Lecture 9-II

Luttinger-liquid

(拉廷格液体)

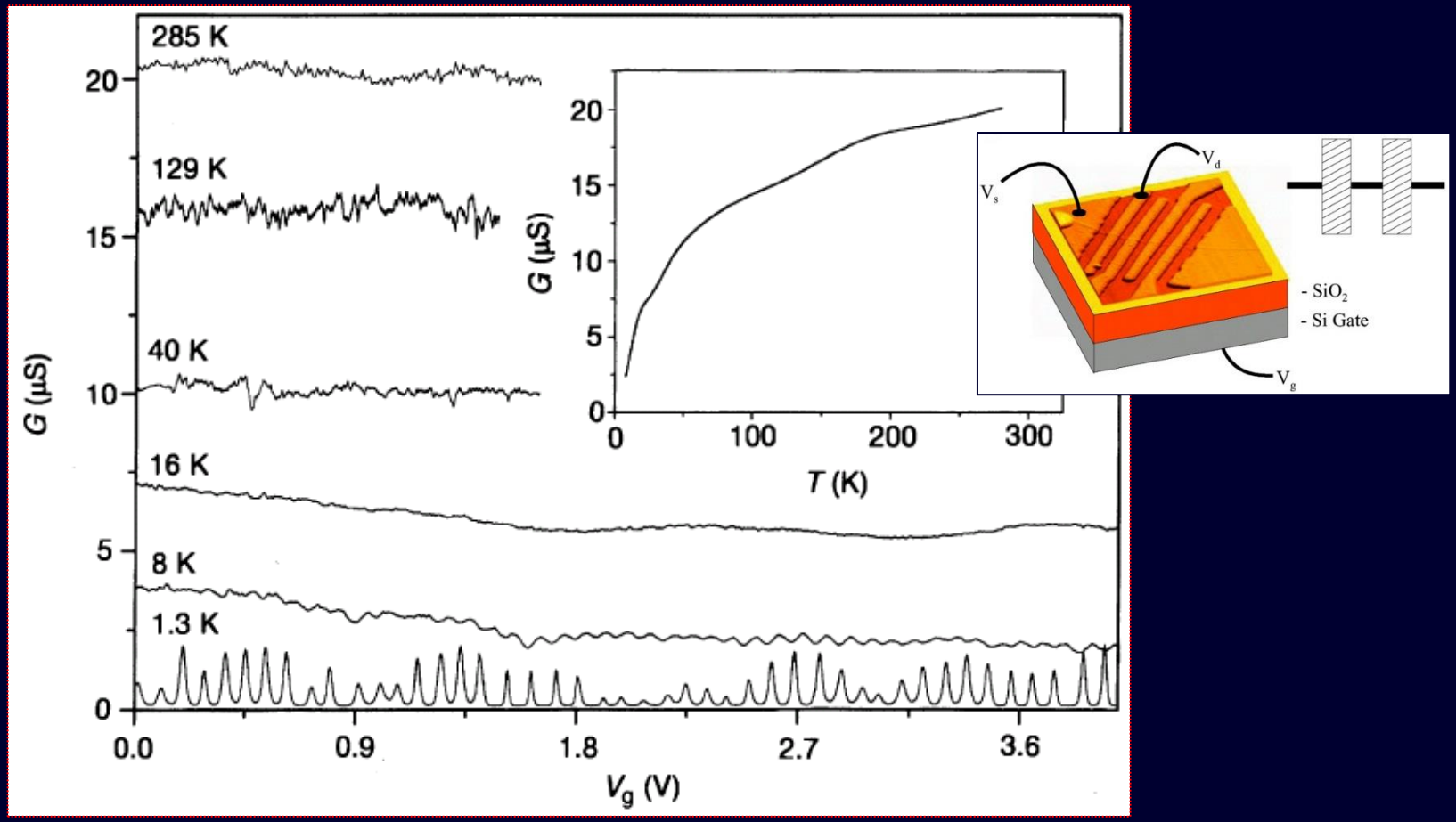
Outline

- Evidences for the Luttinger-liquid
- Brief introduction of Fermi-liquid
- Concept of Luttinger-liquid
- Characterizations of Luttinger Liquid

Outline

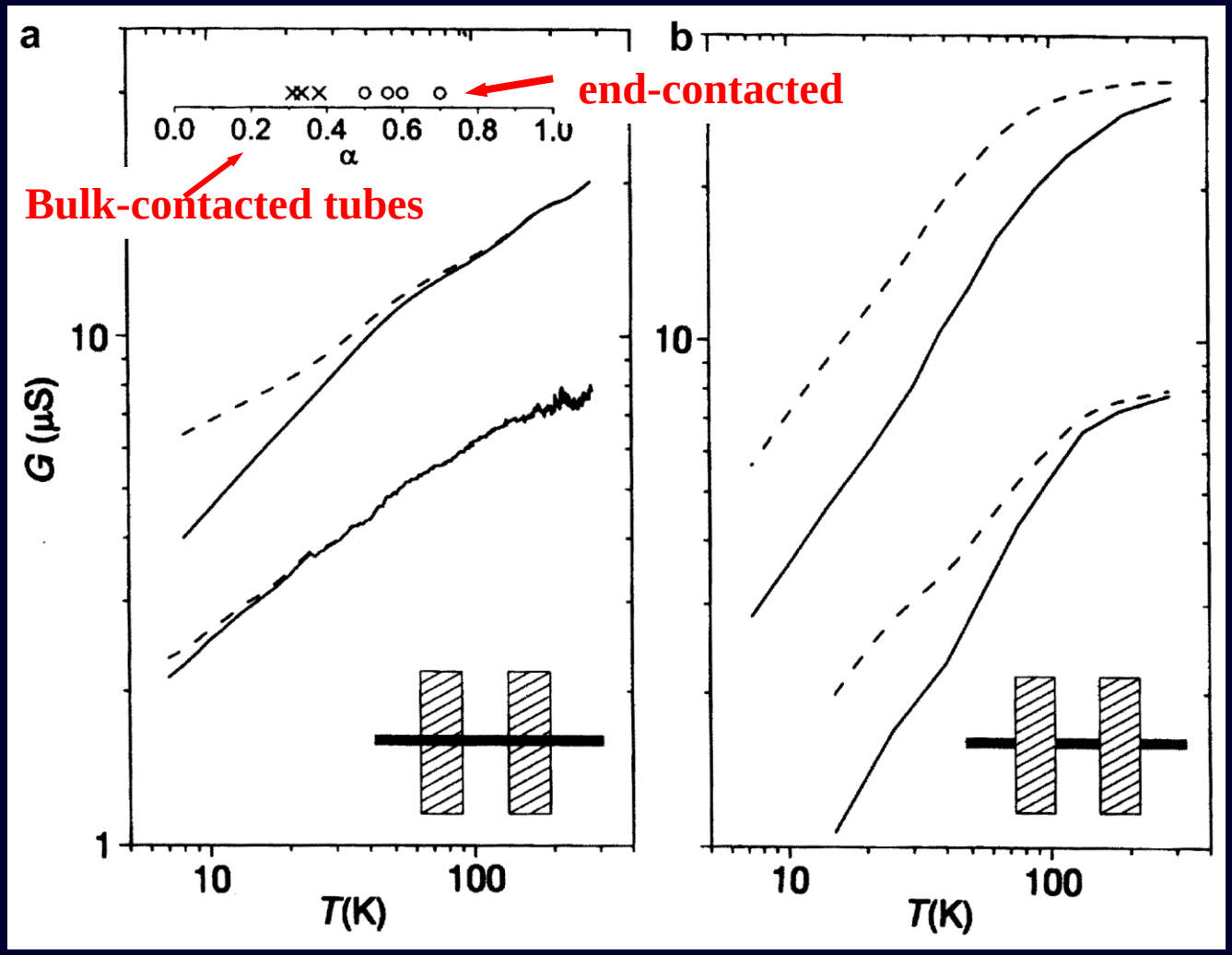
- Evidences for the Luttinger-liquid
- Brief introduction of Fermi-liquid
- Concept of Luttinger-liquid
- Characterizations of Luttinger Liquid

Single-Electron Transistor



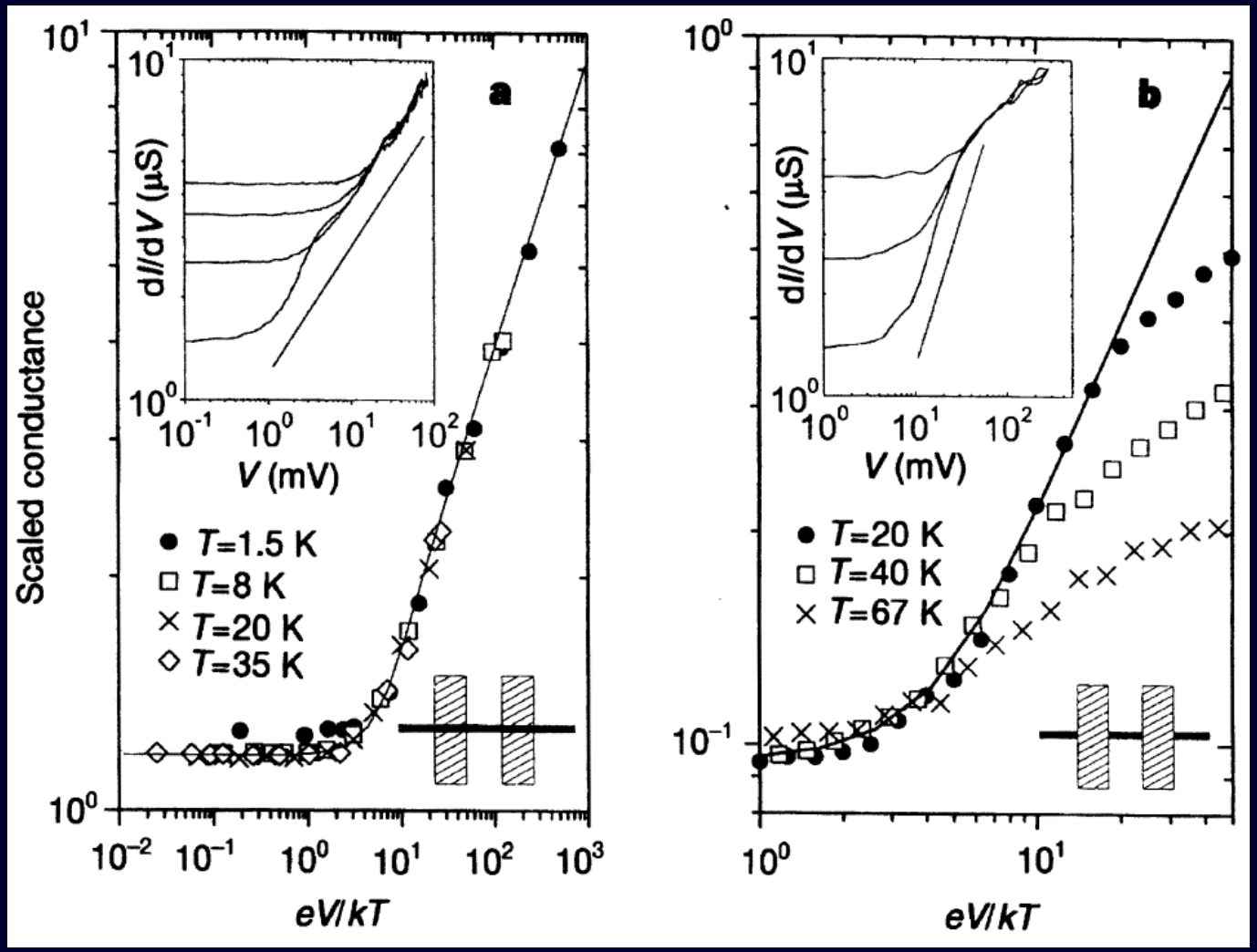
Single-Electron Transport in Ropes of Carbon Nanotubes

At temperatures above 20 K, the thermal energy exceeds the charging energy (i.e., $k_B T > U$). The Coulomb oscillations being nearly completely 'washed out', rendering the conductance independent of gate voltage.



Power law behavior as a function of temperature:

$$G \propto T^{\alpha}$$

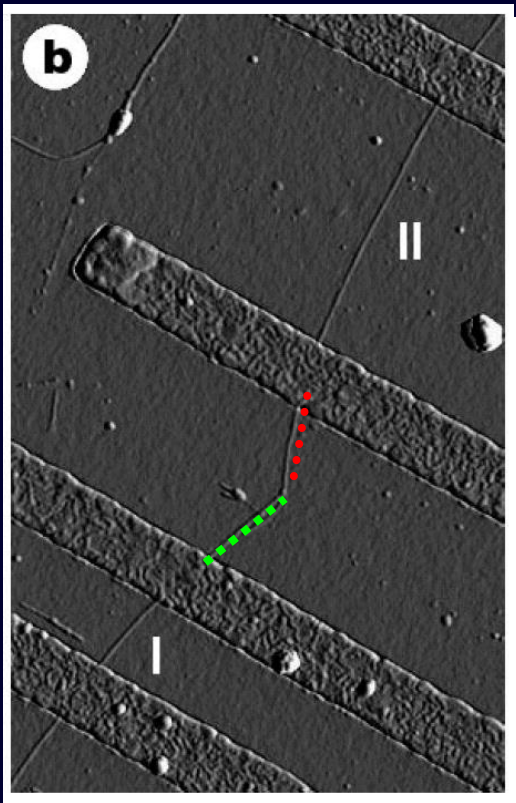


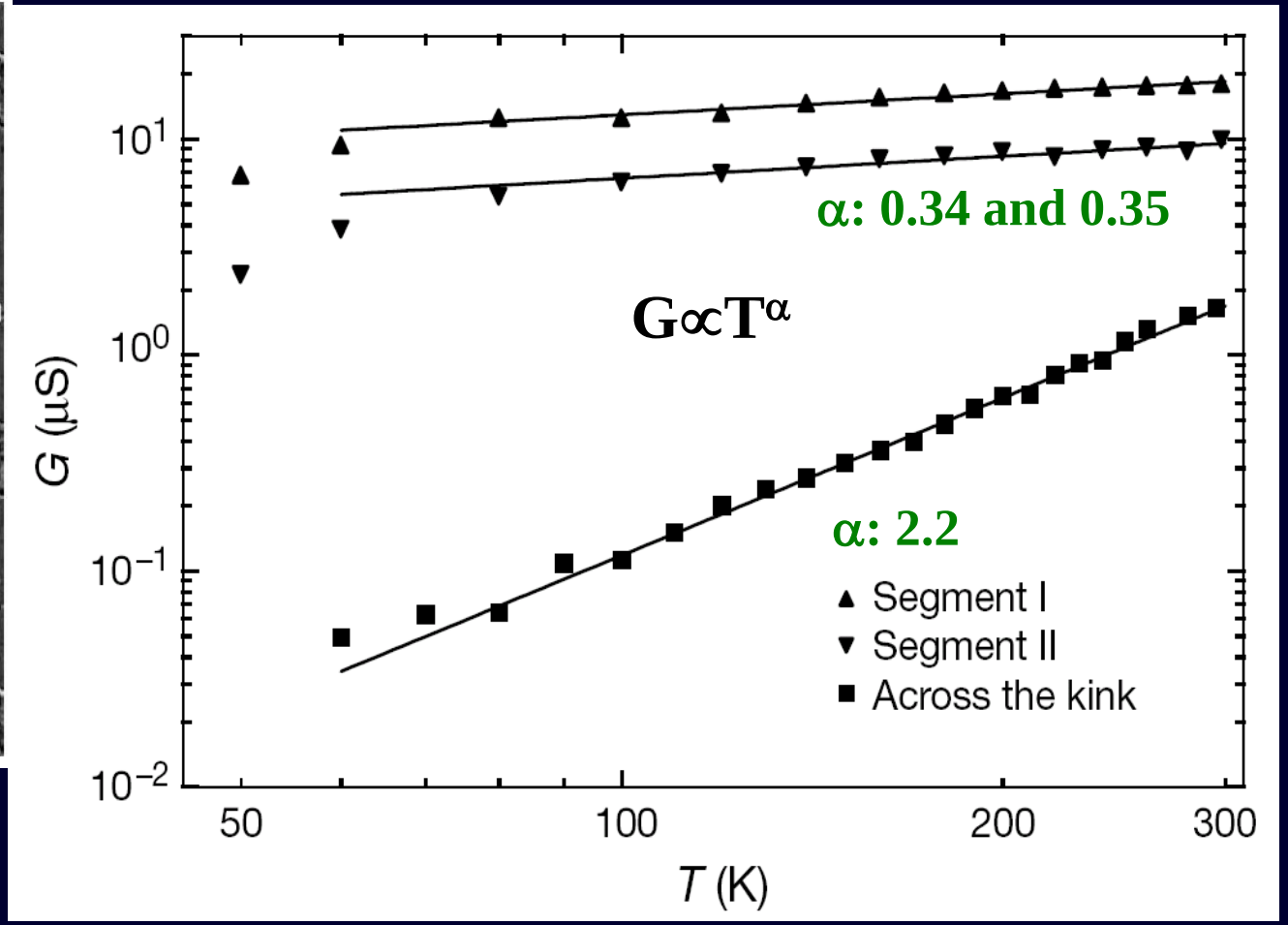
Power law behavior as a function of bias voltage:

$$dI/dV \sim V^{\alpha}$$

Junction

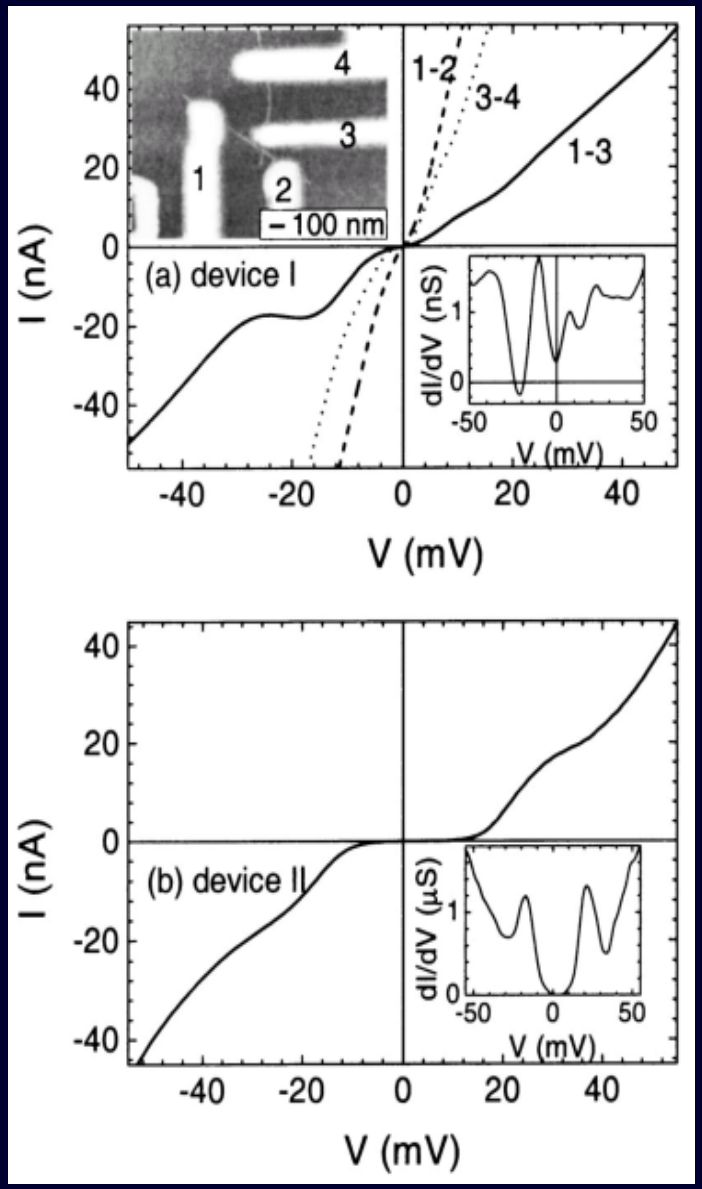
The conductance across the junction is much more temperature dependent than that of the two straight segments





- Power-law behaviour of G versus T was interpreted as a signature for electron-electron correlation.
- The nanotube behaves as **a Luttinger liquid**

Electrical transport through crossed carbon nanotube junctions (MM)



The crossed junction acts as tunneling junction at high temperatures, but it exhibits different behaviors at very low temperatures.

At 4.2K:
Step-like structure in I-V curves

Possible reason: formation of localized states at the crossing, since the resonant tunneling through the localized states can enhance the conductance of the junction.

J. W. Park et al., J. Appl. Phys. 93, 4191(2003)

Electrical transport through carbon nanotube junctions created by mechanical manipulation

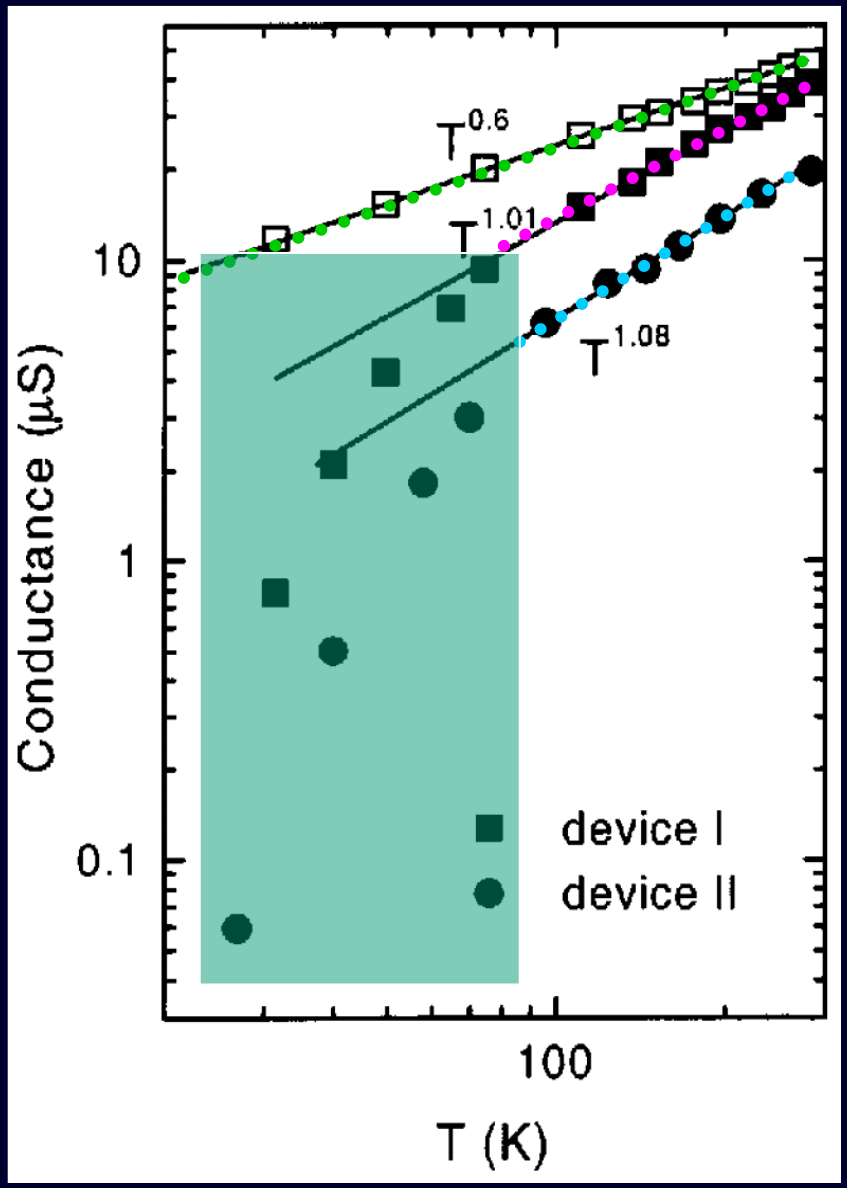
$G \propto T^{\alpha}$

Consistent with the theoretical predictions for tunneling into **Luttinger liquid**

拉廷格液体

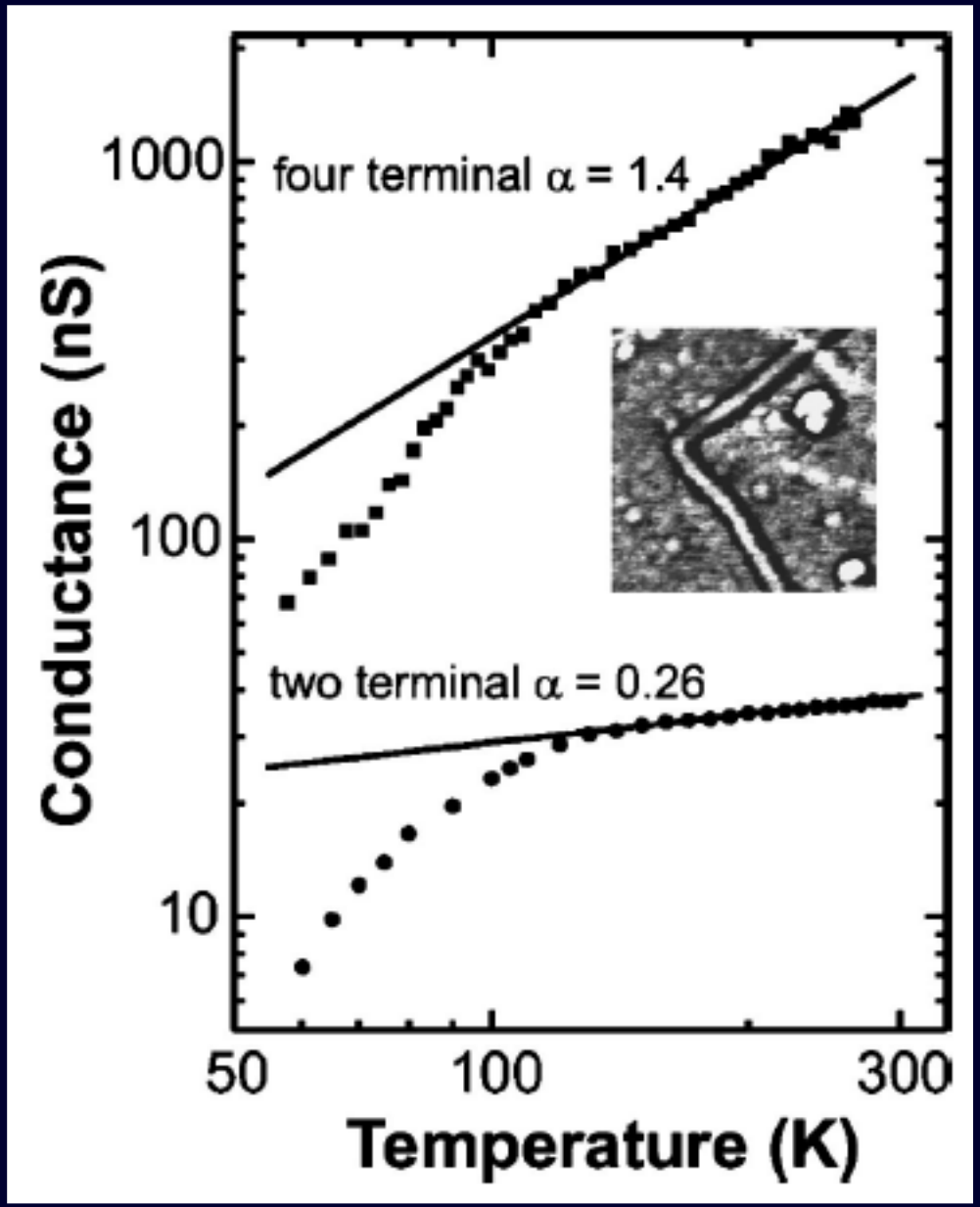
Different α values for junction or individual SWNT

The origin of this deviation is speculated to be associated with localized states formed at the junction.



G-T curve for the junction at zero-bias voltage for devices I and II.

J. W. Park et al., J. Appl. Phys. 93, 4191(2003)



Conductance of a nanotube buckle as a function of temperature in a four- and two-terminal measurement. The straight solid lines on this log-log plot indicate the power-law behavior with the exponent a as denoted.

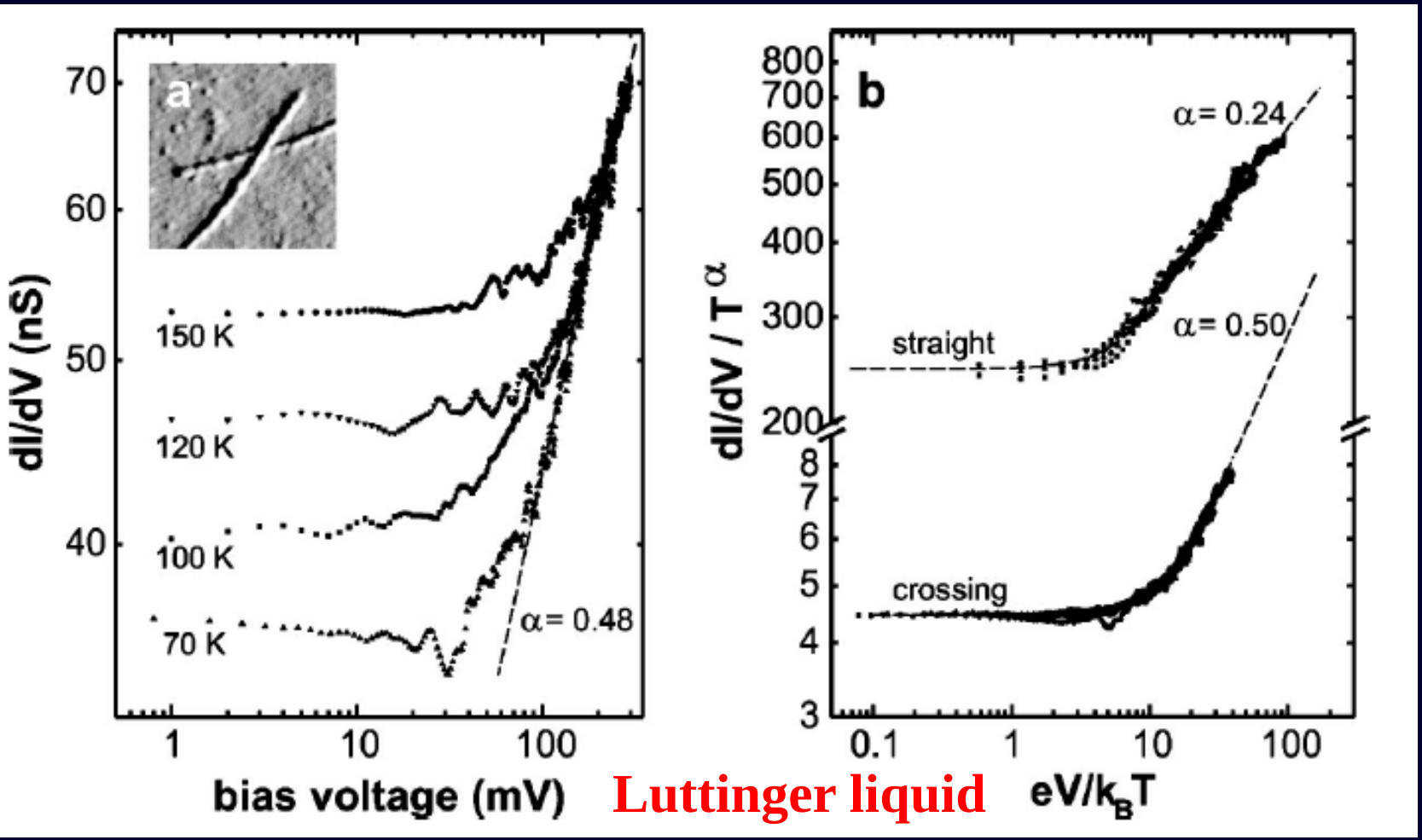
$G \propto T^{\alpha}$

Consistent with the theoretical predictions for tunneling into **Luttinger liquid**

Electrical transport through carbon nanotube junctions

Henk W. Ch. Postma et al., PRB 62, R10 653(2000)

Electrical transport through carbon nanotube junctions



At high voltages, the differential conductance crosses over to a power-law dependence on bias voltage:

$dI/dV \sim V^{\alpha}$

Henk W. Ch. Postma et al., PRB 62, R10 653(2000)

Outline

- Evidences for the Luttinger-liquid
- **Brief introduction of Fermi-liquid**
- Concept of Luttinger-liquid
- Characterizations of Luttinger Liquid

Density of states tells us how many states exist at a given energy E.

Fermi function f(E) specifies how many of the existing states at the energy E will be filled with electrons. The function f(E) specifies, under equilibrium conditions, the probability that an available state at an energy E will be occupied by an electron. It is a probability distribution function。

Fermi statistics： The corresponding Fermi distribution function, written in terms of the energy *E*,

$$f(\mathrm{E}) = \frac{1}{1 + exp[(\mathrm{E} - \mathrm{E_F})/\mathrm{kT}]}$$

E_F: Fermi energy or Fermi level；

$E_F = k_B T_F$

k: Boltzmann's constant(8.6×10^{-5} eV/K)

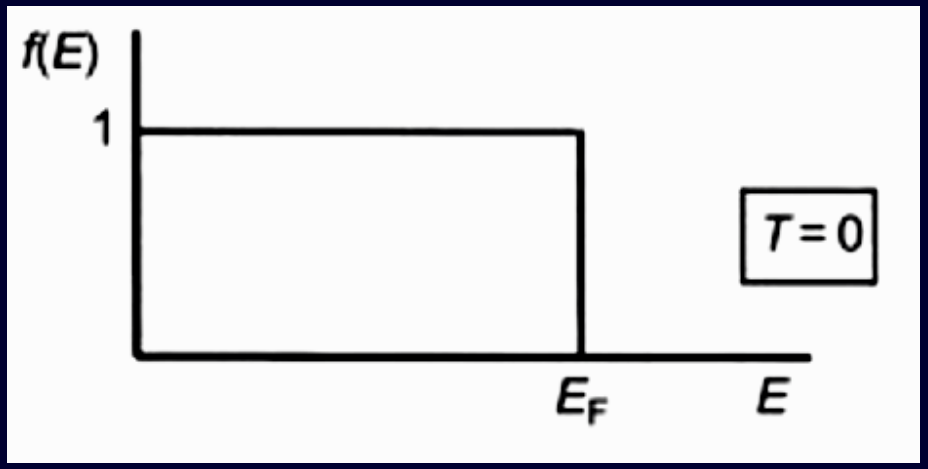
T: temperature

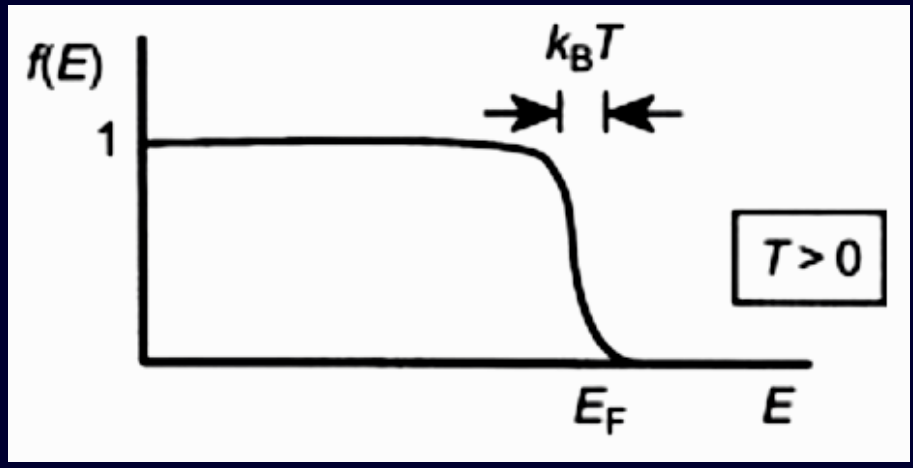
(T_F: Fermi temperature defined as E_F/k_B,which is typically 10^4–10^5 K.)

The Fermi temperature, T_F, can be thought of as the temperature at which thermal effects are comparable to quantum effects associated with Fermi statistics,

Fermi liquid (费米液体)：

遵从费米统计的粒子(费米子)所构成的量子液体。





Fermi–Dirac distribution function $f(E)$ for electrons: (a) at T=0 K and (b) above 0 K.

The distribution function $f(E)$ is 1 for energies below E_F and zero above E_F, and assumes intermediate values only in a region $k_B T$ wide near E_F,

费米统计适用对象：

热平衡时, 自旋量子数为半奇数的粒子(费米子)

电子：自旋量子数为1/2，是费米统计最普遍的应用对象。

费米液体(Fermi liquid)： **遵从费米(Fermi statistics)统计的粒子(费米子)所构成的量子液体。**

费米子(fermion): 自旋为半奇数(1/2，3/2…)的粒子; 满足泡利不相容原理，即两个以上的费米子不能出现在相同的量子态中。(体系的一个量子态只容许容纳一个粒子)

中子、质子: 自旋为1/2, 都是费米子

轻子、核子和超子：自旋都是1/2，是费米子

共振粒子：自旋为3/2、5/2、7/2等也是费米子

*** 中子: 一个带2e/3正电荷(上夸克)和两个带1e/3负电荷(下夸克)组成，两种夸克的电荷相互抵消，中子不显电性; 自由中子是不稳定的，可通过弱相互作用衰变为质子，放出一个电子和一个反中微子，中子自旋为1/2，磁矩为-1.91304275 单位核磁子。**

(标准模型预言中子具有微小、非零的电偶极矩)

**** 根据自旋倍数的不同，基本粒子分为玻色子和费米子:**

费米子: 像电子一样的粒子，有半奇数自旋(如1/2，3/2，5/2…)

玻色子(bosons): 像光子一样的粒子，有整数自旋(如0，1，2…)

费米液体(**Fermi liquid**)最初用于解释^{3}He液体在低温下的行为
(粘滞系数、磁化率、热导率、自旋扩散系数等)

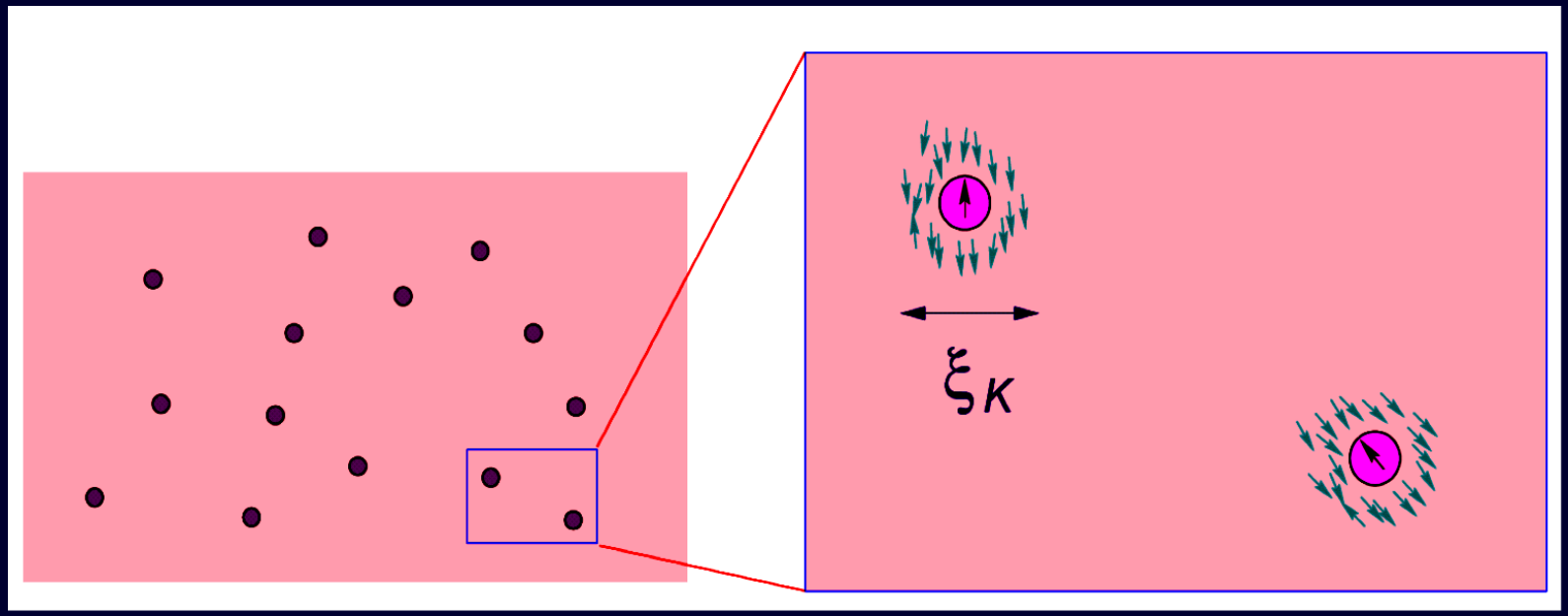
按朗道(Landau)费米液体理论，费米液体是相互作用费米粒子的多粒子系统，不能单独讨论粒子本身：如^{3}He液体，不能单独讨论^{3}He原子，而应讨论液体中的粒子加上与其相互作用的近邻粒子“云雾”一起组成的准粒子。

^{3}He液体的^{3}He原子共有奇数5个费米粒子构成:

外层有2个电子，原子核中有2个质子和1个中子；

作为复合粒子的原子整体仍是费米子，遵从费米-狄拉克(F-D)统计，故^{3}He量子液体是费米液体。

Example of Fermi liquid:

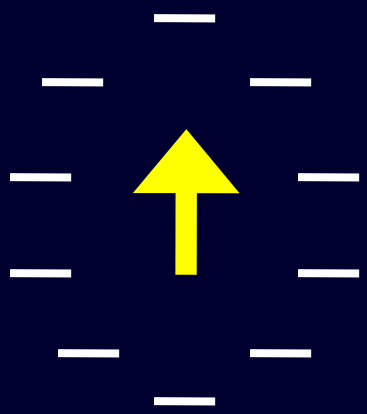


Kondo Effect in Dilute Magnetic Alloys

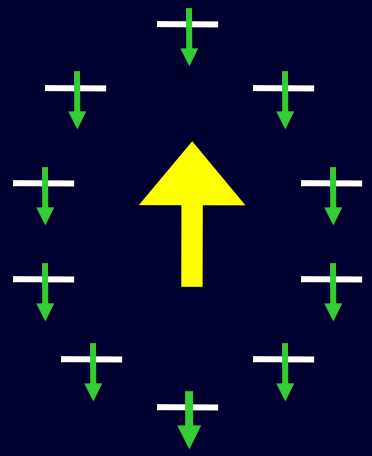
反铁磁的耦合使磁性杂质原子被自旋取向相反的传导电子所包围，从而对杂质磁矩起屏蔽抵消作用，这种作用随温度降低、热涨落的减小而增大, 在接近0K时呈Kondo singlet ground state（杂质原子和聚集在其周围的自旋反向的传导电子形成组合状态，总自旋为0）

-Many body effect which involves interaction between localized magnetic impurity and the free electrons in the surrounding metal

Local Fermi liquid:



$T >> T_K$



$T << T_K$

- Impurity spin is progressively screened below $T_K \propto e^{-1/\rho J}$
- A (local) **Fermi liquid** is formed for $T << T_K$

费米液体理论：建立在精确可解的自由费米气体模型上。

费米气体(Fermi gas)：不存在相互作用的费米子系统(non-intercting fermions)

费米液体理论适用于(三维情形)：

- **^{3}He**
- **Conventional metals**
- **Semiconductor Heterostructures**

理论成功之处：

(1)解决了一个困扰物理学家的问题: 为何在一些存在相互作用的系统中，费米子的性质却和理想费米气体(Ideal Fermi gas, non-intercting fermions)如此相似？

(2)提出了材料中准粒子(quasiparticles)的概念：可类比成自由电子，准粒子携带相同的电荷、自旋和动量，但因电子之间、电子和晶格的相互作用，需重新定义材料中电子的有效质量

核心内容是：在有相互作用时，单粒子激发过渡为准粒子。

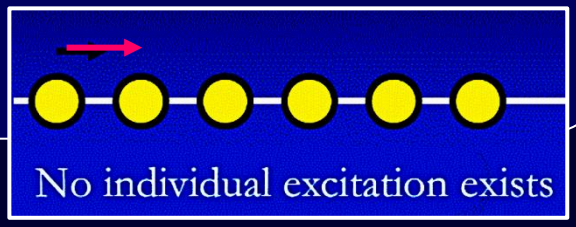
一维情形：费米液体理论失效：

费米液体对于一维的费米子系统：只要存在一点相互作用，

不管多微弱都会导致费米液体理论的失效。

(原因: 一维的费米子排成一条链，电子只能在一维链上运动，因Pauli不相容原理，两个费米子永远不能无限接近; LL中不存在单粒子激发，只存在集体激发，相互作用强)

In one dimension, as shown in Figure 2, an electron that tries to propagate has to push its neighbors because of electron-electron interactions. So no individual motion is possible. Any individual excitation has to become a collective one. Thus, no single particle excitations can exist.



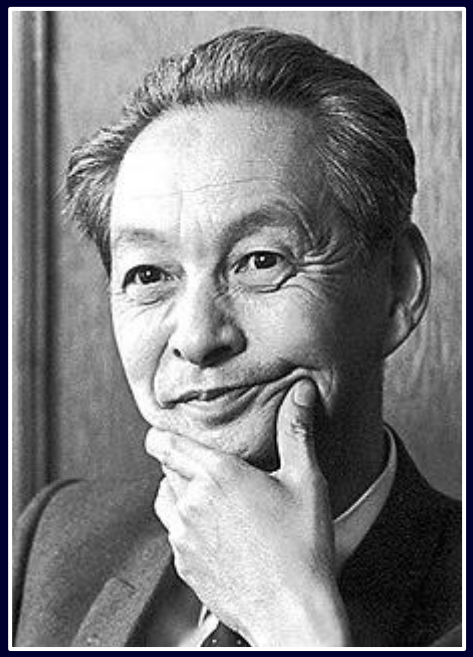
Luttinger liquid(LL)：仍是描述相互作用的费米子，但特指一维体系的费米子系统

Luttinger Liquid模型适用于:

碳纳米管；Quantum wire

量子霍尔效应边界态; 准一维原子陷阱

Tomonaga-Luttinger liquid



Sin-Itiro Tomonaga
(1906–1979)

Joaquin M. Luttinge
(1923–1997)

65年朝永振一郎因“重正化理论”获诺贝尔物理学奖

Tomonaga-Luttinger liquid (TLL): for a one-dimensional (1D) system, Luttinger, building on an approximation scheme of Tomonaga, constructed a soluble 1D model with infinite linear dispersion and a restricted set of interactions

1. Luttinger, J. M. An exactly soluble model of a many-fermion system. J. Math. Phys. 4:1154(1963)
2. Sin-Itiro Tomonaga, Remarks on Bloch's Method of Sound Waves applied to Many-Fermion Problems , Prog. Theor. Phys. 5:544(1950)

Outline

- Evidences for the Luttinger-liquid
- Brief introduction of Fermi-liquid
- **Concept of Luttinger-liquid**
- Characterizations of Luttinger Liquid

Luttinger liquids (LLs)

Basic concepts

- Luttinger liquid behavior describes low-energy physics of correlated 1D electron systems
- manifests in power law scaling of variety of physical observables **(幂次率变化规律)**
- scaling exponents only depend on **Luttinger parameter**
  标度指数

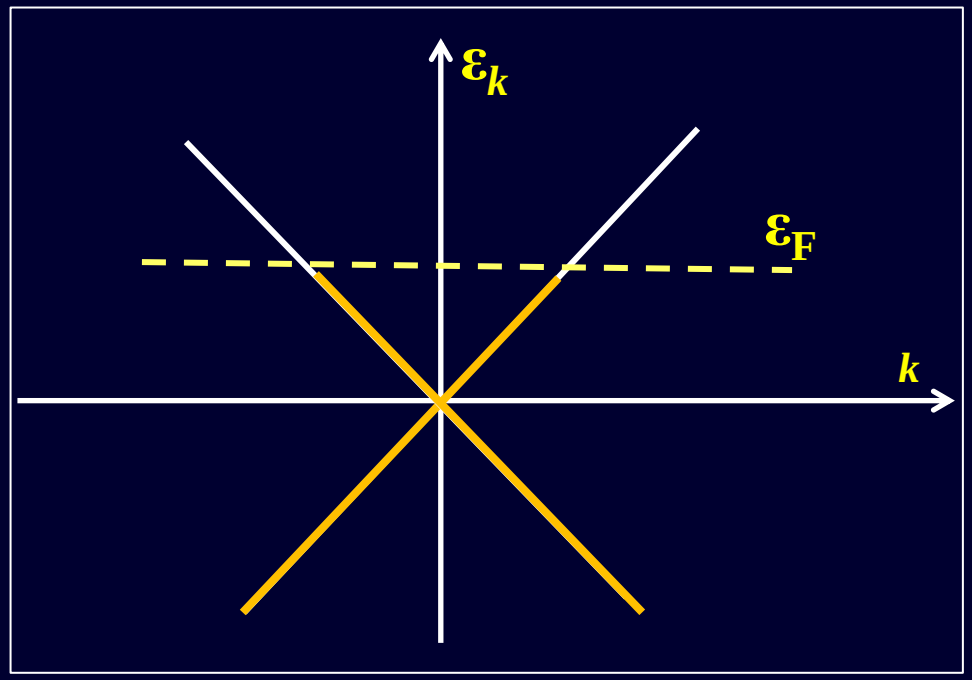
Physical realizations now emerging:
nanotubes, quantum wire, semiconductor wires, long chain molecules, ……

Luttinger liquids模型要点：

(1) 单粒子态的能量色散关系是严格线性的，不受ε_k和k取值大小及正负的限制；

$$\Delta\varepsilon_k = \varepsilon_k - \varepsilon_F = \hbar v_F (k - k_F)$$

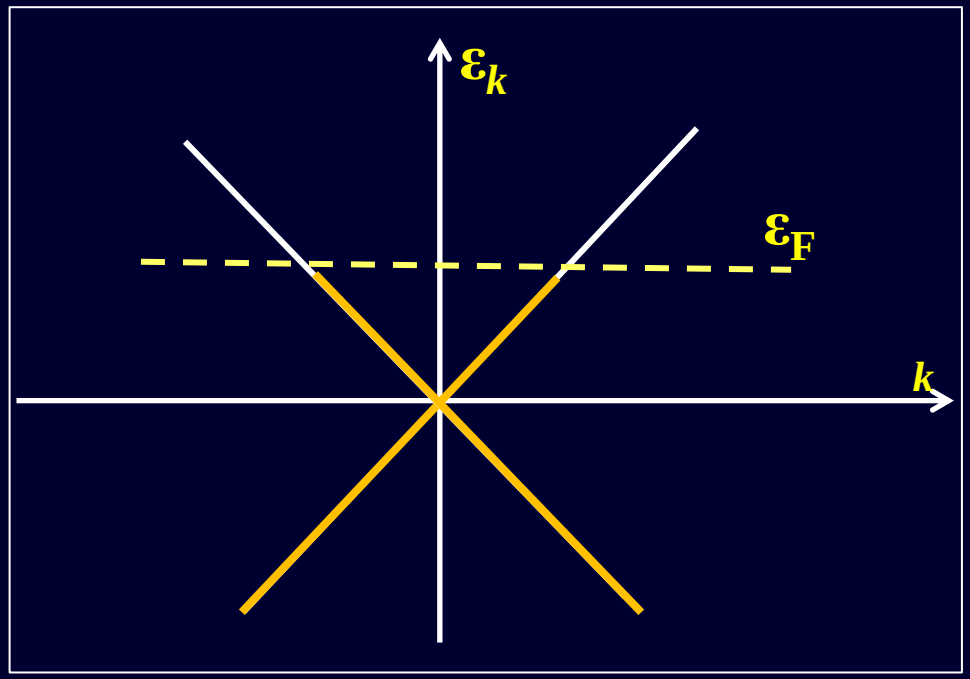
ε_F 为能量零点，k态电子的能量$\Delta\varepsilon_k$与$k - k_F$有线性依赖关系



Luttinger模型的单粒子能谱 (橘黄色为占据态)

相应的准粒子的多种称谓:
- 空穴子(holon)
- 电荷子(chargeon)
- 等离子体子(plasmon)
- 负电荷子(eon)

(2)电子-电子相互作用：前向散射(forward scattering)



单粒子能谱: 正斜率的一支向右运动
负斜率的一支向左运动

前向散射：散射前向右(或向左)的电子，散射后依然向右(或向左)

(3)电子-电子相互作用：背向散射

除前向散射，还需考虑自旋交互作用，即背向散射

散射前后电子的运动方向不变，向右(或向左)的仍向右(或向左)，相邻之间只交换位置，但自旋方向不翻转

Example 1:

Luttinger-liquid behavior in carbon nanotubes

- Fermi-liquid theory assumes that the energy states of the electrons near the Fermi level E_F are not qualitatively altered by Coulomb interactions. (3-D systems)

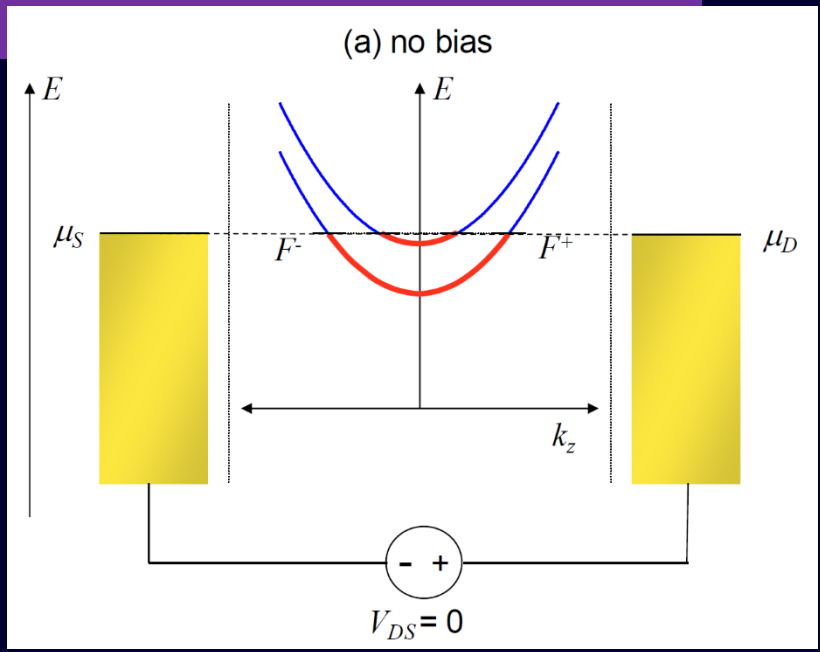
- Luttinger liquid : In 1-dimensional systems, even weak Coulomb interactions cause strong perturbations.

- The resulting system is predicted to be distinctly different from its three-dimensional counterparts

Metallic carbon nanotubes:

A good candidate to study Luttinger-liquid (LL)

- The elastic mean free path of electron in metallic tubes is long. (~several hundreds nm long; several micrometers)
- There are only 2 conduction modes at the Fermi liquid (FL);
- The conductance is $4e^2/h$ for a perfect device without disorder and perfect interfaces



Luttinger liquid

- **An LL can characterized by a interaction parameter g that measures the strength of the interaction between electrons.**

g<<1 for strong interactions

(SWNTs: long-range Coulomb interaction between electrons)

g=1 for non-interacting electron gas

- **For a finite length tube or rope, the Luttinger parameter g is given by:**

$$g = \left[1 + \frac{2U}{\Delta}\right]^{-1/2}$$

U: charging energy of the tube;

Δ: single particle level spacing .

From previous measurements and theoretical estimates **$U/\Delta \approx 6$,** yielding an expected **Luttinger parameter $g_{theory} \approx 0.28$**

g<<1 for strong interactions

Characterizations of Luttinger Liquid (I)

The conductance and differential conductance scale as power laws with respect to temperature and bias voltage, respectively, and that the functional forms and the exponents are in good agreement with theoretical predictions.

For a clean LL, with at small biases (eV << $k_B T$)

$$\mathbf{G(T) \propto T^{\alpha}}$$

with V at large biases (eV >>$k_B T$):

$$\mathbf{dI/dV \propto V^{\alpha}}$$

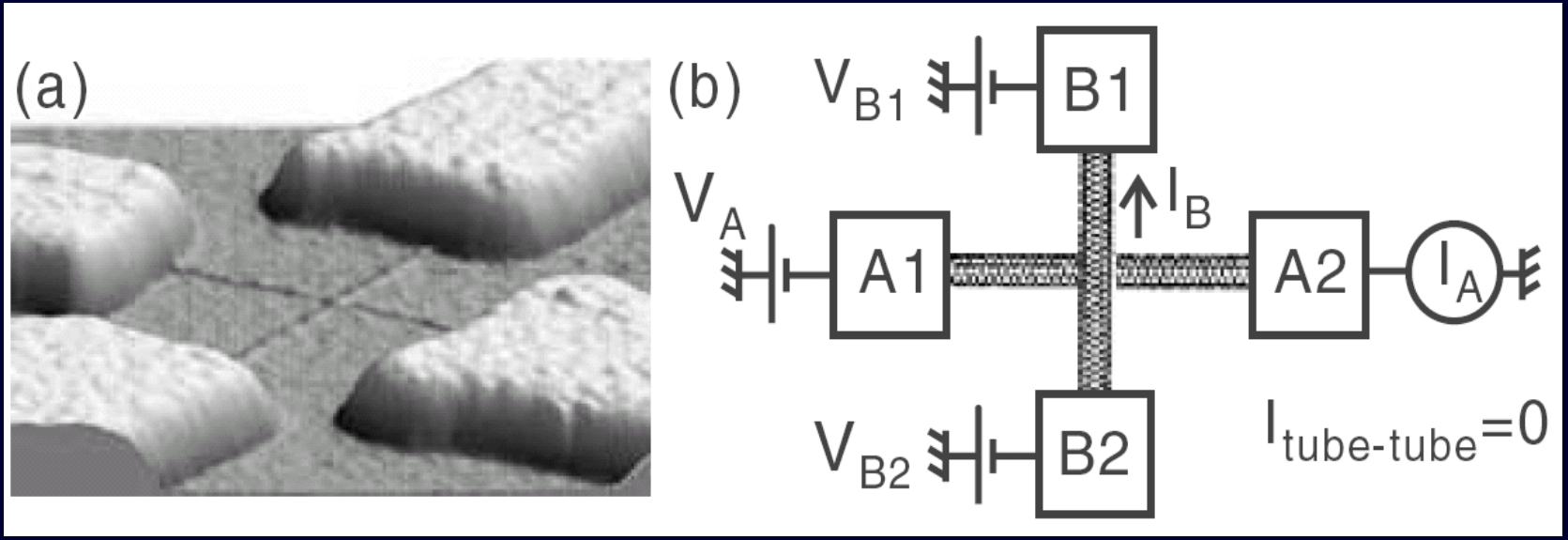
Geometry dependent exponent:

$$\alpha_{bulk} = \left(g + 1/g - 2\right)/8$$

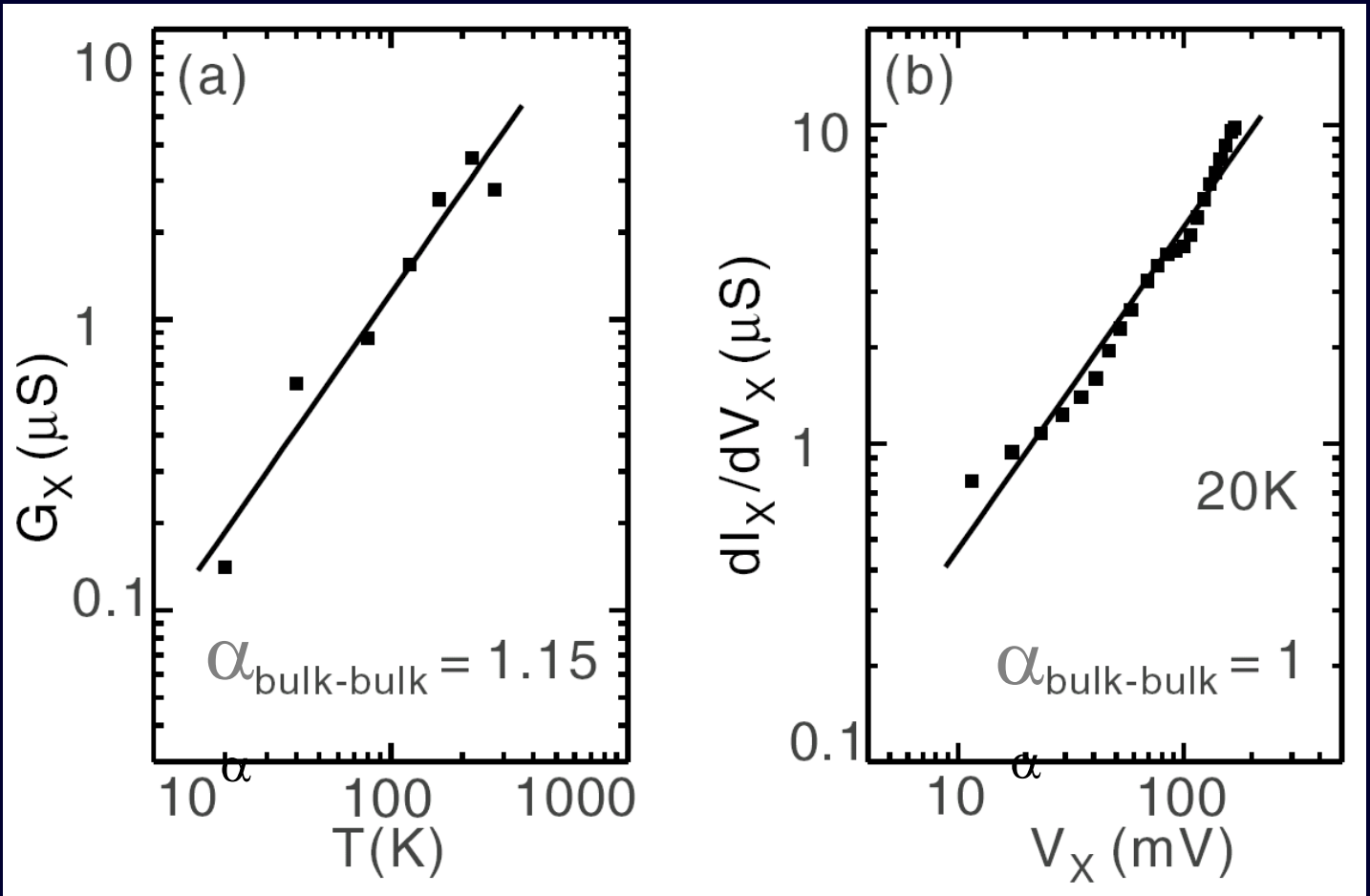
$$\alpha_{end} = \left(1/g - 1\right)/4$$

Luttinger parameter (g=0.28 for nanotube)

Evidence for Luttinger-Liquid Behavior in Crossed Metallic SWNTs



(a) AFM image of a crossed SWNT junction. The electrode height is 45 nm. (b) Scheme of the device together with the measurement setup. The AFM image cannot discriminate which tube lies on top of the other

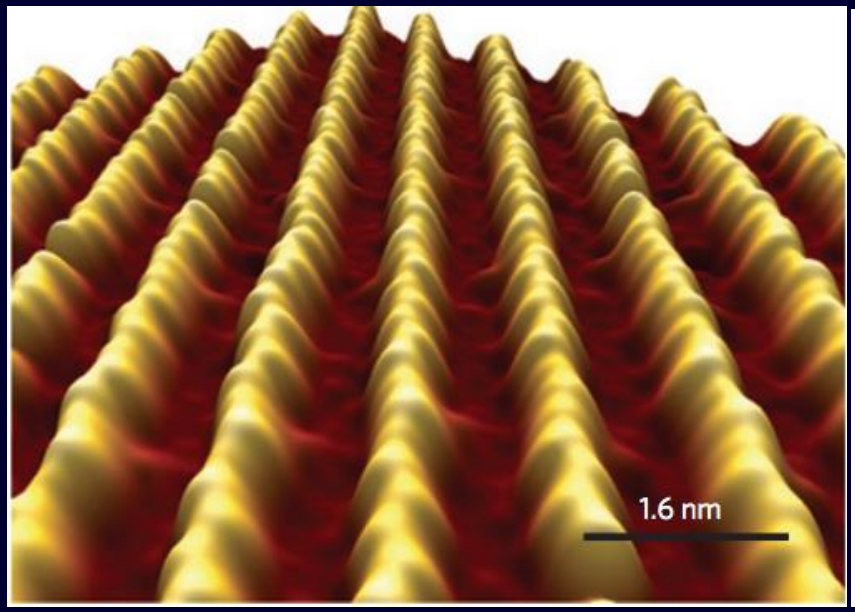


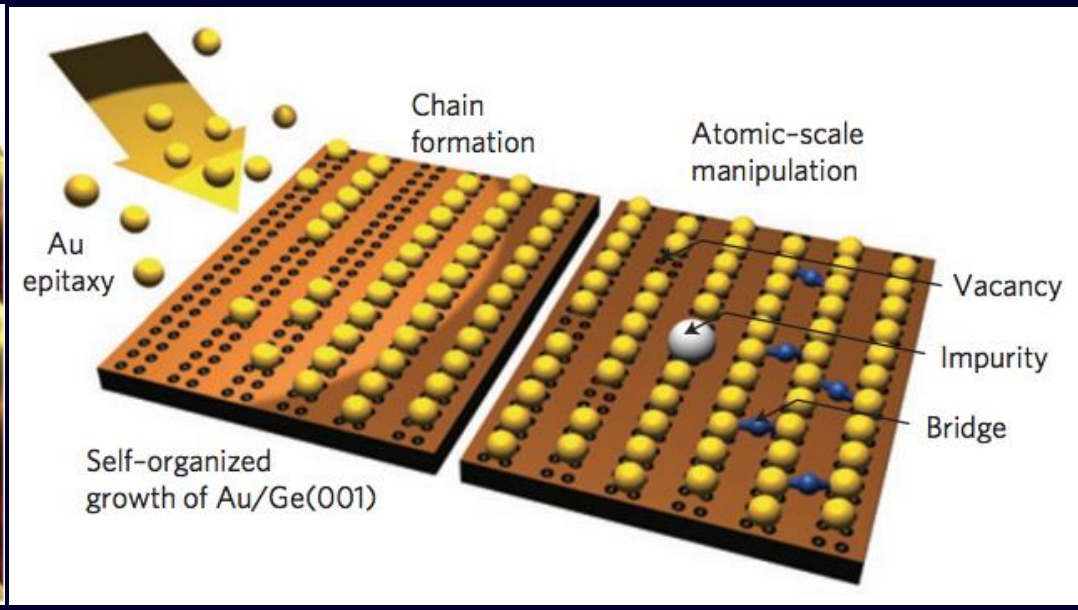
Tunneling measurement on the tube-tube junction in a four-probe configuration. (a) Linear conductance $G_X(V_X=0, T)$. (b) Differential conductance $dI_X/dV_X(V_X, T = 20\ K)$

$\alpha_{bulk} = (g+g^{-1}-2)/8$ ⟹ $g \approx 0.16$

（interaction parameter）

Example 2: **Atomically controlled quantum chains hosting a Tomonaga–Luttinger liquid**

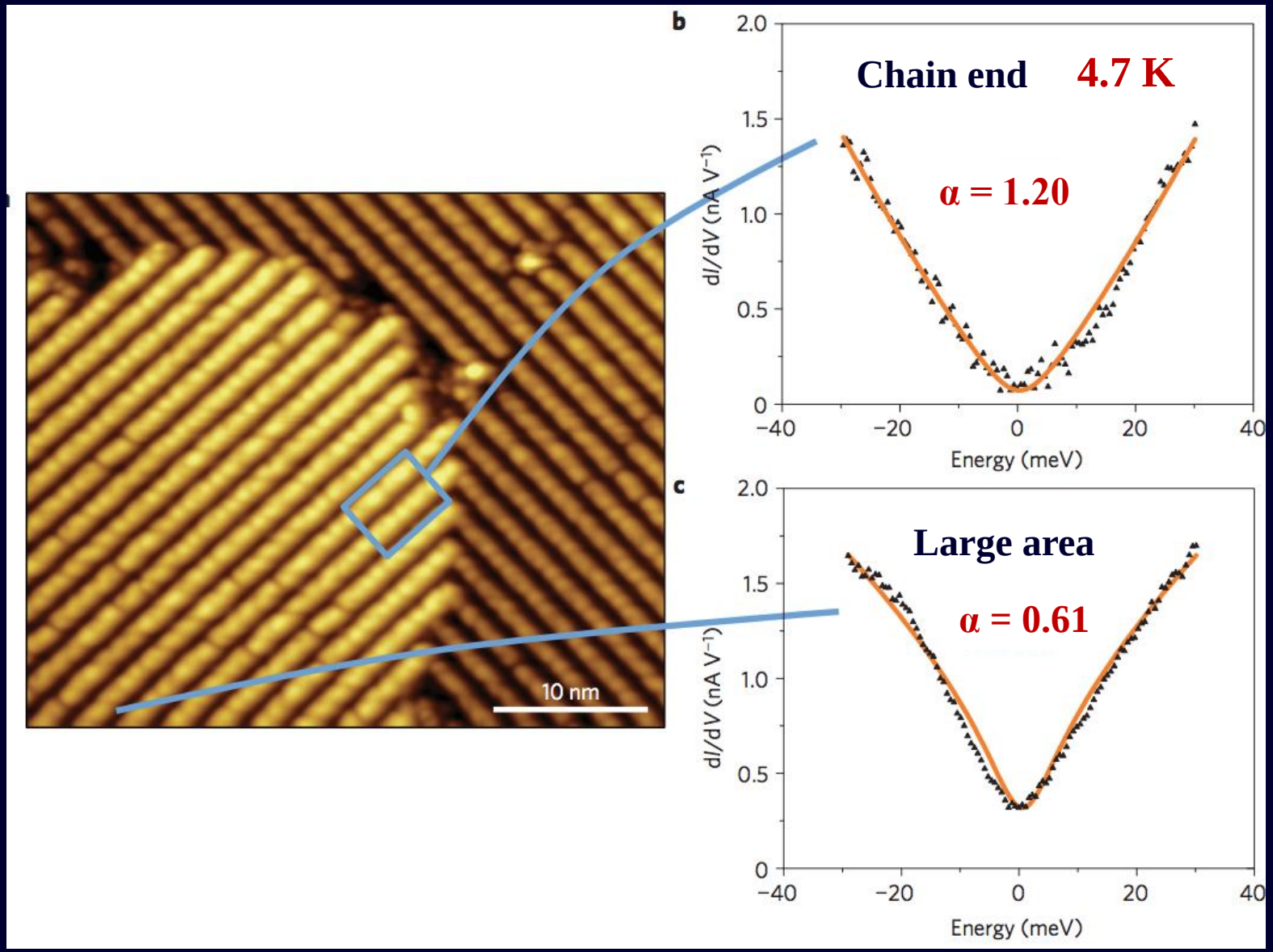




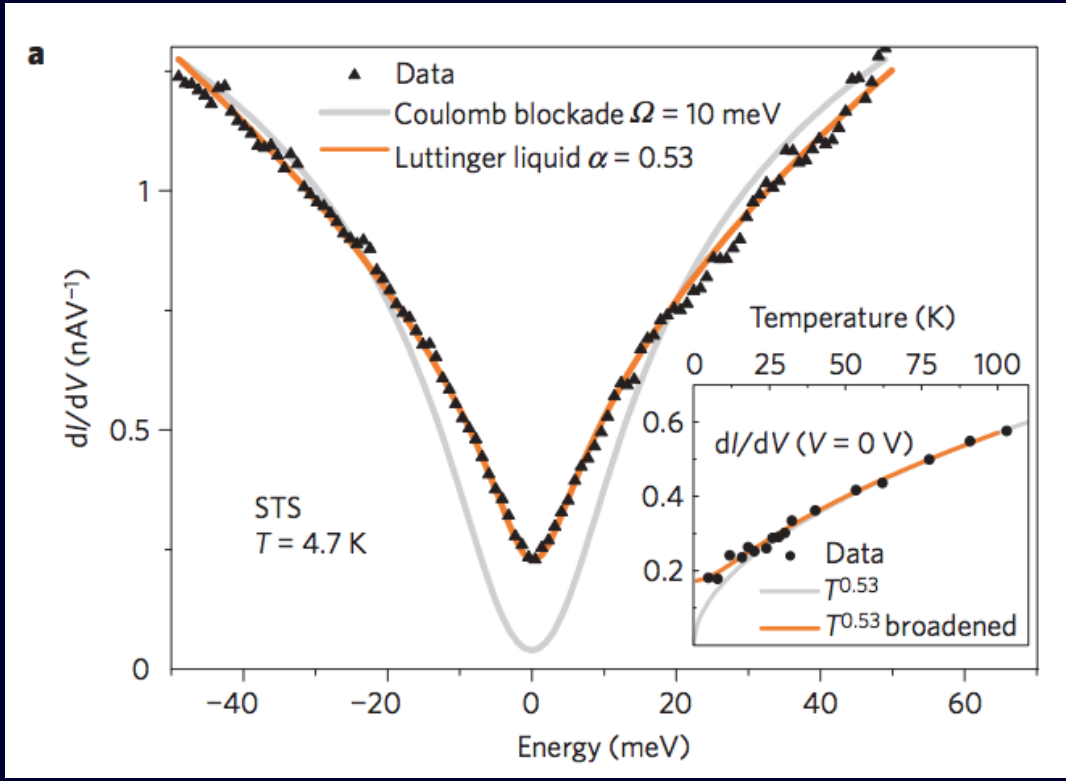
Atomic arrangement of self-organized gold chains on Ge(001).
Topography from STM, showing parallel alignment of Au-induced chains, spaced by 1.6 nm (bias −1.0 V, 0.4 nA). Schematic representation of chain formation and atomic modification possibilities.

C. Blumenstein et al., Nature Physics 7,776(2011)

The low-energy spectra univocally show power-law behaviour



Tunneling spectra near chain end with increased exponent.



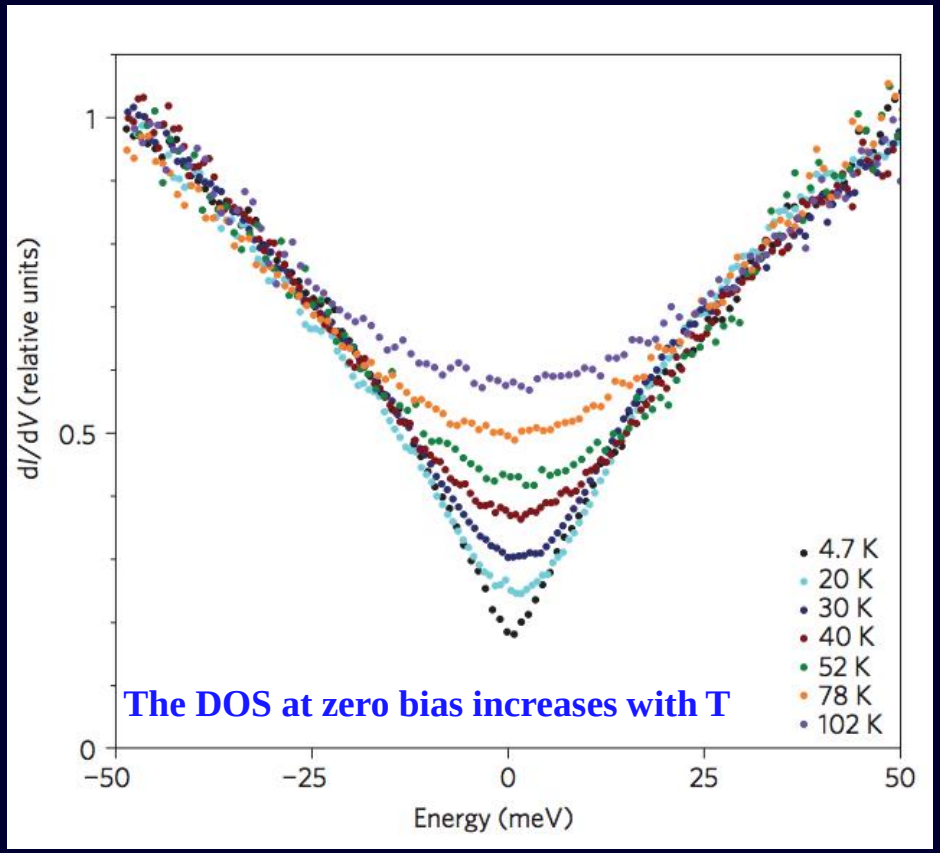
Spectroscopy of the DOS at low energies

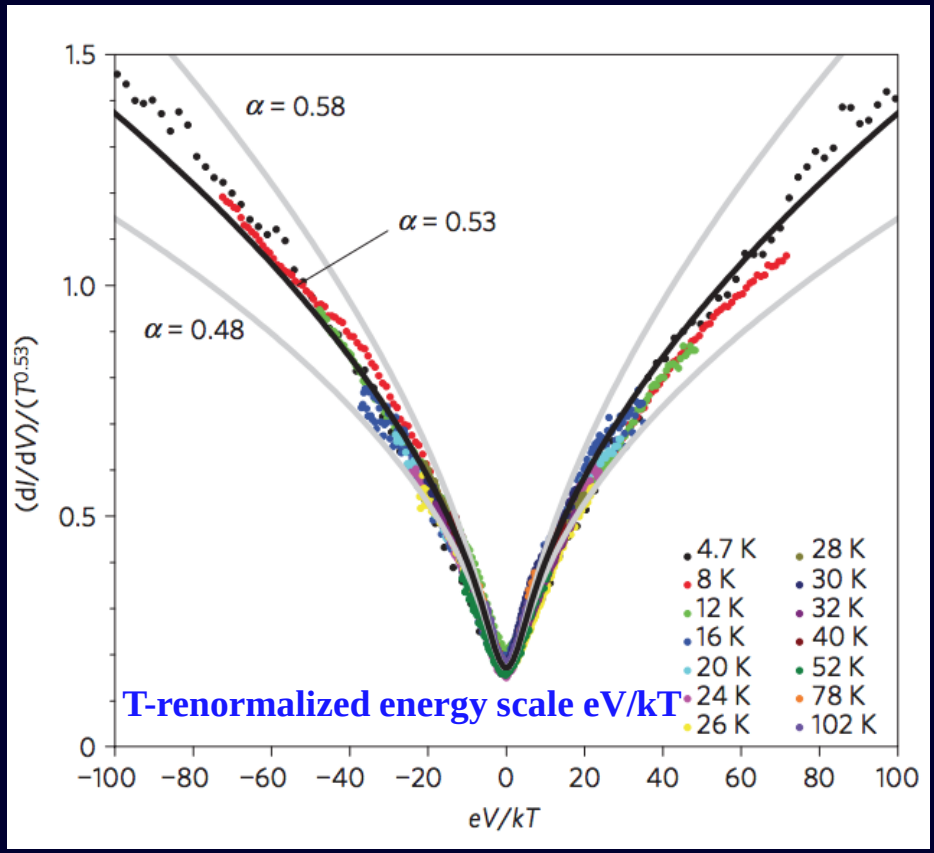
$$\mathrm{DOS}(E,T) \propto T^{\alpha} \cosh\left(\frac{E}{2kT}\right)\left|\Gamma\left(\frac{1+\alpha}{2}+i\frac{E}{2\pi kT}\right)\right|^2$$

The tunnelling DOS follows a universal scaling relation as a function of E and T, where **Γ** is the gamma function

The differential tunnelling conductivity dI/dV recorded on extended nanowires at 4.7 K. This property is a good measure of the electronic density of states (DOS), because variations of the tunnelling-matrix element are negligible for the small voltages used.

The density of states (DOS) obeys universal scaling with energy and temperature





The fit curve accounts for thermal broadening in the TLL description as in equation above, the T dependence of the Fermi distribution for the tip, and is convoluted with the experimental resolution. Black curve: TLL power law with $\alpha = 0.53$. Grey: for ± 0.05 variation of α to test statistical confidence

C. Blumenstein et al., Nature Physics 7,776(2011)

Characterizations of Luttinger Liquid (II)

LL prediction: Spin-Charge Separation

除了前向散射，还要考虑自旋交互作用(背向散射)

散射前后电子的运动方向不变，向右(或向左)的仍向右(或向左)，只是自旋取向有所交换

Hamiltonian separates into charge and spin parts:

$$
\begin{aligned}
H &= H_0 + V_1 + V_2 = H_\rho + H_\sigma \\
H_\sigma &= \sum_{p>0} (\bar{\omega}_p - \frac{pV_{1p}}{2\pi})(\Sigma_p^\dagger \Sigma_p + \bar{\Sigma}_p^\dagger \bar{\Sigma}_p) \\
H_\rho &= \sum_{p>0} [(\bar{\omega}_p + \frac{pV_{1p}}{2\pi})(R_p^\dagger R_p + \bar{R}_p^\dagger \bar{R}_p) + \frac{pV_{2p}}{\pi}(R_p \bar{R}_p + R_p^\dagger \bar{R}_p^\dagger)] \\
\bar{\omega} &= \omega_p + \frac{V_{1p}' p}{2\pi}.
\end{aligned}
$$

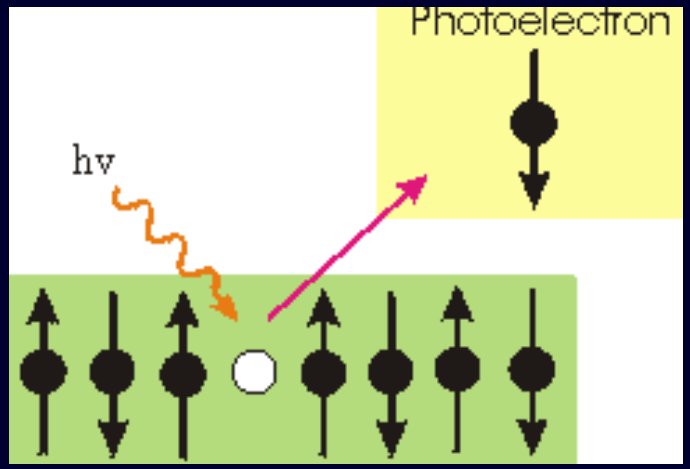
相应的准粒子：
电荷子(chargeon)
自旋子(spinon)

Spin and charge have different speeds –

spin-charge separation!

$$v_c \neq v_s$$

经过一段时间，电荷和自旋将局域在空间的不同处，是完全分离的，此现象称自旋-电荷分离



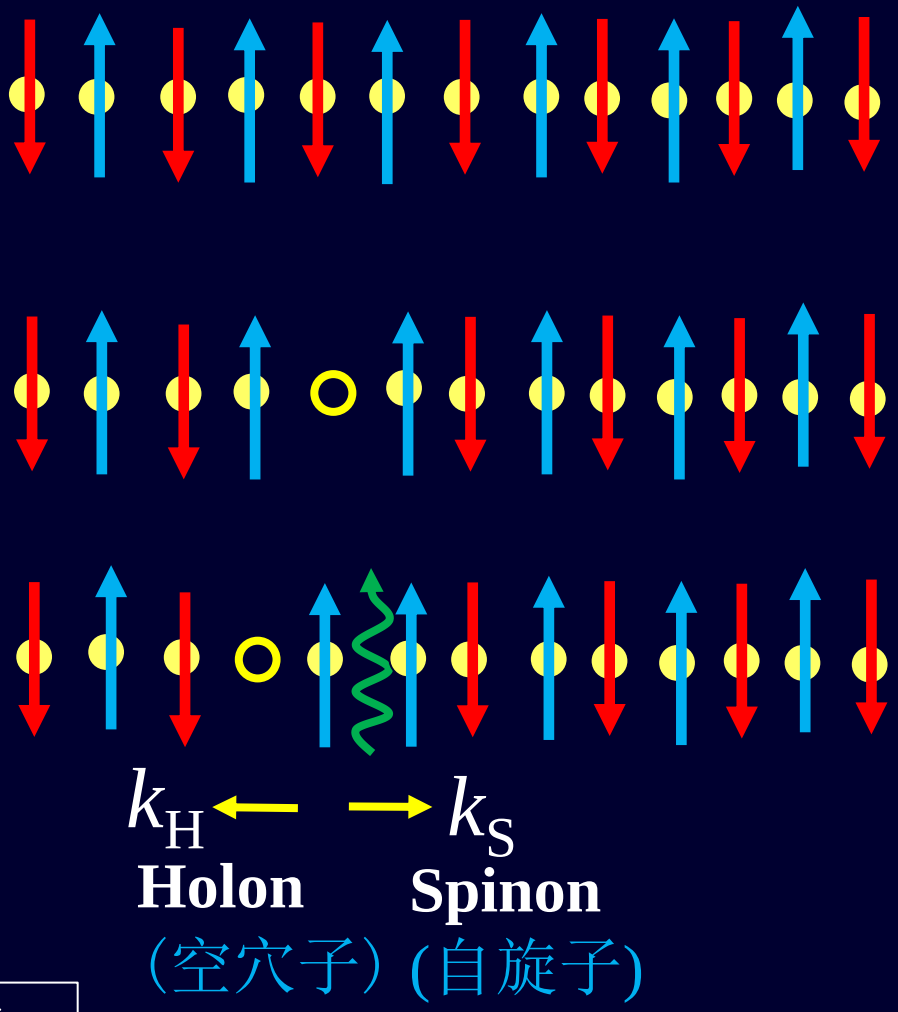
(1)能量照射致使电子逃逸：

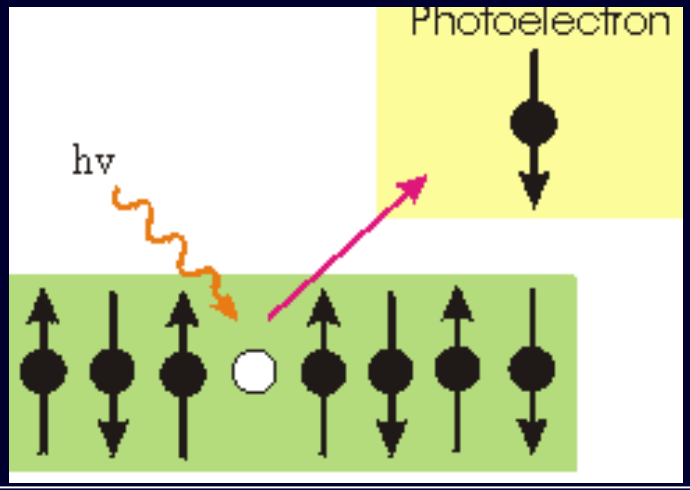
逃逸电子：电荷-e和自旋-1/2

空穴：电荷e和自旋1/2

(2)分离、换位：在电子与相邻空穴或电子换位时自旋取向不变(因哈伯德(Hubbard)模型哈密顿量的动能项不涉及自旋的反转)

最初同时具有电荷自旋属性的空穴，已降解成分别只有自旋属性的自旋子和只有电荷属性的空穴子(因与背景的反铁磁链相比，它们额外具有的，分别是电荷+e和自旋1/2)





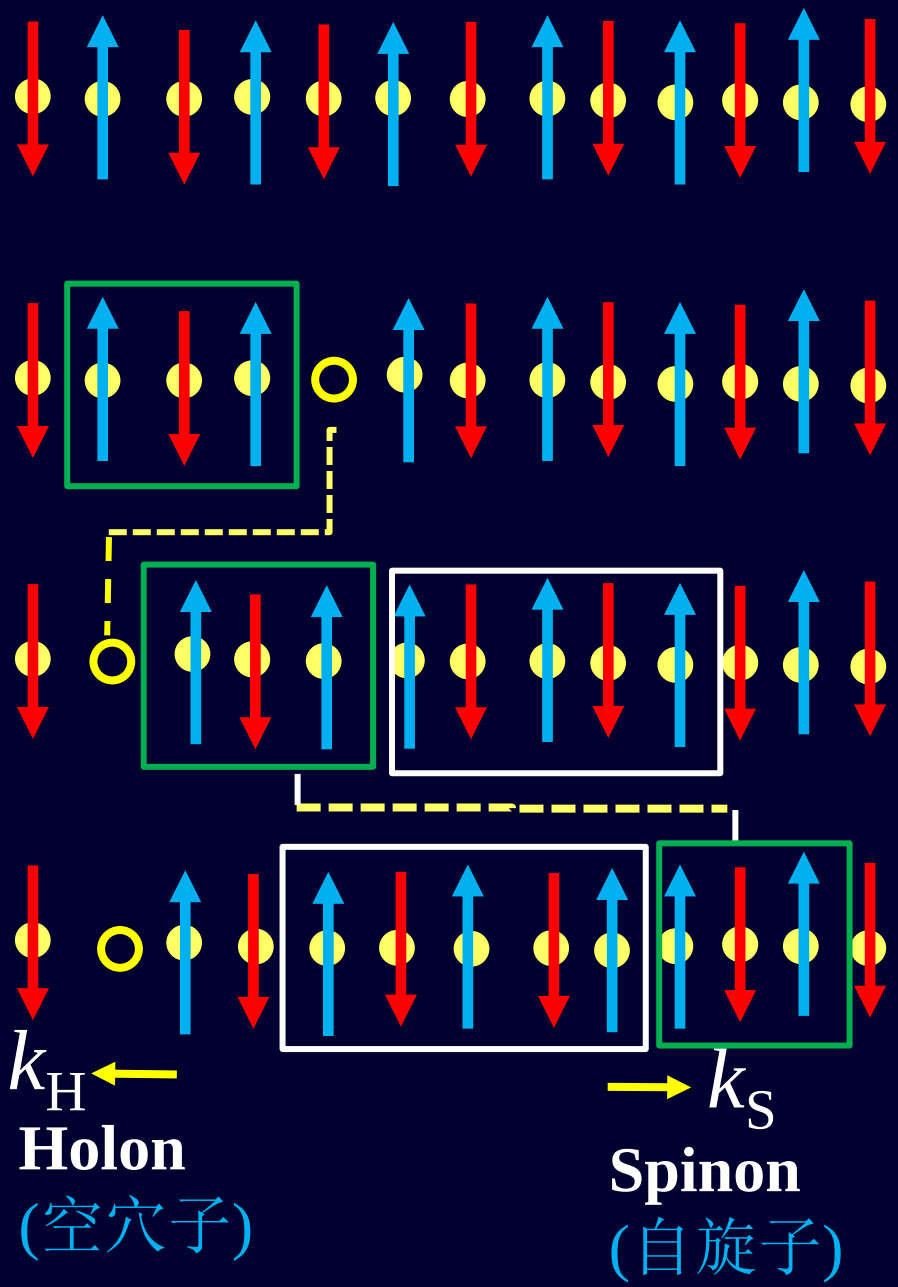
(1)能量照射致使电子逃逸：

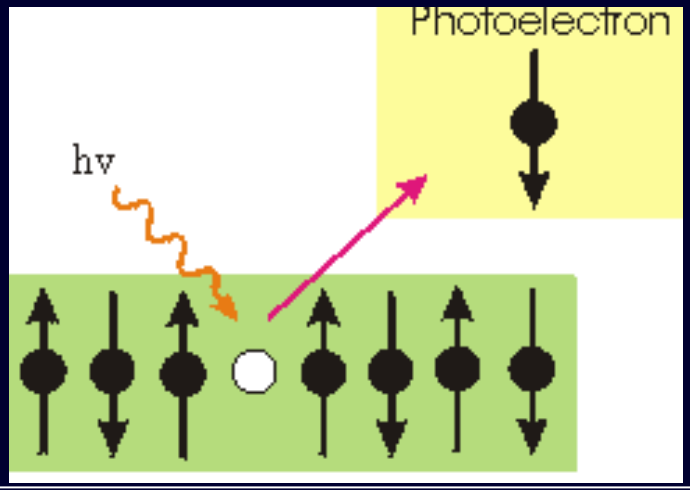
逃逸电子： 电荷-e和自旋-1/2

空穴： 电荷e和自旋1/2

(2)分离、换位：在电子与相邻空穴或电子换位时自旋取向不变 (因哈伯德(Hubbard)模型哈密顿量的动能项不涉及自旋的反转)

最初同时具有电荷自旋属性的空穴，已降解成分别只有自旋属性的自旋子和只有电荷属性的空穴子 (因与背景的反铁磁链相比，它们额外具有的，分别是电荷+e和自旋1/2)





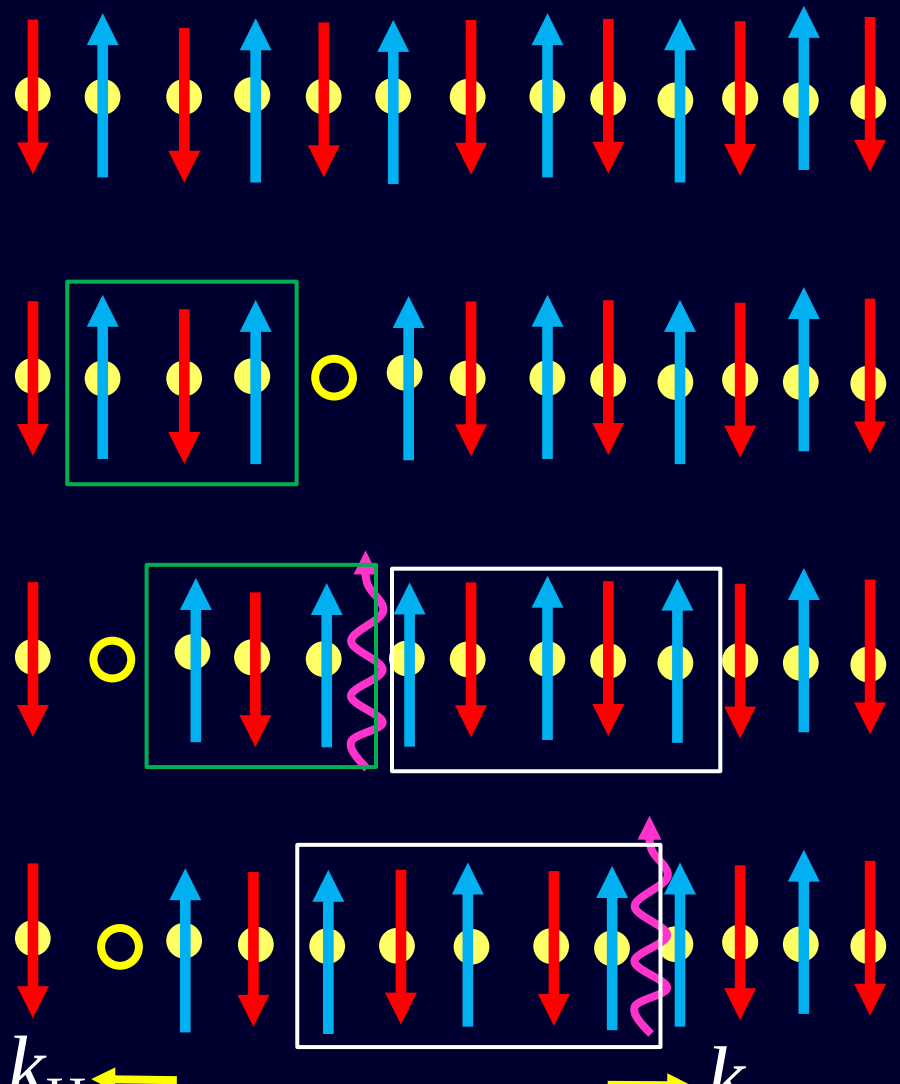
(1)能量照射致使电子逃逸：

逃逸电子： 电荷-e和自旋-1/2

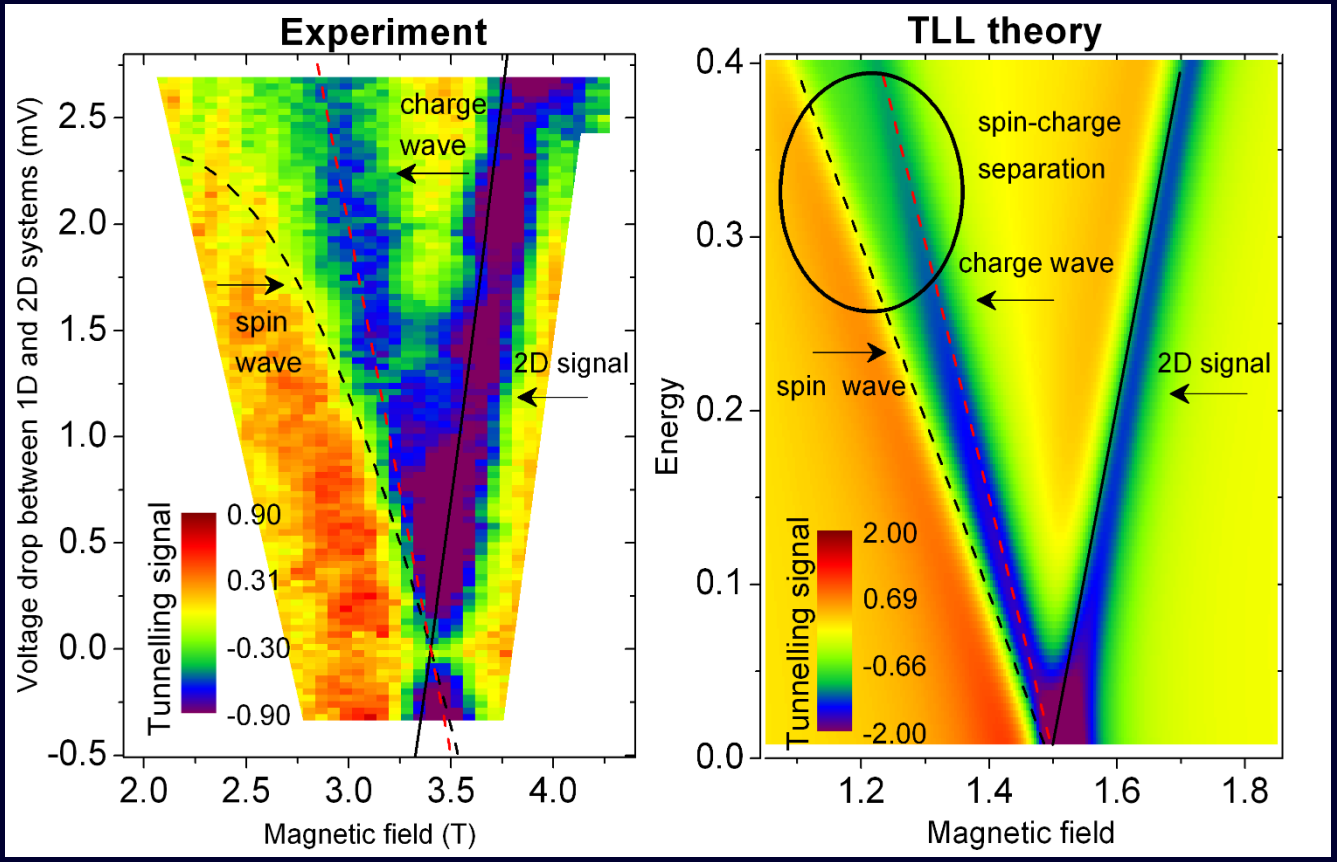
空穴： 电荷e和自旋1/2

(2)分离、换位：在电子和相邻空穴或电子换位时自旋取向不变(因Hubbard模型哈密顿量的动能项不涉及自旋的反转)

最初同时具有电荷自旋属性的空穴，已降解成分别只有自旋属性的自旋子和只有电荷属性的空穴子 (因与背景的反铁磁链相比，它们额外具有的，分别是电荷+e和自旋1/2)



Example 3: Spin-charge separation has been probed in quantum wires



Lines corresponding to charge and spin waves (holons and spinons), for experiment and theory. The different slopes for each show that they travel at different speeds. The spin line is curved in experiment but not in theory, because of the approximation in the theory that makes the calculation possible.

Science **325**, 597-601 (2009)

Summary:

- **Fermi liquid**
- **Luttinger liquids (LLs)**

  Basic concepts

  Keys to the model of Luttinger liquids
- **Characterizations of Luttinger Liquid:**

  (I) power law scaling of variety of physical observables

  $G(T) \propto T^{\alpha}$ with at small biases ($eV << k_B T$):

  $dI/dV \propto V^{\alpha}$ with V at large biases ($eV >> k_B T$)

  (II)Spin-Charge Separation

  Holon & Spinon