

## Homework Assignment 2

Bring your homework to the classroom before the lecture begins

Due date: November 10, Wednesday, 2021

You can choose to answer the questions in English or Chinese, whichever you feel more comfortable with. You are encouraged to discuss with your classmates, but please keep in mind that for the final exam you have to do it alone. You will suffer 20% deduction on the grade of this assignment for each delayed day of submission.

Please motivate your answers with details.

1. Rosa likes strong coffee, the stronger the better. But she can't distinguish small differences. Over the years, her husband has discovered that if he changes the amount of coffee by more than one teaspoon per cup, Rosa can tell that he did it. But she cannot distinguish differences smaller than one teaspoon per cup.

Suppose that Rosa is offered cups A, B, and C. Cup A was brewed using 14 teaspoons of coffee in the pot. Cup B was brewed using 14.75 teaspoons of coffee in the pot and cup C was brewed using 15.5 teaspoons of coffee in the pot. For each of the following expressions determine whether it is true or false.

- (a) (5 points) Is Rosa's "at-least-as-good-as" relation,  $\succeq$ , transitive?
- (b) (5 points) Is Rosa's "can't-tell-the-difference" relation,  $\sim$ , transitive?
- (c) (5 points) Is Rosa's "better-than" relation,  $\succ$ , transitive?

2. Xiaoming's utility function  $u(x_A, x_B) = (x_A + 2)(x_B + 1)$ .
- (a) (3 points) Write an equation for Xiaoming's indifference curve.
  - (b) (5 points) Draw a series of indifference curves (at least three) on a graph.
  - (c) (2 points) Mark the indifference curve with  $U = 36$  on your figure.
3. Ning has utility function  $u(x_A, x_B) = x_A x_B$ . The price of  $x_A$  is  $p_A = 1$ , the price of  $x_B$  is  $p_B = 2$ . Ning's income is 40.
- (a) (3 points) Please write down Ning's budget constraint.
  - (b) (5 points) What is the best bundle that Ning can afford?
  - (c) (2 points) What is the highest utility Ning can achieve?
  - (d) (10 points) Now that the price of good  $x_B$  rose to  $p_B = 3$ . Calculate the substitution effect and the income effect of this price change.
4. Frank consumes only beef and tomatoes. He sells ice cream for a living, which provides unstable income depending on the weather. But he doesn't mind; he never thinks of tomorrow, so each week he spends as much as he earns.
- One week, when the price of beef and tomatoes were each \$2 a pound, Frank consumed 7.5 pound of each.
- The next week the price of beef rose to \$4 a pound, but the price of tomatoes remained at \$2 a pound. By chance, Frank's income had changed so that his old consumption bundle of (7.5, 7.5) was just affordable at the new prices.
- (a) (5 points) Please draw a graph of the budget lines of the two weeks, marking clearly which is the first week and which is the second.
  - (b) (5 points) Please shade the area with the bundles of goods under Frank's new budget line that he definitely will not purchase with this budget.

5. (10 points) There are two commodities  $x_1$  and  $x_2$ . We have observed that at three different price vectors  $p^0, p^1$  and  $p^2$ , the consumer has the following consumptions:

- when  $p^0 = (p_1^0, p_2^0) = (1, 2)$ ,  $x^0 = (x_1^0, x_2^0) = (1, 2)$
- when  $p^1 = (p_1^1, p_2^1) = (2, 1)$ ,  $x^1 = (x_1^1, x_2^1) = (2, 1)$
- when  $p^2 = (p_1^2, p_2^2) = (1, 1)$ ,  $x^2 = (x_1^2, x_2^2) = (2, 2)$

Did this consumer's choices violate the Weak Axiom of Revealed Preference (WARP)?

6. (2018 年期末考题) 一个人拥有 16 单位的初始资产，但他面临火灾风险：一种发生概率为 5% 的火灾会使其损失 7；另一种发生概率为 5% 的火灾会使其损失 12。他的效用函数为  $u(w) = \sqrt{w}$ 。若他买保险，保险公司要求他自己承担前  $x$  单位 ( $x < 7$ ) 的损失 (若火灾发生)，即此  $x$  部分为自付，保险公司只赔付在此之上部分的损失。

- (a) (5 points) 用  $R$  表示保费，投保人购买保险的期望效用为？
- (b) (5 points) 投保人不购买保费的期望效用为？
- (c) (5 points) 投保人愿意支付的最高保费为多少？列出等式即可，不用计算具体值。

7. A farmer thinks that in the next season, the probability of raining normally and raining rarely is the same. Assume the utility of the farmer is  $u(w) = \sqrt{w}$  where  $w$  is the income. We assume this farmer maximize utility using Expected Utility Model under risk. Here is the prospect: if he plants A, he will get 2500 yuan if the rain is normal, and 400 if the rain is rare; if he plants B, then he will get 1600 if the rain is normal, and 900 if the rain is rare.

- (a) (10 points) How to combine the plant of A and B (with proportion of  $\alpha$  and  $1 - \alpha$ ) to get maximized expected utility?
- (b) (10 points) There is an insurance for farmers who only plant A. It costs 400 yuan. In case of raining rarely, the insurance will pay 800 yuan. Can the farmer be better off by switching from the decision from (a) to only plant A and buy insurance?