Ultrafast Optics

Review:

*. Anology between diffraction and dispersion.

Dispersion: $\begin{cases} H(\omega) = e^{-j\pi \Lambda u^2 z} \\ h(t) = \sqrt{\dfrac{1}{j\Lambda z}}\, e^{j\frac{\pi}{\Lambda z}\cdot t^2} \end{cases}$ $\qquad \Lambda = -2\pi k''$

Diffraction: $\begin{cases} H(\omega) = e^{-j\pi \lambda u^2 z} \\ h(x) = \dfrac{1}{\sqrt{j\lambda z}}\, e^{j\frac{\pi}{\lambda z} x^2} \end{cases}$

$\Lambda \sim \lambda$

$t \sim x$

$\omega \sim k_x$

$-k'' \sim \dfrac{1}{k}$

example: Time lens.

$$T = e^{j\frac{\omega}{2f_t} t^2}$$

$f_t \sim f$ (focal length)

chirp: at different temoporal position, ω is different | at differene spatial position, k is different

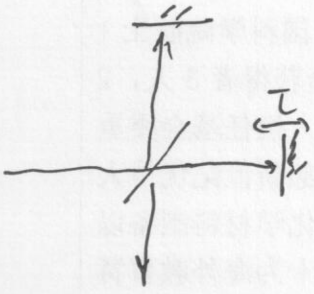
*. General frame: Temporal and spatial evolution of a pulse

$$E(x,y,0,t) \xrightarrow{F.T.} \tilde{E}(x,y,0,\omega) \xrightarrow{\otimes} h(x,y,L)$$

$$\longrightarrow \tilde{E}(x,y,L,\omega) \xrightarrow{I.F.T.} \tilde{E}(x,y,L,t)$$

*. Femtosecond optics.

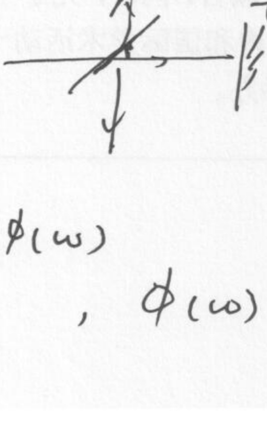
1. short pulse interferometry



$$I(\tau) \rightarrow \tilde{I}(\omega) = |E(\omega)|^2$$

Spectrum of the pulse

Today. special case



$\tilde{E}_1(\omega)$

$$\tilde{E}_2(\omega) = \tilde{E}_1(\omega)\cdot e^{j\phi(\omega)}, \quad \phi(\omega) = 2d\frac{\omega}{c}(n-1)$$

why $\xrightarrow{} \boxed{n}_{d} \to$ extra: $\frac{\omega}{c} nd - \frac{\omega}{c} d$

$$\phi(\omega) = \phi(\omega_0) + \phi'(\omega_0)\cdot\Delta\omega + \frac{1}{2}\phi''(\omega_0)\cdot\Delta\omega^2$$

$$\phi'(\omega_0) = \frac{2d}{c/n_g} - \frac{2d}{c} = \Delta\tau$$

where: n_g is group index: $n_g = n + \omega_0 \frac{dn}{d\omega}\Big|_{\omega=\omega_0}$

$c/n_g \to$ group velocity

$$\phi'' = k'' 2d.$$

8. when a material is inserted: $\tilde{E}_1^*(\omega)\tilde{E}_2(\omega)$ ①

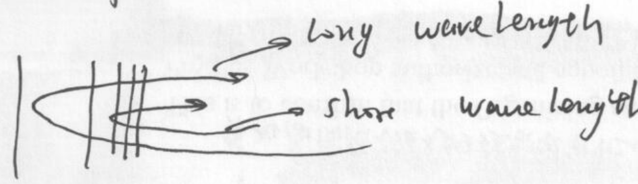
w/o : $\tilde{E}_1^*(\omega)\tilde{E}_1(\omega)$ ②

$$\frac{①}{②} = e^{i\phi(\omega_0)} \cdot e^{i\Delta\tau(\omega-\omega_0)} \cdot e^{i\frac{1}{2}k'' 2d(\omega-\omega_0)^2}$$

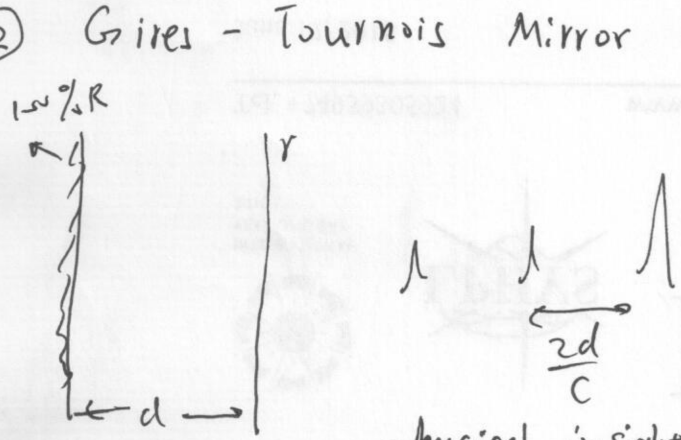
A. method to find second and high order dispersion of a bulk sample

2. Mirror: $R(\omega) e^{i\phi(\omega)}$

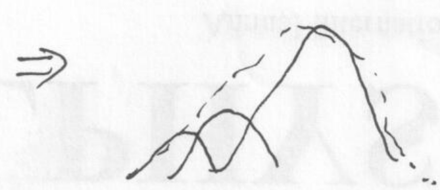
① Chirp mirror:



② Gires-Tournois Mirror (G-T Mirror)



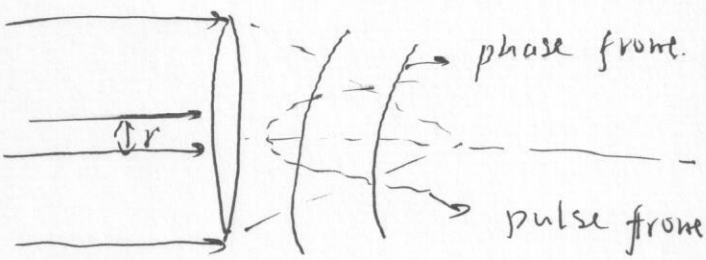
if d is small



physical insight:

Today continue to ~~discuss~~ introduce key optical elements.

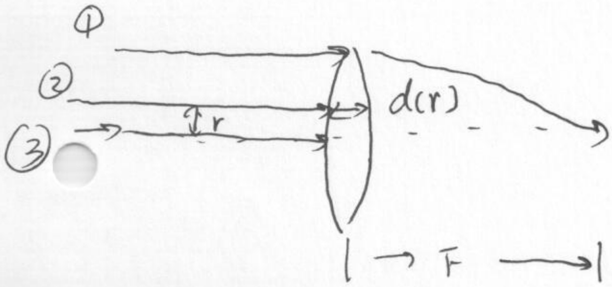
V. Lens. Basically phase plate. Fermat's principle (least time principle)



$V_g = \frac{c}{n_g}$, $V_p = \frac{c}{n}$

at the edge. $V_g = V_p$

not true in center part.



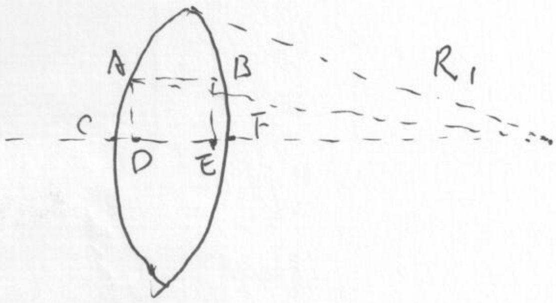
For Ray 2

$T_{pulse} - T_{phase}$

$$= \left(\frac{d(r)}{V_g} - \frac{d(r)}{V_p} \right) = \Delta t$$

$$T_{phase}\Big|_{2} = T_{phase}\Big|_{1} = T_{pulse}\Big|_{1}$$

Now find function for $d(r)$



$$d(r) = AB = DE = CF - CD - EF$$

$$CD = R_1 - \sqrt{R_1^2 - r^2}$$

$$\approx \frac{r^2}{2R_1}$$

Same: $EF \approx \frac{r^2}{2R_2}$

$$d(r) = d_0 - \frac{r^2}{2}\left(\frac{R_2 - R_1}{R_1 R_2}\right)$$

$$V_g = \frac{c}{n_g}, \quad n_g = n + \omega_0 \frac{dn}{d\omega}\Big|_{\omega=\omega_0} = n - \lambda \cdot \frac{dn}{d\lambda}.$$

$$V_p = \frac{c}{n}, \quad \frac{1}{V_g} - \frac{1}{V_p} = \frac{1}{c}(n_g - n) = \frac{-\lambda}{c} \cdot \frac{dn}{d\lambda}.$$

knowing lens aperture a.

$$d(a) = 0 \Rightarrow d_0 = \frac{a^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \Delta t = d(r)\left(\frac{1}{v_g} - \frac{1}{v_p}\right)$$

$$= \left[\frac{a^2}{2}\left(\frac{R_2 - R_1}{R_1 R_2}\right) - \frac{r^2}{2}\left(\frac{R_2 - R_1}{R_1 R_2}\right)\right] \cdot \frac{-\lambda}{c} \cdot \frac{dn}{d\lambda}$$

and we know for singlet lens:

$$f = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \Delta t = -\left(\frac{a^2}{2} - \frac{r^2}{2}\right) \cdot \frac{1}{c} \lambda \frac{d}{d\lambda}\left[(n-1)\cdot\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right]$$

$$= -\frac{a^2 - r^2}{2c} \lambda \frac{d}{d\lambda}\left(\frac{1}{f}\right)$$

physical picture: pulse is ~~broan~~ broadened by

chromatic abberation: $\frac{d}{d\lambda}\left(\frac{1}{f}\right)$

*. Comment: Have not touched GVD yet.

Example: For a beam w/ radius b

$$\Delta t' = \Delta t\big|_{r=0} - \Delta t\big|_{r=b} = -\frac{b^2}{2c} \cdot \lambda \frac{d}{d\lambda}\left(\frac{1}{f}\right)$$

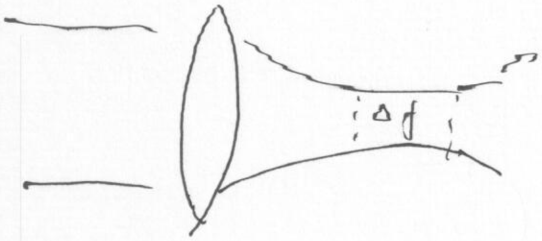
eg: 50 fs, $\lambda = 248$ nm, $\lambda \frac{dn}{d\lambda} = -0.17$.

$f = 30$ mm, $b = 4$ mm.

$\Rightarrow \Delta t' = 300$ fs

other than pulse front ~~titting~~ tilting

Spatial effect of a lens.



Gaussian beam.

$$\Delta f = -f^2 \frac{d}{d\lambda}\left(\frac{1}{f}\right)\cdot \Delta\lambda$$

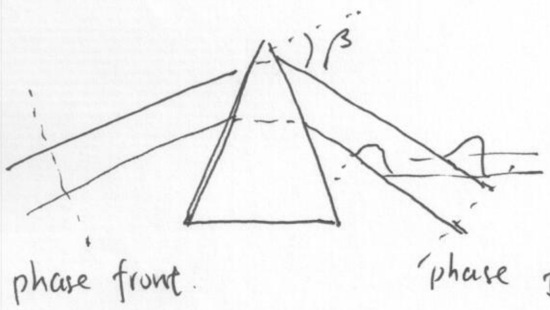
$$W = W_0\sqrt{1+\left(\frac{z}{z_0}\right)^2} \approx W_0 \frac{z}{z_0} = W_0 \cdot \frac{\frac{\Delta f}{2}}{z_0}$$

$\Downarrow$

spatial broadening of the beam due to chromatic abberation.

To really solve focusing effect of pulses by a lens. have to use the full spatial and temporal method.

*. prism.

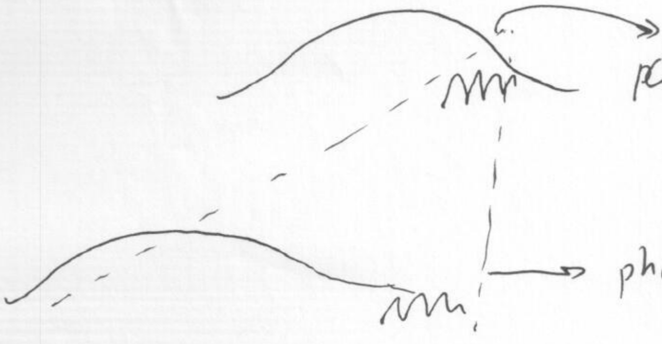


pulse front $\beta = \beta(\omega) \rightarrow$ angular dispersion.

$\rightarrow$ tilt of pulse front.

condition: consider a large enough beam size

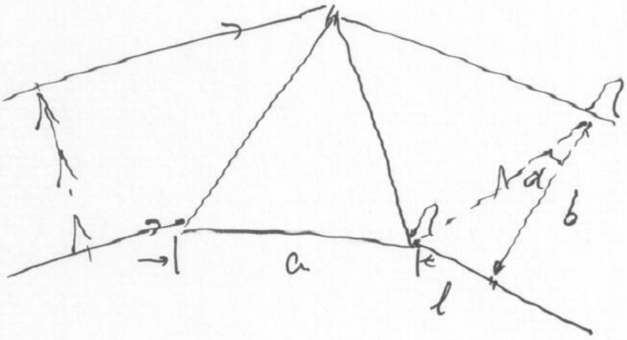
$$\gg L \cdot \frac{d\beta}{d\omega}$$



$\rightarrow$ pulse front : envelope front

$\rightarrow$ phase front : carrier's phase front

Let's do it quantitatively.



$$\tan\alpha = \frac{l}{b}$$

$$T_{phase} = \frac{a}{c/n}$$

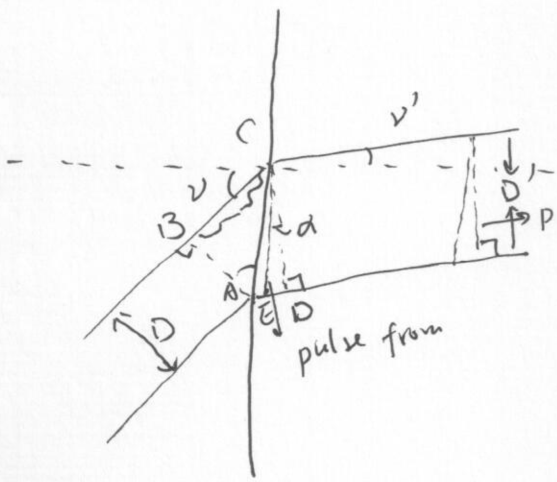
$$T_{pulse} = \frac{a}{c/n_g}$$

$$\Delta t = a\left(\frac{n}{c} - \frac{n_g}{c}\right)$$

$$\Rightarrow l = \Delta t \cdot c = a(n - n_g)$$

$$\Rightarrow \tan\alpha = \frac{a(n-n_g)}{b} = \frac{a}{b}\cdot\lambda\cdot\frac{dn}{d\lambda}$$

Similar example: refraction



$\cancel{\tan\alpha}$

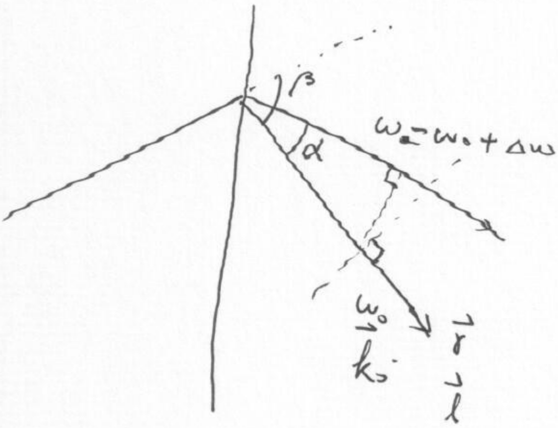
$$\tan\alpha = \frac{DG}{D'}$$

$$\tau = \frac{\tan\nu \cdot D}{c}$$

$$DG = \frac{c}{n}\cdot\tau - V_g\cdot\tau$$

$$\Rightarrow \tan\alpha = \frac{\left(\frac{c}{n}\cdot\tau - V_g\tau\right)}{\frac{D}{\cos\nu}\cdot\cos\nu'}$$

Application of prism: angular dispersion



$\beta(\omega) \sim \frac{d\beta}{d\omega}$

phase:

$$\phi(\omega) = \vec{k}\cdot\vec{r} = k\cdot l\cdot\cos\alpha = \frac{\omega}{c}\cdot l\cdot\cos\alpha$$

$$\phi'(\omega) = \frac{l}{c}\cdot\cos\alpha - \frac{\omega}{c}\cdot l\cdot\sin\alpha\cdot\alpha'$$

$$\phi''(\omega) = \frac{l}{c}\cdot(-\sin\alpha)\alpha' - \left[\frac{1}{c}\cdot\sin\alpha\cdot\alpha' + \frac{\omega}{c}\cdot l\cos\alpha(\alpha')^2 + \frac{\omega}{c}l\cdot\sin\alpha\cdot\alpha''\right]$$

$$= -\frac{2l}{c}\sin\alpha\cdot\alpha' - \frac{\omega}{c}l\cos\alpha(\alpha')^2 - \frac{\omega}{c}l\cdot\sin\alpha\cdot\alpha''$$

α is small $\Rightarrow \simeq -\frac{\omega}{c}\cdot l\cdot\cos\alpha\,(\alpha')^2$ < always 0

* Comments:

① GVD of prism is always negtive

② $\propto l$ and $\propto \left(\frac{d\alpha}{d\omega}\right)^2$, i.e. $\left|\frac{d\beta}{d\omega}\right|^2$

$$\frac{d\alpha}{d\omega} = \frac{d\alpha}{dn}\cdot\frac{dn}{d\omega}$$

$\frac{d\alpha}{dn}$ proved to be -2

in λ unit: $\frac{dn}{d\omega} = \frac{dn}{d\lambda}\cdot\frac{d\lambda}{d\omega} = \left(-\frac{\lambda^2}{2\pi c}\right)\cdot\frac{dn}{d\lambda}$

$$\Rightarrow \phi'' = -\frac{\omega}{c}\cdot l\cdot\cos\alpha\cdot\left(-2\cdot\frac{-\lambda^2}{2\pi c}\cdot\frac{dn}{d\lambda}\right)^2$$

$$= -\frac{2\pi c}{\lambda}\cdot\frac{1}{c}\cdot l\cdot\cos\alpha\left(-2\cdot\frac{-\lambda^2}{2\pi c}\cdot\frac{dn}{d\lambda}\right)^2$$

$$= \frac{-2l\cdot\lambda^3}{\pi c^2}\left(\frac{dn}{d\lambda}\right)^2$$

is this all we need to include ?

~~Dow~~ not forget material dispersion

GVD from material.

$$\phi_m = \frac{\omega}{c}\cdot nL$$

$$\cancel{\phi_m' = \frac{1}{c}\cdot n\cdot L + \frac{\omega}{c}L\cdot\frac{dn}{d\omega}}$$

$$\phi_m'' = \cancel{\frac{L}{c}\frac{dn}{d\omega} + \frac{1}{c}\cdot L\cdot\frac{dn}{d\omega}} + \frac{\omega}{c}\cdot L\cdot\frac{d^2n}{d\omega^2}$$

know: $\frac{d}{d\omega} = \left(-\frac{\lambda^2}{2\pi c}\right)\cdot\frac{d}{d\lambda}$

$$\phi_m'' = \frac{2\pi c}{\lambda}\cdot\frac{1}{c}\cdot L\cdot\frac{\lambda^{4}}{4\pi^2 c^2}\left(\frac{d^2n}{d\lambda^2}\right)$$

$$= \frac{\lambda^3}{2\pi c^2}\cdot L\cdot\frac{d^2n}{d\lambda^2}$$

$$\phi''_{total} = \phi''_{angular} + \phi''_{Material}$$

$$= -\frac{2l\lambda^3}{\pi c^2}\left(\frac{dn}{d\lambda}\right)^2 + \frac{\lambda^3}{2\pi c^2}L\cdot\frac{d^2n}{d\lambda^2}$$