Homework Assignment 4

Bring your homework to the classroom before the lecture begins.

Due date: Dec 8, 2021

You can choose to answer the questions in English or Chinese, whichever you feel more comfortable with. You are encouraged to discuss with your classmates, but please keep in mind that for the final exam you have to do it alone. You will suffer 20% deduction on the grade of this assignment for each delayed day of submission.

Please motivate your answers with details.

1. [10 points] A monopolist sells in two markets. The demand curve for the monopolist's product is $x_1 = a_1 - b_1 p_1$ in market 1 and $x_2 = a_2 - b_2 p_2$ in market 2, where x_1 and x_2 are the quantities sold in each market, and p_1 and p_2 are the prices charged in each market. The monopolist has zero marginal cost. Note that although the monopolist can charge different prices in the two markets, it must sell all units within a market at the same price.
   Under what conditions on the parameters (a_1, b_1, a_2, b_2) will the monopolist optimally choose not to price discriminate (assume interior solutions)?

2. A parent has two children A and B. She has a total wealth of 1000 to give to them.

   (a) [5 points] The parent's utility function is $U(a, b) = \sqrt{a} + \sqrt{b}$ where a is the amount of money she gives to A and b is the amount of money she gives to B. How will she choose to divide the money?

(b) [5 points] Suppose that her utility is $U(a,b) = -\frac{1}{a} - \frac{1}{b}$. How will she choose to divide the money?

(c) [5 points] Suppose that her utility is $U(a,b) = \min\{a,b\}$. How will she choose to divide the money?

(d) [5 points] Suppose that her utility is $U(a,b) = \max\{a,b\}$. How will she choose to divide the money?

(e) [5 points] Suppose that her utility is $U(a,b) = a^2 + b^2$. How will she choose to divide the money?

3. A parent has two children A and B. She has a total wealth of 1000 to give to them. Now that A is a much more efficient shopper than B so that A is able to get twice as much consumption goods as B can for every unit of money that he spends. Let a be the amount of consumption goods that A gets and b the amount that B gets. We will measure consumption goods so that one unit of consumption goods costs 1 for A and 2 for B. Thus the parent's budget constraint is $a + 2b = 1000$.

(a) [5 points] If the mother's utility function is $U(a,b) = a + b$, how will the parent choose to divide the money?

(b) [5 points] If the mother's utility function is $U(a,b) = -a - b$, how will the parent choose to divide the money?

(c) [5 points] If the mother's utility function is $U(a,b) = a \times b$, how will the parent choose to divide the money?

4. (2019 年期末考题) 在一个纯交换经济中有两种商品 1 和 2，有两个人 A 和 B。这两个人的效用函数与初始禀赋如下：

$$u^A(x_1,x_2) = (x_1^A x_2^A)^2,\ w^A = (18,4)$$

$$u^B(x_1,x_2) = ln(x_1^B) + 2ln(x_2^B),\ w^B = (3,6)$$

(a) [5 points] 描绘出帕累托集的特征（写出该集用 x_1^A 和 x_2^A 表示的特征函数式）。

(b) [10 points] 求出瓦尔拉斯均衡。