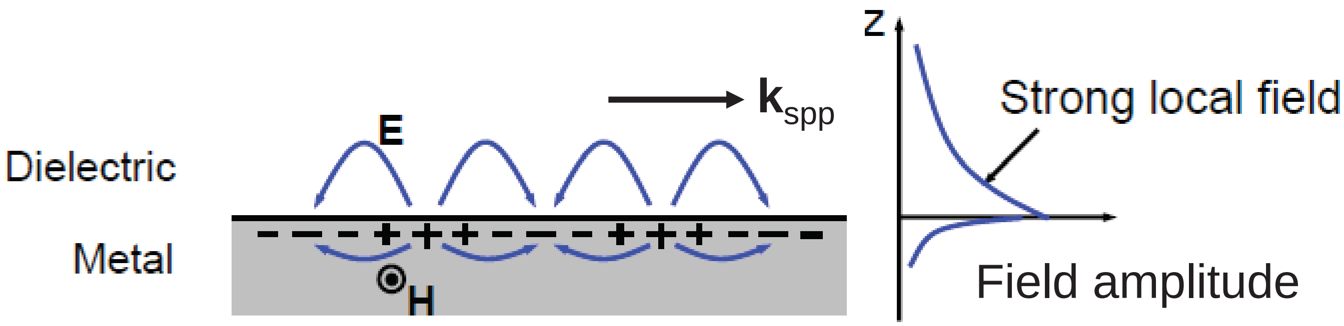
Content of this lecture

1. Surface plasmon polaritons (SPPs)
   - Dispersion relation of SPPs
   - Plasmon dispersion in full spectrum
   - Plasmon dispersion of real metals
   - SPP: transverse or longitudinal wave?
   - Wavelength of SPPs
   - Propagation length and loss of SPPs

2. SPPs in multilayer system
   - Dispersion relation of coupled SPP modes
   - IMI & MIM heterostructures

1. Surface plasmon polaritons (SPPs)

- What we have learnt so far:
  - Compare: electron "gas" in a metal vs. real gas composed of molecules
  - Metals allow for electron density waves – plasmons
  - At ω_p, longitudinal oscillations inside metal – volume plasmons
- The second type of plasmons in metal:
  - Surface plasmons – plasmons at metal-dielectric interface
  - When surface plasmon **couple with a photon** – surface plasmon polariton
  - SPP is a surface wave – propagate along the interface (wavevector $\mathbf{k}_{spp}$)
    – evanescently confined in the normal direction

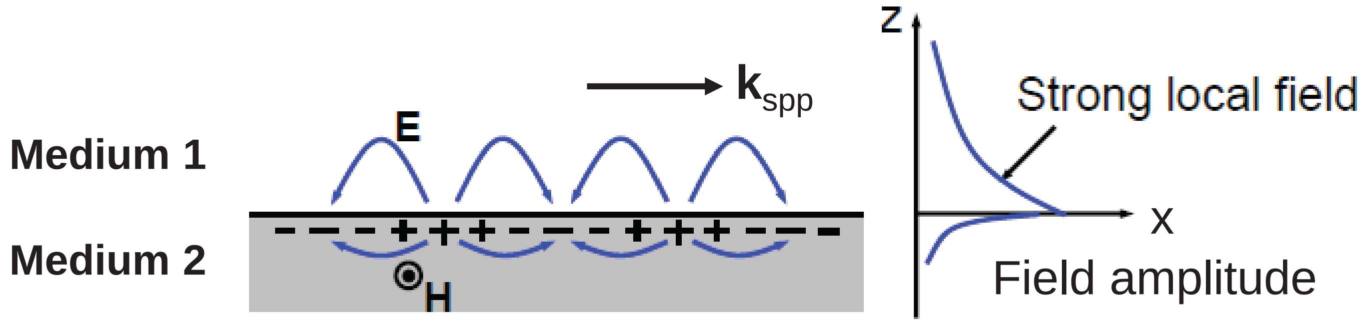


surface plasmon polariton: 表面等离激元，evanescent: 倏逝的，confine: 约束，normal: 法线

Dispersion relation of SPP

Let's solve Maxwell's equations with boundary conditions.

We are looking for solutions described below:



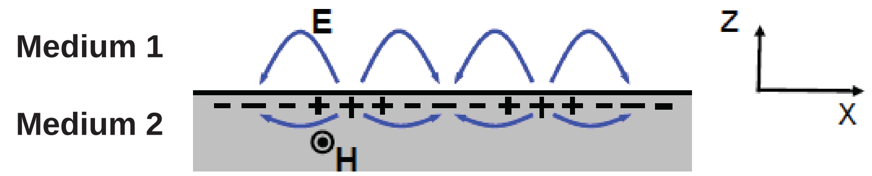
EM wave propagating in x direction and decaying in z direction should have the form:

$$\mathbf{E}(x,y,z) = \mathbf{A}\exp(\pm k_z z)\exp(i\beta x)$$

- "+" in Medium 2
- "–" in Medium 1

- β : the x component of the wave vector $\mathbf{k}$ (also called propagation constant, $\beta' \rightarrow$ phase velocity, $\beta'' \rightarrow$ loss)
- ik_z: the z component of the wave vector $\mathbf{k}$, k_z is positive real

We start from Maxwell's equations:



$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}, \quad \nabla \times \mathbf{H} = \varepsilon_0 \varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

$$\partial_t \to -i\omega$$

SPP wave:

$$\mathbf{E}(x, y, z) = \mathbf{A} \exp(\pm k_z z) \exp(i\beta x)$$

$$\partial_x \to i\beta, \partial_y = 0, \partial_z \to \pm k_z$$

$$\pm k_z E_y = -i\omega\mu_0 H_x$$

$$\pm k_z E_x - i\beta E_z = i\omega\mu_0 H_y$$

$$i\beta E_y = i\omega\mu_0 H_z$$

$$\pm k_z H_y = i\omega\varepsilon_0\varepsilon E_x$$

$$\pm k_z H_x - i\beta H_z = -i\omega\varepsilon_0\varepsilon E_y$$

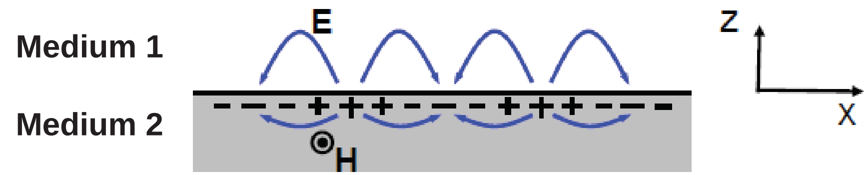
$$i\beta H_y = -i\omega\varepsilon_0\varepsilon E_z$$

Two sets of independent solutions:

TE solution, with E_y, H_x, H_z

TM solution, with H_y, E_x, E_z

For TM solution:



$$\pm\square_z E_x - \square\beta E_z = \square\omega\mu_0 H_y$$

$$\pm\square_z H_y = \square\omega\varepsilon_0\varepsilon E_x \;\longrightarrow\; \begin{cases} -\square_{z1} H_{y1} = \square\omega\varepsilon_0\varepsilon_1 E_{x1} & \text{(in Medium 1)} \\ \square_{z2} H_{y2} = \square\omega\varepsilon_0\varepsilon_2 E_{x2} & \text{(in Medium 2)} \end{cases}$$

$$\square\beta H_y = -\square\omega\varepsilon_0\varepsilon E_z$$

Boundary conditions:

$$H_{y1} = H_{y2}, \quad E_{x1} = E_{x2} \longrightarrow \frac{\square_{z1}}{\square_{z2}} \cdot \frac{H_{y1}}{H_{y2}} = -\frac{\varepsilon_1}{\varepsilon_2} \cdot \frac{E_{x1}}{E_{x2}} \Rightarrow \boxed{\frac{\square_{z1}}{\square_{z2}} = -\frac{\varepsilon_1}{\varepsilon_2}} \quad (*)$$

- ε_1 and ε_2 must have opposite signs – a metal and a dielectric

According to the wave equation: $(\partial_x \to \square\beta, \partial_y = 0, \partial_z \to \pm\square_z)$

$$\nabla^2\mathbf{E} + k_0^2\varepsilon\mathbf{E} = 0 \longrightarrow (k_z^2 - \beta^2 + k_0^2\varepsilon)\mathbf{E} = 0 \longrightarrow k_z^2 = \beta^2 - k_0^2\varepsilon$$

Substitute it into (*) and we arrive at:

$$\boxed{\beta = k_0\sqrt{\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}} = \frac{\omega}{c}\sqrt{\frac{\varepsilon_m\varepsilon_d}{\varepsilon_m+\varepsilon_d}}}$$

← **dispersion relation of SPP!**

- For TE solution:
  - It can be proven that no such confined surface mode is supported.

Homework

SPPs can only be excited by TM-polarized field!!!

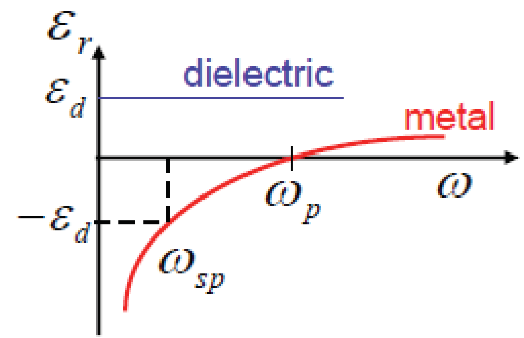
Plot the dispersion curve of SPP

$$\beta = \frac{\omega}{c}\sqrt{\frac{\varepsilon_{\square}\varepsilon_{\square}}{\varepsilon_{\square}+\varepsilon_{\square}}}$$

Two reasonable premises:

1. Neglect the dispersion of the dielectric: ε_d = constant

2. Drude metal without damping: $\varepsilon_m(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$

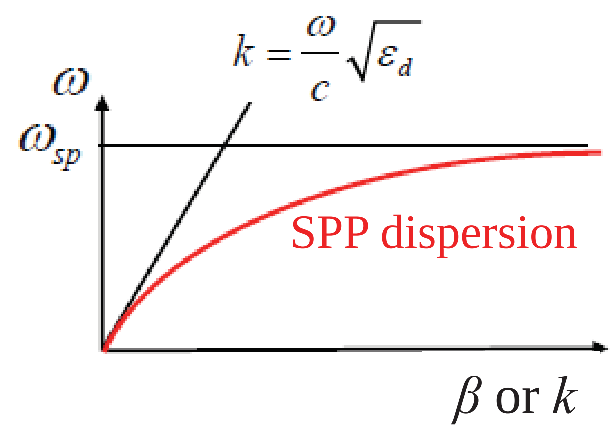


- At very low ω: $\varepsilon_m \to -\infty$

$$\beta = \frac{\omega}{c}\lim_{\varepsilon_m \to -\infty}\sqrt{\frac{\varepsilon_m\varepsilon_d}{\varepsilon_m+\varepsilon_d}} \approx \frac{\omega}{c}\sqrt{\varepsilon_d}$$ (tends to light line in the dielectric)

- At an ω where $\varepsilon_m = -\varepsilon_d$:

$\beta \to \infty$ (short-wavelength limit)

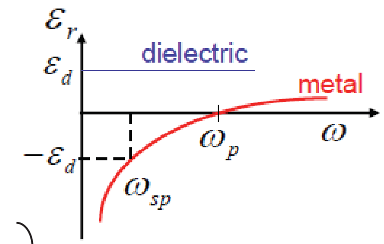


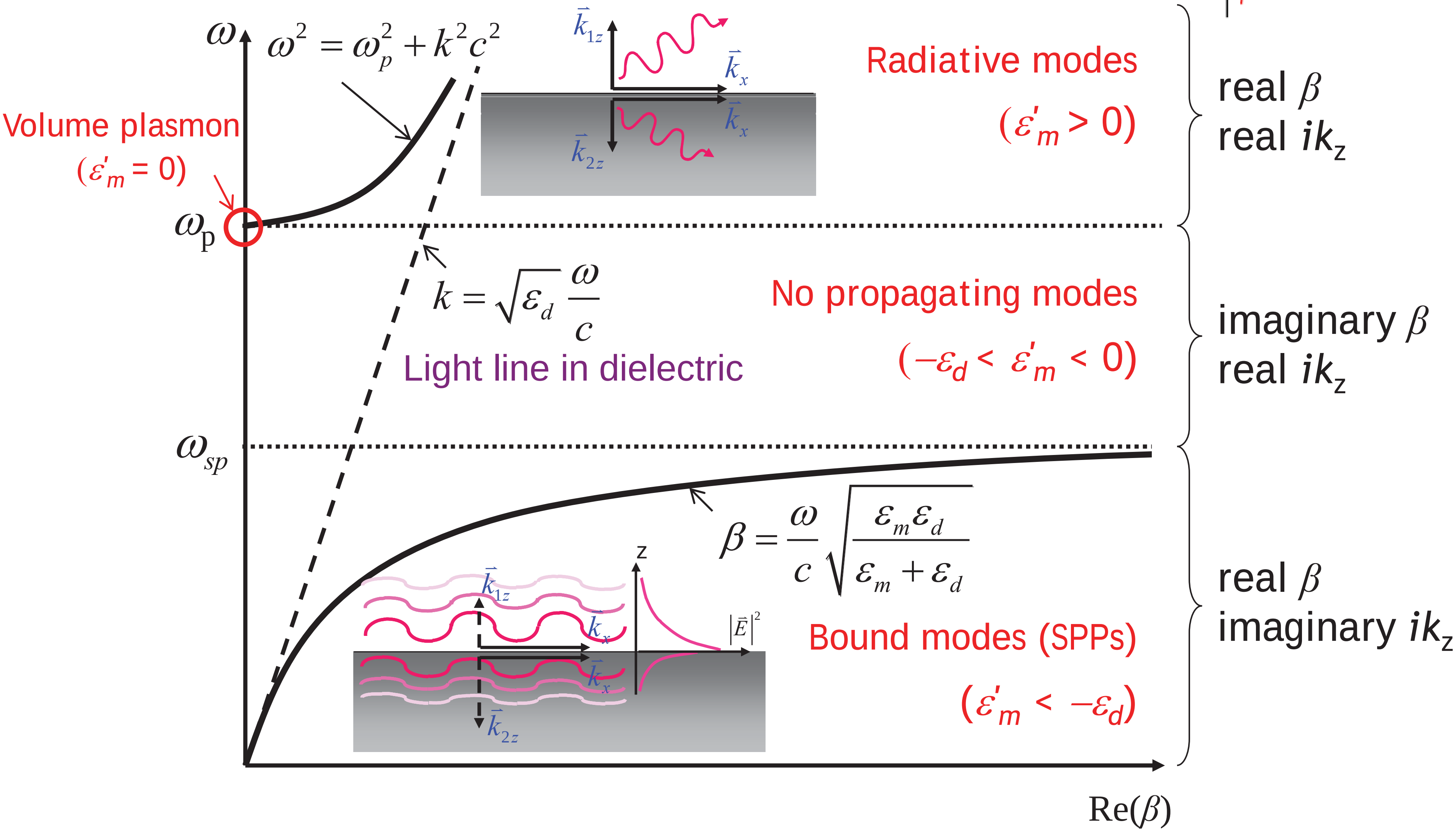
This frequency is called the characteristic **surface plasmon frequency** ω_{sp}:

$$\boxed{\omega_{sp} = \frac{\omega_p}{\sqrt{1+\varepsilon_d}}}$$

(Derive it by yourself)

Plasmon dispersion in full spectrum

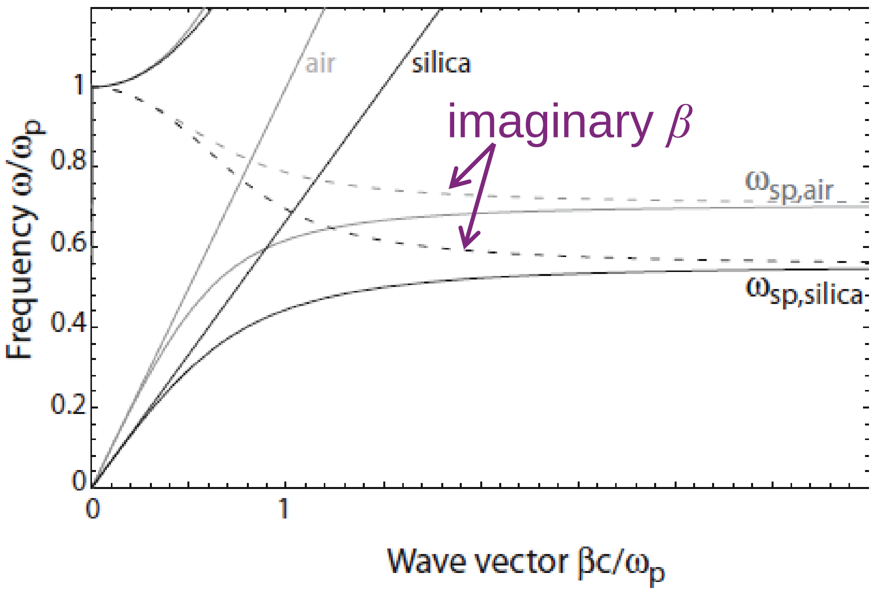




spectrum: 光谱，radiative: 辐射的，bound: 约束的，imaginary: 虚数的

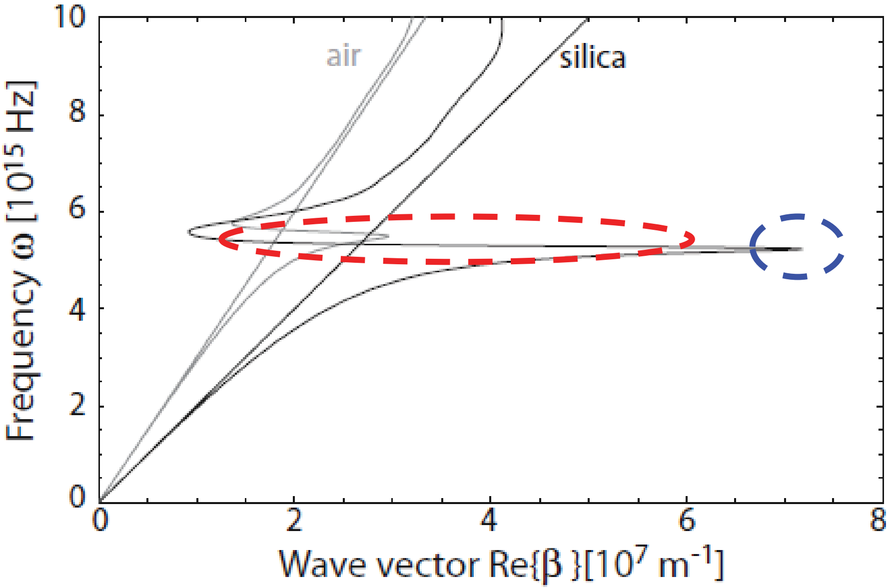
Plasmon dispersion of real metals

Drude model



damping neglected: $\gamma = 0$

Measured on Ag



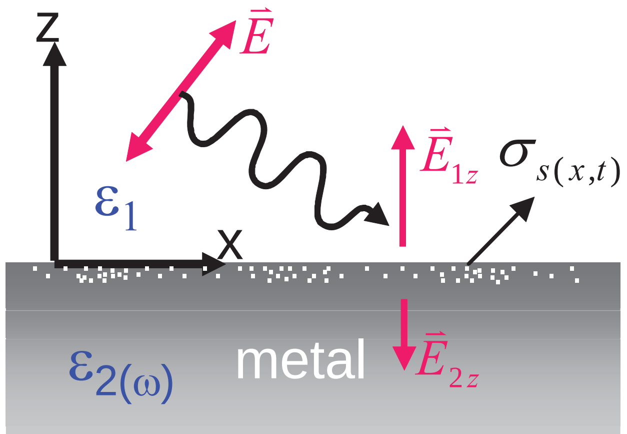
non-negligible damping! $\gamma \neq 0$

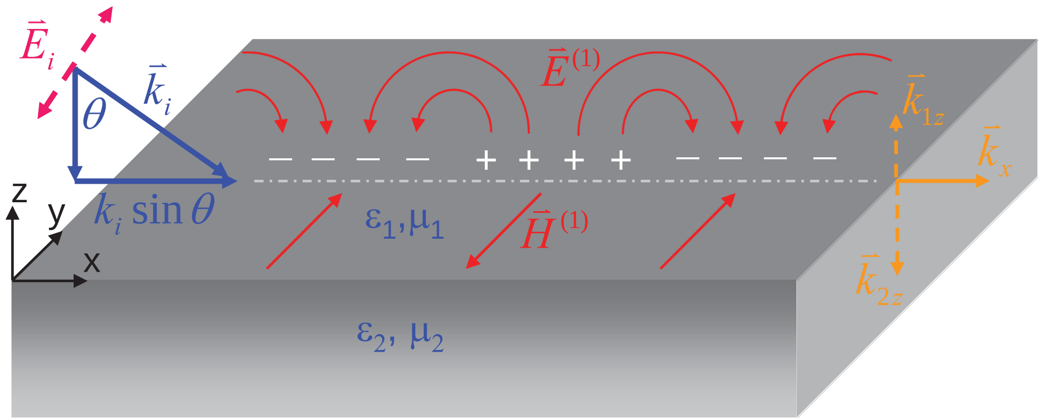
- **β approaches a finite limit at ω_{sp}**
- **Quasibound modes between ω_{sp} and ω_p is allowed**

damping: 衰减

SPP: transverse or longitudinal wave?

Understand why must TM excitation:



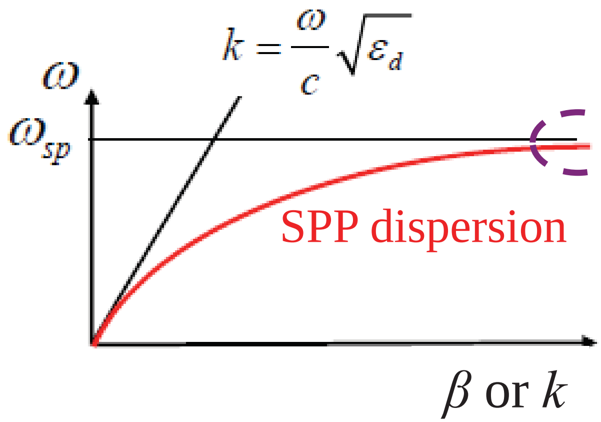


- Discontinuous E_z on interface $\longrightarrow$ accumulation of surface charges
- E_x component $\longrightarrow$ "push" the electrons to oscillate
- TE excitation $\longrightarrow$ continuous $\mathbf{E}$ $\longrightarrow$ no surface charge $\longrightarrow$ no SPP

So, is the SPP wave transverse or longitudinal?

Both!!!

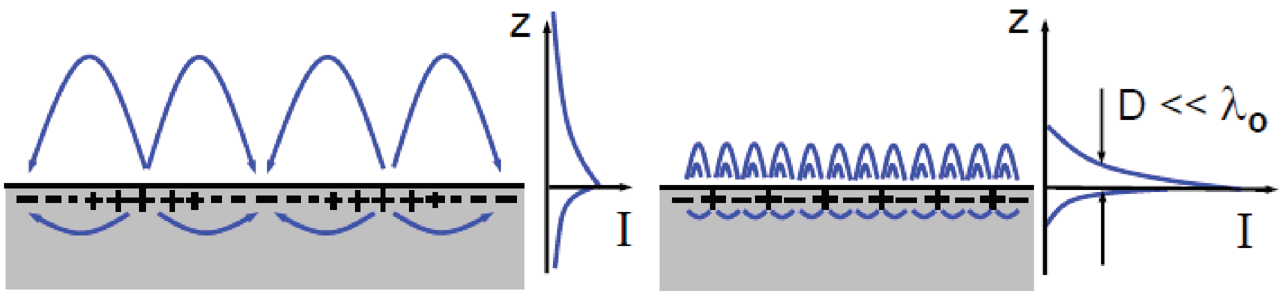
But, let's consider the special case of short-wavelength limit:



$$\beta \to \infty \quad (\lambda \to 0)$$

$$\therefore k_z = \sqrt{\beta^2 - k_0^2 \varepsilon} \to \infty$$

field decays quickly in *z* direction



Long wavelength   Short wavelength

- The field is confined in a very small region near the metal surface
- In this small region, the effective permittvity $\varepsilon = \varepsilon_m + \varepsilon_d = 0$

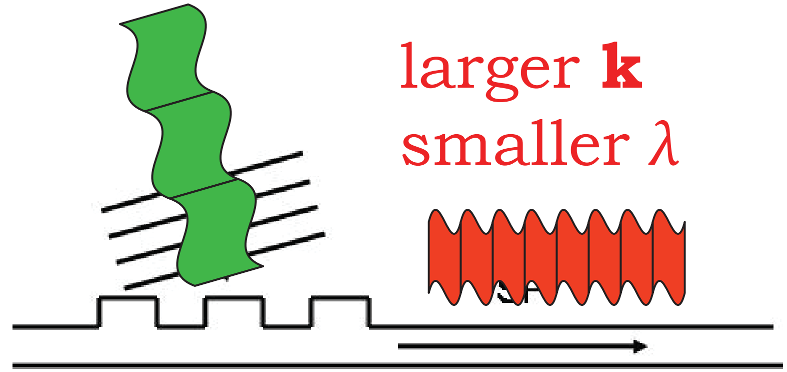
  $\rightarrow$ Longitudinal wave!

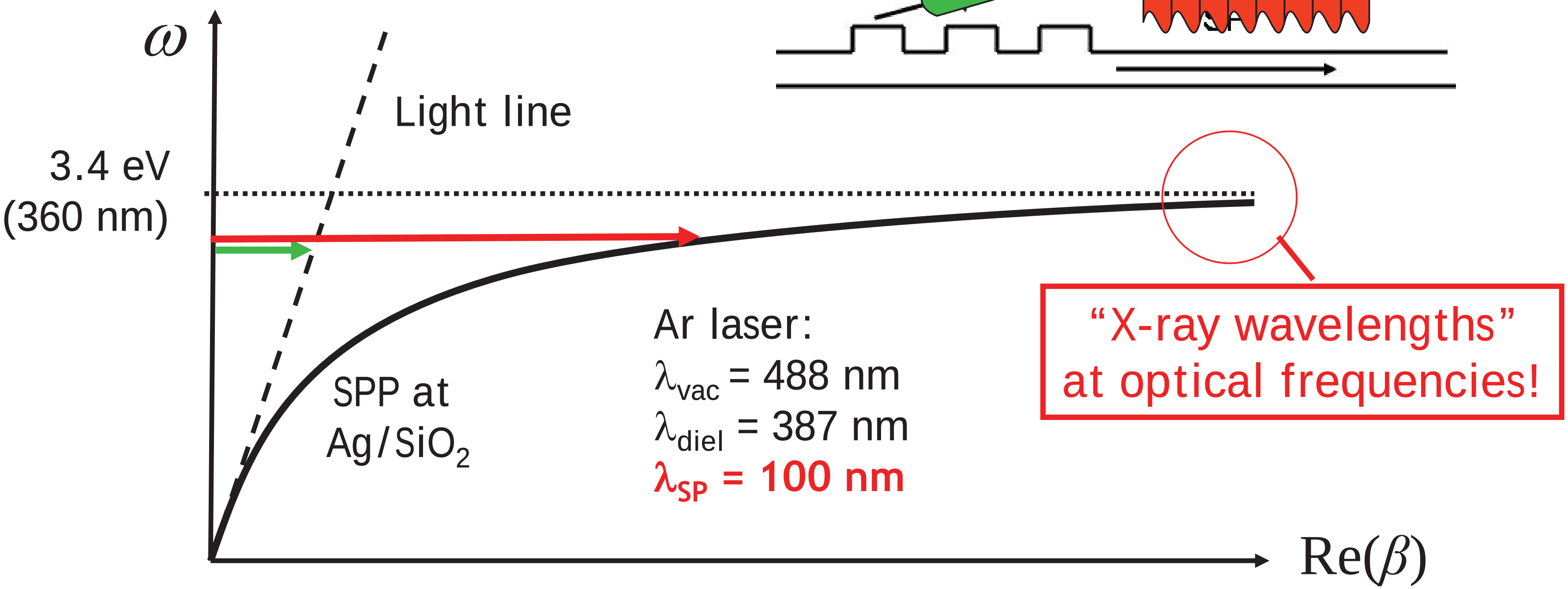
- $v_g \sim 0 \rightarrow$ non-propagating, quasi-static surface modes

Name: surface plasmon ~~polariton~~

Wavelength of SPP

$$\lambda_{spp} = \frac{2\pi}{k_{spp}} = \frac{2\pi}{\beta}$$



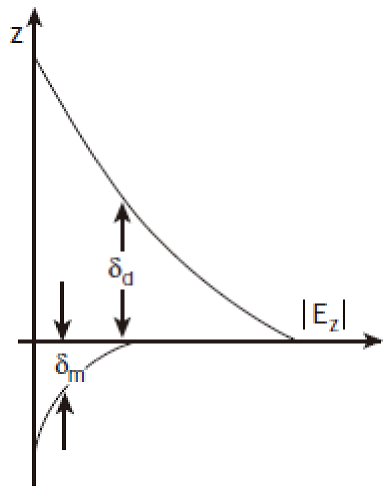


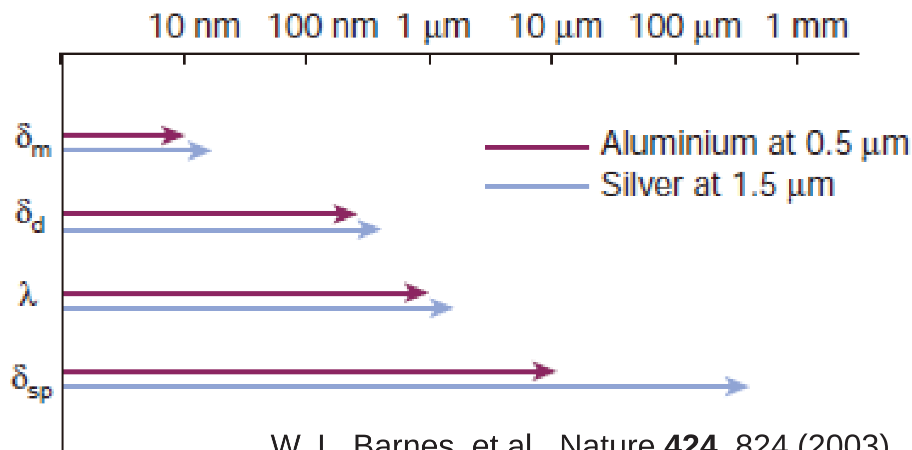
- SPP wavelength can reach nanoscale at optical frequencies!
- Light cannot excite SPPs on planar metal surface directly.

How to excite? – next lecture

Propagation length & loss of SPPs

Three characteristic length scales (important!):





W. L. Barnes, et al., Nature **424**, 824 (2003).

$$\delta_m = \frac{1}{2k_{zm}}$$

$$\delta_d = \frac{1}{2k_{zd}}$$

$$\delta_{sp} = \frac{1}{2\beta''}$$

δ_m: decay length in metal → field confinement

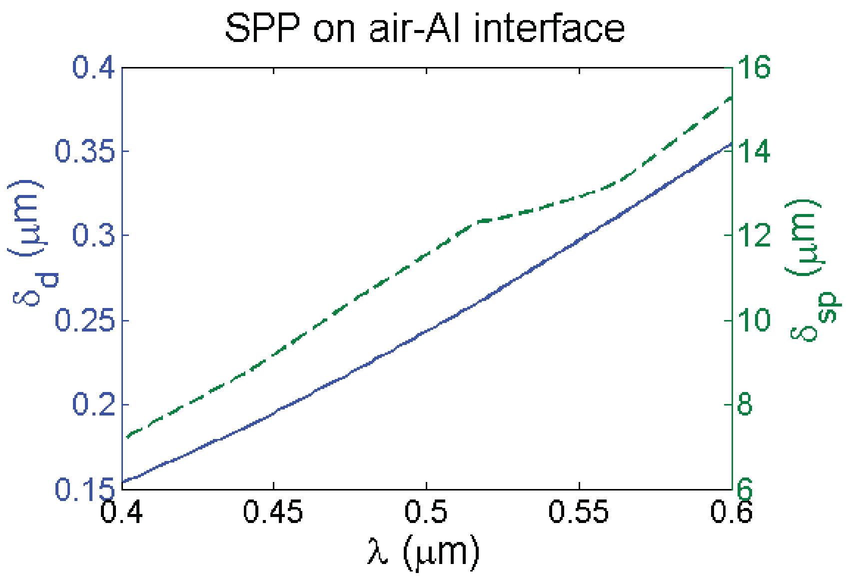
δ_d: decay length in dielectric ($\sim\lambda/2$)

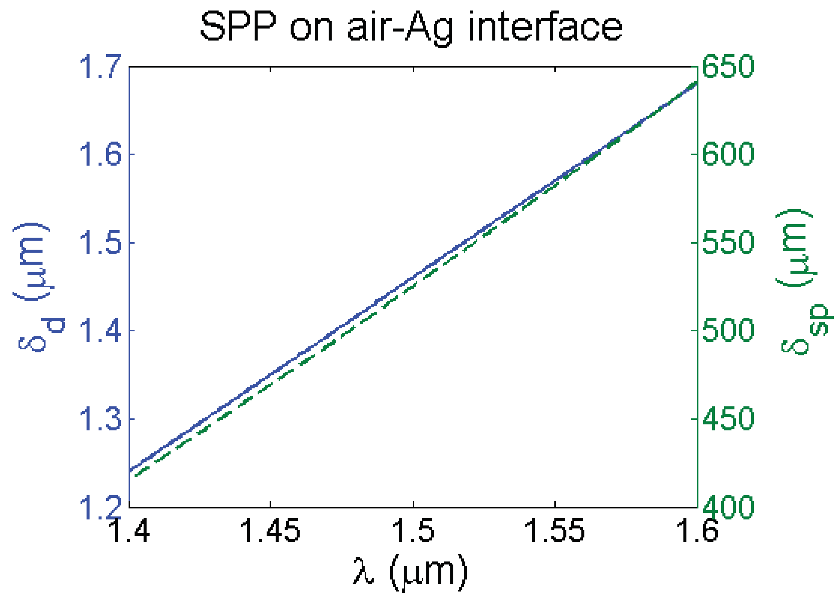
δ_{sp}: propagation length of SPPs → loss

- We expect long propagation length and tight field confinement.
- Therefore, δ_{sp}/δ_d is a key measure for plasmonic devices, which is expected to be as large as possible!

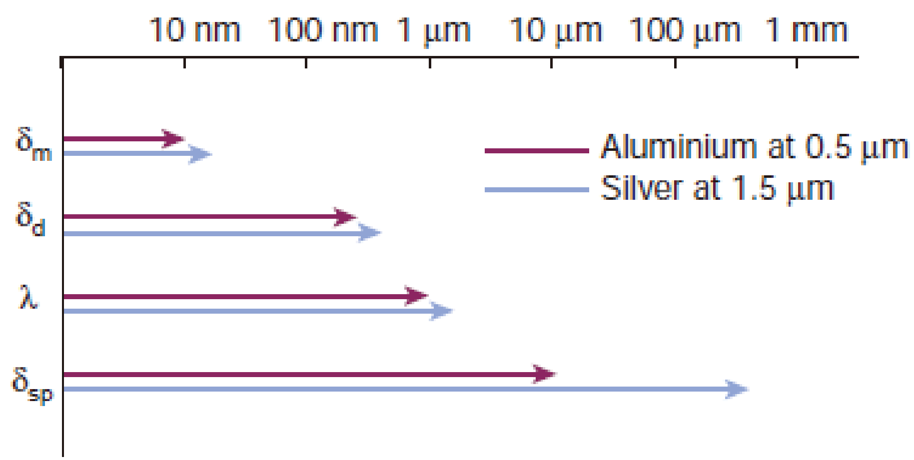
loss: 损耗， characteristic: 特征的

Numerical verification:

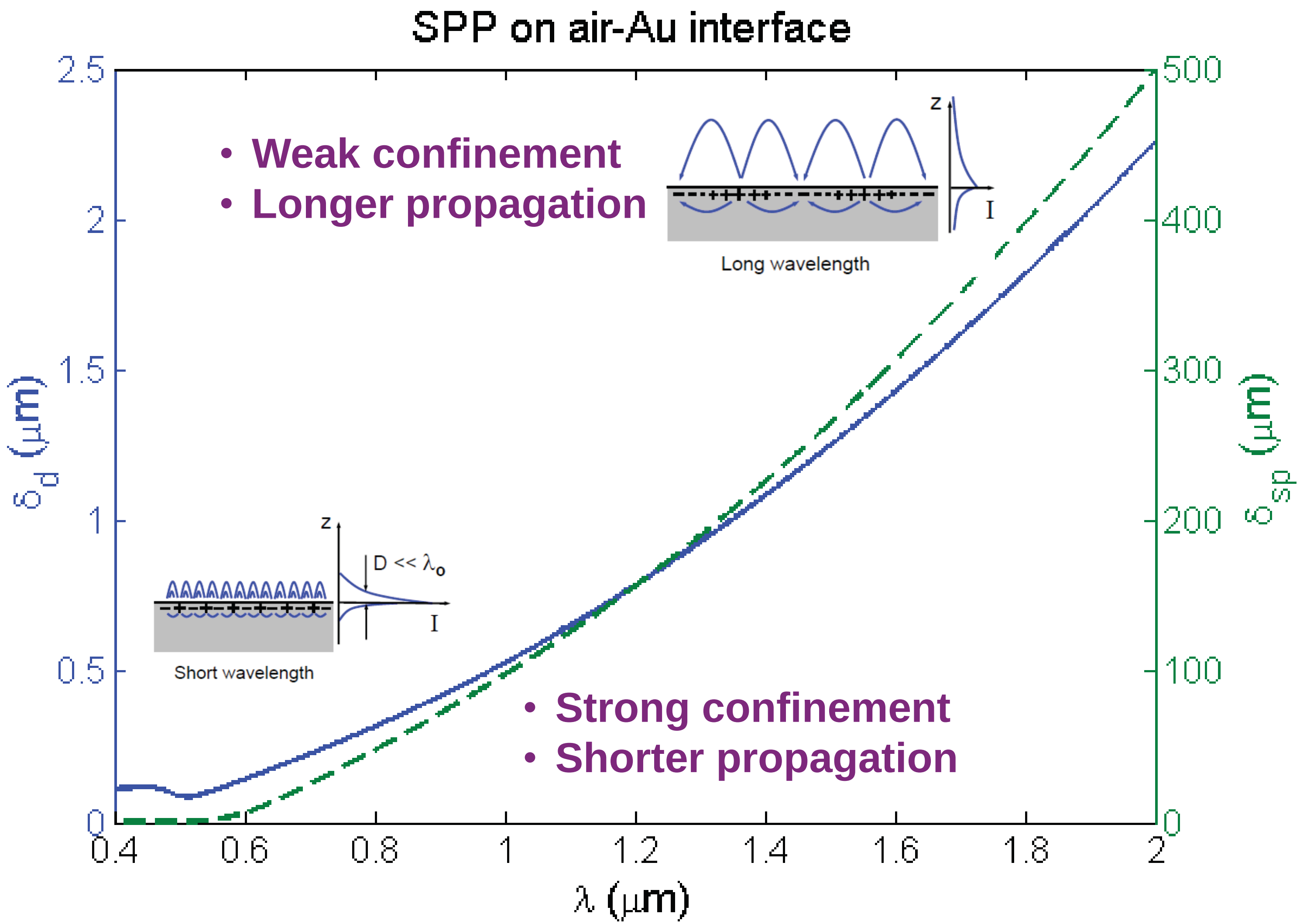




Understand the dependence of $\boldsymbol{\delta_m}$, $\boldsymbol{\delta_d}$, and $\boldsymbol{\delta_{sp}}$ on wavelength λ!

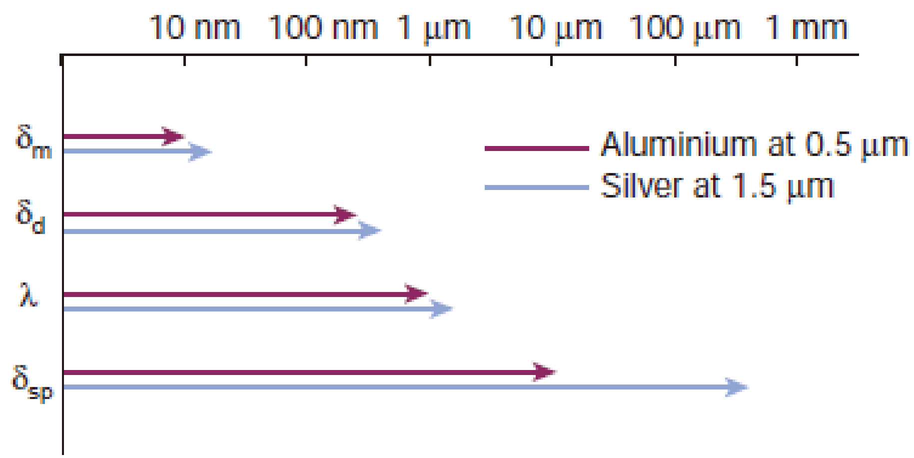


δ_d and δ_{sp} of SPPs on gold surface:



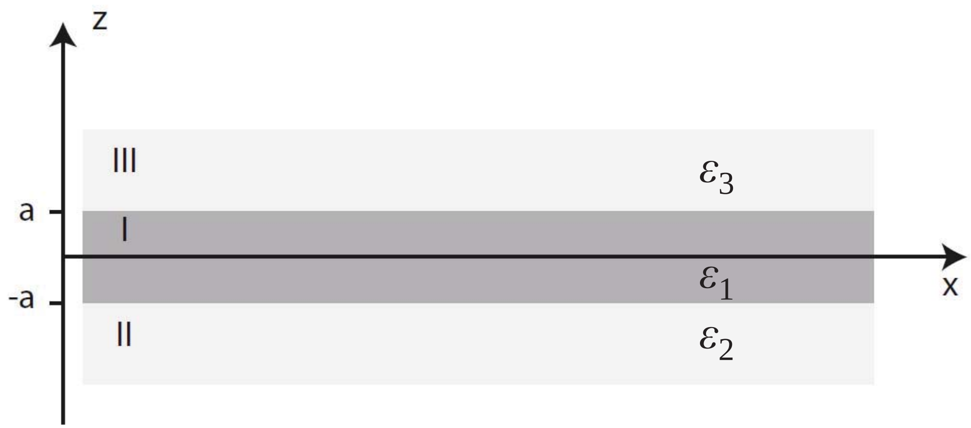
The trade-off between **confinement** and **loss** is typical for plasmonics!

trade-off: 折衷，平衡

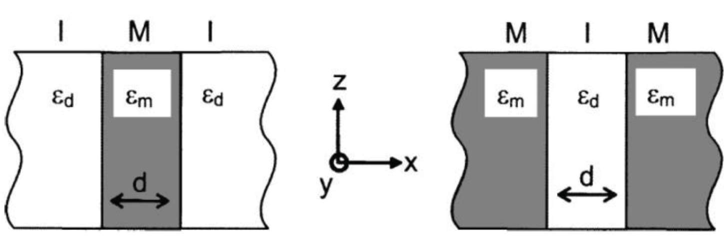


- The propagation length of SPP is on the order of µm at visible frequency.
- How to increase δ_{SP} while maintaining the confinement?
- This is crucial for many applications such as sensing and SPP waveguiding in plasmonic circuitry (cm scale δ_{SP} expected).
- Let's see SPPs in multilayer system…

2. SPPs in multilayer system (waveguide)

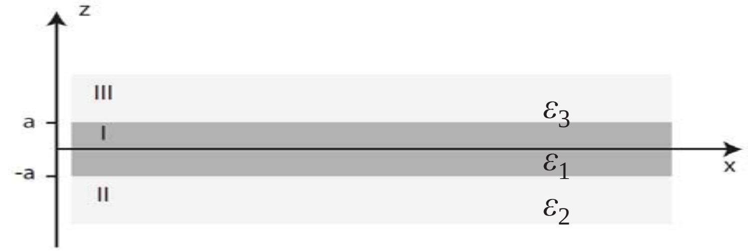


- Consist of alternating metal and insulator layers
- Each single interface can sustain SPPs
- When $2a < \delta_d$ (δ_m), interaction of SPPs → coupled SPP modes
- Two typical three-layer heterostructures:
  - Insulator-metal-insulator (IMI)
  - Metal-insulator-metal (MIM)



multilayer: 多层的，alternating: 交替的，coupled: 耦合的，insulator: 绝缘体，heterostructure: 复合结构

Dispersion of coupled SPP modes



For TM waves:

→ Write the expressions of H_y, E_x, E_z in three spatial regions

→ Match boundary conditions: continuity of H_y and E_x

→ Satisfy the wave equation in three regions:

$$\boxed{k_i^2 = \beta^2 - k_0^2 \varepsilon_i} \qquad k_i \equiv k_{z,i}$$

→ Dispersion relation linking β and ω:

$$\boxed{e^{-4k_1 a} = \frac{k_1/\varepsilon_1 + k_2/\varepsilon_2}{k_1/\varepsilon_1 - k_2/\varepsilon_2} \frac{k_1/\varepsilon_1 + k_3/\varepsilon_3}{k_1/\varepsilon_1 - k_3/\varepsilon_3}} \quad (*)$$

read the textbook and derive it by yourself

Note: for infinite thickness ($a \to \infty$), equation (*) reduces to $\frac{k_1}{k_{2,3}} = -\frac{\varepsilon_1}{\varepsilon_{2,3}}$

two uncoupled SPP modes at the respective interfaces

infinite: 无限的， continuity: 连续性

- Consider identical materials above and below the middle layer:

$$\varepsilon_2 = \varepsilon_3, \quad k_2 = k_3$$

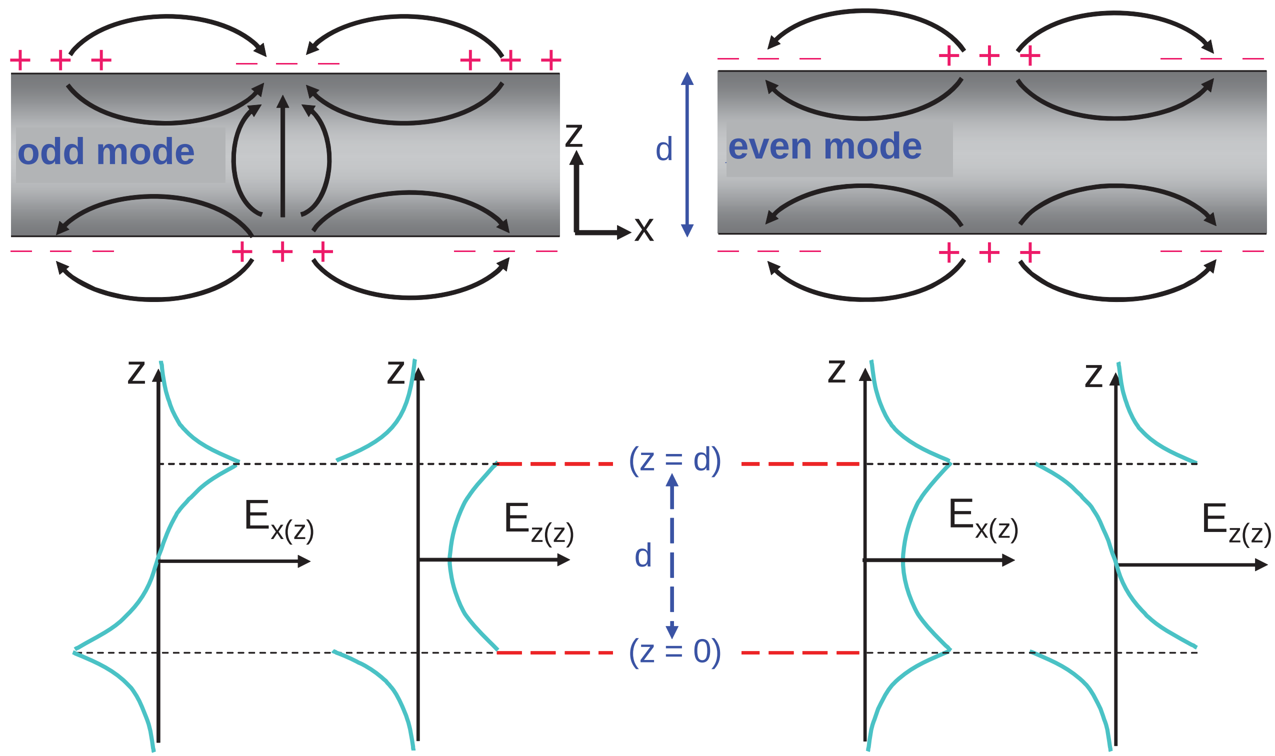
- Equation (*) splits into two equations:

$$\tanh k_1 a = -\frac{k_2 \varepsilon_1}{k_1 \varepsilon_2}$$

Odd modes
(E_x odd, H_y and E_z even)

$$\tanh k_1 a = -\frac{k_1 \varepsilon_2}{k_2 \varepsilon_1}$$

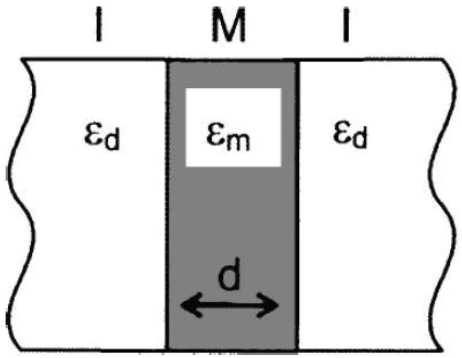
Even modes
(E_x even, H_y and E_z odd)



Do it by yourself:
read through pages 30-34 of the textbook and derive these equations

odd: 奇数的， even: 偶数的

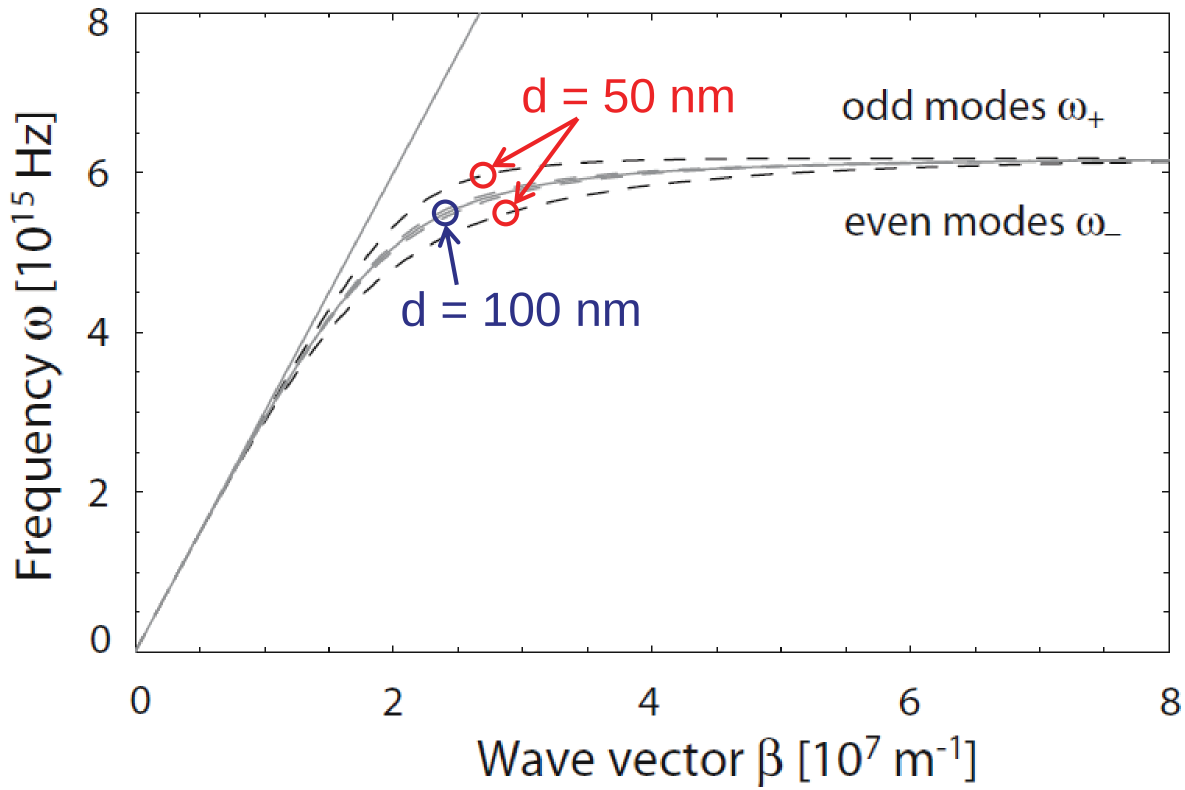
IMI geometry



$$\varepsilon_{\mathrm{d}} > 0,\ \ \varepsilon_{\mathrm{m}} = 1 - \frac{\omega_p^2}{\omega^2} < 0$$

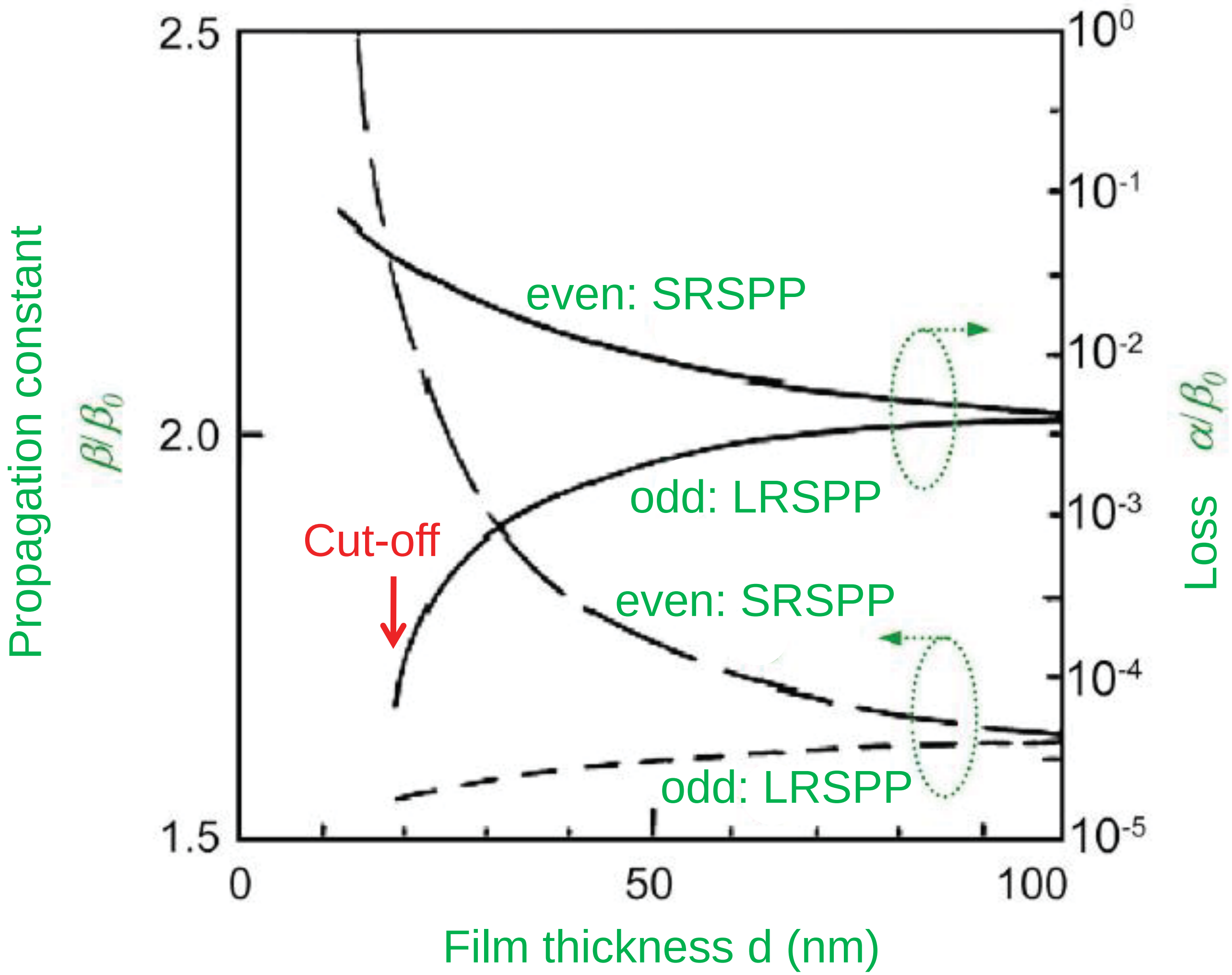
$$\Downarrow$$

$$\mathrm{Im}(\beta) = 0$$



- When $\beta \to \infty$: $\omega_+ = \dfrac{\omega_\mathrm{p}}{\sqrt{1+\varepsilon_2}}\sqrt{1 + \dfrac{2\varepsilon_2 \mathrm{e}^{-2\beta a}}{1+\varepsilon_2}}$ $\quad \omega_- = \dfrac{\omega_\mathrm{p}}{\sqrt{1+\varepsilon_2}}\sqrt{1 - \dfrac{2\varepsilon_2 \mathrm{e}^{-2\beta a}}{1+\varepsilon_2}}$
- When d decreases:
  - odd modes → closer to light line → $\delta_{\mathrm{SP}}\uparrow$, confinement↓ — long-range SPP (LRSPP)
  - even modes → farther from light line → $\delta_{\mathrm{SP}}\downarrow$, confinement↑ — short-range SPP (SRSPP)

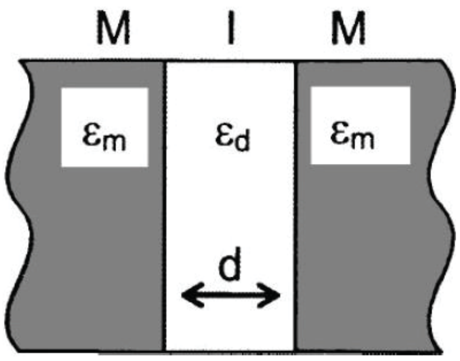
For asymmetric IMI structure ($\varepsilon_2 \neq \varepsilon_3$) at λ_0 = 632.8 nm:



- For asymmetric IMI, LRSPP is bound only for *d* above cut-off
- For symmetric IMI, there is no cut-off *d* (LRSPP is still bound for *d*→0)
- Symmetric IMI is preferred!
- LRSPP is preferred!

Further reading on LRSPP: Berini, Adv. Opt. Photon. **1**, 484 (2009).

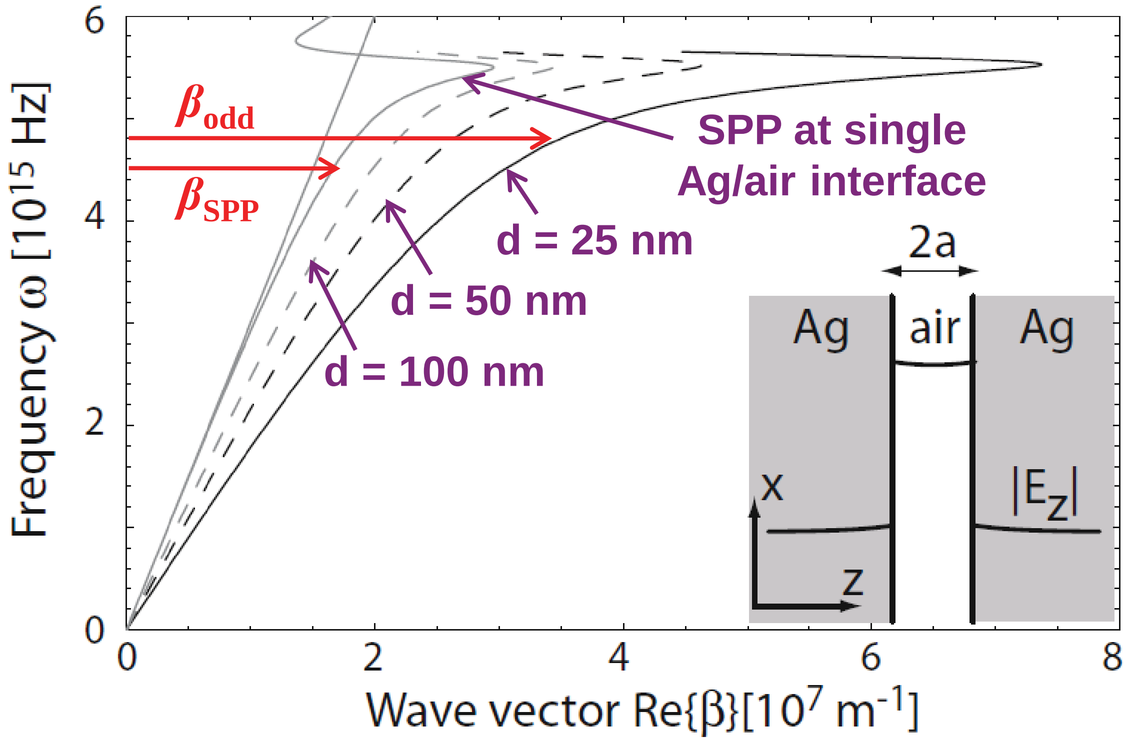
MIM geometry



$\varepsilon_d > 0$, real metal ε_m

$\mathrm{Im}(\beta) \neq 0$

Dispersion of the odd mode



- Usually only odd mode exists; even mode is not supported.

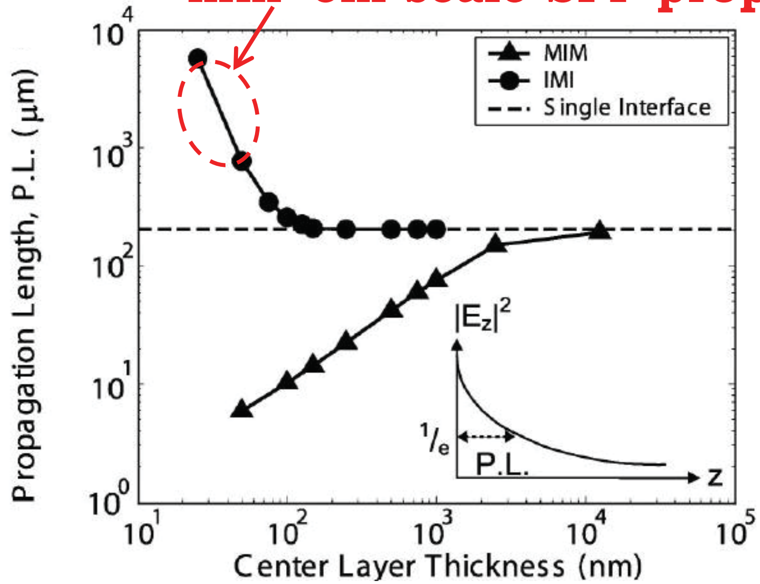
  For more details, see Prade et al., PRB **44**, 13556 (1991).

- $\beta_{odd} > \beta_{SPP} \rightarrow$ large β can be excited at lower $\omega \rightarrow$ better confinement

IMI vs. MIM

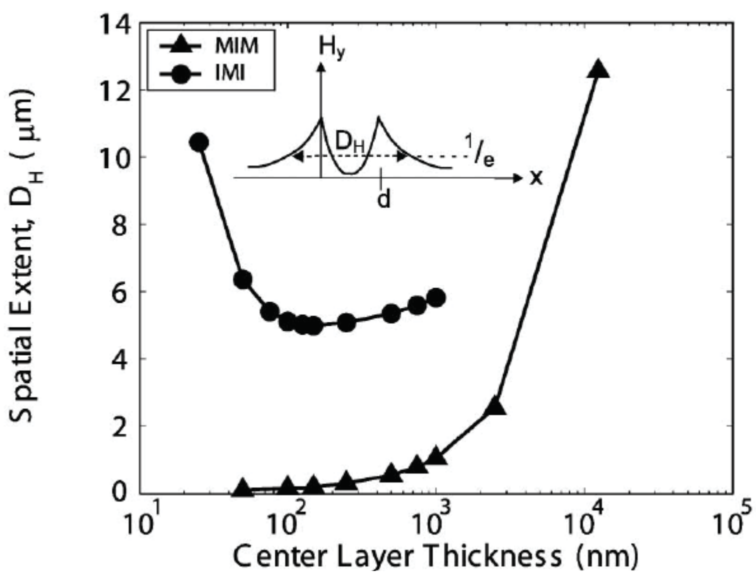
Au–air, λ = 1.55 μm

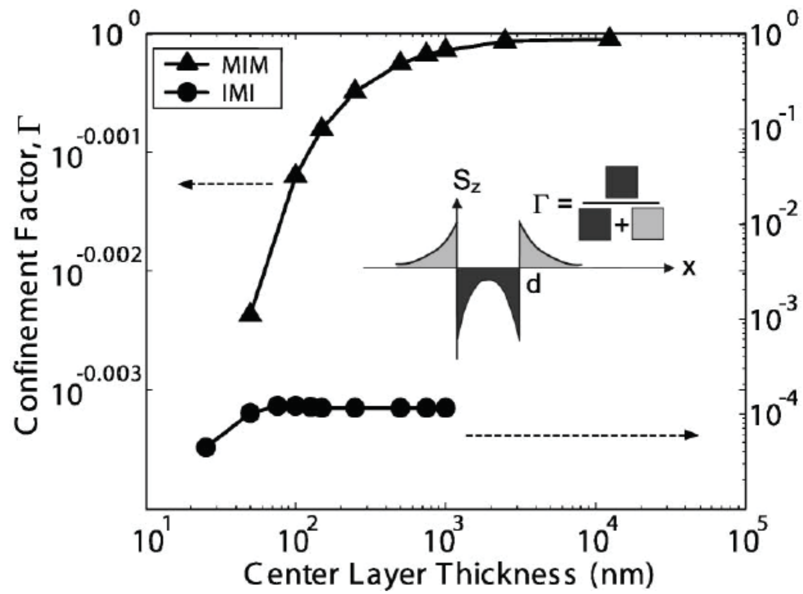
mm~cm scale SPP propagation!



Propagation length:
IMI > MIM

Confinement:
IMI < MIM





Zia et al., JOSA A **21**, 2442 (2004).

Summary

- Surface plasmons polaritons (SPPs):

  Confined surface wave; Transverse and longitudinal oscillations;

  TM excitation; Dispersion relation;

  Short-wavelength limit at ω_{sp}; Plasmon dispersion in full spectrum;

  Dispersion of real metals; SPP wavelength ("x-ray" at optical ω);

  Three characteristic lengths; Trade-off between localization and loss.

- SPPs in multilayer system:

  Dispersion relation of coupled SPP modes;

  IMI & MIM heterostructures;

  Odd & even modes (LRSPP & SRSPP);

  Properties of the coupled modes.