Problem 1:

sol: $D = \frac{d}{d\lambda}\left(\frac{1}{v_g}\right)$, $v_g = \frac{dw}{dk}$, $n_g = n - \lambda\frac{dn}{d\lambda} \Rightarrow D = \frac{d}{d\lambda}\left(\frac{n_g}{c}\right) = -\frac{\lambda}{c}\frac{d^2n}{d\lambda^2}$, $\lambda = \frac{2\pi c}{w}$

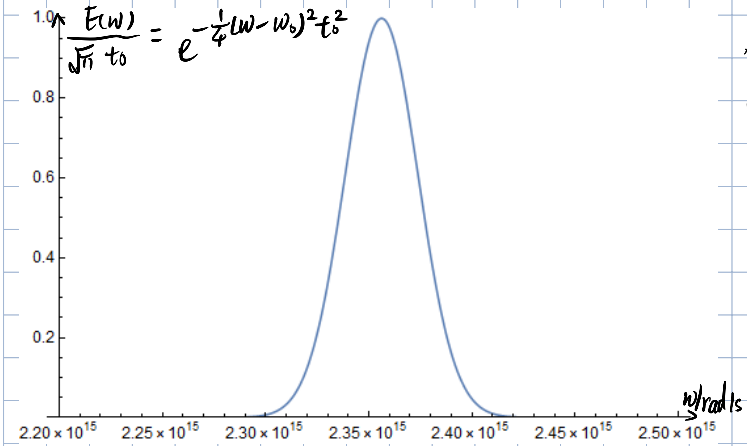
$$k = \frac{nw}{c} \Rightarrow \frac{dk}{dw} = \frac{n}{c} + \frac{w}{c}\frac{dn}{d\lambda}\cdot\frac{d\lambda}{dw} = \frac{n}{c} - \frac{w}{c}\frac{dn}{d\lambda}\frac{2\pi c}{w^2} = \frac{n}{c} - \frac{\lambda}{c}\frac{dn}{d\lambda} \Rightarrow \frac{d^2k}{dw^2} = \frac{1}{c}\frac{dn}{d\lambda}\cdot\frac{d\lambda}{dw} - \frac{1}{c}\frac{dn}{d\lambda}\cdot\frac{d\lambda}{dw} - \frac{\lambda}{c}\frac{d}{dw}\left(\frac{dn}{d\lambda}\right)$$

$$\Rightarrow \frac{d^2k}{dw^2} = -\frac{\lambda}{c}\frac{d^2n}{d\lambda^2}\cdot\frac{d\lambda}{dw} = D\cdot\frac{d\lambda}{dw} = -\frac{2\pi c}{w^2}\cdot D = -\frac{\lambda}{w}D \Rightarrow \frac{d^2k}{dw^2} = -\frac{\lambda}{w}D \Rightarrow D = -\frac{2\pi c}{\lambda^2}\frac{d^2k}{dw^2}$$

problem 2:

(1) $E(w) = F.T.\{E(t)\} = \int_{-\infty}^{+\infty} e^{-\frac{t^2}{t_0^2}} e^{-jw_0t} e^{jwt}\,dt = \int_{-\infty}^{+\infty} e^{-\frac{1}{t_0^2}\left[\left(t+\frac{j}{2}(w-w_0)t_0^2\right)\right]^2 - \frac{1}{4}(w-w_0)^2t_0^2}\,dt$

$= e^{-\frac{1}{4}(w-w_0)^2t_0^2} \cdot \int_{-\infty}^{+\infty} e^{-\frac{t^2}{t_0^2}}\,dt = \sqrt{\pi}\,t_0\,e^{-\frac{1}{4}(w-w_0)^2t_0^2}$. $t_0 = 80\times10^{-15}\,s$



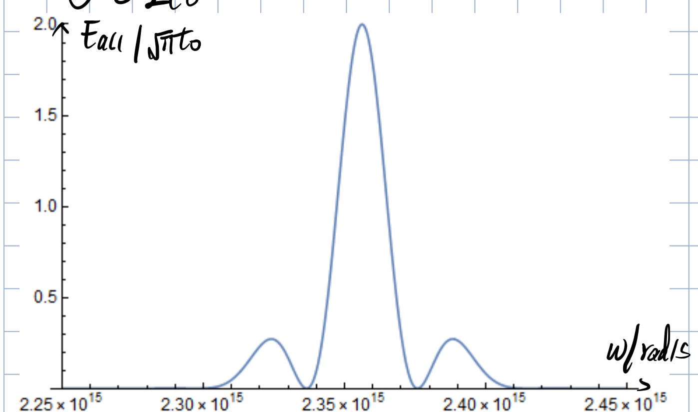
, Because $E(w)$ is too small, I take the coefficients out of the front ($\sqrt{\pi}\,t_0$)

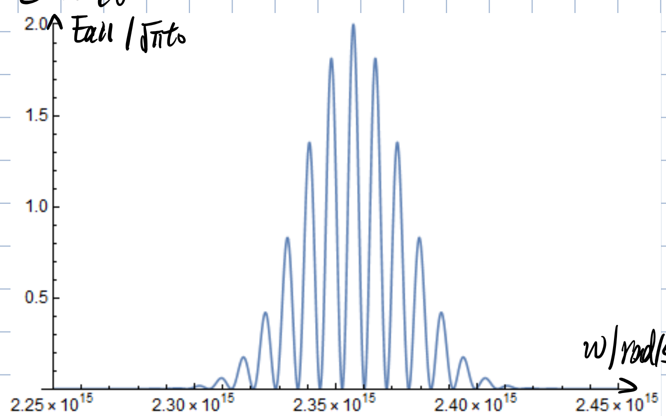
(2)

$E(t-\tau) = e^{-\frac{(t-\tau)^2}{t_0^2}} e^{-jw_0(t-\tau)}$ let $\tau_1 = t_0$, $\tau_2 = 2t_0$

$\Rightarrow E_{all} = e^{-\frac{1}{4}(w-w_0)^2t_0^2}\,(1+e^{jw\tau})\,\sqrt{\pi}\,t_0 = 2\sqrt{\pi}\,t_0\cos\frac{w\tau}{2}\,e^{-\frac{1}{4}(w-w_0)^2t_0^2}$

$\Rightarrow$ ① $\tau = 2t_0$ ② $\tau = 10t_0$





If the two pulses have certain time decay, the two pulses just seem like having a interference, only in suitable frequency, the Amplitude will be bigger. otherwise it will be zero if we have a wrong frequency. of course the new wave will have a few peaks not only one (zero time decay). And because the two pulses can't be the largest at the same time, so except $w=0$, the other peaks will be smaller.