i^T\beta)^2}{2\xi_2^2\tilde{v}_r}+\tilde{v}_r\right)+b\right)$$

$$\bullet\ [\eta^2|-] \propto \prod_{k=1}^{p}\frac{\eta^2}{2}\exp\left(-\frac{\eta^2}{2}s_k\right)\cdot(\eta^2)^{c-1}\exp(-d\eta^2)$$

$$\propto (\eta^2)^{p+c-1}\exp\left\{-\eta^2\left(\sum_{k=1}^{p}\frac{s_k}{2}+d\right)\right\}$$

$$\bullet\ [\pi_0|-] \propto \prod_{k=1}^{p}\left[(1-\pi_0)\frac{1}{\sqrt{2\pi s_k}}\exp\left(-\frac{\beta_k^2}{2s_k}\right)I(\beta_k\neq 0)+\pi_0\,\delta_0(\beta_k)\right]$$

$$\times\ \pi_0^{c-1}(1-\pi_0)^{d-1}$$

if $\beta_k = 0$, $[\pi_0|-] \propto \pi_0^{c-1}(1-\pi_0)^{d-1}\,\pi_0^{p} = \pi_0^{p+c-1}(1-\pi_0)^{d-1}$

$$\sim \text{Beta}(p+c,\ d)$$

if $\beta_k \neq 0$, $[\pi_0|-] \propto (1-\pi_0)^{p}\,\pi_0^{c-1}(1-\pi_0)^{d-1}$

$$= \pi_0^{c-1}(1-\pi_0)^{p+d-1} \sim \text{Beta}(c,\ p+d)$$

let $Z_k = \begin{cases} 1, & \text{if } \beta_k \neq 0 \\ 0, & \text{if } \beta_k = 0 \end{cases}$

$$[\pi_0|-] \sim \text{Beta}\left(p+c-\sum_{k=1}^{p}Z_k,\ d+\sum_{k=1}^{p}Z_k\right)$$