1、请分别从无质量粒子（如，光子）和有质量粒子在 Schwarzschild 时空中的运动方程出发，求其圆轨道（即 $e = 0$）时的半径 $R^{(\text{massless})}$ 和 $R^{(\text{massive})}$，结果用 $L$ 和 $M$ 表示（$G$ 和 $c$ 可以带着，也可以取为 1）。

2、请从上题所得结果出发，讨论其所意味着的最小圆轨道半径。 [Hint：$R_{\min}^{(\text{massless})} = 3M$、$R_{\min}^{(\text{massive})} = 6M$]

3、[from Schutz's book] A clock is in a circular orbit at $r = 10M$ in a Schwarzschild metric.

(a) How much time elapses on the clock during one orbit? (Integrate the proper time $\mathrm{d}\tau = \sqrt{|\mathrm{d}s^2|}$ over an orbit.)

(b) It sends out a signal to a distant observer once each orbit. What time interval does the distant observer measure between receiving any two signals?

(c) A second clock is located at rest at $r = 10M$ next to the orbit of the first clock. (Rockets keep it there.) How much time elapses on it between successive passes of the orbiting clock?

(d) Calculate (b) again in seconds for an orbit at $r = 6M$ where $M = 14\,M_\odot$. This is the minimum fluctuation time we expect in the X-ray spectrum of Cyg X-1（天鹅座 X-1）：why?

(e) If the orbiting 'clock' is the twin Alice, in the orbit in (d), how much does she age during the time her twin Bob lives 40 years far from the black hole and at rest with respect to it?

4、证明在 Kruskal-Szekeres 坐标下，线元为

$$\mathrm{d}s^2 = \frac{32M^3}{r}\mathrm{e}^{-r/2M}\left(-\mathrm{d}v^2 + \mathrm{d}u^2\right) + r^2\mathrm{d}\Omega^2\,.$$