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测量误差作业

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1 数据处理练习题

1.1 测量钢筒体积

1.1.1 计算结果

表 1: 习题一

项目	D/cm	d/cm	H/cm
零点读数	$D_0 = 0.000$	$d_0 = 0.000$	$H_0 = 0.000$
1	2.506	1.690	4.201
2	2.508	1.691	4.206
3	2.504	1.688	4.203
4	2.505	1.692	4.202
5	2.508	1.688	4.202
6	2.505	1.688	4.200
平均值	2.506	1.690	4.202
平均值的标准差	0.0007	0.0008	0.0009
考虑仪器允差后的标准差	0.0013	0.0014	0.0015
修正零点后的平均值	2.506	1.690	4.202

$\bar{D} \pm \sigma_{\bar{D}} = (2.5060 \pm 0.0013)\, cm$
$\bar{d} \pm \sigma_{\bar{d}} = (1.6900 \pm 0.0014)\, cm$
$\bar{H} \pm \sigma_{\bar{H}} = (4.2020 \pm 0.0015)\, cm$
$V = \dfrac{\pi}{4}\left(\bar{D}^2 - \bar{d}^2\right)\bar{H} = 11.300 cm^3$
$$\sigma_V = V\sqrt{\left(\frac{2\bar{D}\sigma_{\bar{D}}}{\bar{D}^2-\bar{d}^2}\right)^2 + \left(\frac{2\bar{d}\sigma_{\bar{d}}}{\bar{D}^2-\bar{d}^2}\right)^2 + \left(\frac{\sigma_{\bar{H}}}{\bar{H}}\right)^2} = 0.027 cm^3$$
$V \pm \sigma_V = (11.300 \pm 0.027)\, cm^3$

1.1.2 计算过程

$\bar{D} = \frac{\sum_{i=1}^{6} D_i}{6} = 2.506 cm$
$\bar{d} = \frac{\sum_{i=1}^{6} d_i}{6} = 1.690 cm$
$\bar{H} = \frac{\sum_{i=1}^{6} H_i}{6} = 4.202 cm$
$V = \dfrac{\pi}{4}\left(\bar{D}^2 - \bar{d}^2\right)\bar{H} = 11.300 cm^3$
$$\sigma_{\bar{D}A} = \sqrt{\frac{\sum_{i=1}^{6}\left(D_i - \bar{D}\right)^2}{6 \times 5}} = 0.0007 cm$$
$\sigma_{\bar{D}B} = \sigma_{\bar{H}B} = \sigma_{\bar{d}B} = \frac{\Delta}{\sqrt{3}}$
$\sigma_{\bar{D}} = \sqrt{\sigma_{\bar{D}A}^2 + \sigma_{\bar{D}B}^2} = 0.0013 cm$

同理,$\sigma_{\bar{d}A} = \sqrt{\dfrac{\sum_{i=1}^{6}\left(d_i - \bar{d}\right)^2}{6 \times 5}} = 0.0008cm$

$\sigma_{\bar{d}} = \sqrt{\sigma_{\bar{d}A}^2 + \sigma_{\bar{d}B}^2} = 0.0014cm$

$\sigma_{\bar{H}A} = \sigma_{\bar{H}A} = \sqrt{\dfrac{\sum_{i=1}^{6}\left(H_i - \bar{H}\right)^2}{6 \times 5}} = 0.0009cm$

$\sigma_{\bar{H}} = \sqrt{\sigma_{\bar{H}A}^2 + \sigma_{\bar{H}B}^2} = 0.0015cm$

$\sigma_V = V\sqrt{\left(\dfrac{2\bar{D}\sigma_{\bar{D}}}{\bar{D}^2 - \bar{d}^2}\right)^2 + \left(\dfrac{2\bar{d}\sigma_{\bar{d}}}{\bar{D}^2 - \bar{d}^2}\right)^2 + \left(\dfrac{\sigma_{\bar{H}}}{\bar{H}}\right)^2} = 0.027cm^3$

1.2 钢球体积

1.2.1 已知数据表

表 2: 习题二

n	1	2	3	4	5	6	平均值
$d(cm)$	3.252	3.254	3.252	3.250	3.252	3.252	3.252

零点值 $d_0 = 0.002cm$, 修正后 $\bar{d} = 3.2500cm$

$\sigma_{\bar{d}} = 0.0013cm$

1.2.2 测量结果

$\bar{d} \pm \sigma_{\bar{d}} = (3.2500 \pm 0.0013)\, cm$

$V = \dfrac{\pi}{6}\bar{d}^3 = 17.9742cm^3$

$\sigma_V = V\sqrt{\left(\frac{3\sigma_{\bar{d}}}{\bar{d}}\right)^2} = 0.022cm^3$

$V \pm \sigma_V = (17.974 \pm 0.022)\, cm^3$

1.3 分析与讨论

1.3.1 钢筒体积误差分析

根据计算结果可知, 未定系统误差即仪器的允差对于实验过程中的不确定度影响更大, 随机误差影响更小.

1.3.2 钢球体积误差分析

在测量钢球体积实验中, 系统误差主要来源于零点误差, 而零点值产生系统误差大于测量过程中的随机误差, 所以随机误差影响更小.

1.3.3 系统误差与随机误差

系统误差为相同条件下多次测量一个物理量时测量值对真值的偏离总是相同的, 来源有: 理论公式的近似性, 仪器结构不完善, 环境条件改变等多方面, 还有一类系统误差为仪器的允差. 系统误差通常而言较大, 而且不能通过增加测量次数的方法来减小.

随机误差是由于不确定因素导致的每次测量值的无规律涨落, 测量值对真值偏差时大时小且无法预测. 来源有：测量者感官分辨率的涨落, 环境条件的微小波动等等. 随机误差最大的特点是随机性, 因此只要增加测量次数, 就可以减小随机误差, 这类误差通常较系统误差而言较小.

2 教材课后习题

1.(1)1 位 (2)4 位 (3)2 位 (4)6 位

2.(1)c=$\frac{ab}{b-a} = 10.0cm(\sigma_c = \sqrt{(\frac{b^2\sigma_a}{(b-a)^2})^2 + (\frac{a^2\sigma_b}{(b-a)^2})^2} = 0.1cm)$

(2)$y = e^{-x^2} = 8 \times 10^{-38}(|\sigma_y| = 2x \times 0.01 \times e^{-x^2} = 2 \times 10^{-38})$

(3)$y = lnx = 4.037(\sigma_y = \frac{0.1}{x} = 0.002)$

(4)$y = cosx = 0.98657(\sigma_y = sin9.4^\circ \times \frac{1}{60} \times \frac{\pi}{180} = 0.00005)$

3.$(b)\sigma_\rho = \dfrac{m_1}{m_1 - m_2}\rho_0\sqrt{\left(\dfrac{m_2\sigma_{m1}}{m_1(m_1-m_2)}\right)^2 + \left(\dfrac{\sigma_{m2}}{m_1-m_2}\right)^2}$

$(c)\sigma_y = \sqrt{\left(\dfrac{b\sigma_a}{a(a+b)}\right)^2 + \left(\dfrac{a\sigma_b}{b(a+b)}\right)^2}$

4.

$L = \dfrac{L_1 + L_2}{2}$

$\sigma_L = \sqrt{(\dfrac{\sigma_{L_1}}{2})^2 + (\dfrac{\sigma_{L_1}}{2})^2} = 0.6\mu m$

5.

$S = L_1L_2 - \frac{\pi}{4}(d_1^2 + d_2^2)$

$\dfrac{\sigma_S}{S} = \sqrt{(\frac{L_2\sigma_{L_1}}{S})^2 + (\frac{L_1\sigma_{L_2}}{S})^2 + (\frac{\pi d_1\sigma_{d_1}}{2S})^2 + (\frac{\pi d_2\sigma_{d_2}}{2S})^2}$

要求 $\dfrac{\sigma_S}{S_{max}} = 0.5\%$

代入数据可得:$\sigma_{d_2} = 1cm$, 因此, 测量小孔直径用游标卡尺即可

7.

(1)$g_0 = \frac{2h}{t^2}$

$\sigma_g = g_0\sqrt{(\dfrac{\sigma_h}{h})^2 + (\dfrac{2\sigma_t}{t})^2} = g_0\sqrt{10^{-8} \times 2} = 0.1cm/s^2$

$g = g_0 + \sigma_g = 980.1cm/s^2$

(2)$(a)\frac{1}{4}sin^2(\frac{\theta}{2}) + \frac{9}{64}sin^4(\frac{\theta}{2}) \leqslant 0.5\%$

可得:$\theta \leqslant 16.17^\circ$

$(b)\frac{1}{4}sin^2(\frac{\theta}{2}) + \frac{9}{64}sin^4(\frac{\theta}{2}) \leqslant 0.05\%$

可得:$\theta \leqslant 5.12°$

10.

经过线性拟合过后,$y_i = b_0 + \lambda i$

$b_0 = 18.716mm, \lambda = 8.753mm, r = 0.999969$

$\sigma_{\lambda A} = \lambda\sqrt{\dfrac{\frac{1}{r^2}-1}{n-2}} = 0.024mm$

$\sigma_{\lambda B} = \frac{\sqrt{e^2+e_{y_i}^2}}{\sqrt{3}} = 0.006mm$

$\sigma_\lambda = \sqrt{\sigma_{\lambda A}^2 + \sigma_{\lambda B}^2} = 0.025mm$

$c = f\lambda = 346.44m/s$

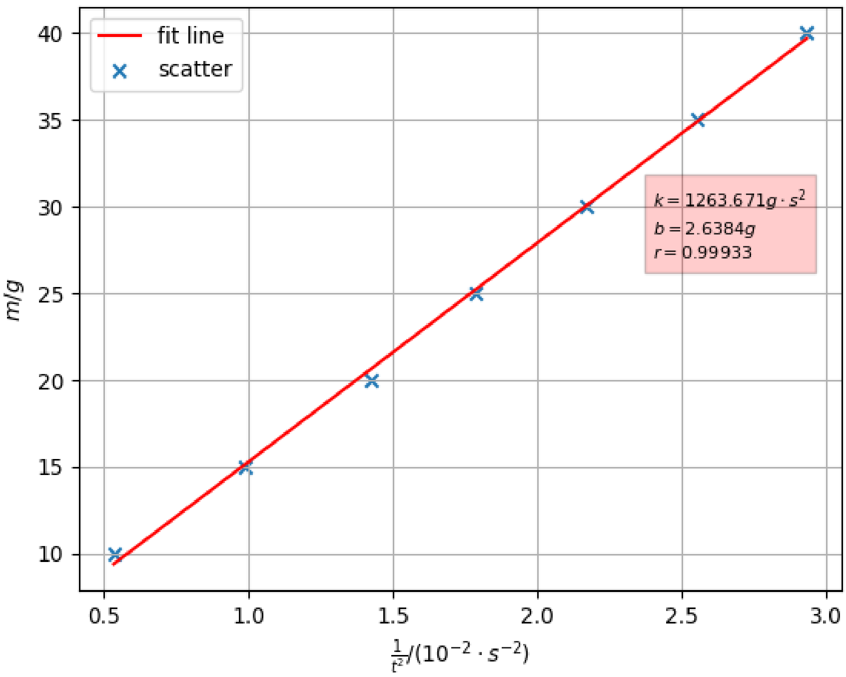
$\sigma_c = c\sqrt{(\frac{\sigma_\lambda}{\lambda})^2 + (\frac{\sigma_f}{f})^2} = 1.0m/s$

$c = (346.4 \pm 1.0)m/s$

11.

(1)

图 1: m~$\frac{1}{t^2}$ 关系图



因此观察图形可知此关系为线性关系

(2)

$\bar{x} = 25g, \bar{y} = 1.770 \times 10^{-2}s^{-2}$

$\bar{xy} = 52.14 \times 10^{-2}g \cdot s^{-2}, \bar{x^2} = 725g^2, \bar{y^2} = 3.757 \times 10^{-4}s^{-4}$

$k_2 = \dfrac{\bar{xy} - \bar{x}\bar{y}}{\bar{x^2} - (\bar{x})^2} = 7.89 \times 10^{-4}/(g \cdot s^2)$

$b_1 = \bar{y} - k_2\bar{x} = -2.03 \times 10^{-3}s^{-2}$

$r_1 = \dfrac{\bar{xy} - \bar{x}\bar{y}}{\sqrt{[\bar{x^2} - (\bar{x})^2][\bar{y^2} - (\bar{y})^2]}} = 0.99933$

(3)

由 (2) 中的数据, 可得 x 与 y 交换位置即可:

$k_1 = \dfrac{\bar{xy} - \bar{x}\bar{y}}{\bar{y^2} - (\bar{y})^2} = 1.26 \times 10^3 g \cdot s^2$

$b_1 = \bar{x} - k_2\bar{y} = 2.62g$

$r_2 = \dfrac{\bar{xy} - \bar{x}\bar{y}}{\sqrt{[\bar{x^2} - (\bar{x})^2][\bar{y^2} - (\bar{y})^2]}} = 0.99933$

相关系数相同, 因为相关系数对于 x 和 y 是对称的, 所以两种情况将 x 和 y 互换不影响相关系数大小, 关系式为:$k_1k_2 = r^2$