**Microeconomics** Group45 吴熙楠

# Homework04

## Contents

# 1 Problem 1

**Answer:**

The total revenue that the monopolist can get is that $u = x_1p_1 + x_2p_2$

Because the monopolist has zero marginal cost, We have no limit on $x_1$ and $x_2$.

$\frac{\partial u}{\partial p_1} = \frac{\partial u}{\partial p_2} = 0 \Rightarrow p_1 = \frac{a_1}{2b_1}, p_2 = \frac{a_2}{2b_2}$

The monopolist will not choose to price discriminate, so $p_1 = p_2$.

Then we can get that $a_1b_2 = a_2b_1$ or $\frac{a_1}{b_1} = \frac{a_2}{b_2}$.

# 2 Problem 2

**Answer:**

(a)We know that $b = 1000 - a$, so $U(a) = \sqrt{a} + \sqrt{1000 - a}$.

In order to maximize $U$, $\frac{dU}{da} = \frac{1}{2\sqrt{a}} - \frac{1}{2\sqrt{1000 - a}} = 0 \Rightarrow a = 500$.

Because $b = 1000 - a = 500$, $a = b = 500$, $U_{max} = 20\sqrt{5}$. It's easy to show that $U$ is the largest in this range.

(b)We know that $b = 1000 - a$, so $U(a) = -(\frac{1}{a} + \frac{1}{1000 - a})$.

In order to maximize $U$, $\frac{dU}{da} = \frac{1}{a^2} - \frac{1}{(1000 - a)^2} = 0 \Rightarrow a = 500$.

Because $b = 1000 - a = 500$, $a = b = 500$, $U_{max} = -\frac{1}{250}$. It's easy to show that $U$ is the largest in this range.

(c)In this situation, if $a = b = 500$, then $U = min(a, b) = 500$.

If $a \neq b$, we assume that $a < b$. Then we can get that $a < 500$ and $b > 500$, so $U = min(a, b) = a < 500$.

So $U_{max} = 500$, and $a = b = 500$.

(d)In this situation, if $a = b = 500$, then $U = max(a, b) = 500$.

If $a \neq b$, we assume that $a < b$. Then we can get that $a < 500$ and $b > 500$, so $U = max(a, b) = b > 500.(0 \leq a, b \leq 1000)$

So $b = 1000$ and $a = 0$, or $a = 1000$ and $b = 0$, $U_{max} = 1000$. So she will give all of their money to A or all of their money to B.

(e)We know that $b = 1000 - a$, so $U(a) = a^2 + (1000 - a)^2 = 2a^2 - 2000a + 1000^2$.

In order to maximize $U$, we know that this is a parabolic function.

So $b = 1000$ and $a = 0$, or $a = 1000$ and $b = 0$, $U_{max} = 1000000$. It's easy to show that $U$ is the largest in this range.

# 3 Problem 3

**Answer:**

(a)We know that $a = 1000 - 2b$, so $U(b) = 1000 - b.(0 \le a \le 1000, 0 \le b \le 500)$

We find that the bigger $b$ is, the smaller the utility function $U$ is. So in order to maximize $U$, $b = 0$.

Because $a = 1000 - 2b = 1000$, $a = 1000, b = 0$, $U_{max} = 1000$. It's easy to show that $U$ is the largest in this range.

(b)We know that $a = 1000 - 2b$, so $U(b) = b - 1000.(0 \le a \le 1000, 0 \le b \le 500)$

We find that the bigger $b$ is, the bigger the utility function $U$ is. So in order to maximize $U$, $b = 500$.

Because $a = 1000 - 2b = 0$, $a = 0, b = 500$, $U_{max} = -500$. It's easy to show that $U$ is the largest in this range.

(c)We know that $a = 1000 - 2b$, so $U(b) = 1000b - 2b^2.(0 \le a \le 1000, 0 \le b \le 500)$

In order to maximize $U$, $\frac{dU}{db} = 1000 - 4b = 0 \Rightarrow b = 250$.

Because $a = 1000 - 2b = 500$, $a = 500, b = 250$, $U_{max} = 125000$. It's easy to show that $U$ is the largest in this range.

# 4 Problem 4

**Answer:**

(a)$\mathcal{L} = u^A + \lambda(u^B - \bar{u}) + \mu_1(x_1^A + x_1^B - w_1) + \mu_2(x_2^A + x_2^B - w_2)$

$\frac{\partial \mathcal{L}}{\partial x_1^A} = \frac{\partial u^A}{\partial x_1^A} + \mu_1 = 0 \qquad \frac{\partial \mathcal{L}}{\partial x_2^A} = \frac{\partial u^A}{\partial x_2^A} + \mu_2 = 0 \Rightarrow MRS_A = \frac{\mu_1}{\mu_2}$

Also we can get that $MRS_B = \frac{\mu_1}{\mu_2} = MRS_A$

$MRS_A = \frac{\partial u^A / \partial x_1^A}{\partial u^A / \partial x_2^A}|_{x_1^{A\star}, x_2^{A\star}} = \frac{x_2^{A\star}}{x_1^{A\star}} \qquad MRS_B = \frac{\partial u^B / \partial x_1^B}{\partial u^B / \partial x_2^B}|_{x_1^{B\star}, x_2^{B\star}} = \frac{x_2^{B\star}}{2x_1^{B\star}}$

From the Lagrangian procedure, we know that $MRS_A = MRS_B \Rightarrow \frac{x_2^{A\star}}{x_1^{A\star}} = \frac{x_2^{B\star}}{2x_1^{B\star}}$

$\Rightarrow \frac{x_2^{A\star}}{x_1^{A\star}} = \frac{10 - x_2^{A\star}}{2(21 - x_1^{A\star})}$ or $\frac{x_2^{B\star}}{2x_1^{B\star}} = \frac{10 - x_2^{B\star}}{21 - x_1^{B\star}}$

(b)We know that in order to achieve the Walrath equilibrium,

we have $\frac{\partial u^A / \partial x_1^A}{\partial u^A / \partial x_2^A}|_{x_1^{A\star}, x_2^{A\star}} = \frac{\partial u^B / \partial x_1^B}{\partial u^B / \partial x_2^B}|_{x_1^{B\star}, x_2^{B\star}} = \frac{p_1}{p_2} \Rightarrow \frac{x_2^{A\star}}{x_1^{A\star}} = \frac{x_2^{B\star}}{2x_1^{B\star}} = \frac{p_1}{p_2}$

Also we have $x_1^{A\star} + x_1^{B\star} = 21, x_2^{A\star} + x_2^{B\star} = 10$.

$p_1 x_1^{A\star} + p_2 x_2^{A\star} = p_1 w_1^A + p_2 w_2^A, p_1 x_1^{B\star} + p_2 x_2^{B\star} = p_1 w_1^B + p_2 w_2^B$

We assume that $p_2 = 1$.

Then we can solve the equations, and we can get that:

$x_1^{A\star} = \frac{29}{2}, x_2^{A\star} = \frac{58}{11}, x_1^{B\star} = \frac{13}{2}, x_2^{B\star} = \frac{52}{11}, p_1 = \frac{4}{11}, p_2 = 1$