Review:

① pulse propagation

2nd order dispersion

$$\frac{\partial \xi}{\partial z} + k' \frac{\partial \xi}{\partial t} + \frac{1}{2} k'' j \frac{\partial \xi}{\partial t^2} = 0$$

retarded time frame: $\tau = t - \frac{z}{v_g}$ where $\frac{1}{v_g} = \frac{dk}{d\omega}$

$$\frac{\partial \xi}{\partial z} + \frac{1}{2} j k'' \frac{\partial \xi}{\partial \tau^2} = 0 \Rightarrow H(\Omega) = e^{j \frac{k''}{2} \Omega^2 z}$$

Analogy to diffraction: $H(u,v) = e^{-j\pi\lambda(u^2+v^2)z}$

② Gaussian pulse propagation

$$\xi(t,0) = e^{-\frac{t^2}{t_0^2}}$$

$$\Rightarrow \xi(t,z) = e^{-\frac{1}{\pi} z(1+jb)\frac{t^2}{t_0'^2}}$$

where $t_0' = t_0\sqrt{1+b^2}$ analogy to spatial propagation

$$b = \frac{2k''z}{t_0^2} \qquad W = W_0\sqrt{1+\frac{z^2}{z_0^2}}$$

$$z_0 = \frac{\pi w_0^2}{\lambda}$$

③ Chirping

instant f: $\omega(t) = \omega_0 \frac{\partial \phi(t)}{\partial t} = \omega_0 + 2b\frac{t}{t_0^2}$

bandwidth: $\Delta\omega = \frac{2\sqrt{1+b^2}}{t_0}$, not change in linear system (i.e. linear chirping)

$z=0$. $a=0$. chirping-free

(Transform limited pulse)

in general $\Delta\omega = \frac{2\sqrt{1+a^2}}{t_0} = \frac{2}{t_0}$

$z=z$ $\quad \Delta\omega = \frac{2\sqrt{1+b^2}}{t_0'} = \frac{2\sqrt{1+b^2}}{t_0\sqrt{1+b^2}} = \frac{2}{t_0}$

$$b = \frac{2k'' z}{t_0^2}$$

(1) linear system will not create new frequency

$a\uparrow \Rightarrow t_0\uparrow \Rightarrow$ keep $\Delta\omega$ unchanged

(2) to obtain new frequency $\Rightarrow$ use nonlinear.

(4) High order dispersion such as k'''

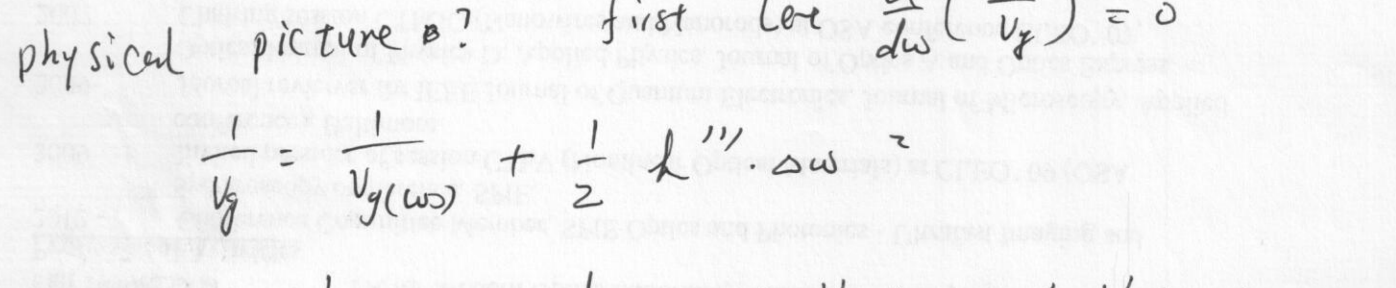
GVD $k'' \sim \frac{d}{d\omega}\left(\frac{1}{v_g}\right) \Rightarrow$ third order: $\frac{d}{d\omega}(k'')$

$$\frac{1}{v_g} = \frac{1}{v_g(\omega_0)} + \frac{d}{d\omega}\left(\frac{1}{v_g}\right)\cdot\Delta\omega + \frac{1}{2}\frac{d^2}{d\omega^2}\left(\frac{1}{v_g}\right)\cdot\Delta\omega^2$$

physical picture: first let $\frac{d}{d\omega}\left(\frac{1}{v_g}\right) = 0$

$$\frac{1}{v_g} = \frac{1}{v_g(\omega_0)} + \frac{1}{2}k'''\Delta\omega^2$$

$k''' > 0$, $\quad \frac{1}{v_g(\omega)} > \frac{1}{v_g(\omega_0)} \Rightarrow v_g(\omega) < v_g(\omega_0)$ $\quad k''' < 0$



today continue to discuss chirping L[illegible]4

$$e^{-(1+ja)\frac{t^2}{t_0^2}} \quad @ \ z=0$$

if $a \neq 0$, $t_0' = t_0\sqrt{1+b^2}$

$$b = a + \frac{2k''z}{t_0^2(1+a^2)}$$

means that we can find a z by let $b=0$

$$z_c = \frac{-a t_0^2(1+a^2)}{2k''z}$$

$\therefore$ $b=0$,

$\therefore$ t_0 is the smallest pulse width.
(shortest)

$\Rightarrow$ i.e. For a transform limited pulse, t_0

means the shortest pulse width supported by the bandwidth

Nonlinear chirping:

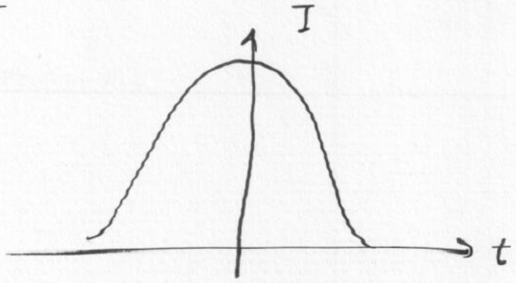
Kerr effect: $\Delta n = n_2 I$

$$\phi_{NL} = \frac{\omega}{c}\cdot \Delta n L \Rightarrow A_0 e^{-\frac{t^2}{t_0^2} - j\omega t} e^{j\phi_{NL}} \cdot e^{j\phi_L}$$

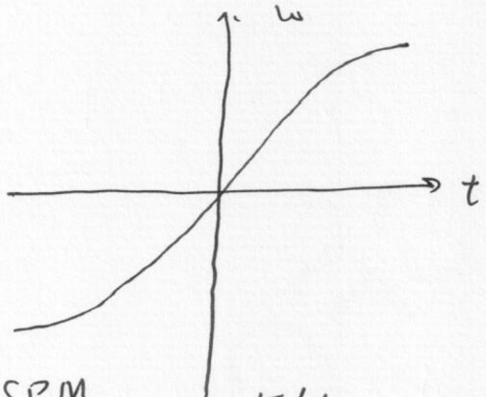
$$\Rightarrow \phi = \omega_0 t - \phi_{NL} - \phi_L$$

instant $\omega(t) = \omega_0 - \frac{\partial \phi_{NL}}{\partial t} = \omega_0 - \frac{\omega_0}{c} n_2 L \cdot \frac{dI}{dt}$

So look at a pulse



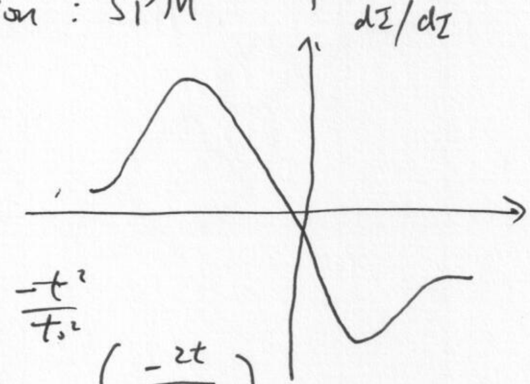
$$\omega(t) = \omega_0 - \frac{\omega_0}{c}\cdot n_2 L \cdot \frac{dI}{dt}$$



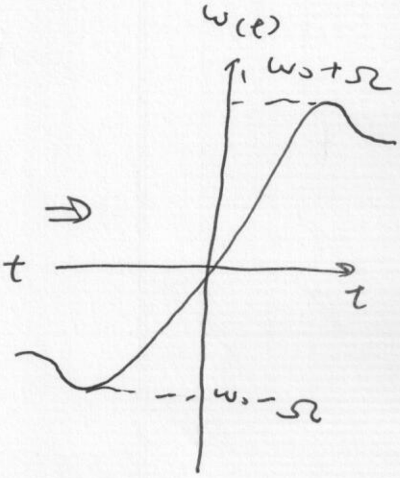
know: $I = I_0 e^{-\frac{t^2}{t_0^2}}$

$$\frac{dI}{dt} = I_0 e^{-\frac{t^2}{t_0^2}}\left(-\frac{2t}{t_0^2}\right)$$

Self-phase-modulation : SPM



⇒



Let $\frac{d\omega}{dt} = 0$ ⇒ find Ω

$$\omega(t) = \omega_0 - \frac{\omega}{c}\cdot n_2 L\cdot I_0 e^{-\frac{t^2}{t_0^2}}\left(\frac{-2t}{t_0^2}\right)$$

$$\frac{d\omega}{dt} = -\frac{\omega}{c}\cdot n_2 L I_0\left(-\frac{2t}{t_0^2}\right)^2\cdot e^{\frac{-t^2}{t_0^2}} - \frac{\omega n_2 L}{c}\cdot I_0\left(-\frac{2t}{t_0^2}\right)\cdot e^{-\frac{t^2}{t_0^2}} = 0$$

$$\Rightarrow \Omega = \omega(t) = \omega\left(\pm\frac{t_0}{\sqrt{2}}\right)$$

$$\Omega = \omega_0 \pm \left(\frac{\omega}{c}\cdot n_2 I_0\right)\cdot z\cdot\frac{1}{t_0}\cdot C_1$$

C_1: constant number

⇒ refractive index change at peak position

$$= \omega_0 \pm \frac{\phi_{NL}}{t_0}\cdot C_1$$

$$\Delta\omega \sim \frac{\frac{\omega_0}{c}\cdot n_2 I_0 z}{t_0} \Rightarrow \frac{\Delta\omega}{\omega_0} \sim \frac{n_2 I_0 z}{c\, t_0}$$

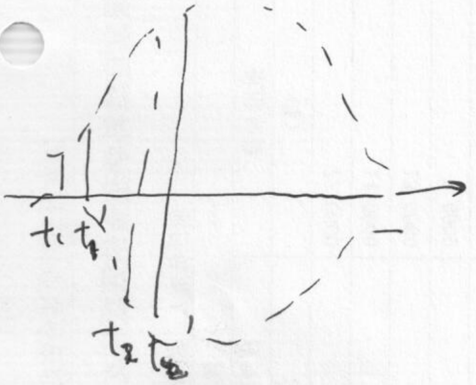
given: $n_2 \sim 3\times10^{-20}\ \frac{m^2}{W}$

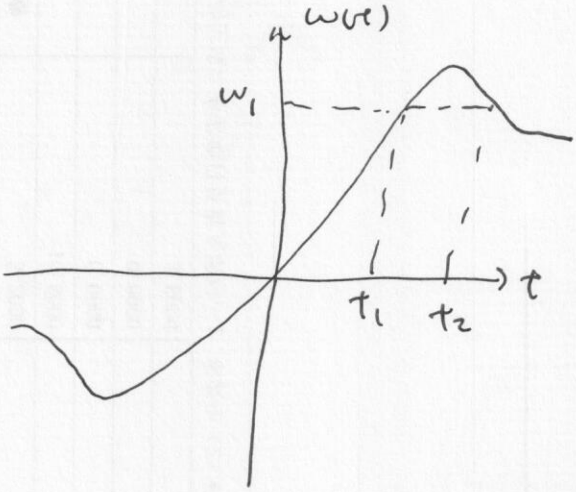
$1nJ$, $1\mu m^2$, $100 fs$, $1cm$.

$$\Rightarrow \frac{\Delta\omega}{\omega_0} \sim 8\times10^{-20} \cdot \frac{10^{-9}\times 1\times10^{-2}}{(1\times10^{-6})^2\times100\times10^{-15}\times3\times10^{8}\times100\times10^{-15}}$$

~ 0.1

Spectrum of the pulse:



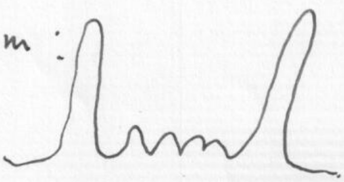


$$F(\omega_1) = \int_{-\infty}^{\infty} (\ ) e^{+j\phi(t)} \cdot e^{j\omega_1 t} \cdot dt$$

$$= \int_{-\infty}^{t_1} + \int_{t_1}^{t_1'} + \int_{t_1'}^{t_2} + \int_{t_2}^{t_2'} + \int_{t_2'}^{+\infty}$$

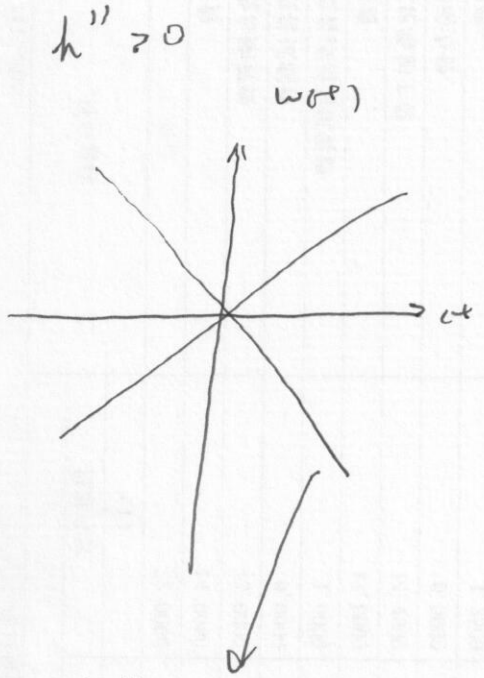
$$= \underbrace{\int_{t_1}^{t_1'} + \int_{t_2}^{t_2'}}_{\text{delay}}$$ two pulse: same f but delay

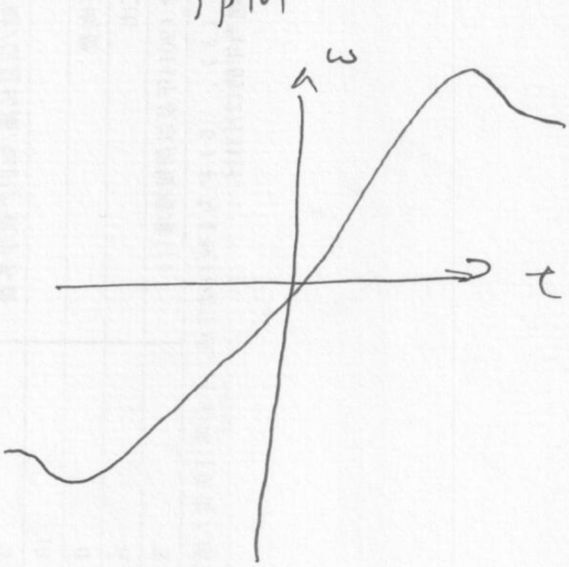
$\Rightarrow$ in spectrum:



Now let's review for both linear chirp (dispersion)

and nonlinear chirp (SPM)





So far, discussed pulse propagation in general dispersion material and included a little bit of Kerr effect.

Here: optical component such as mirror, grating

how to model: spatial and temporal.

both

easier to work with $F(\omega)$

phase modulation of linear Optics

e.g. mirror, greting, lens.

$$\mathcal{E}_{in}(\omega) \longrightarrow \boxed{H(\omega)} \longrightarrow \mathcal{E}_{out}(\omega)$$

$$H(\omega) = R(\omega)\cdot e^{j\phi(\omega)}$$

$$\phi(\omega) = \phi(\omega_0) + \phi'(\omega_0)\cdot\Delta\omega + \frac{1}{2}\phi''\Delta\omega^2$$

$$\mathcal{E}_{out}(t) = \frac{1}{2\pi}\int \mathcal{E}_{in}(\omega)\cdot H(\omega)\cdot e^{-j\omega t}\cdot d\omega$$

$$\mathcal{E}_{in}(\omega) = \hat{A}(\Omega)\cdot e^{-j\omega_0 t}$$

$\phi(\omega_0)$: carrier frequency

$\phi'(\omega_0)$: delay of pulse envelope

$\phi''(\omega_0)$: group velocity dispersion

So far. only on z-direction

$$(\nabla^2 - \mu\varepsilon\frac{\partial^2}{\partial t^2})E = 0$$

$$\downarrow$$

$\frac{\partial^2}{\partial z^2}$, physical: simple mode

Now we only study a ~~fig~~ single frequency. $\frac{\partial}{\partial t} \to -j\omega$

but consider x, y, z.

$$(\nabla^2 + \omega^2 \mu_0 \varepsilon)E = 0$$

$$\Rightarrow (\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2)E = 0$$

$$E \sim \underbrace{\mathcal{E}(x,y,z)}_{\text{slowly varying along } z}\cdot e^{jkz} \qquad \frac{\partial\mathcal{E}}{\partial z} \ll jk\mathcal{E}$$

$$\frac{\partial^2}{\partial x^2}\mathcal{E} = \frac{\partial^2 \mathcal{E}}{\partial x^2}\cdot e^{jkz}$$

$$\frac{\partial^2}{\partial y^2}\mathcal{E} = \frac{\partial^2 \mathcal{E}}{\partial y^2}\cdot e^{jkz}$$

$$\frac{\partial}{\partial z}\mathcal{E} = \frac{\partial \mathcal{E}}{\partial z}\cdot e^{jkz} + jk\,\mathcal{E}\cdot e^{jkz}$$

$$\frac{\partial^2}{\partial z^2}\mathcal{E} = \frac{\partial^2}{\partial z^2} e^{jkz} + 2jk\frac{\partial \mathcal{E}}{\partial z}\cdot e^{jkz} - k^2\cdot \mathcal{E} e^{jkz}$$

$\because \quad \frac{\partial \mathcal{E}}{\partial z} \ll jk\,\mathcal{E}$

$\therefore \quad \frac{\partial}{\partial z}\left(\frac{\partial \mathcal{E}}{\partial z}\right) \ll jk\left(\frac{\partial \mathcal{E}}{\partial z}\right)$

$\therefore$ So. $2jk\frac{\partial \mathcal{E}}{\partial z}\cdot \cancel{e^{jkz}} - k^2\mathcal{E} + \frac{\partial^2}{\partial x^2}\mathcal{E} + \frac{\partial^2}{\partial y^2}\mathcal{E} + k^2\mathcal{E} = 0$

$$\Rightarrow \frac{\partial \mathcal{E}}{\partial z} - \frac{j}{2k}\left(\frac{\partial^2 \mathcal{E}}{\partial x^2} + \frac{\partial^2 \mathcal{E}}{\partial y^2}\right) = 0$$

recall: $\frac{\partial \mathcal{E}}{\partial z} + \frac{1}{2}jk''\frac{\partial^2 \mathcal{E}}{\partial t^2} = 0$

$E(x,y,z) = A(x,y,z)\cdot e^{jkz}$ (base on slowly varying envelope)

$\frac{\partial}{\partial z} \ll jk.$

$jk \sim \frac{\partial}{\partial z}\cdot e^{jkz}$
$\sim jk e^{jkz}$

means ~~th~~ the change at the wavelength ~~set~~ scale

Another way to get equation:

$$k(\omega) = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

now we focus a single frequency: so $k(\omega)$ is fixed

beam is along z, so

$$k = k_z \sqrt{1 + \frac{k_x^2 + k_y^2}{k_z^2}}$$

$$\approx k_z \left(1 + \frac{1}{2} \frac{k_x^2 + k_y^2}{k_z}\right)$$

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2}$$

$$= k \left(1 - \frac{1}{2} \frac{k_x^2 + k_y^2}{\cancel{k_z} k}\right)$$

$$= k - \frac{k_x^2 + k_y^2}{2k}$$

$$\frac{\partial}{\partial z} \sim j(k_z - k_0) \sim jk$$

$$\frac{\partial}{\partial x} \sim jk_x \qquad \Rightarrow \frac{\partial}{\partial z} - \frac{j}{2k}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) = 0$$

$$\frac{\partial}{\partial z} \sim jk_y$$

Now. Solve: $\frac{\partial A}{\partial z} - \frac{j}{2k}\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}\right) = 0$

$x \sim u, \qquad y \sim v.$

$\frac{\partial}{\partial x} \sim j2\pi u, \qquad \frac{\partial}{\partial y} \sim j2\pi v$

F.T. on x, y space ~ spatial frequency

$$\frac{\partial \tilde{A}}{\partial z} - \frac{j}{2k}(-4\pi^2)(u^2+v^2)\tilde{A} = 0$$

$$\Rightarrow \frac{\partial \tilde{A}}{\partial z} + j\pi\lambda(u^2+v^2)\tilde{A} = 0$$

$$\tilde{A}(u,v,z) = \tilde{A}(u,v,0)\underbrace{e^{-j\pi\lambda z(u^2+v^2)}}_{H(u,v)}$$

$$H(u,v) \sim e^{-j\pi\lambda z(u^2+v^2)}$$

$$h(x,y) \sim e^{j\pi\frac{1}{\lambda z}(x^2+y^2)}$$

e.g. Gaussian beam propagation: $e^{-\frac{x^2+y^2}{w_0^2}} \otimes h(x,y)$

$$e^{j\frac{\pi}{\lambda q_0}(x^2+y^2)} \Rightarrow j\frac{\pi}{\lambda q_0} = \frac{-1}{w_0} \Rightarrow q_0 = \frac{\pi w_0^2}{j\lambda}$$

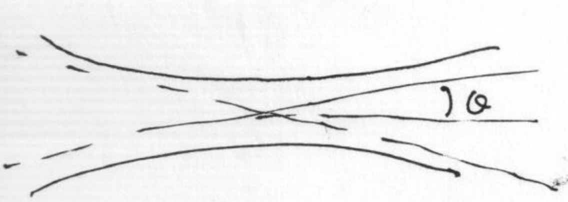
So $e^{j\frac{\pi}{\lambda q_0}(x^2+y^2)} \otimes h(x,y) \sim e^{j\frac{\pi}{\lambda z}(x^2+y^2)}$

$\Downarrow$

two free space system:

$$= e^{j\frac{\pi}{\lambda(q_0+z)}(x^2+y^2)}$$

Another way. conventional way. F.T. → · IFT.



$$\theta = \frac{W(z)}{\lambda} = \frac{w_0}{z_0} = \frac{\lambda}{\pi w_0}$$

DOF: $z_0 = \frac{\pi w_0^2}{\lambda}$ $w_0 = \frac{\lambda}{NA}$

$W^2(z) = w_0^2 (1 + \frac{z^2}{z_0^2})$; $a + jb = r \cdot e^{j\phi}$

$z_0 = \frac{\pi w_0^2}{\lambda}$

$z \to \infty \gg z_0$, $W(z) \approx w_0 \frac{z}{z_0}$

if z is small: $\cancel{\phi = \tan\phi \Rightarrow \phi}$ $\phi \approx \tan\phi = \frac{z}{z_0} \Rightarrow$ plane wave

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Dispersion. $\frac{\partial A}{\partial z} + \frac{1}{2} j k'' \frac{\partial^2 A}{\partial t^2} = 0$

L.T. $\to \boxed{H(\omega)} \to \Omega = 2\pi u$

$H(u) = e^{j\frac{1}{2}k''\Omega^2 z} = e^{-j\pi(-2\pi k'')u^2 z}$

$e^{-j\pi \Lambda z u^2}$

$e^{-\pi(\sqrt{j\Lambda z}\, u)^2}$

$h(t) = \sqrt{\frac{1}{j\Lambda z}}\, e^{j\frac{\pi}{\Lambda z}t^2}$

$\frac{1}{\sqrt{j\Lambda z}} e^{-\pi(\frac{t}{\sqrt{j\Lambda z}})^2}$

Diffraction:

1D: $\frac{\partial A}{\partial z} - j\frac{1}{2k}\frac{\partial^2 A}{\partial x^2} = 0$

$H(u) = e^{-j\pi\lambda z \cdot u^2}$

$h(x) = \sqrt{\frac{1}{j\lambda z}}\, e^{j\frac{\pi}{\lambda z}x^2}$

$t \to x$, $\Omega \to k_x$. $\Lambda \sim \lambda$, $-k'' \to \frac{1}{k}$. $\Lambda \sim -2\pi k''$