光华管理学院
Guanghua School of Management

Microeconomics

微观经济学

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About the exam: Five Problems

1. Consumer preferences
2. Risky Choices (Expected utility)
3. Perfect Competition
4. Monopoly and Price Discrimination
5. Games: NE

Refresh: Dominance

Strictly dominant 严格占优

Player i's strategy s_i' strictly dominates player i's strategy s_i if $u_i(s_i', s_{-i}) > u_i(s_i, s_{-i})$ for all s_{-i}.

Strictly dominated 严格被占优

Player i's strategy s_i' is strictly dominated by player i's strategy s_i if $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$ for all s_{-i}.

		pair α	pair β
me	α	0,0	3,-1
me	β	-1,3	1,1

Never choose a strictly dominated strategy.

Refresh: Best response

Best response (BR)

1. Player i's strategy $\hat{s}_i$ is a best response to the strategy s_{-i} to other players if $u_i(\hat{s}_i, s_{-i}) \geq u_i(s_i^{'}, s_{-i})$ for all $s_i^{'}$ is S_i.
   - or $\hat{s}_i$ solves $\max_{s_i} u_i(s_i, s_{-i})$.
2. Player $i's$ strategy $\hat{s}_i$ is a best response to the belief p about the other player's choices if $Eu_i(\hat{s}_i, p) \geq Eu_i(s_i^{'}, p)$ for all $s_i^{'}$ is S_i.
   - or $\hat{s}_i$ solves $\max_{s_i} Eu_i(s_i, p)$.

Never choose a strategy that is not best response to any strategy of your opponent.

Refresh: Nash equilibrium

Nash equilibrium

A strategy profile $(s_1^*, s_2^*, ..., s_N^*)$ is a Nash equilibrium (NE) if for each i, her choice s_i^* is a best response to other player's choices s_{-1}^*.

1. No individual can do strictly better by *deviating*, holding others fixed.
2. **self-fulfilling 自我实现 beliefs**： If everyone believes that everyone else is going to playing their part of NE, then everyone will actually play their part of NE.

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NE: applications

An application: Cournot Duopoly

An application: Cournot Duopoly 古诺双寡头

The firms competing in the same market. This situation lies between perfect competition and monopoly.

- **players**: two firms
- **strategies**: quantities of an identical product each of them produces: q_1, q_2.
- **payoffs**:
  - firms aim to maximize profits
  - cost of production is $TC = cq_i$ (constant $MC = c$).
  - price of the market: $p = a - b(q_1 + q_2)$ ($a > 0$ and $b > 0$) and $p > c$.

An application: Cournot Duopoly

As one of the two firms, firm i, how do you determine your quantity?

$$\max_{q_i} \; p(q_i, q_j)q_i - cq_i = [a - b(q_i + q_j)]q_i - cq_i$$

F.O.C.,

$$a - 2bq_i - bq_j - c = 0$$

$$BR_i = q_i^* = \frac{a-c}{2b} - \frac{q_j}{2}$$

Therefore,

$$BR_1 = q_1^* = \frac{a-c}{2b} - \frac{q_2}{2} \text{ and } BR_2 = q_2^* = \frac{a-c}{2b} - \frac{q_1}{2}$$

An application: Cournot Duopoly

As one of the two firms, firm i, how do you determine your quantity?

$$\max_{q_i} \; p(q_i, q_j)q_i - cq_i = [a - b(q_i + q_j)]q_i - cq_i$$

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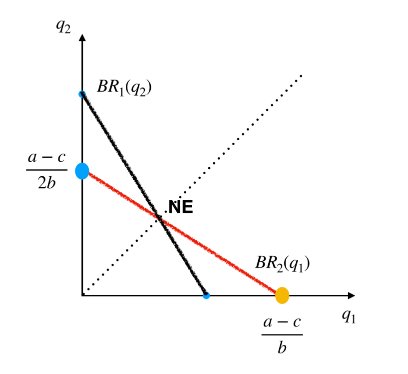
Therefore,

$$BR_1 = q_1^* = \frac{a-c}{2b} - \frac{q_2}{2} \text{ and } BR_2 = q_2^* = \frac{a-c}{2b} - \frac{q_1}{2}$$

- the more likely "I" was to invest, the less "you" want to invest (strategic substitutes **战略性替代**).
- this game is different from the partnership game where the more likely "I" was to invest, the more "you" want to invest (strategic complements **战略性互补**)

An application: Cournot Duopoly 古诺双寡头



NE is when each player is playing the best response against each other('s best response)

$$q_1^* = q_2^* = \frac{a-c}{3b}$$

An application: Cournot Duopoly 古诺双寡头

- Cournot equilibrium output pair (q_1^*, q_2^*) occurs at the interaction of the BR functions, i.e.,

$$(q_1^*, q_2^*) = (\frac{a-c}{3b}, \frac{a-c}{3b})$$

- Aggregate output becomes

$$q^* = q_1^* + q_2^* = \frac{2(a-c)}{3b}$$

which is larger than under monopoly, $q^M = \frac{a-c}{2b}$, but smaller than under perfect competition, $q^C = \frac{a-c}{b}$.

An application: Cournot Duopoly

- The equilibrium price becomes

$$p(q^*) = a - bq^* = a - b[\frac{2(a-c)}{3b}] = \frac{a+2c}{3}$$

which is lower than under monopoly, $p^M = \frac{a+c}{2}$, but higher than under perfect competition, $p^C = c$.

- Finally, the equilibrium profits of every firm i

$$\pi_i^* = p(q^*)q_i^* - cq_i^* = \frac{(a-c)^2}{9b}$$

which are lower than under monopoly, $\pi^M = \frac{(a-c)^2}{4b}$, and higher than under perfect competition, $\pi^C = 0$.

An application: Cournot Duopoly

	Monopoly		Duopoly		Perfect competitive
industry quantity	$\frac{a-c}{2b}$	$<$	$2 \times \frac{(a-c)}{3b}$	$<$	$\frac{a-c}{b}$
industry profit	$\frac{(a-c)^2}{4b}$	$>$	$2 \times \frac{(a-c)^2}{9b}$	$>$	0

Cournot Duopoly vs. Cartel

Apparently, there is space for Pareto improvement for the industry. What if the two firms decide to act as one monopolist and share the profits? Can they reach an agreement to obtain Pareto improvement for the industry?

- They have incentive to deviate.
- What if they actually reached the agreement (Cartel **卡特尔**)? For example, OPEC (**石油输出国组织**) is the most famous Cartel in the world.
    - other firms will enter.
    - it is only possible for special industries, like electricity generation, drug industry, petroleum, cell phone service in China.

Another application: Bertrand competition

Another application: Bertrand competition 伯川德竞争

The firms competing in the same market.

- **players**: two firms producing identical product
- **strategies**: setting prices p_1, p_2, assume $S_i: \ 0 \leq p_i \leq 1$
- **payoffs**:
  - firms aim to maximize profits
  - cost of production is $TC = cq_i$ (constant $MC = c$).
  - total quantity demanded by the market $Q(p) = 1 - p$ where p is the lower of the prices
    - if $p_1 < p_2$, $q_1 = 1 - p_1$
    - if $p_1 > p_2$, $q_1 = 0$
    - if $p_1 = p_2$, $q_1 = \frac{1-p_1}{2}$

Intuition: The firm with lower price gets the whole market. Two firms share the market if the price coincides.

Another application: Bertrand competition

As one of the two firms, how do you determine your price?

$$\max_{p_i} \; p_i q_i(p_i, p_j) - c q_i = [p_i - c] q_i(p_i, p_j)$$

Take the example of firm 1:

- in case $p_2 < c$, it is better for firm 1 to drop out of the market by setting $p_1 > p_2$.
- in case $p_2 > c$, firm 1 will set a price $p_1 < p_2$, occupy the whole market and be a monopolist
  - if $c < p_2 \leq p^M$, then $p_1 = p_2 - \varepsilon$
  - if $c < p_2$ and $p_2 > p^M$, then $p_1 = p^M$
- in case $p_2 = c$, firm 1 will also keep $p_1 = c$ and obtain zero profit, or just drop out off the market by setting $p_1 > c$.

Another application: Bertrand competition

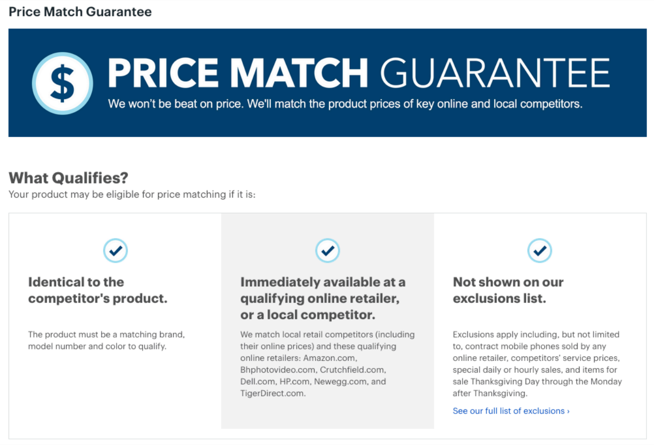
There is a unique **NE**:

$$BR_1 = BR_2 \Rightarrow p_1 = p_2 = c$$

A surprising result: when two firms are competing on prices, the equilibrium is the same as perfect competitive market.

- The same setting as Cournot, but with a different strategy set (a different way of thinking what they are doing), leads to a very different outcome.
  - Cournot: two firms hold the same price
  - Bertrand: two firms undercut each other's price
- Examples of Bertrand competition would be the airlines, cell phone service in most places of the world, etc.

"Price match" is a mechanism that prevents undercutting.



Mixed strategies

Rock, paper, scissors

	R	S	P
R	0, 0	1, -1	-1, 1
S	-1, 1	0, 0	1, -1
P	1, -1	-1, 1	0, 0

There is no NE in "pure strategies", where pure strategies $= \{R, P, S\}$.
To find NE, let's think about how do you usually play this game.

Rock, paper, scissors

	R	S	P
R	0, 0	1, -1	-1, 1
S	-1, 1	0, 0	1, -1
P	1, -1	-1, 1	0, 0

There is no NE in "pure strategies", where pure strategies $= \{R, P, S\}$.
To find NE, let's think about how do you usually play this game.

- with no priors, we assume your opponent will randomize those strategies by $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$
- let's check if you playing $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is the BR for his/her strategy
- actually given your opponent is a randomization machine with $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, whatever you play, your expected payoff is 0.
- but (unfortunately) your opponent can adjust his/her strategy, so only both of you playing $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is NE.

Mixed strategy

Mixed strategy

A mixed strategy p_i is a randomization of $i's$ pure strategies. $p_i(s_i)$ is the probability that p_i assigns to the pure strategy s_i

- $p_i(s_i)$ could be zero, say, $(\frac{1}{2}, \frac{1}{2}, 0)$
- $p_i(s_i)$ could be 1, in this case it is a pure strategy.

Mixed strategy

Payoffs from mixed strategy

The expected payoff of the mixed strategy p_i is a weighted average of the expected payoffs of each of the pure strategies in the mix.

Example:

		player 2	
		a	b
player 1	A	2, 1	0, 0
	B	0, 0	1, 2

Suppose $p_1 = (\frac{1}{5}, \frac{4}{5})$ and $p_2 = (\frac{1}{2}, \frac{1}{2})$. What is p_1's expected payoff?

$$EU_1(p_1, p_2) = \frac{1}{5} \times \frac{1}{2} \times 2 + \frac{4}{5} \times \frac{1}{2} \times 1 = \frac{3}{5}$$

Mixed strategy

Mixed strategy NE

A mixed strategy profile $(p_1^*, p_2^*, ..., p_N^*)$ is a mixed strategy NE if for each player i, p_i^* is a BR to p_{-i}^*.

- if $p_i^*(s_i) > 0$ (a mixed strategy is a BR and pure strategy s_i is played with positive prob.), then s_i^* is also a BR_i to p_{-i}^*.
- think about the "rock paper scissors" game, if $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is a BR against a randomization machine, then playing pure strategies are also BR.
- Intuitively, if mixing (R,S,P) with $p > 0$ is a best response, then any pure strategy should lead to the same expected payoff. Otherwise, you should drop it from your mixing profile.
- this idea will help you in finding NE.

Mixed strategy: an example

player 2

		l	r
player 1	L	50, $\underline{50}$	$\underline{80}$, 20
	R	$\underline{90}$, 10	20, $\underline{80}$

There is no pure-strategy NE. Let's try mixed-strategy NE.

- Assume player 1 is mixing L and R with $(p, 1-p)$ at NE (against player 2's mixed strategy $(q, 1-q)$)), then L and R must have the same payoffs.
  - $50q + 80(1-q) = 90q + 20(1-q) \rightarrow q = 0.6$
- Same idea holds for player 2
  - $50p + 10(1-p) = 20p + 80(1-p) \rightarrow p = 0.7$

So $NE = [(0, 7, 0.3), (0.6, 0.4)]$.

Mixed strategy: an example

		player 2: l	r
player 1	L	50, $\underline{50}$	$\underline{80}$, 20
	R	$\underline{90}$, 10	20, $\underline{80}$

$NE = [(0,7,0.3),(0.6,0.4)]$

- Suppose player 2 tend to go to *l* more than 60% of the time, how should player 1 respond? Always go to R! But then player 2 will go to r all the time...
- Suppose player 2 tend to go to *l* less than 60% of the time, how should player 1 respond? Always go to L! And player 2 will respond accordingly... and player 1 will ...

Mixed strategy: an example

Just for understanding: let's check $[(0,7,0.3),(0.6,0.4)]$ is really NE: no strictly profitable deviation for each player.

		player 2	
		l	r
player 1	L	50, $\underline{50}$	$\underline{80}$, 20
	R	$\underline{90}$, 10	20, $\underline{80}$

- If player 1's strategy $(0,7,0.3)$ is really a BR to player 2's $(0.6,0.4)$?
  - payoff(L)=payoff(R)=payoff $(0,7,0.3)$, so any of the pure strategy is not strictly profitable
  - (check by yourself that the above three strategies yield the same payoff for player 1, given player 2 is playing NE)
  - then any linear combination of L and R yield the same payoff, therefore is not strictly profitable
- For player 2 the same logic applies.

Mixed strategy: tax payer and audit office

players: the tax payer and the audit officer
strategies: the tax payer choose to be honest (H) or cheat (C), and the audit officer choose to audit (A) or not (N).
payoffs:

		tax payer H	tax payer C
audit office	A	2, $\underline{0}$	$\underline{4}$, -10
audit office	N	$\underline{4}$, 0	0, $\underline{4}$

No pure-strategy NE. Mix-strategy NE is $[(\frac{2}{7}, \frac{5}{7}), (\frac{2}{3}, \frac{1}{3})]$.
Interpretation:

- In reality, audit officer is indeed randomizing.
- As a tax payer, you do not toss a coin to randomize. Rather, it is the proportion of tax payers who choose to be honest.

Mixed strategy: tax payer and audit office

How to assure randomness is common knoweldge?

National Institute of Standards and Technology
U.S. Department of Commerce

Some features of a beacon, as defined by the new reference:

- Periodically pulsates randomness (e.g., once a minute).
- Each pulse has a fresh 512-bit random string, cryptographically combining entropy from at least two separate random number generators (RNGs).
- Each pulse is indexed, time-stamped and signed.
- Any past pulse is publicly accessible.
- The sequence of pulses forms a hash chain.
- Far-apart pulses can be efficiently verified via a short chain (skiplist).
- A pre-commitment of local randomness enables securely combining randomness from multiple beacons.

Mixed strategy: tax payer and audit office

As the policy maker, you want to raise the punishment from -10 to -20.

audit office (rows), tax payer (columns)

	H	C
A	2, 0	4, -20
N	4, 0	0, 4

What will happen?

- At NE, tax compliance will not change at all!!
- Mix-strategy NE is $\left[\left(\frac{1}{6}, \frac{5}{6}\right), \left(\frac{2}{3}, \frac{1}{3}\right)\right]$
  - what determines the mix strategy for the column player is the row player's payoff, which we did not change.
- At NE, the audit rate will go down, which is good for the audit officer and the society (because audit is costly).
- Then what to do if we want to change the compliance rate? We need to change the payoff of the auditor!

Mixed strategy: tax payer and audit office

How do we get higher compliance rate?

- surprisingly, punishment harder does not work.
- instead, we have to audit more often (say, by making the cost of auditing smaller, or give an extra incentive to the auditor) .
- or set audit rate higher outside NE. Imagine setting the audit rate to 1, then cheating is not profitable.

Repeated interactions

Repeated games

Repeated games are very common in real life:

- Counrnot competition is repeated over time by the same group of firms
- OPEC cartel is also repeated over time

In addition, players' interaction in a repeated game can help us rationalize cooperation...

Prisoner's dilemma: Revisited

- the threat of future punishment may sometimes provide incentive for good behavior today
- most relationships do not have contract, or mafias
- but most of them are repeated

When can a threat of punishment work, and how they work?

SPE: a new solution concept 子博弈精炼 NE

We want to rule out NE that instruct people to not to play NE in a subgame...

Subgame perfect equilibrium (SPE)

A NE $(s_1^*, s_2^*, ..., s_N^*)$ is a subgame perfect equilibrium (SPE) if it induces a NE in every subgame of the game.

Prisoner's dilemma: Revisited

The NE is (D, D) in a one-shot prisoner's dilemma:

	Cooperate	Defect
Cooperate	2, 2	-1, 3
Defect	3, -1	0, 0

How about playing twice? The second stage NE is (D, D). The payoff is (0, 0).

	Cooperate	Defect
Cooperate	2+0, 2+0	-1+0, 3+0
Defect	3+0, -1+0	0+0, 0+0

The SPE is [(D, D), (D, D)]
It seems that repeating the game won't help.

Prisoner's dilemma: Revisited

Insight: If the state game we face has a unique NE, then there is a unique SPE in the finitely-repeated game in which all players behave as in the stage-game equilibrium during all T rounds of play.

- you cannot convince the other player that you will play a strategy other than NE in any subgame.
- it seems that games with finite rounds will always unraveling from the back

Is there any hope for finite game?

Achieving a better outcome

One-shot game:

		player 2 A	player 2 B	player 2 C
player 1	A	4, 4	0, $\underline{5}$	0, 0
player 1	B	$\underline{5}$, 0	$\underline{1}$, $\underline{1}$	0, 0
player 1	C	0, 0	0, 0	$\underline{3}$, $\underline{3}$

NE: (B, B), (C, C) and mixed-strategy NE.
(A, A) is a Pareto improvement. How can we enforce that?

Achieving a better outcome

How about play it twice?

		player 2		
		A	B	C
player 1	A	4, 4	0, $\underline{5}$	0, 0
	B	$\underline{5}$, 0	$\underline{1}$, $\underline{1}$	0, 0
	C	0, 0	0, 0	$\underline{3}$, $\underline{3}$

Achieving a better outcome

How about play it twice?

		player 2		
		A	B	C
player 1	A	4, 4	0, $\underline{5}$	0, 0
	B	$\underline{5}$, 0	$\underline{1}$, $\underline{1}$	0, 0
	C	0, 0	0, 0	$\underline{3}$, $\underline{3}$

Consider a strategy:
Play A and then play C if (A, A) was played in the previous stage, and B otherwise.
If both players are playing this strategy, is it a SPE?

Achieving a better outcome

Given the other player is playing this strategy:

- if a player follows this strategy, the payoff is $4+3=7$.
- if a player deviate from (A, A) to (A, B), the payoff is $5+1=6$.

The temptation to defect today is smaller than the total payoff of cooperation.

Think
How do we know a threat/promise is credible? What is the difference between this game and the prisoner's dilemma?

Achieving a better outcome

Given the other player is playing this strategy:

- if a player follows this strategy, the payoff is 4+3=7.
- if a player deviate from (A, A) to (A, B), the payoff is 5+1=6.

The temptation to defect today is smaller than the total payoff of cooperation.

Think
How do we know a threat/promise is credible? What is the difference between this game and the prisoner's dilemma?

- the prisoner's dilemma has only one NE
- this game has several NEs
- when there are multiple NEs in the stage game (the game that is been repeated), we can use one as reward, and the other as punishment

Infinite Prisoner's dilemma

Is there any hope for the Prisoner's dilemma?
How about a game that can go on forever (theoretically)?

- We flip two coins: if both times we have "head", the game ends; otherwise the game continues ($\delta = \frac{3}{4}$).

	Cooperate	Defect
Cooperate	2, 2	-1, 3
Defect	3, -1	0, 0

This time, there is no (definite) last stage.
Let's try to play this game!

Infinite Prisoner's dilemma

	Cooperate	Defect
Cooperate	2, 2	-1, 3
Defect	3, -1	0, 0

- both players always defect is a SPE (**ALLD**).
- we introduce another strategy: play C if no one played D, then D forever (**GRIM**).

If both players are playing this strategy, will it help to sustain cooperation?
Given that you will play this strategy, for the other player:

- the payoff of defect is 3+0+0+...
- the payoff of cooperation is $2 + 2\delta + 2\delta^2 + 2\delta^3 + ...$

The condition for not defect is $\delta \geq \frac{1}{3}$

Infinite Prisoner's dilemma

Given $\delta \geq \frac{1}{3}$, is it a SPE (play C if no one played D, then D forever)?

- given that the other player playing this strategy, is it the best response for you to play the same?
  - compare to playing D forever
  - compare to playing C forever
  - compare to playing C at the beginning, and D afterwards even if the other is still playing C
  - compare to playing D at the beginning, and C afterwards at some point...

Trigger strategy in infinite Prisoner's dilemma

- we can get cooperation in Prisoner's dilemma using the **Trigger strategy 触发策略** (as a SPE) provided $\delta \geq \frac{1}{3}$.
- for an ongoing relationship, to provide incentives for good behavior today, it helps for there to be a high probability that the relationship will continue.
- in a game with yourself (take a diet, exercise, etc.), your willingness to maintain a good behavior depends on the weight you put on the future (patience).

Think
Once the other person defects, you "break up" forever. Can we think of a way to "forgive"?
What if the other person made a mistake (trembling hand)?

Infinite Prisoner's dilemma

How about a shorter punishment, say, play (C, C),...,(C, C), **(C, D), (D, D)**, (C, C), ...

	Cooperate	Defect
Cooperate	2, 2	-1, 3
Defect	3, -1	0, 0

Will it help to sustain cooperation?
Given that you will play this strategy, for the other player:

- the payoff of defect is $3 + 0\delta + 2\delta^2 + 2\delta^3 + ... = 3 + \frac{2}{1-\delta} - (2 + 2\delta)$
- the payoff of cooperation is $2 + 2\delta + 2\delta^2 + 2\delta^3 + ... = \frac{2}{1-\delta}$

The condition for not defect is $\delta \geq \frac{1}{2}$

Infinite Prisoner's dilemma

Given $\delta \geq \frac{1}{2}$, is both players playing it (punish for one period) a SPE?

- given that the other player playing this strategy, is it the best response for you to play the same? [try to think of some situations]
- if you use a shorter punishment, you need hight δ to sustain cooperation (longer relationship).

Infinite Prisoner's dilemma

Strategies involving forgiveness:

- Tit-for-tat: starts with C, then does whatever the other player has done in the previous round.
- Generous Tit-for-tat: choose C with a positive probability when the opponent has defected.

The evolution of cooperation

Robert Axelrod: computer tournaments for the Prisoner's Dilemma

- The strategies were submitted by game theorists in economics, sociology, political science, and mathematics.
- Game lengths of 200 moves.
- In the first tournament, 14 strategies plus a random strategy (as the baseline) were paired with each other in each round. So in total $15 \times 15 = 225$ rounds.
- The highest average score was attained by the simplest of all strategies submitted: TIT-FOR-TAT (TFT) **针锋相对**

The evolution of cooperation

- In the second tournament, 62 strategies were involved. TFT won again.
    - Players submitted more malicious strategies to "screw up" kind strategies
    - It turns out that the 15 worst-performing strategies include 14 malicious ones.
- But what happens if more of the people are "bad" guys? Can you still earn money by being "good"?

The evolution of cooperation

- In the third tournament, the distribution of each strategy can change in the next “generation”.
    - Almost all malicious strategies ”die out” after 200 rounds.
    - After 1000 rounds, the environment is stable.
    - Including TFT and other five strategies that start with C but also retaliates are successful.
    - At the end, since malicious strategies die out, we cannot distinguish ”tit-for-tat” and ”always C” since they behave the same.

  Cooperation is sustained by a long-term relationship, not trust.

The evolution of cooperation

Four features of strategies that make you earn more money:

- it was never the first to defect " **善良**"
- it was provocable into retaliation by a defection of the other " **不懦弱**"
- it was forgiving after just one act of retaliation " **宽恕**"
- it was never envious " **不嫉妒**"

Questions?