$$\overset{n\times 1}{y_i} = \overset{p\times 1}{x_i^T}\overset{1\times p}{\beta} + \xi_2 \tau^{-\frac{1}{2}} \sqrt{\tilde{v}_i}\, z_i ; \qquad x_i = (x_{i1}, \cdots, x_{ip})^T$$ p1

$$\tilde{v} \mid \tau \sim \prod_{i=1}^{n} \tau \exp(-\tau \tilde{v}_i) ; \ \exp(\tau) \qquad \tilde{v} = (\tilde{v}_1, \cdots, \tilde{v}_n)$$

$$z \sim \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2} z_i^2) ; \qquad z = (z_1, \cdots, z_n) \quad N(0,1)$$

$$\beta \mid s_1, s_2, \cdots, s_p \sim N_p(0, D_s), \quad D_s = \mathrm{Diag}(s_1, \cdots, s_p)$$

$$s_1, \cdots, s_p \sim \prod_{k=1}^{p} \frac{\eta^2}{2} e^{-\frac{\eta^2}{2} s_k} ds_k, \quad s_1, \cdots, s_p > 0 .$$

$$\left(\beta, s \mid \eta^2 \sim \prod_{k=1}^{p} \frac{1}{\sqrt{2\pi s_k}} \exp\left(-\frac{\beta_k^2}{2 s_k}\right) \prod_{k=1}^{p} \frac{\eta^2}{2} \exp\left(-\frac{\eta^2}{2} s_k\right) \right)$$

$$\tau, \eta^2 \sim \tau^{a-1} \exp(-b\tau) (\eta^2)^{c-1} \exp(-d\eta^2)$$

$$\tau \sim \mathrm{gamma}(a, b), \quad \eta^2 \sim \mathrm{gamma}(c, d)$$

$$\pi(\beta \mid \eta) = \prod_{k=1}^{p} \frac{\eta}{2} \exp\{-\eta |\beta_k|\} .$$

$$\beta_k \mid \eta^2, s_k \sim (1-\pi_0) N(0, s_k) + \pi_0 \delta_0(\beta_k), \quad k = 1, \cdots, p$$

$$f(y \mid x, \tilde{v}, \beta, s, \tau, \eta^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi \tau^{-1} \xi_2^2 \tilde{v}_i}} \exp\left\{ -\frac{(y_i - x_i^T \beta - \xi_1 \tilde{v}_i)^2}{2\tau^{-1} \xi_2^2 \tilde{v}_i} \right\}$$

$$* f(\beta_k \mid \overset{n\times 1}{x}, \overset{n\times p}{y}, \tilde{v}, \overset{p\times 1}{\beta_{-k}}, \cancel{s}, \cancel{\tau}, \cancel{\eta^2}) \propto f(y \mid x, \tilde{v}, \beta, s, \tau, \eta)\, \pi(\beta_k \mid s_k)$$

$$\propto \exp\left\{ -\frac{1}{2} \sum_{i=1}^{n} \frac{\overset{(y - x^T\beta)^T (y - x^T\beta)}{(y_i - x_i^T \beta - \cancel{\xi_1 \tilde{v}_i})^2}}{\tau^{-1} \xi_2^2 \tilde{v}_i} \right\} \left((1-\pi_0) \frac{1}{\sqrt{2\pi s_k}} \exp\left(-\frac{\beta_k^2}{2 s_k}\right) \underset{I(\beta_k \neq 0)}{\wedge} + \pi_0 \delta_0(\beta_k) \right)$$

$$\propto \exp\left[-\frac{1}{\cancel{2}\tau^{-1} \xi_2^2 \tilde{v}_i} \left(\underset{n\times 1}{y} - x_k \beta_k - \overset{X_{(k)}\beta_{(k)}}{\sum_{j \neq k} x_j \beta_j} \right)^T \left(y - x_k \beta_k - \sum_{j \neq k} x_j \beta_j \right) \right]$$

$$x_k = (x_{1k}, \cdots, x_{nk})^T \quad n\times 1 .$$

$$\left[(1-\pi_0)(2\pi s_k)^{-\frac{1}{2}} \exp\left(-\frac{1}{2 s_k} \beta_k^T \beta_k\right) I(\beta_k \neq 0) + \pi_0 \delta_0(\beta_k) \right)$$

$X_{(k)} = (X_1, \cdots, X_{k-1}, X_{k+1}, \cdots, X_p)_{n\times(p-1)}$

$\beta_{(k)} = (\beta_1, \cdots, \beta_{k-1}, \beta_{k+1}, \cdots, \beta_p)^T$

$\exp(-\frac{1}{2S_k}\beta_k^T\beta_k)$

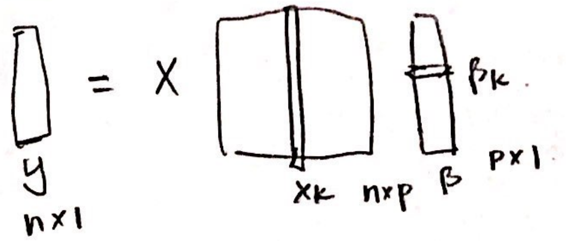
$$= (1-\pi_0)(2\pi S_k)^{-\frac{1}{2}} \exp\left(-\frac{1}{2\tau^{-1}\xi_2^2\tilde{v}_i}\left(\beta_k^T X_k^T X_k \beta_k + \frac{\tau^{-1}\xi_2^2\tilde{v}_i}{\cancel{\#}S_k}\beta_k^T\beta_k - 2\beta_k^T X_k^T(y - X_{(k)}\beta_{(k)}) + \|y - \cancel{X_{(k)}}\beta_{(k)}\|_2^2\right)\right) +$$

$$\exp\left(-\frac{1}{2\tau^{-1}\xi_2^2\tilde{v}_i}\|y - \cancel{X_{(k)}}\beta_{(k)}\|_2^2\right)\cdot \pi_0 \, \cancel{\delta_0(\beta_k)}. \quad \beta_k = 0.$$

$$= (1-\pi_0)(2\pi S_k)^{-\frac{1}{2}} \exp\left(-\frac{1}{2\tau^{-1}\xi_2^2\tilde{v}_i}\left[\left(X_k^T X_k + \frac{\tau^{-1}\xi_2^2\tilde{v}_i}{S_k}\right)\beta_k^T\beta_k - 2\beta_k^T X_k^T(y - X_{(k)}\beta_{(k)}) + \|y - X_{(k)}\beta_{(k)}\|^2\right]\right) + *$$

$$= (1-\pi_0)(2\pi S_k)^{-\frac{1}{2}} \exp\left[\frac{-1}{2}\left[\underbrace{\left(\frac{X_k^T X_k}{\tau^{-1}\xi_2^2\tilde{v}_i} + \frac{1}{S_k}\right)}_{M}\beta_k^T\beta_k - 2\,\frac{X_k^T(y - X_{(k)}\beta_{(k)})}{\tau^{-1}\xi_2^2\tilde{v}_i}\beta_k^T + \underbrace{\|y - X_{(k)}\beta_{(k)}\|_2^2\cdot\frac{1}{\tau^{-1}\xi_2^2\tilde{v}_i}}_{N}\right]\right] + *$$

$$\Sigma_k = \left(\frac{X_k^T X_k}{\tau^{-1}\xi_2^2\tilde{v}_i} + \frac{1}{S_k}\right)^{-1}, \qquad \mu_k = \frac{X_k^T(y - X_{(k)}\beta_{(k)})\cdot\Sigma_k}{\tau^{-1}\xi_2^2\tilde{v}_i}$$



$X_k = (x_{1k}, \cdots, x_{nk})^T_{n\times 1}$

$X_{(k)}: n\times(p-1)$ $\quad \beta_k: (p-1)\times 1$

$$= (1-\pi_0)(2\pi S_k)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\frac{(\beta_k - \mu_k)^2}{\Sigma_k} + N + \boxed{T}\right)$$

$$+ \exp(\quad)\cdot\pi_0\cdot\delta_0(\beta_k)$$

$$\Sigma_k = \tilde{\sigma}_k^{*2} = \left(\sum_{i=1}^n \frac{\tau x_{ik}^2}{\xi_2^2\tilde{v}_i} + \frac{1}{S_k}\right)^{-1}, \qquad \tilde{\mu}_k = \frac{\sum_{i=1}^n \frac{\tau\cdot(y_i - \sum_{j=1, j\neq k}^p x_{ij}\beta_j)x_{ik}}{\xi_2^2\tilde{v}_i}}{\sum_{i=1}^n \frac{\tau x_{ik}^2}{\xi_2^2\tilde{v}_i} + \frac{1}{S_k}}$$

$$= (1-\pi_0)\, s_k^{-\frac{1}{2}} (2\pi)^{-\frac{1}{2}} \cdot |\Sigma_k|^{-\frac{1}{2}} \underbrace{\exp\left(-\frac{1}{2\Sigma_k}(\beta_k-\mu_k)^T(\beta_k-\mu_k)\right)}_{N(\mu_k,\Sigma_k)}$$

$$\cdot |\Sigma_k|^{\frac{1}{2}} \exp\left(\frac{1}{2\Sigma_k}\mu_k^T\mu_k\right) \cdot \cancel{\exp(}$$

$$\cdot \exp\left(-\cancel{\frac{1}{2\tau^{-1}\xi_2^2\tilde{v}_i}} \|y - X_{(k)}\beta_{(k)}\|_2^2\right)$$

$$+ \exp\left(-\cancel{\frac{1}{2\tau^{-1}\xi_2^2\tilde{v}_i}} \|y - X_{(k)}\beta_{(k)}\|_2^2\right) \cdot \pi_0\, \delta_0(\beta_k)$$

$$= (1-\pi_0)\, s_k^{-\frac{1}{2}} \cdot |\Sigma_k|^{\frac{1}{2}} \exp\left\{\frac{1}{2}\left\| \frac{\Sigma_k^{\frac{1}{2}} \cancel{(y - X_{(k)}\beta_{(k)})} X_k^T(y - X_{(k)}\beta_{(k)})}{\tau^{-1}\xi_2^2\tilde{v}_i}\right\|_2^2\right\} \times N(\mu_k, \Sigma_k)$$

$$+ \pi_0\, \delta_0(\beta_k)$$

$$l_k = P(\beta_k = 0 \mid \text{rest})$$

$$= \frac{\pi_0}{\pi_0 + (1-\pi_0)\, s_k^{-\frac{1}{2}} |\Sigma_k|^{\frac{1}{2}} \exp\left\{\frac{1}{2}\left\|\frac{\Sigma_k^{\frac{1}{2}} X_k^T(y - X_{(k)}\beta_{(k)})}{\tau^{-1}\xi_2^2\tilde{v}_i}\right\|_2^2\right\}}$$

$$\frac{X_k^T(y - X_{(k)}\beta_{(k)})}{\tau^{-1}\xi_2^2\tilde{v}_i} \downarrow \sum_{i=1}^{n} \frac{\tau\left(y_i - \sum_{j=1, j\neq k}^{p} x_{ij}\beta_j\right) x_{ik}}{\xi_2^2 v_i}$$

$$\sim (1-l_k)\, N(\mu_k, \Sigma_k) + l_k\, \delta_0(\beta_k)$$

BB

$$[\pi_0 \mid \cdot\,] \propto \prod_{k=1}^{p} \left[(1-\pi_0) \frac{1}{\sqrt{2\pi s_k}} \exp\left(-\frac{\beta_k^2}{2s_k}\right) I(\beta_k \neq 0) + \pi_0 \delta_0(\beta_k) \right] \cdot$$

$$\times\ \pi_0^{\,c-1} (1-\pi_0)^{d-1}$$

对 $\beta_k = 0$, $[\pi_0 \mid \cdot\,] \propto \pi_0^{\,c-1} (1-\pi_0)^{d-1} \pi_0^{\,p} = \pi_0^{\,p+c-1} (1-\pi_0)^{d-1}$

$$\sim \text{Beta}(p+c,\ d)$$

对 $\beta_k \neq 0$, $[\pi_0 \mid \cdot\,] \propto (1-\pi_0)^{p} \pi_0^{\,(c-1)} (1-\pi_0)^{d-1} = \pi_0^{\,c-1} (1-\pi_0)^{p+d-1}$

$$\sim \text{Beta}(c,\ p+d)$$

$$z_k = \begin{cases} 1, & \text{对 } \beta_k \neq 0 \\ 0, & \text{对 } \beta_k = 0 \end{cases}$$

$$[\pi_0 \mid \cdot\,] \sim \text{Beta}\left(p+c-\sum_{k=1}^{p} z_k,\ d+\sum_{k=1}^{p} z_k\right)$$

对 $\beta_k = 0$.

$$[s_k \mid \cdot\,] \propto \frac{1}{\sqrt{s_k}} \exp\left(-\frac{1}{2}\eta^2 s_k\right) = s_k^{-\frac{1}{2}} e^{-\frac{1}{2}\eta^2 s_k}$$

$$= s_k^{\frac{1}{2}-1} e^{-\frac{1}{2}\eta^2 s_k}$$

$$\sim \text{gamma}\left(\frac{1}{2},\ \frac{1}{2}\eta^2\right)$$

Scanned with CamScanner