Lecture 12-II

Quantum Interference Effect

(II)

Outline

- Weak Localization
- Universal Conductance Fluctuations
- Ballistic transport

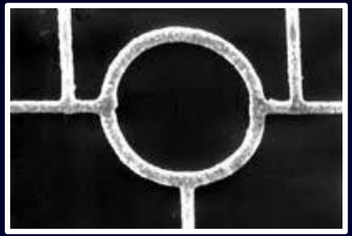
Outline

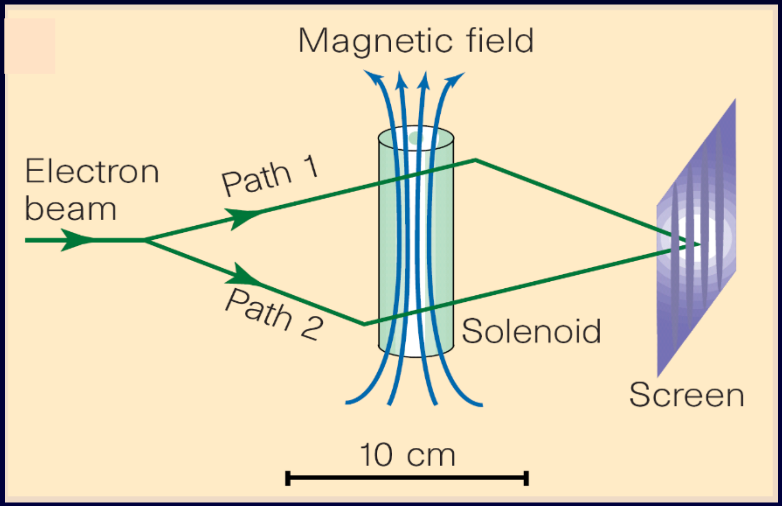
- Weak Localization
- Universal Conductance Fluctuations
- Ballistic transport

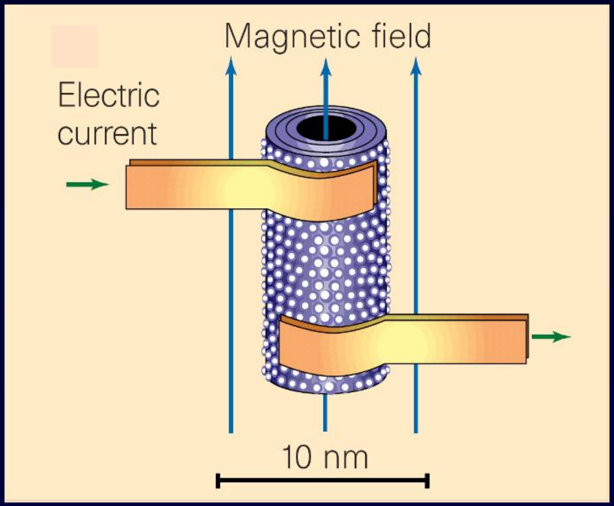
Aharonov-Bohm (A-B) Effect:

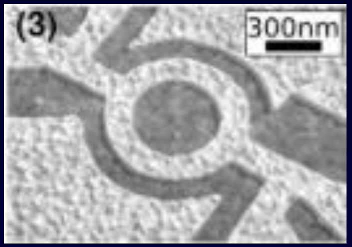
带电粒子在零磁场区域(但非零矢势)运动时产生的相位效应;

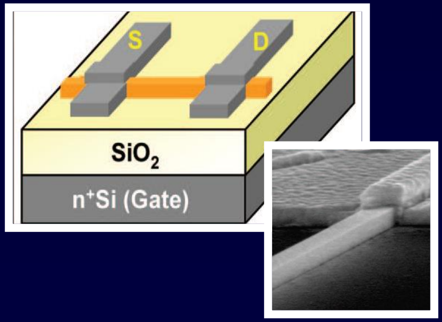
由电磁场矢势和标势引起的量子干涉效应





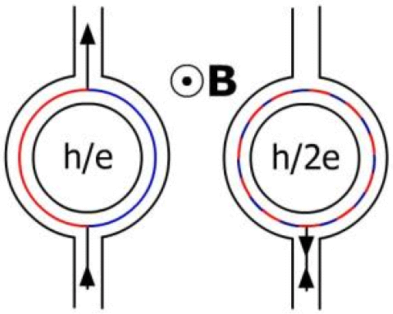






Observing the Aharonov–Bohm effect— the quantum mechanical influence of a magnetic field on the motion of electrons.

AB & AAS oscillations



↑ AB　　↑ AAS

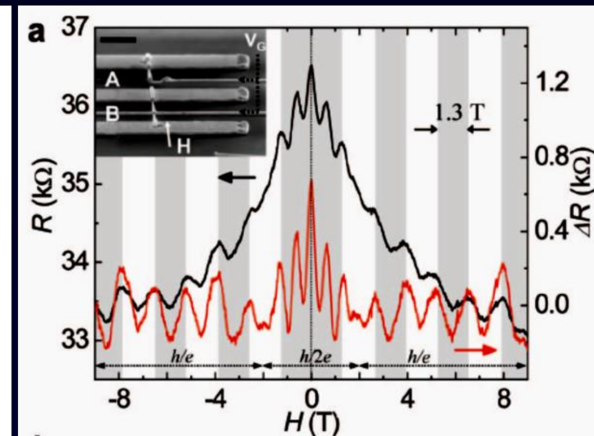
- AAS = Altshuler-Aronov-Spivak
（前苏联学者在81年预言的新效应）
- interference of two electron waves going one full circle clockwise and counterclockwise
- identical paths
$\Rightarrow$ phase difference at $B = 0$ is 0

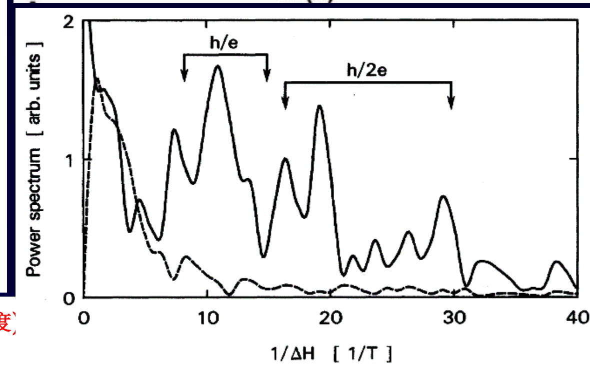
- possible oscillations:

round trips	period (in Φ)	type
1/2	h/e	AB
1	$h/2e$	AAS
3/2	$h/3e$	AB
...		

- number of round trips limited by L_φ

L_ϕ **Phase-coherence length**(相位相干长度)
Distance over which the electrons maintain their phase coherence





Φ_0= h/e

Flux quantum
磁通量子

金属薄壁的磁致电阻随磁通量变化呈现周期为 $\Phi_0/2$ 振荡—AAS effect

AAS oscillations, Why?

- **Weak localization**
- **Interference of scattered electron waves propagating along different paths**

弱定域化(Weak Localization):

在低温下，金属导体中电子在弹性散射占主导时，在传导途中由于陷入闭合路径，但不形成稳定的定域态，仍能继续参与导电，从而使电导率减小的现象。

弹性散射占主导

金属导体中电子

传导途中由于陷入闭合路径

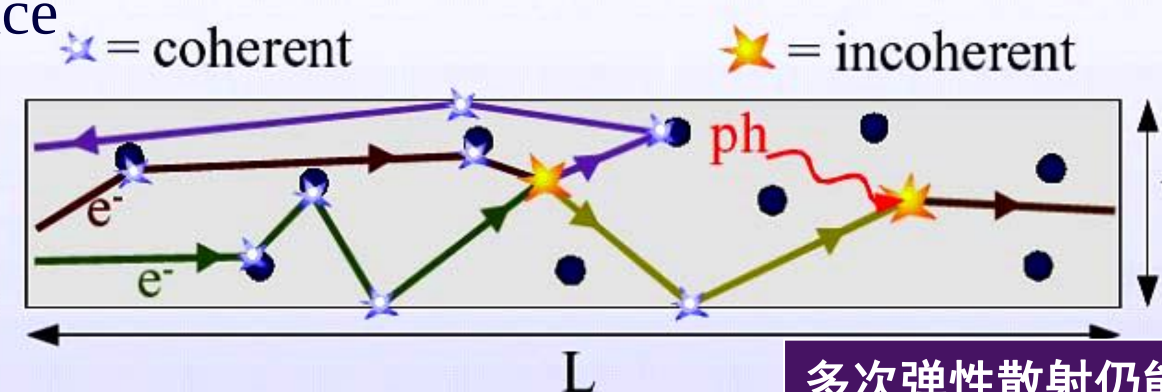
Scattering mechanisms & lengths

Elastic scattering & Inelastic scattering

(改变电子运动方向、电子能量保持不变，散射波与入射波之间的相位有确定的关系)
(电子在散射前后运动方向和能量都发生变化)

mechanism		charact. length	typical (wire)
specimen border	coherent	$L \times W \times t$	$(500 \times 50 \times 25)\ \mathrm{nm}^3$
e^--impurity/**defect**	coherent	ℓ	$10\ \mathrm{nm} - \ \mu\mathrm{m}$
e^--phonon-scatt.	incoherent	L_{ph}	$3\ \mu\mathrm{m}\ (T/\mathrm{K})^{-2}$
e^--e^--scatt.	incoherent	L_{ee}	$3\ \mu\mathrm{m}\ (T/\mathrm{K})^{-1/3}$
$\hookrightarrow$ dephasing	incoherent	L_φ	$1\ \mu\mathrm{m}\ (1\ \mathrm{K})$

e^--lattice



多次弹性散射仍能记住原来相位，
此对电导能力有重要影响

金属中参与导电的是费米能附近的电子(λ_F, v_F)：

在准经典的描述中，金属中的电子看作一个粒子，

其轨迹为经典的无规则行走路径。

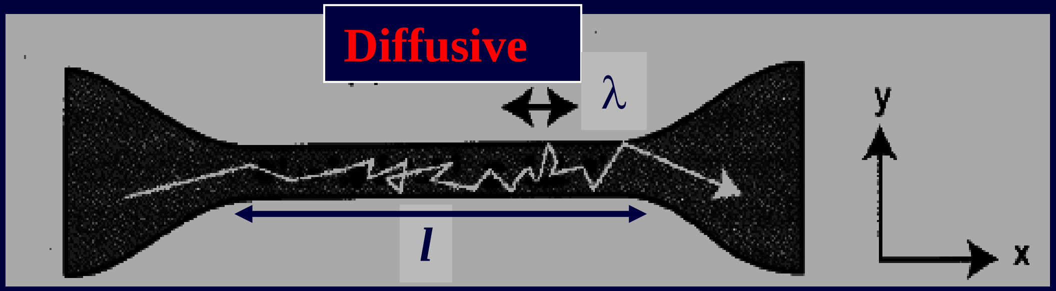
电子传导电流是作为无规则行走的扩散流

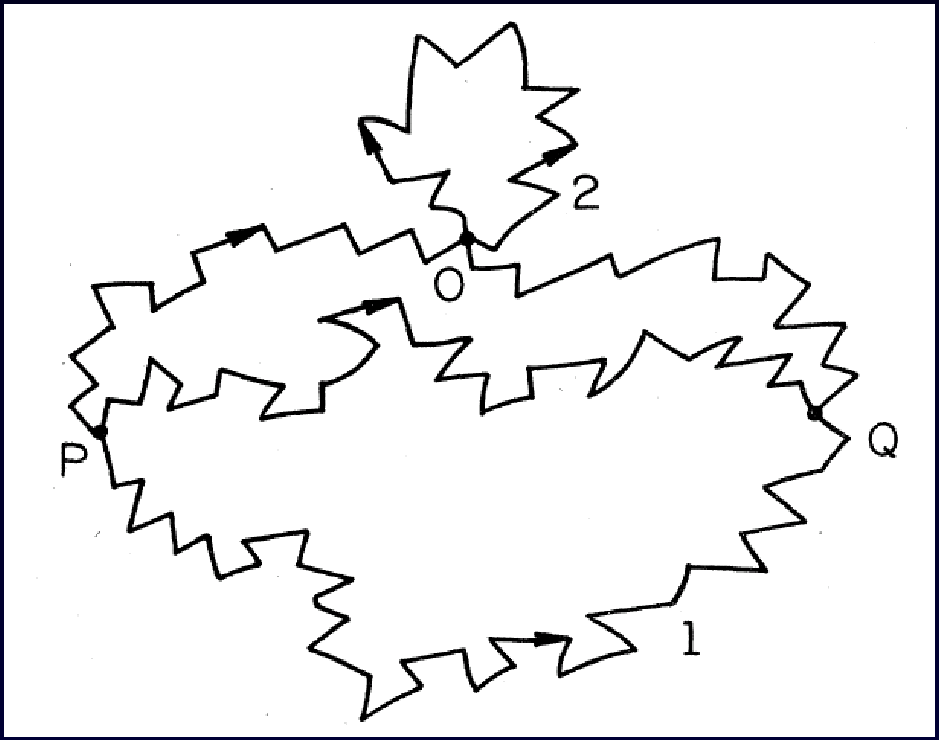
Quasiclassical condition:

$$\lambda \ll l$$

λ-electron wavelength;

l-mean free path





Different type of quasiclassical particle trajectories connecting P and Q

Point O is the trajectory self-crossing point.

In the classical theory of transport phenomena the total probability for a particle to transfer from point P to point Q is the sum of probabilities of such a transfer over all possible trajectories.

Feynman path

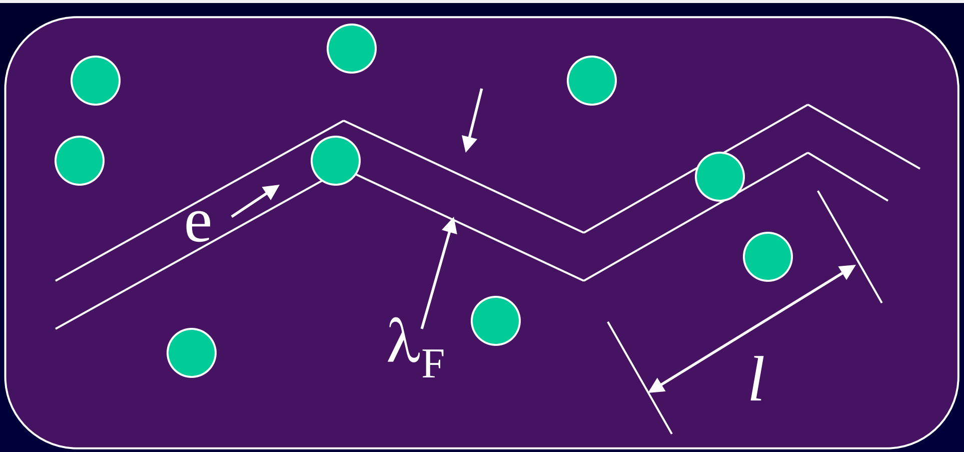
按照Feynman 对量子力学的表述，粒子在两点之间的输运，要考虑所有可能的路径，各路径对两点间总传播几率幅的贡献有相同的权重

Feynman特点：

(1)路径是直径为 λ_F 的管子(通道)；

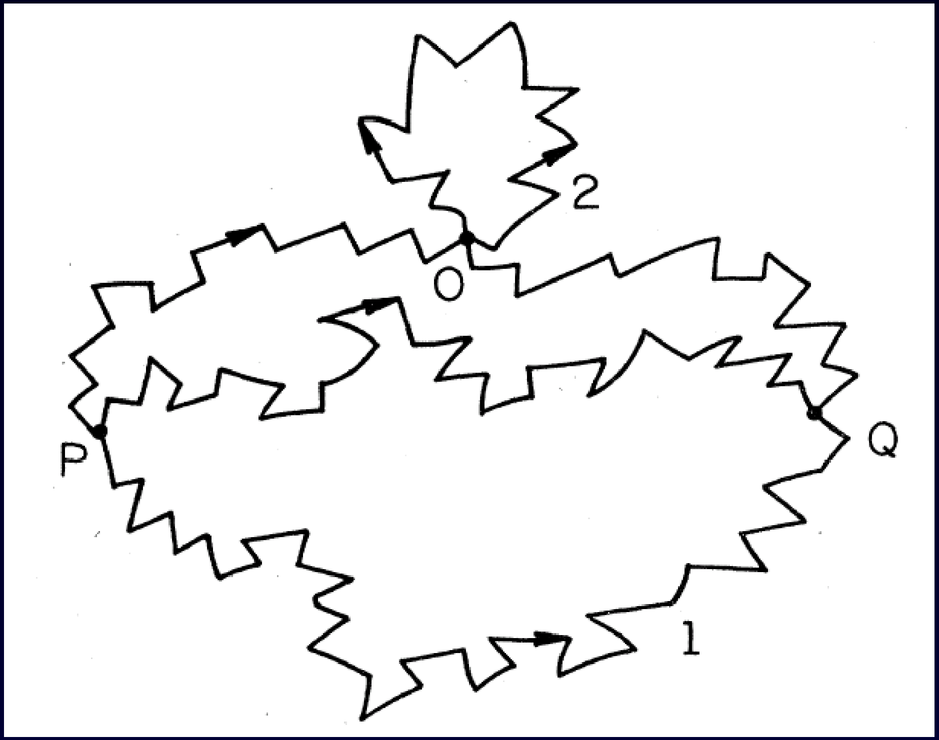
(2)管内所有路径相位大致相同，几率幅相加时不会抵消

(3)管外路径，稍有差别的路径间相位有明显改变，总贡献相消



按其解释，经典路径是极值路径，管内路径稍有变化(偏离 $< \lambda_F$)，相位不变的。

依据费因曼对量子力学用路径观念的表述，量子效应表现在相位因子依赖于电子行走的时间和路径



Different type of quasi-classical particle trajectories connecting P and Q

Point O is the trajectory self-crossing point

电子从P点到Q点可循不同路径无规则向前行走；

设第(i)条路径行走的概率幅为：

$$A_i = |A_i| e^{i\varphi_i}$$

φ_i: 与第i条路径相关的相位因子

In quantum mechanics:

should include interference of scattered electron waves propagating along different paths：

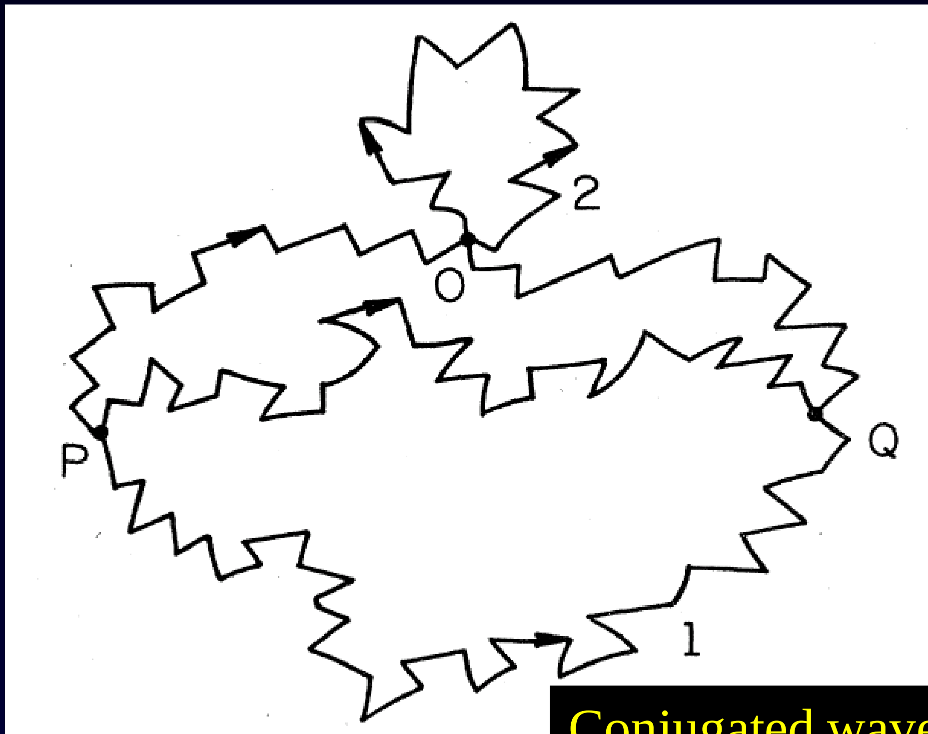
总的渡越概率P是所有可能路径几率幅相加的绝对值的平方

$$P(r,r',t)=\left|\sum_i A_i\right|^2=\sum_i |A_i|^2+\sum_{i\neq j} A_i A_j^*$$

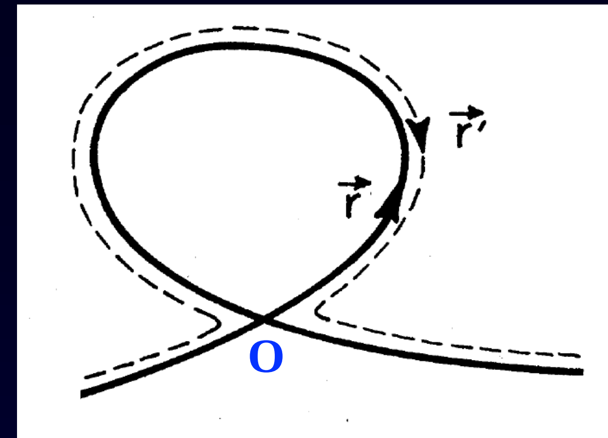
Classical probability

Interference contribution

$\sum_{i\neq j} A_i A_j^*$ – Vanishes for most paths since phases are almost random



Conjugated waves



Propagation along solid and dashed paths can interfere.

电子传导途中在O点陷入一个闭合路径，电子可顺时针或逆时针方向经多次散射回到O点；电子沿着$\vec{r}$和$\vec{r}'$过程相当于时间反演电子从***k***态到-***k***态的背散射。

For self-crossing trajectories (trajectory 2), <u>the wave interference turns out to be essential</u>. <u>The two waves propagating along such trajectories in two opposite directions</u> <u>accumulate the same phase difference</u>.

电子由O点出发经顺时针和逆时针两种方式回到O点的概率幅绝对值相等；在两种方式中相应区段，对应于的背散射，具有时间反演对称性，整个闭合路径中方向相反两过程的相移相同

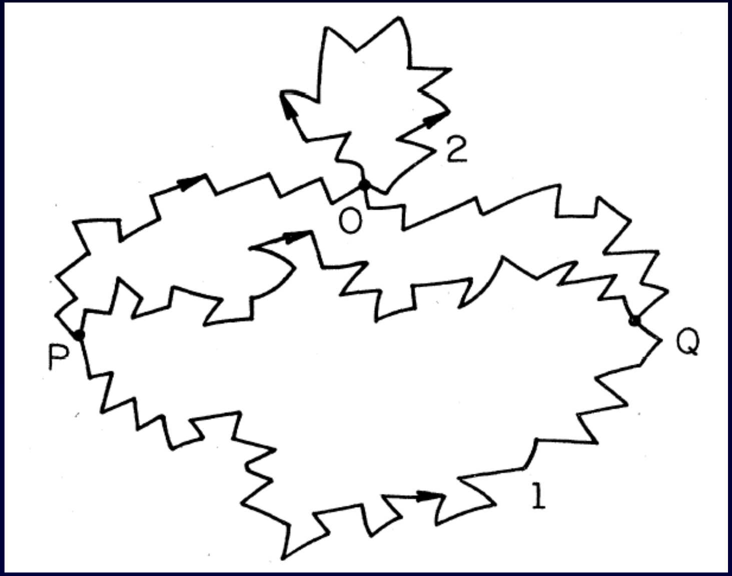
第(i)条路径行走的概率幅为：

$$A_i = |A_i| e^{i\varphi_i}$$

φ_i: 与第i条路径相关的相位因子

For $i \to i+1$ and $i+1 \to i$:

$$|A_-| = |A_+| = |A| \qquad \Delta\varphi_+ = \Delta\varphi_- ;$$



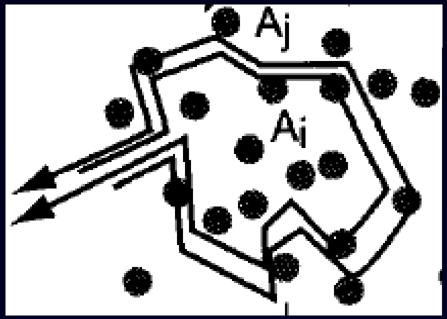
$$P(r, r', t) = \left|\sum_i A_i\right|^2 = \sum_i |A_i|^2 + \sum_{i \neq j} A_i A_j^*$$

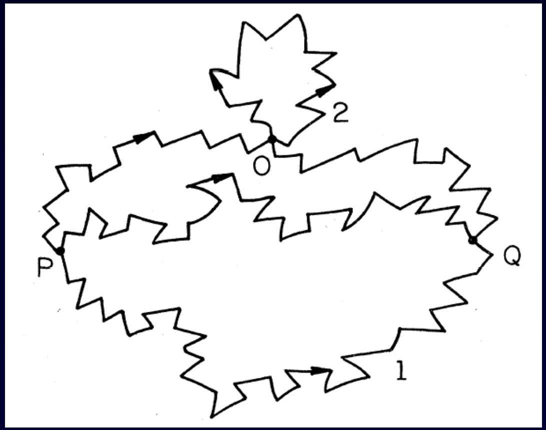
The contribution of these trajectories to the probability of coming to self-intersecting point will be:

$$|A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2\,\mathrm{Re}A_1^* A_2 = 4|A_1|^2$$

A higher probability of returning back to point O means a lower probability of transfer from point P to point Q. Thus the interference of conjugated waves favors particle localization

The amplitude A_j is just a time reversal of A_i:





$$\left|\mathbf{A}_i+\mathbf{A}_j\right|^2=\left|\mathbf{A}_i+\mathbf{A}_j^*\right|^2=\mathbf{4}\left|\mathbf{A}_i\right|^2$$

The backscattering probability is enhanced by a factor of 2 of the classical probability ($2|\mathbf{A}_i|^2$)!

概率增大源于闭合路径中顺、逆时针两过程波函数的干涉效应。

电子在途中逗留的概率增加了，意味在终点Q找到电子的概率下降了，在宏观现象上表现为电导有所减小。

弱定域化(weak localization):

在低温下，金属导体中电子<u>在弹性散射占主导时</u>，在传导途中由于陷入<u>闭合路径</u>，但不形成稳定的定域态，仍能继续参与导电，从而使电导率减小的现象。

弱定域化的物理图像：

闭合路径中，两时间反演电子波的相位相干现象

是对载流子输运过程的量子力学修正，或者说对经典电导率的量子力学修正(与路径自相交的几率成比例)，是量子力学波函数叠加原理导致宏观可观察量变化的独特范例。

弱定域化发生在金属扩散区：$l < L < \xi$

L:体系的尺度; l: 电子所受弹性散射的平均自由程;
ξ: 定域化长度

弱定域中弱的含义：

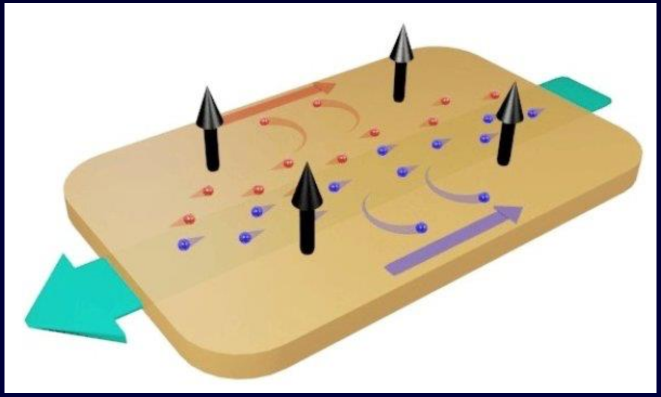
(1) This effect is called the weak localization since the relative number of closed loops is small.

(2) The effect is sensitive to very weak magnetic fields.

Magnetoresistance induced by weak localization

Magnetoresistance is the tendency of a material to change the value of its electrical resistance in an externally-applied magnetic field.

磁致电阻：研究输运现象的一个重要工具，电子在磁场中运动时由于受到洛伦兹力的作用，产生回旋运动，从而增加了电子受到散射的几率，导致电阻上升。



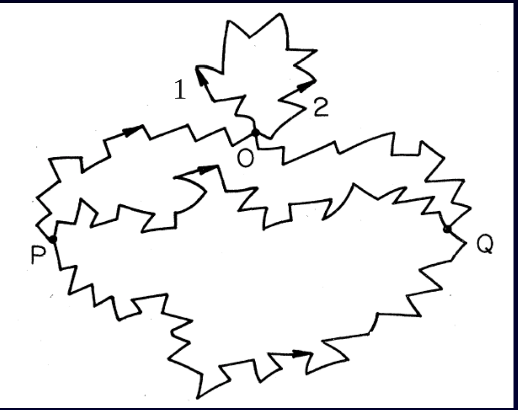
The weak localization effect is sensitive to very weak magnetic fields.

Magnetoresistance induced by weak localization

If the sample is placed in a magnetic field, then the amplitudes of the probability of completing the loop on contour 2 of the Fig. (right below), clockwise and counterclockwise, acquire additional phase factors:

$$A_1 \rightarrow A_1 \exp\left[\frac{ie}{\hbar c}\oint \mathbf{A}\, dl\right] = A_1 \exp\left[i\frac{2\pi\phi}{\phi_0}\right]$$

$$A_2 \rightarrow A_2 \exp\left[-i\frac{2\pi\phi}{\phi_0}\right]$$



where the magnetic flux through the loop is $\phi = HS$

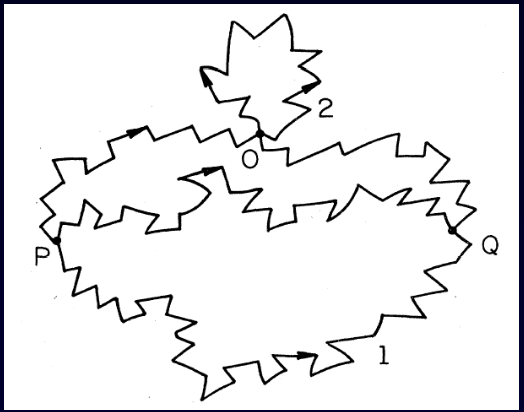
S： the projection of the loop area on the plane perpendicular to the magnetic field direction； H： magnetic field

Equation above implies that the phase difference between the conjugated waves is:

$$\Delta\varphi = 2\pi\frac{2\phi}{\phi_0} = \frac{2\pi\emptyset}{\emptyset_0/2}$$

The probability for a particle coming to O point :

$$\boxed{P(O) = 2|A|^2\left[1 + \cos\left(\frac{2\pi\Phi}{\Phi_0/2}\right)\right]}$$



(Where Φ_0=h/e, magnetic quantum flux 磁量子通量)

Compared with the case without applying the magnetic field:

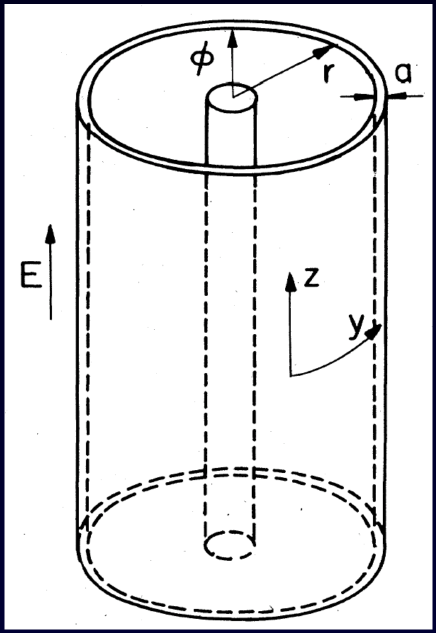
$$\boxed{\left[1 + \cos\left(\frac{2\pi\Phi}{\Phi_0/2}\right)\right] \le 2}$$

磁场破坏了时间反演对称性，导致电子回到闭合路径始点的概率有所下降、并随磁场变化而振荡(周期h/2e)—导致负磁阻

此效应在正常磁阻可忽略的很弱的磁场下即可出现，是弱定域化存在的重要证据。

Example 1

Magnetoresistance oscillation



Schematic of experiment with a solenoid carrying magnetic flux ϕ inside a cylinder with wall thickness a.

A hollow metal cylinder with a hole enclose a long solenoid carrying a magnetic flux so that magnetic field is zero everywhere outside that solenoid. However, the vector potential *A* within the sample should be non zero

If mean free path *l* is smaller than the cylinder circumference, the size effects of pure metals may be neglected.

Longitudinal magnetoresistance $\Delta R(H)$ at $T=1.1$ K for a cylindrical lithium film evaporated onto a 1-cm-long quartz filament. $R_{4.2}=2$ kΩ, $R_{300}/R_{4.2}=2.8$. Solid line: averaged from four experimental curves. Dashed line: calculated for $L_\varphi=2.2$ μm, $\tau_\varphi/\tau_{so}=0$, filament diameter $d=1.31$ μm, film thickness 127 nm. Filament diameter measured with scanning electron microscope yields $d=1.30\pm0.03$ μm

magnetic field scales:	diameter	(h/2e)
classic experiment	1 μm	~ 2 mT
multiwall nanotubes	15 nm	~ 10 T
single wall nanotubes	3 nm	~ 400 T

Experiment: Sharvin & Sharvin 1981
Theory: Altshuler, Aronov & Spivak 1981

$$B_0=\Phi_0/\pi r^2$$

It was predicted however that, for the particular geometry of a small, thin-walled hollow metallic cylinder in an external magnetic field, the A–B effect should cause the electrical conductance to oscillate with the magnetic flux through the cylinder's bore.

When researchers evaporated a normal metal film (Li) onto a quartz fiber they found exactly this behaviour.

Metal film (20 nm thick) deposited on the surface of a cylinder (1μm in diameter);

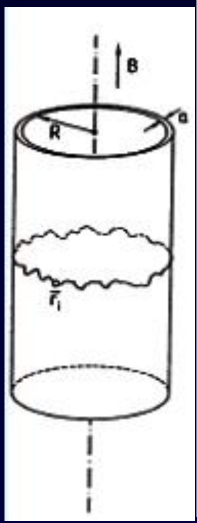
Quantum interference in thin metallic cylinders

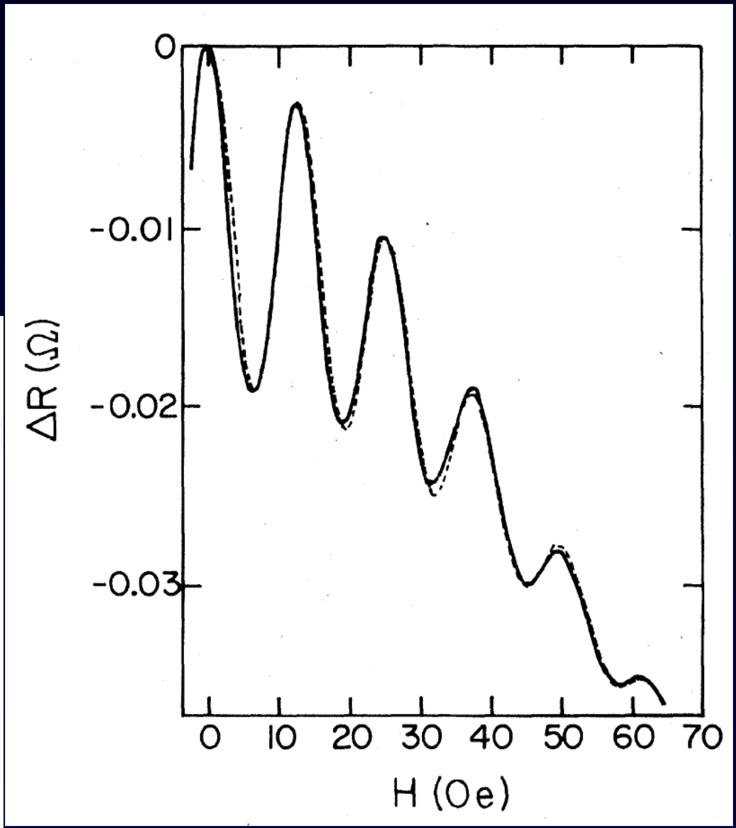
--> resistance oscillations with a period of h/2e

Conductance of metallic cylinders
vs magnetic field:

$\Phi_0/2$ ---oscillations

(Where Φ_0=h/e -- magnetic quantum flux量子通量)





奥斯特

(1) The oscillation phase was successfully reversed and a negative sign achieved for the monotonic magnetoresistance(MR).

(2) The phase difference accumulated by conjugated waves going around the cylinder is

$$\Delta\varphi = 2\pi\frac{2\phi}{\phi_0} = \frac{2\pi\emptyset}{\emptyset_0/2}$$

i.e., it is the same for all conjugated waves running around the cylinder. As a result, the cylinder resistivity will oscillate with the period $\phi_0/2$.

Solid line: averaged from four experimental curves. Dashed line: calculated for $L_\varphi=2.2$ μm, $\tau_\varphi/\tau_{so}=0$, filament diameter $d=1.31$ μm, film thickness 127 nm. Filament diameter measured with scanning electron microscope yields $d=1.30\pm0.03$ μm

(3) 磁场增大时，磁阻幅度变小: 由于金属薄膜有一定的厚度，可能使闭合路径包围的面积不严格相等，磁场加强时，不同回路磁阻振荡的不同步性加剧。

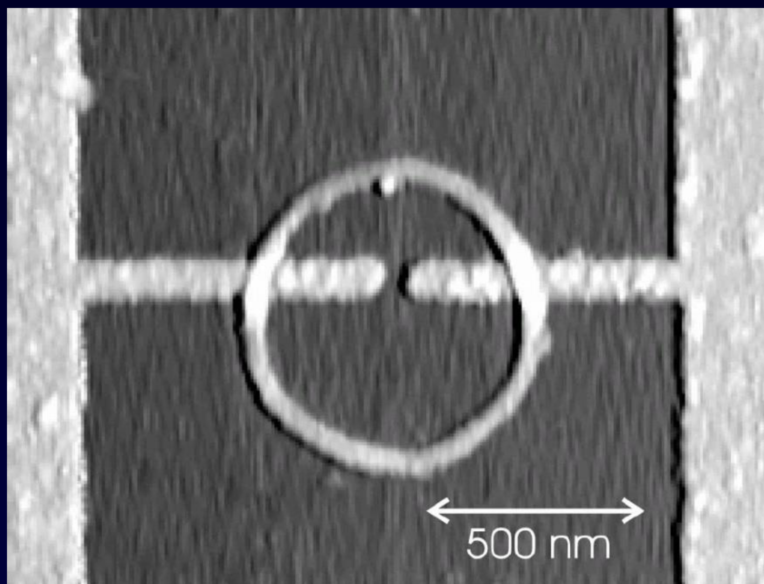
(4) 磁场很强时，与闭合路径相联系的电子弱定域化遭受破坏，ASS效应消失。

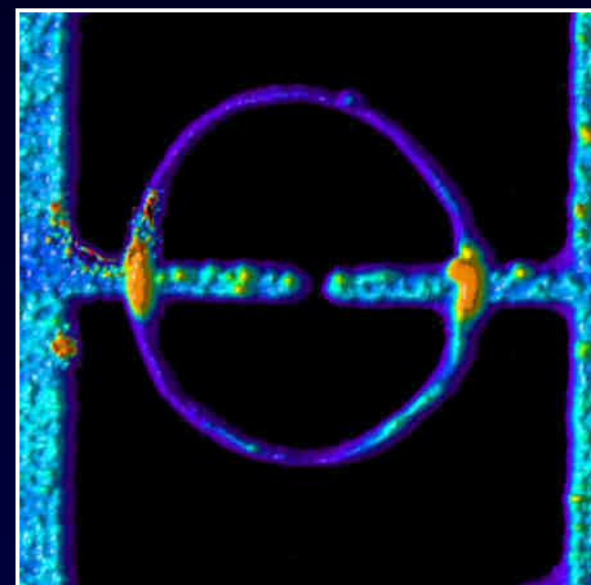
Negative MR is characteristic of materials in a state of weak localization

Example 2

One-Dimensional Localization

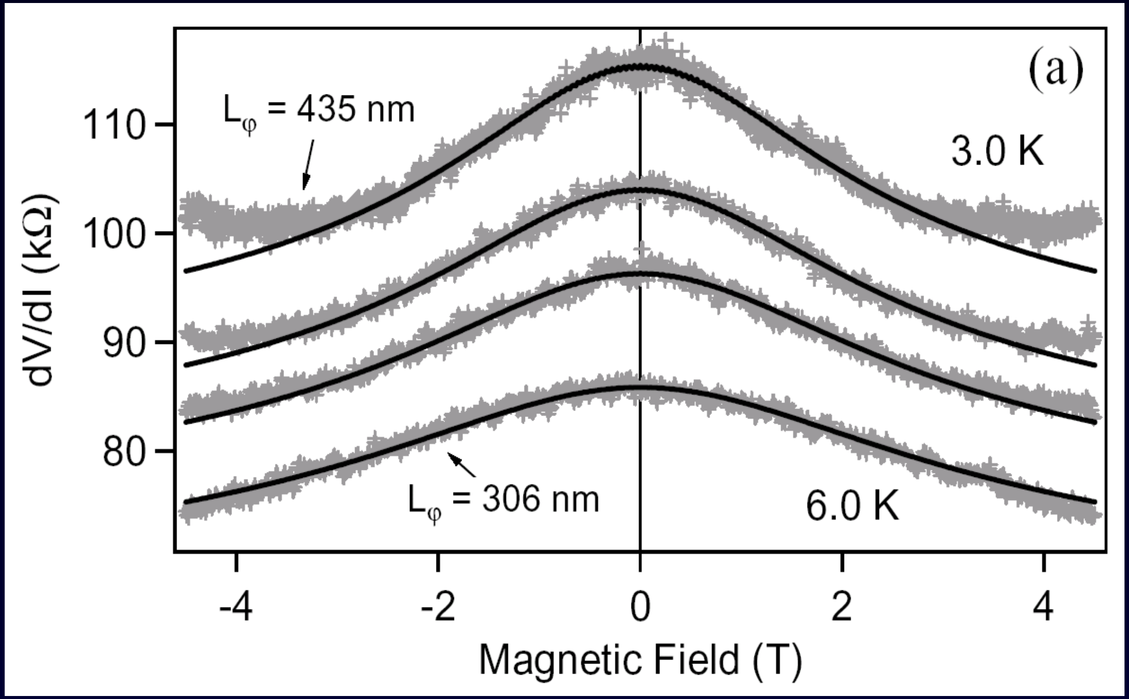
The closed ring geometry in principle may provide an additional path for interference.





AFM image of a nanotube ring spanning two gold electrodes

Ti/Au electrodes (25 nm thick) are patterned by *e*-beam lithography on 100 nm of SiO_2 grown on degenerately doped Si. Typical ring resistances range from 20 to 50 kV at 300 K.



From the effect of the field on the resistance, the coherence length $L\varphi$ can thus be obtained.

Negative MR is characteristic of materials in a state of weak localization---one-dimensional weak localization (1D-WL).

The solid black lines are fits to the 1D weak localization theory using w= 1.4 nm. Similar fits were performed using different wall thicknesses w between 1.4 and 20 nm. For example, the obtained values of L_φ at 3.00 K are 342 nm for w=2 nm and 216 nm for w=3 nm.

Weak localization was observed which allowed the coherence length $L\varphi$ in MWNTs to be determined：

The change of the conductance ΔG(H) of a unit length along the circumference of a metallic ring of radius *R* and of wall thickness *w* smaller than $L\varphi$ can be written as:

$$\Delta G(H) = -\frac{2e^2 L_\varphi(H)}{h} \frac{\sinh\left(\frac{2\pi R}{L_\varphi(H)}\right)}{\cosh\left(\frac{2\pi R}{L_\varphi(H)}\right) - \cos\left[2\pi \frac{2\Phi}{\Phi_0}\right]} \qquad (1)$$

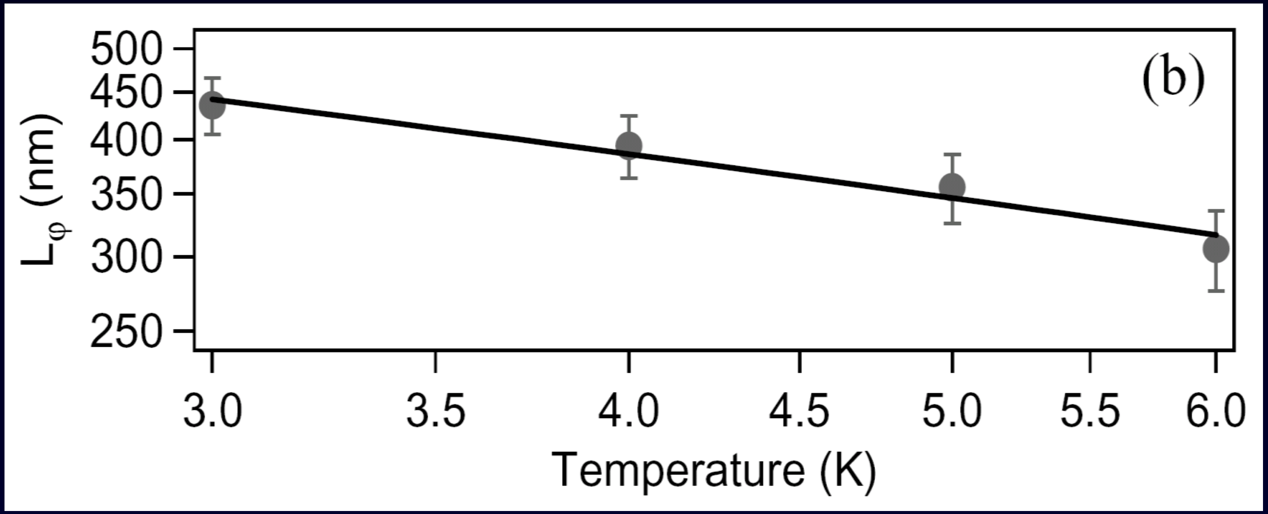
Where: *H*: magnetic field (perpendicular to the ring through which a flux Φ passes);

$\Phi_0 = h/e$ (flux quantum 量子通量);

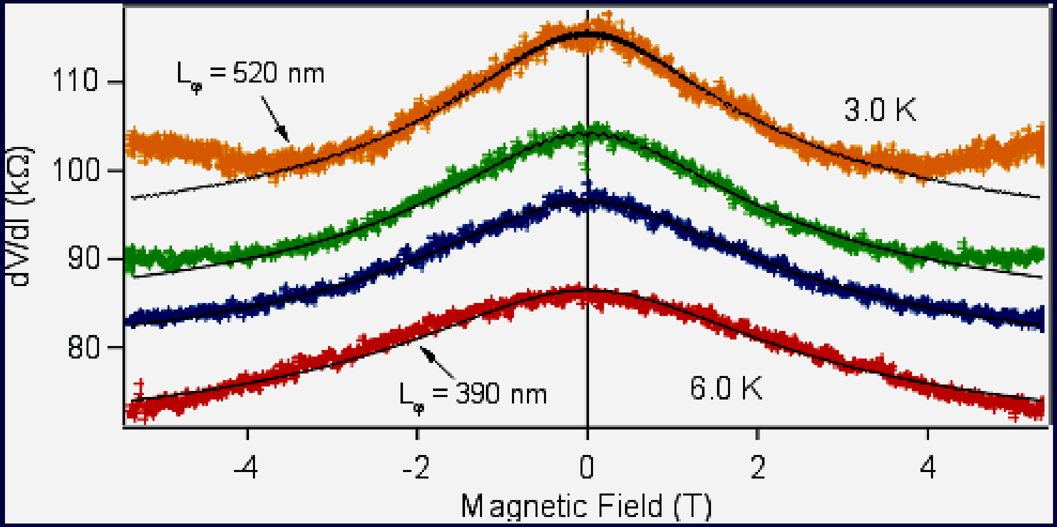
$L_\varphi(H)$: coherence length (magnetic field dependent);

w: wall thickness;

$$\frac{1}{L_\varphi^2(H)} = \frac{1}{L_\varphi^2} + \frac{1}{3}\left(\frac{2\pi w H}{\Phi_0}\right)^2 \qquad (2)$$

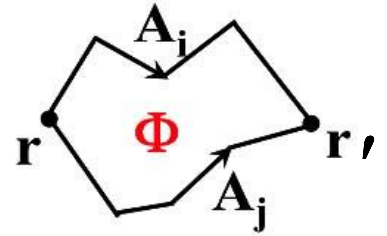


Coherence length from Eqs. (1) and (2) vs temperature (w =1.4 nm). The line is a fit to $L_\varphi \propto T^{-1/3}$.



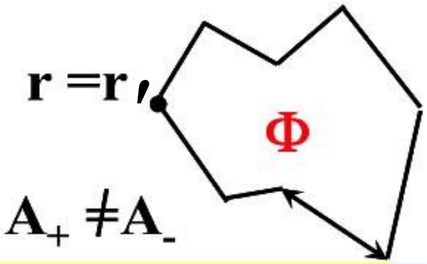
signatures of coherent backscattering in *disordered* quantum wires:

$$P(r,r',t) = \left|\sum_i A_i\right|^2 = \sum_i |A_i|^2 + \sum_{i\neq j} A_i A_j^*$$



Closed loop of time reversed paths:

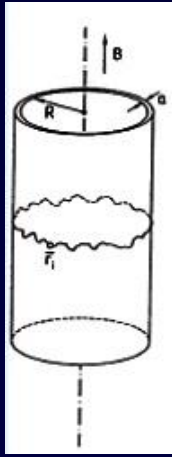
$$P(r,r,t) = |A_+ + A_-|^2 = 4|A|^2$$

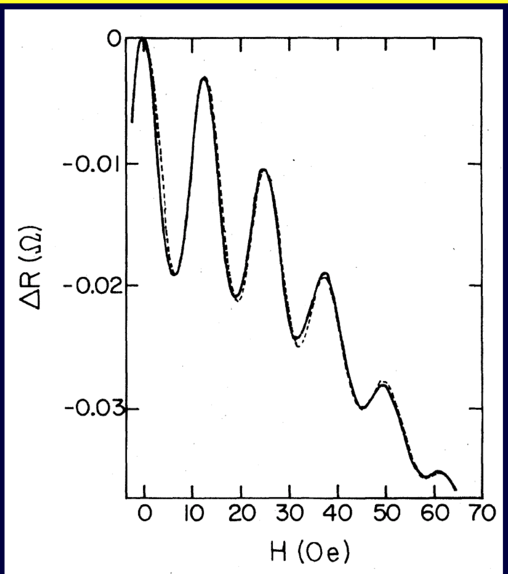


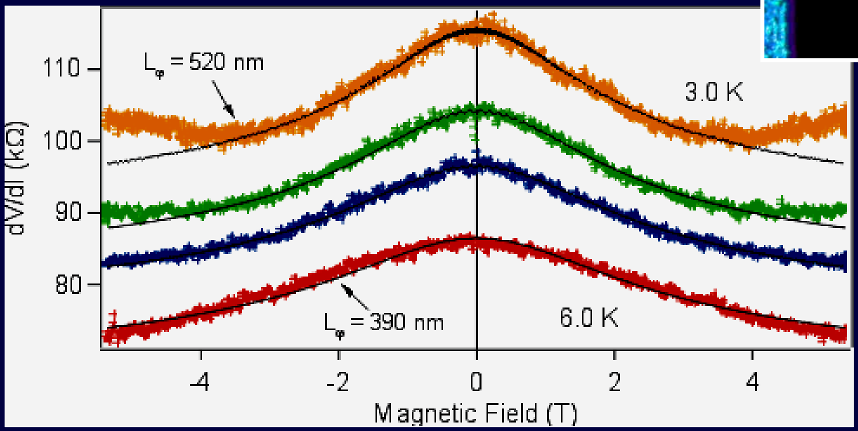
⟹ enhanced backscattering probability!

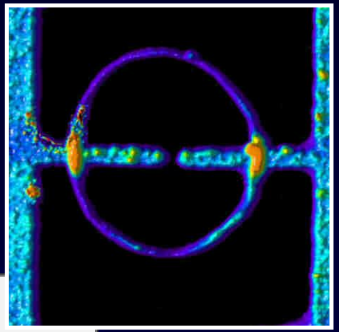
Magnetic field breaks time-reversal symmetry:

- coherent backscattering suppressed by magnetic field: negative magnetoresistance near B=0
- reproducible fluctuation pattern specific for impurity configuration: “magneto-fingerprints”



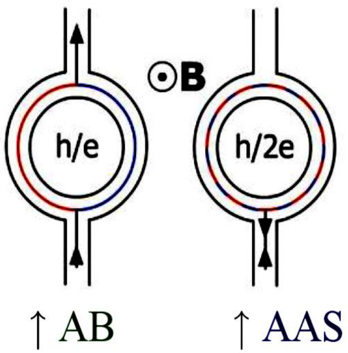






No periodic oscillation appears in CNT ring, why?

AB & AAS oscillations



- AAS = Altshuler-Aronov-Spivak
- interference of two electron waves going one full circle clockwise and counterclockwise
- identical paths
  $\Rightarrow$ phase difference at $B = 0$ is 0

- possible oscillations:

round trips	period (in Φ)	type
1/2	h/e	AB
1	$h/2e$	AAS
3/2	$h/3e$	AB
...		

- number of round trips limited by L_φ

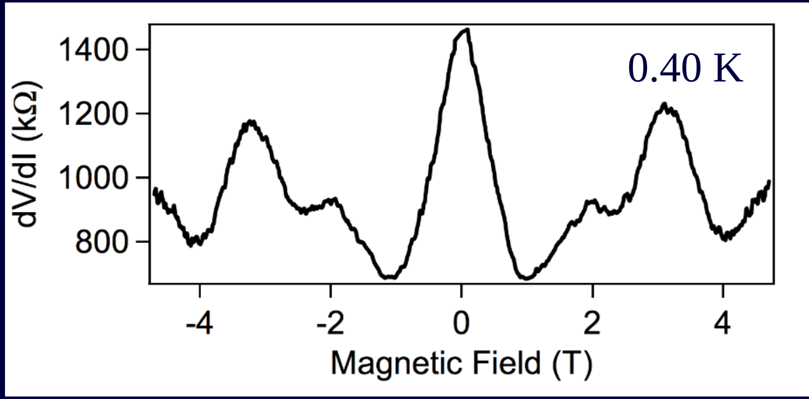
Phase-coherence length(相位相干长度):
Distance over which the electrons maintain their phase coherence

Phase-coherence length：

$$L_\phi = \sqrt{D\tau_\phi}$$

D: the elastic diffusion constant

τ_ϕ: phase coherence time describes the combined effect of the inelastic and magnetic scattering.



At higher fields, new peaks due to conductance fluctuations appear. These are universal conductance fluctuations that result from scattering by defects

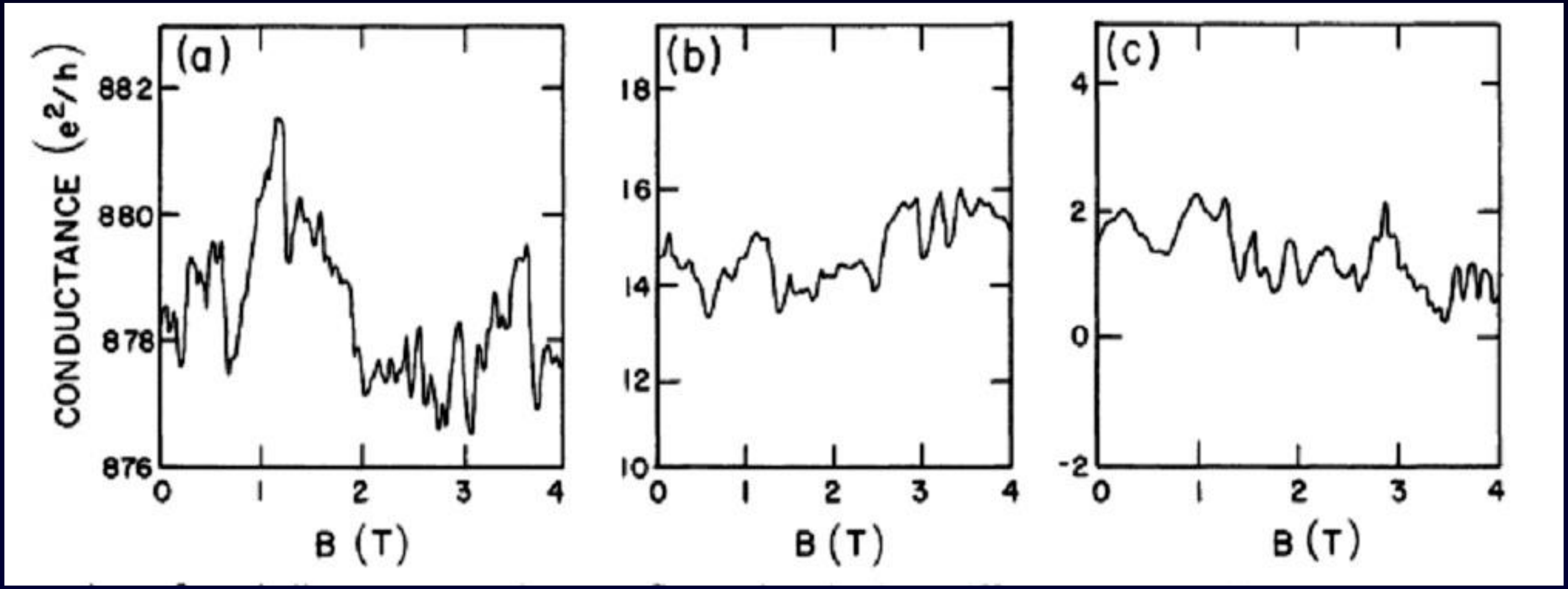
Outline

Weak Localization

Universal Conductance Fluctuations

Ballistic transport

Universal Conductance Fluctuations
普适电导涨落

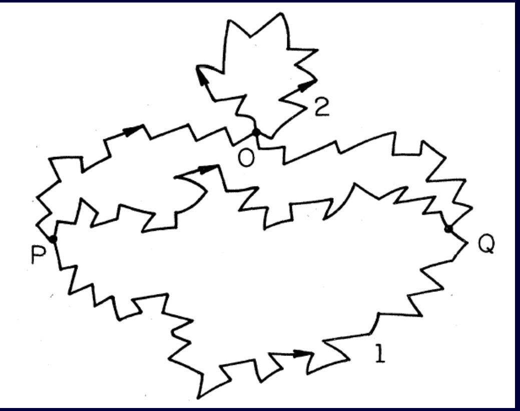


(a) 800nm-diam gold ring (A-B oscillation filtered out)

(b) Quasi-1D silicon MOSFET

(c) Numerical calculation of g(B) for an Anderson model

介观尺度的样品中，电子从一端到另外一端输运的非闭合路径干涉，与宏观样品不同，是不可忽略的。



即使样品的宏观参数（材料、杂质浓度、形状和尺寸）完全相同，由于微观上杂质分布的具体组态（或位形）不同，从样品到样品，电导会有显著的涨落。

P. A. Lee *et al*. Phys. Rev. B 35, 1039 (1987)

从实验的角度，制作一系列宏观参数相同的样品比较困难，更方便的是用同一样品，研究电导涨落随磁场的变化。

由于在磁场中无规则行走的电子，其波函数会得到附加的相位，磁场的改变对干涉项的影响，类似于样品中杂质位形的变化。

磁场中不同样品中电子无规则行走，其波函数相位不同，导致来自干涉项对概率贡献各异。各个样品有各自特有的电导涨落模式：样品拥有自己的指纹，且可长时间重现。

Features of Universal Conductance Fluctuations (UCF)

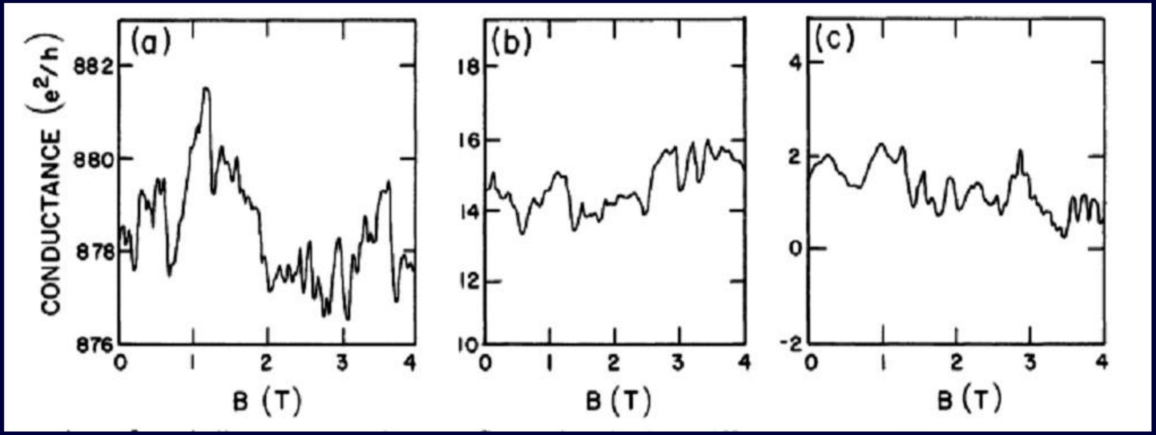
(1) Their zero-temperature amplitude should be independent of the system size or the degree of disorder

…different in each individual sample ("magnetic fingerprint")

… can be reproducible if we keep the macro-conditions constant.

(2) At low temperatures the conductance fluctuations exhibit a universal amplitude of order e^2/h;

(3) *Not* heat noise (time independent)



Features of Universal Conductance Fluctuations (UCF)

(4) Instead of periodic oscillations the magneto-resistance of these structures showed noise-like but reproducible fluctuations at low temperatures

(5) The amplitude of these fluctuations decreased with increasing temperature suggesting that they were related to an intrinsic effect

要求：介观尺度，处于正常导电区域，即满足：

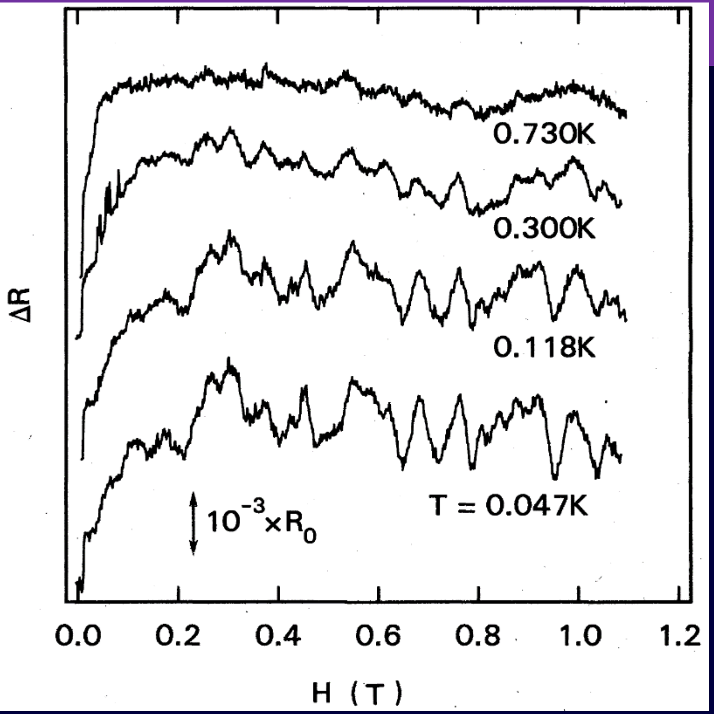
$$\lambda_F << l << L << L\varphi << \xi$$

L: 体系的尺度;

l: 电子所受弹性散射的平均自由程;

λ_F费米波长

ξ: 定域化长度；　$L\varphi$：相干长度



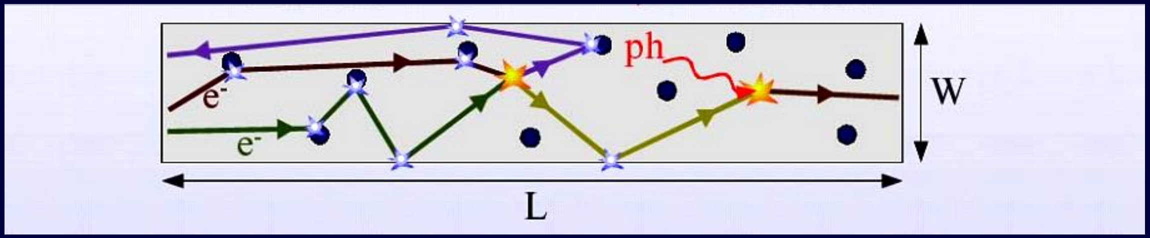
C. P. Umbach et al. Phys. Rev. B 30, 4048 (1984)

The reasons for UCF:

Due to irregular quantum interference
(border & impurity scattering)

根据Landauer理论，电导正比于总透射几率，从样品一侧到另外一侧的透射几率是诸多费曼路径相应的几率幅之和。

电子通过样品时经历多次与边界或杂质的散射，其费曼路径是无规则行走式的准经典轨迹，不同路径之间的相位差不同，导致随机干涉效应，使电导呈现非周期的不规则涨落。



在有磁场的情况下，每一条路径将获得一附加的相位因子：

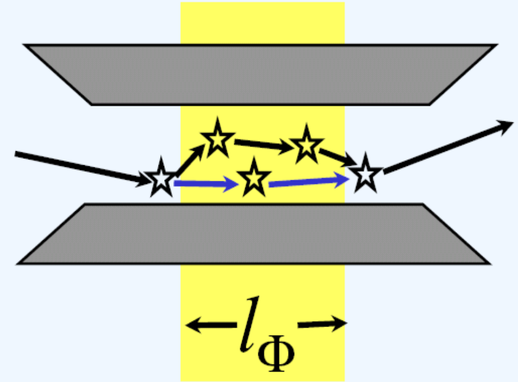
$$e/\hbar \oint A dl$$

两条不同的费曼路径之间由磁场引起的相位差由这两条路径所包围的磁通Φ决定，因路径是无规则的，两条路径所包围的磁通也无规则，导致随机干涉效应及相应的电导涨落。

$$A_1 \rightarrow A_1 \exp\left[\frac{ie}{\hbar c}\oint \mathbf{A}\, dl\right] = A_1 \exp\left[i\boxed{\frac{2\pi\phi}{\phi_0}}\right]$$

$$A_2 \rightarrow A_2 \exp\left[-i\frac{2\pi\phi}{\phi_0}\right]$$

Interference of many diffusion paths lead to aperiodic fluctuation pattern in the conductance:



⇨ Ensemble averaging of conductance fluctuations ΔG if L < l_Φ

综平均电导波动

$$\Delta G_{RMS} = \sqrt{12}\,\frac{e^2}{h}\left(\frac{l_\Phi}{L}\right)^{3/2}$$

L_ϕ - phase-coherence length

⇨ vary interference pattern by applying electric or magnetic fields

⇨ determine phase coherence length l_Φ at different temperatures

The sample size $L < L_\phi$ (phase-coherence length); $L_\phi = \sqrt{D\tau_\phi}$

$L < L_T$ (thermal diffusion length): $L_T = \sqrt{\hbar D / k_B T}$

D is the elastic diffusion constant and the τ_ϕ phase coherence time describes the combined effect of the inelastic and magnetic scattering.

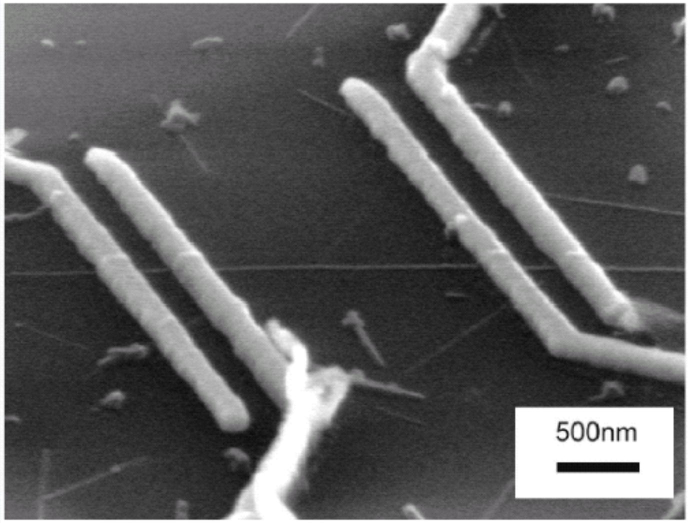
The rms amplitude of the fluctuations δG reaches the "universal" value rms $[\delta G\,] \cong e^2/h$

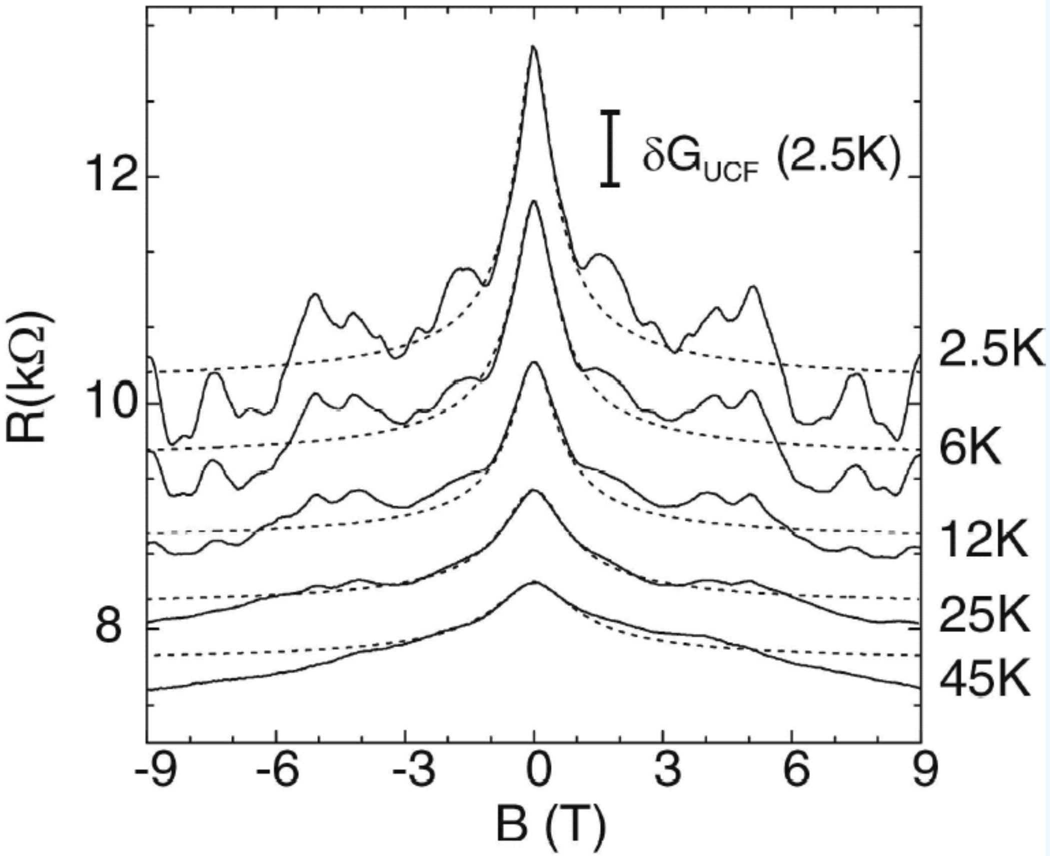
When either L_ϕ or L_T becomes smaller than L, the shorter of these two length scales governs the amplitude of the fluctuations

rms: 均方根振幅

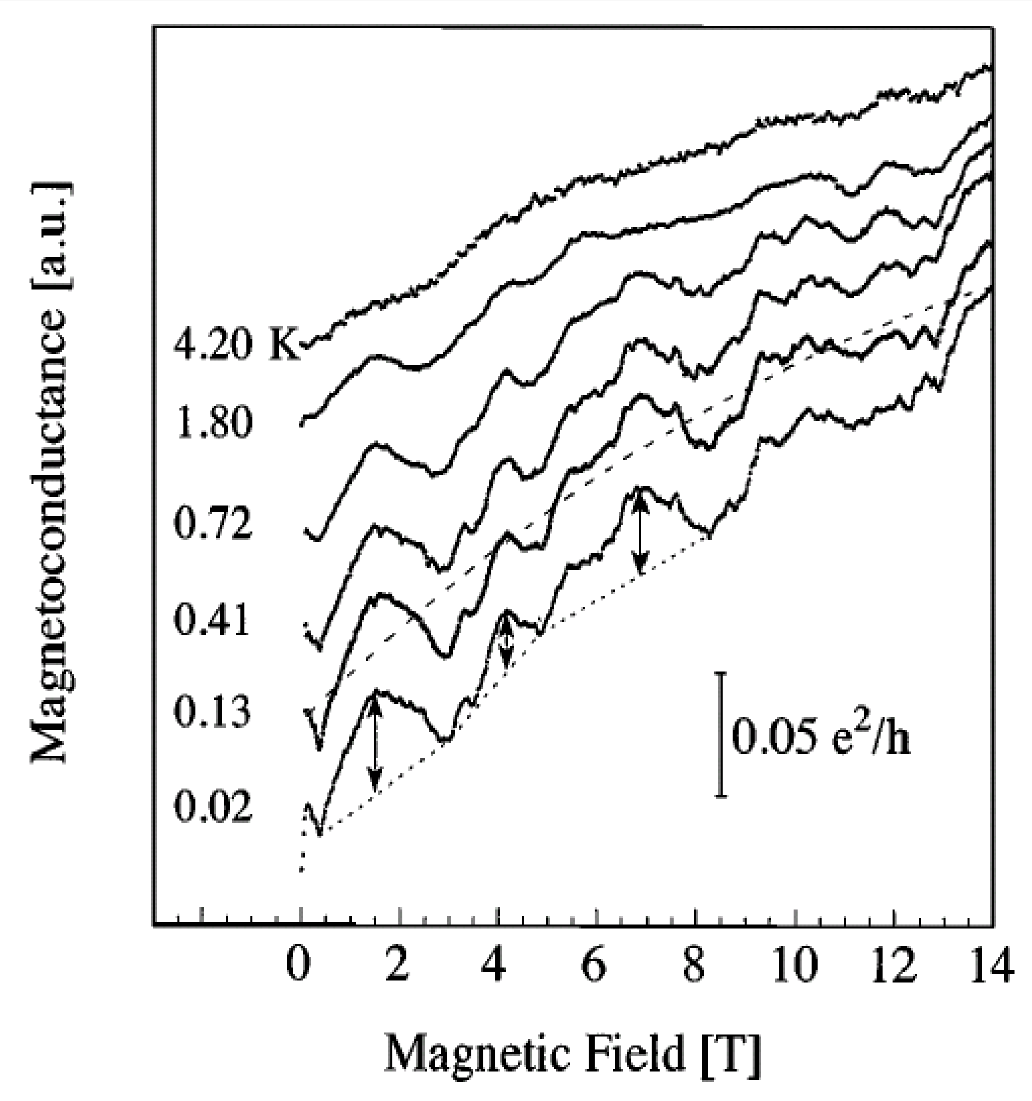
Example 1

Perpendicular magnetoresistance





Analysis of central peak yields L_ϕ~300 nm at low T
symmetric UCF pattern as expected for good contacts
temperature dependent background resistance



Quantum Transport in a Multiwalled Carbon Nanotube

At low temperature, reproducible, aperiodic fluctuations of the conductance appear

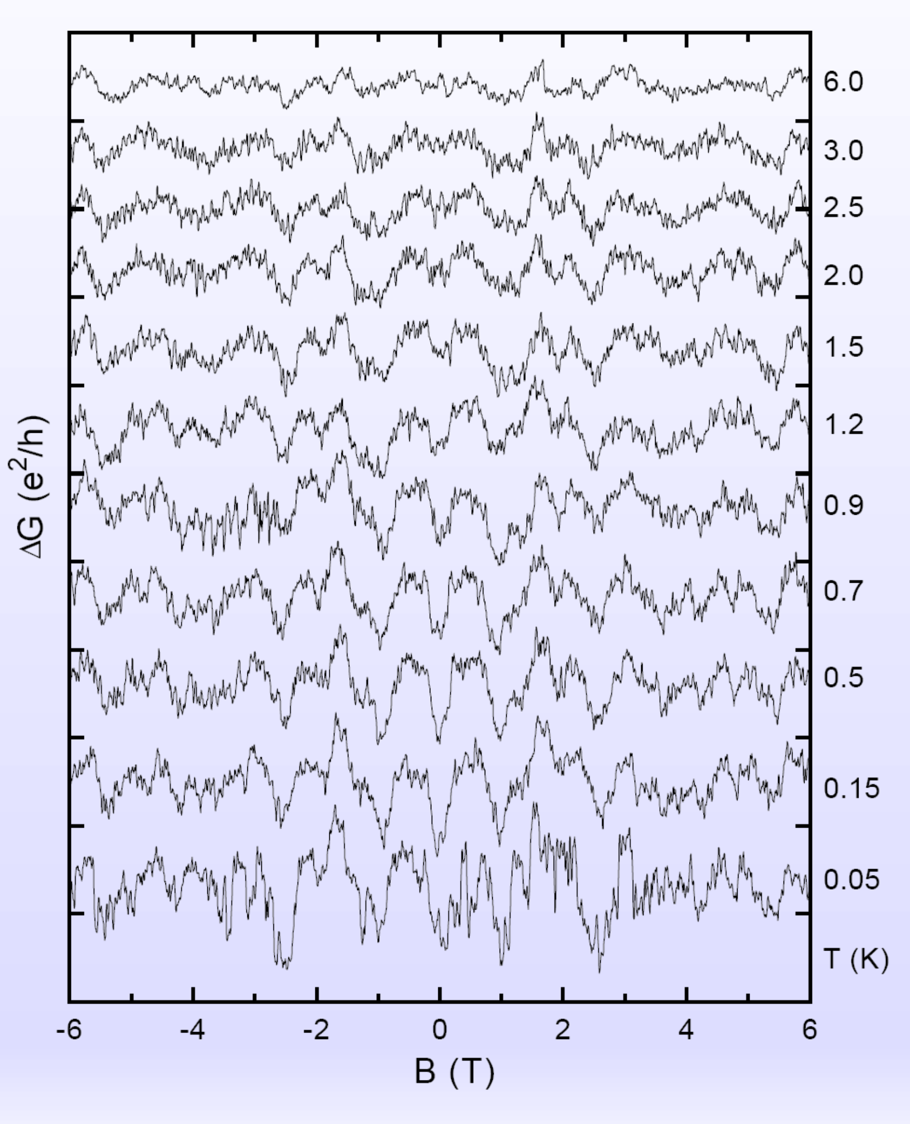
The statistical self-averaging of the UCF results in fluctuations with rms amplitude:

$$\mathrm{rms}[\delta G] = 0.61\,\frac{e^2}{h}\left(\frac{n\pi d}{L_\phi}\right)^{1/2}\left(\frac{L_\phi}{L}\right)^{3/2}$$

均方根

Magnetic field dependence of the magnetoconductance at different temperatures.

Example 2



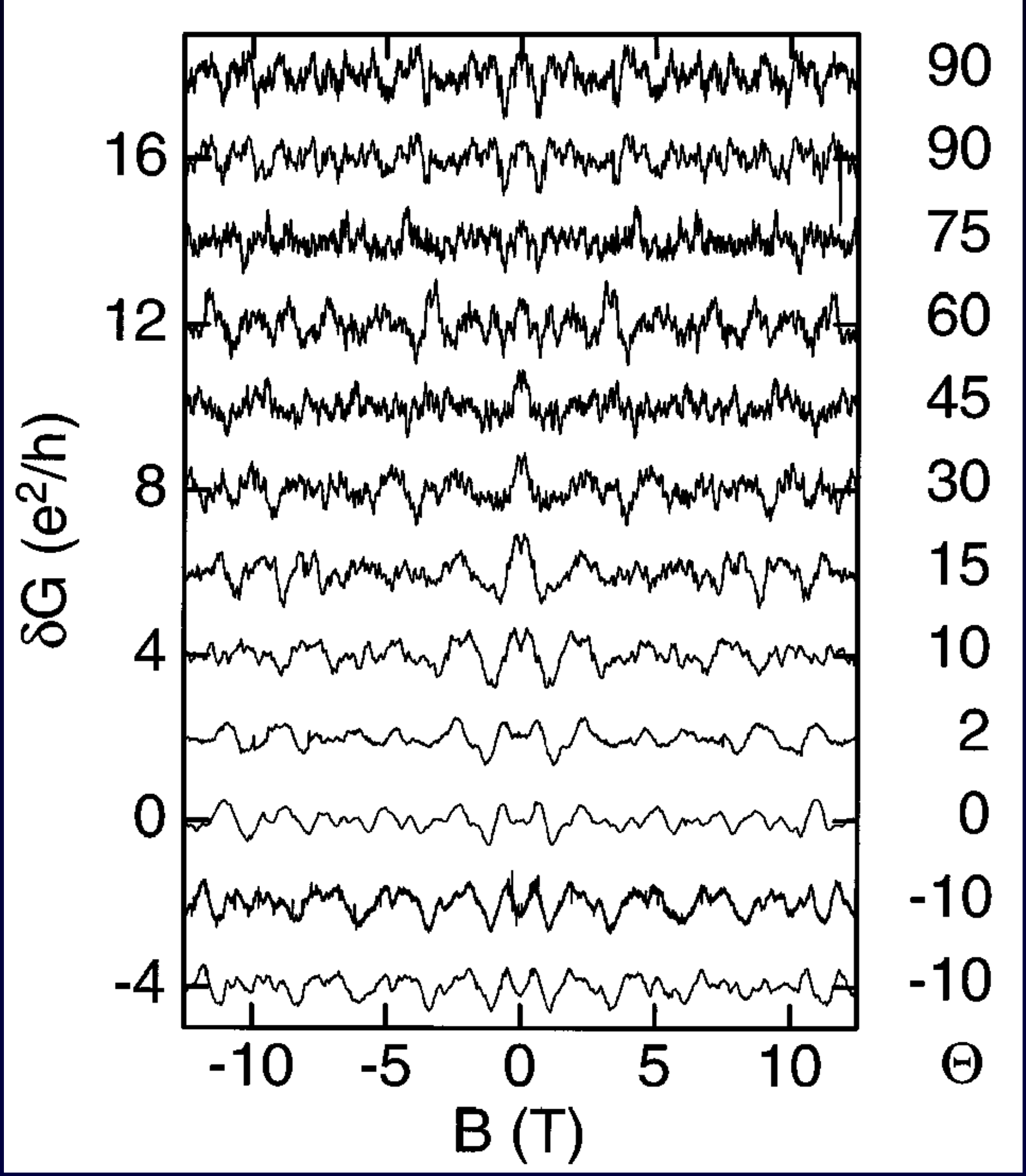
UCF temperature dependence

UCF in gold nanowire

(L = 600 nm, W = 60 nm)

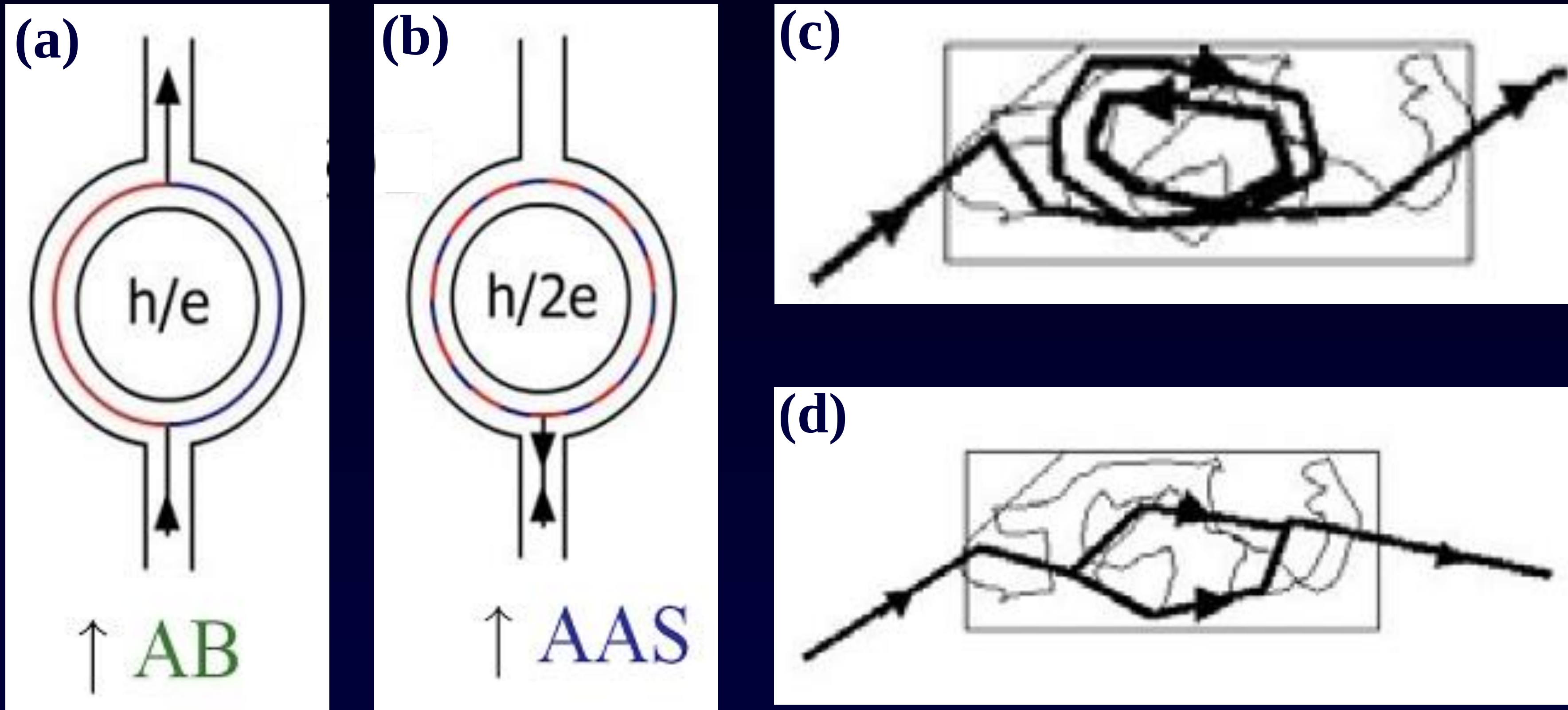
H. Hegger, Dissertation, Uni Koeln (1997)

Angular Dependence of Universal Conductance Fluctuations in Noble-Metal Nanowires



Conductance fluctuations of sample 1 with w = 45 nm, L = 500 nm, and d =25 nm for various angles between magnetic field and current direction

Summary:



The schematics represent the following interference effects: (a) A-B effect, (b) AAS effect, (c) weak localization and (d) universal conductance fluctuations.

Interfering pair of paths can involve either real paths (a and d) or time-reversed paths (b and c); In case a and b corresponding to the characteristic flux scale of h/e or h/2e respectively.