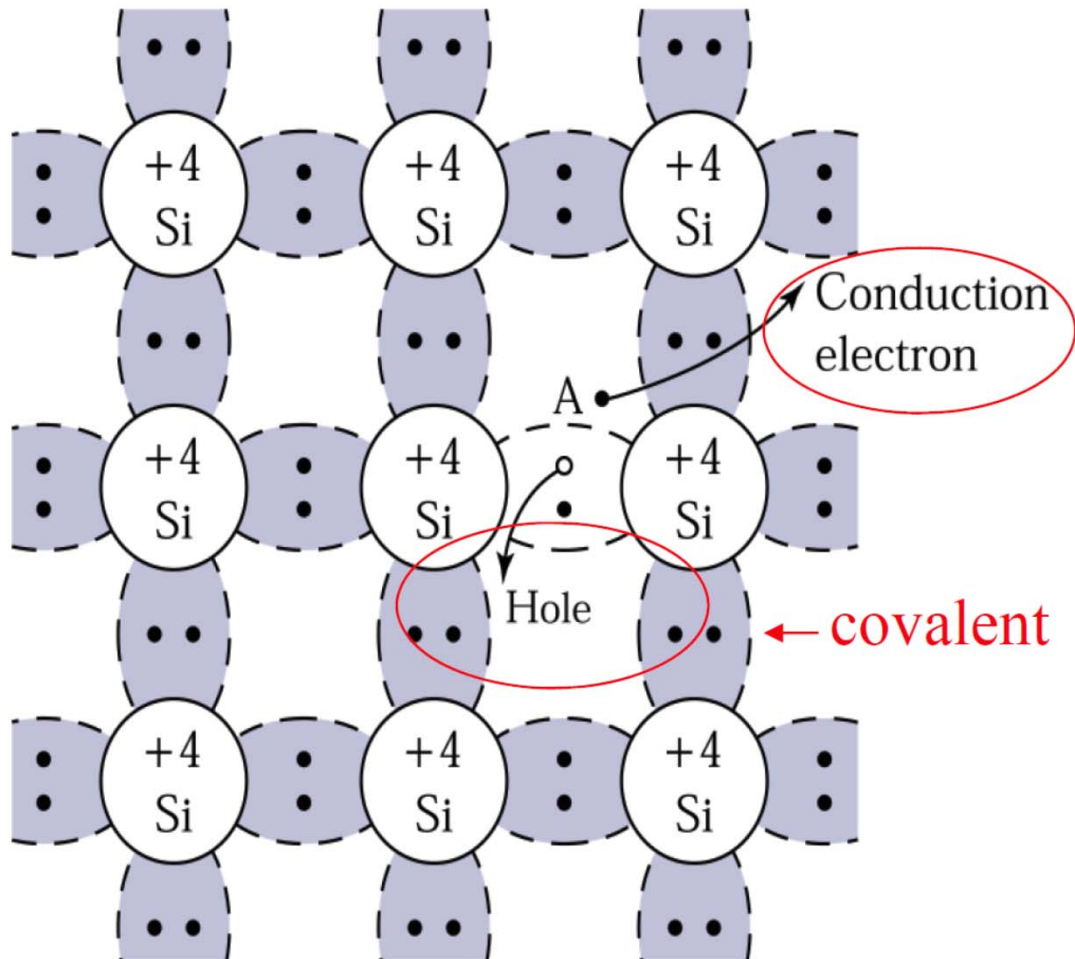


Carrier concentration

Si : 10^{22} #/cm³

$\left\{ \begin{array}{l} n \\ p \end{array} \right.$ Intrinsic $n=p=n_i \doteq 1.5 \times 10^{10}$



A broken bond at Position A, resulting in a conduction electron and a hole.

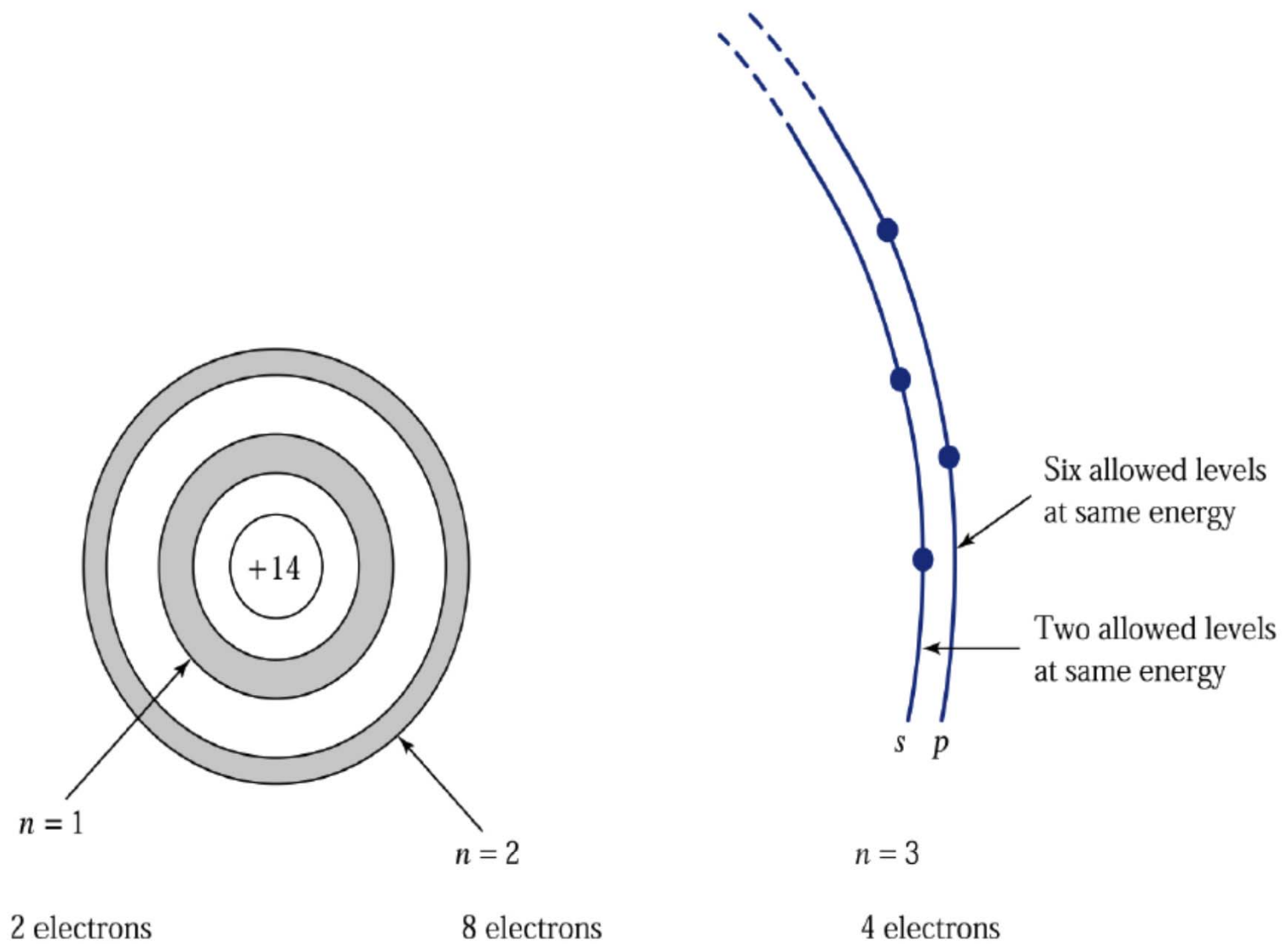
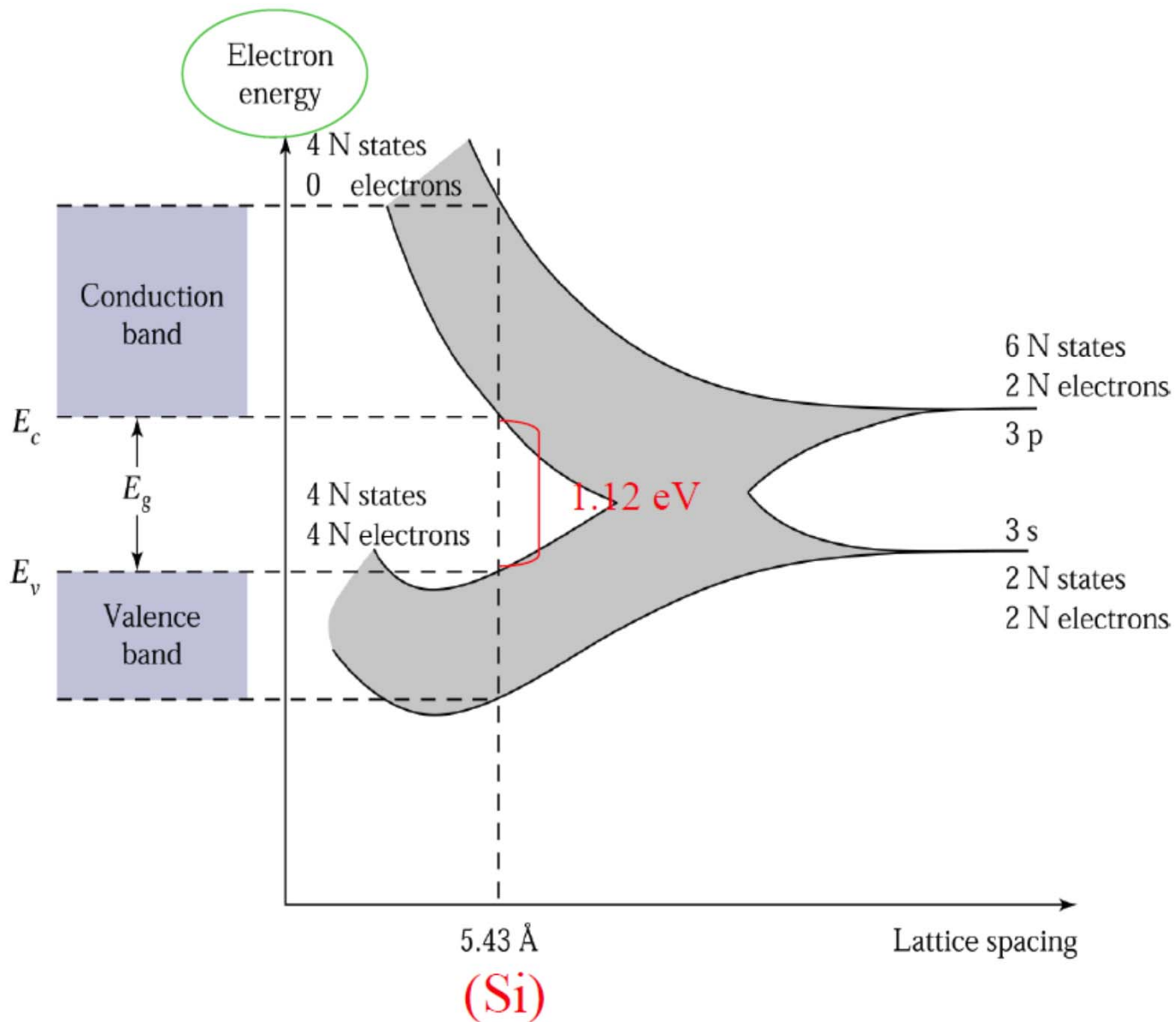
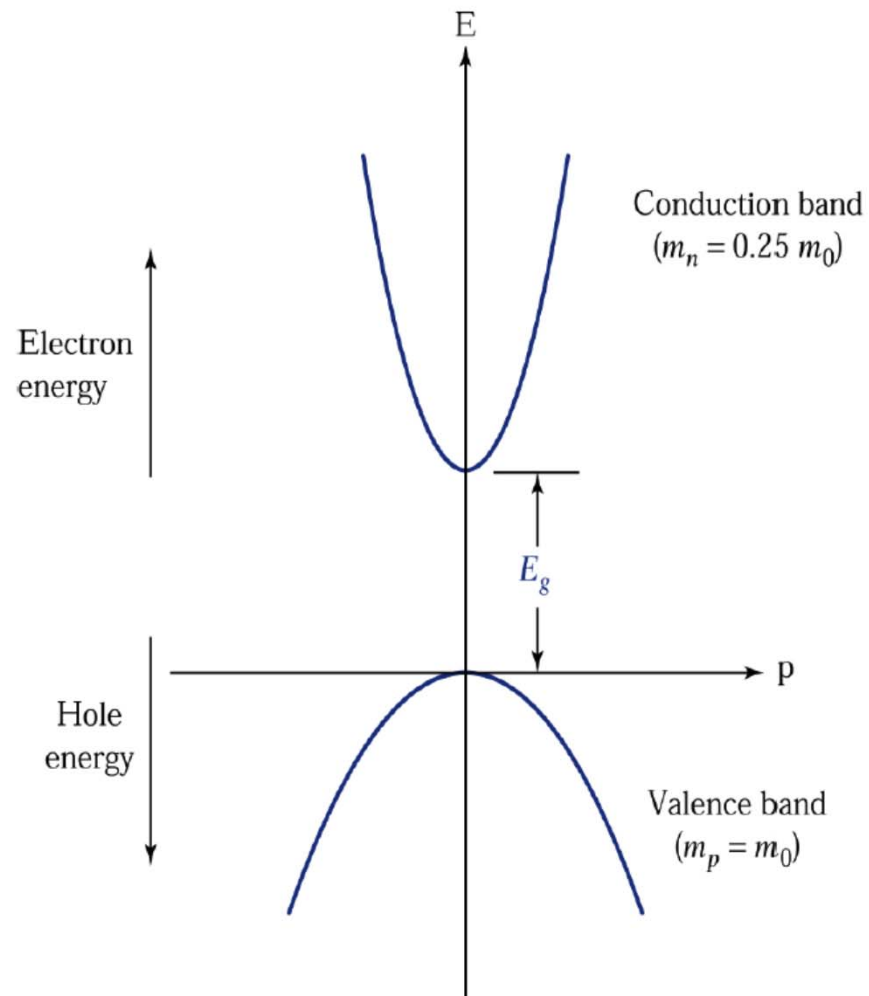


Figure 1.10 Schematic representation of an isolated silicon atom.
 $1S^2$ $2S^2$ $2P^6$ $3S^23P^2$

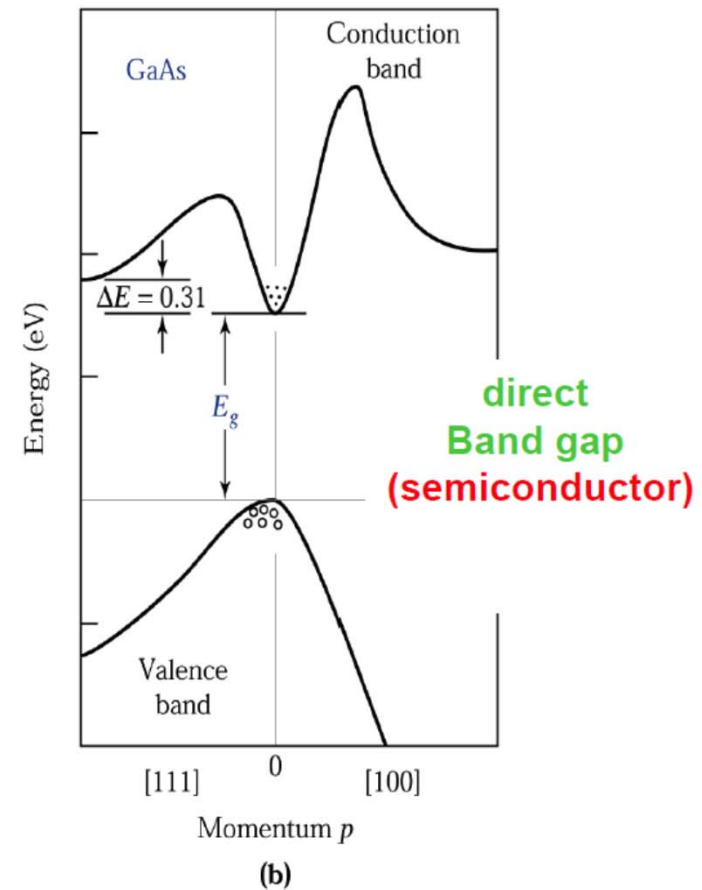
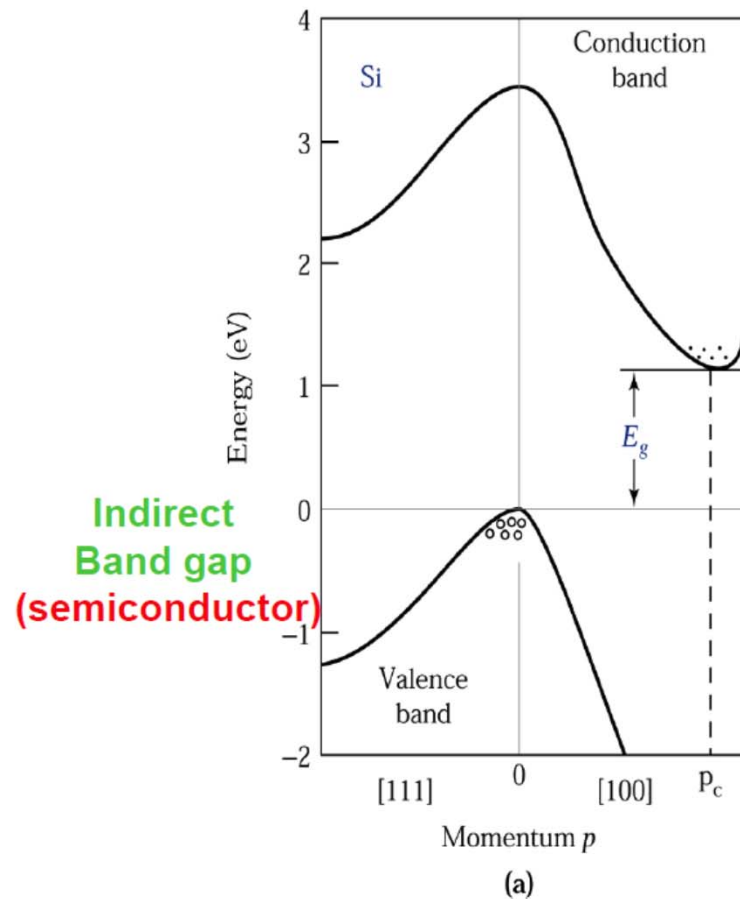


Formation of energy bands as a diamond lattice crystal is formed by bringing isolated silicon atoms together.

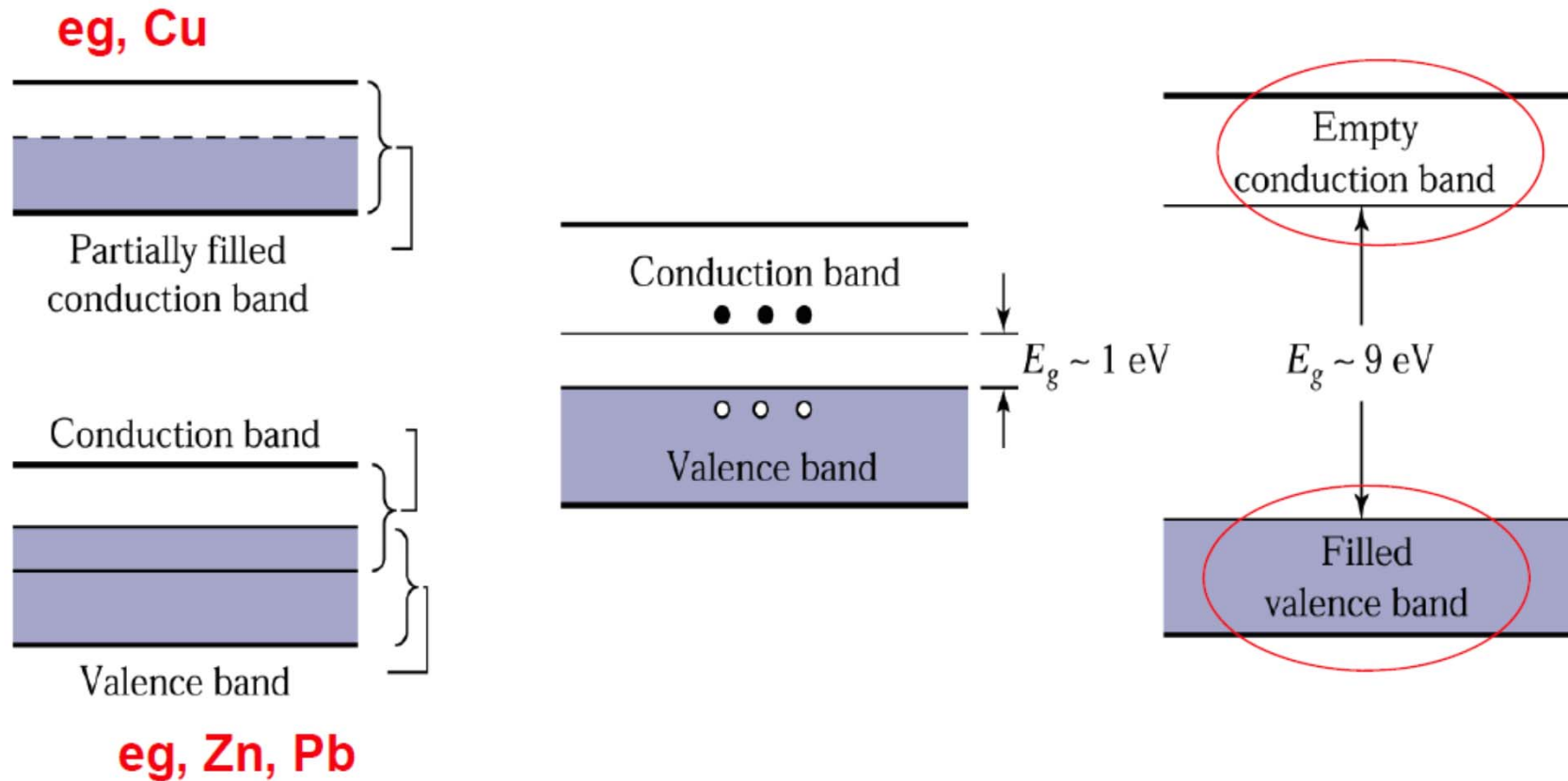


A schematic **energy-momentum** diagram for a special semiconductor with

$m^n = 0.25 m^0$ and $m^p = m^0$.



Energy band structures of Si and GaAs. Circles (°) indicate holes in the valence bands and dots (•) indicate electrons in the conduction bands.



Schematic energy band representations of (a) a conductor with two possibilities (either the partially filled conduction band shown at the upper portion or the overlapping bands shown at the lower portion), (b) a semiconductor, and (c) an insulator.

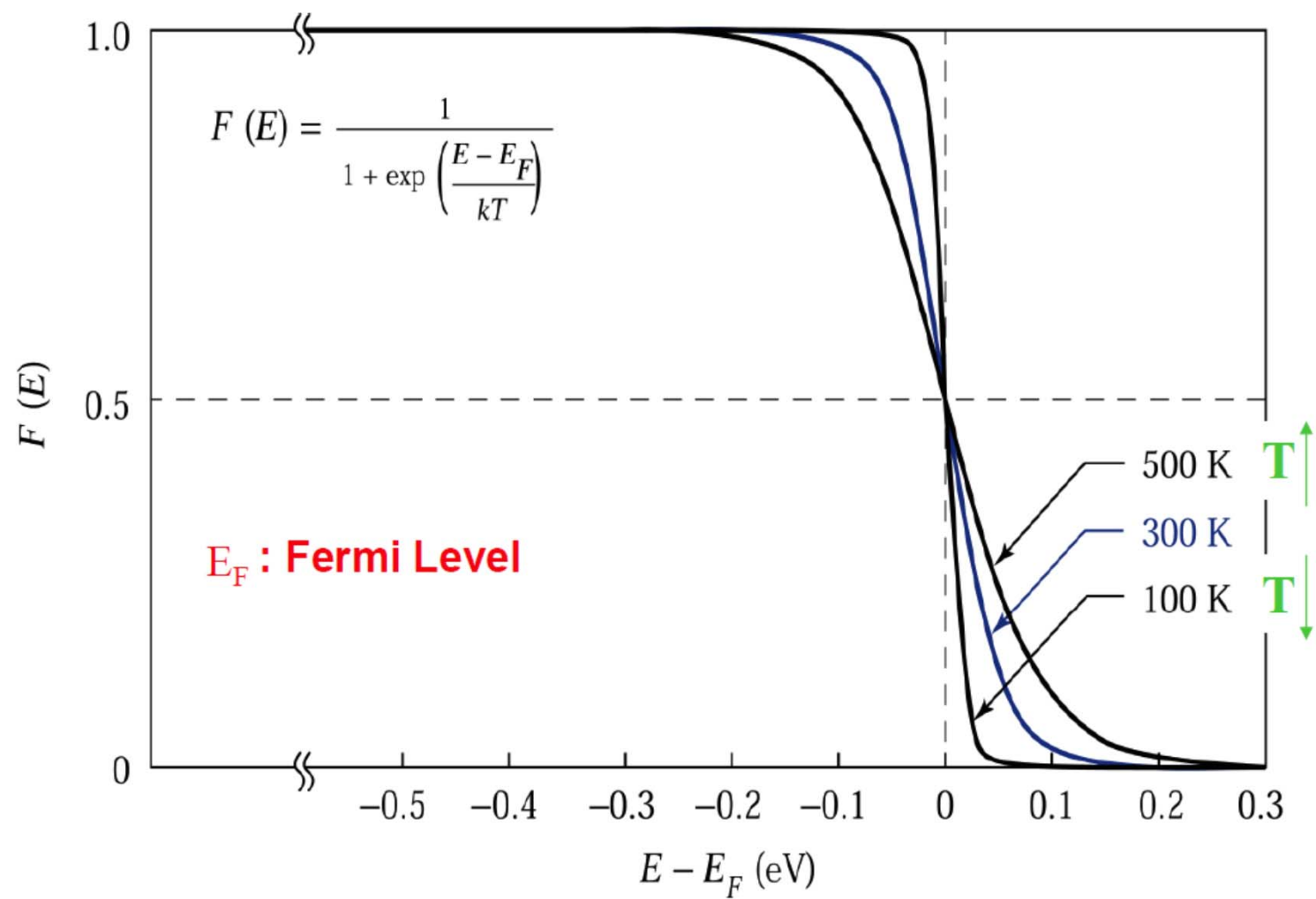
Intrinsic semiconductor : impurity e,p << thermal e,p

**Fermi-Dirac distribution
function**

$$n = \int_0^{E_{top}} \underline{n(E)} dE = \int_0^{E_{top}} \underline{N(E)} F(\overset{\swarrow}{E}) dE \quad (9)$$

$$F(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \quad (10)$$

Density of state $N(E) \sim E^{1/2}$



Density of carriers

- The density of electrons (n_o) can be found precisely if we know
 1. the *number* of allowed energy states in a small energy range, dE : $S(E)dE$

“the density of states”
 2. the *probability* that a given energy state will be occupied by an electron: $f(E)$

“the distribution function”

$$n_o = \int_{band} S(E)f(E)dE$$

For quasi-free electrons in the conduction band:

1. We must use the effective mass (averaged over all directions)
2. the potential energy E_p is the edge of the conduction band (E_C)

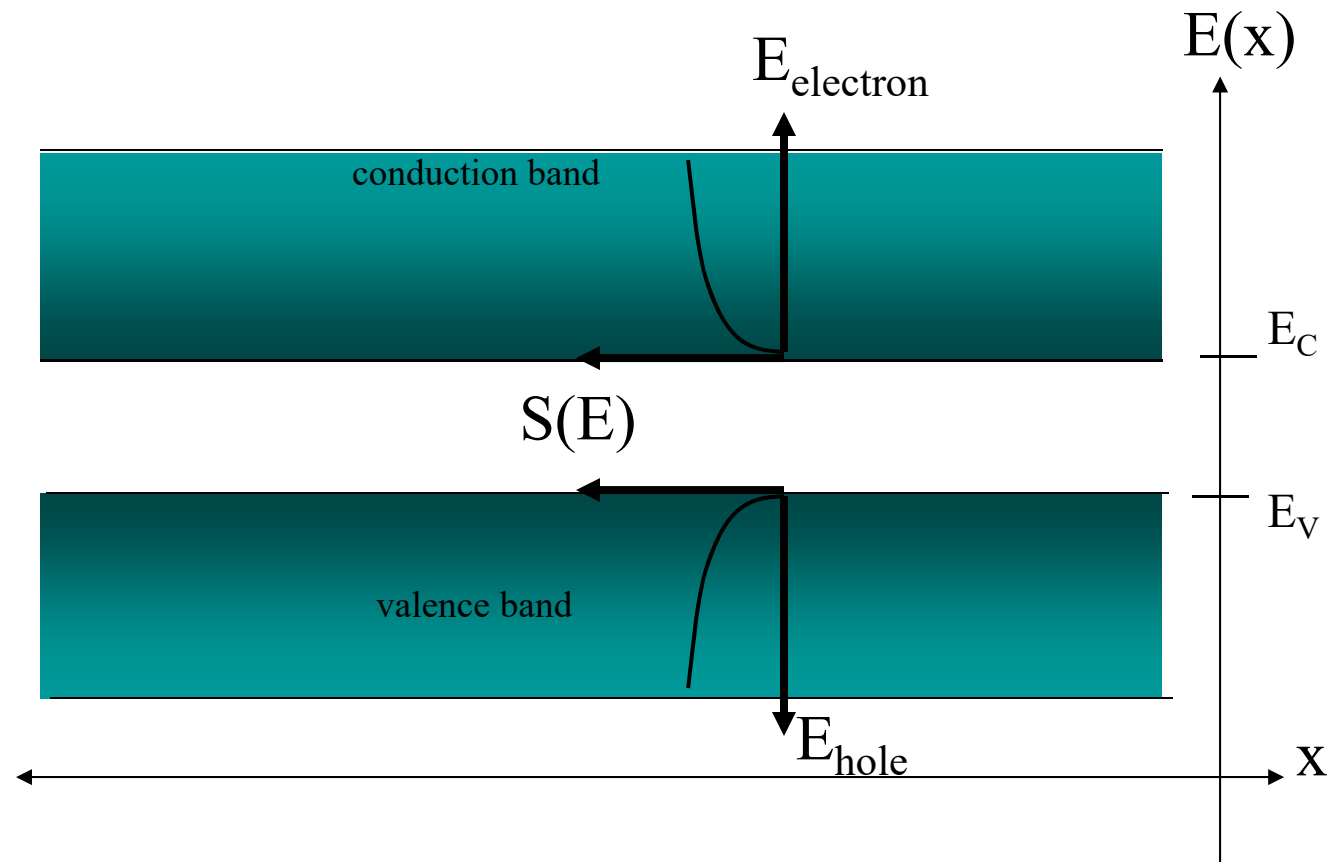
$$S(E) = \frac{1}{2\pi^2} \left(\frac{2m_{dse}^*}{\hbar^2} \right) (E - E_C)^{\frac{1}{2}}$$

For holes in the valence band:

1. We still use the effective mass (averaged over all directions)
2. the potential energy E_p is the edge of the valence band (E_V)

$$S(E) = \frac{1}{2\pi^2} \left(\frac{2m_{dsh}^*}{\hbar^2} \right) (E_V - E)^{\frac{1}{2}}$$

Energy Band Diagram



note: increasing electron energy is 'up', but increasing hole energy is 'down'.

Reminder of our GOAL:

- The density of electrons (n_o) can be found precisely if we know
 1. the *number* of allowed energy states in a small energy range, dE : $S(E)dE$

“the density of states”
 2. the *probability* that a given energy state will be occupied by an electron: $f(E)$

“the distribution function”

$$n_o = \int_{band} S(E)f(E)dE$$

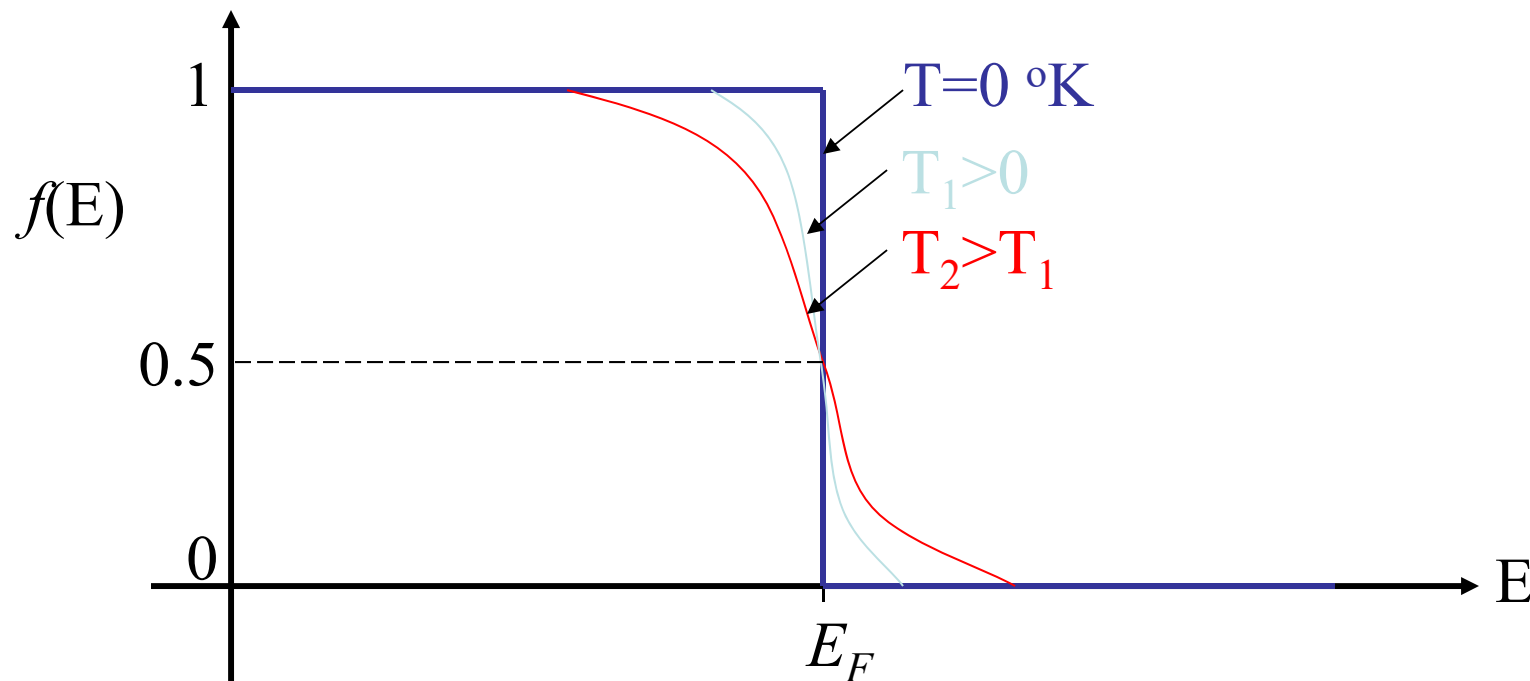
Fermi-Dirac Distribution

The probability that an electron occupies an energy level, E , is

$$f(E) = 1/\{1 + \exp[(E - E_F)/kT]\}$$

- where T is the temperature (Kelvin)
- k is the Boltzmann constant ($k = 8.62 \times 10^{-5} \text{ eV/K}$)
- E_F is the Fermi Energy (*in eV*)
- (Can derive this – statistical mechanics.)

$$f(E) = 1 / \{ 1 + \exp[(E - E_F) / kT] \}$$



All energy levels are filled with e⁻'s below the Fermi Energy *at 0 °K*

Fermi-Dirac Distribution for holes

Remember, a hole is an energy state that is NOT occupied by an electron.

Therefore, the probability that a state is occupied by a hole is the probability that a state is NOT occupied by an electron:

$$\begin{aligned} f_p(E) &= 1 - f(E) = 1 - 1/\{1+\exp[(E-E_F)/kT]\} \\ &= \{1+\exp[(E-E_F)/kT]\} / \{1+\exp[(E-E_F)/kT]\} - \\ &\quad 1/\{1+\exp[(E-E_F)/kT]\} \\ &= \{\exp[(E-E_F)/kT]\} / \{1+\exp[(E-E_F)/kT]\} \\ &= 1/\{\exp[(E_F - E)/kT] + 1\} \end{aligned}$$

The Boltzmann Approximation

If $(E-E_F) > kT$ such that $\exp[(E-E_F)/kT] \gg 1$ then,

$$\begin{aligned} f(E) &= \{1 + \exp[(E-E_F)/kT]\}^{-1} \cong \{\exp[(E-E_F)/kT]\}^{-1} \\ &\cong \exp[-(E-E_F)/kT] \quad \dots \text{the Boltzmann approx.} \end{aligned}$$

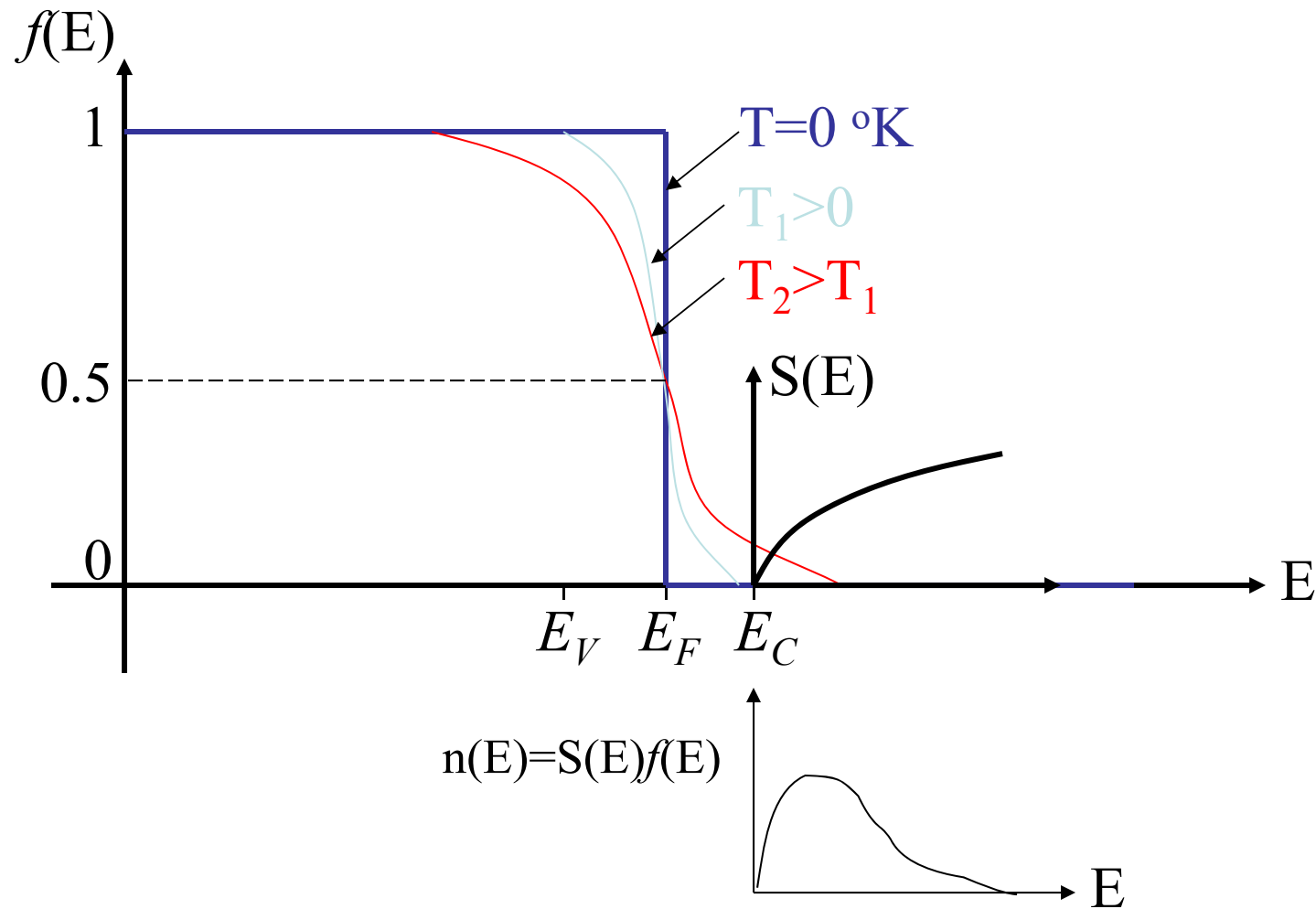
similarly, $f_p(E)$ is small when $\exp[(E_F - E)/kT] \gg 1$:

$$\begin{aligned} f_p(E) &= \{1 + \exp[(E_F - E)/kT]\}^{-1} \cong \{\exp[(E_F - E)/kT]\}^{-1} \\ &\cong \exp[-(E_F - E)/kT] \end{aligned}$$

If the Boltz. approx. is valid, we say the semiconductor is **non-degenerate**.

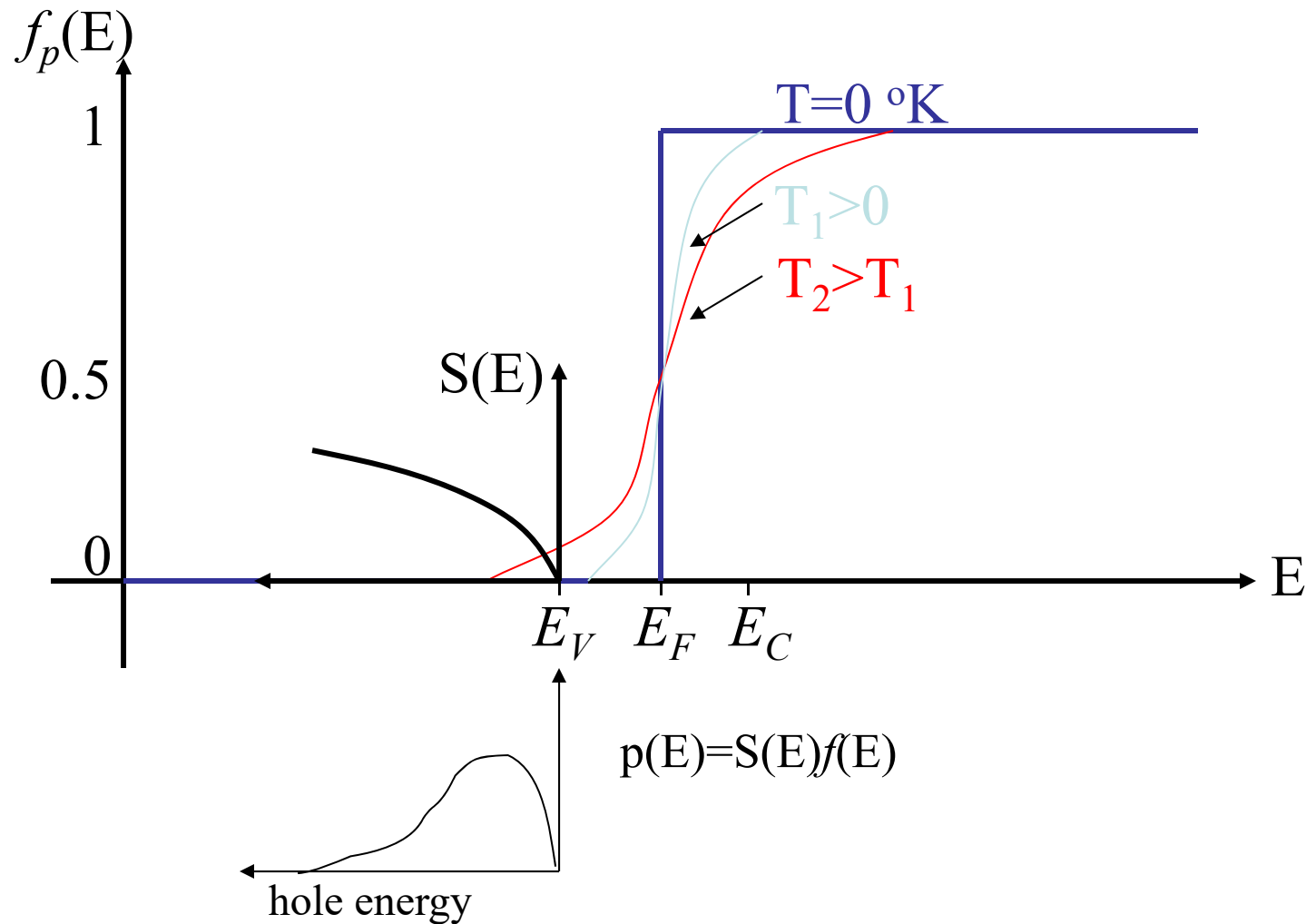
Putting the pieces together:

for electrons, $n(E)$



Putting the pieces together:

for holes, $p(E)$



Finding n_o and p_o

$$n_0 = \int_{E_c}^{E_c(\max)} S(E) f(E) dE \approx \frac{1}{2\pi^2} \left(\frac{2m_{dse}^*}{\hbar^2} \right)^{3/2} \int_{E_c}^{\infty} \sqrt{E - E_c} e^{-[(E - E_F)/kT]} dE$$

$$= N_C \exp[-(E_C - E_F)/kT] \quad \dots \text{where } N_C = 2 \left(\frac{m_{dse}^* kT}{2\pi\hbar^2} \right)^{3/2}$$

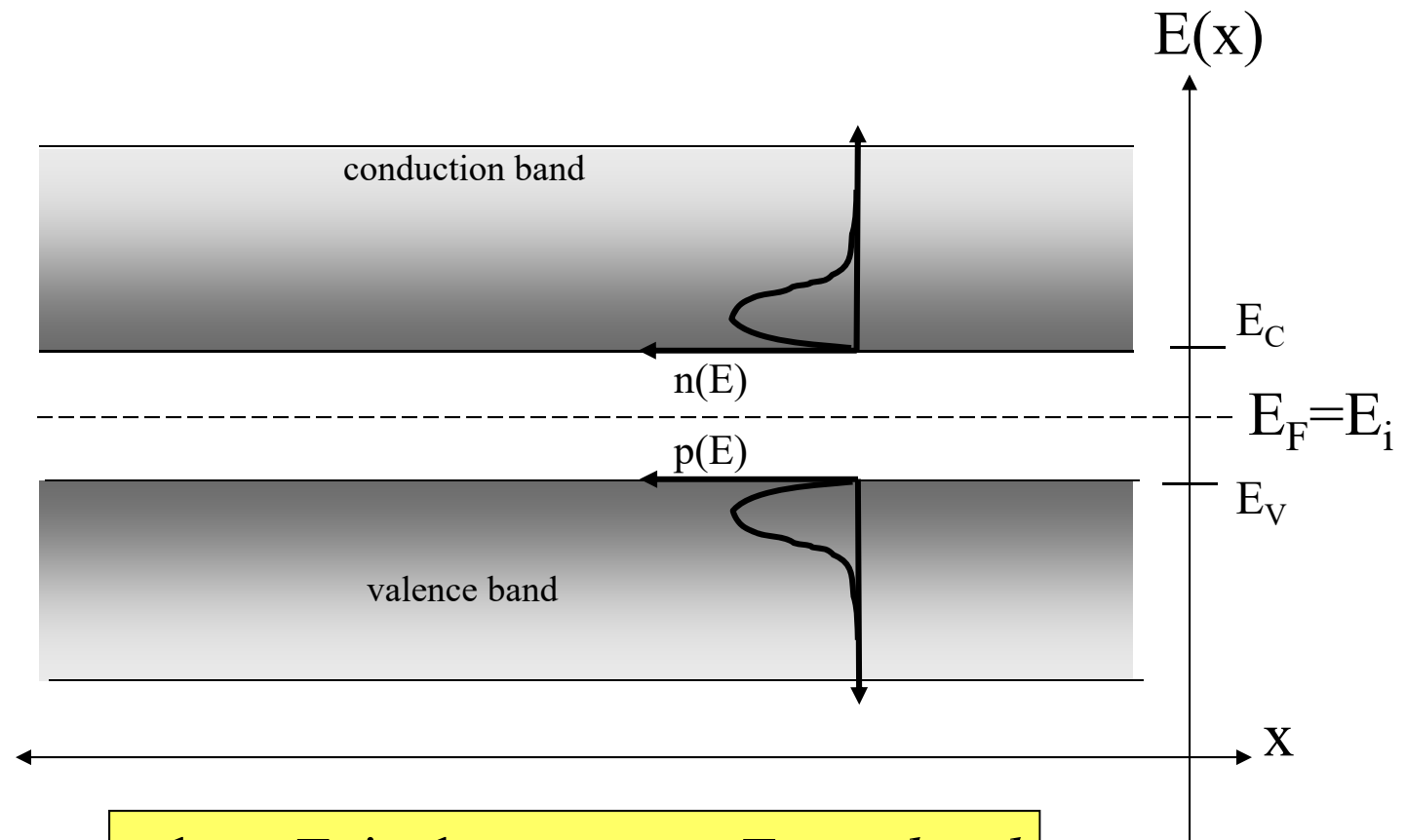
the effective density of states
in the conduction band

$$p_0 = \int_{E_v(\min)}^{E_v} S(E) f_p(E) dE \approx \frac{1}{2\pi^2} \left(\frac{2m_{dsh}^*}{\hbar^2} \right)^{3/2} \int_{-\infty}^{E_v} \sqrt{E_v - E} e^{-[(E_F - E)/kT]} dE$$

$$= N_V \exp[-(E_F - E_V)/kT] \quad \dots \text{where } N_V = 2 \left(\frac{m_{dsh}^* kT}{2\pi\hbar^2} \right)^{3/2}$$

Energy Band Diagram

intrinsic semiconductor: $n_o = p_o = n_i$

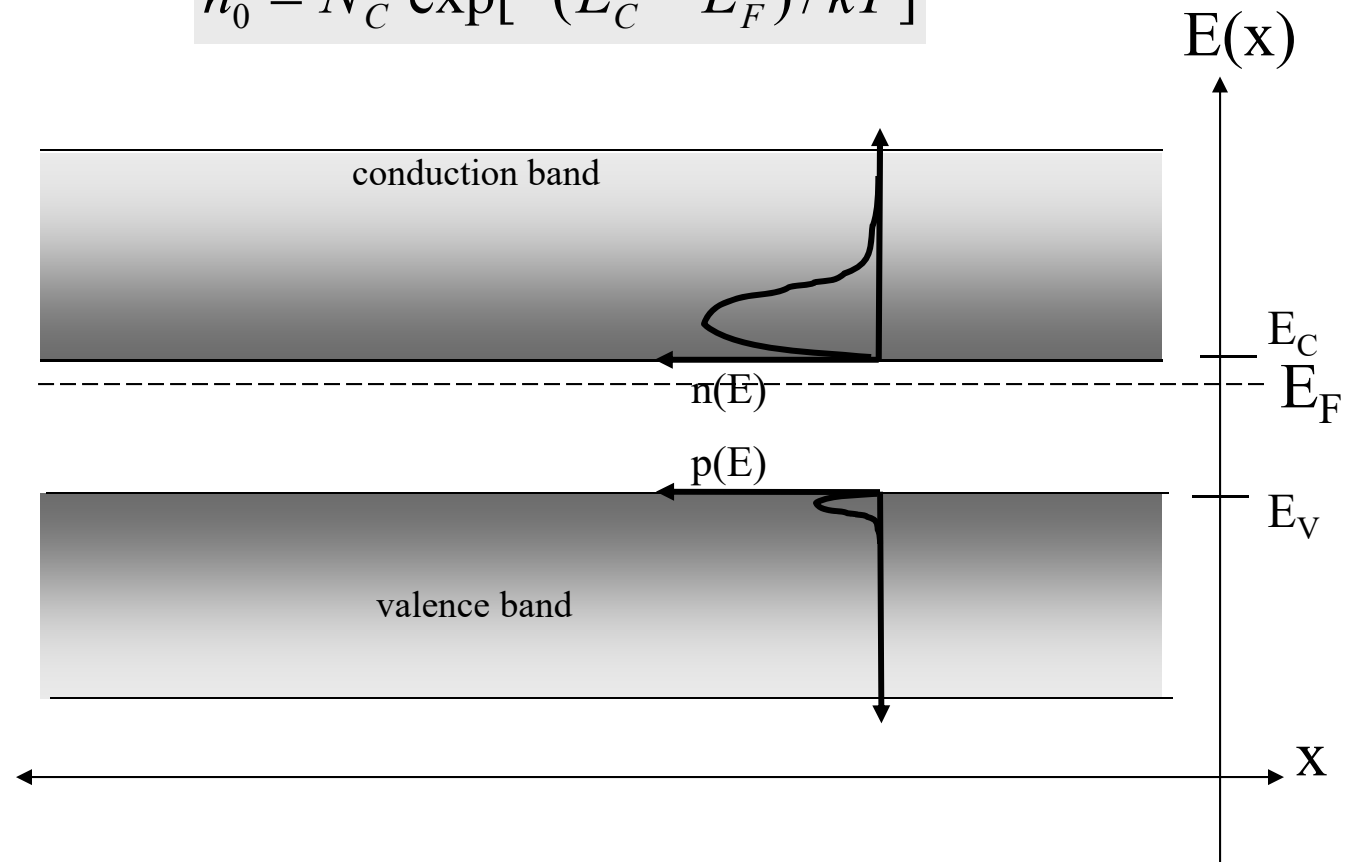


where E_i is the *intrinsic Fermi level*

Energy Band Diagram

n-type semiconductor: $n_o > p_o$

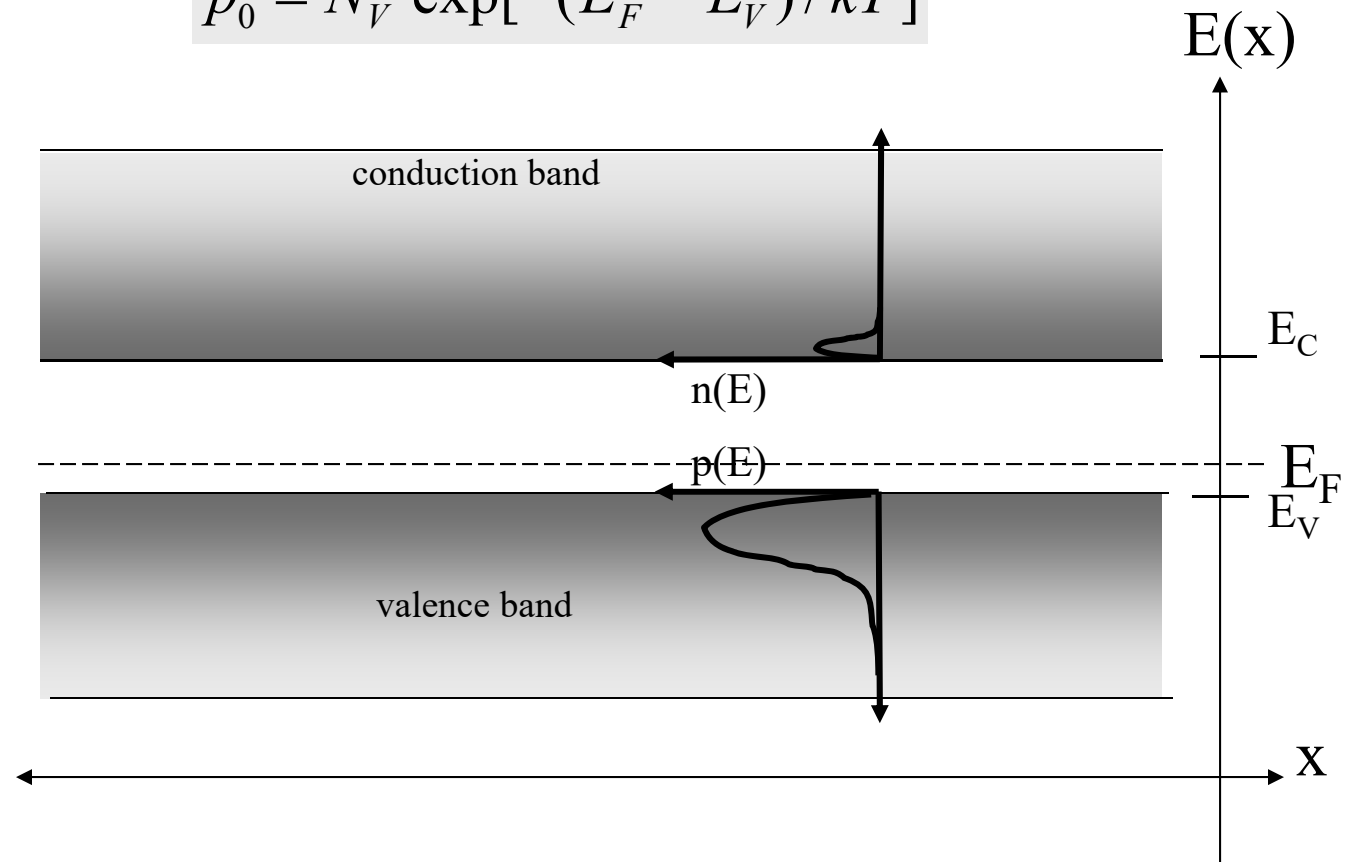
$$n_0 = N_C \exp[-(E_C - E_F)/kT]$$



Energy Band Diagram

p-type semiconductor: $p_o > n_o$

$$p_0 = N_V \exp[-(E_F - E_V)/kT]$$



A very useful relationship

$$\begin{aligned}n_0 p_0 &= N_C \exp[-(E_C - E_F)/kT] \times N_V \exp[-(E_F - E_V)/kT] \\&= N_C N_V e^{-(E_C - E_V)/kT} = N_C N_V e^{-E_g/kT}\end{aligned}$$

...which is independent of the Fermi Energy

Recall that $n_i = n_o = p_o$ for an intrinsic semiconductor, so

$$n_o p_o = n_i^2$$

for all non-degenerate semiconductors.

(that is as long as E_F is not within a few kT of the band edge)

$$\begin{aligned}n p &= N_C N_V e^{-E_g/kT} = n_i^2 \\n_i &= \sqrt{N_C N_V} e^{-E_g/2kT}\end{aligned}$$

The intrinsic carrier density

$$n_0 p_0 = N_C N_V e^{-E_g / kT} = n_i^2$$

$$n_i = \sqrt{N_C N_V} e^{-E_g / 2kT}$$

is sensitive to the *energy bandgap*, *temperature*, and m^*

$$N_C = 2 \left(\frac{m_{dse}^* kT}{2\pi\hbar^2} \right)^{3/2}$$

The intrinsic Fermi Energy (E_i)

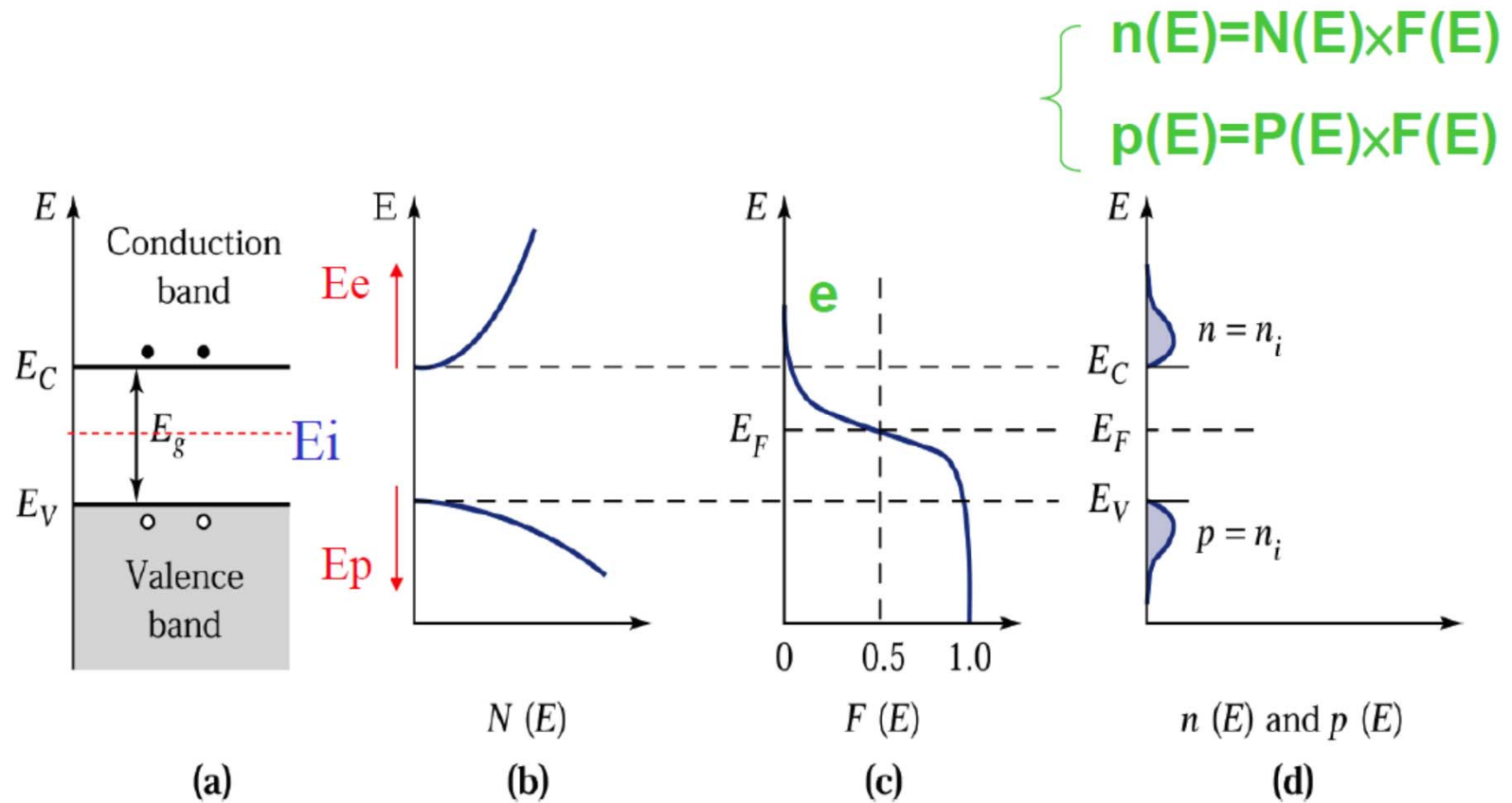
For an intrinsic semiconductor, $n_o = p_o$ and $E_F = E_i$

$$N_C \exp[-(E_C - E_i)/kT] = N_V \exp[-(E_i - E_V)/kT]$$

which gives

$$E_i = (E_C + E_V)/2 + (kT/2) \ln(N_V/N_C)$$

so the intrinsic Fermi level is approximately in the middle of the bandgap.



Intrinsic semiconductor. (a) Schematic band diagram. (b) Density of states. (c) Fermi distribution function. (d) Carrier concentration.

effective density of state

$$N_C \equiv 2(2\pi m_n kT / h^2)^{3/2} \quad (13a)$$

$2.86 \times 10^{19} \text{ cm}^{-3}$
For silicon

$$\star \quad n = N_C \cdot \exp\left[-\frac{(E_C - E_F)}{kT}\right] \quad (16)$$

$$N_V \equiv 2(2\pi m_p kT / h^2)^{3/2} \quad (18)$$

$2.66 \times 10^{19} \text{ cm}^{-3}$
For silicon

$$\star \quad p = N_V \cdot \exp\left[-\frac{(E_F - E_V)}{kT}\right] \quad (17)$$

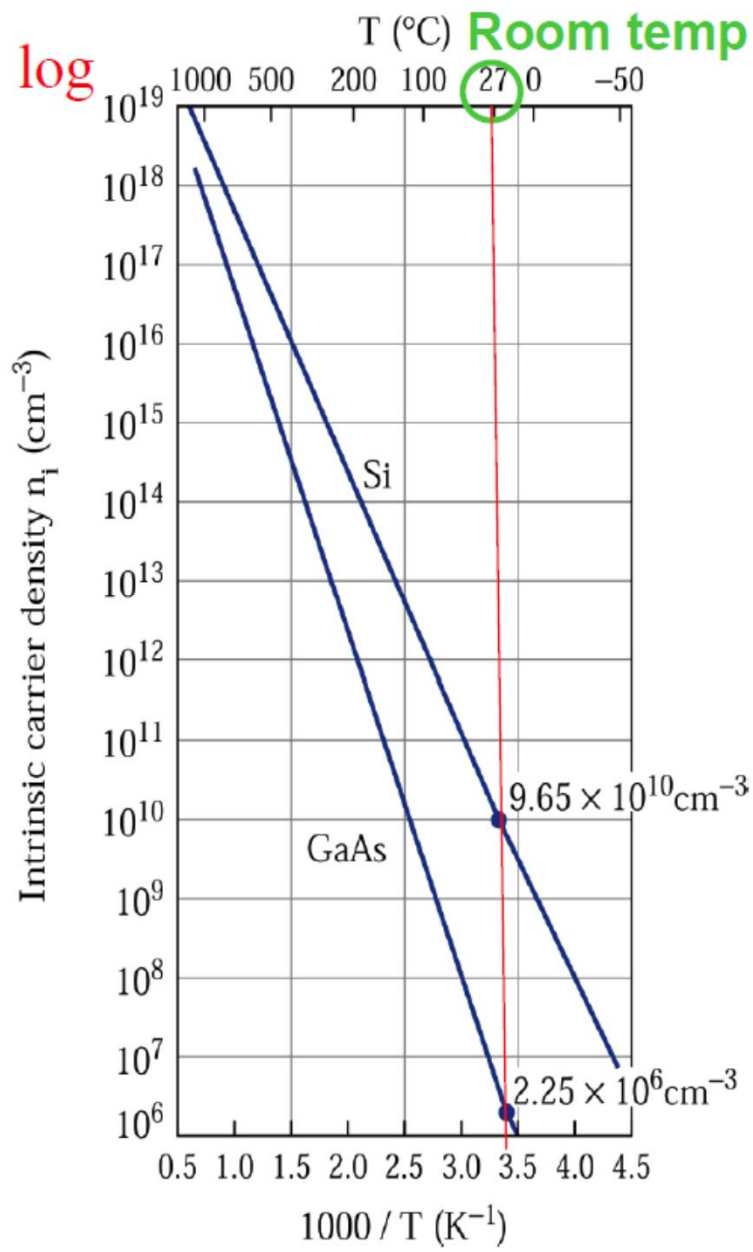
{ Ei : intrinsic fermi level
 ni : intrinsic carrier density \longrightarrow (Si) $1.45 \times 10^{10} \text{ #/cm}^3$
 Intrinsic semiconductor $n=p=n_i$

$$np = n_i^2 \quad \text{Mass action law (20)} \quad \left\{ \begin{array}{l} \text{Intrinsic,} \\ \text{Extrinsic} \end{array} \right.$$

$$n_i^2 = N_C N_V \exp \left[-\frac{E_g}{kT} \right] \quad (21)$$

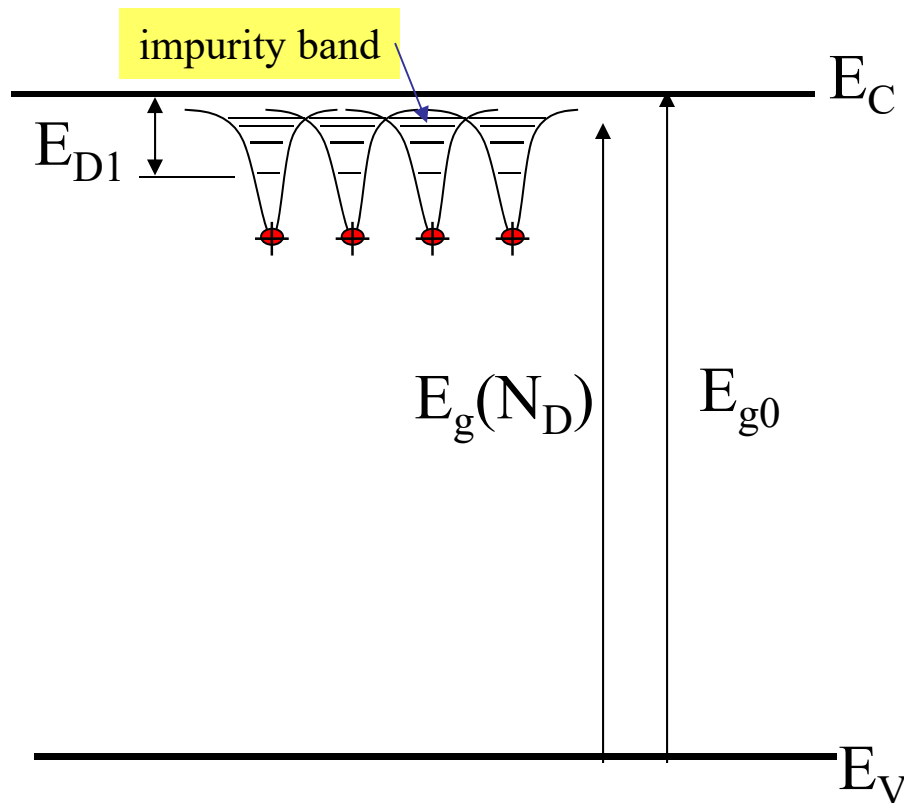
← $E_g = E_c - E_v$

$$n_i = \sqrt{N_C N_V} \exp \left[-\frac{E_g}{2kT} \right] \quad (22)$$



Intrinsic carrier densities in Si and GaAs as a function of the reciprocal of temperature.

Degenerate Semiconductors



for $N_D > 10^{18} \text{ cm}^{-3}$ in Si

1. The doping concentration is so high that E_F moves within a few kT of the band edge (E_C or E_V).

Boltzman approximation not valid.

2. High donor concentrations cause the allowed donor wavefunctions to overlap, creating a band at E_{dn} .

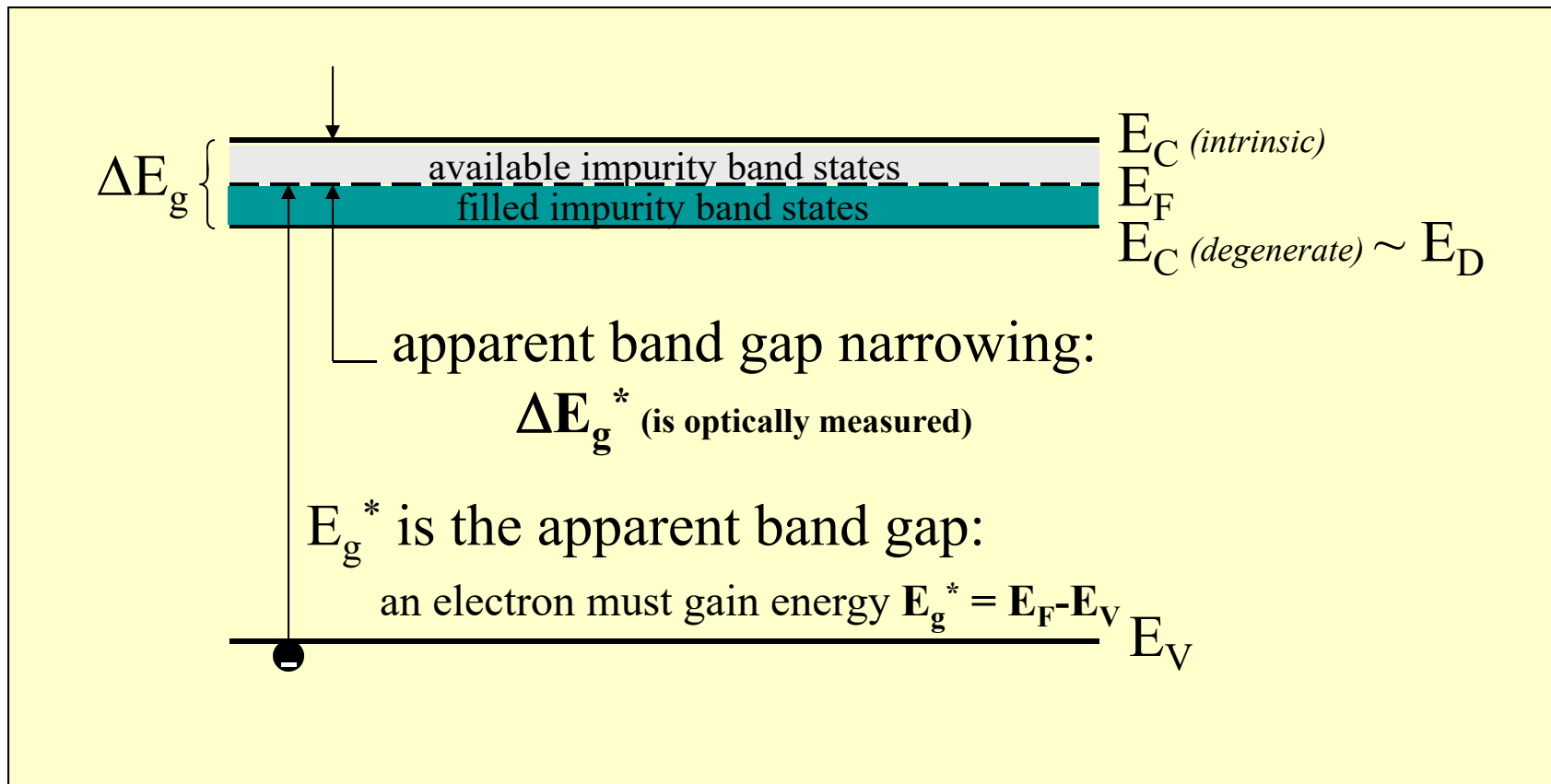
First only the high states overlap, but eventually even the lowest state overlaps.

This effectively decreases the bandgap by

$$\Delta E_g = E_{g0} - E_g(N_D)$$

Degenerate Semiconductors

As the doping conc. increases more, E_F rises above E_C



Electron Concentration

in degenerately doped n-type semiconductors

The donors are fully ionized: $n_o = N_D$

The holes still follow the Boltz. approx. since $E_F - E_V \gg kT$

$$\begin{aligned} p_o &= N_V \exp[-(E_F - E_V)/kT] &&= N_V \exp[-(E_g^*)/kT] \\ &&&= N_V \exp[-(E_{go} - \Delta E_g^*)/kT] \\ &&&= N_V \exp[-E_{go}/kT] \exp[\Delta E_g^*/kT] \end{aligned}$$

$$\begin{aligned} n_o p_o &= N_D N_V \exp[-E_{go}/kT] \exp[\Delta E_g^*/kT] \\ &= (N_D/N_C) N_C N_V \exp[-E_{go}/kT] \exp[\Delta E_g^*/kT] \\ &= (N_D/N_C) n_i^2 \exp[\Delta E_g^*/kT] \end{aligned}$$

Summary

non-degenerate:

$$n_o p_o = n_i^2$$

degenerate n-type:

$$n_o p_o = n_i^2 (N_D/N_C) \exp[\Delta E_g^*/kT]$$

degenerate p-type:

$$n_o p_o = n_i^2 (N_A/N_V) \exp[\Delta E_g^*/kT]$$