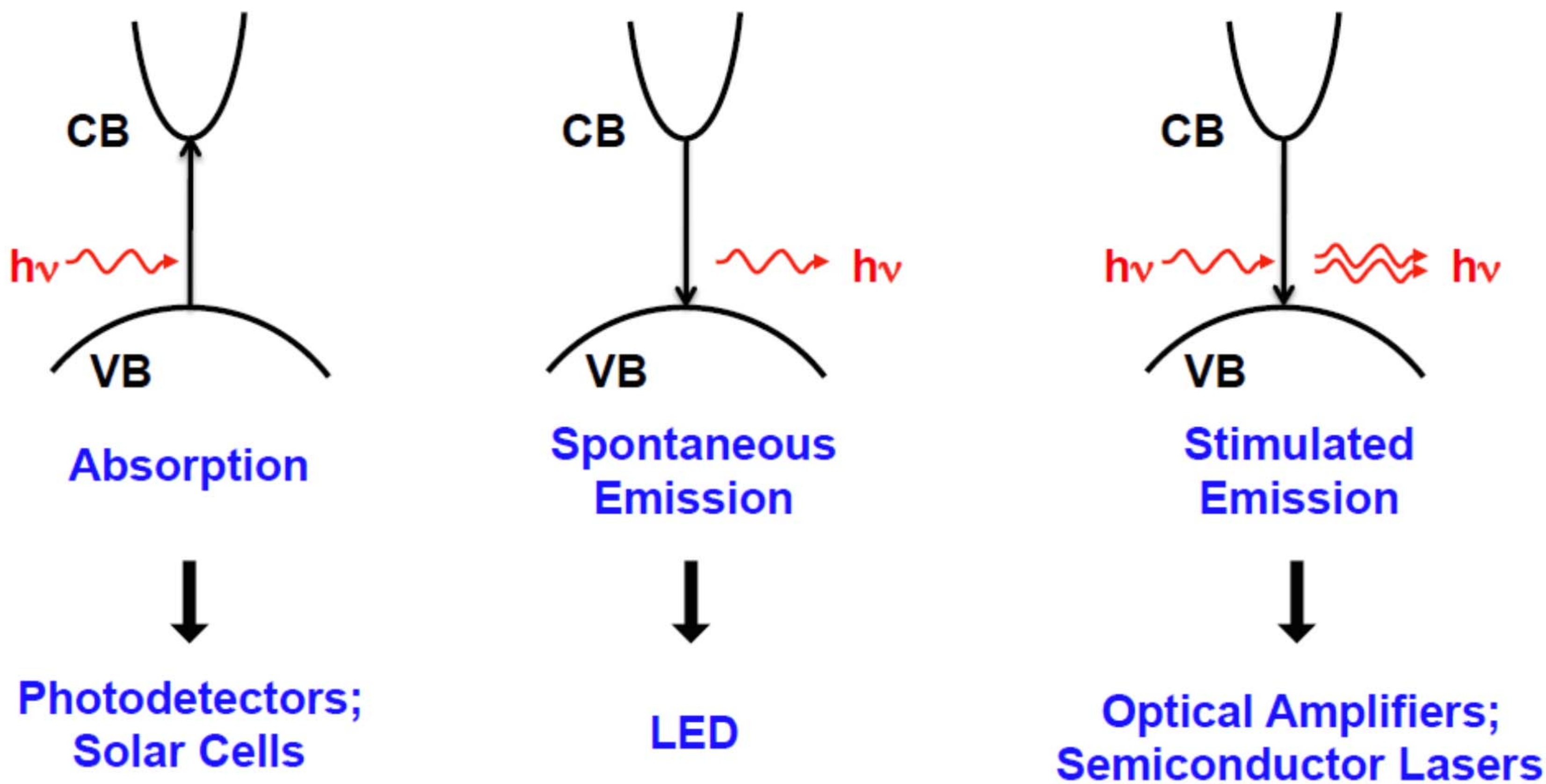
Absorption Coefficient of semiconductor

Band-to-Band Transition



Joint Density of States for Semiconductor

The electron states associated with optical transition are related by conservation of mementum: $\vec{k}_b \approx \vec{k}_a \approx \vec{k}$

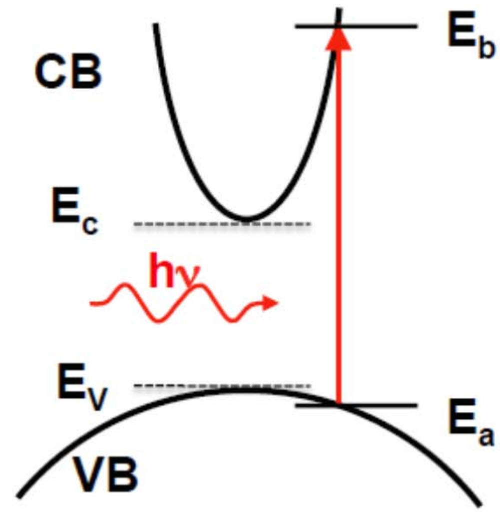
$$E_b = E_C + \frac{\hbar^2 k^2}{2m_e^*} \qquad E_a = E_V - \frac{\hbar^2 k^2}{2m_h^*}$$

$$E_b - E_a = (E_C - E_V) + \frac{\hbar^2 k^2}{2}\left(\frac{1}{m_e^*} + \frac{1}{m_h^*}\right)$$

$$h\omega = E_g + \frac{\hbar^2 k^2}{2m_r^*}$$

Joint density of states for the pair of electron states:

$$\rho_r(\hbar\omega - E_g) = \frac{1}{2\pi^2}\left(\frac{2m_r^*}{\hbar^2}\right)^{3/2}\sqrt{\hbar\omega - E_g}$$



Absorption Coefficient

CB completely full, VB completely empty, total upward transition at $\hbar\omega$:

$$R_0(\hbar\omega) = \frac{2}{V}\sum_k \left[\frac{2\pi}{\hbar}\left|H_{ba}^{'}\right|^2 \delta(E_b - E_a - \hbar\omega)\right] = \frac{2\pi}{\hbar}\left|H_{ba}^{'}\right|^2 \int \frac{2d\vec{k}}{(2\pi)^3}\delta(E_g + \frac{\hbar^2 k^2}{2m_r^*} - \hbar\omega)$$

$$\boxed{R_0(\hbar\omega) = \frac{2\pi}{\hbar}\left|H_{ba}^{'}\right|^2 \rho_r(\hbar\omega - E_g)} \quad \text{unit: } [\frac{1}{m^3 s}]$$

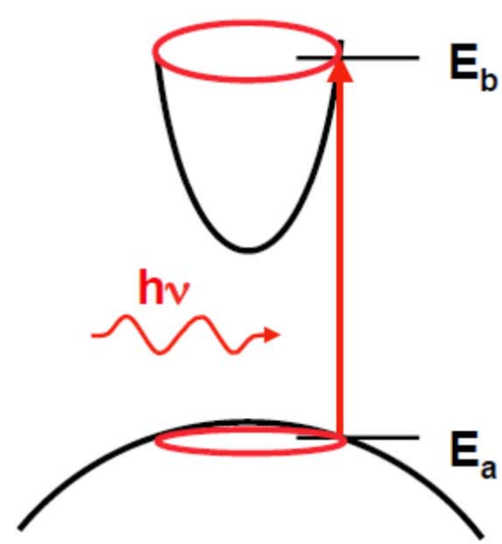
Absorption coefficient

$$\alpha_0(\hbar\omega) = \frac{R(\hbar\omega)}{\text{photon flux}} = \frac{R(\hbar\omega)}{\frac{\varepsilon_0\varepsilon_r E_0^2}{2}\frac{c}{n_r}\frac{1}{\hbar\omega}} = \frac{R(\hbar\omega)}{\frac{\varepsilon_0 n_r \omega^2 A_0^2 c}{2\hbar\omega}} \quad \text{unit: } [\frac{1}{m}]$$

$$H_{ba}^{'} = -\frac{eA_0}{2m_0}\hat{e}\cdot\vec{P}_{cv}$$

$$\boxed{\alpha_0(\hbar\omega) = C_0\left|\hat{e}\cdot\vec{P}_{cv}\right|^2 \rho_r(\hbar\omega - E_g)}$$

$$C_0 = \frac{\pi e^2}{n_r c\varepsilon_0 m_0^2 \omega} \approx 7\times 10^9 \qquad \text{unit: } \left[\frac{m^2}{kg}\right]$$



Typical Values of Optical Matrix Element

$$H'_{ba} = -\frac{eA_0}{2m_0}\hat{e}\cdot\vec{P}_{cv} = -e\vec{E}\cdot\vec{r}_{ba} = -ei\omega\frac{A_0}{2}\hat{e}\cdot\vec{r}_{ba} \Rightarrow \left|\vec{r}_{ba}\right| = \frac{\left|\vec{P}_{cv}\right|}{m_0\omega}$$

The optical matrix element is often expressed in E_p :

$$\left|\hat{e}\cdot\vec{P}_{cv}\right|^2 = \frac{m_0}{6}E_p$$

Typical values of E_p (Table K.2 on p.709 of Chuang textbook)

GaAs: $E_p = 25.7$ eV

The corresponding $\left|\vec{r}_{ba}\right| \sim 0.4$ nm

AlAs: $E_p = 21.1$ eV

InAs: $E_p = 22.2$ eV

InP: $E_p = 20.7$ eV

GaP: $E_p = 22.2$ eV

Absorption Coefficient for GaAs

$$\alpha_0(\hbar\omega) = C_0 \left|\hat{e}\cdot\vec{P}_{cv}\right|^2 \rho_r(\hbar\omega - E_g)$$

$nr := 3.5$ $\quad \varepsilon 0 := 8.854\cdot 10^{-12}\,\frac{F}{m}$ $\quad h\_bar = 1.055\times 10^{-34}\,\frac{m^2\cdot kg}{s}$ $\quad 1eV = 1.6\times 10^{-19}\,J$

$Eg := 1.42eV$ $\quad \omega := \frac{Eg}{h\_bar}$ $\quad \omega = 2.154\times 10^{15}\,\frac{1}{s}$

$$C0 := \frac{\pi\cdot q^2}{m0^2\cdot\omega\cdot\varepsilon 0\cdot c\cdot nr} \qquad C0 = 4.842\times 10^{9}\,\frac{m^2}{kg}$$

$Ep := 25.7eV$ $\quad \frac{m0}{6}\cdot Ep = 6.243\times 10^{-49}\,kg\cdot J$

$$m\_r := \frac{0.067\cdot 0.5}{0.067 + 0.5}\cdot m0$$

$$\rho r(hv) := \frac{1}{2\pi^2}\cdot\left(\frac{2\cdot m\_r}{h\_bar^2}\right)^{\frac{3}{2}}\cdot\sqrt{hv - Eg} \qquad \rho r(1.43eV) = 6.102\times 10^{43}\,\frac{1}{m^3\cdot J}$$

$$\alpha 0(hv) := C0\cdot\frac{m0}{6}\cdot Ep\cdot\rho r(hv) \qquad \alpha 0(1.43eV) = 1.845\times 10^{5}\,\frac{1}{m} \qquad \alpha 0(1.43eV) = 1.845\times 10^{3}\,\frac{1}{cm}$$

Gain

Absorption coefficient when bands are partially filled

Absorption Coefficient

CB completely full, VB completely empty, total upward transition at $\hbar\omega$:

$$R_0(\hbar\omega) = \frac{2}{V}\sum_k \left[\frac{2\pi}{\hbar}\left|H'_{ba}\right|^2 \delta(E_b - E_a - \hbar\omega)\right] = \frac{2\pi}{\hbar}\left|H'_{ba}\right|^2 \int \frac{2d\vec{k}}{(2\pi)^3}\delta(E_g + \frac{\hbar^2k^2}{2m_r^*} - \hbar\omega)$$

$$\boxed{R_0(\hbar\omega) = \frac{2\pi}{\hbar}\left|H'_{ba}\right|^2 \rho_r(\hbar\omega - E_g)} \quad \text{unit: } [\frac{1}{m^3 s}]$$

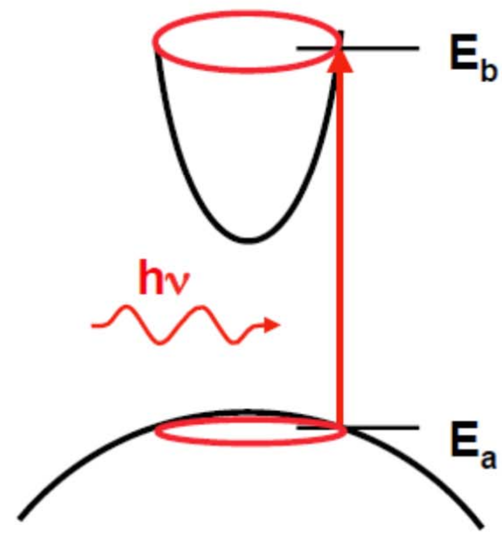
Absorption coefficient

$$\alpha_0(\hbar\omega) = \frac{R(\hbar\omega)}{\text{photon flux}} = \frac{R(\hbar\omega)}{\frac{\varepsilon_0\varepsilon_r E_0^2}{2}\frac{c}{n_r}\frac{1}{\hbar\omega}} = \frac{R(\hbar\omega)}{\frac{\varepsilon_0 n_r \omega^2 A_0^2 c}{2\hbar\omega}} \quad \text{unit: } [\frac{1}{m}]$$

$$H'_{ba} = -\frac{eA_0}{2m_0}\hat{e}\cdot\vec{P}_{cv}$$

$$\boxed{\alpha_0(\hbar\omega) = C_0\left|\hat{e}\cdot\vec{P}_{cv}\right|^2 \rho_r(\hbar\omega - E_g)}$$

$$C_0 = \frac{\pi e^2}{n_r c\varepsilon_0 m_0^2\omega} \approx 7\times 10^9 \qquad \text{unit: } \left[\frac{m^2}{kg}\right]$$



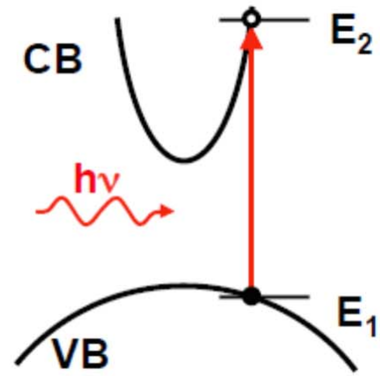
Absorption Coefficient

When CB and VB are partially filled:

Absorption Condition: VB is occupied, CB is empty

Absorption probability $= f_V(E_1)\left(1 - f_C(E_2)\right)$

$$R_{12}(\hbar\omega) = \frac{2}{V}\sum_k \left[\frac{2\pi}{\hbar}\left|H'_{ba}\right|^2 \delta(E_b - E_a - \hbar\omega)\right] f_V\left(1 - f_C\right)$$

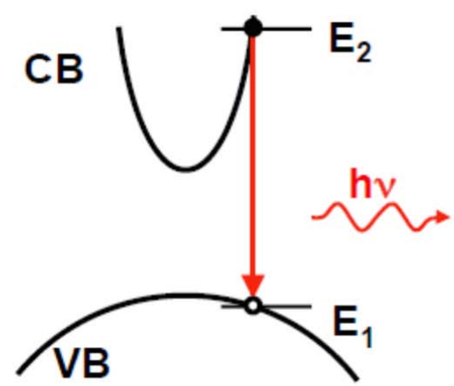


$$R_{12} \propto f_V\left(1 - f_C\right)$$

Emission Condition: CB is occupied, VB is empty

Emission probability $= f_C(E_2)\left(1 - f_V(E_1)\right)$

$$R_{21}(\hbar\omega) = \frac{2}{V}\sum_k \left[\frac{2\pi}{\hbar}\left|H'_{ba}\right|^2 \delta(E_b - E_a - \hbar\omega)\right] f_C\left(1 - f_V\right)$$



$$R_{21} \propto f_C\left(1 - f_V\right)$$

Absorption Coefficient

Net absorption rate:

$$R(\hbar\omega) = \frac{2}{V}\sum_{k}\left[\frac{2\pi}{\hbar}\left|H'_{ba}\right|^2 \delta(E_b - E_a - \hbar\omega)\right]\left[f_V\left(1 - f_C\right) - f_C\left(1 - f_V\right)\right]$$

$$= \frac{2\pi}{\hbar}\left|H'_{ba}\right|^2 \int \frac{2d\vec{k}}{(2\pi)^3}\delta(E_g + \frac{\hbar^2 k^2}{2m_r^*} - \hbar\omega)\left[f_V(k) - f_C(k)\right]$$

$$= \frac{2\pi}{\hbar}\left|H'_{ba}\right|^2 \rho_r(\hbar\omega - E_g)\left[f_V\left(-\left(\hbar\omega - E_g\right)\frac{m_r^*}{m_h^*}\right) - f_C\left(E_g + \left(\hbar\omega - E_g\right)\frac{m_r^*}{m_e^*}\right)\right]$$

$$\boxed{\alpha(\hbar\omega) = C_0\left|\hat{e}\cdot\vec{P}_{cv}\right|^2 \rho_r(\hbar\omega - E_g)\left[-f_g(\hbar\omega - E_g)\right]}$$

Fermi Inversion Factor :

$$\boxed{f_g(\hbar\omega - E_g) = f_C\left(E_g + \left(\hbar\omega - E_g\right)\frac{m_r^*}{m_e^*}\right) - f_V\left(-\left(\hbar\omega - E_g\right)\frac{m_r^*}{m_h^*}\right)}$$

Fermi Inversion Factor

$$f_g(\hbar\omega - E_g) = f_C\left(E_2\right) - f_V\left(E_1\right)$$

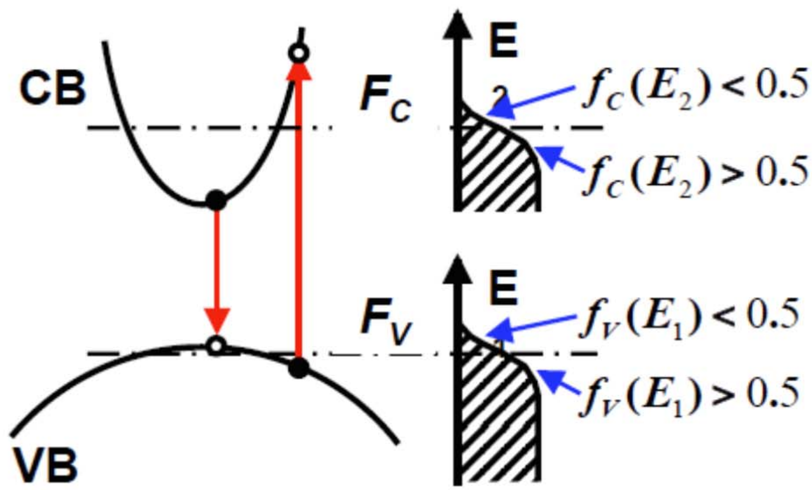
$$\begin{cases} E_2 = E_g + \left(\hbar\omega - E_g\right)\dfrac{m_r^*}{m_e^*} \\ E_1 = -\left(\hbar\omega - E_g\right)\dfrac{m_r^*}{m_h^*} \end{cases}$$

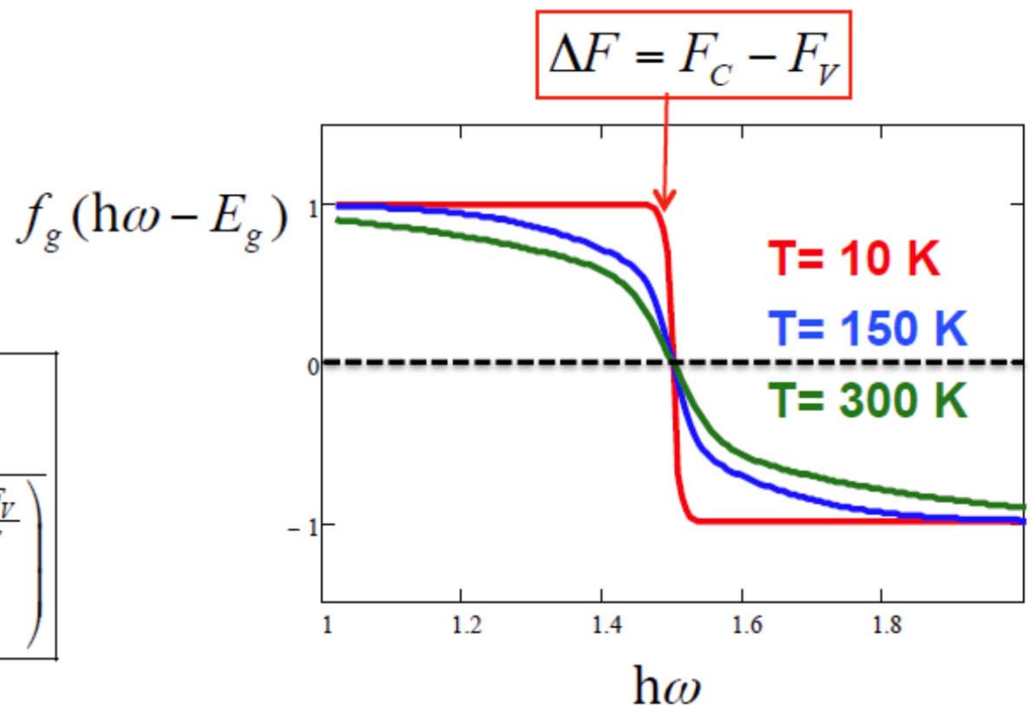
And $\quad k_1 = k_2$

$$f_C\left(E_2\right) = \frac{1}{1 + e^{\frac{E_2 - F_C}{k_B T}}}$$

$$f_V\left(E_1\right) = \frac{1}{1 + e^{\frac{E_1 - F_V}{k_B T}}}$$

$$\boxed{f_g(\hbar\omega - E_g) = \frac{e^{\frac{E_1 - F_V}{k_B T}} - e^{\frac{E_2 - F_C}{k_B T}}}{\left(1 + e^{\frac{E_2 - F_C}{k_B T}}\right)\left(1 + e^{\frac{E_1 - F_V}{k_B T}}\right)}}$$





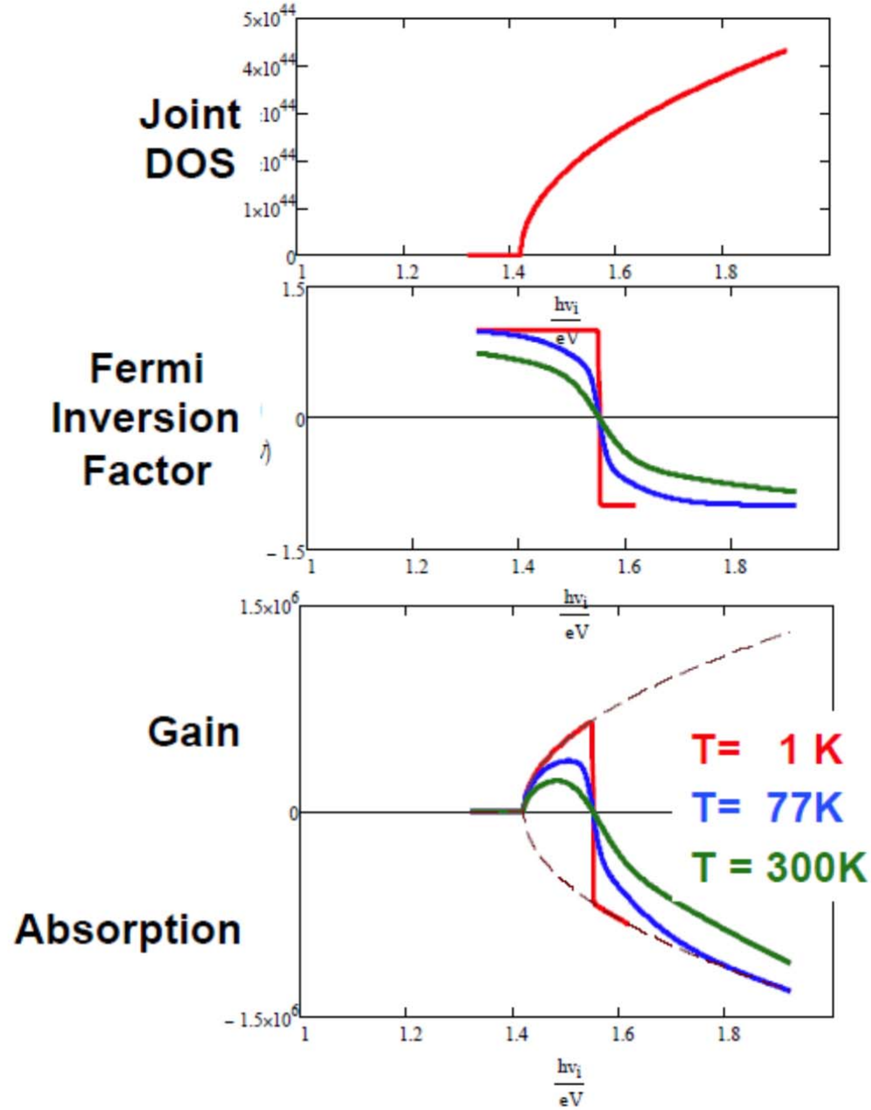
Optical Gain Coefficient

$$\alpha_0(\hbar\omega) = C_0 \left| \hat{e} \cdot \overrightarrow{P}_{cv} \right|^2 \rho_r(\hbar\omega - E_g)$$

$$\boxed{\alpha(\hbar\omega) = \alpha_0(\hbar\omega)\left[-f_g(\hbar\omega - E_g)\right]}$$

$$g(\hbar\omega) = -\alpha(\hbar\omega)$$

$$\boxed{g(\hbar\omega) = \alpha_0(\hbar\omega) f_g(\hbar\omega - E_g)}$$



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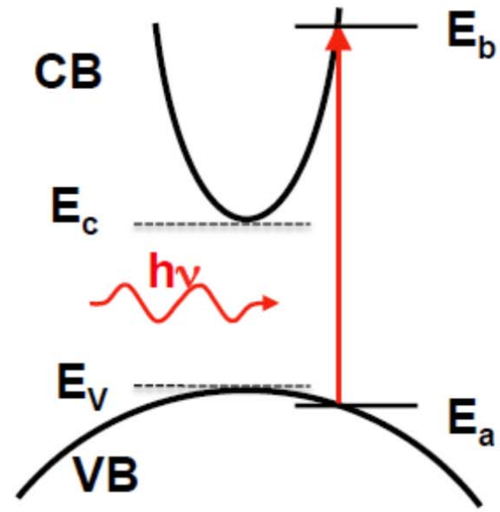

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$$E_b - E_a = (E_C - E_V) + \frac{\hbar^2 k^2}{2}\left(\frac{1}{m_e^*} + \frac{1}{m_h^*}\right)$$

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Joint density of states for the pair of electron states:

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Optical Gain Coefficient versus Bias

$$g(\mathrm{h}\omega) = \alpha_0(\mathrm{h}\omega) f_g(\mathrm{h}\omega - E_g)$$

Bernard-Duraffourg

Inversion Condition

$$\boxed{\Delta F = F_C - F_V > E_g}$$

Spectral Range of Gain

$$\boxed{E_g < \mathrm{h}\omega < \Delta F = F_C - F_V}$$