

$$y_i = \underset{1 \times 1}{x_i^T} \underset{1 \times 1}{\beta} + \underset{1 \times p}{\xi_2} \underset{1 \times p}{\tau^{-\frac{1}{2}} \sqrt{\tilde{v}_i}} \underset{1 \times 1}{z_i}$$

$$\tilde{v} | \tau \sim \prod_{i=1}^n \tau \exp(-\tau \tilde{v}_i) \quad \tilde{v} = (\tilde{v}_1, \dots, \tilde{v}_n)$$

$$z \sim \prod_{i=1}^n \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2} z_i^2) \quad z = (z_1, \dots, z_n)$$

$$\beta_k | S_k \sim \frac{1}{\sqrt{2\pi S_k}} \exp(-\frac{\beta_k^2}{2 S_k}) \quad N(0, S_k), \quad k=1, \dots, p$$

$$S_k | \eta^2 \sim \frac{\eta^2}{2} \exp(-\frac{\eta^2}{2} S_k)$$

$$\tau \sim \text{gamma}(a, b), \quad \eta^2 \sim \text{gamma}(c, d)$$

$$\tau, \eta^2 \sim \tau^{a-1} \exp(-b\tau) (\eta^2)^{c-1} \exp(-d\eta^2)$$

Spike and slab:

$$\beta_k | S_k \sim (1 - \pi_0) N(0, S_k) + \pi_0 \delta_0(\beta_k), \quad k=1, \dots, p$$

$$f(y | -) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi \tau^{-1} \xi_2^2 \tilde{v}_i}} \exp \left\{ -\frac{(y_i - x_i^T \beta - \xi_1 \tilde{v}_i)^2}{2 \tau^{-1} \xi_2^2 \tilde{v}_i} \right\}, \quad \xi_1 = 0$$

$$[\beta_k | -] \propto \exp \left\{ -\frac{1}{2} \sum_{i=1}^n \frac{(y_i - x_i^T \beta)^2}{\tau^{-1} \xi_2^2 \tilde{v}_i} \right\} \times$$

$$\left\{ (1 - \pi_0) \frac{1}{\sqrt{2\pi S_k}} \exp(-\frac{\beta_k^2}{2 S_k}) I(\beta_k \neq 0) + \pi_0 \delta_0(\beta_k) \right\} \\ \propto \exp \left\{ -\sum_{i=1}^n \frac{1}{2 \tau^{-1} \xi_2^2 \tilde{v}_i} (y_i - x_{ik} \beta_k - \sum_{j=1, j \neq k}^p x_{ij} \beta_j)^2 \right\} \times$$

$$\left\{ (1 - \pi_0) \frac{1}{\sqrt{2\pi S_k}} \exp(-\frac{\beta_k^2}{2 S_k}) I(\beta_k \neq 0) + \pi_0 \delta_0(\beta_k) \right\}$$

$$\text{let } \tilde{y}_{ik} = y_i - \sum_{j=1, j \neq k}^p x_{ij} \beta_j$$

$$\propto \exp \left\{ -\sum_{i=1}^n \frac{1}{2\tau^{-1} \sigma_k^2 \tilde{v}_i} (x_{ik}^2 \beta_k^2 - 2x_{ik} \beta_k \cdot \tilde{y}_{ik} + \tilde{y}_{ik}^2) \right\} \times$$

$$\left\{ (1-\pi_0) \frac{1}{\sqrt{2\pi\sigma_k}} \exp\left(-\frac{\beta_k^2}{2\sigma_k}\right) I(\beta_k \neq 0) + \pi_0 \delta_0(\beta_k) \right\}$$

$$\propto (1-\pi_0) \frac{1}{\sqrt{2\pi\sigma_k}} \exp \left\{ -\frac{1}{2} \left[ \left( \sum_{i=1}^n \frac{\tau x_{ik}^2}{\sigma_k^2 \tilde{v}_i} + \frac{1}{\sigma_k} \right) \beta_k^2 - 2 \sum_{i=1}^n \frac{\tau \tilde{y}_{ik} x_{ik}}{\sigma_k^2 \tilde{v}_i} \beta_k \right. \right.$$

$$\left. + \sum_{i=1}^n \frac{\tau}{\sigma_k^2 \tilde{v}_i} \cdot \tilde{y}_{ik}^2 \right\} I(\beta_k \neq 0) +$$

$$\pi_0 \exp \left\{ -\sum_{i=1}^n \frac{\tau}{2\sigma_k^2 \tilde{v}_i} \tilde{y}_{ik}^2 \right\} \delta_0(\beta_k)$$

$$\propto (1-\pi_0) \frac{1}{\sqrt{2\pi\sigma_k}} \exp \left\{ -\frac{1}{2} \left[ \left( \sum_{i=1}^n \frac{\tau x_{ik}^2}{\sigma_k^2 \tilde{v}_i} + \frac{1}{\sigma_k} \right) \beta_k^2 - 2 \sum_{i=1}^n \frac{\tau \tilde{y}_{ik} x_{ik}}{\sigma_k^2 \tilde{v}_i} \beta_k \right] \right\}$$

$$\cdot \exp \left\{ -\sum_{i=1}^n \frac{\tau}{2\sigma_k^2 \tilde{v}_i} \tilde{y}_{ik}^2 \right\} \cdot I(\beta_k \neq 0) + \pi_0 \delta_0(\beta_k) \cdot \exp \left\{ -\sum_{i=1}^n \frac{\tau}{2\sigma_k^2 \tilde{v}_i} \tilde{y}_{ik}^2 \right\}$$

$$\propto (1-\pi_0) \frac{1}{\sqrt{2\pi\sigma_k}} \exp \left\{ -\frac{1}{2} \left[ \left( \sum_{i=1}^n \frac{\tau x_{ik}^2}{\sigma_k^2 \tilde{v}_i} + \frac{1}{\sigma_k} \right) \beta_k^2 - 2 \sum_{i=1}^n \frac{\tau \tilde{y}_{ik} x_{ik}}{\sigma_k^2 \tilde{v}_i} \beta_k \right] \right\}$$

$$+ \pi_0 \delta_0(\beta_k)$$

$$\sigma_k^2 = \left( \sum_{i=1}^n \frac{\tau x_{ik}^2}{\sigma_k^2 \tilde{v}_i} + \frac{1}{\sigma_k} \right)^{-1}, \quad \tilde{\mu}_k = \frac{\sum_{i=1}^n \frac{\tau \tilde{y}_{ik} x_{ik}}{\sigma_k^2 \tilde{v}_i}}{\sum_{i=1}^n \frac{\tau x_{ik}^2}{\sigma_k^2 \tilde{v}_i} + \frac{1}{\sigma_k}}$$

$$\propto (1-\pi_0) \sigma_k^{-\frac{1}{2}} \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp \left( -\frac{1}{2\sigma_k^2} (\beta_k - \tilde{\mu}_k)^2 \right) \cdot (\sigma_k^2)^{\frac{1}{2}} \cdot N(\tilde{\mu}_k, \sigma_k^2)$$

$$\exp \left( \frac{1}{2} \left( \sum_{i=1}^n \frac{\tau \tilde{y}_{ik} x_{ik}}{\sigma_k^2 \tilde{v}_i} \right)^2 \cdot \sigma_k^2 \right) \rightarrow \left( \exp \left( +\frac{1}{2\sigma_k^2} \cdot \tilde{\mu}_k^2 \right) \right)$$

$$+ \pi_0 \delta_0(\beta_k)$$

$$\propto (1-\pi_0) S_k^{-\frac{1}{2}} \cdot (\sigma_k^2)^{\frac{1}{2}} \cdot \exp \left( -\frac{1}{2} \left( \sum_{i=1}^n \frac{\tau \tilde{y}_{ik} x_{ik}}{\sum_{i=1}^n \tilde{u}_i} \right)^2 \cdot \sigma_k^2 \right) \cdot N(\tilde{\mu}_k, \sigma_k^2) + \pi_0 \delta_0(\beta_k)$$

$$l_k = P(\beta_k = 0 \mid \text{rest})$$

$$= \frac{\pi_0}{\pi_0 + (1-\pi_0) S_k^{-\frac{1}{2}} (\sigma_k^2)^{\frac{1}{2}} \exp \left( -\frac{1}{2} \left( \sum_{i=1}^n \frac{\tau \tilde{y}_{ik} x_{ik}}{\sum_{i=1}^n \tilde{u}_i} \right)^2 \cdot \sigma_k^2 \right)}$$

$$\tilde{y}_{ik} = y_i - \sum_{j=1, j \neq k}^p x_{ij} \beta_j$$

$$\sim (1-l_k) N(\tilde{\mu}_k, \sigma_k^2) + l_k \delta_0(\beta_k)$$

•  $[S_k | -]$

$$\text{If } \beta_k \neq 0, [S_k | -] \propto \frac{1}{\sqrt{2\pi} \delta_k} \exp \left( -\frac{\beta_k^2}{2\delta_k} \right) \cdot \exp \left( -\frac{\eta^2}{2} S_k \right)$$

$$\frac{1}{S_k} \propto \frac{1}{\sqrt{2\pi} \delta_k} \exp \left\{ -\frac{1}{2} \left[ \eta^2 S_k + \beta_k^2 \cdot S_k^{-1} \right] \right\}$$

$$\sim \text{inv. Gaussian} \left( \mu = \sqrt{\frac{\eta^2}{\beta_k^2}}, \lambda = \eta^2 \right),$$

$$\text{If } \beta_k = 0, [S_k | -] \propto \exp \left( -\frac{\eta^2}{2} S_k \right) \sim \exp \left( -\frac{\eta^2}{2} \right)$$

•  $[\tilde{u}_i | -] \propto \frac{1}{\sqrt{\tilde{u}_i}} \exp \left( -\frac{(y_i - x_i^T \beta_H)^2}{2\tau^{-1} \sum_{i=1}^n \tilde{u}_i} \right) \exp(-\tau \tilde{u}_i)$

$$\propto \frac{1}{\sqrt{\tilde{u}_i}} \exp \left\{ -\frac{1}{2} \left[ 2\tau \tilde{u}_i + \frac{\tau (y_i - x_i^T \beta)^2}{\sum_{i=1}^n \tilde{u}_i} \right] \right\}$$

$$\frac{1}{\tilde{u}_i} \sim \text{inv. Gaussian} \left( \mu = \sqrt{\frac{2\tau \cdot \sum_{i=1}^n \tilde{u}_i}{\tau (y_i - x_i^T \beta)^2}}, \lambda = 2\tau \right)$$

$$\bullet [\tau | -] \propto \tau^{\frac{n}{2}} \exp \left\{ -\frac{1}{2} \sum_{i=1}^n \frac{(y_i - x_i^T \beta)^2}{\tau^{-1} \sum_{j=1}^2 \tilde{v}_r} \right\} \tau^n \exp \left( -\tau \sum_{i=1}^n \tilde{v}_r \right) \cdot \tau^{a-1} \exp(-b\tau)$$

$$\propto \tau^{a + \frac{3n}{2} - 1} \exp \left\{ -\tau \left[ \sum_{i=1}^n \left( \frac{(y_i - x_i^T \beta)^2}{2 \sum_{j=1}^2 \tilde{v}_r} + \tilde{v}_r \right) + b \right] \right\}$$

$$\sim \text{Gamma} \left( a + \frac{3}{2}n, \sum_{i=1}^n \left( \frac{(y_i - x_i^T \beta)^2}{2 \sum_{j=1}^2 \tilde{v}_r} + \tilde{v}_r \right) + b \right)$$

$$\bullet [\eta^2 | -] \propto \prod_{k=1}^p \frac{\eta^2}{2} \exp \left( -\frac{\eta^2}{2} s_k \right) \cdot (\eta^2)^{c-1} \exp(-d\eta^2)$$

$$\propto (\eta^2)^{p+c-1} \exp \left\{ -\eta^2 \left( \sum_{k=1}^p \frac{s_k}{2} + d \right) \right\}$$

$$\bullet [\pi_0 | -] \propto \prod_{k=1}^p \left[ (1-\pi_0) \frac{1}{\sqrt{2\pi\delta_k}} \exp \left( -\frac{\beta_k^2}{2\delta_k} \right) I(\beta_k \neq 0) + \pi_0 \delta_0(\beta_k) \right] \times \pi_0^c (1-\pi_0)^{d-1}$$

$$\nmid \beta_k = 0, [\pi_0 | -] \propto \pi_0^{c-1} (1-\pi_0)^{d-1} \pi_0^p = \pi_0^{p+c-1} (1-\pi_0)^{d-1} \sim \text{Beta}(p+c, d)$$

$$\nmid \beta_k \neq 0, [\pi_0 | -] \propto (1-\pi_0)^p \pi_0^{c-1} (1-\pi_0)^{d-1} = \pi_0^{c-1} (1-\pi_0)^{p+d-1} \sim \text{Beta}(c, p+d)$$

$$\text{let } z_k = \begin{cases} 1, & \nmid \beta_k \neq 0 \\ 0, & \nmid \beta_k = 0 \end{cases}$$

$$[\pi_0 | -] \sim \text{Beta} \left( p+c - \sum_{k=1}^p z_k, d + \sum_{k=1}^p z_k \right)$$