Lecture 13-0

# Ballistic Transport

A conductor will show **ohmic behavior** as long as its dimensions are much larger than

(1) the **electron de Brogilie wavelength**
(2) the **mean free path**
(3) the **phase-relaxation length**

**Mean-free path :** the average distance that an electron travel before it experiences elastic scattering which destroys its initial momentum.

**Phase-relaxation length:** the average distance that an electron travels before it experiences inelastic scattering which destroy its initial coherent state;

| | |
|---|---|
| specimen border | coherent |
| $e^-$-impurity | coherent |
| $e^-$-phonon-scatt. | incoherent |
| $e^-$-$e^-$-scatt. | incoherent |
| $\hookrightarrow$ dephasing | incoherent |

**Elastic scattering & Inelastic scattering**

(改变电子运动方向、电子能量保持不变，散射波与入射波之间的相位有确定的关系)

(电子在散射前后运动方向和能量都发生变化)

Datta, S. Electronic Transport in Mesoscopic Systems. Cambridge, (1995).

**Ballistic transport：** the transport of electrons in a medium having negligible electrical resistivity caused by scattering.

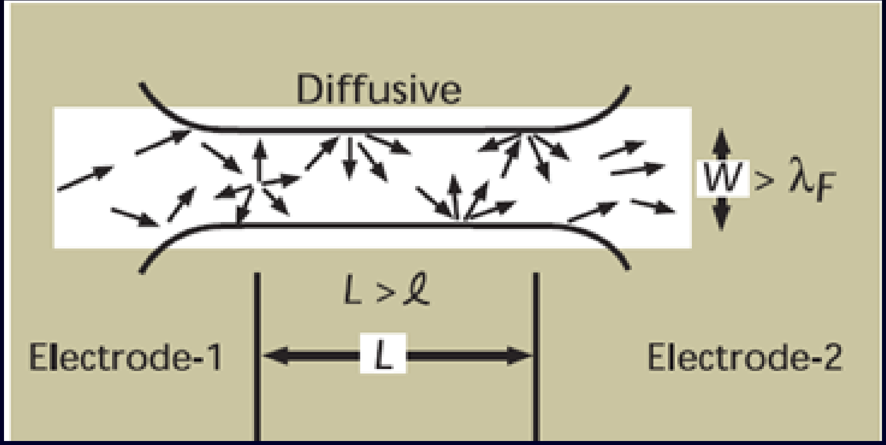
In general, the resistivity exists because an electron, while moving inside a medium, is scattered by impurities, defects, the atoms/molecules composing the medium that simply oscillate around their equilibrium position (in a solid).

Ballistic transport is observed when the mean free path of the electron is (much) longer than the dimension of the medium through which the electron travels.

If the sample length L larger than mean free path $L_m$

Diffusive transport:

$$L > L_m$$




扩散区局域电导率(Einstein relation)

$$\sigma = e^2 Z(E_F) D$$

$Z(E_F)$: 费米能级$E_F$处的能态密度；D：扩散系数

电子散射的平均自由程和体系尺度相比甚小，电子会与无序分布的杂质发生散射，路径无规则

介观体系中的普适电导涨落(UCF)发生在量子的扩散区

当电子平均自由程大于或接近体系的尺度相当时，进入弹道输运区，限制电流大小的是样品的边界散射，杂质散射可以忽略。

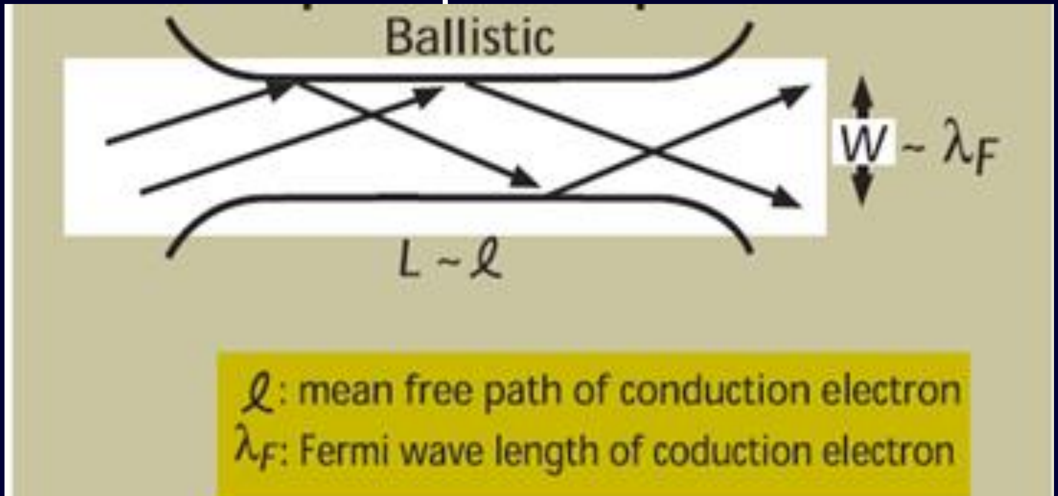
电子输运行为类似于波导

## $L << L_m$: Ballistic transport

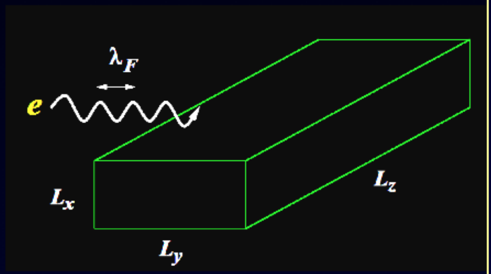
Electrons moves elastically from one contact to the other, or are scattered back to the source

## $L \sim L_m$: Quasi-ballistic transport

This regime exhibits features of both diffusive and ballistic transport.



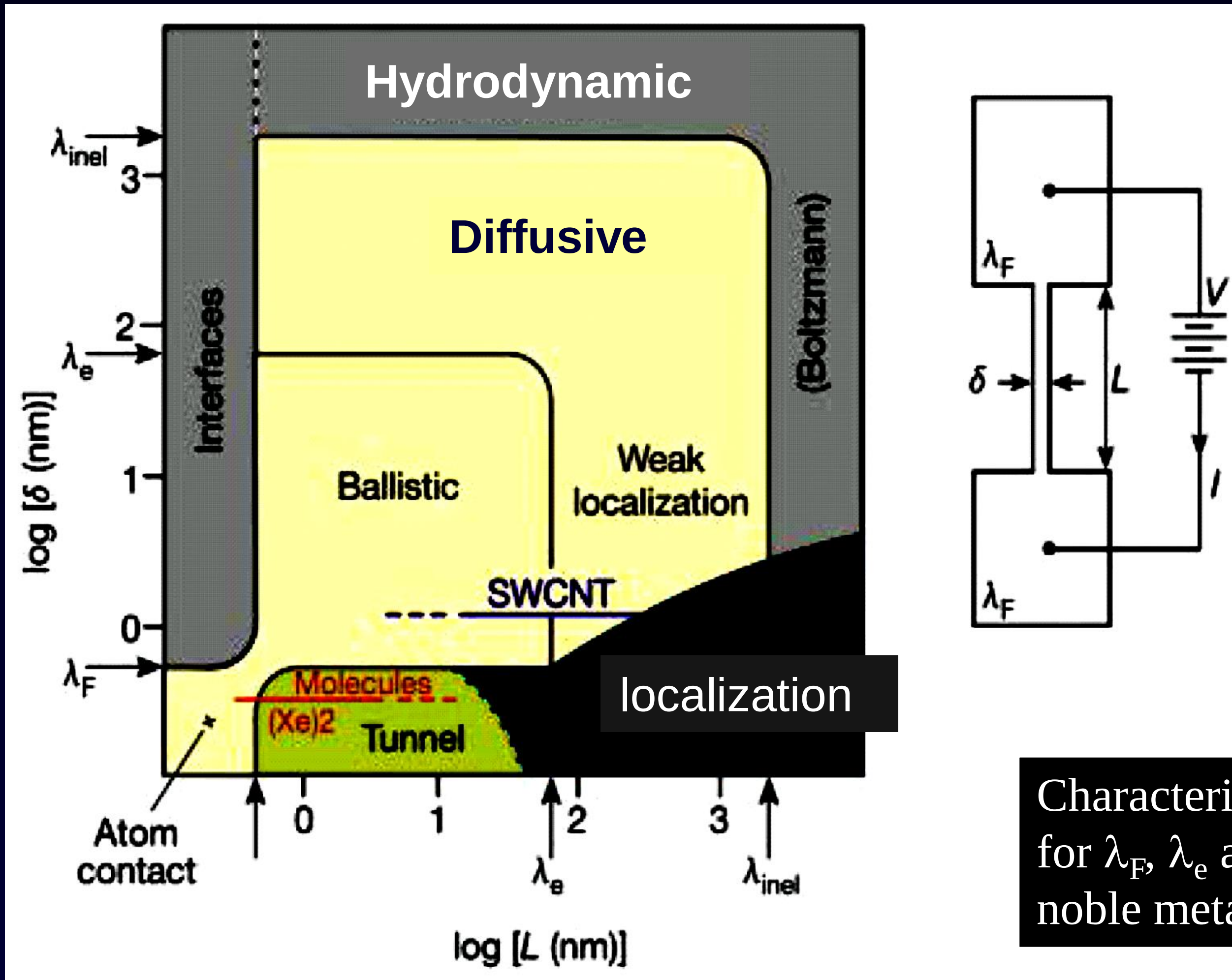

**quasi-1D: $W \sim \lambda_F$ (Fermiwavelength)**




1-D structures

$L_x < L_y < \lambda_F << L_z$

( If $L_x < L_y < L_z$ )




Characteristic orders of magnitude for $\lambda_F$, $\lambda_e$ and $\lambda_{intel}$ are taken for noble metals at low temperature.

**Regimes of electronic transport as a function of the wire width d and length L**

$\lambda_F$-de Broglie carrier wavelength in the contact electrodes (away from the constriction)

**$\lambda_e$-elastic mean free path in the wire**

**$\lambda_{intel}$-inelastic mean free path in the wire**

## Ballistic transport regime：(Landauer formula )

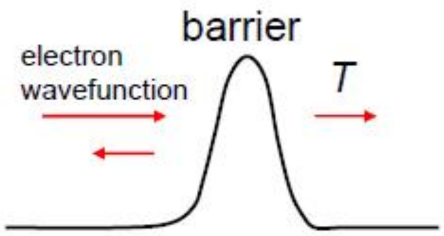
$$G = \frac{2e^2}{h}\sum_{n=1}^{N} T_n$$

N: 许可的传播模式数量或通道数

$T_n$: 第n个电导通道的电子穿透概率

$$T_n = \sum_{m=1}^{N} \left| t_{nm} \right|^2$$

$t_{nm}$：从第m个模式过渡到第n个模式穿透几率幅




Rolf Landauer (1927-1999);
- *G* controversial issue in 80'ies

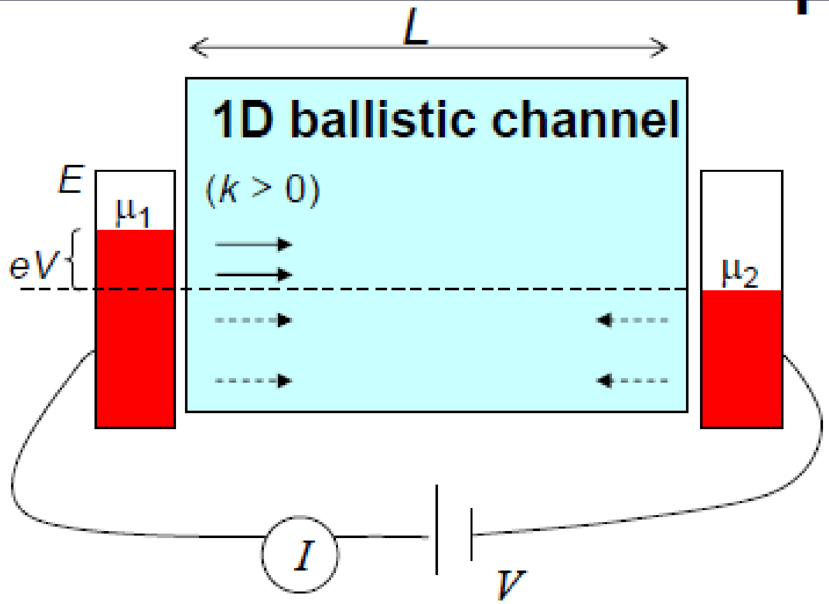
**Landauer formula:**

$$G = \frac{2e^2}{h} T \quad \text{(transmission probability } T\text{)}$$

$$\boxed{\frac{2e^2}{h} = \frac{1}{12.9k\Omega}}$$

where e: the electron charge;
h: Planck constant.

# Conductance of 1D quantum wire:




Contacts: 'Ideal reservoirs'

Chemical potential $\mu \sim E_F$ (Fermi level)

Channel: 1D, ballistic (transport without scattering)

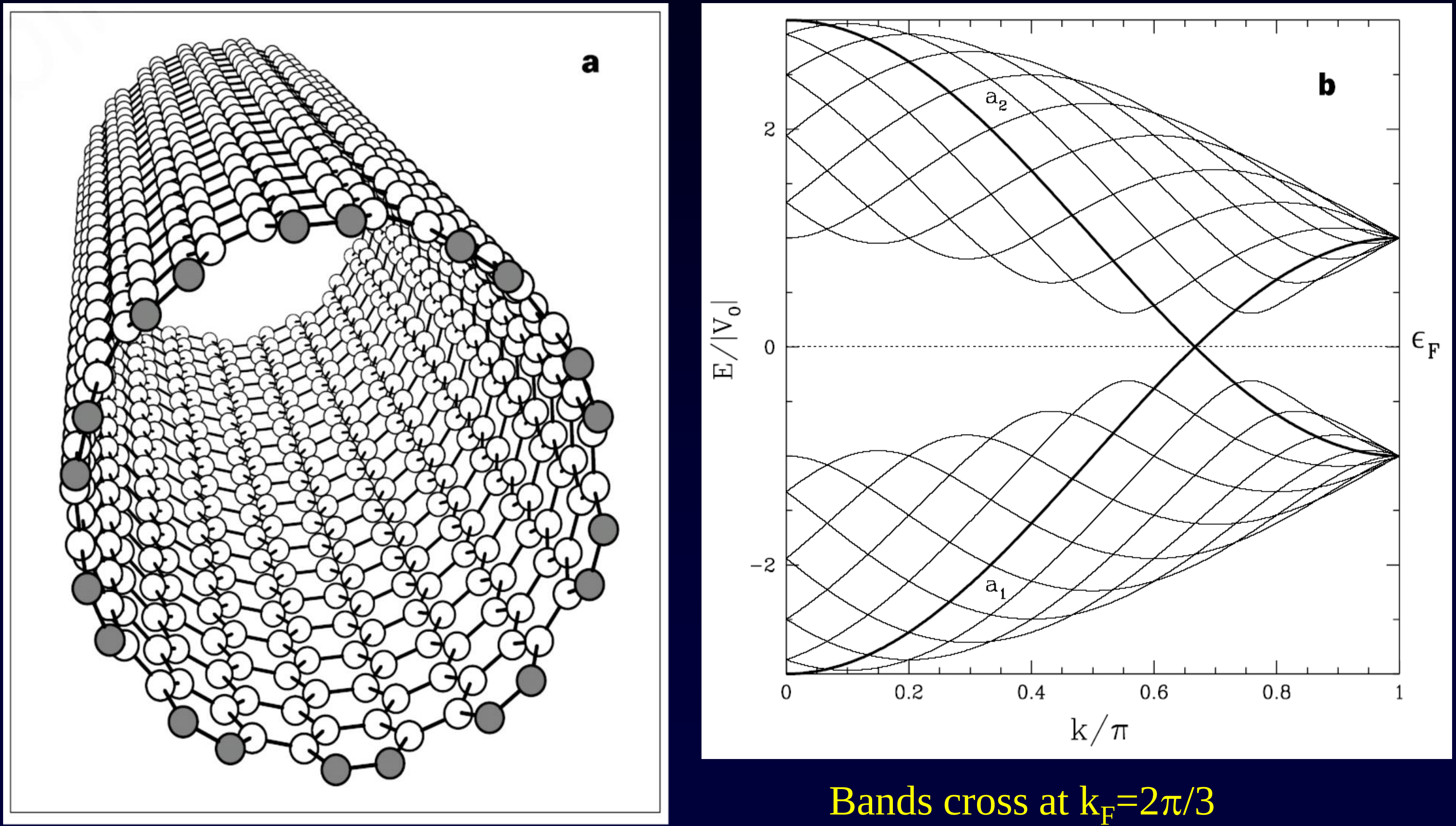
$$I = \int_{\mu_1}^{\mu_2} e \overset{\text{velocity}}{v(E)} \left( \underset{\text{spin}}{2} \overbrace{\underset{k>0}{\frac{1}{2}} g_{1D}(E)}^{\text{1D density}} \right) \mathrm{d}E = \int_{\mu_1}^{\mu_2} e v(E) \left( \frac{2}{h v(E)} \right) \mathrm{d}E$$

$$= \frac{2e}{h}(\mu_2 - \mu_1) = \frac{2e}{h}(eV)$$

$$\boxed{G = I/V = \frac{2e^2}{h}}$$

Conductance is fixed, regardless of length $L$, no well defined conductivity $\sigma$

# Ex. 1: Carbon nanotubes as long ballistic conductors



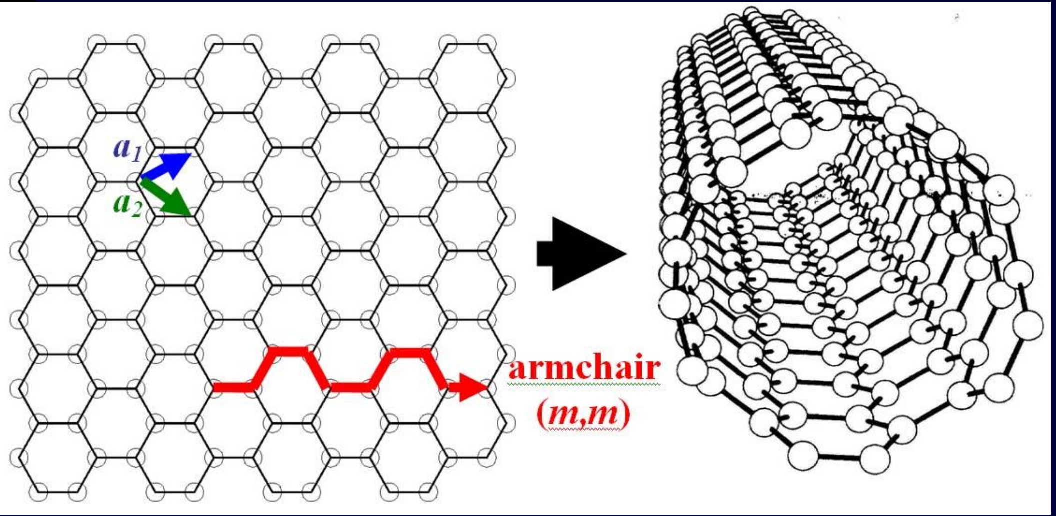

Bands cross at $k_F=2\pi/3$

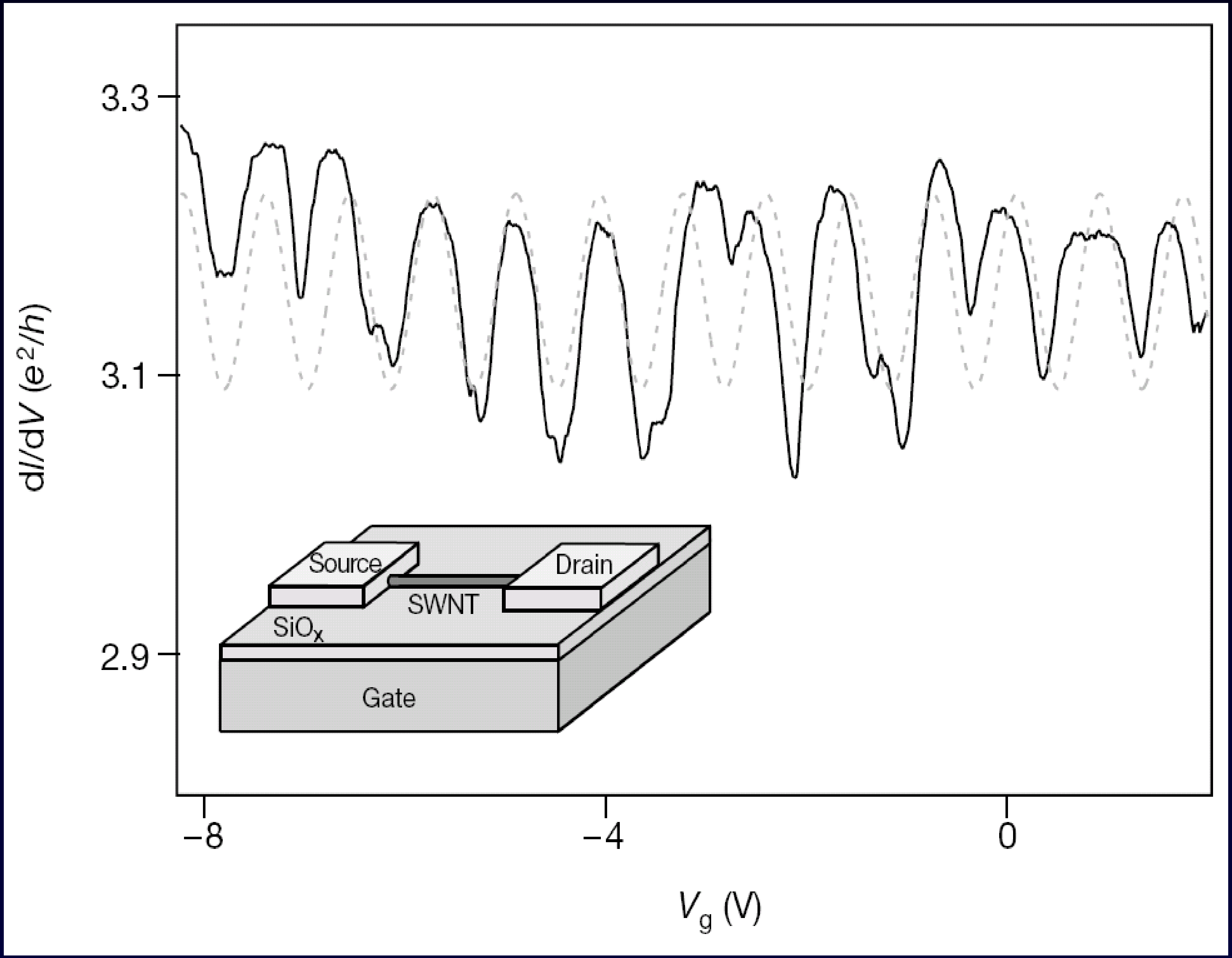
**Geometry and band structure of the [10,10] armchair nanotube.**

Unlike normal metallic wires, conduction electrons in armchair nanotubes experience an effective disorder averaged over the tube's circumference, leading to electron mean free paths that increase with nanotube diameter.

This increase should result in exceptional ballistic transport properties and localization lengths of 10μm or more for tubes with the diameters that are typically produced experimentally.

碳纳米管对无序的敏感度是很弱的，并不像单列原子中局部缺陷对局域化那样有明显影响，此源于其分子结构及相关的电子结构(仅两个子带)；因碳纳米管的环状结构，电子波函数即使受到无序影响，可平均到整个圆周上，同样的缺陷对碳纳米管的波函数影响并不明显






Zero-bias differential conductance of a 200-nm SWNT device plotted against gate voltage. Isolated SWNTs were synthesized on a degenerately doped silicon wafer with a 1-μm oxide layer by chemical vapour deposition. (Au/Cr electrodes)

In the absence of disorder, symmetry can be used to block diagonalize the full model to the point that the $a_1$ and $a_2$ bands are described exactly (within an unimportant constant) by the hamiltonian:

$$H_0 = V_0 \sum_{j=1}^{2} (-1)^{(j+1)} \left[ \sum_{m} \left[ |a_{jm}\rangle\langle a_{jm}| + \left( |a_{jm}\rangle\langle a_{j(m+1)}| + \mathrm{H.c.} \right) \right] \right] \quad (1)$$

In its most general form $H$ is given by:

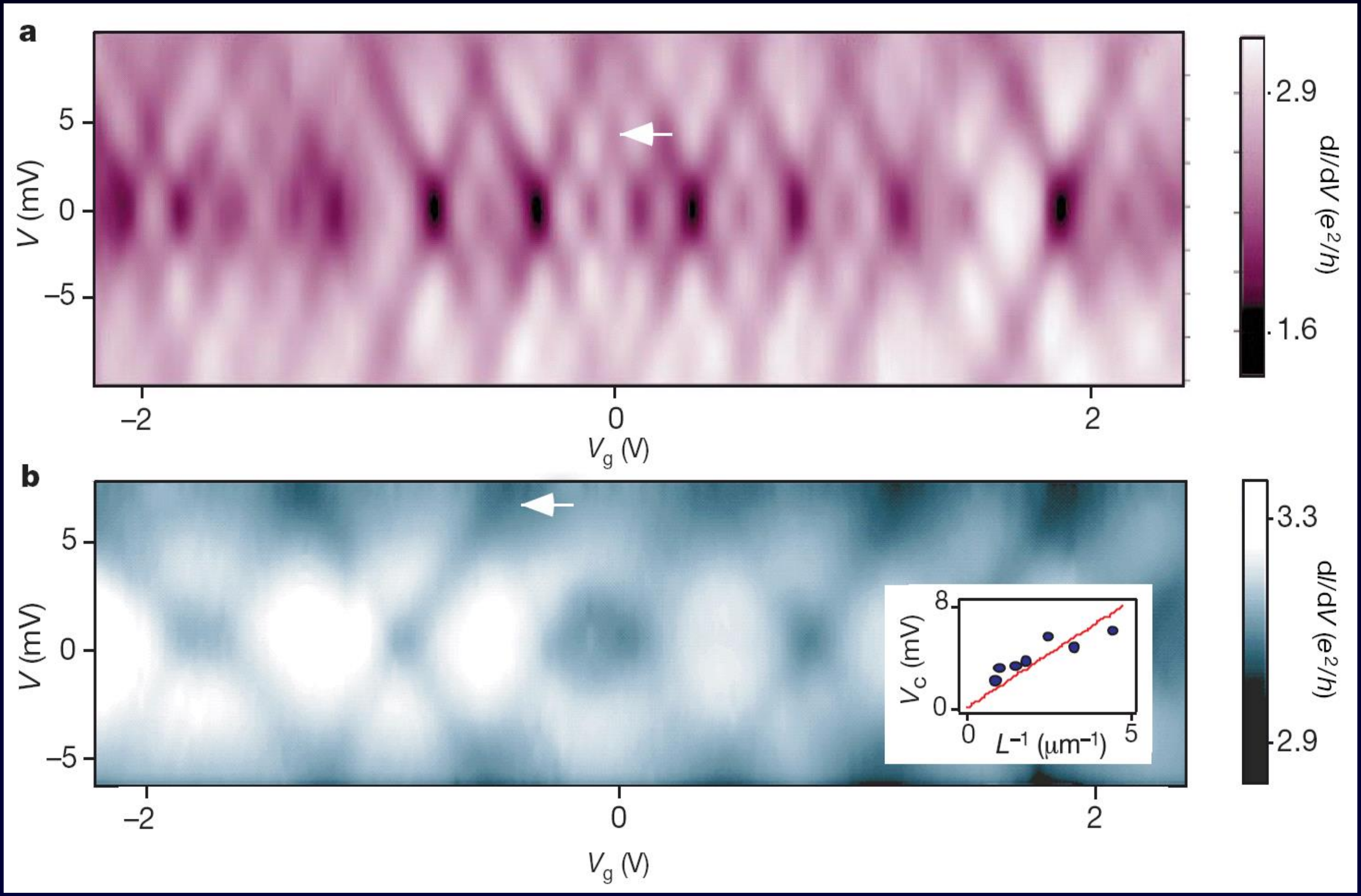
$$\begin{aligned} H = \sum_{m} \Bigg\{ \sum_{j=1}^{2} \Big\{ & (\bar{\epsilon}_m + (-1)^{(j+1)} \widehat{V}_m) |a_{jm}\rangle\langle a_{jm}| \\ & + [(-1)^{(j+1)} \overline{V}_m |a_{jm}\rangle\langle a_{j(m+1)}| + \mathrm{H.c.}] \Big\} \\ & + [\tilde{\epsilon} |a_{1m}\rangle\langle a_{2m}| + \widetilde{V}_m (|a_{1m}\rangle\langle a_{2(m+1)}| - |a_{2m}\rangle\langle a_{1(m+1)}|) + H.c.] \Bigg\} \quad (2) \end{aligned}$$

For the two-band model in the weak scattering limit $l$ :

$$l = \frac{6V_0^2}{(2\sigma_\epsilon^2 + 9\sigma_V^2)} N_{\mathrm{B}}. \quad (3)$$

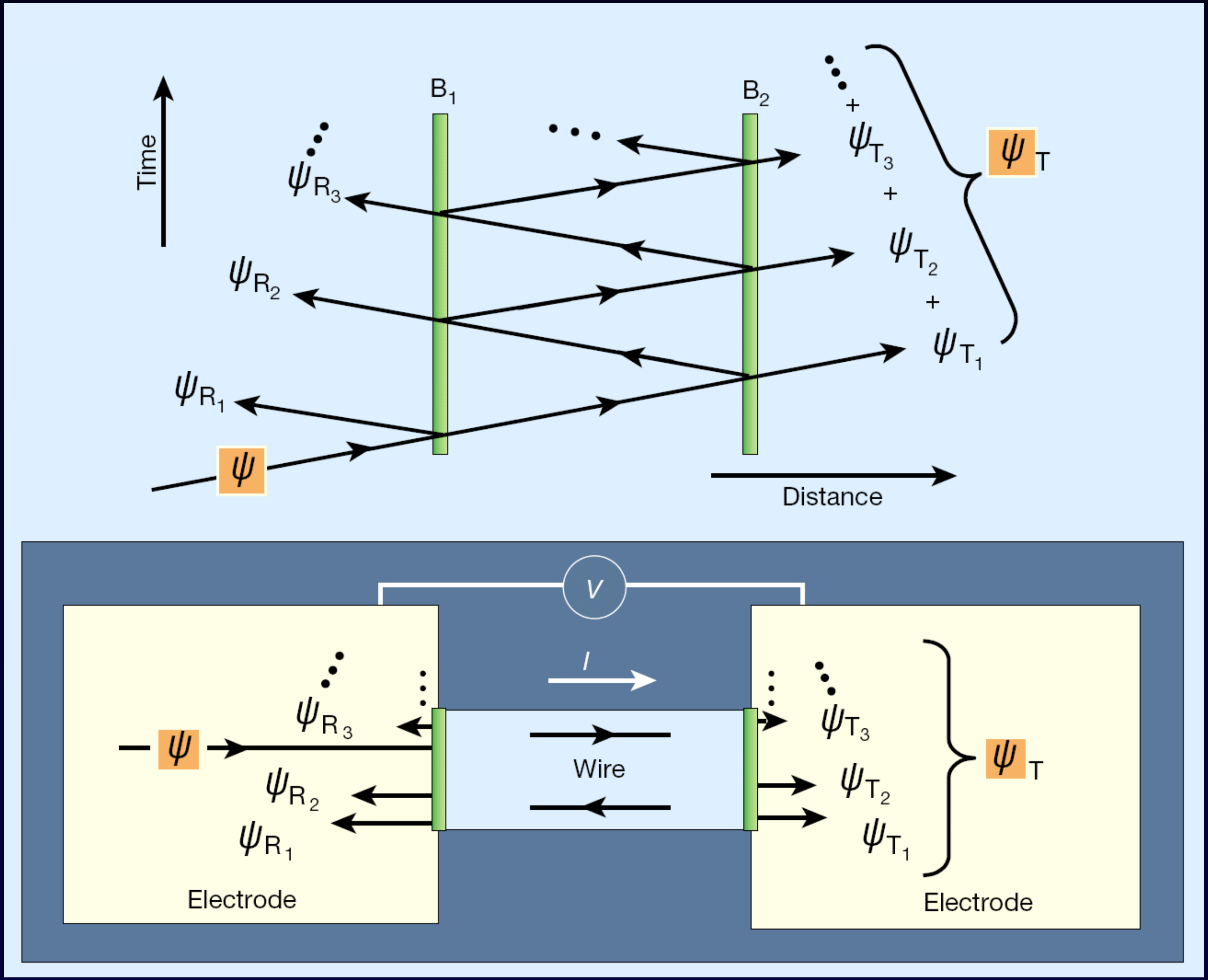
**Equation (3) predicts that weakly disordered small-diameter armchair tubules differ fundamentally from normal metallic wires** which have electron mean free paths along the wire independent of the wire's transverse size.

# Nanotubes go ballistic *(largely free from scattering )*

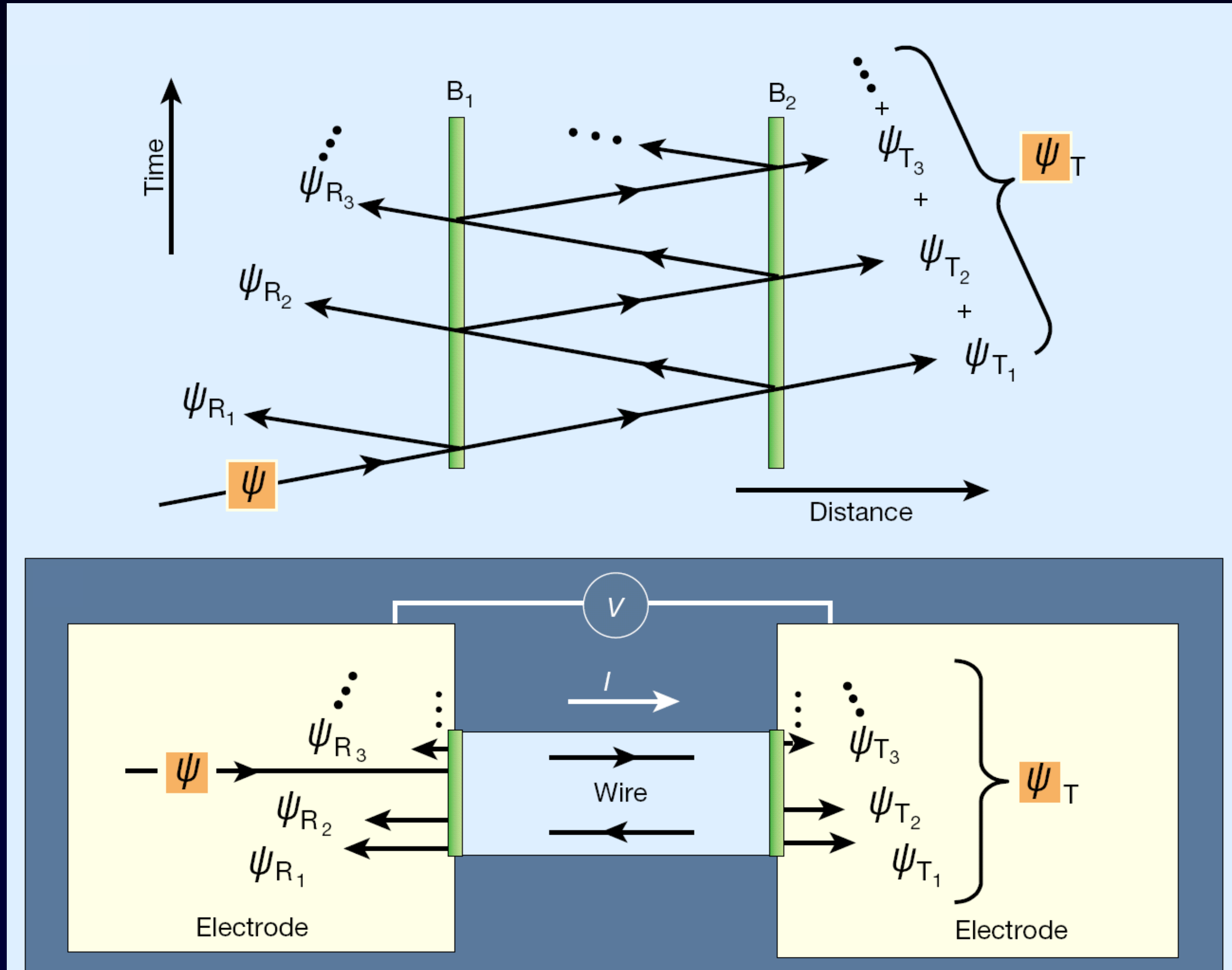



**Two-dimensional ∂I/ ∂V plots as a function of V and Vg measured at 4 K**. [Data from a 530-nm (a) and 220-nm (b) SWNT device]

Both plots show a quasi-periodic pattern of crisscrossing dark lines that correspond to the ∂I/∂V dips as V and Vg are varied



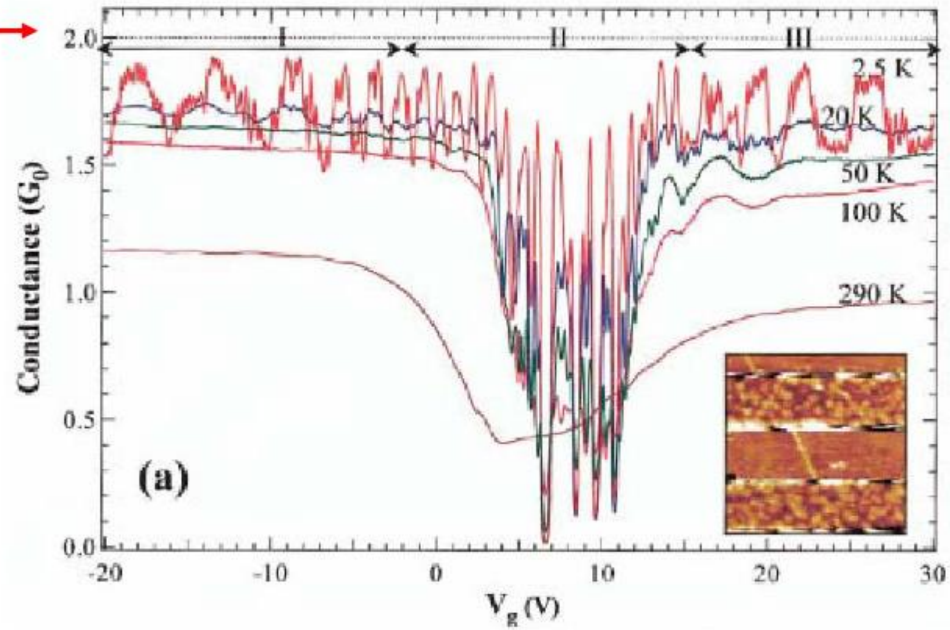

Each internal reflection reduces the amplitude of the wave by a fixed fractional amount and causes a fixed change in phase.




The round trip between the barriers adds a further phase change of $2\pi L/\lambda$ (where $L$: the length of the round trip and $\lambda$: the wavelength of the wave between $B_1$ and $B_2$). The total transmitted wave, T, is a periodic oscillatory function of $1/\lambda$ , with a period determined by $L$.

# Ballistic transport in metal tubes

$4e^2/h$ ✓



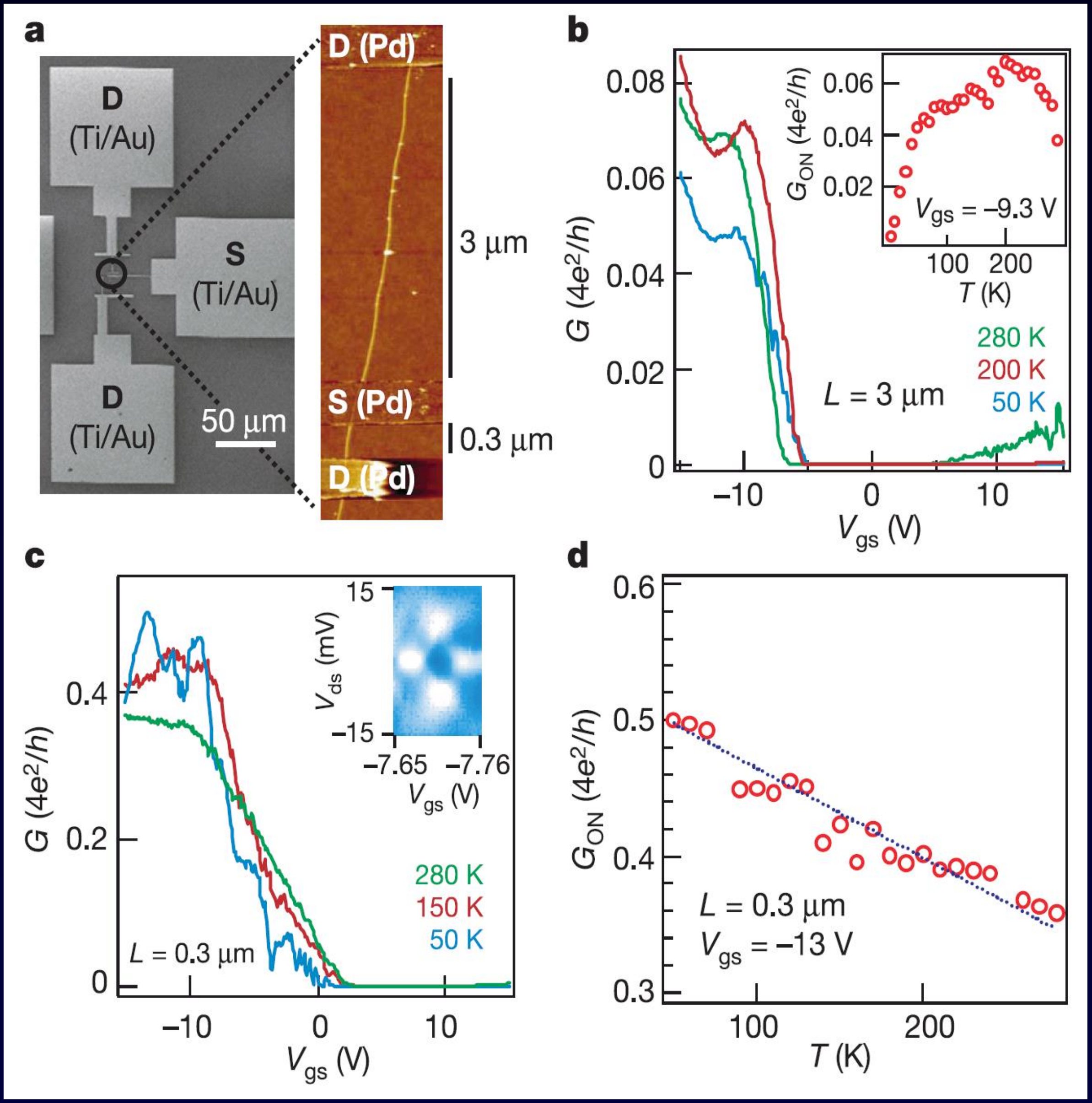

1D conductor

- Near the theoretical limit $4e^2/h$ (with two subbands)
- Close to Fermi level backscattering is suppressed in armchair (metal) tubes by symmetry

(Ballistic transport also possible in very short semiconducting tubes, otherwise mostly diffusive)

Kong et al, PRL 87, 106801 (2001)
McEuen et al, PRL 83, 5098 (1999)

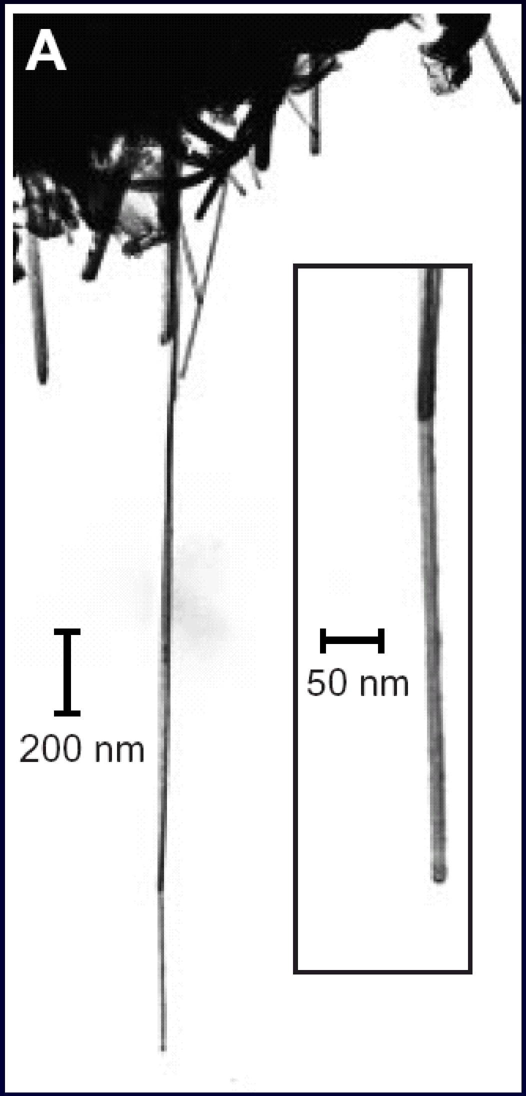
# Ballistic carbon nanotube field-effect transistors

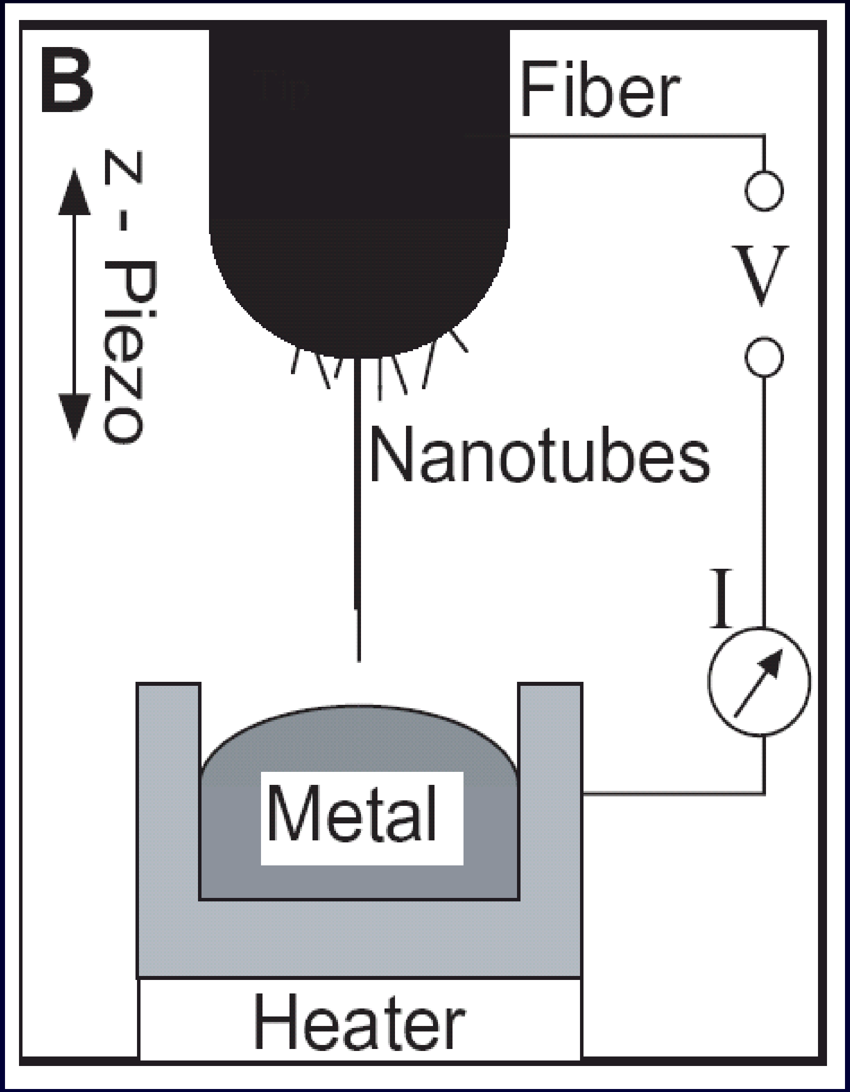



Pd, a noble metal with high work function and good wetting interactions with nanotubes, greatly reduces or eliminates the barriers for transport through the valence band of nanotubes.

Nature 424, 654(2003)

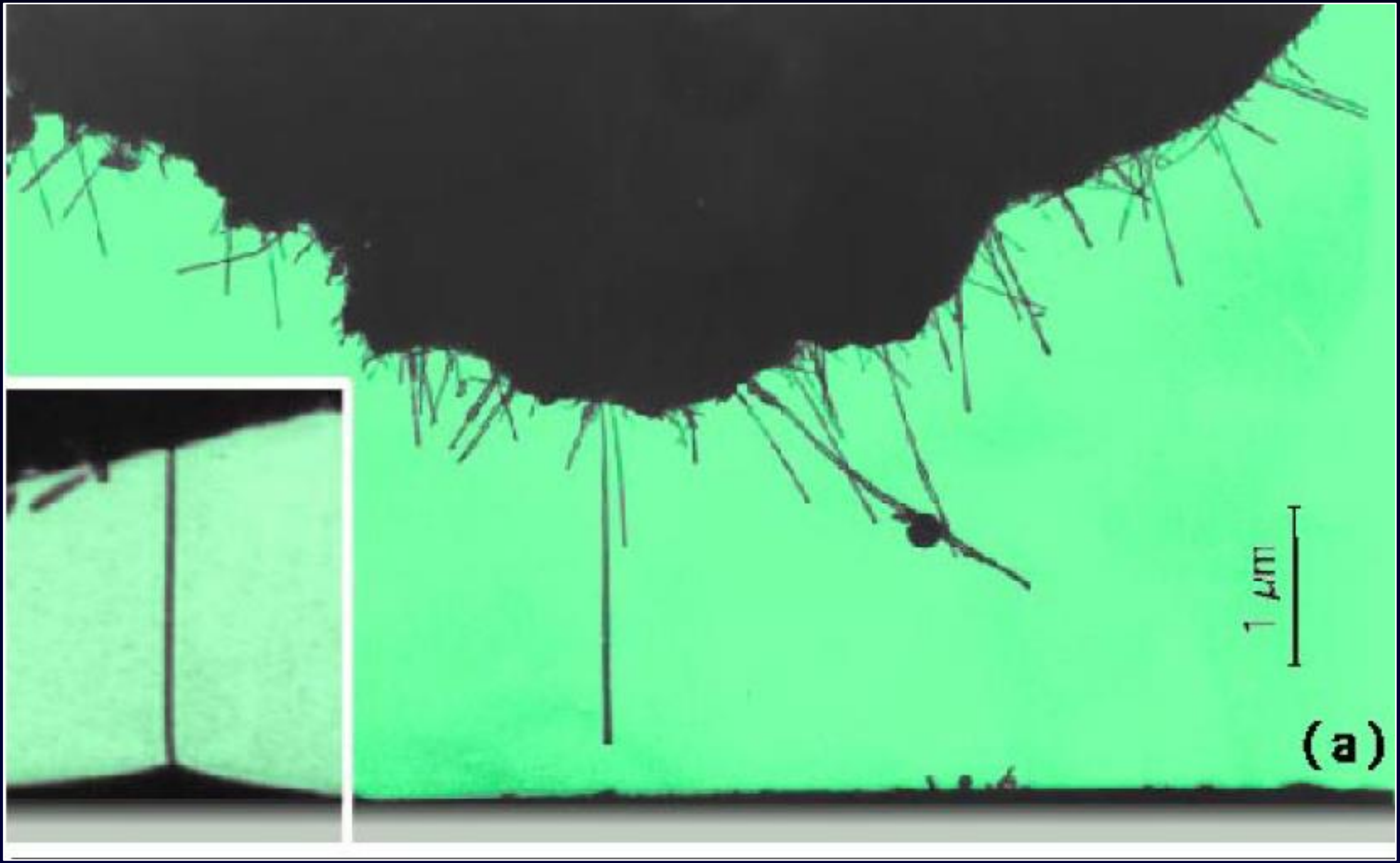
# Ex. 2: Room temperature ballistic conduction in Carbon Nanotube



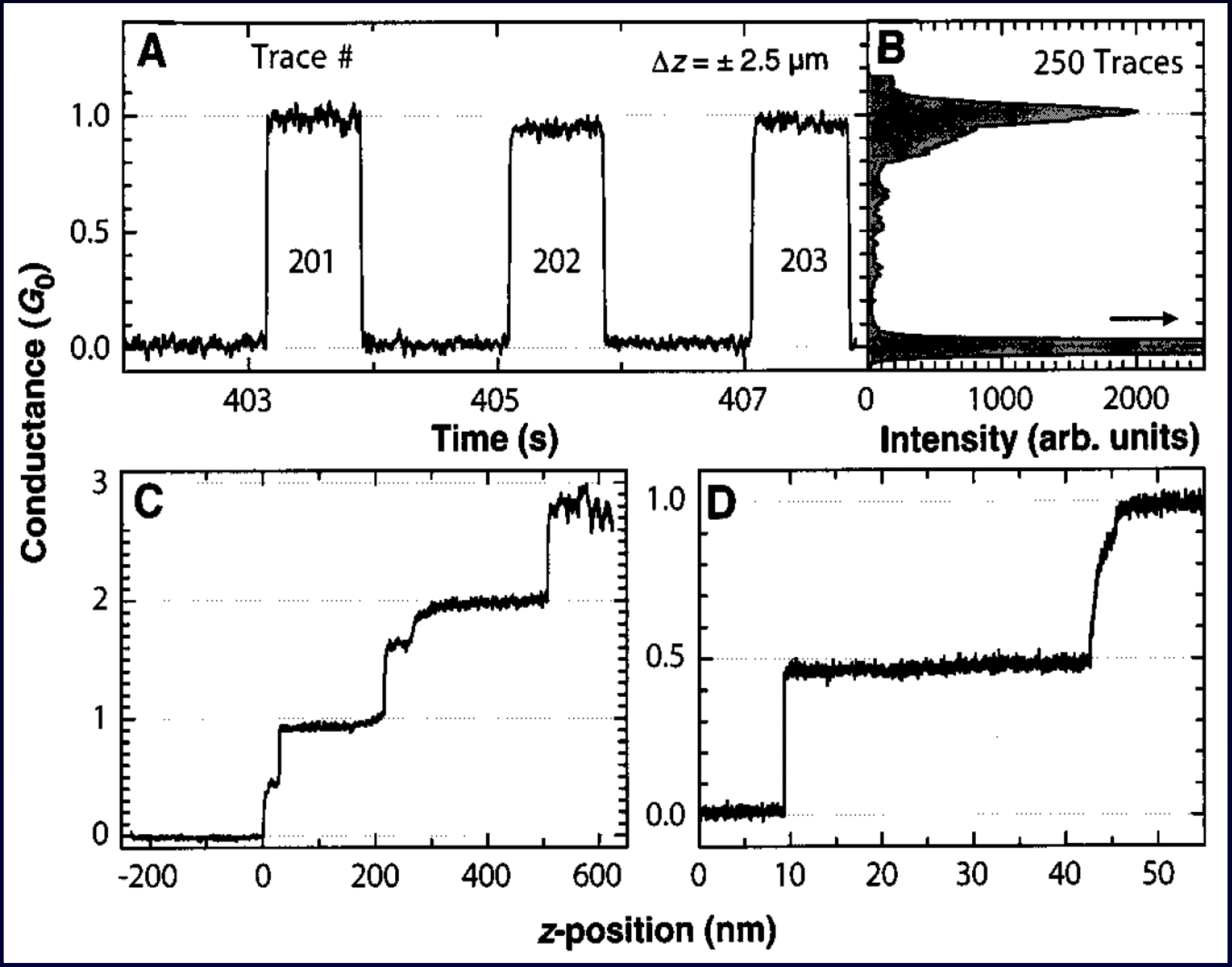




Liquid metal (Mercury)

# Room temperature ballistic conduction in Carbon Nanotube




J. Phys. Chem. B 2002, 106, 12104-12118
*Science* 280 (5370), 1744-1746.

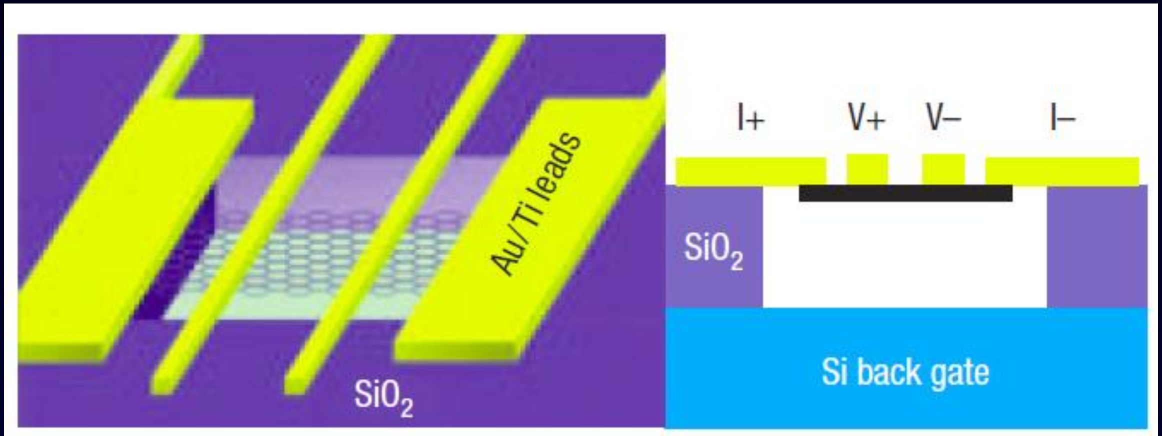


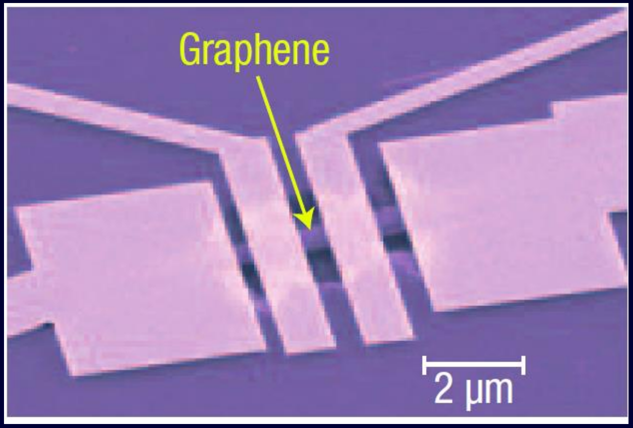

Conductance changes in units of $2e^2/h$ (12.9 kilohms)

**If $L_e$>L, a two channels ballistic conductor $G=4e^2/h$**

*Science* **280** (5370), 1744-1746.

# Ex. 3: Approaching ballistic transport in suspended graphene

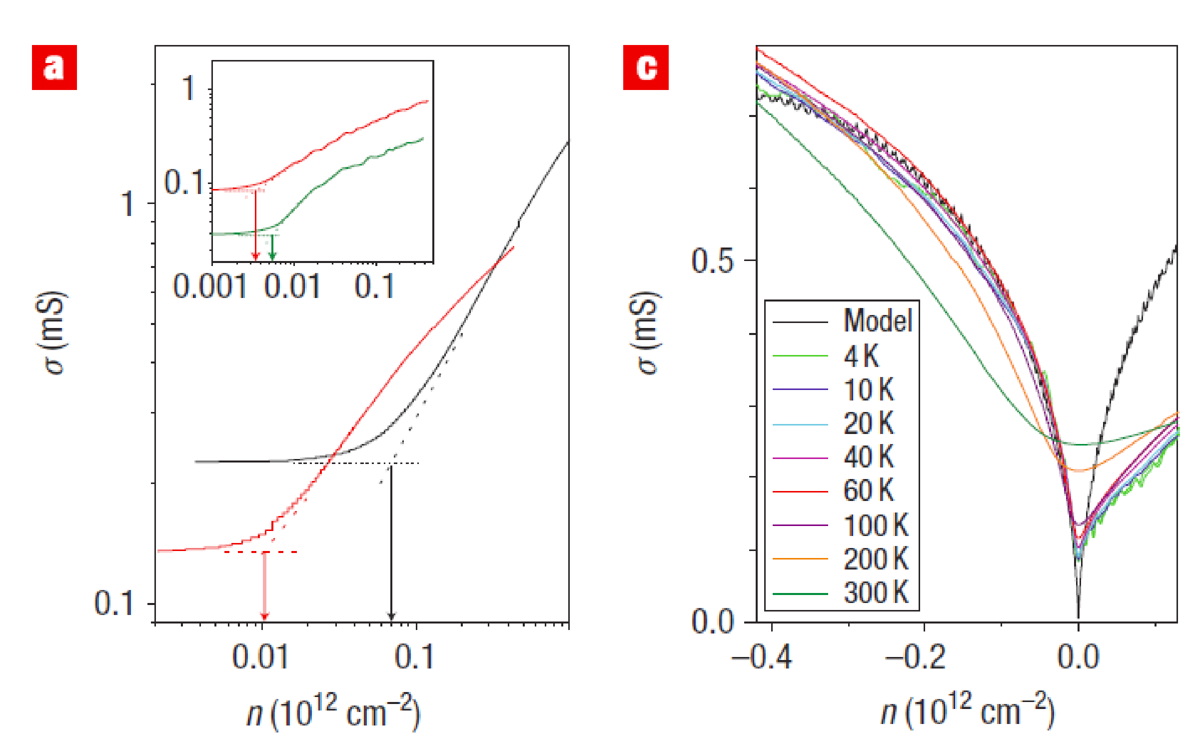



Suspended graphene (SG)
sample characterization

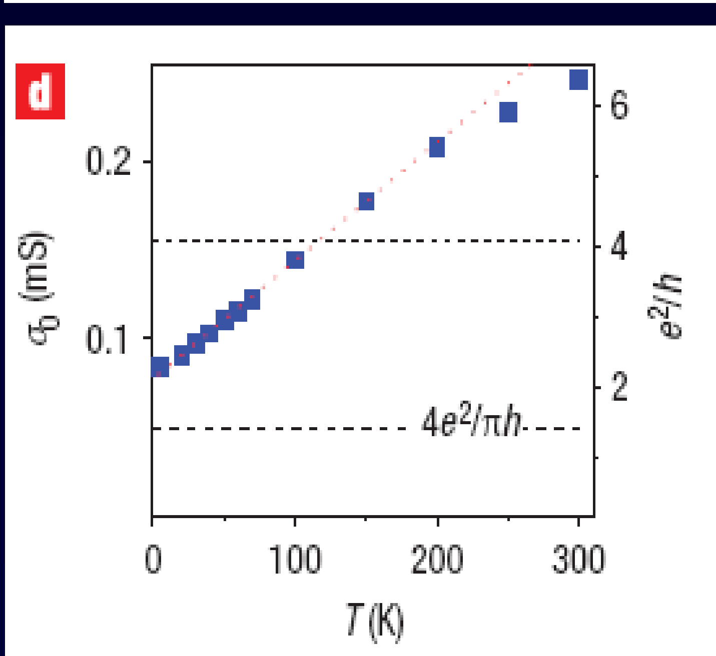
non-suspended graphene (NSG)

Nature Nanotechnology 3, 491(2008)




Comparison of the carrier density dependence of conductivity for SG (red curve) and NSG (black curve) devices

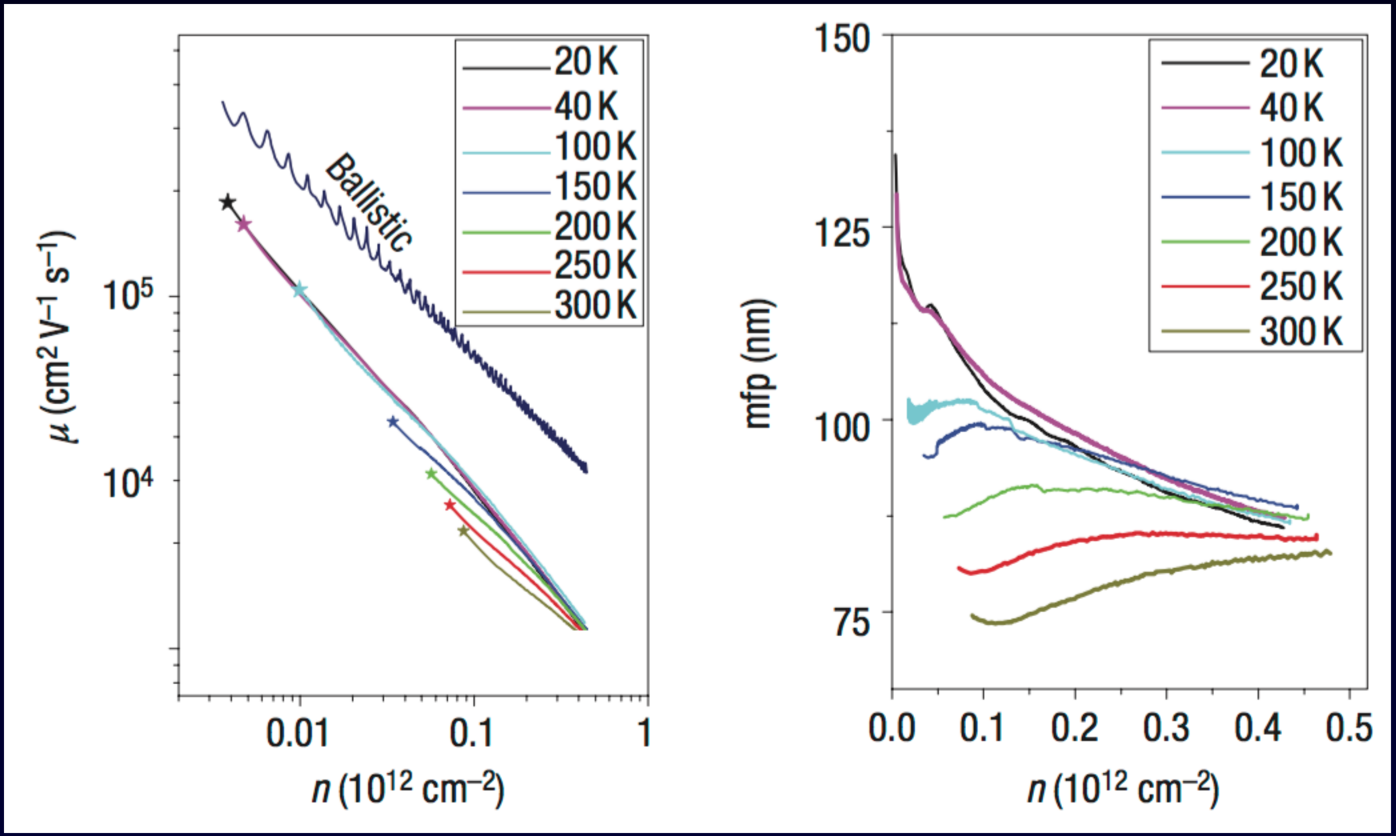
Temperature dependence of the charge inhomogeneity for two SG samples and one NSG sample




Close to the Dirac point the minimum conductivity $\sigma_0$ is roughly linear in T for T <100 K with a finite intercept at T =0

$$\sigma_0 = 84\,\mu\mathrm{S} = 1.7\,\frac{4e^2}{\pi h}$$

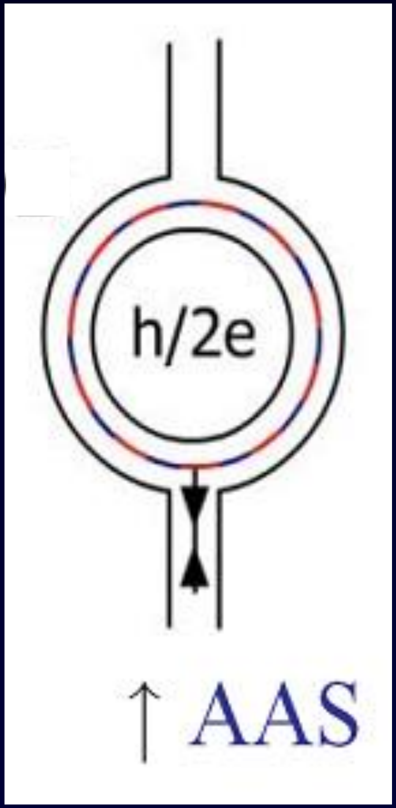
Here $4e^2/\pi h$ is the theoretically predicted value at the Dirac point for ballistic transport mediated by evanescent modes.

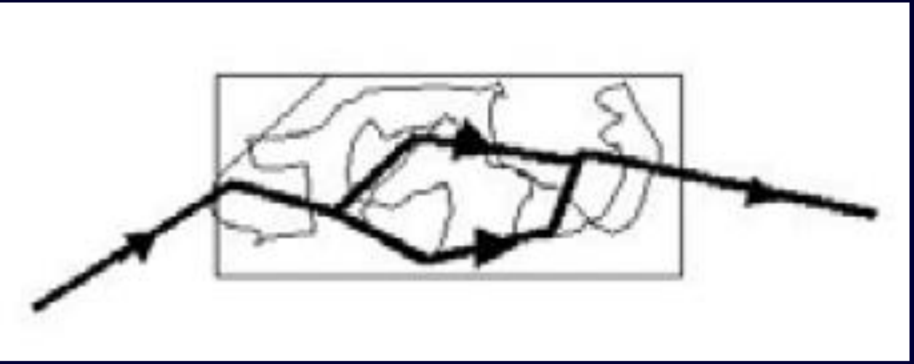



**Mobility and mean free path (mfp) of hole branch carriers.**

# Summary

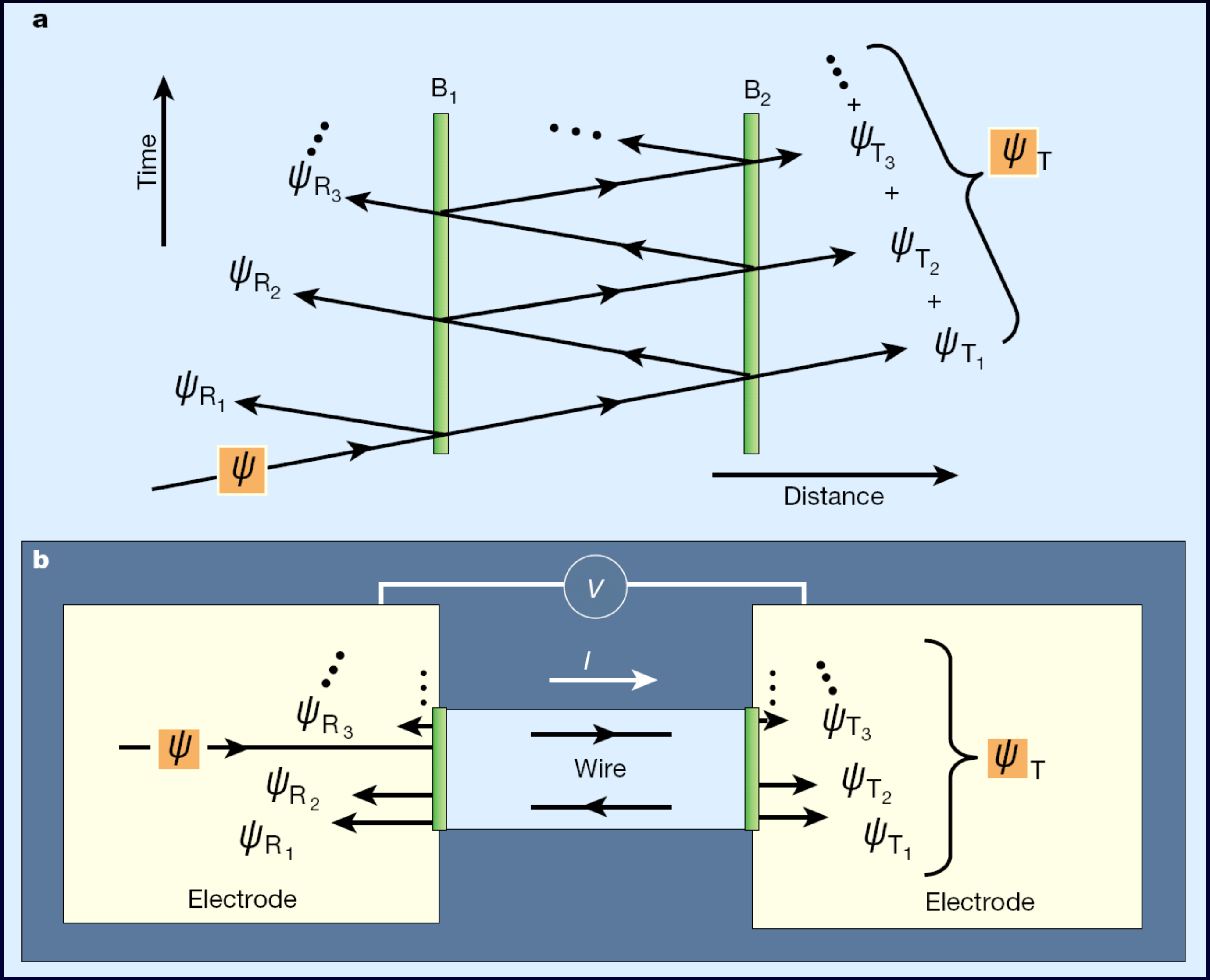
## Weak Localization
## Universal Conductance Fluctuations

**Interfering pair of paths can involve either real paths (a and d) or time-reversed paths (b and c)**

# Ballistic Transport



**Collective (AAS, weak localization) and individual (AB, UCF) interference of transmission modes** causes conductance variations of magnitude $\sim e^2/h$ in cold nanostructures.

**Possible future technological applications:**

Electronic components based on

- interference controlled by geometry of nano wire network
- signal encoding / decoding by constructive interference

**Ballistic transport:** increase the conductance in the 'ON' state, and the current delivery capability—a key determinant of device performance