Microeconomics Group45 吴熙楠

Homework03

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1 Problem 1

Answer:

(a)$MU_x = \dfrac{\partial u(x,y)}{\partial x} = 3, MU_y = \dfrac{\partial u(x,y)}{\partial y} = 1$

(b)Yes, because $MU_x = 3 > 0$ and $MU_y = 1 > 0$

(c)$MRS_{x,y} = -\dfrac{MU_x}{MU_y} = -3$, it means that having one book for his friends is as good as having three for himself. In other words, the utility keeps the same while he has 3 more y and 1 less x.

(d)No, because MRS is constant no matter what x and y are.

2 Problem 2

Answer:

(a)We suppose that his utility function is $Eu = \pi u(w_g) + (1-\pi)u(w_b)$
$= \pi u(w + xr_g) + (1-\pi)u(w + xr_b)$

$\dfrac{dEu}{dx} = \pi r_g \dfrac{du(z)}{dz}|_{z=w+xr_g} + (1-\pi)r_b \dfrac{du(z)}{dz}|_{z=w+xr_b}$

① for $0 \le x \le w$, if $\dfrac{dEu}{dx} > 0$, then the more he invests, the more utility he can get.
The best strategy is that he invests all his wealth($x = w$).

② for $0 \le x \le w$, if $\dfrac{dEu}{dx} < 0$, then the more he invests, the less utility he can get.
The best strategy is that he invests zero($x = 0$).

③ for $0 \le x \le w$, if $\dfrac{dEu}{dx} == 0$, then no matter what he invests, the utility is a constant.
Then no matter what he invests, the utility is a constant.

④for $0 \le x \le w$, if Eu increases first but decreases later.

$$\frac{dEu}{dx} = \begin{cases} > 0, & \text{for } 0 \le x < x_0, \\ \le 0, & \text{for } x_0 \le x \le w, \end{cases}$$

Then for $x = x_0$, we can get the best utility. So the best strategy is that he invests x_0.

⑤for $0 \le x \le w$, if Eu decreases first but increases later.

$$\frac{dEu}{dx} = \begin{cases} < 0, & \text{for } 0 \le x < x_1, \\ \ge 0, & \text{for } x_1 \le x \le w, \end{cases}$$

Then if $Eu(x = w) > Eu(x = 0)$, then the best strategy is that he invests w; if $Eu(x = w) < Eu(x = 0)$, then the best strategy is that he invests 0; if $Eu(x = w) == Eu(x = 0)$, then he can choose to invest 0 or w.

(b)Similarly we should change the utility:$Eu' = \pi u(w_g) + (1-\pi)u(w_b)$
$= \pi u(w + (1-t)xr_g) + (1-\pi)u(w + (1-t)xr_b)$

$\dfrac{dEu'}{dx} = \pi(1-t)r_g \dfrac{du(z)}{dz}|_{z=w+(1-t)xr_g} + (1-\pi)(1-t)r_b \dfrac{du(z)}{dz}|_{z=w+(1-t)xr_b} = (1-t)\dfrac{dEu(x')}{dx'}|_{x'=(1-t)x}$

① for $0 \le x \le w$, if $\dfrac{dEu'}{dx} > 0$, then the more he invests, the more utility he can get.
The best strategy is that he invests all his wealth($x = w$).

② for $0 \le x \le w$, if $\dfrac{dEu'}{dx} < 0$, then the more he invests, the less utility he can get.
The best strategy is that he invests zero($x = 0$).
③ for $0 \le x \le w$, if $\dfrac{dEu'}{dx} == 0$, then no matter what he invests, the utility is a constant.
Then no matter what he invests, the utility is a constant.
④for $0 \le x \le w$, if Eu' increases first but decreases later.($x_0' = \dfrac{x_0}{1-t}, x_0 < (1-t)w$)

$$\frac{dEu'}{dx} = \begin{cases} > 0, & \text{for } \ 0 \le x < x_0', \\ \le 0, & \text{for } \ x_0' \le x \le w, \end{cases}$$

Then for $x = x_0'$, we can get the best utility. So the best strategy is that he invests x_0'.
⑤for $0 \le x \le w$, if Eu' decreases first but increases later.($x_1' = \dfrac{x_1}{1-t}, x_1 < (1-t)w$)

$$\frac{dEu'}{dx} = \begin{cases} < 0, & \text{for } \ 0 \le x < x_1', \\ \ge 0, & \text{for } \ x_1' \le x \le w, \end{cases}$$

Then if $Eu'(x = w) > Eu'(x = 0)$, then the best strategy is that he invests w; if $Eu'(x = w) < Eu'(x = 0)$, then the best strategy is that he invests 0; if $Eu'(x = w) == Eu'(x = 0)$, then he can choose to invest 0 or w.

We should notice that $x_0' = \dfrac{x_0}{1-t}, x_1' = \dfrac{x_1}{1-t}$.

3 Problem 3

Answer:

Assume that he borrows x yuan in the first period, then $c_1 = x$ and $c_2 = 8800 - 1.1x$. So $u(x) = x(8800 - 1.1x)$, and $\dfrac{du}{dx} = 8800 - 2.2x$. Because this is a quadratic function, we can set $\dfrac{du}{dx}$ equal to zero and then we can get the maximum utility function.
So $\dfrac{du}{dx} = 8800 - 2.2x = 0 \to x = c_1 = 4000, c_2 = 4400$, and $u_{max} = 17600000$

4 Problem 4

Answer:

(a)$\epsilon_{x,I} = \dfrac{\partial x}{\partial I}\dfrac{I}{x} = \dfrac{2-2I}{p_x}\dfrac{p_x}{2-I} = \dfrac{2(1-I)}{2-I}$

(b)If beef is a normal good, then $0 < \epsilon_{x,I} \to 0 < \dfrac{2(1-I)}{2-I}$.

So we can solve this inequality, and we can get that $I < 1$ or $I > 2$.

But we also know that $x \ge 0$, so we can get that $0 \le I \le 2$.

So if $0 \le I < 1$, the beef is a normal good.

5 Problem 5

Answer:

(a)We can know that $Q^D = Q^S \rightarrow 150 - 50p = 60 + 40p$, So we can solve that $p = 1, Q = 100$.

(b)$Q^{S\prime} = 60 + 40(p - 0.5) = 40 + 40p$,then $Q^D = Q^{S\prime} \rightarrow 150 - 50p = 40 + 40p$.
We can solve that $p' = \frac{11}{9}, Q' = \frac{800}{9}$

(c)$CS_1 = \int_0^{100}(\frac{150-q}{50})dq - 1 \times 100 = 200 - 100 = 100$

$CS_2 = \int_0^{\frac{800}{9}}(\frac{150-q}{50})dq - \frac{11}{9} \times \frac{800}{9} = \frac{6400}{81}$

So $\Delta CS = CS_2 - CS_1 = \frac{6400}{81} - 100 = -\frac{1700}{81}$

(d)$PS_1 = 1 \times 100 - \int_0^{100}(\frac{q-60}{40})dq = 100 + 25 = 125$

$PS_2 = \frac{11}{9} \times \frac{800}{9} - \int_0^{\frac{800}{9}}(\frac{q-40}{40})dq = \frac{8000}{81}$

$\Delta PS = PS_2 - PS_1 = \frac{8000}{81} - 125 = -\frac{2125}{81}$

(e)government benefit=$0.5 \times \frac{800}{9} = \frac{400}{9}$

(f)the dead-weight loss=$0.5 \times (100 - \frac{800}{9}) \times 0.5 = \frac{25}{9}$

We can find that $\Delta CS + \Delta PS$ + government benefit + the dead-weight loss=0.