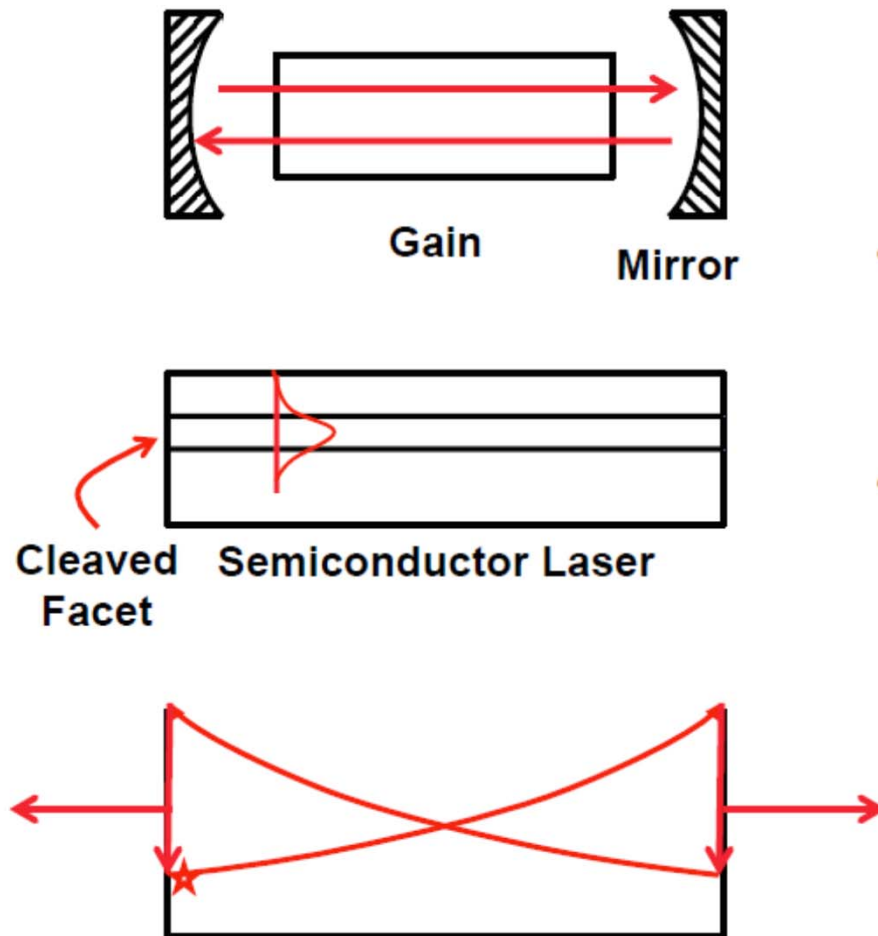
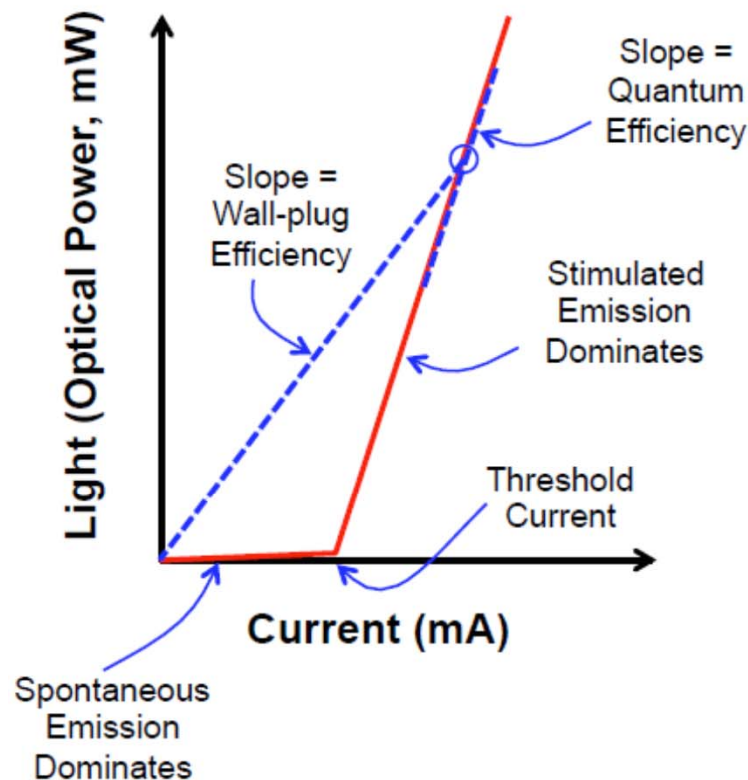


Basic Concept of Lasers



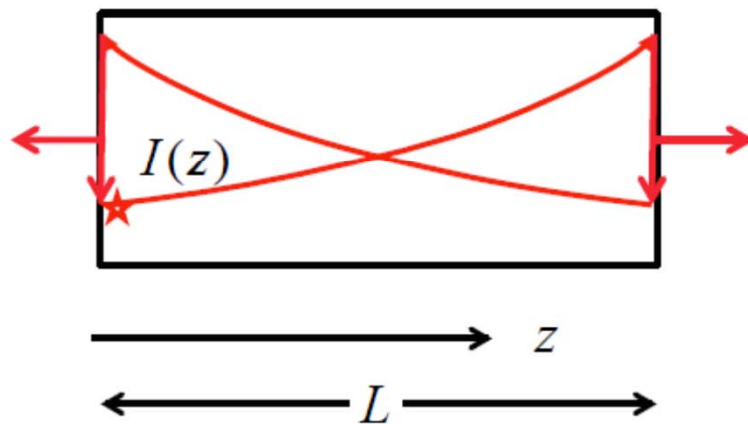
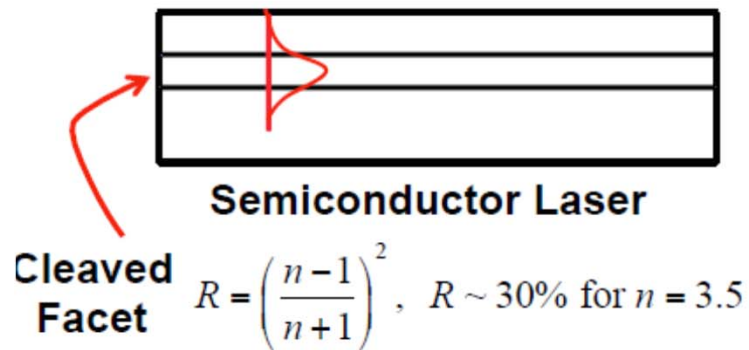
- **Laser:**
 - Light Amplification by Stimulated Emission of Radiation
- **Basic elements:**
 - Gain media
 - Optical cavity
- **Threshold condition:**
 - Bias point where laser starts to “lase”
 - Gain (nearly) equals loss

L-I Curve of Semiconductor Lasers



- Distinctive threshold (at least in classical lasers)
- Semiconductor laser is a forward-biased p-n junction, so mainly a current-biased device
- Threshold current :
 - Minimum current at which the laser starts to “lase”
- Quantum efficiency
 - “Differential” electrical-to-optical conversion efficiency, i.e., how many photons generated by injected electrons beyond threshold
- Wall-plug efficiency
 - Total electrical-to-optical conversion efficiency

“Edge-Emitting” Semiconductor Lasers



g : gain coefficient [cm^{-1}]

Light amplification: $I(z) = I_0 e^{\Gamma g z}$

Γ : confinement factor

(fraction of energy in gain media)

Threshold condition:

Round-trip gain = 1

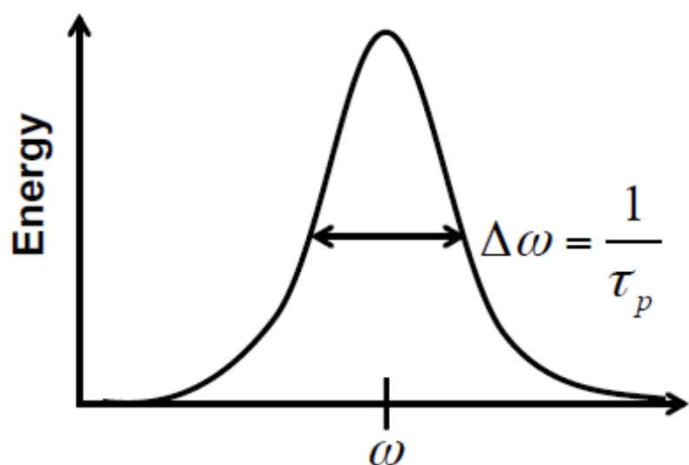
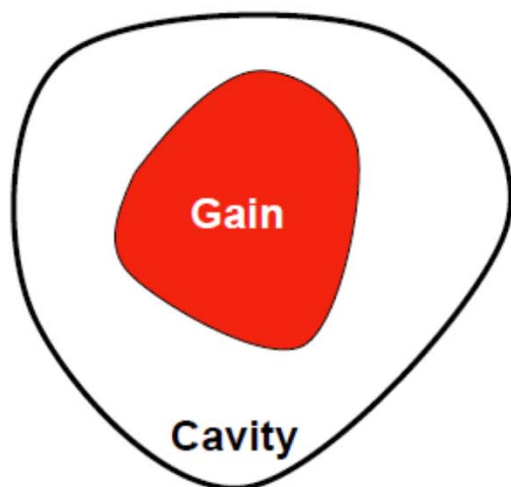
$$e^{\Gamma g L - \alpha_i L} R_1 e^{\Gamma g L - \alpha_i L} R_2 = 1$$

$$g = g_{th} = \frac{\alpha_i}{\Gamma} + \frac{1}{2\Gamma L} \ln \left(\frac{1}{R_1 R_2} \right) = \frac{\alpha_i + \alpha_m}{\Gamma}$$

α_i : intrinsic loss

$\alpha_m = \frac{1}{2L} \ln \left(\frac{1}{R_1 R_2} \right)$: mirror loss (i.e., output light)

Generic Description of Optical Cavity



Quality Factor:

$$Q = \frac{\text{Energy Stored}}{\text{Energy Dissipated per Cycle}}$$

$$Q = \frac{\omega}{\Delta\omega}$$

$$\Delta\omega = \frac{1}{\tau_p}$$

τ_p : photon lifetime [sec]

$$\frac{1}{\tau_p} = \alpha \frac{c}{n} \quad \left(\begin{array}{l} \alpha: \text{loss rate per cm} \\ 1/\tau_p: \text{loss rate per sec} \end{array} \right)$$

$$Q = \omega\tau_p$$

Photon Lifetime and Spectral Width

Decay of optical energy when input is turned off
(ring-down measurement):

$$I(t) = I_0 e^{-t/\tau_p} \quad \text{for } t \geq 0$$

Electrical (optical) field:

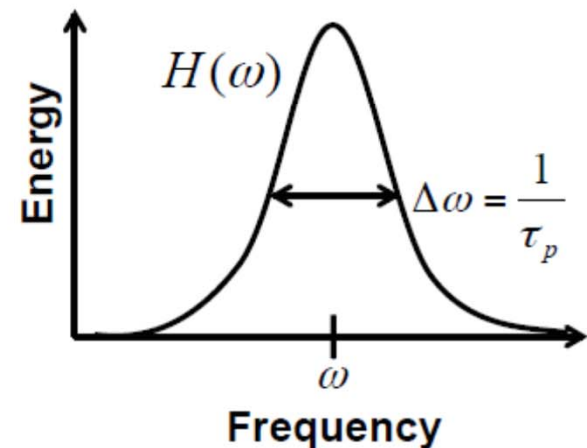
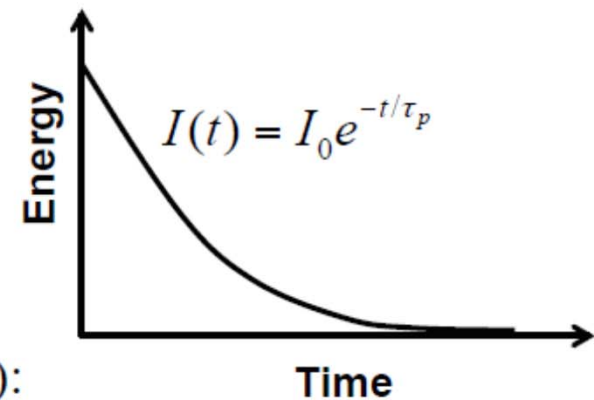
$$E(t) = E_0 e^{j\omega_0 t} e^{-t/2\tau_p} \quad \text{for } t \geq 0$$

Frequency domain response (Fourier transform):

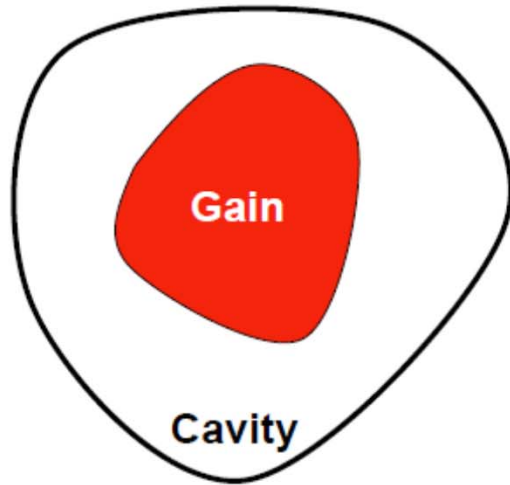
$$H(\omega) = \int_0^{\infty} e^{j\omega_0 t} e^{-t/2\tau_p} e^{-j\omega t} dt = \frac{1}{j(\omega - \omega_0) + 1/2\tau_p}$$

$$\text{FWHM of } |H(\omega)|^2 : \quad \omega - \omega_0 = \pm \frac{1}{2\tau_p}$$

$$\boxed{\Delta\omega = \frac{1}{\tau_p}}$$



Threshold Condition of Generic Lasers

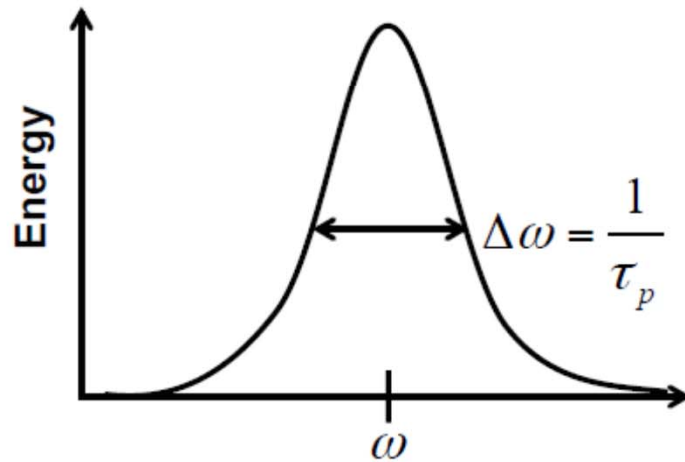


Gain = Loss

(rate of gain = rate of loss)

$$\Gamma g_{th} \frac{c}{n} = \frac{1}{\tau_p} = \frac{\omega}{Q}$$

$$g_{th} = \frac{\omega}{Q} \frac{n}{\Gamma c}$$



Quantum efficiency:

$$\eta = \frac{\alpha_m}{\alpha_m + \alpha_i} = \frac{Q_{rad}^{-1}}{Q_{rad}^{-1} + Q_{loss}^{-1}} = \frac{Q_{rad}^{-1}}{Q^{-1}}$$

$$\eta = \frac{Q}{Q_{rad}}$$

Typical Q of Semiconductor Laser

Edge-emitting laser:

$$L = 100\mu m, R = 30\%, \omega \sim 100THz, \tau_p \sim 1ps, Q \sim 600$$

Vertical Cavity Surface-Emitting Laser (VCSEL)

$$L = 1\mu m, R = 99\%, Q \sim 700$$

Microdisk (Whispering Gallery Mode or WGM) Laser

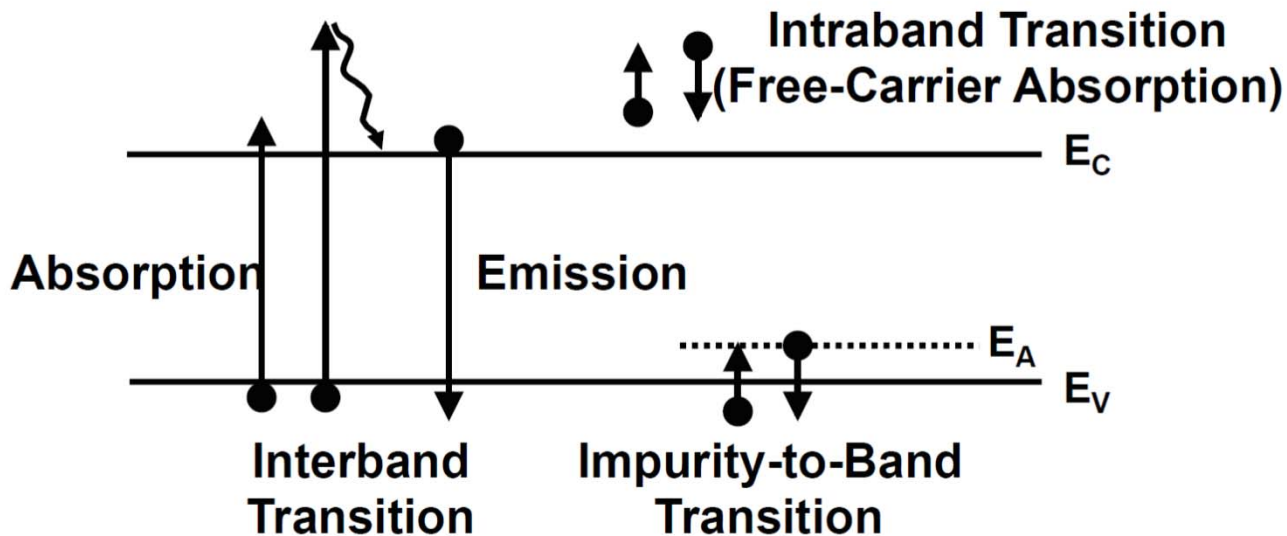
$$Q \sim 1000 \text{ (up to } 10^{11} \text{ possible in low loss materials)}$$

Photonic crystal laser: $Q \sim 1000$ (up to 10^6 possible)

Metal cavity laser (plasmonic laser): $Q \sim 10$ to 100

The gain medium

Optical Properties of Semiconductors



Optical transitions

- Absorption: exciting an electron to a higher energy level by absorbing a photon
- Emission: electron relaxing to a lower energy state by emitting a photon

Band-to-Band Transition

Since most electrons and holes are near the band-edges, the photon energy of band-to-band (or interband) transition is approximately equal to the bandgap energy:

$$h\nu = E_g$$

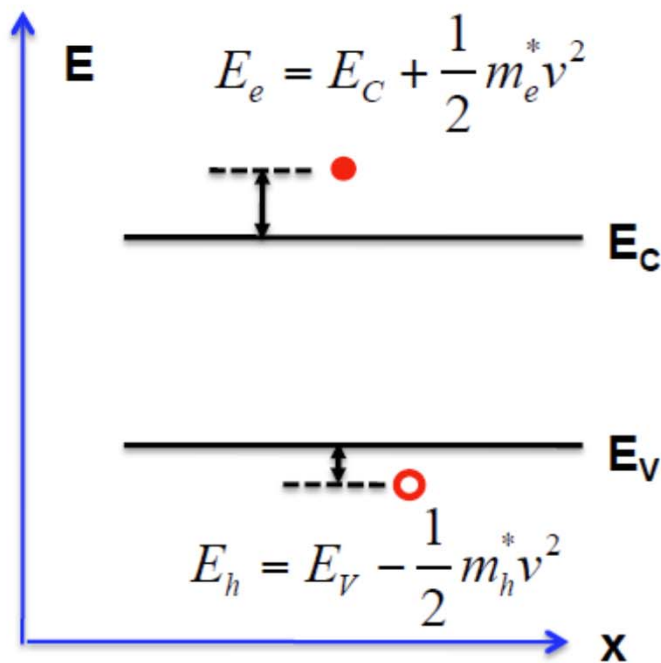
The optical wavelength of band-to-band transition can be approximated by

$$\lambda = \frac{c}{\nu} = \frac{hc}{E_g} \approx \frac{1.24}{E_g}$$

λ : wavelength in μm

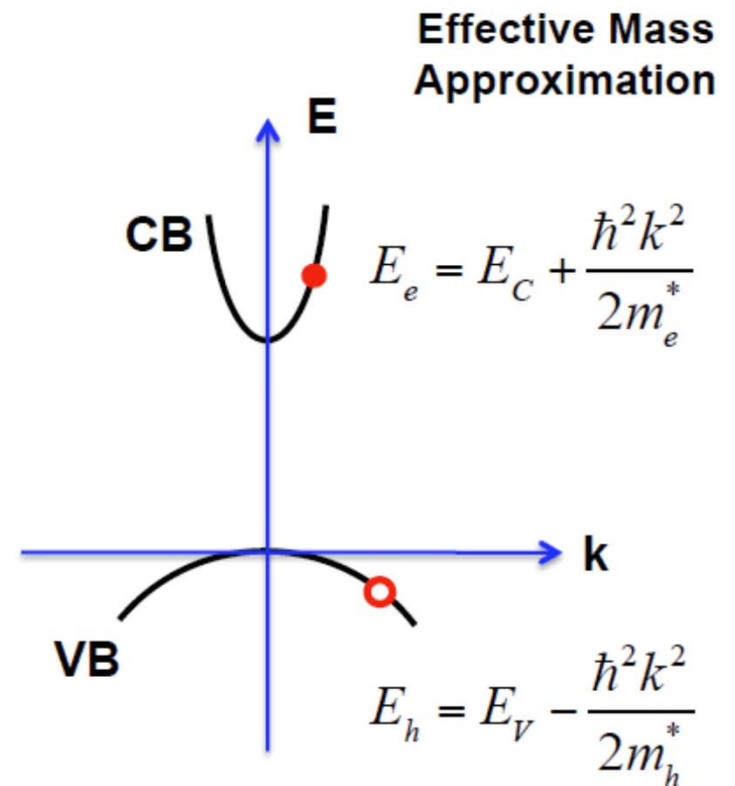
E_g : energy bandgap in eV

Energy Band Diagram in Real Space and k-Space



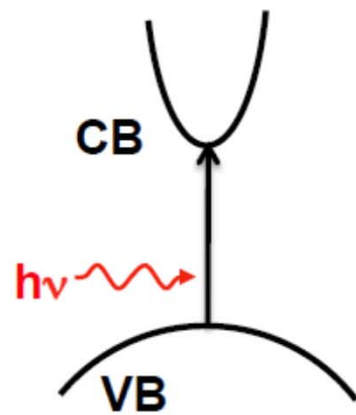
Real Space

Momentum:
 $\hbar k = m_e^* v_e$



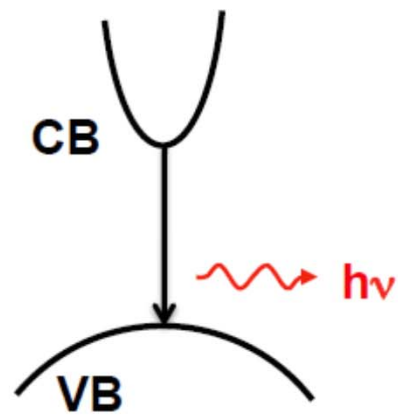
K-Space

Band-to-Band Transition



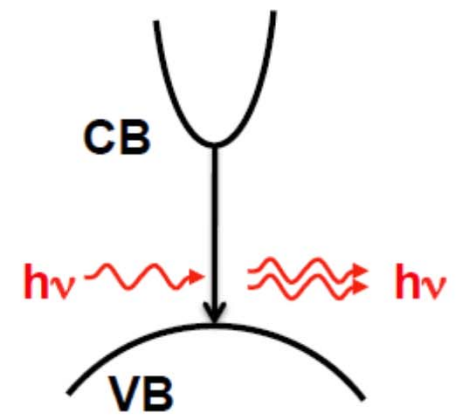
Absorption

Photodetectors;
Solar Cells



Spontaneous
Emission

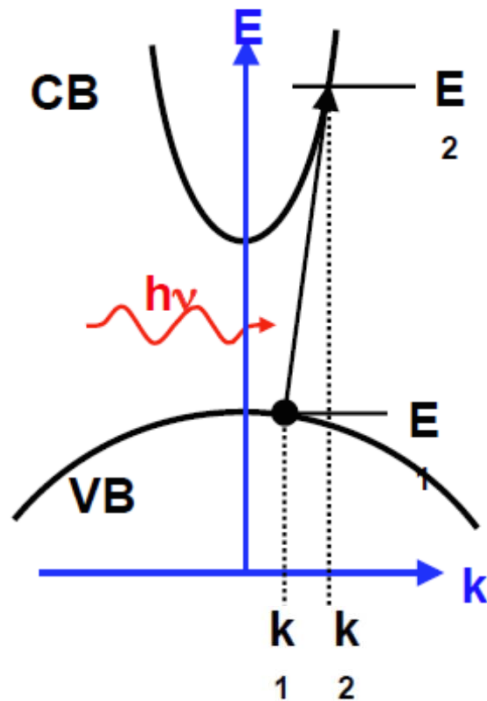
LED



Stimulated
Emission

Optical Amplifiers;
Semiconductor Lasers

Conservation of Energy and Momentum



Optical transitions are
“vertical” lines

- Conditions for optical absorption and emission:
 - Conservation of energy

$$E_2 - E_1 = h\nu$$

- Conservation of momentum

$$k_2 - k_1 = k_{hv}$$

$$k_2, k_1 \sim \frac{2\pi}{a}$$

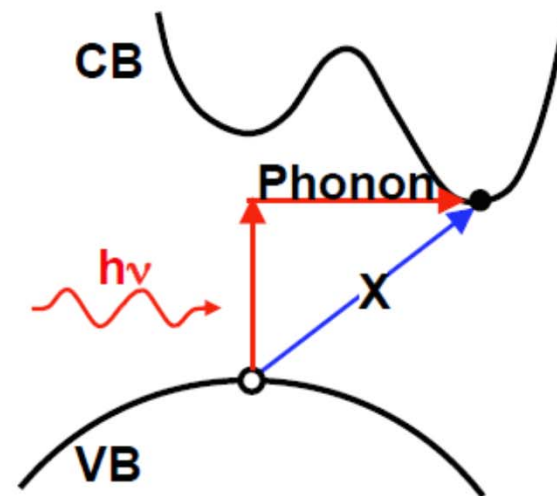
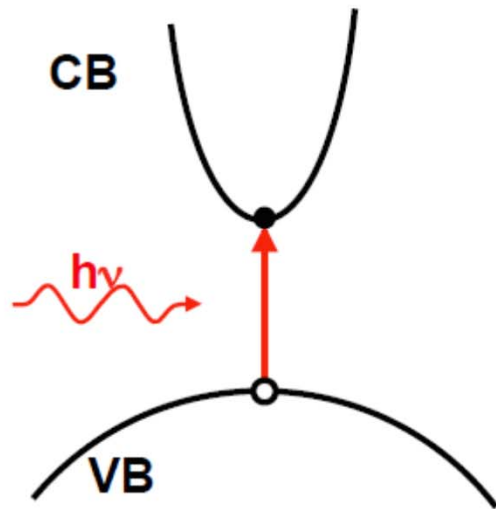
$$k_{hv} \sim \frac{2\pi}{\lambda}$$

$$(a \sim 0.5nm) \ll (\lambda \sim 1\mu m)$$

Lattice
Constant

$$\Rightarrow k_2 = k_1$$

Direct vs Indirect Bandgaps



- Direct bandgap materials
 - CB minimum and VB maximum occur at the same k
 - Examples
 - GaAs, InP, InGaAsP
 - $(\text{Al}_x\text{Ga}_{1-x})\text{As}$, $x < 0.45$
- Indirect bandgap materials
 - CB minimum and VB maximum occur at different k
 - Example
 - Si, Ge
 - $(\text{Al}_x\text{Ga}_{1-x})\text{As}$, $x > 0.45$
 - Not “optically active”