Active Quantum Steering Research Proposal

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Inspired by professor Yuval Gefen's lecture, We hope to deeply understand the measurement-induced quantum steering. We have briefly read some literature and we are more interested in the situation of quantum steering in many-body Hilbert space. In this research proposal, We briefly introduced the background and progress of this research, then we stated a research goal, and we proposed our research plan, including the method, timeline, personnel and the expected results.

I. INTRODUCTION & BACKGROUND

Quantum state preparation is a prominent routine in quantum information processing toolbox[1]. There are multiple traditional ways to constructing such protocols, for instance, designing the Hamiltonian of the system such that its unitary evolution leads to a designated state[2] or cooling system to extreme low temperature to make the equilibrium of the system to a target ground state, which, however, require knowledge of the starting state and fine-tuning of the system Hamiltonian, or some extreme laboratory condition.

Based a sequence of generalized measurements[3], Prof. Yuval Gefen proposed both passive[4] and active[1] version of quantum steering protocol that can bring the system towards the target state via measurement back-action and circumvent the traditional limitations proposed above.

A. Passive Quantum Steering

The main goal of the Passive Quantum Steering (PQS) is to circumvent the extreme conditions needed in traditional quantum state preparation process, that is, we can always steer the system to our target state no matter how its initial state is and the protocol only involves simple quantum evolutions without too much external intervention and adjustment[5, 6].

For PQS proposed by Yuval Gefen's measurement-induced method, we steer the state of a quantum system from any arbitrary initial state by coupling it to a auxiliary degree of freedom. The protocol requires multiple repetitions of an elementary step: During each step, the system evolves for a fixed time while coupled to auxiliary degrees of freedom (which we term "detector qubits") that have been prepared in a specified initial state. The detectors are discarded at the end of the step, or equivalently, their state is determined by a projective measurement with an unbiased average over all outcomes. The steering harnesses backaction of the detector qubits on the system, arising from entanglement generated during the coupled evolution.

The sketch map is shown in FIG1, where the ρ_s represents the density matrix of the system and ρ_{s-d} represents the density matrix of the coupled system and detector. H_{s-d} represents the coupling Hamiltonian and as long as we make a clever choice of the coupling Hamiltonian, the system will be "rotated" into the target state, which we regarded as the measurement back-action of the detector.

The general expression of the coupling Hamiltonian has the form:

$$H_{\mathrm{s-d}} = \sum_n \left(O_{\mathrm{d}}^{(n)} \left|\Phi_{\mathrm{d}}\right\rangle \left\langle \Phi_{\mathrm{d}}\right| \right) \otimes U_{\mathrm{s}}^{(n)} + \text{ H.c.} \qquad (1)$$

where n labels the detector qubit. Since $O_{\mathrm{d}}^{(n)}$ connects the state $|\Phi_{\mathrm{d}}\rangle$ to its orthogonal subspace, it satisfies $\left\langle \Phi_{\mathrm{d}} \middle| O_{\mathrm{d}}^{(n)} \middle| \Phi_{\mathrm{d}} \right\rangle = 0$. Therefore we can see the coupling process set such a trend that the system state is "rotated" from states orthogonal to the target state and the detector (qubit) will "click", which means the qubit will jump from its original state to the other state.

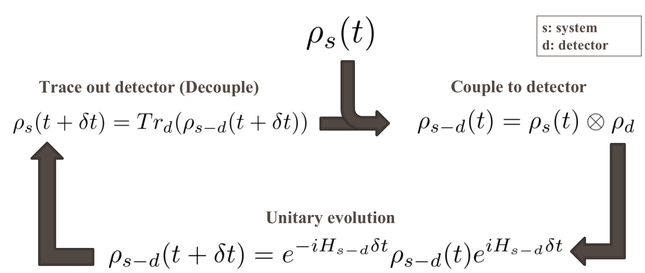

FIG. 1. Elementary step of Passive Quantum Steering (PQS), the system is coupled with the detector at first Then the system-detector coupling Hamiltonian makes the system evolve for a short period of time δt. In the end, the detector is decoupled from the system, equivalent to tracing out the detector degree of freedom in the total density matrix.

After an elementary step, the density matrix is traced over the detectors' degree of freedom, so that the detector is decoupled and all the possibility of "click" and "non-click" will all be considered with an unbiased average of all outcome, which is exactly the back-action of the

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measurement process of the detector. As every step steer the system towards the target state a little bit, we can conduct a series of such measurement done by a series of detectors, all with the same initial state and the same way of coupling to the system, we can expect such a series of measurement will finally lead us to our target state no matter how the initial state of the system is.

We can illustrate the PQS protocol more specifically using an example in which we want to steer a two-spin-1/2 system into a singlet state from any arbitrary state. The coupling Hamiltonian has a form:

$$H_{\mathrm{s-d}} = J \sum_{i=1}^{3} \left(\sigma_{\mathrm{d}_i}^{-} \rho_{\mathrm{d}} \otimes U_i + \mathrm{H.c.}\right) \quad \begin{cases} U_1 = |S_0\rangle \langle T_+| \\ U_2 = |S_0\rangle \langle T_-| \\ U_3 = |S_0\rangle \langle T_0| \end{cases} \tag{2}$$

Here, the $|S_0\rangle$ represents singlet state and $|T_I\rangle$'s represent 3 triplet state. In this example, we can see more detailedly how the coupling Hamiltonian makes the system evolve in a way so that system state's component of triplet states are steered to singlet state with a "click" on detector (qubit). And we can regard these $U_i's$ as navigating tools in the system Hilbert space, which guide the system state to move in several directions in the Hilbert space towards the target state.

In PQS protocol, we indiscriminately adopt all navigating tools to make sure the system state won't evolve in an undesired direction. However, we will see such a method might lead to some redundancy and that's why AQS protocol has been proposed[7–9].

B. Active Quantum Steering

The protocol of PQS has already provided a feasible way to steer a system from any arbitrary state to any target state and the most important thing is to find appropriate navigating tools. In PQS protocol, we indiscriminately adopt all navigating tools to evade undesired evolution, but that leads to some redundancy when the system has evolved into a rather small subspace in Hilbert space and a lot of redundant navigating tool only works outside this subspace.

Also, we find the term "measurement-induced (passive) quantum steering" doesn't include much measurement, since all the outcomes ("click" and "nonclick" condition) are unbiasedly averaged over and only the so-called "measurement back-action" plays a role. It's natural to ask whether we can read out the measurement result of the detector after each elementary step and make use of that measurement as a feedback to adjust our steering process more actively. That's how the concept of active quantum steering (AQS) is introduced.

Particularly, among all the active quantum steering protocols proposed by Prof. Gefen, two techniques which had been termed Greedy Orienteering Policy and Quantum State Machine (QSM), exhibit eminent speedup and fidelity improvement in both theoretical prediction and numerical simulation. More detailedly, a speedup factor of around 10 had been observed in Gefen's work.

1. *Greedy Orienteering Policy (GOP)*

The first AQS method is GOP based on cost function, the cost function has a form:

$$R\left(\rho_s^{(\mathrm{fin})}, |\psi_0\rangle\right) \equiv 1 - \left\langle \psi_0 \middle| \rho_s^{(\mathrm{fin})} \middle| \psi_0 \right\rangle \tag{3}$$

As we see, the cost function stands for how much the system's density matrix is in our target state, like the intersection part. And of course we want this cost function to reach 0. So such a cost function defined on Hilbert space of the system and the steering process is just like what we do in a gradient descent process, where we find the maximum gradient to get down to the minimum cost function in the fastest way. And the coupling Hamiltonian we choose can be regarded as the direction we choose in orur protocol, so we don't use all coupling Hamiltonian at the same time equally and we don't use all qubit at the same time equally, instead, we choose them with the guidance of cost function's maximum gradient. As long as the guiding direction is close to the "gradient" in the Hilbert space, then we can approximately assure a fastest descent towards the target state.

2. *Quantum State Machine (QSM)*

The second is QSM, where we give every qubit and coupling Hamiltonian a map, telling how the system will evolve in conditions that detector clicked or did not click. We denote the possibility of each system state developing direction and get an panorama graph as shown in the graph. And them we only need to choose the shortest path and use the corresponding detector on this very path.

To put it more specifically, we look into an example of steering a 3-qubit system towards the W-state[10]:

$$W = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$$

For the steering coupling Hamiltonian, we choose the following family of couplings (assuming labels A, B, and C for the qubits):

$$\begin{aligned} V_1 &= \sigma_A^+ - \sigma_C^+ \\ V_2 &= \sigma_A^- \sigma_C^- \\ V_3 &= \sigma_A^- \sigma_B^+ - P_A^0 P_B^1 \\ V_4 &= \sigma_B^+ \sigma_C^- - P_B^1 P_C^0 \end{aligned}$$

Here, $\sigma^{\pm} = \frac{1}{2}\left(\sigma^x \pm i\sigma^y\right)$ and $P^a = |a\rangle\langle a|, a = 0, 1$.A passive version of the protocol would amount to blindly alternating between the steering actions with different V_i, which does yield the target state, given that the steering is applied a sufficient number of times.

The map in FIG2 shows how the system state evolve when detector i clicked or didn't click, once we give each edge a corresponding possibility and characteristic time, the whole steering problem can be transformed into a "shortest route problem" in graph analysis.

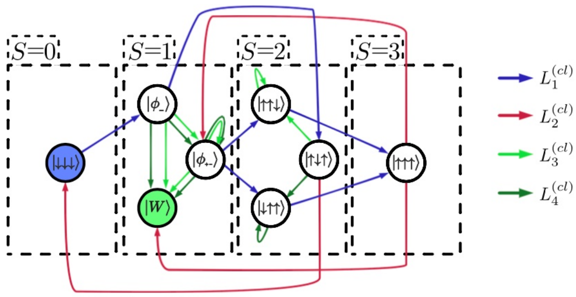

FIG. 2. Quantum State Machine (QSM) graph. The vertices represent basis in system Hilbert space and the edges connecting them represent how system state evolve given different outcomes of each detector.

II. MOTIVATION

Although, AQS has provided a considerably acceleration for QS protocol, several obstacles remains in both these two techniques.

A. Possible improvements on GOP

For Greedy Orienteering Policy (GOP), the steering is actively oriented by a specific cost function defined in the Hilbert space, thus it's obvious that the choice of cost functio will definitely affect our AQS speed and error-tolerance.

The system is steered towards the state of which the cost function is at its global minimum, and the specific basis and metric choice will affect the expression of the cost function and the optimum way to accelerate the steering process involves a shrewd choice of that basis and metric which gives the "steepest decent". Beside the speedup by choice of basis and metric , we can adopt a more error- reducing way of orienting by forecasting more steps in advance rather than using this greedy policy that only considers a local "steepest decent", which might cause the system state to be trapped a local minimum and add more errors to our steering[11].

Also, a better "landscape" of cost function might accelerate the steering process as well. Therefore the design of a refined cost function is analogous to a "steepest descent problem" as well.

B. Possible Improvements on QSM

For Quantum State Machine (QSM), the active steer- ing strategy is tantamount to a shortest route choosing problem based on graph analysis, in which we draw a corresponding map of both "click" and "non-click" out- comes of the detector, indicating how the system would react to these outcomes, and then we combine all maps of our detector to generate a panorama map called QSM graph to orient our steering by finding the fastest route towards target state on this "map". Also, the routes and characteristic duration of each route depends heavily on the basis choosing, thus a clever strategy to choose ap- propriate basis for generating the QSM graph is of great significance.

Besides, the QSM doesn't transform the AQS problem into a classical graph analysis because of the quantum nature of the system. As shown in FIG2, if there are more than one edges coming out of or into one vertex, there are quantum effects of superposition and interference, we have to consider a multi-layer graph analysis, which, with the increase of system complexity, becomes impossible to deal with. Thus a clever choice of basis also helps circumvent this problem.

C. Other aspects of possible improvements

At the same time, the practical application of active quantum steering is limited to some low-body entan- glement states or low-degree multi-body entanglement states. The improvement of strategy may bring forward the possibility of more complex multi-body entanglement state quantum steering methods.

III. RESEARCH GOAL

Due to the abundant room for improvement in the active quantum steering protocols proposed by Prof. Yuval Gefen's group, we'd like to improve the strategy of active quantum steering protocols in the way mentioned in our Motivation part. Also, we would like to design some configurations, or find some special many-body entangled states, and use the active quantum steering method mentioned above to extend the field of application of the active quantum steering, at least theoretically.

IV. RESEARCH PLAN

We wanted to find some special many-body entangled states as target states[12], design both passive and active (especially the improved active) methods, and compare the effects of various steering protocols.

A. Method

Numerical Simulation: First, we find some non-trivial many-body entangled state as our target state. Then we construct both passive and active quantum steering plan just as Gefen did in his papers.

Then, we can make use of com- puter software (such as Mathematica, Matlab, Python, etc.) to simulate such PQS and AQS protocols and compare the active quantum steering compared with the passive quantum speedup of steering.

Preliminary Analysis: Firstly, we can find out the general basis choosing strategy of Quantum State Ma- chine (QSM) and Greedy Orienteering Policy to improve the efficiency of strategic selection and the speed of steer- ing. Then, we can improve on the Greedy Orienteering Policy where considering only one step rather than mul- tiple steps might lead to errors, or where system state is limited to a local minimum of a cost function. Finally improve the fault tolerance ability of steering.

Machine Learning: We can use Q-Learing[13], SARSA[14], or variational scoring methods to help us find the best base selection and determine the best number of steps to take in the Greedy Orienteering Policy.

Specifically, variational method can be implemented to refine the cost function of GOP method so as to obtain a better "landscape" in AQS.

Also, Q-learning is a model-free reinforcement learning algorithm to learn the value of an action in a particular state. It does not require a model of the environment (hence "model-free"), and SARSA is its modified version, such a model-free ML algorithm might help us in finding better strategy in basis choosing, etc.

B. Timeline

02/10-03/21: Literature research and talk with Yuval Gefen.

03/22-03/31: Find target state and propose remitive QS method.

04/01-04/10: Numerical simulations of premitive QS method and do some preliminary analysis.

04/10-04/30: Discuss how ML can be implemented in cost function designing in GOP and basis choosing in QSM.

05/01-06/31: Implement our analysis and ML method, compare the speed error tolerance of refined method to our original QS protocols. Making adjustments if necessary.

C. Personnel

In our group, Yiyang Jiang is responsible for analytical work and literature research, Xinan Wu is responsible for numerical simulation work, and Yunlong Wu is responsible for machine learning work. Every other week or so, we talk to each other about our work and maybe make changes to the timeline in real time based on our work. We may also choose some time to ask Prof. Yuval Gefen some questions.

D. Expected Results

We hope to find an effective way to improve the active quantum steering speed and fault tolerance rate. It can be based on some strategies found in analytical analysis or machine learning. At the same time, we hope to provide quantum steering strategy for preparing more complex entanglement states.

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