Ultrafast Optics

Review.

Optical elements:

*. lens

temporal effect

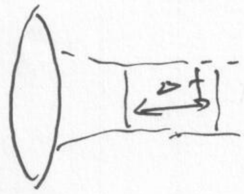
pulse front ~~tit~~ vs. phase front.

$$\tau = \frac{-(a^2 - r^2)}{2c} \cdot \lambda \frac{d}{d\lambda}\left(\frac{1}{f}\right)$$

$\Rightarrow$ pulse broadening.

Spatial effect.

$$\Delta f = -f^2 \frac{d}{d\lambda}\left(\frac{1}{f}\right) \cdot \Delta\lambda$$



$z \to \Delta f$

Gaussian beam: $W = W_0 \dfrac{z}{z_0}$

$\Rightarrow$ beam broadening

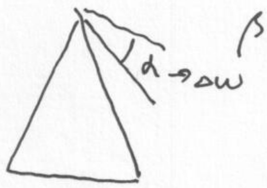
*. Mirror: $R(\omega) \cdot e^{j\phi(\omega)}$

Chirped mirror, G.-T mirror,

both for dispersion controlling

*. Angular dispersion:

prism:



$$\phi(\omega) = \frac{\omega}{c} \cdot l \cdot \cos\alpha$$

$$\phi''_{angular} = -\frac{\omega}{c} l \left(\frac{d\alpha}{d\omega}\right)^2$$

$$= -\frac{2 l \lambda^3}{\pi c^2} \left|\frac{dn}{d\lambda}\right|^2 < 0$$

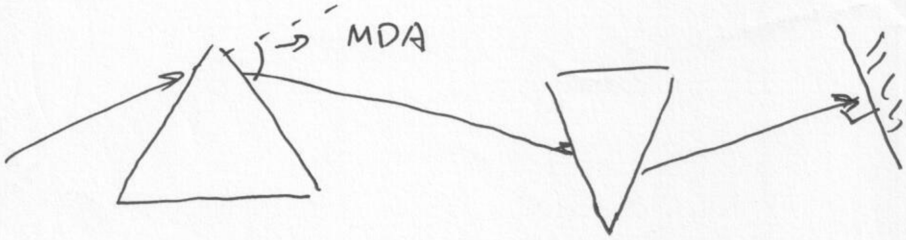
$$\phi_{material} = \frac{\omega}{c} \cdot nl$$

$$\phi''_m(\omega) = \frac{\lambda^3}{2\pi c^2} L \cdot \frac{d^2 n}{d\lambda^2}$$

$$\therefore \phi''_{total} = \phi''_{angular} + \phi''_m$$

$$= \frac{\lambda^3}{2\pi c^2} L \cdot \frac{d^2 n}{d\lambda^2} - \frac{2l\lambda^3}{\pi c^2} \left| \frac{dn}{d\lambda} \right|^2$$

Application of prism pair : dispersion control



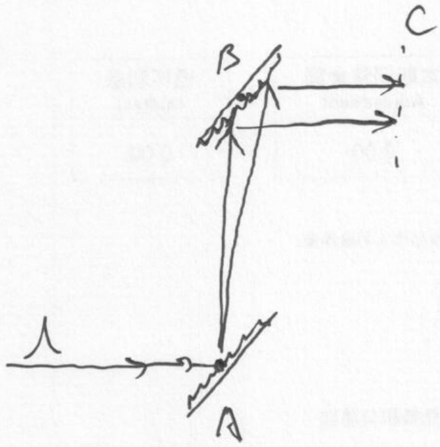
- MDA: minimum deviation angle.
- No astigmation.

in practice, usually use dual conditions: Brewster's angle + MDA

how to derive Brewster's angle ?

Today: grating , introduction to ML. (Mode lock

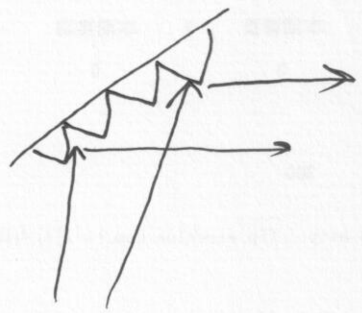
* grating



optical path length: $\overline{ABC}$

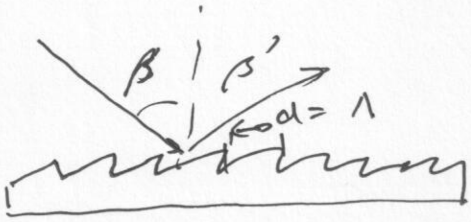
$$\phi(\omega) = \frac{\omega}{c}\overline{ABC} + \phi_{grating}$$

why: B is not a mirror



grating carries extra phase.

grating equation?



treat grating as an optical scattor.

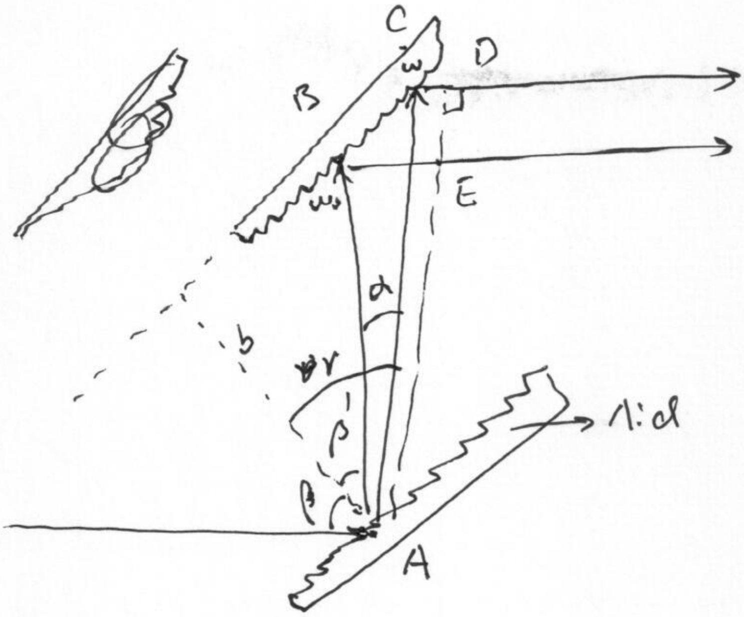
for photon: $p = \frac{E}{c} = \frac{\hbar\omega}{c} = \frac{hc}{\lambda}\cdot\frac{1}{c} = \frac{h}{\lambda}$

Λ is like the wavelongh of a photon

⇒ grating's momentum: $\frac{h}{\Lambda}$

momentum conservation

$$\frac{h}{\lambda}\cdot\sin\beta' = \frac{h}{\lambda}\cdot\sin\beta + \frac{h}{\Lambda}$$



$$\gamma = \beta' + \alpha$$

$$\begin{cases} \sin\beta' - \sin\beta = \dfrac{2\pi}{d}\cdot\dfrac{c}{\omega_0} \\[2ex] \sin\gamma - \sin\beta = \dfrac{2\pi}{d}\cdot\dfrac{c}{\omega} \end{cases}$$

$$\overline{ACD} = AC + CD = \frac{b}{\cos\gamma} + \frac{b}{\cos\gamma}\cos(\beta+\gamma)$$

$$= \frac{b}{\cos\gamma}\left[1 + \cos(\beta+\gamma)\right]$$

$$\phi(\omega) = \frac{\omega}{c}\cdot\overline{ACD} + \phi_y = \frac{\omega}{c}\cdot\overline{ACD} - \frac{2\pi}{d}\cdot b\cdot \tan\gamma$$

$$\phi_y \Downarrow e^{-jk\cdot x}$$

k: grating vector

$$\therefore\ \phi'(\omega) = \frac{1}{c}\overline{ACD} + \underbrace{\frac{\omega}{c}\cdot\frac{d\,opl}{d\omega}}_{①} - \underbrace{\frac{2\pi}{d}\cdot b\cdot\frac{1}{\cos^2\gamma}\cdot\frac{d\gamma}{d\omega}}_{②}$$

$$① = \frac{\omega}{c}\cdot\frac{d}{d\omega}\left[\frac{b}{\cos\gamma}\left(1+\cos(\beta+\gamma)\right)\right]$$

$$= \frac{\omega}{c}\cdot\frac{\sin\gamma}{\cos^2\gamma}\cdot\frac{d\gamma}{d\omega}\left[1+\cos(\beta+\gamma)\right]$$

$$+\frac{\omega}{c}\cdot\frac{b}{\cos\gamma}\left(-\sin(\beta+\gamma)\right)\cdot\frac{d\gamma}{d\omega}$$

$$= \frac{\omega}{c}\;\frac{b}{\cos^2\gamma}\cdot\frac{d\gamma}{d\omega}\left[\sin\gamma\cdot\left[1+\cos(\beta+\gamma)\right]-\cos\gamma\left(\sin(\beta+\gamma)\right)\right]$$

$$= \frac{\omega}{c}\cdot\frac{b}{\cos^2\gamma}\cdot\frac{d\gamma}{d\omega}\left[\sin\gamma-\sin\beta\right]$$

recall : $\frac{2\pi}{d}\cdot\frac{c}{\omega}$

$$= \frac{2\pi}{d}\;\frac{b}{\cos^2\gamma}\cdot\frac{d\gamma}{d\omega}$$

$$② = -\frac{2\pi}{d}\cdot\frac{b}{\cos^2\gamma}\cdot\frac{d\gamma}{d\omega}$$, so $\Rightarrow \phi'(\omega) = \frac{1}{c}\,\overline{ACD}$

$$\Rightarrow \phi''(\omega) = \frac{1}{c}\cdot\frac{d\,\overline{ACD}}{d\omega}$$

$$= \cancel{\frac{1}{c}\cdot\frac{c}{\omega}\cdot\frac{2\pi}{d}\,b\cdot\frac{1}{\cos^2\gamma}}\cdot\frac{d\gamma}{d\omega}$$

$$= \frac{b}{c}\;\frac{d}{d\omega}\left[\frac{1+\cos(\beta+\gamma)}{\cos\gamma}\right]$$

$$= \frac{b}{c}\frac{1}{\cos^2\nu}\left[\sin\nu\left[1+\cos(\beta+\nu)\right]-\cos\nu\cdot\sin(\beta+\nu)\right]\frac{d\nu}{d\omega}$$

~~From $\sin\nu - \sin\beta = \frac{2\pi}{d}\frac{\omega}{c}\frac{c}{\omega}$~~

~~$\cos\nu \cdot \frac{d\nu}{d\omega} =$~~

$$= \frac{b}{c}\frac{1}{\cos^2\nu}\left[\sin\nu+\left(\sin\nu\cdot\cos(\beta+\nu)-\cos\nu\cdot\sin(\beta+\nu)\right)\right]\frac{d\nu}{d\omega}$$

$$= \frac{b}{c}\frac{1}{\cos^2\nu}\left[\sin\nu-\sin\beta\right]\cdot\frac{d\nu}{d\omega}$$

$$= \frac{b}{c}\cdot\frac{1}{\cos^2\nu}\cdot\frac{2\pi}{d}\cdot\frac{c}{\omega}\frac{d\nu}{d\omega}$$

From $\sin\nu - \sin\beta = \frac{2\pi}{d}\cdot\frac{c}{\omega}$

$$\cos\nu\frac{d\nu}{d\omega} = \frac{2\pi}{d}\frac{-1}{\omega^2}\cdot c$$

$$= \frac{b}{c}\cdot\frac{1}{\cos^3\nu}\cdot\frac{2\pi}{d}\cdot\frac{c}{\omega}\cdot\frac{1}{\cos\nu}\frac{-1}{\omega^2}\cdot c\cdot\frac{2\pi}{d}$$

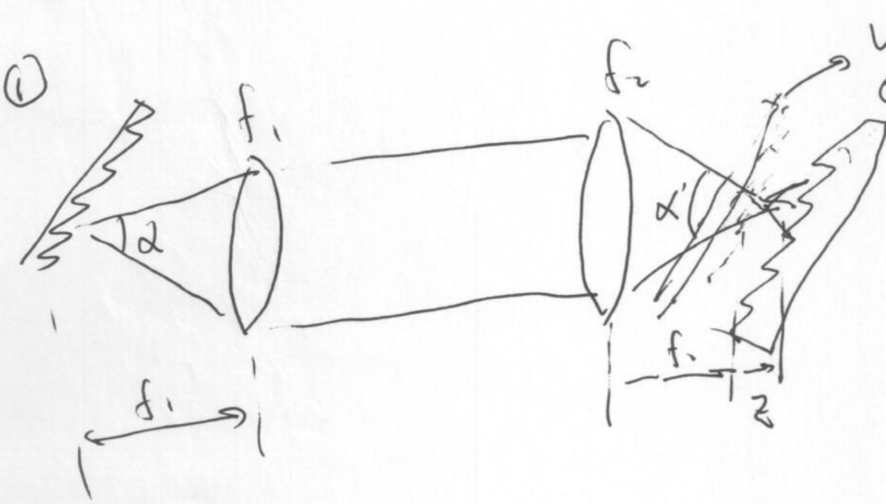
$$= -\frac{4\pi^2 b c}{\cos^3\nu\, d^2\cdot\omega^3}.$$

comment: always < 0

Then how to tune dispersion?

~~co~~ Combine with focusing elements such as telescope:

①



$$\frac{\alpha'}{\alpha} = \frac{f_1}{f_2}$$

$$\phi''(\omega) = -\frac{4\pi^2 b c}{\cos^3\nu\cdot d^2\omega^3}$$

Road map so far:

1. Complex Analytical Signal

2. Law of propagation: { spatial / temporal

3. Nonlinear

4. fs optics: optical elements

Next: 5. Mode locking { qualitative / quantitative / chapter #5 in book

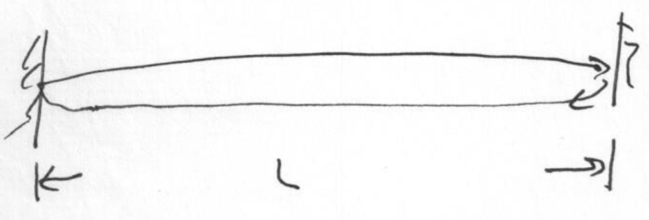
6. manipulatie.

7. diagnostics: autocorrelation, phase retrieval. etc.

8. Application

Mode locking

1. laser cavity mode



$$\frac{\omega}{c} \cdot 2L = m \cdot 2\pi$$

$$\omega_n = \frac{m\pi c}{L} = \frac{m \cdot 2\pi}{T_R} \qquad T_R = \frac{2L}{c} \text{ Round trip time}$$

2. Superposition of multiple mode

$$\sum A_n \cdot e^{-j[(\omega_0 + n\cdot\Delta\omega)t - \phi_n]}$$

assume: $A_n = 1$, $\phi_n = 0$. N modes (odd number)

$$\sum_{-\frac{N-1}{2}}^{\frac{N-1}{2}} e^{-j(\omega_0 + n\cdot\Delta\omega)t}$$

$$= e^{-j\omega_0 t} \sum_{-\frac{N-1}{2}}^{\frac{N-1}{2}} e^{-jn\cdot\Delta\omega t}$$

$$= e^{-j\omega_0 t} \frac{e^{+j\frac{N-1}{2}\Delta\omega t}\left(1 - e^{-j\Delta\omega\cdot t\cdot N}\right)}{1 - e^{-j\Delta\omega t}}$$

$$= e^{-j\omega_0 t} \cdot \frac{\sin(\frac{N}{2}\Delta\omega t)}{\sin(\frac{1}{2}\Delta\omega t)}$$

$$= A(t) \cdot e^{-j\omega_0 t}$$

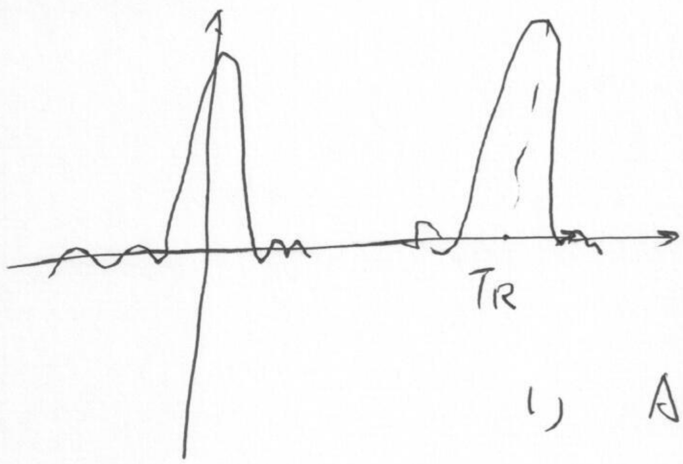
where $A(t) = \dfrac{\sin(\frac{N}{2}\cdot\Delta\omega t)}{\sin(\frac{\Delta\omega}{2} t)}$

$A(0) = N$,

$A(t) = 0 \Rightarrow \frac{N}{2}\Delta\omega = \pm\pi \Rightarrow$ find node.

$$t = \pm \frac{2\pi}{\Delta\omega N} = \pm \frac{T_R}{N}$$

recall $\omega_n = \dfrac{m\cdot 2\pi}{T_R} \Rightarrow \cancel{\Delta\omega =} \dfrac{2\pi}{\Delta\omega} = T_R$



@ $t = \pm \frac{T_R}{N}$

Recall.

$\omega_m = \frac{m \cdot 2\pi}{T_R}$. $\frac{2\pi}{\Delta\omega} = T_R$

Comments:

1) $A(0) = N$

~~2)~~ $\cancel{\tau \sim \frac{T_R}{N} \Rightarrow}$

2) $A(t + T_R) = A(t)$

3) $\tau \sim \frac{T_R}{N} \Rightarrow N = \frac{\delta\omega}{\Delta\omega}$ bandwidth / mode spacing

$\tau \sim \frac{T_R \, \Delta\omega}{\delta\omega} = \frac{2\pi}{\delta\omega} = \frac{1}{\Delta f}$