

data generating model:

$$Y = Cb + E\alpha + G\beta + W\eta + \varepsilon$$

C: clinical factor

$C = (C_1, \dots, C_{q_1})$ ,  $b = (b_1, \dots, b_{q_1})^T$ ,  $q_1=3$ ,  $b$  is generated from unif (1, 2.2)

E: environmental factor

$E = (E_1, \dots, E_{q_2})$ ,  $\alpha = (\alpha_1, \dots, \alpha_{q_2})^T$ ,  $q_2=4$ ,  $\alpha$  is generated from unif (1.2, 2.5)

E have 2 continuous variables and 2 discrete variables.

G: genes

$G = (G_1, \dots, G_P)$ ,  $\beta = (\beta_1, \dots, \beta_P)^T$ ,  $P=100$ , the nonzero  $(\beta_1, \dots, \beta_8)$  is generated from unif(1, 2.5) and other  $\beta$  is 0.

#nonzero: 8

W: GxE interactions

$W = (G_1 \times E_1, \dots, G_1 \times E_{q_2}, \dots, G_p \times E_1, \dots, G_p \times E_{q_2}, \dots)$ ,  $\eta = (\eta_1, \dots, \eta_{PXq_2})^T$

the nonzero  $\eta_1, \eta_{10}, \eta_{19}, \eta_{32}, \eta_{57}, \eta_{70}, \eta_{96}, \eta_{97}, \eta_{138}, \eta_{144}, \eta_{157}, \eta_{170}$  are generated from unif(1.8, 2.5) and other  $\eta$  is 0.

#nonzero: 12

Estimate the coefficients of  $\beta$  and  $\eta$  with marginal model:

$$Y = Cb + E\alpha + X\beta + W'\eta' + \varepsilon$$

$X = G_j$ ,  $W' = (X \times E_1, \dots, X \times E_{q_2})$ ,  $\eta' = (\eta'_1, \dots, \eta'_{q_2})^T$

## Simulation Results

n=300, p=200, error distribution: N(0,1)

### Bayesian Lasso

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	8	12	7	41.53
sd	0	2.86	1.64	9.77

### Bayesian Lasso Spike and Slab

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	8	3	6.47	13.9
sd	0	1.46	1.83	5.46

### LAD Bayesian Lasso

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	8	8.1	5.9	17.97
sd	0	4.05	1.65	6.9

### LAD Bayesian Lasso spike and slab

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	8	37.37	10.67	181.27
sd	0	8.52	1.09	38.04

n=300, p=200, error distribution: t(2)

#### Bayesian Lasso

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	8	11.9	6.9	42.1
sd	0	3.145	1.73	10.79

#### Bayesian Lasso Spike and Slab

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	8	3.4	6.4	12.6
sd	0	1.77	6.4	1.072

#### LAD Bayesian Lasso

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	8	8.5	6.4	17.13
sd	0	3.5	1.75	5.96

#### LAD Bayesian Lasso spike and slab

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	8	38.47	10.53	186.53
sd	0	8.85	1.43	35.73

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E: environmental factor

$E = (E_1, \dots, E_{q_2})$ ,  $\alpha = (\alpha_1, \dots, \alpha_{q_2})^T$ ,  $q_2=4$ ,  $\alpha$  is generated from unif (1.2, 2.5)

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$G = (G_1, \dots, G_P)$ ,  $\beta = (\beta_1, \dots, \beta_P)^T$ ,  $P=100$ , the nonzero  $(\beta_1, \dots, \beta_8)$  is generated from unif(1, 2.5) and other  $\beta$  is 0.

#nonzero: 8

W: GxE interactions

$W = (G_1 \times E_1, \dots, G_1 \times E_{q_2}, \dots, G_p \times E_1, \dots, G_p \times E_{q_2})$ ,  $\eta = (\eta_1, \dots, \eta_{PXq_2})^T$

the nonzero  $(\eta_1, \eta_2, \eta_3), \eta_8, (\eta_9, \eta_{10}), \eta_{16}, (\eta_{17}, \eta_{18}), \eta_{24}, (\eta_{25}, \eta_{26})$  are generated from unif(1.8, 2.5) and other  $\eta$  is 0.

#nonzero: 12

Estimate the coefficients of  $\beta$  and  $\eta$  with marginal model:

$$Y = Cb + E\alpha + X\beta + W'\eta' + \varepsilon$$

$X = G_j$ ,  $W' = (X \times E_1, \dots, X \times E_{q_2})$ ,  $\eta' = (\eta'_1, \dots, \eta'_{q_2})^T$

## Simulation Results

n=300, p=200, error distribution:  $N(0,1)$

### Bayesian Lasso (95% confidence interval)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	8	9.36	11.17	50.7
sd	0	3.71	0.98	11.16

### Bayesian Lasso Spike and Slab (MPM)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	7.96	1.83	11.7	24.2
sd	0.18	1.15	0.467	6.58

### LAD Bayesian Lasso (95% confidence interval)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	8	5.57	11.7	19.46
sd	0	2.67	0.53	5.21

### LAD Bayesian Lasso spike and slab (MPM)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	8	20.46	12	102.63
sd	0	5.41	0	23.25

## Simulation Results

n=300, p=200, error distribution: t(2)

### Bayesian Lasso (95% confidence interval)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	7.5	8.03	11	53.67
sd	0.73	3.03	1.08	12.32

### Bayesian Lasso Spike and Slab (MPM)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	7.17	1.33	11.13	15.37
sd	1.05	1.12	0.97	5.013

### LAD Bayesian Lasso (95% confidence interval)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	7.43	5.07	11.17	23.47
sd	0.77	3.17	0.79	7

### LAD Bayesian Lasso spike and slab (MPM)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	7.9	20.1	11.97	104.37
sd	0.25	5.58	0.18	26.6

null model

data generating model:

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G: genes

$G = (G_1, \dots, G_P)$ ,  $\beta = (\beta_1, \dots, \beta_P)^T$ ,  $P=100$ , all  $\beta$  are 0.

#nonzero: 0

W: GxE interactions

$W = (G_1 \times E_1, \dots, G_1 \times E_{q_2}, \dots, G_P \times E_1, \dots, G_P \times E_{q_2})$ ,  $\eta = (\eta_1, \dots, \eta_{P \times q_2})^T$

the nonzero  $(\eta_1, \eta_2, \eta_3), \eta_8, (\eta_9, \eta_{10}), \eta_{16}, (\eta_{17}, \eta_{18}), \eta_{24}, (\eta_{25}, \eta_{26})$  are generated from unif(1.8, 2.5). All  $\eta$  are 0.

#nonzero: 0

Estimate the coefficients of  $\beta$  and  $\eta$  with marginal model:

$$Y = Cb + E\alpha + X\beta + W'\eta' + \varepsilon$$

$X = G_j$ ,  $W' = (X \times E_1, \dots, X \times E_{q_2})$ ,  $\eta' = (\eta'_1, \dots, \eta'_{q_2})^T$

## Simulation Results

n=300, p=200, error distribution:  $N(0,1)$

### Bayesian Lasso (95% confidence interval)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	0	8.67	0	31.6
sd	0	2.35	0	8.93

### Bayesian Lasso Spike and Slab (MPM)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	0	0.8	0	2.5
sd	0	0.85	0	2.19

### LAD Bayesian Lasso (95% confidence interval)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	0	14.4	0	51.27
sd	0	4.06	0	9.77

### LAD Bayesian Lasso spike and slab (MPM)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	0	2.3	0	7.27
sd	0	1.67	0	3.73



## Simulation Results

n=300, p=200, error distribution: t(2)

### Bayesian Lasso (95% confidence interval)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	0	8.9	0	44.27
sd	0	4.24	0	29.76

### Bayesian Lasso Spike and Slab (MPM)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	0	1.57	0	11.03
sd	0	1.96	0	12.98

### LAD Bayesian Lasso (95% confidence interval)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	0	4.93	0	15.83
sd	0	3.04	0	8.55

### LAD Bayesian Lasso spike and slab (MPM)

	TP(main)	FP(main)	TP(interaction)	FP(interaction)
mean	0	0.67	0	1.8
sd	0	0.88	0	2.31