LAD - BLSS
marginal

$$\underset{n\times 1}{Y} = \underset{n\times q_1}{C}\cdot \underset{q_1\times 1}{b} + \underset{n\times q_2}{E}\cdot \underset{q_2\times 1}{\alpha} + \overset{n\times 1}{X}\overset{1\times 1}{\beta} + \underset{n\times q_2}{W}\underset{q_2\times 1}{\eta} + \underset{n\times 1}{\varepsilon}$$

$$y_i = C_i\cdot b + E_i\cdot\alpha + X_i\beta + W_i\eta + \xi_2\tau^{-\frac{1}{2}}\sqrt{\tilde{v}_i}\, z_i\,, \quad i=1,\cdots,n$$

$$\tilde{v}\mid\tau \sim \prod_{i=1}^{n}\tau\exp(-\tau\tilde{v}_i)\,, \quad \tilde{v}_i \sim \exp(\tau)$$

$\alpha, b, \tilde{v}_r, \beta, \eta, S_1, S_2, \tau, \tilde{\eta}_1^2, \tilde{\eta}_2^2, \pi_1, \pi_2$

$$z \sim \prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}z_i^2\right), \quad z_i \sim N(0,1)$$

$$\beta\mid S_1 \sim \frac{1}{\sqrt{2\pi S_1}}\exp\left(-\frac{\beta^2}{2S_1}\right), \quad N(0, S_1)$$

$$S_1\mid\tilde{\eta}_1^2 \sim \frac{\tilde{\eta}_1^2}{2}\exp\left(-\frac{\tilde{\eta}_1^2}{2}S_1\right)$$

$$\eta_k\mid S_{2k} \sim \frac{1}{\sqrt{2\pi S_{2k}}}\exp\left(-\frac{\eta_k^2}{2S_{2k}}\right)$$

$$S_{2k}\mid\tilde{\eta}_2^2 \sim \frac{\tilde{\eta}_2^2}{2}\exp\left(-\frac{\tilde{\eta}_2^2}{2}S_{2k}\right)$$

$$\tau \sim \text{gamma}(a, b)$$

$$\tilde{\eta}_1^2 \sim \text{gamma}(c_1, d_1)\,, \quad \tilde{\eta}_2^2 \sim \text{gamma}(c_2, d_2)$$

$$\alpha \sim \prod_{k=1}^{q_2}\frac{1}{\sqrt{2\pi\alpha_0}}\exp\left(-\frac{\alpha_k^2}{2\alpha_0}\right)$$

$$b \sim \prod_{i=1}^{q_1}\frac{1}{\sqrt{2\pi b_0}}\exp\left(-\frac{b_i^2}{2b_0}\right)$$

$$\beta\mid S_1, \pi_1 \sim (1-\pi_1)\,N(0, S_1) + \pi_1\,\delta_0(\beta)$$

$$\eta_k\mid S_{2k}, \pi_2 \sim (1-\pi_2)\,N(0, S_{2k}) + \pi_2\,\delta_0(\eta_k)\,, \quad k=1,\cdots q_2$$

$$\pi_1 \sim \text{Beta}(r_1, u_1)\,, \quad \pi_2 \sim \text{Beta}(r_2, u_2)$$

$$f(y|-) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\tau^{-1}\xi_2^2 \tilde{v}_r}} \exp\left\{-\frac{(y_i - C_i \cdot b - E_i \cdot \alpha - x_i\beta - w_i\eta)^2}{2\tau^{-1}\xi_2^2 \tilde{v}_r}\right\}$$

P2

$$= \exp\left\{-\frac{1}{2}\sum_{i=1}^{n}\frac{(y_i - C_i b - E_i\alpha - x_i\beta - w_i\eta)^2}{\tau^{-1}\xi_2^2 \tilde{v}_r}\right\} \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\tau^{-1}\xi_2^2 \tilde{v}_r}}$$

- $$b_j|- \propto \exp\left\{-\frac{1}{2}\sum_{i=1}^{n}\frac{(y_i - C_i b - E_i\alpha - x_i\beta - w_i\eta)^2}{\tau^{-1}\xi_2^2 \tilde{v}_r}\right\} \exp\left(-\frac{b_j^2}{2b_0}\right), \quad j=1,\cdots,q_1$$

$$\propto \exp\left\{-\frac{1}{2}\left[\left(\sum_{i=1}^{n}\frac{\tau C_{ij}^2}{\xi_2^2 \tilde{v}_r} + \frac{1}{b_0}\right) b_j^2 - 2\sum_{i=1}^{n}\frac{\tau \tilde{y}_{ij} C_{ij}}{\xi_2^2 \tilde{v}_r} b_j\right]\right\}$$

$$\tilde{y}_{ij} = y_i - E_i\alpha - x_i\beta - w_i\eta - \sum_{j'=1,\, j\neq j'}^{q_1} \cancel{x_{ij'}\beta_{j'}}\ C_{ij'} b_{j'}$$

$$\sigma_j^2 = \left(\sum_{i=1}^{n}\frac{\tau C_{ij}^2}{\xi_2^2 \tilde{v}_r} + \frac{1}{b_0}\right)^{-1}, \quad \mu_j = \left(\sum_{i=1}^{n}\frac{\tau \tilde{y}_{ij} C_{ij}}{\xi_2^2 \tilde{v}_r}\right)\cdot \sigma_j^2$$

$\sim N(\mu_j, \sigma_j^2)$ same as LAD-BL

- $$\alpha_k|- \propto \exp\left\{-\frac{1}{2}\sum_{i=1}^{n}\frac{(y_i - C_i b - E_i\alpha - x_i\beta - w_i\eta)^2}{\tau^{-1}\xi_2^2 \tilde{v}_r}\right\} \exp\left(-\frac{\alpha_k^2}{\alpha_0}\right), \quad k=1,\cdots q_2$$

$$\propto \exp\left\{-\frac{1}{2}\left[\left(\sum_{i=1}^{n}\frac{\tau E_{ik}^2}{\xi_2^2 \tilde{v}_r} + \frac{1}{\alpha_0}\right)\alpha_k^2 - 2\sum_{i=1}^{n}\frac{\tau \tilde{y}_{ik} E_{ik}}{\xi_2^2 \tilde{v}_r}\alpha_k\right]\right\}$$

$$\tilde{y}_{ik} = y_i - C_i b - x_i\beta - w_i\eta - \sum_{j=1, j\neq k}^{q_2} E_{ij}\alpha_j$$

$$\sigma_k^2 = \left(\sum_{i=1}^{n}\frac{\tau E_{ik}^2}{\xi_2^2 \tilde{v}_r} + \frac{1}{\alpha_0}\right)^{-1}, \quad \mu_k = \left(\sum_{i=1}^{n}\frac{\tau \tilde{y}_{ik} E_{ik}}{\xi_2^2 \tilde{v}_r}\right)\cdot \sigma_k^2$$

$\sim N(\mu_k, \sigma_k^2)$ same as LAD-BL

- $\tilde{v}_i|- \propto f(y|-)\,\pi(\tilde{v}_r|\tau)$ same as LAD-BL

$$\propto \frac{1}{\sqrt{\tilde{v}_r}} \exp\left\{-\frac{(y_i - C_i b - E_i\alpha - x_i\beta - w_i\eta)^2}{2\tau^{-1}\xi_2^2 \tilde{v}_r}\right\} \exp(-\tau\tilde{v}_r)$$

$$\propto \frac{1}{\sqrt{\tilde{v}_r}} \exp\left\{-\frac{1}{2}\left[(2\tau)\tilde{v}_r + \frac{\tau(y_i - C_i b - E_i\alpha - x_i\beta - w_i\eta)^2}{\xi_2^2}\cdot v_i^{-1}\right]\right\}$$

$$\frac{1}{\tilde{v}_r} \sim \text{inv. Gaussian}\left(\mu = \sqrt{\frac{2\tau\cdot\xi_2^2}{\tau(y_i - C_i b - E_i\alpha - x_i\beta - w_i\eta)^2}},\ \lambda = 2\tau\right)$$

$$\bullet\ \beta \mid - \propto \exp\left\{-\frac{1}{2}\sum_{i=1}^{n}\frac{(y_i - X_i\beta - C_i b - E_i\alpha - W_i\eta)^2}{\tau^{-1}\xi_2^2\tilde{v}_i}\right\} \times$$

$$\left\{(1-\pi_1)\frac{1}{\sqrt{2\pi s_1}}\exp\left(-\frac{\beta^2}{2s_1}\right) I(\beta \neq 0) + \pi_1\delta_0(\beta)\right\}$$

$$\tilde{y}_i = y_i - C_i b - E_i\alpha - W_i\eta$$

$$\mu = \left(\sum_{i=1}^{n}\frac{\tau\tilde{y}_i X_i}{\xi_2^2\tilde{v}_i}\right)\cdot\sigma^2, \qquad \sigma^2 = \left(\sum_{i=1}^{n}\frac{\tau X_i^2}{\xi_2^2\tilde{v}_i} + \frac{1}{s_1}\right)^{-1}$$

$$l_1 = \frac{\pi_1}{\pi_1 + (1-\pi_1)s_1^{-\frac{1}{2}}(\sigma^2)^{\frac{1}{2}}\exp\left(\frac{1}{2}\left(\sum_{i=1}^{n}\frac{\tau\tilde{y}_i X_i}{\xi_2^2\tilde{v}_i}\right)^2\cdot\sigma^2\right)}$$

$$\sim (1-l_1)N(\mu,\sigma^2) + l_1\delta_0(\beta)$$

$k=1,\cdots,q_2$

$$\bullet\ \eta_k \mid - \propto \exp\left\{-\frac{1}{2}\sum_{i=1}^{n}\frac{(y_i - X_i\beta - C_i b - E_i\alpha - W_i\eta)^2}{\tau^{-1}\xi_2^2\tilde{v}_i}\right\} \times$$

$$\left\{(1-\pi_2)\frac{1}{\sqrt{2\pi s_{2k}}}\exp\left(-\frac{\eta_k^2}{2s_{2k}}\right) I(\eta_k \neq 0) + \pi_2\delta_0(\eta_k)\right\}$$

$$\tilde{y}_{ik} = y_i - X_i\beta - C_i b - E_i\alpha - \sum_{j=1, j\neq k}^{q_2} W_{ij}\eta_j$$

$$\mu_k = \left(\sum_{i=1}^{n}\frac{\tau\tilde{y}_{ik}W_{ik}}{\xi_2^2\tilde{v}_i}\right)\cdot\sigma_k^2, \qquad \sigma_k^2 = \left(\sum_{i=1}^{n}\frac{\tau W_{ik}^2}{\xi_2^2\tilde{v}_i} + \frac{1}{s_{2k}}\right)^{-1}$$

$$l_{2k} = \frac{\pi_2}{\pi_2 + (1-\pi_2)s_{2k}^{-\frac{1}{2}}(\sigma_k^2)^{\frac{1}{2}}\exp\left(\frac{1}{2}\left(\sum_{i=1}^{n}\frac{\tau\tilde{y}_{ik}W_{ik}}{\xi_2^2\tilde{v}_i}\right)^2\cdot\sigma_k^2\right)}$$

$$\sim (1-l_{2k})N(\mu_k,\sigma_k^2) + l_{2k}\delta_0(\eta_k)$$

• $[S_1 | -]$

if $\beta = 0$, $[S_1 | -] \propto \exp\left(-\frac{\tilde{\eta}_1^2}{2} S_1\right) \quad \sim \exp\left(\frac{\tilde{\eta}_1^2}{2}\right)$

if $\beta \neq 0$, $[S_1 | -] \propto \frac{1}{\sqrt{2\pi S_1}} \exp\left(-\frac{\beta^2}{2S_1}\right) \exp\left(-\frac{\tilde{\eta}_1^2}{2} S_1\right)$

$$\propto \frac{1}{\sqrt{S_1}} \exp\left\{-\frac{1}{2}\left[\tilde{\eta}_1^2 S_1 + \beta^2 \cdot S_1^{-1}\right]\right\}$$

$\frac{1}{S_1} \sim$ inv. gaussian $\left(\mu = \sqrt{\frac{\tilde{\eta}_1^2}{\beta^2}},\ \lambda = \tilde{\eta}_1^2\right)$

$k = 1, \cdots, q_2$

• $[S_{2k} | -]$

if $\eta_k = 0$, $[S_{2k} | -] \propto \exp\left(-\frac{\tilde{\eta}_2^2}{2} S_{2k}\right) \sim \exp\left(\frac{\tilde{\eta}_2^2}{2}\right)$

if $\eta_k \neq 0$, $[S_{2k} | -] \propto \frac{1}{\sqrt{2\pi S_{2k}}} \exp\left(-\frac{\eta_k^2}{2S_{2k}}\right) \cdot \exp\left(-\frac{\tilde{\eta}_2^2}{2} S_{2k}\right)$

$$\propto \frac{1}{\sqrt{S_{2k}}} \exp\left\{-\frac{1}{2}\left[\tilde{\eta}_2^2 S_{2k} + \eta_k^2 \cdot S_{2k}^{-1}\right]\right\}$$

$\frac{1}{S_{2k}} \sim$ inv. Gaussian $\left(\mu = \sqrt{\frac{\tilde{\eta}_2^2}{\eta_k^2}},\ \lambda = \tilde{\eta}_2^2\right)$

• $f(\tau | -) \propto f(y | -)\, \pi(\tilde{v} | \tau)\, \pi(\tau)$

$$\propto \tau^{\frac{n}{2}} \exp\left\{-\frac{1}{2} \sum_{i=1}^{n} \frac{(y_i - C_i b - E_i \alpha - x_i \beta - W_i \eta)^2}{\tau^{-1} \xi_2^2 \tilde{v}_r}\right\} \cdot \tau^n \exp\left(-\tau \sum_{i=1}^{n} \tilde{v}_r\right)$$

$$\cdot\, \tau^{a-1} \exp(-b\tau)$$

$$\propto \tau^{a + \frac{3}{2}n - 1} \exp\left\{-\tau\left[\underbrace{\sum_{i=1}^{n}\left(\frac{(y_i - C_i b - E_i \alpha - x_i \beta - W_i \eta)^2}{2 \xi_2^2 \tilde{v}_r} + \tilde{v}_r\right) + b}\right]\right\}$$

$\sim$ gamma $\left(a + \frac{3}{2}n,\ \quad\right)$ same as LAD-BL

- $f(\tilde{\eta}_1^2 \mid -) \propto \pi(S_1 \mid \tilde{\eta}_1^2)\, \pi(\tilde{\eta}_1^2)$

$$\propto \frac{\tilde{\eta}_1^2}{2} \exp\left(-\frac{\tilde{\eta}_1^2 S_1}{2}\right) (\tilde{\eta}_1^2)^{c_1-1} \exp(-d_1 \tilde{\eta}_1^2) \qquad \text{same LAD-BL}$$

$$\propto (\tilde{\eta}_1^2)^{1+c_1-1} \exp\left(-\tilde{\eta}_1^2 \left(\frac{S_1}{2} + d_1\right)\right) \sim \text{gamma}\left(1+c_1, \frac{S_1}{2} + d_1\right)$$

- $f(\tilde{\eta}_2^2 \mid -) \propto \pi(S_2 \mid \tilde{\eta}_2^2)\, \pi(\tilde{\eta}_2^2)$

$$\propto \prod_{k=1}^{q_2} \frac{\tilde{\eta}_2^2}{2} \exp\left(-\frac{\tilde{\eta}_2^2}{2} S_{2k}\right) (\tilde{\eta}_2^2)^{c_2-1} \exp(-d_2 \tilde{\eta}_2^2)$$

$$\propto (\tilde{\eta}_2^2)^{q_2+c_2-1} \exp\left(-\tilde{\eta}_2^2 \left(\sum_{k=1}^{q_2} \frac{S_{2k}}{2} + d_2\right)\right)$$

$$\sim \text{gamma}\left(q_2 + c_2 - 1, \sum_{k=1}^{q_2} \frac{S_{2k}}{2} + d_2\right) \qquad \text{same LAD-BL}$$

- $[\pi_1 \mid -] \propto \left[(1-\pi_1) \frac{1}{\sqrt{2\pi S_1}} \exp\left(-\frac{\beta^2}{2S_1}\right) I(\beta \neq 0) + \pi_1 \delta_0(\beta)\right] \times$

$$\pi_1^{r_1-1} (1-\pi_1)^{u_1-1} \qquad \sim \text{Beta}(1 + r_1 - Z_1, u_1 + Z_1)$$

let $Z_1 = \begin{cases} 1, & \text{if } \beta \neq 0 \\ 0, & \text{if } \beta = 0. \end{cases}$ same as BL-SS

- $[\pi_2 \mid -] \propto \prod_{k=1}^{q_2} \left[(1-\pi_2) \frac{1}{\sqrt{2\pi S_{2k}}} \exp\left(-\frac{\eta_k^2}{2 S_{2k}}\right) I(\eta_k \neq 0) + \pi_2 \delta_0(\eta_k)\right]$

$$\times\ \pi_2^{r_2-1} (1-\pi_2)^{u_2-1}$$

let $Z_{2k} = \begin{cases} 1, & \text{if } \eta_k \neq 0 \\ 0, & \text{if } \eta_k = 0 \end{cases}$ same as BL-SS

$$\sim \text{Beta}\left(q_2 + r_2 - \sum_{k=1}^{q_2} Z_{2k},\ u_2 + \sum_{k=1}^{q_2} Z_{2k}\right)$$