Homework Assignment 3

Bring your homework to the classroom before the lecture begins

Due date: Dec 1, Wednesday, 2021

You can choose to answer the questions in English or Chinese, whichever you feel more comfortable with. You are encouraged to discuss with your classmates, but please keep in mind that for the final exam you have to do it alone. You will suffer 20% deduction on the grade of this assignment for each delayed day of submission (Deadline: due date 6:40 PM sharp).

Please motivate your answers with details.

1. [20 points] Peter's utility function is

$$u(x, y) = 4x - (x - y)$$

where x is the number of books he owns, and y is the number of books his friend owns. That is, he suffers from guilt when he owns more books than his friend ($x > y$).

   (a) Find Peter's marginal utility for the books he owns, MU_x, and for his friend's, MU_y.

   (b) Are his preferences monotonic (i.e., weakly increasing in both goods)?

   (c) Find Peter's MRS between x and y. Interpret your results.

   (d) Are his preferences strictly convex? (Hint: convex preferences indicate diminishing MRS).

2. [20 points] Assume a risk averse consumer has wealth w, and he plans to invest x on risky asset. Under the "good" situation, the risky asset gives return of $r_g > 0$, and under the "bad" situation, it gives return of $r_b < 0$. Therefore, the wealth of the consumer under the "good" and "bad" situations are

$$w_g = (w - x) + x(1 + r_g) = w + xr_g$$
$$w_b = (w - x) + x(1 + r_b) = w + xr_b$$

Assume the probability that the "good" situation is realized is π, and the probability that the "bad" situation is realized is $1 - \pi$.

(a) What is this consumer's best strategy on investment?

(b) Assume there is a tax on the consumer's risky asset gain, which is t, i.e., the payment from investing in risky asset is $(1 - t)r_g$ or $(1 - t)r_b$ after tax. What is this consumer's best investment level in risky asset compared to in (a)?

3. [10 points] There is a decision that concerns two periods for Li. In the first period, he is in college without any income. In the second period, he is working with income 8800 yuan. In the first period he can borrow money from the university with interest rate of 10%. The consumption of the first and second period is c_1 and c_2, respectively. His consumption utility is $u(c_1, c_2) = c_1 \cdot c_2$.
Assume there is no inflation. How much should he borrow from the university to maximize his utility from consumption?

4. [20 points] Suppose that the demand for beef (good x) can be expressed as $x = \frac{2I - I^2}{p_x}$.

(a) Calculate the income elasticity for beef. [Hint: $\varepsilon_{x,I} = \frac{\partial x}{\partial I}\frac{I}{x}$]

(b) For what values of I is beef a normal good?

5. [30 points] Suppose that the demand and supply of gas are

$$Q^D = 150 - 50p,\ Q^S = 60 + 40p$$

(a) Find the market equilibrium in price and quantity of gas.

(b) Suppose that the government imposes quantity tax on per unit sold: $t = 0.5$. Find the market price and quantity.

(c) Calculate the change in consumer surplus before and after tax.

(d) Calculate the change in producer surplus before and after tax.

(e) Calculate government benefit from taxing.

(f) Calculate the dead-weight loss of a tax.