Ultrafast Optics

Review: { Mode locking
general model.

* Active mode locking:

$\alpha_m = \alpha_0 (1 - \cos \omega_m t)$

* Passive mode locking: → SA (Saturable absorper)
realizations of SA effect:

(1). SA material (CNT., 2-D material, dye ...)

(2) Nonlinear effects: { Self focusing
Nonlinear polarization rotation (kerr)

(3) Additive phase ~~modulation~~ mode lock (APM)

* General model: (1) Loss / phase shift; (2) Gain (bandwidth limited)
(3) GVD; (4) SPM (kerr); (5) SA.

Today: pulse measurement

$E(t) = \varepsilon(t) \cdot e^{j\phi(t)} \cdot e^{-j\omega_0 t}$

goal: (1) $\varepsilon(t)$? Amplitude (2) $\phi(t)$: phase. (3). ω_0: easy to obtain

1. Intensity Autocorrelation: $A_c(\tau) = \int I(t) \cdot I(t-\tau) \cdot dt$

where $I(t) = |\varepsilon(t)|^2$

perform F.T. $\tilde{A}_c(\omega) = |\tilde{I}(\omega)|^2$

* difficult to get $\tilde{I}(\omega)$ from $\tilde{A}(\omega)$

*. have to know the pulse shape.

Assuming Gaussian pulse:

$$I(t) = e^{-\frac{t^2}{t_0^2}}$$

$$A_c(\tau) = \int I(t) \cdot I(t-\tau) \cdot dt = \int e^{-\frac{t^2}{t_0^2}} e^{-\frac{(t-\tau)^2}{t_0^2}} \cdot dt$$

$$= e^{-\frac{\tau^2}{2t_0^2}} \int e^{-\frac{2(t-\frac{\tau}{2})^2}{t_0^2}} \cdot dt$$

$$\propto e^{-\frac{\tau^2}{(\sqrt{2}\,t_0)^2}}$$

$\Rightarrow$ That is what we usually do.

* measure the $A_c(\tau)$ vs. delay τ.
* find $\frac{1}{e}$ Intensity
* $/\sqrt{2} = t_0$

How to implement?

Second Harmonic Generation (SHG)

A little touch on Nonlinear Optics here.

principle: $P^{(2)} = \varepsilon_0 \chi^{(2)} \vdots E_1 E_2$

how to generate: Must be Non-centric symmetry.

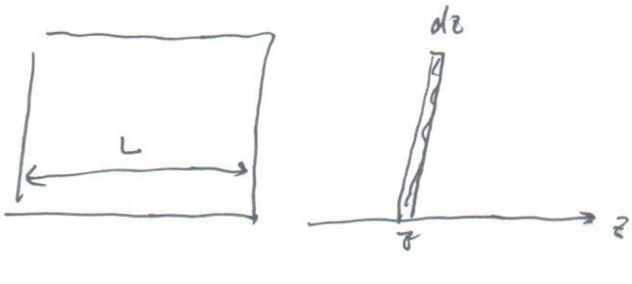
Argurement: $P^{(2)-} = \varepsilon_0 \chi^{(2)} \vdots E_1^- E_2^-$

$$= \varepsilon_0 \chi^{(2)} \vdots E_1^+ E_2^+ = P^{(2)+}$$

$$\Rightarrow \chi^{(2)} = 0$$

$$P = \varepsilon_0 \chi^{(1)} \vdots \vec{E} + \varepsilon_0 \chi^{(2)} \vdots \vec{E}\vec{E}$$

Assume: a crystal:



$$SHG \propto \left(\bar{E} e^{jk(\omega)\cdot z}\right)^2 \cdot \chi^{(2)} \cdot dz$$

$\Rightarrow$ propagate: $\chi^{(2)} \cdot \left(\bar{E} e^{jk(\omega)\cdot z}\right)^2 \cdot dz \cdot e^{jk(2\omega)\cdot(L-z)}$

$\Rightarrow$ To sum: $\int_0^L \chi^{(2)} \left(\bar{E} e^{jk(\omega)\cdot z}\right)^2 \cdot e^{jk(2\omega)\cdot(L-z)} \cdot dz$

$$\propto \chi^{(2)} \cdot \int_0^L \bar{E}^2 \cdot e^{j[2k(\omega)-k(2\omega)]z} \cdot dz$$

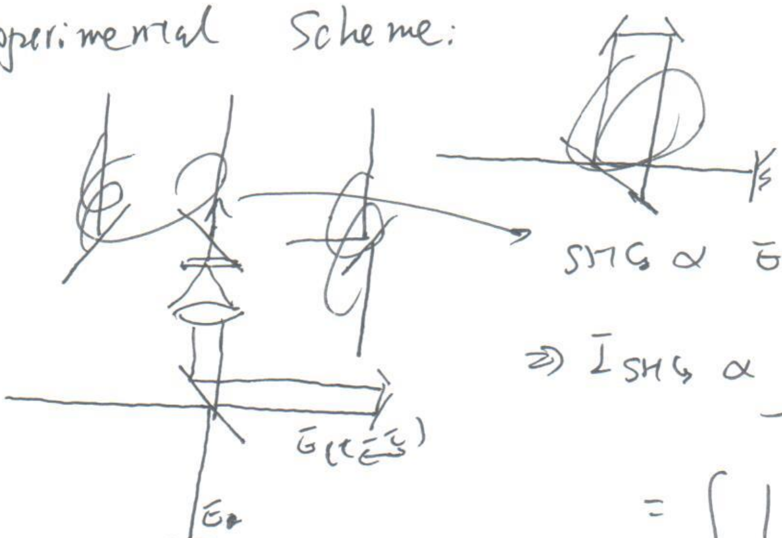
$$= \chi^{(2)} \cdot \bar{E}^2 \cdot F.T\left[\mathrm{rect}\left(\frac{z - L/2}{L}\right)\right] \text{ at } 2k(\omega) - k(2\omega)$$

$$\propto \chi^{(2)} \cdot \bar{E}^2 \cdot \mathrm{sinc}\left(L \cdot \frac{2k(\omega) - k(2\omega)}{2\pi}\right)$$

Notice a ~~mon~~ commonly used term:

phase mismatch: $\Delta k = 2k(\omega) - k(2\omega)$

~~Exp~~ Experimental Scheme:



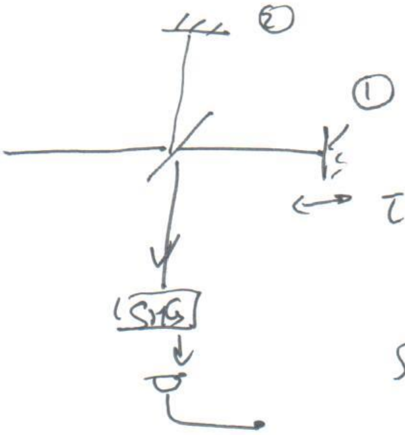
$SHG \propto \bar{E}(t), \bar{E}(t-\tau)$

$$\Rightarrow I_{SHG} \propto \int \left|\bar{E}(t)\cdot\bar{E}(t-\tau)\right|^2 \cdot dt$$

$$= \int \left|\bar{E}(t)\right|^2 \cdot \left|\bar{E}(t-\tau)\right|^2 \cdot dt$$

$$= \int I(t) \cdot I(t-\tau) \cdot dt$$

2. Interferometric Auto correlator.



$E_1(t-\tau) + E_2(t)$

$$SHG \propto \int \left| \left[E_1(t-\tau) + E_2(t) \right]^2 \right|^2 \cdot dt$$

$$E_1 = E_2 = E$$

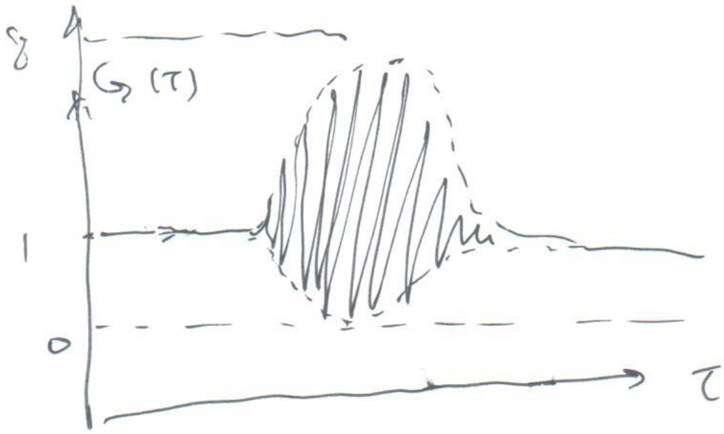
- Let's look at some simple property of this:

$$\tau = \pm\infty, \quad G(\tau) = \int \left\{ |E(t-\tau)|^4 + |E(t)|^4 \right] \cdot dt$$

$$= 2\int |E(t)|^4 \cdot dt$$

$$G_{min} = 0. \quad G_{max} = 16\int |E(t)|^4 \cdot dt.$$

How to know the pulse is chirped?



Now let's look at $G(\tau)$ carefully:

$$G(\tau) = \int \left| E_1^2(t-\tau) + E_1^2(t) + 2E_1(t-\tau)\cdot E_2(t) \right|^2 \cdot dt$$

$$= \left[E_1^2(t-\tau) + E_2^2(t) + 2E_1(t-\tau)\cdot E_2(t) \right] \times$$

$$\left\{ E_1^{*2}(t-\tau) + E_2^{*2}(t) + 2\cdot E_1^*(t-\tau)\cdot E_2^*(t) \right] dt$$

a term

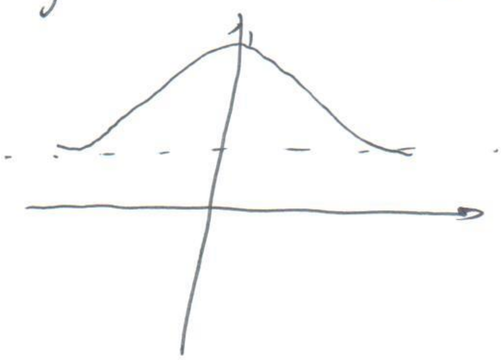
$$= \begin{cases} \text{A term: } \int \left[|E_1(t-\tau)|^4 + |E_2(t)|^4 + 4|E_1(t-\tau)|^2 \cdot |E_2(t)|^2 \right] dt \\ + \int \left[2E_1^2(t-\tau)\cdot E_1^*(t-\tau)\cdot E_2^*(t) + 2E_1^{*2}(t-\tau)\cdot E_1(t-\tau)\cdot E_2(t) \right. \\ \left. + 2E_1(t-\tau)\cdot E_2(t)\cdot E_2^*(t) + 2E_1^*(t-\tau)\cdot E_2^*(t)\cdot E_2^2(t) \right] \cdot dt \quad \} \rightarrow \text{B term} \end{cases}$$

$$+ \int \left[E_1^2(t-\tau) E_2^{*2}(t) + c.c. \right] \cdot dt \rightarrow \text{C term}$$

Comment:

* A term: is ~~An~~ Intensity Autocorrelation curve ~~with~~ with background.

$$= \int |E_1(t-\tau)|^4 dt + \int |E_2(t)|^4 dt + 4\int I_1(t-\tau) I_2(t)\cdot dt$$



* B term:

$$\int \left[2E_1(t-\tau) |E_1(t-\tau)|^2 E_2(t) \right.$$
$$\left. + 2E_1(t-\tau)\cdot |E_2(t)|^2 \cdot E_2^*(t) + c.c \right] \cdot dt$$

$$= \int \Big\{ 2\, \varepsilon_1^3(t-\tau) \cdot e^{j\phi_1(t-\tau)} \cdot e^{-j\omega_0(t-\tau)} \cdot \varepsilon_2(t) \cdot e^{-j\phi_2(t)}\, e^{j\omega_0 t}$$

$$+ 2\, \varepsilon_1(t-\tau) \cdot e^{j\phi_1(t-\tau)} \cdot e^{-j\omega_0(t-\tau)} \cdot \varepsilon_2^3(t) \cdot e^{-j\phi_2(t)} \cdot e^{j\omega_0 t} + c.c. \Big] \cdot dt$$

Notice carrier frequency: $e^{-j\omega_0\tau}$

$$= \int \Big\{ 2\, \varepsilon_1(t-\tau) \cdot \varepsilon_2(t) \cdot \left[\varepsilon_1^2(t-\tau) + \varepsilon_2^2(t) \right] \cdot e^{j[\phi_1(t-\tau) - \phi_2(t)]}\, e^{j\omega_0\tau}$$

$$+ c.c. \Big\} \cdot dt$$

$$= 4 Re \Big\{ \int \varepsilon_1(t-\tau) \cdot \varepsilon_2(t) \cdot \left[\varepsilon_1^2(t-\tau) + \varepsilon_2^2(t) \right] \cdot e^{j[\phi_1(t-\tau) - \phi_2(t)]}\, e^{j\omega\tau_0} \cdot dt$$

$$\Rightarrow 4 Re \Big\{ B(\tau) \cdot e^{j\omega_0\tau} + c.c \Big\}$$

Where: $B(\tau) \equiv \int \varepsilon_1(t-\tau) \cdot \varepsilon_2(t) \left[\varepsilon_1^2(t-\tau) + \varepsilon_2^2(t) \right] \cdot e^{j[\phi_1(t-\tau) - \phi_2(t)]} \cdot dt.$

C term:

$$\int \left[\varepsilon_1^2(t-\tau)\, \varepsilon_2^2(t) \cdot e^{j[2\phi_1(t-\tau) - 2\phi_2(t)]}\, e^{j2\omega_0\tau} + c.c. \right] dt$$

$$= 2 Re \Big\{ C(\tau) \cdot e^{j2\omega_0\tau} \Big\}$$

where: $C(\tau) = \int \mathcal{E}_1^2(t-\tau)\cdot\mathcal{E}_2^2(\tau)\cdot e^{j[2\phi(t-\tau) - 2\phi_2(\tau)]} \cdot dt$

$\Downarrow$

field Autocorrelation of SHG Signal

i.e. F.T of $C(\tau)$ is spectrum of SHG signal.

In brief,

$$G(\tau) = A + 4\,\mathrm{Re}\{B(\tau)\cdot e^{j\omega_0\tau}\} + 2\,\mathrm{Re}\{C(\tau)\cdot e^{j2\omega_0\tau}\}$$

Comments:

* Three frequencies.

* A. B. C. can be extracted by a doing F.T. of data.

filtering. $\Rightarrow$ I.F.T.

14.5.13 here

Now let's look at a practical case:

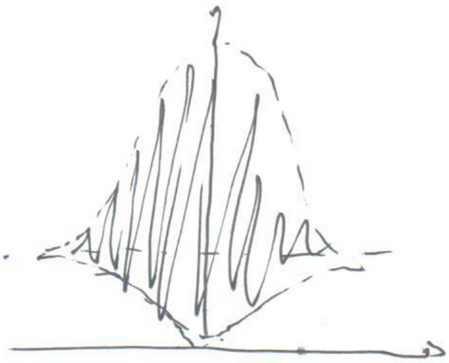
$$E(t) = e^{-\frac{t^2}{T_G^2}(1+ja)}\, e^{-j\omega_0 t}$$ Gaussian Pulse with

Simple chirping

Perform simulation:

$$G_2(\tau) = \left\{1 + 2e^{-\frac{\tau^2}{(\sqrt{2}\,T_G)^2}} + 4e^{-\frac{a^2+3}{4}\left(\frac{\tau}{T_G}\right)^2} \cdot \cos\frac{a}{2}\left(\frac{\tau}{T_G}\right)^2 \cdot \cos\omega_0\tau + 2e^{-(1+a^2)\left(\frac{\tau}{T_G}\right)^2} \cdot \cos 2\omega_0\tau\right\}$$

$$G(\tau) = A(\tau) + 4Re\{B(\tau)\cdot e^{j\omega_0\tau} + c.c\} + 2Re\{C(\tau)\cdot e^{j2\omega_0\tau} + c.c\}$$



$A(\tau)$: Intensity Auto Correlation w/ background.

$B(\tau)$:

$C(\tau)$: field Autocorrelation of SHG signal.

3). why Interferometric Autocorrelation?

indicating : chirping or Non-chirping.

To precise get phase : 2-D ~~retriev retreived~~. retrieval.

~~Today: Basics of FROG~~

~~— one of the methods for 2D retrieval~~

~~Today~~. Basics of FROG

- One of the methods for 2D retrieval.

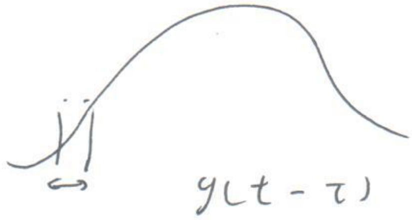
Frequency Resolved Optical Gating

Complex envelope:

$$\varepsilon(t) = A(t) \cdot e^{j\phi(t)}$$

Basic Idea: use gate to sample $\varepsilon(t)$.

$$\varepsilon_s(t) = \varepsilon(t) \cdot g(t-\tau)$$



$$S(\omega, \tau) = \left| F.T \{ \varepsilon_s(t) \} \right|^2$$

$$= \left| \int \varepsilon_s(t) \cdot e^{j\omega t} \cdot dt \right|^2$$

Can we use $S(\omega, \tau) \longrightarrow \varepsilon(t)$?

1). $\varepsilon_s(t) \Rightarrow \varepsilon(t)$ Yes.

But 2). $S(\omega, \tau) \Rightarrow \varepsilon_s(t, \tau)$? $\Rightarrow$ lack of phase

How to get phase information base on magnitude $S(\omega, \tau)$

2-D Retrieval, Notice: 1-D retrieval can't do it.

$$S(\omega, \tau) = \left| \int \varepsilon_s(t, \tau) \cdot e^{j\omega t} \cdot dt \right|^2$$

$$= \left| \iint \left(\int \tilde{\mathcal{E}}_s(t, \omega_\tau) \cdot e^{-j\omega_\tau \cdot \tau} \cdot d\tau \right) \cdot e^{j\omega t} \cdot dt \right|^2$$

$$= \underbrace{S(\omega, \tau)}_{} \rightarrow \text{experiments}$$

initial $\mathcal{E}(t)$ with Random guess

$$\downarrow \text{gate}$$

$$\mathcal{E}_s(t, \tau) = \mathcal{E}(t) \cdot g(t - \tau)$$

$$\downarrow \text{F.T.}$$

$$\hat{\mathcal{E}}_s(\omega, \tau)$$

$$\downarrow \text{Force magnitude} = \sqrt{S(\omega, \tau)}$$

$$\tilde{\mathcal{E}}_s'(\omega, \tau)$$

$$\downarrow \text{I.F.T}$$

$$\mathcal{E}_s'(t, \tau)$$

$$\downarrow$$

$$\mathcal{E}(t)$$