Ultrafast Optics   L#5 ①

Review.

1. Chirping effect

* Linear chirping: GVD

$$E(t,0) \rightarrow e^{-(1+aj)\frac{t^2}{t_0^2}}$$

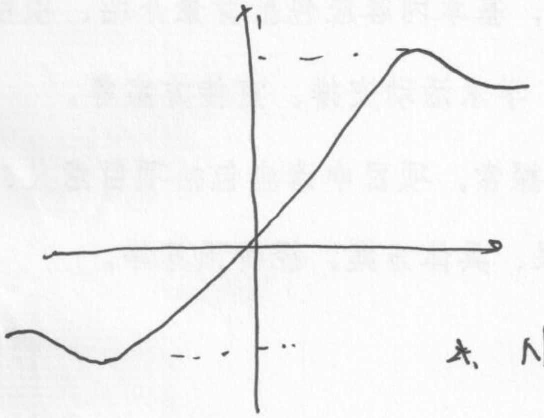
after dispersive propagation z

new chirping: $b = a + \dfrac{2k''z}{t_0^2(1+a^2)}$

* Nonlinear chirping:

SPM ~ Kerr effect: $\Delta n = n_2 I$

by using Gaussian pulse:



$$\rightarrow \frac{\Delta\omega}{\omega_0} \sim \frac{n_2 I_0 z}{c\, t_0}$$

* New frequency
* Complex spectral structure

Instant phase change $\phi(t)$ ~~as~~ is dependence on pulse shape.

Comments:

① NL $\phi_{NL}(t)$ does not change pulse profile
only changes spectrum

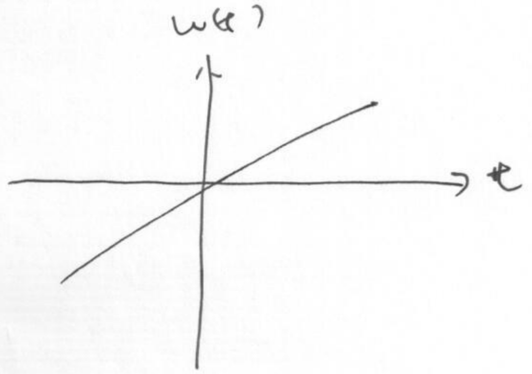
② Combined with GVD or higher order dispersion

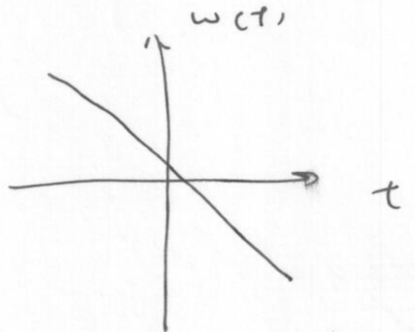
*. Nonlinear + Dispersion

Entangled together.

useful example: soliton.

SPM $\qquad\qquad\qquad\qquad$ $k'' < 0$





Make it possible to achive balance between
SPM and Dispersion

2. Spatial-temporal problem.

Dispersion: $\dfrac{\partial A}{\partial z} + \dfrac{1}{2} j k'' \dfrac{\partial^2 A}{\partial t^2} = 0$

$$H(u) = e^{-j\pi \Lambda u^2 z} \qquad \Lambda = -2\pi k''$$

$$h(t) = \sqrt{\frac{1}{j\Lambda z}}\, e^{j\frac{\pi}{\Lambda z} t^2}$$

Diffraction: $\dfrac{\partial A}{\partial z} - j\dfrac{1}{2k}\dfrac{\partial^2 A}{\partial x^2} = 0$

$\Lambda \leftrightarrow \lambda$

$t \leftrightarrow x$

$\Omega \leftrightarrow k_x$

$-k'' \leftrightarrow \frac{1}{k}$

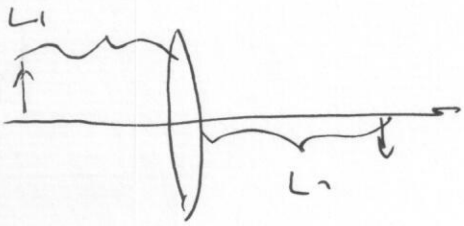
$$H(u) = e^{-j\pi\lambda u^2 z}$$

$$h(x) = \frac{1}{\sqrt{j\lambda z}}\, e^{j\frac{\pi}{\lambda z} x^2}$$

This Lecture

* finishing up phase modulation of linear system
* Femtosecond optics

Solving dispersion problem by performing diffraction experiments: (time lens)



give lens. phase function

$$g_1 \quad g_2 \quad e^{-j\frac{k}{2f}(x^2+y^2)} = t(x,y)$$

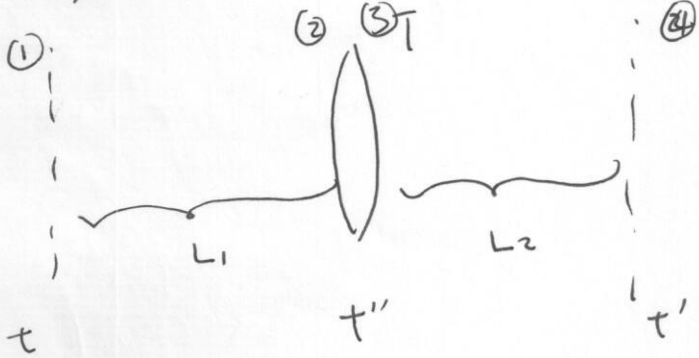
$$g_2 = g_1 \cdot t(x,y)$$

$$\frac{1}{L_1} + \frac{1}{L_2} = \frac{1}{f}$$

Assume. we have certain system (Dispersion)

$$T = e^{-j\frac{\omega_0}{2f_t}\cdot t^2}$$

give initial pulse $f(t)$



① $f(t)$

② $f(t) \otimes \sqrt{\frac{1}{j\lambda_1 L_1}}\, e^{j\frac{\pi}{\lambda_1 L_1}t^2}$

③ $[\quad ② \quad] \cdot e^{-j\frac{\omega_0}{2f_t}t^2}$

④ $[\qquad] \otimes \sqrt{\frac{1}{j\lambda_2 L_2}}\, e^{j\frac{\pi}{\lambda_2 L_2}t^2}$

① $f(t)$

$$\Rightarrow ② \quad \sqrt{\frac{1}{j\lambda_1 L_1}} \cdot \int dt \cdot f(t) \cdot e^{j\frac{\pi}{\lambda_1 L_1}(t-t'')^2}$$

$$= \frac{1}{\sqrt{j\lambda_1 L_1}} \cdot \int dt\, f(t) \cdot e^{j\frac{\pi}{\lambda_1 L_1}t^2} \cdot e^{j\frac{\pi}{\lambda_1 L_1}t''^2} \cdot e^{-j\frac{\pi}{\lambda_1 L_1}\cdot 2tt''}$$

③
$$\frac{1}{\sqrt{j\lambda_1 L_1}}\int dt\, f(t)\cdot e^{j\frac{\pi}{\lambda_1 L_1}t^2}\cdot e^{j\frac{\pi}{\lambda_1 L_1}t''^2}\cdot e^{-j\frac{\pi}{\lambda_1 L_1}2tt''}\cdot e^{-j\frac{\omega_0}{2f_t}\cdot t''^2}$$

$$=\frac{1}{\sqrt{j\lambda_1 L_1}}\int dt\, f(t)\, e^{j\frac{\pi}{\lambda_1 L_1}t^2}\cdot e^{-j\frac{\pi}{\lambda_1 L_1}2tt''}\cdot e^{j\left(\frac{\pi}{\lambda_1 L_1}-\phi\frac{\omega_0}{2f_t}\right)t''^2}$$

④
$$\frac{1}{\sqrt{j\lambda_1 L_1}}\cdot\frac{1}{\sqrt{j\lambda_2 L_2}}\cdot\int dt\cdot f(t)\, e^{j\frac{\pi}{\lambda_1 L_1}t^2}\int dt''\cdot e^{-j\frac{\pi}{\lambda_1 L_1}2tt''}\cdot e^{j\left(\frac{\pi}{\lambda_1 L_1}-\frac{\omega_0}{2f_t}\right)t''^2}\cdot e^{j\frac{\pi}{\lambda_2 L_2}(t'-t'')^2}$$

$$=\frac{1}{\sqrt{-\lambda_1 L_1\lambda_2 L_2}}\cdot\int dt\, f(t)\, e^{j\frac{\pi}{\lambda_1 L_1}t^2}\int dt''\, e^{-j\frac{\pi}{\lambda_1 L_1}2tt''}\cdot e^{j\left(\frac{\pi}{\lambda_1 L_1}-\frac{\omega_0}{2f_t}\right)t''^2}\cdot e^{j\frac{\pi}{\lambda_2 L_2}t'^2}\cdot e^{j\frac{\pi}{\lambda_2 L_2}t''^2}\cdot e^{-j\frac{2\pi}{\lambda_2 L_2}t't''}$$

$$=\frac{1}{\sqrt{C}}\cdot\int dt\cdot f(t)\cdot e^{j\frac{\pi}{\lambda_1 L_1}t^2}\cdot e^{j\frac{\pi}{\lambda_2 L_2}t'^2}\int dt''\cdot\left[e^{j\left(\frac{\pi}{\lambda_1 L_1}+\frac{\pi}{\lambda_2 L_2}-\frac{\omega_0}{2f_t}\right)t''^2}\cdot e^{-j2\pi\left(\frac{t}{\lambda_1 L_1}+\frac{t'}{\lambda_2 L_2}\right)\cdot t''}\right]$$

if we can find a time lens.

$$\frac{\pi}{\lambda_1 L_1}+\frac{\pi}{\lambda_2 L_2}=\frac{\omega_0}{2f_t}$$

$$④\Rightarrow\frac{1}{\sqrt{-A}}\cdot\int dt\cdot f(t)\cdot e^{j\frac{\pi}{\lambda_1 L_1}t^2}\, e^{j\frac{\pi}{\lambda_2 L_2}t'^2}\cdot\delta\left(\frac{t}{\lambda_1 L_2}+\frac{t'}{\lambda_2 L_2}\right)$$

$$\propto f\left(-\frac{t'}{M}\right)\quad \text{where } M=\frac{\lambda_2 L_2}{\lambda_1 L_1}$$

Comments on $e^{-j\frac{\omega_0}{2f_t}\cdot t^2}$

Dispersion — Diffraction

$\omega_0 \sim$ frequency — $k \sim$ spatial frequency (angular)

$f_t \iff f$ (focal lens)

chirp: at Different time position. ω is differenc

→ at Different spatial position. k is different.

How to implementation of time lens

$\Delta n \sim n_2 I$

$$e^{j\frac{\omega}{c} n_2 I_0 \frac{t^2}{t_0^2} z} \sim e^{j\frac{\omega}{c} n_2 I_0 \frac{t^2}{t_0^2} z}$$

$\downarrow$

f_t

Temporal and Spatial evolution of a pulse

$E(x,y,0,t) \xrightarrow{F.T} \tilde{E}(x,y,0,\omega) \to \otimes \frac{e^{j\lambda L}}{j\lambda L} \cdot e^{j\frac{\pi}{\lambda L}(x^2+y^2)}$

$\Rightarrow \tilde{E}(x,y,L,\omega)$

I.F.T

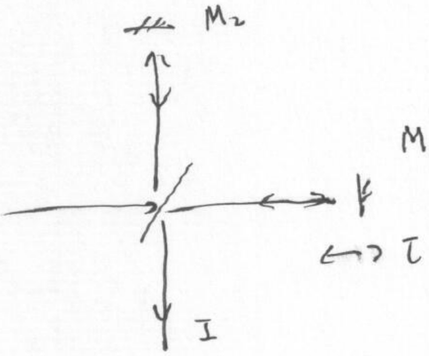
$\to E(x,y,L,t)$

$z=0$ — $z=L$

Notice phase term. here.

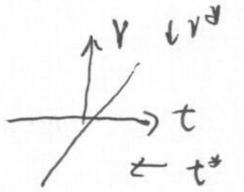
Femtosecond Optics

1. Short Pulse Interferometry



$$I(\tau) = \left| E_1(t-\tau) + E_2(t) \right|^2$$

about B.S. (Beamsplitter)



$r^*, t^* \Rightarrow$ time reversal

$$r^* t + t^* r = 0$$

base on causality

down port should be zero

$$\Rightarrow r^* t + (t \cdot r^*)^* = 0$$

$$\Rightarrow r^* t = i(\quad) \quad \text{pure imaginary}$$

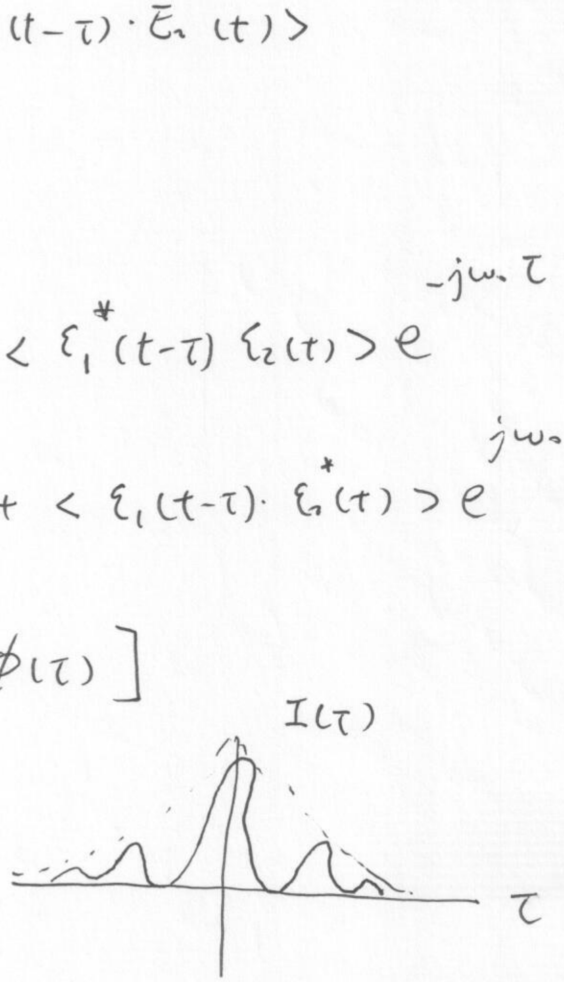
$$I(\tau) = \langle |E_1|^2 \rangle + \langle |E_2|^2 \rangle + \langle E_1^*(t-\tau) \cdot E_2(t) \rangle$$
$$+ \langle E_1(t-\tau) \cdot E_2^*(t) \rangle$$

Let
$$\begin{cases} E_1(t) = \varepsilon_1(t) \cdot e^{-j\omega_0 t} \\ E_2(t) = \varepsilon_2(t)\, e^{-j\omega_0 t} \end{cases}$$

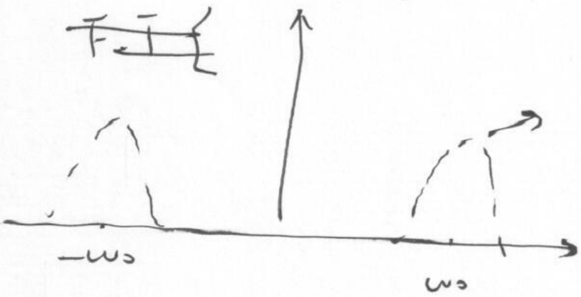
$$I(\tau) = \langle |\varepsilon_1|^2 \rangle + \langle |\varepsilon_2| \rangle^2 + \langle \varepsilon_1^*(t-\tau)\, \varepsilon_2(t) \rangle e^{-j\omega_0 \tau}$$
$$+ \langle \varepsilon_1(t-\tau) \cdot \varepsilon_2^*(t) \rangle e^{j\omega_0 \tau}$$

$$\propto 2|A(\tau)| \cos[\omega_0 \tau - \phi(\tau)]$$

$$A(\tau) = |A| \cdot e^{j\phi}$$



perform F.T. to $I(\tau) \rightsquigarrow \tilde{I}(\omega)$



$$\text{F.T}\left[\langle \varepsilon_1^*(t-\tau)\cdot \varepsilon_2(t)\rangle \cdot e^{-j\omega_0\tau}\right]$$

$$= \cancel{\text{F.T.}\left[\int_{-\frac{T}{2}}^{\frac{T}{2}} \varepsilon_1^*(t-\tau)\cdot \varepsilon_2(t)\cdot dt\right]}$$

$$= \cancel{\text{F.T}\left[\int_{-\infty}^{\infty} \varepsilon_1^*(t-\tau)\cdot \varepsilon_2\right.}$$

$$= \int_{-\infty}^{\infty}\left(\int_{-\frac{T}{2}}^{\frac{T}{2}} \varepsilon_1^*(t-\tau)\cdot \varepsilon_2(t)\cdot dt\right) e^{-j\omega_0\tau}\cdot e^{+j\omega\tau}\cdot d\tau$$

$$= \int_{-\infty}^{\infty}\left(\int_{-\frac{T}{2}}^{\frac{T}{2}} \varepsilon_1^*(t-\tau)\cdot \varepsilon_2(t)\cdot dt\right)\cdot e^{+j(\omega-\omega_0)\tau}\cdot d\tau$$

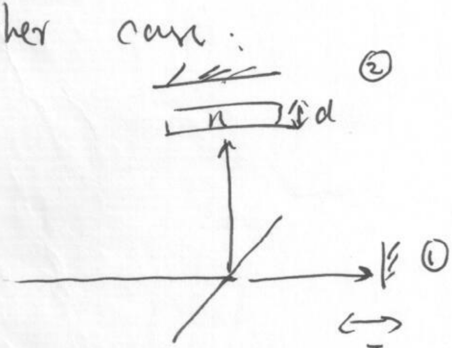
$$= \tilde{\varepsilon}_1^*(\Omega)\cdot \tilde{\varepsilon}_2(\Omega)$$ ~~given:~~

$$\Rightarrow \tilde{I}(\omega_0+\Omega) = \tilde{\varepsilon}_1^*(\Omega)\cdot \tilde{E}_2(\Omega)$$

if two pulses are identical. $\Rightarrow \tilde{I}(\omega_0+\Omega) = |\varepsilon(\Omega)|^2$

Now we got the spectrum of the pulse

Another case:

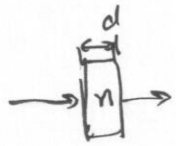


Now: $\tilde{E}_2(\omega) = \tilde{E}_1(\omega)\cdot e^{j\phi(\omega)}$,

$\tilde{E}_1(\omega) = \tilde{E}_1(\omega)$

$$\phi(\omega) = \frac{\omega}{c}(n-1)\cdot d \times 2$$

why:



$$\frac{\omega}{c}\cdot nd - \frac{\omega}{c}\cdot d.$$

$$\phi(\omega) = \phi(\omega_0) + \phi'(\omega_0)\cdot\Delta\omega + \frac{1}{2}\phi''(\omega_0)\cdot\Delta\omega^2 + \cdots$$

$$\phi(\omega_0) = \frac{\omega_0}{c}\cdot(n-1)\cdot d$$

$$\phi'(\omega_0) = \frac{d}{d\omega}\left[\frac{\omega}{c}\cdot n\cdot d - \frac{\omega}{c}\cdot 2d\right]\Bigg|_{\omega=\omega_0}$$

$$= \frac{2d}{c/n_g} - \frac{2d}{c} = \Delta\tau$$

→ Stop here on 2016.3.22

$$n_g = n + \omega_0\cdot\frac{dn}{d\omega}\Bigg|_{\omega=\omega_0}$$

group index. c/n_g → group velocity

$$\phi''(\omega_0) = \frac{d^2}{d\omega^2}\left[\frac{\omega 2dn}{c} - \frac{\omega}{c}\cdot 2d\right]$$

$$= \frac{d^2}{d\omega^2}\left[\frac{\omega}{c}\cdot n\cdot 2d\right] = k''\cdot 2d \quad , k = \frac{\omega}{c}\cdot n(\omega)$$

$$\tilde{E}_2(\omega) = \tilde{E}_1(\omega)\cdot e^{j\phi(\omega_0)}\cdot e^{j\Delta\tau\cdot(\omega-\omega_0)}\cdot e^{\frac{1}{2}jk''2d(\omega-\omega_0)^2}$$

First let ignore k'' (GVD, from material)

take I.F.T

$$E_2(t) = E_1(t-\Delta\tau)\cdot e^{j\phi(\omega_0)}\cdot e^{j\Delta\tau\cdot\omega_0}$$

$\Downarrow$ envelope

define. $\Delta\phi = \phi(\omega_0) - \omega_0\cdot\Delta\tau = \frac{\omega_0}{c}(n-n_g)\cdot 2d$

$$= E_1(t-\Delta\tau)\cdot e^{j\Delta\phi}$$

~~Envo~~ Envelope is ~~de~~ delayed by $\Delta\tau = \phi'(\omega_0)$