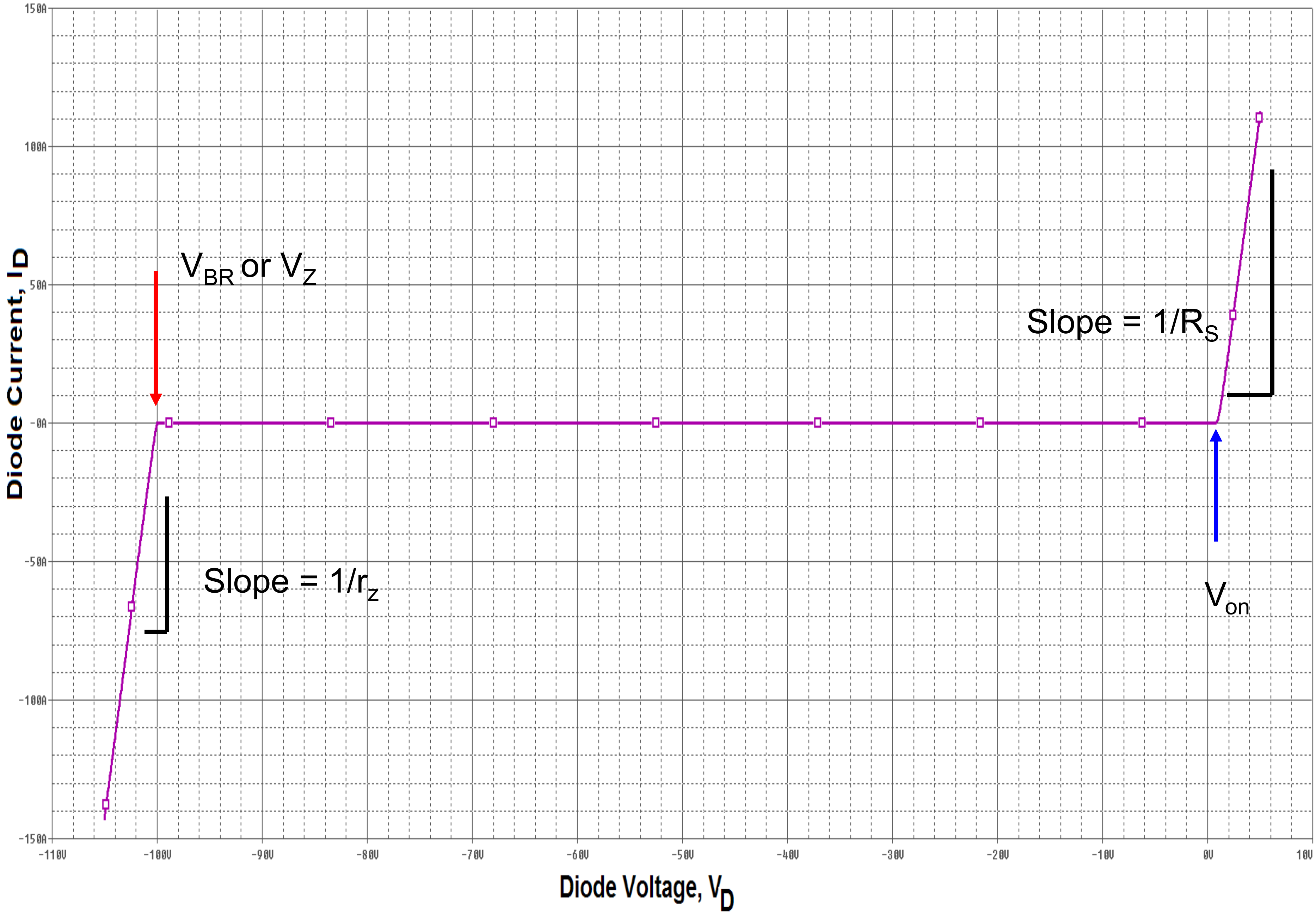
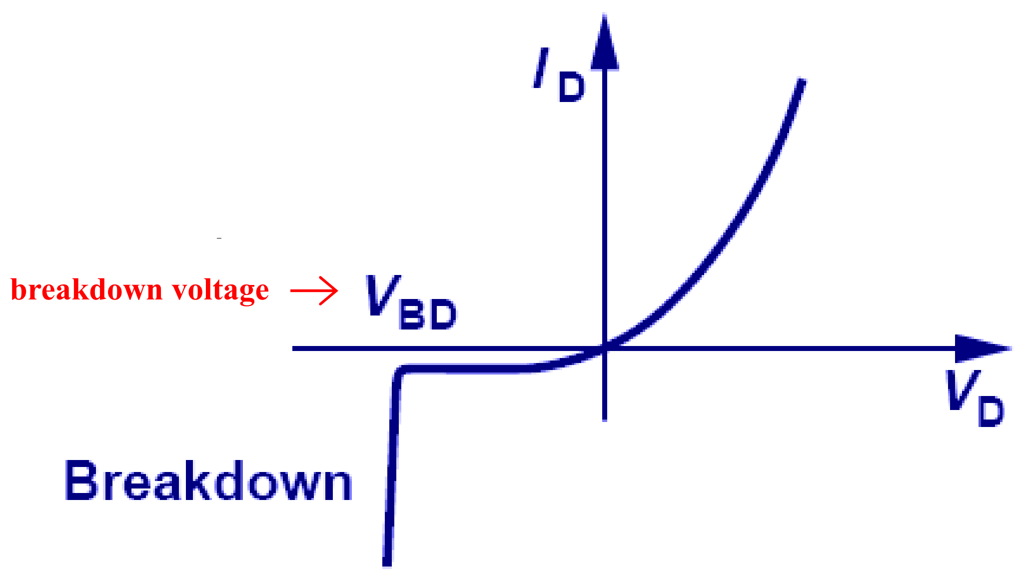

150A
100A
50A
-0A
-50A
-100A
-150A
-110V
-100V
-90V
-80V
-70V
-60V
-50V
-40V
-30V
-20V
-10V
0V
10V
Diode Current, I_D
Diode Voltage, V_D
V_{BR} or V_Z
Slope = $1/R_S$
Slope = $1/r_z$
V_{on}

Reverse Breakdown

- As the reverse bias voltage increases, the electric field in the depletion region increases. Eventually, it can become large enough to cause the junction to break down so that a large reverse current flows:



Reverse Breakdown Mechanisms

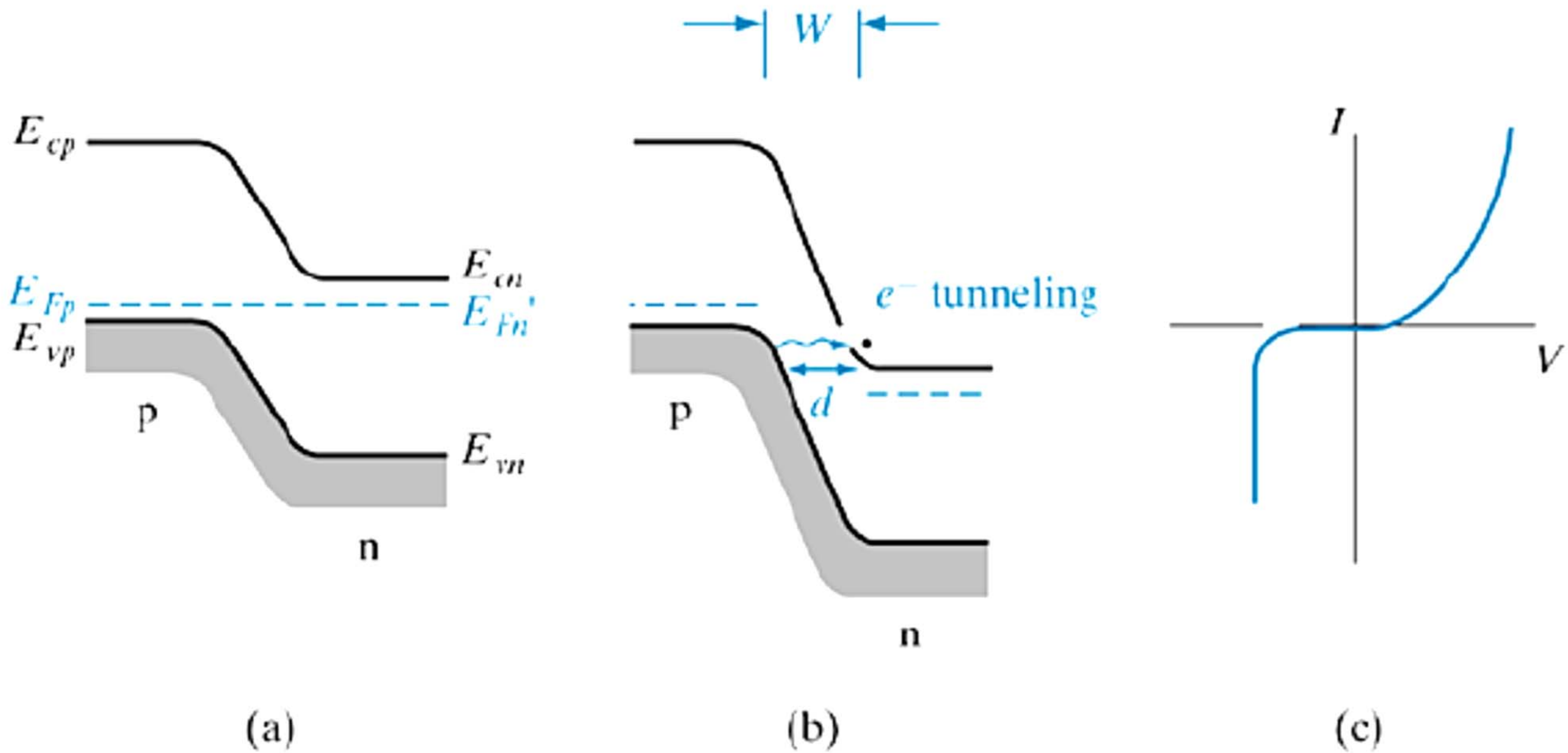
Zener breakdown

occurs when the electric field is sufficiently high to pull an electron out of a covalent bond (to generate an electron-hole pair).

Avalanche breakdown

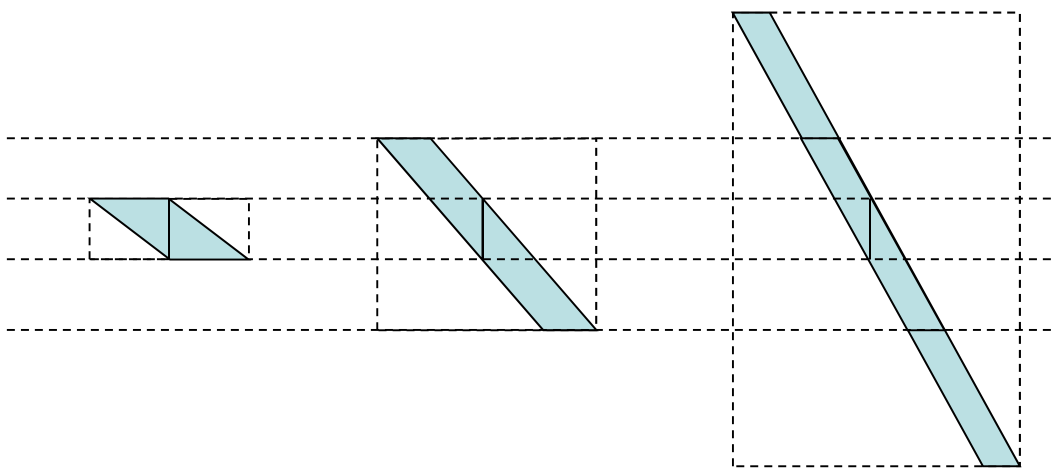
occurs when electrons and holes gain sufficient kinetic energy (due to acceleration by the E-field) in-between scattering events to cause electron-hole pair generation upon colliding with the lattice.

Zener breakdown

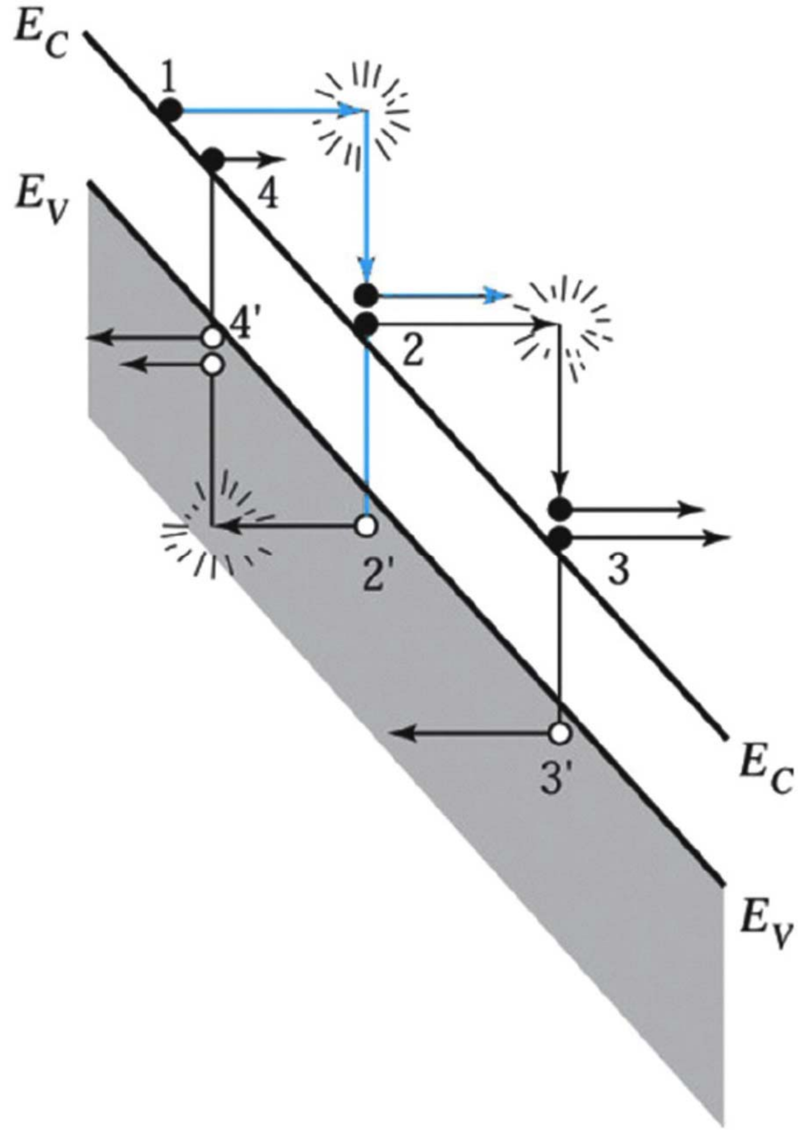


For Zener breakdown (tunneling)

- Extends only a very short distance W from each side of the junction
  - The junction be sharp
  - The doping high
- The tunneling distance d may be too large for appreciable tunneling. However,
  - d becomes smaller as the reverse bias is increased, because the higher electric fields result in steeper slopes for the band edges
    - This assumes that the transition region width W does not increase appreciably with reverse bias
      - For low voltages and heavy doping on each side of the junction, this is a good assumption
- If Zener breakdown does not occur with reverse bias of a few volts, avalanche breakdown will become dominant



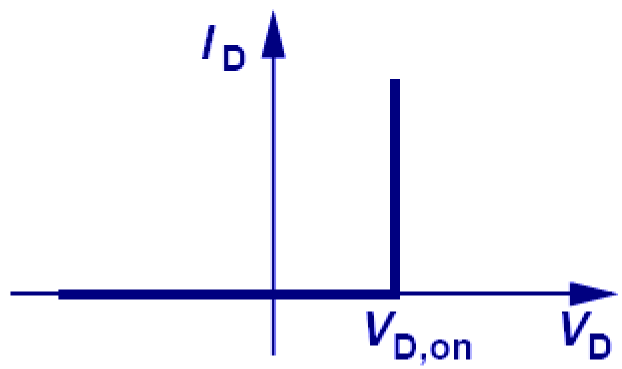
Energy band diagram for the avalanche process



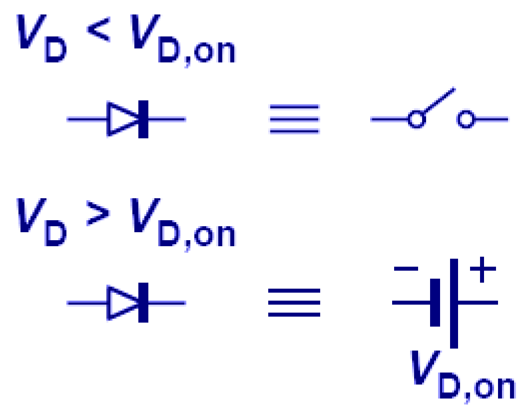
For Avalanche breakdown

- Impact ionization rather than field ionization (Zener)
- Carrier multiplication

Constant-Voltage Diode Model



(a)



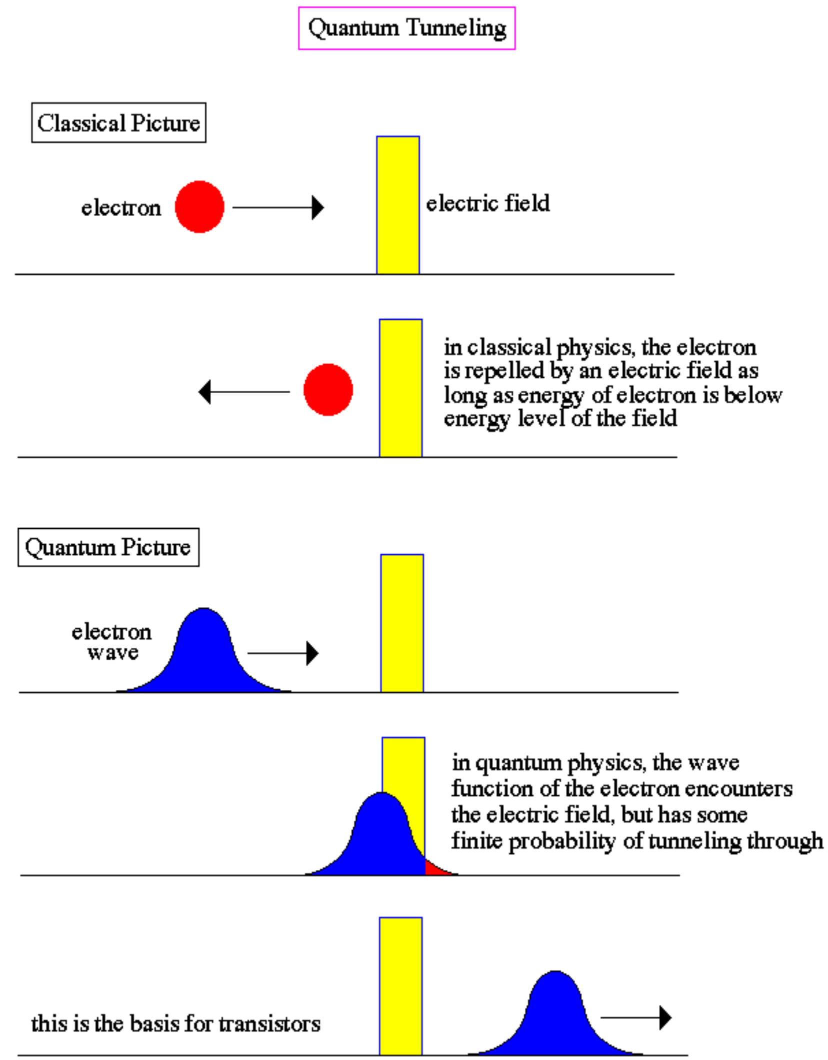
(b)

- If $V_D < V_{D,on}$: The diode operates as an open circuit.
- If $V_D \geq V_{D,on}$: The diode operates as a constant voltage source with value $V_{D,on}$.

Tunneling

The Tunneling of a Particle

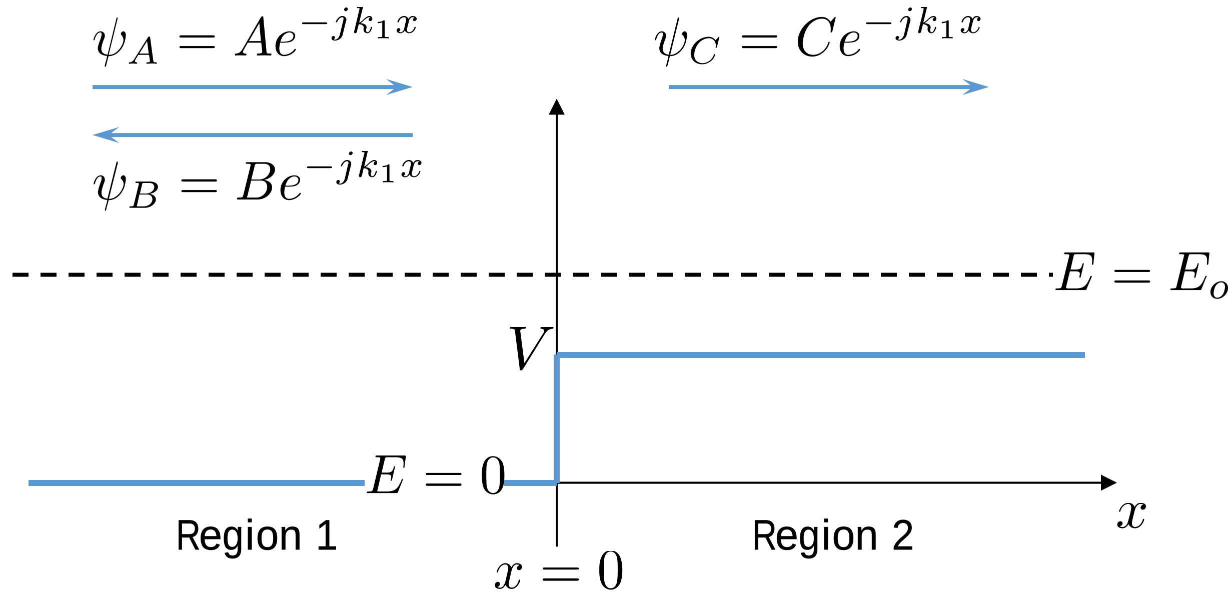
- Tunneling refers to the ability of a particle to overcome and cross a potential energy barrier that it would not be able do based on classical understanding.
- It is only the particle's wave-nature that allows for this phenomenon.
- The probability wave describing the particle's position is an integral that overlaps into the energy barrier, allowing for some finite probability that the particle might actually "tunnel" through.



Images:http://4.bp.blogspot.com/,
abyss.uoregon.edu/.../ quantum_tunneling.gif

A Simple Potential Step

CASE I : $E_o > V$

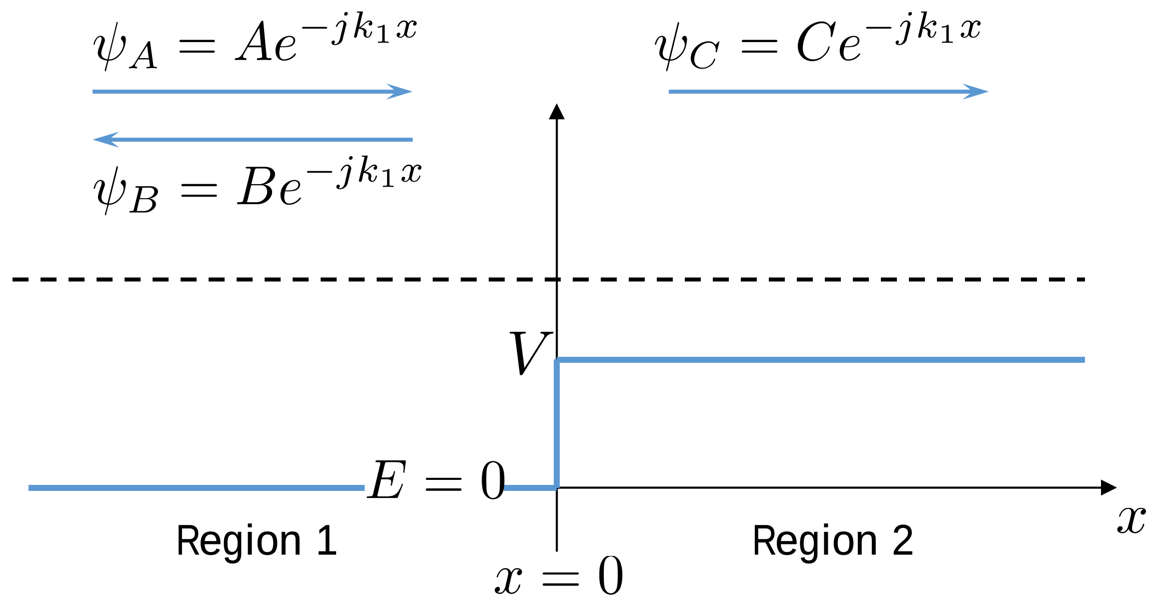


In Region 1: $$E_o\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \quad \Rightarrow \quad k_1^2 = \frac{2mE_o}{\hbar^2}$$

In Region 2: $$(E_o - V)\,\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \quad \Rightarrow \quad k_2^2 = \frac{2m\,(E_o - V)}{\hbar^2}$$

A Simple Potential Step

CASE I : $E_o > V$



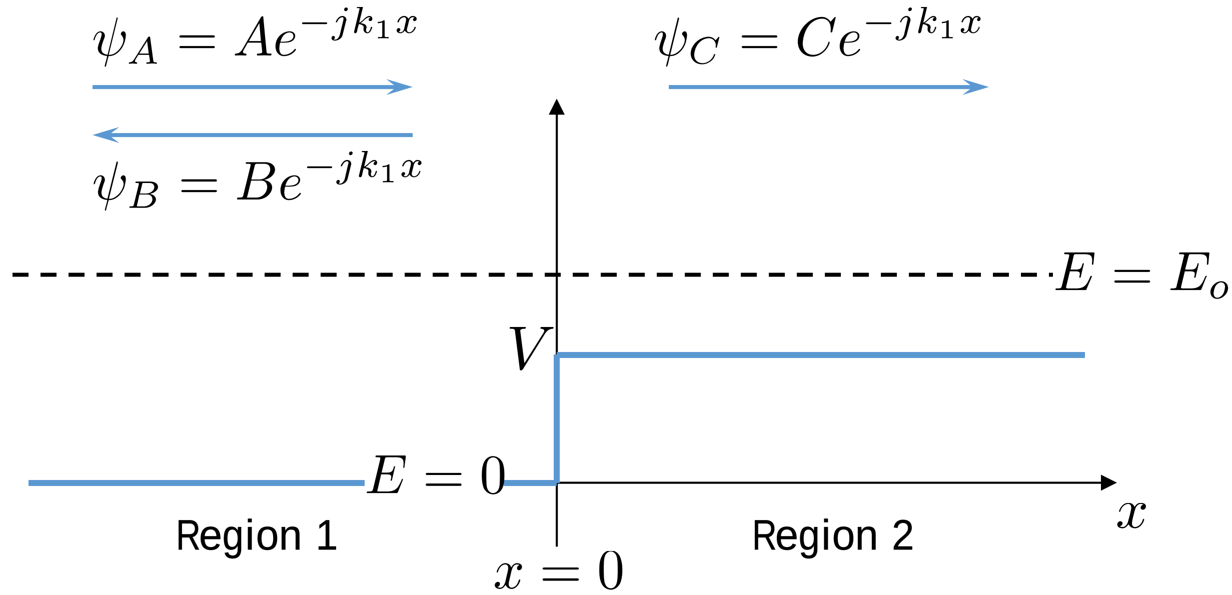
$$\psi_1 = Ae^{-jk_1x} + Be^{jk_1x} \qquad \psi_2 = Ce^{-jk_2x}$$

ψ is continuous:
$$\psi_1(0) = \psi_2(0) \quad \Rightarrow \quad A + B = C$$

$\frac{\partial \psi}{\partial x}$ is continuous:
$$\frac{\partial}{\partial x}\psi(0) = \frac{\partial}{\partial x}\psi_2(0) \quad \Rightarrow \quad A - B = \frac{k_2}{k_1}C$$

A Simple Potential Step

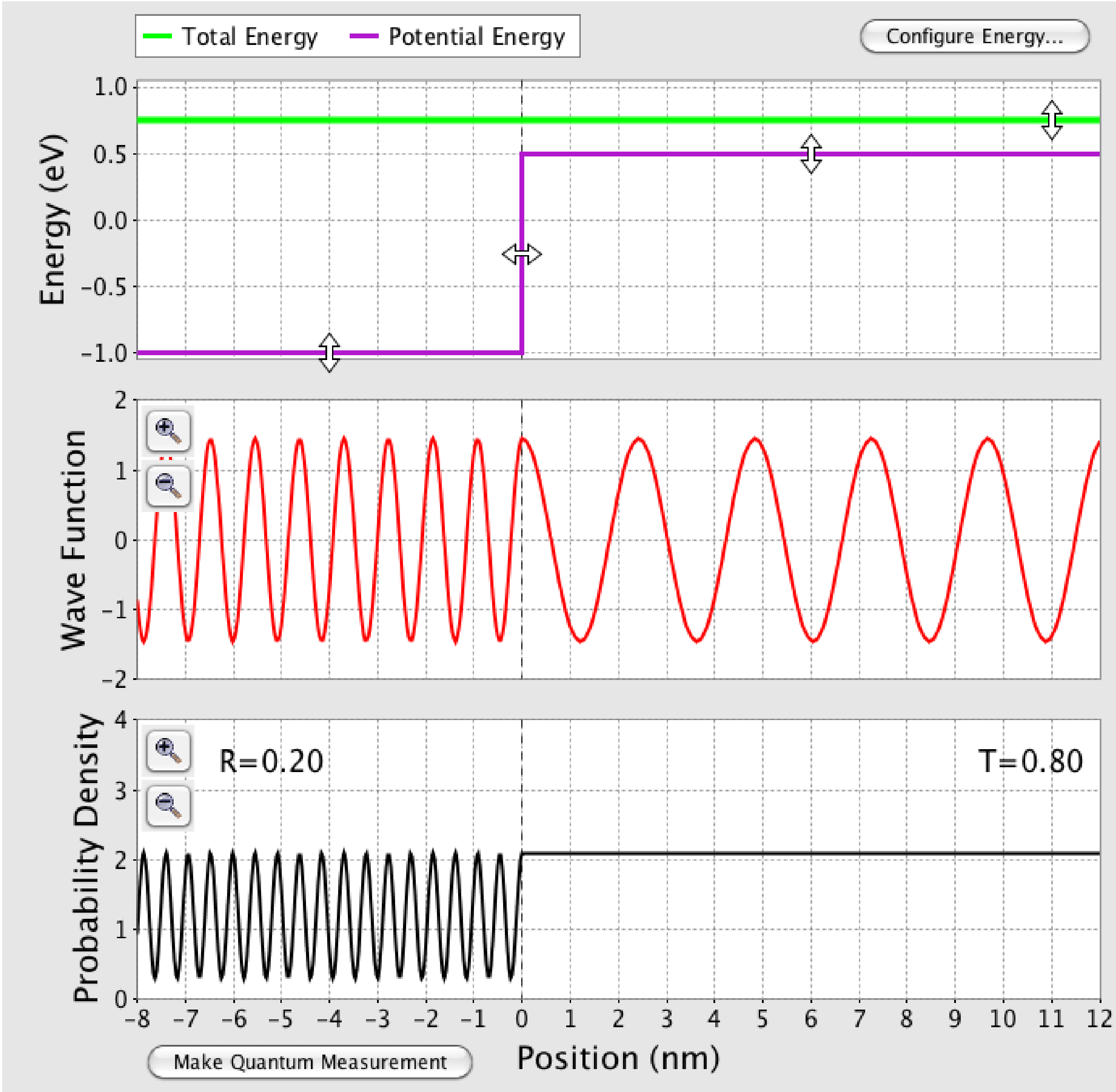
CASE I : $E_o > V$



$$\frac{B}{A} = \frac{1 - k_2/k_1}{1 + k_2/k_1} = \frac{k_1 - k_2}{k_1 + k_2}$$

$$\frac{C}{A} = \frac{2}{1 + k_2/k_1} = \frac{2k_1}{k_1 + k_2}$$

$$\begin{cases} A + B = C \\ A - B = \dfrac{k_2}{k_1} C \end{cases}$$



Example from: http://phet.colorado.edu/en/get-phet/one-at-a-time

Quantum Electron Currents

Given an electron of mass m

that is located in space with charge density $\rho = q\left|\psi(x)\right|^2$

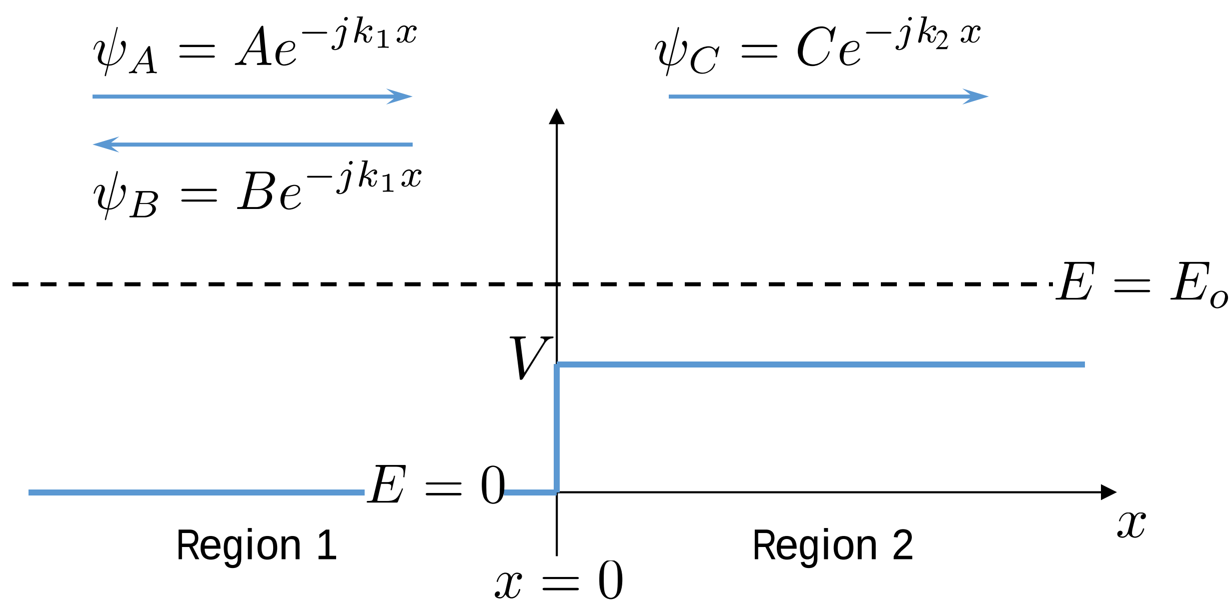
and moving with momentum $<p>$ corresponding to $<v> = \hbar k/m$

... then the current density for a *single electron* is given by

$$\boxed{J = \rho v = q\left|\psi\right|^2 (\hbar k/m)}$$

A Simple Potential Step

CASE I : $E_o > V$



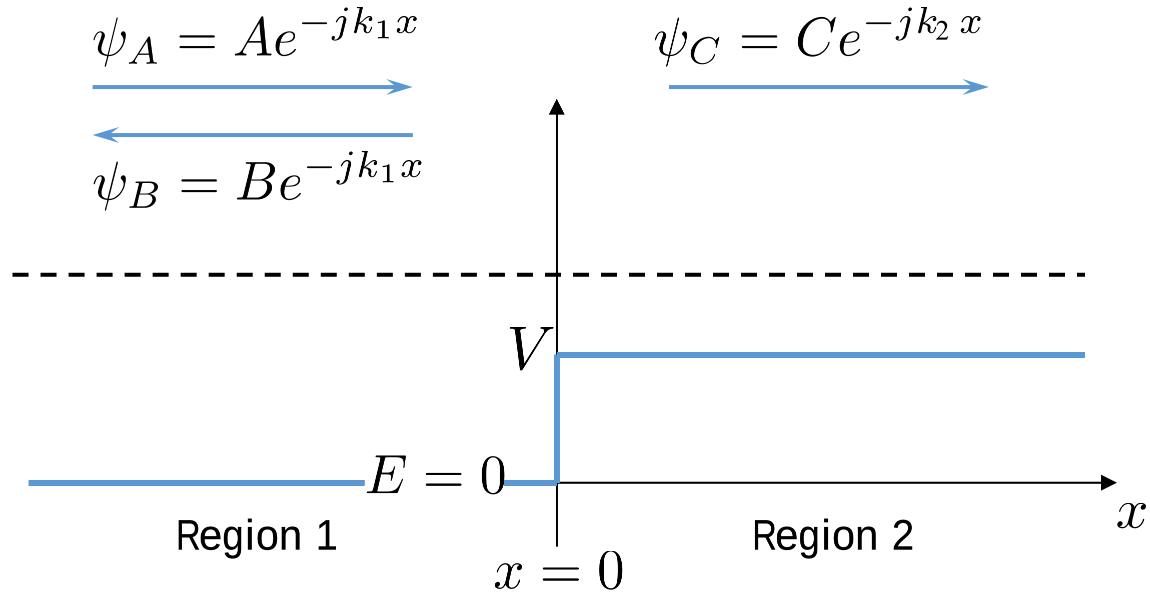
$$\text{Reflection} = R = \frac{J_{reflected}}{J_{incident}} = \frac{J_B}{J_A} = \frac{|\psi_B|^2(\hbar k_1/m)}{|\psi_A|^2(\hbar k_1/m)} = \left|\frac{B}{A}\right|^2$$

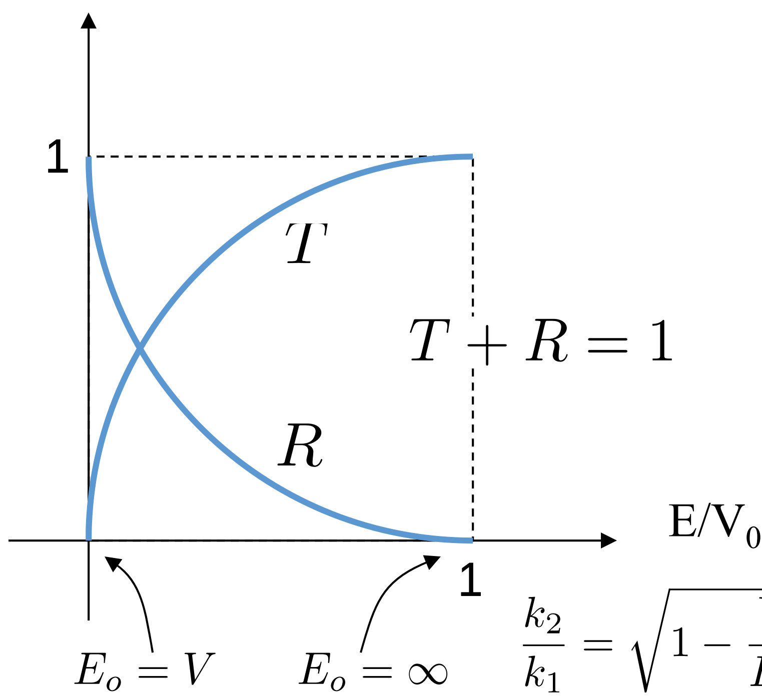
$$\text{Transmission} = T = \frac{J_{transmitted}}{J_{incident}} = \frac{J_C}{J_A} = \frac{|\psi_C|^2(\hbar k_2/m)}{|\psi_A|^2(\hbar k_1/m)} = \left|\frac{C}{A}\right|^2 \frac{k_2}{k_1}$$

$$\frac{B}{A} = \frac{1 - k_2/k_1}{1 + k_2/k_1} \qquad \frac{C}{A} = \frac{2}{1 + k_2/k_1}$$

A Simple Potential Step

CASE I : $E_o > V$





$$\frac{k_2}{k_1} = \sqrt{1 - \frac{V}{E_o}}$$

$$\text{Reflection} = R = \left|\frac{B}{A}\right|^2 = \left|\frac{k_1 - k_2}{k_1 + k_2}\right|^2$$

$$\text{Transmission} = T = 1 - R$$

$$= \frac{4k_1k_2}{|k_1 + k_2|^2}$$

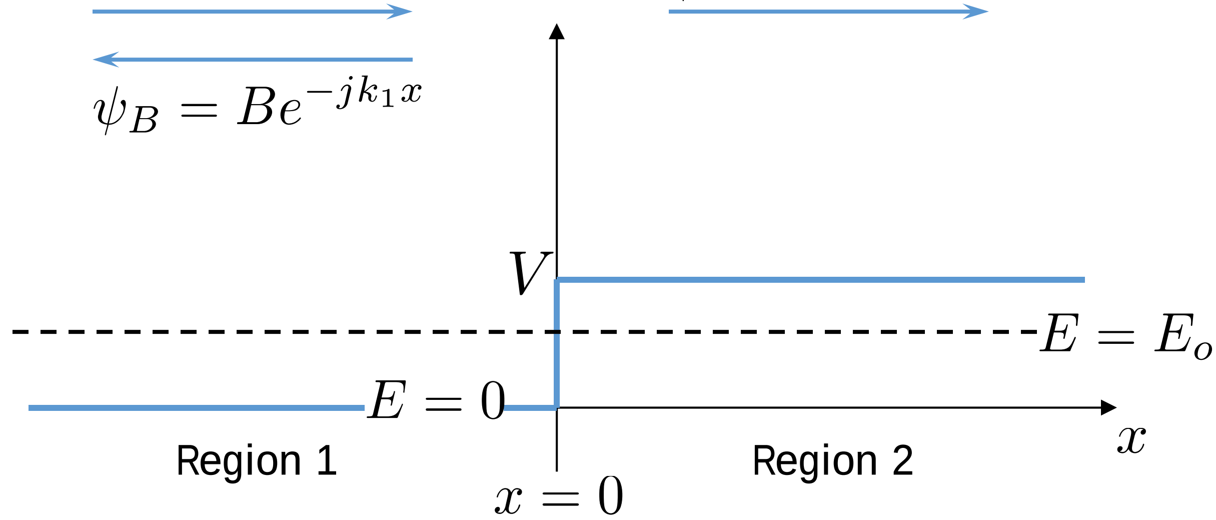
A Simple Potential Step

$$\psi_A = Ae^{-jk_1x}$$

$$\psi_B = Be^{-jk_1x}$$

$$\psi_C = Ce^{-\kappa x}$$

CASE II : $E_o < V$



In Region 1:

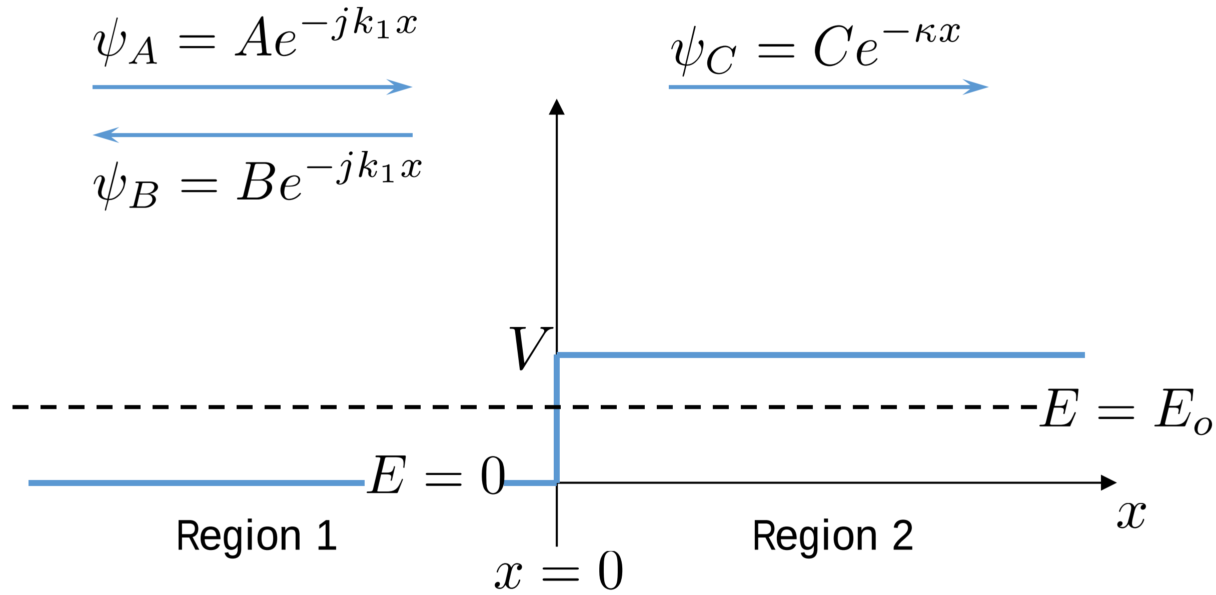
$$E_o\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \Rightarrow k_1^2 = \frac{2mE_o}{\hbar^2}$$

In Region 2:

$$(E_o - V)\,\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \Rightarrow \kappa^2 = \frac{2m\,(E_o - V)}{\hbar^2}$$

A Simple Potential Step

CASE II : $E_o < V$



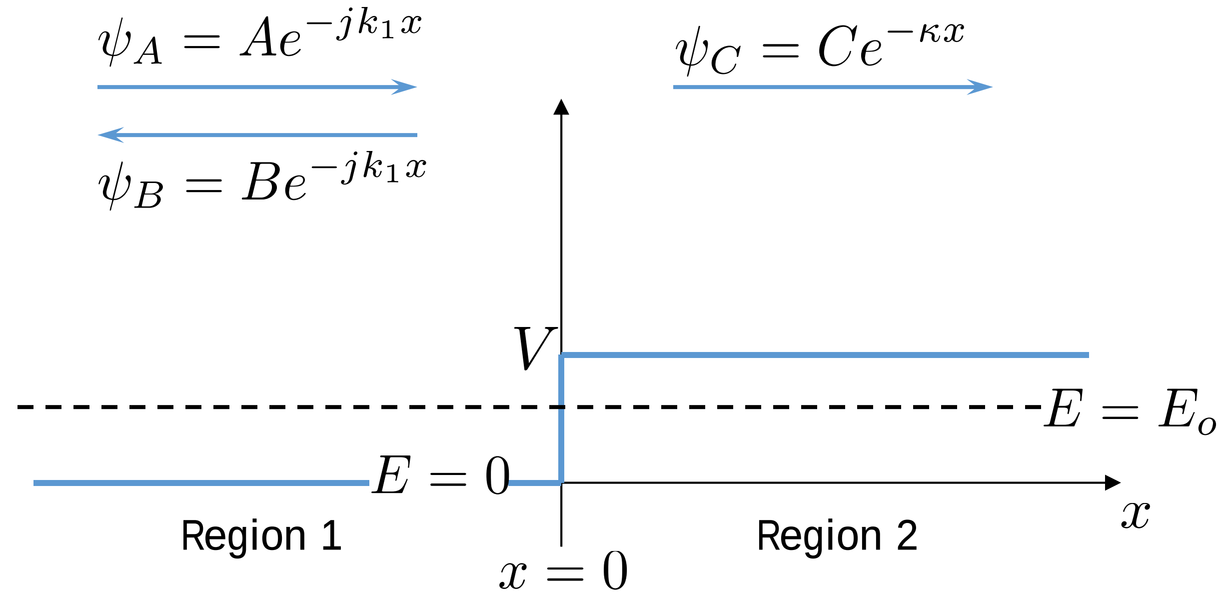
$$\psi_1 = Ae^{-jk_1x} + Be^{jk_1x} \qquad \psi_2 = Ce^{-\kappa x}$$

ψ is continuous: $\quad \psi_1(0) = \psi_2(0) \quad \Rightarrow \quad A + B = C$

$\dfrac{\partial \psi}{\partial x}$ is continuous: $\quad \dfrac{\partial}{\partial x}\psi(0) = \dfrac{\partial}{\partial x}\psi_2(0) \quad \Rightarrow \quad A - B = -j\dfrac{\kappa}{k_1}C$

A Simple Potential Step

CASE II : $E_o < V$



$$\frac{B}{A} = \frac{1 + j\kappa/k_1}{1 - j\kappa/k_1} = \exp(2j\phi), \text{ where } \phi = \kappa/k_1$$

$$\frac{C}{A} = \frac{2}{1 - j\kappa/k_1}$$

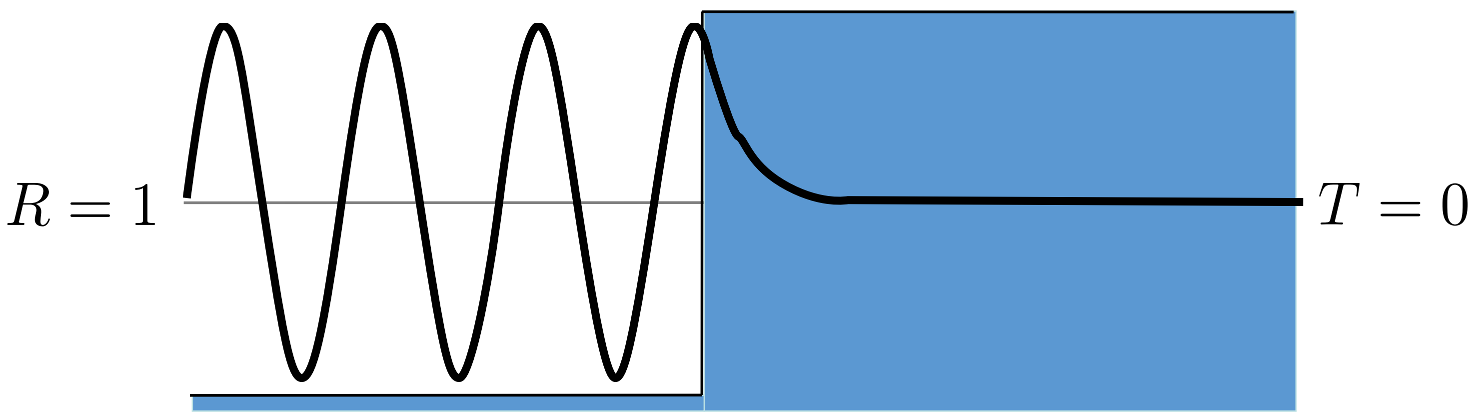
$$\begin{cases} A + B = C \\ A - B = -j\dfrac{\kappa}{k_1}C \end{cases}$$

$$\boxed{R = \left|\frac{B}{A}\right|^2 = 1 \qquad\qquad T = 0}$$

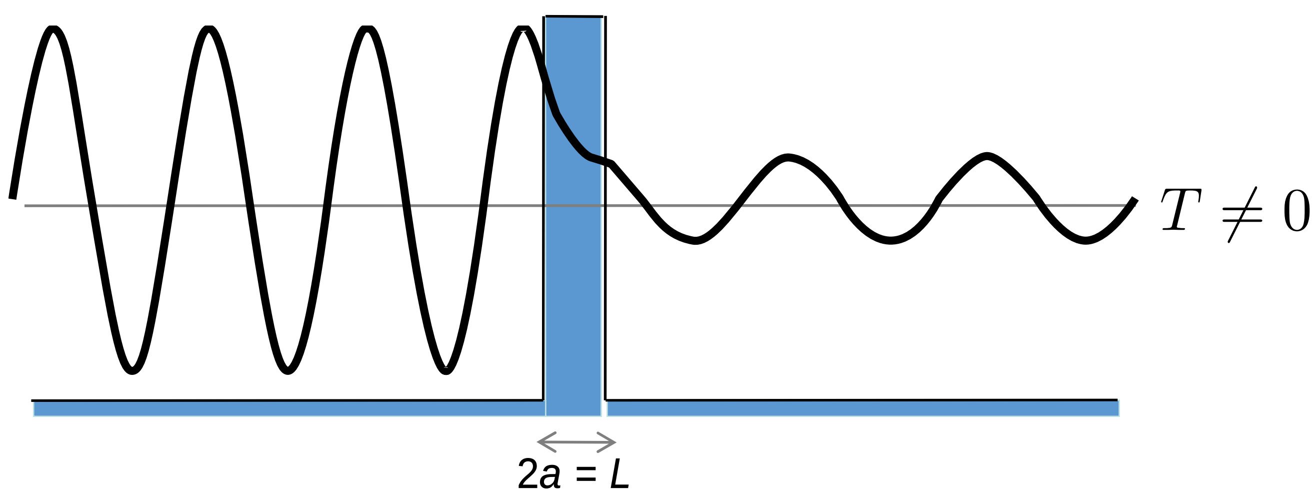
Total reflection → Transmission must be zero

Quantum Tunneling Through a Thin Potential Barrier

Total Reflection at Boundary

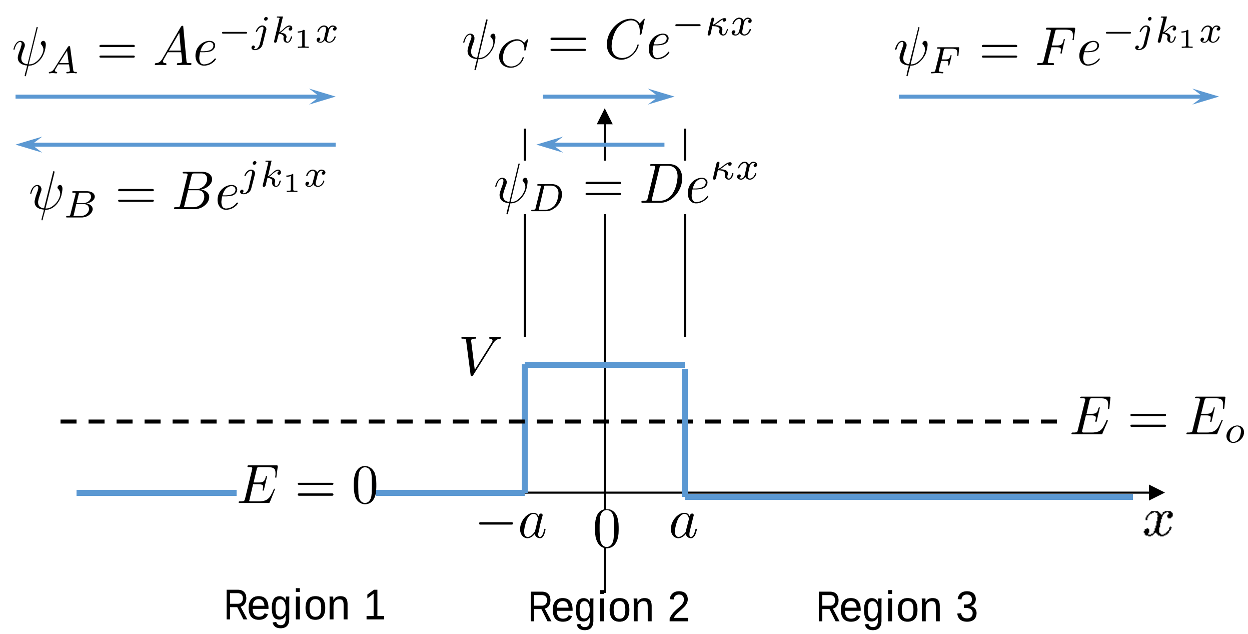


Frustrated Total Reflection (Tunneling)



A Rectangular Potential Step

CASE II : $E_o < V$



In Regions 1 and 3: $$E_o\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \Rightarrow k_1^2 = \frac{2mE_o}{\hbar^2}$$

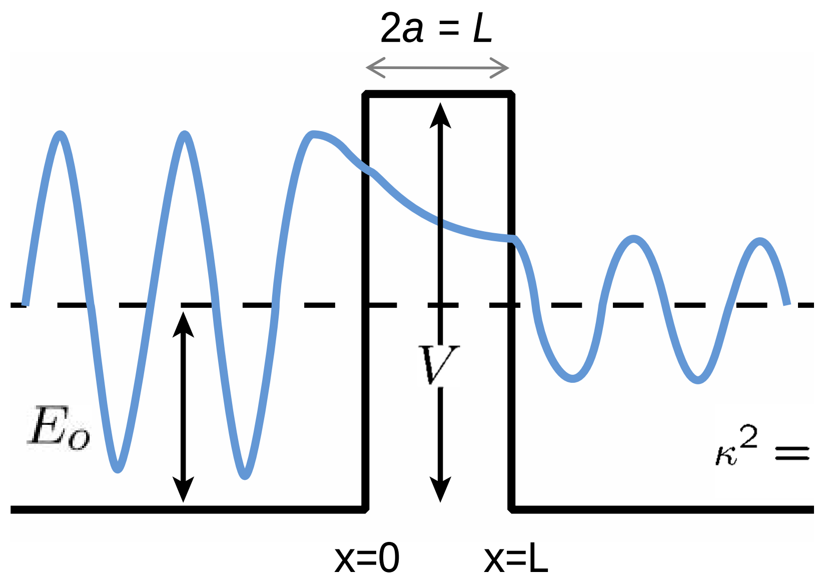
In Region 2: $$(E_o - V)\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \Rightarrow \kappa^2 = \frac{2m(V - E_o)}{\hbar^2}$$

for $E_o < V$:

$$\boxed{T = \left|\frac{F}{A}\right|^2 = \frac{1}{1 + \frac{1}{4}\frac{V^2}{E_o(V-E_o)}\sinh^2(2\kappa a)}}$$

A Rectangular Potential Step

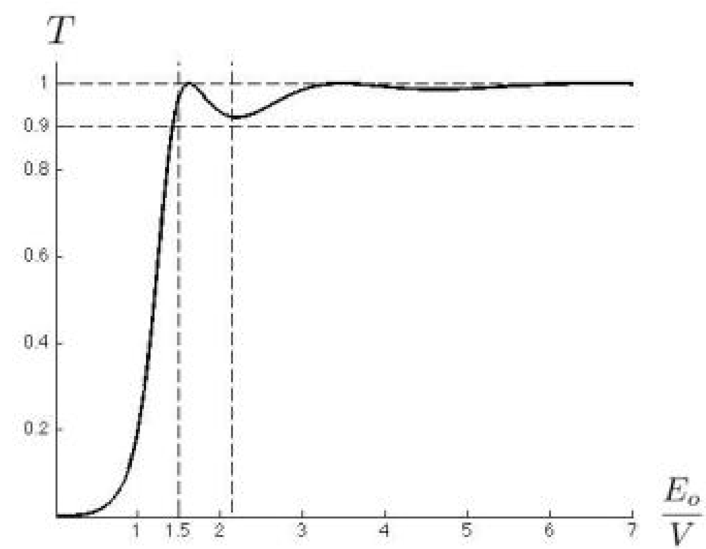
Real part of Ψ for $E_o < V$, shows hyperbolic (exponential) decay in the barrier domain and decrease in amplitude of the transmitted wave.



$$\kappa^2 = \frac{2m\,(V - E_o)}{\hbar^2}$$

for $E_o < V$:

$$T = \left|\frac{F}{A}\right|^2 = \frac{1}{1 + \frac{1}{4}\frac{V^2}{E_o(V-E_o)}\sinh^2(2\kappa a)}$$



Transmission Coefficient versus E_o/V for barrier with $2m(2a)^2V/\hbar = 16$