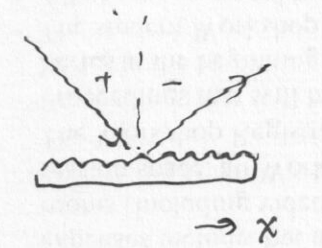
Ultrafast Optics

Review.

+ grating: Extra phase by grating: + angular dispersion



$$\phi_{grating} = -k \cdot x$$

For grating pair, we had

$$\phi''(\omega) = -\frac{4\pi^2 b c}{\cos^3\theta \cdot d^2 \cdot \omega^3}, < 0 \text{ always}$$

+ imaging ~~focus~~ elements: such as 4-f system

⇒ normal or abnormal dispersions

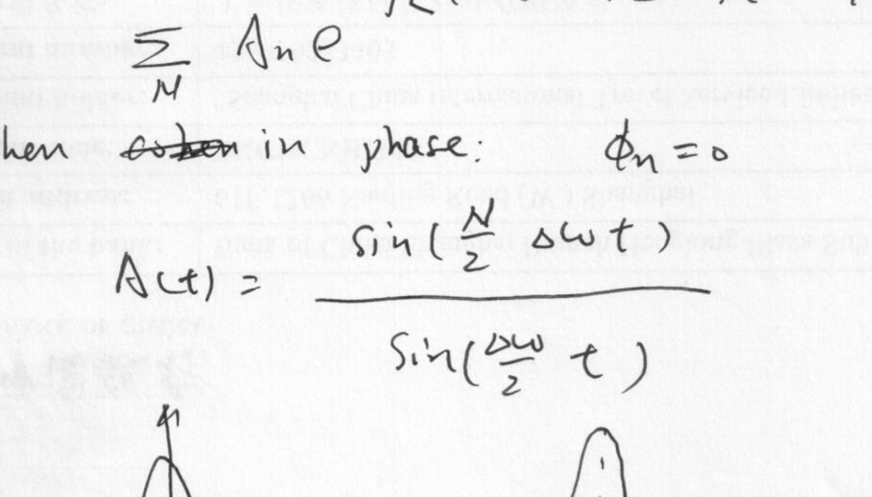
+. general picture to mode lock.

mode locking mechanism: Superposition of multiple modes

$$\sum_N A_n e^{-j[(\omega_0 + n\cdot\Delta\omega)t - \phi_n]}$$

when ~~coher~~ in phase: $\phi_n = 0$

$$A(t) = \frac{\sin(\frac{N}{2}\Delta\omega t)}{\sin(\frac{\Delta\omega}{2} t)}$$



Comments:

① $A(0) \propto N$

② $A(t + T_R) = A(t)$

where $T_R = \frac{2\pi}{\Delta\omega}$

③ $\tau \propto \frac{T_R}{N} \Rightarrow \tau \propto \frac{T_R \cdot \Delta\omega}{\delta\omega} = \frac{1}{\Delta f}$

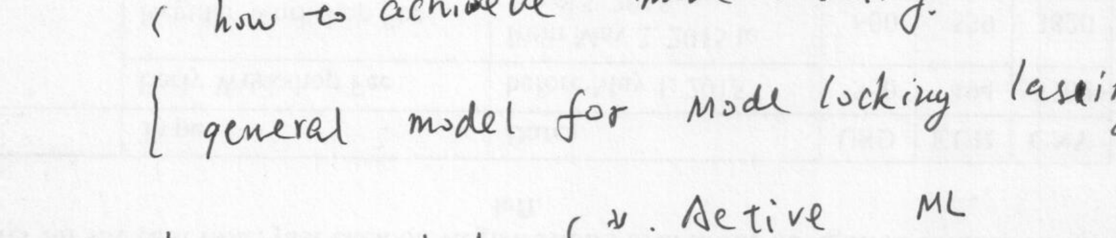
(F.T. transform)

Today:

{ how to achieve mode locking.
{ general model for mode locking lasing

mode locking methods: { • Active ML
{ • passive ML.

first let's think of why single mode for a conventional CW laser?



Active mode locking.

modulate ω_0 at $\Delta\omega$ (mode spacing ~MHz)

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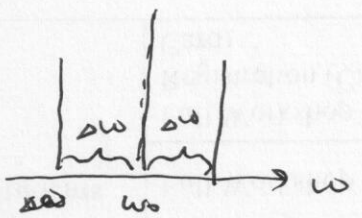
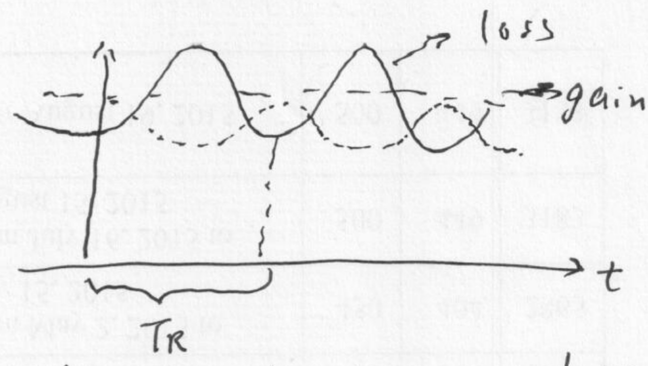
↓

E: $Ae^{-j\omega_s t}\cdot\cos(\Delta\omega t)$

$\alpha_m = \alpha_0(1-\cos\omega_m t)$

Frequency domain: time domain:





the gain ~~widow~~ window is large, why get ML/pulse (ns) narrowed?

assume gain has a Gaussian shape in time.

$e^{-\frac{t^2}{\tau^2}}$ after n round trips

$e^{-\frac{t^2}{(\frac{\tau}{\sqrt{n}})^2}}$ → pulse gets narrowed

i.e. laser shapes the pulse itself, the peak gets amplified more and more.

where n is ~~photon life~~ how many times a photon can travel in the cavity

$$n \sim \frac{\text{cavity photon life}}{\text{round trip time} \sim ns}$$

→ related to linewidth / a factor ~ ms

⇒ n is a large number

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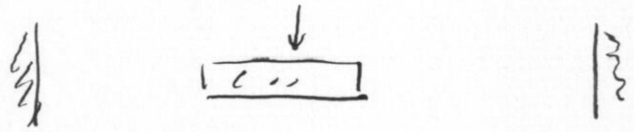
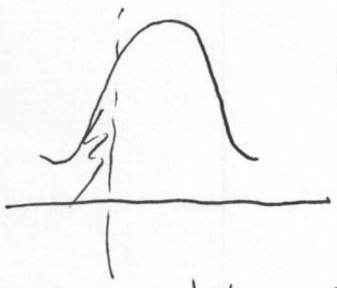
+ passive mode locking

straightforward way:

saturable absorper



but in need of : a fast absorper.

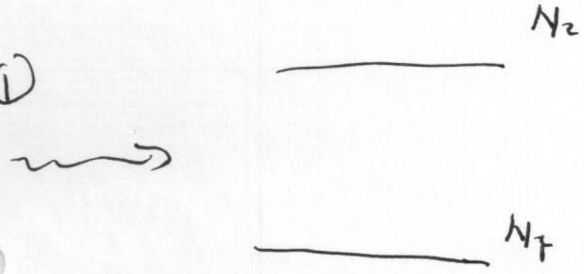


① leading edge saturate the absorper.

② trailing edge is benefited by ①

by cutting the edge, a short pulse forms

realization of SA

① 

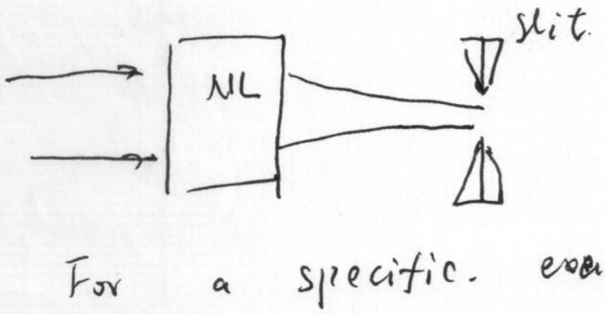
population inversion

$N_2 = N_1 \Rightarrow$ transparent

examples: dye . quantum well

② Nonlinear mechanism.

Self focusing



High power → go through

low power → blocked.

For a specific example: Kerr-lens

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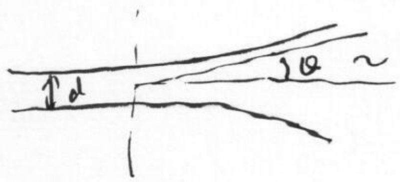
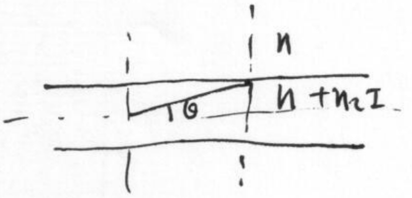
a. Simplified model:



$\theta \sim \dfrac{\lambda/n}{d}$ w/o. Kerr.

$$\left.\begin{array}{l}\Delta p = \dfrac{n}{\lambda}\cdot\theta \\ \Delta x = d\end{array}\right\} \Rightarrow \Delta p\cdot\Delta x \sim 1$$



Treat as total reflection.

$$\cos\theta\cdot(n + n_2 I) = n.$$

$$\Rightarrow \frac{1}{\cos\theta} = 1 + \frac{n_2}{n} I$$

a small θ

$$\Rightarrow \frac{1}{1-\frac{1}{2}\theta^2} \doteq 1 + \frac{1}{2}\theta^2 = 1 + \frac{n_2}{n} I$$

$$\Rightarrow I\cdot\frac{n_2}{n} = \frac{1}{2}\,\frac{\lambda^2}{n^2 d^2}$$

$$\Rightarrow \cancel{P=CI}\ I\cdot d^2 = \frac{\lambda^2}{2 n n_2} \Rightarrow \text{critical power}$$

Comment: 1. only power matters

2. if $P > P_{cr.}$ beam will be focused.

Extending thinking:

smaller than diffraction ~~lit~~ limited spot?

N.B. other nonlinear effects will also consume the pump power.

An example:

$$n_2 : \sim 3\times10^{-20}\ m^2/W. \quad n=1.5, \quad \lambda \sim 800\,nm$$

$$\Rightarrow P_{cr} = \frac{\lambda^2}{2 n n_2} \doteq \frac{(800\times10^{-9})^2}{2\times3\times10^{-20}\cdot1.5}$$

$$\sim \cdot 10^{6}\ (W)$$

fs laser: $\frac{1\,nJ}{100\,fs} \sim 10^{4}\ W$,

actually inside the cavity the power is much stronger.

To solve the above problem precisely.

Let's recall the approach:

For including nonlinearity:

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2} + \underbrace{\frac{2\pi}{\lambda}\cdot n_2 I}_{NL\ \text{term}}$$

$$k_z = k - \frac{k_x^2 + k_y^2}{2k} + \frac{2\pi}{\lambda}\cdot n_2 I$$

$$k_z - k = -\frac{k_x^2 + k_y^2}{2k} + \frac{2\pi}{\lambda}\cdot n_2 I$$

$$j(k - k_z) \sim \frac{\partial}{\partial z}$$

$$jk_x \sim \frac{\partial}{\partial x}$$

$$jk_y \sim \frac{\partial}{\partial y}$$

$$\Rightarrow -j\frac{\partial}{\partial z} = \frac{1}{2k}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) + \frac{2\pi}{\lambda}\cdot n_2 I$$

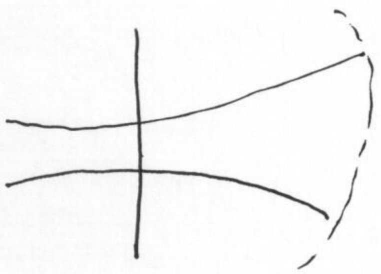
$$-j2k\frac{\partial}{\partial z} = \nabla_T + \frac{2k^2}{n} n_2 I$$

$$\Rightarrow \nabla_T A + j2k\frac{\partial A}{\partial z} + \frac{2k^2}{n} n_2 |A|^2 \cdot A = 0$$

$$\Downarrow$$

$$A = A(x,y,z)\,e^{jkz}$$

$$e^{-j\frac{\pi}{\lambda R}(x^2+y^2)}$$



Kerr effect: $e^{j\frac{2\pi}{\lambda}\cdot n_2 I} \rightarrow I = I_0 e^{-\frac{\pi(x^2+y^2)}{w_0^2}}$

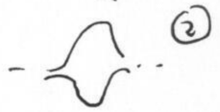
$$\Downarrow$$

$$I_0\left(1-\frac{x^2+y^2}{w_0^2}\right)$$

3) Another mechanism for passive mode-locking

APM: Additive phase mode lock.

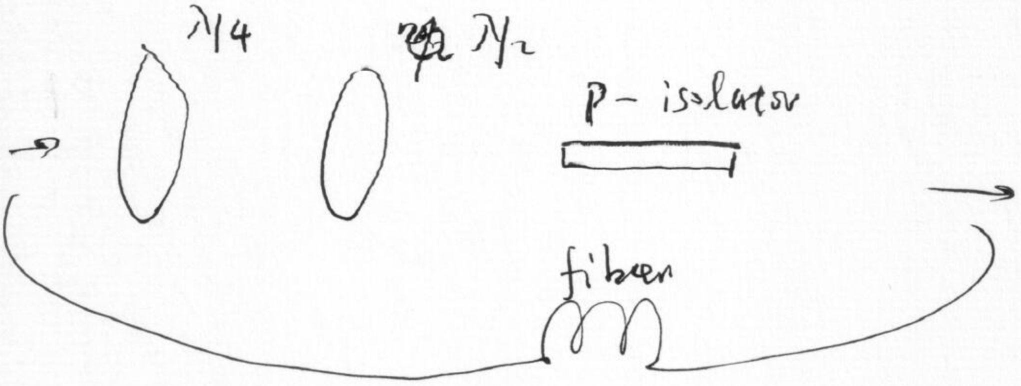
two cavities.



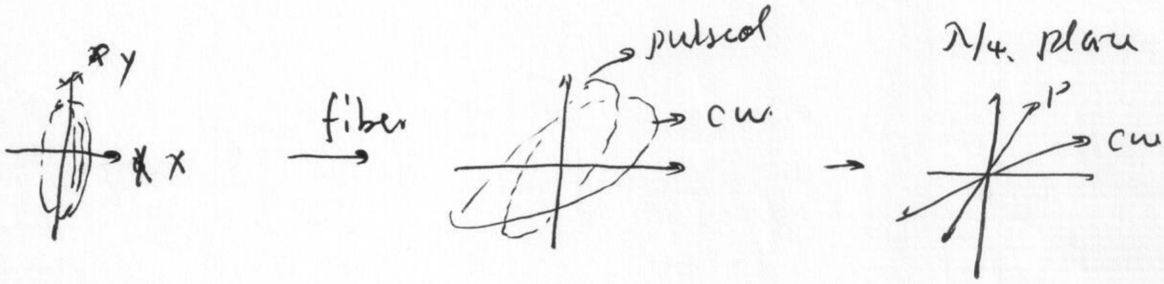
* ② returen pulse: center part is in phase w/ pulse in ①, Amplify the center
* Wing of returned pulse is out of phase to that of ①.

4). Nonlinear polarization Rotation

or (Kerr-gating)



$n = n_0 + n_2 I$ < x, y



why can obtain fs scale by using

NPR ?

Quantitative analysis of passive mode locking

General Round-trip model

① Loss / phase shift:

$$a(t+T_R) = e^{-(\alpha - j\beta)L} \cdot a(t)$$

↓ envelope

$$= e^{-(l-jx)} \cdot a(t)$$

$$\approx [1-(l-jx)] \cdot a(t)$$

$$\Rightarrow \Delta a(t) = -(l-jx)\, a(t)$$

② Gain: $\Delta a = g a$, but we here consider the bandwidth/profile of the gain

$$g(\omega) = \frac{g_0}{1+\frac{(\omega-\omega_0)^2}{\Omega_g^2}} \approx g_0\left[1-\frac{(\omega-\omega_0)^2}{\Omega_g^2}\right]$$

$$\Downarrow \quad \frac{1}{\Omega_g^2}\frac{d^2}{dt^2}$$

in frequency domain: $g(\omega)\cdot \tilde{a}(\omega)$

in time domain: $\Delta a = g\left(1+\frac{1}{\Omega_g^2}\frac{d^2}{dt^2}\right)\cdot a(t)$

③ GVD: $\Delta a = jD\frac{\partial^2}{\partial t^2}$

where $e^{jk(\omega)\cdot L} = e^{j[k_0 + k'\cdot\Delta\omega + \frac{1}{2}k''\Delta\omega^2]\cdot L}$

$$\sim e^{j\frac{1}{2}k''\cdot\Delta\omega^2\cdot L} \sim 1 - \frac{1}{2}k''\cdot\Delta\omega^2\cdot L$$

$$D \Rightarrow -\frac{1}{2}k'' L$$

4 SPM:

$$a'(t) = e^{j\Delta\phi(t)} \cdot a(t)$$

$$= \{1 + j\Delta\phi(t)\} a(t)$$

$$\Rightarrow \Delta a(t) = j\delta \cdot |A|^2 \cdot a(t)$$

5 SA:

$$\Delta a = r|A|^2 a(t)$$

$$\alpha_{loss} = (\alpha - r|A|^2) \Rightarrow e^{-\alpha_{loss}}$$

$$\approx 1 - (\alpha_0 - r|A|^2)$$

$$\propto 1 + r|A|^2$$

For steady state: $\sum \Delta a(t) = 0$

$$\left[-(l - jx) + g\left(1 + \frac{1}{\Omega_g^2}\frac{\partial^2}{\partial t^2}\right) + jD\frac{\partial^2}{\partial t^2} + j\delta|A|^2 + r|A|^2\right] a(t) = 0$$