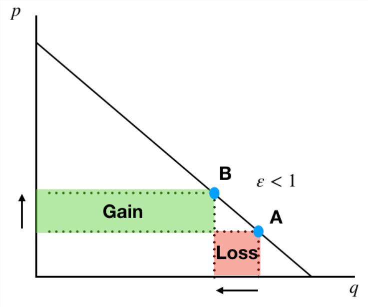
# Microeconomics

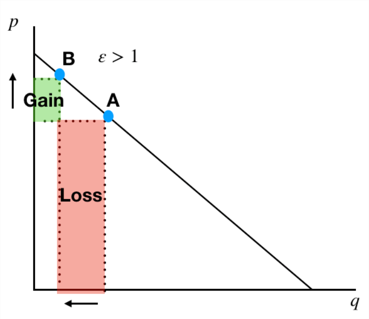
## 微观经济学

Yu Gao

24 November 2021

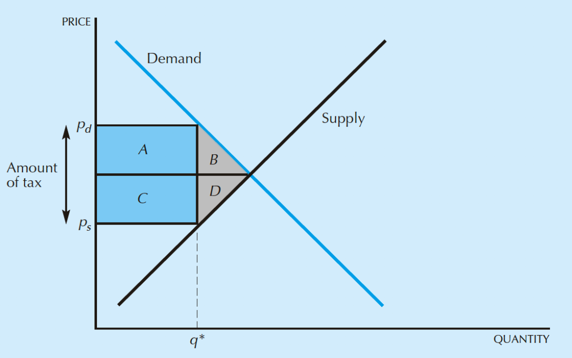
# Refresh: Elasticity




It seems that the net revenue of a price change depends on the elasticity of demand.

# Refresh: Deadweight loss




- which part is the changes in consumer's surplus?
- which part is the changes in producer's surplus?
- which part is collected by the government?
- which part is the social cost (deadweight loss) of the tax?

# Table of contents

# Firm behaviors

# Consumer behavior and firm behavior

Firms are special "consumers". They purchase "inputs". Their "utility" comes from profits.

Output of a production process is observable whereas the "output" of consumption (utility) is not directly observable.

| Consumers | Firms |
|---|---|
| Utility function | Production function |
| Indifference curve | Isoquant curves 等产量线 |
| Utility maximizing | Profit maximizing |

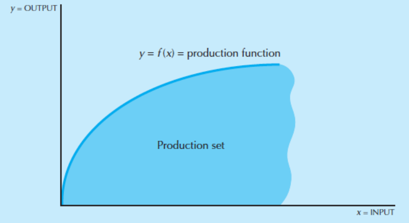
# Technological constraints: production function

# Production function 生产函数

Assuming the firms has only one output, then

$$Q = f(x_1, x_2, ..., x_n) = f(\mathbf{x})$$

where **x** is a vector of inputs, for example workers, computers, machines, land, etc.

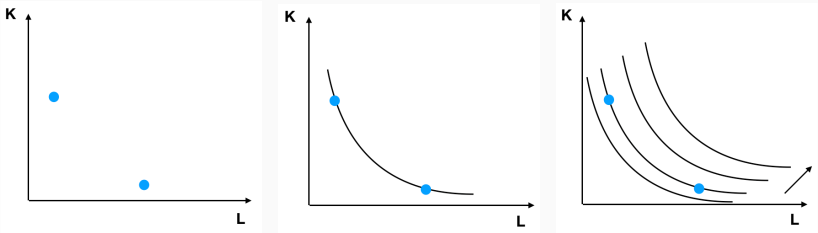



For simplicity, we usually assume two inputs: labor ($L$) and capital ($K$). An example of the production function:

$$f(x_1, x_2) = x_1^a x_2^{1-a}$$

# Isoquant curves 等产量线

In consumer theories, given a utility function, we can draw a series of indifference curves.
Likewise, given a production function, we can draw a series of isoquant curves (for simplicity, assuming two inputs: $K$ and $L$):

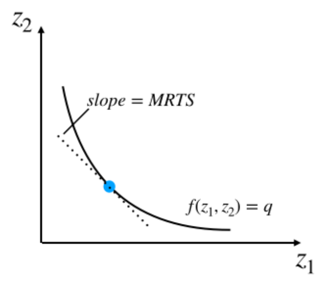



**Think**

What assumptions did we make for this shape of isoquant curves?

# Marginal rate of technical substitution (MRTS)

In the two-dimensional case, we can increase the amount of input 1 and decrease the amount of input 2 so as to maintain a constant level of output.



# Marginal rate of technical substitution (MRTS) 边际技术替代率

Assume a production function $f(x_1, x_2) \equiv y$.
Take the total differential of $f(\mathbf{x})$:

$$dy = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2$$

Since output remains constant, we have

$$0 = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2$$

which can be solved for

$$\frac{dx_2}{dx_1} = -\frac{\partial f / \partial x_1}{\partial f / \partial x_2} = -\frac{MP_1}{MP_2}$$

where $MP_i$ is the marginal production of factor $i$.
Sometimes MRTS is taken as the absolute value of its original form, which gives a positive number.

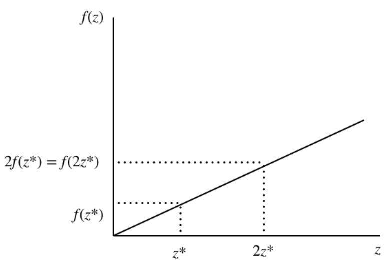
# Returns to scale 规模效应

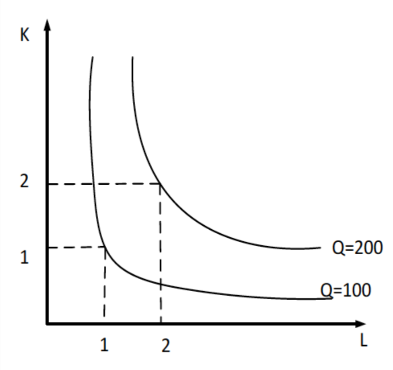
# Returns to scale 规模效应

What will happen to the level of output if we scale all inputs up or down by some amount $t \geq 0$?

**Constant returns to scale (CRS) 规模效应不变**:

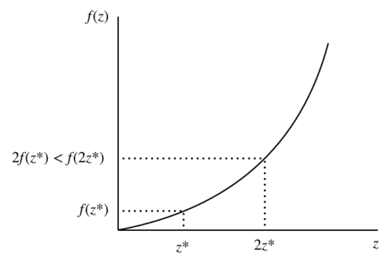
- $f(t\mathbf{x}) = tf(\mathbf{x})$ for all $t \geq 0$; i.e., the production function $f(\mathbf{x})$ is homogeneous of degree 1.

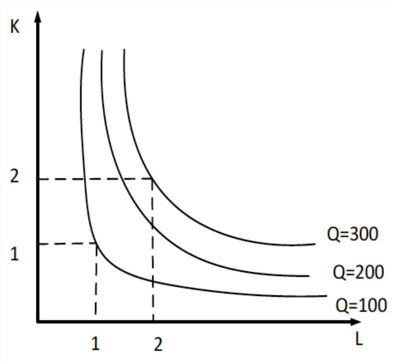


**Increasing returns to scale (IRS) 规模效应递增**:

- $f(t\mathbf{x}) > tf(\mathbf{x})$ for all $t > 1$.

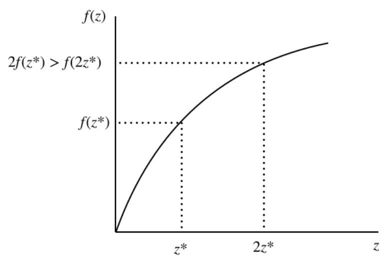


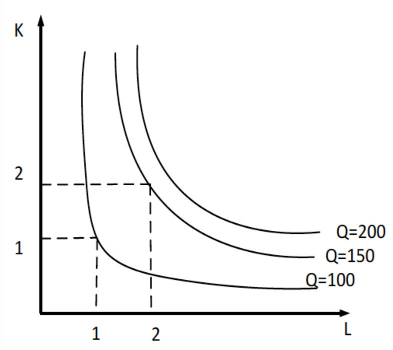



# Returns to scale 规模效应

**Decreasing returns to scale (DRS) 规模效应递减**:

- $f(t\mathbf{x}) < tf(\mathbf{x})$ for all $t > 1$.



# Returns to scale 规模效应

Let us check returns to scale in the Cobb-Douglas production function $f(\lambda x_1, \lambda x_2) = x_1^{\alpha} x_2^{\beta}$. Increasing all arguments by a common factor $\lambda$, we obtain $f(x_1, x_2) = (\lambda x_1)^{\alpha} (\lambda x_2)^{\beta} = \lambda^{\alpha+\beta} x_1^{\alpha} x_2^{\beta}$.

- When $\alpha + \beta = 1$, we have constant returns to scale.
- When $\alpha + \beta > 1$, we have increasing returns to scale.
- When $\alpha + \beta < 1$, we have decreasing returns to scale.

# Returns to scale 规模效应

**Think**

What affects returns to scale?

# Profit maximization

# Firm's problem: profit maximization

Assumptions:

- Firms are price takers, implying that the production plans of every individual firm do not alter market prices $p$.
  - This holds for both output and input.
  - It implies that the firm's share in the product and input market is negligible.
- The production set satisfies free disposal (the producer can dispose of the additional inputs he does not need at no cost).

We consider a very simple case where a firm using several inputs to produce a single output.

# Firm's problem: profit maximization

Profit=Revenue-Cost

**利润 = 收益-成本**

Assume the firm only produces one output, then its profit function can be written as

$$\max_{\mathbf{x}} pf(\mathbf{x}) - \mathbf{wx}$$

where

- $p$ is the price of output
- $\mathbf{w}$ is the vector of factor prices (rent, salary, etc.)
- $\mathbf{x} = (x_1, ..., x_n)$ are inputs

Note: unlike the utility maximization problem of consumers, the PMP does not impose a budget constraint (its objective function already considers the cost of each production plan), and thus becomes an easier unconstrained problem.

# Firm's problem: profit maximization

F.O.C. of interior solutions:

$$p \frac{\partial f(\mathbf{x}^*)}{\partial x_i} = w_i$$

where $MP_i = \frac{\partial f(\mathbf{x}^*)}{\partial x_i}$.

- The market value of the marginal product obtained from using an additional unit of input $i$, $p \cdot MP_i$ must coincide with the price of acquiring an additional unit of the input, $w_i$.

# Firm's problem: profit maximization

Rearranging it as $p = \frac{w_i}{MP_i}$,

$$\frac{w_k}{w_l} = \frac{MP_k}{MP_l} \equiv MRTS_{l,k}(x^*)$$

or

$$\frac{MP_k}{w_k} = \frac{MP_l}{w_l}$$

- The marginal productivity per dollar spent on input $l$ is equal to that spent on input $k$.

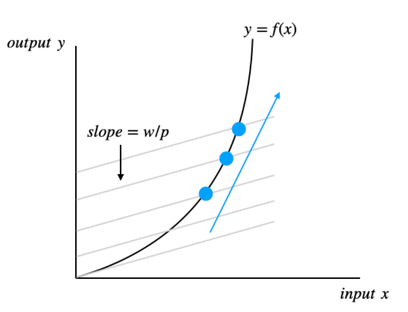
# Firm's problem: profit maximization

Let's assume one input $x$ with price $w$ and one output $f(x) = y$ with price $p$.

Profits are given by $\pi = py - wx$. Then on the figure below we see $y = \pi / p + (w/p)x$ for different levels of $\pi$.

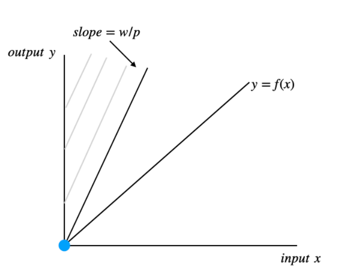
- The intercept increases with $\pi$.
- The slope is fixed to $w/p$.

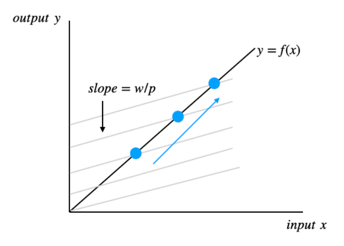
When f(x) is convex: $\frac{d^2 f(x^*)}{dx^2} > 0$



# Firm's problem: profit maximization

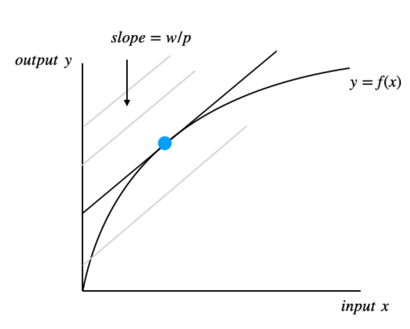
When f(x) is linear: $\frac{d^2 f(x^*)}{dx^2} = 0$



# Firm's problem: profit maximization

When f(x) is concave: $\frac{d^2 f(x^*)}{dx^2} < 0$




The optimal point is $\frac{df(x^*)}{dx} = \frac{w}{p}$.

# Cost minimization

# Cost minimization

What is the difference between a cost minimizing firm and a profit maximizing firm?

- Some “firms” do not aim at maximizing profits (say, governments, NGOs). Instead, they target an output level (say, service) and try to minimize their cost.
- Another way to look at the supply behavior of a firm facing competitive output market

# Cost minimization problem (CMP)

Consider the problem of finding a cost-minimizing way to produce a given level of output:

$$\min_{x} \mathbf{w} \cdot \mathbf{x}$$

$$s.t.\ f(\mathbf{x}) = y$$

Solve by Lagrangian:

$$\mathcal{L} = \mathbf{w} \cdot \mathbf{x} + \lambda(y - f(\mathbf{x}))$$

F.O.C.:

$$w_i - \lambda \frac{\partial f(\mathbf{x}^*)}{\partial x_i} = 0$$

$$f(\mathbf{x}^*) = y$$

# Cost minimization

$$w_i - \lambda \frac{\partial f(\mathbf{x}^*)}{\partial x_i} = 0 \tag{1}$$

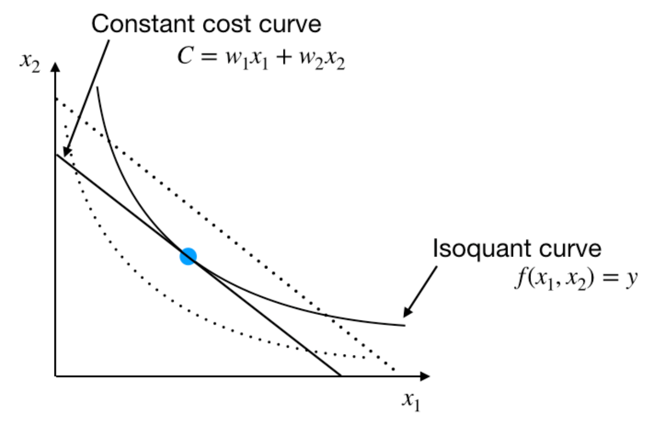
$$w_j - \lambda \frac{\partial f(\mathbf{x}^*)}{\partial x_j} = 0 \tag{2}$$

(1)/(2) gives:

$$\frac{w_i}{w_j} = \frac{\frac{\partial f(\mathbf{x}^*)}{\partial x_i}}{\frac{\partial f(\mathbf{x}^*)}{\partial x_j}}$$

- The left-hand side tells us at what rate factor $j$ can be substituted for factor $i$ while maintaining a constant cost (the economic rate of substitution).
- The right-hand side tells us at which rate factor $j$ can be substituted for factor $i$ while maintaining a constant level of output (the technical rate of substitution).
- They should be equal at the optimum.

# Cost minimization




**Cost minimization**: the isoquant must be tangent to the constant cost line ($x_2 = C/w_2 - (w_1/w_2)x_1$).

# PMP and CMP

Remember the PMP:

$$\max_{x_i} py - \mathbf{w} \cdot \mathbf{x}$$

Using the cost function, we can restate the firm's problem as

$$\max_{y} py - c(\mathbf{w}, y)$$

F.O.C. for $y^*$ to be profit maximizing is then

$$p - \frac{\partial c(w, y^*)}{\partial y} = 0$$

$$p = \frac{\partial c(w, y^*)}{\partial y}$$

In other words, at the optimum, marginal benefit (price) = marginal cost.

# The geometry of cost

# The geometry of cost

We denote the output by $y$ and hold the vector of factor prices constant at $\bar{w} \gg 0$.

- Average cost:

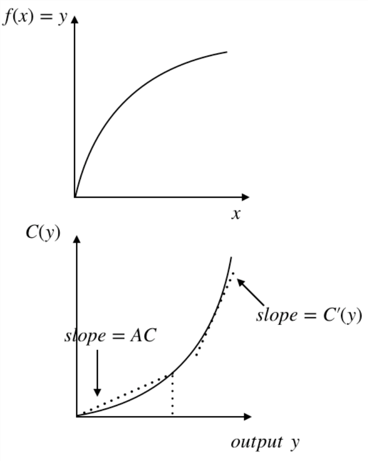
$$AC(y) = \frac{C(y)}{y}$$

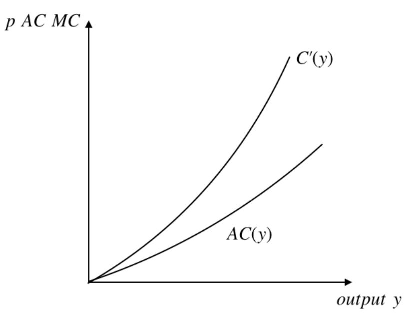
- Marginal cost:

$$C'(y) = \frac{dC(y)}{dy}$$

# The geometry of cost: Case 1

A strictly decreasing returns to scale technology:

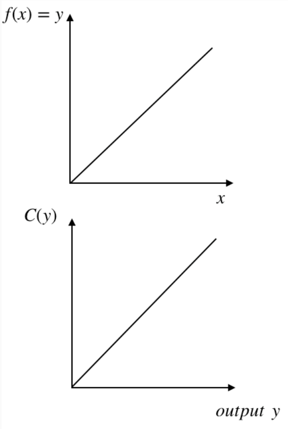


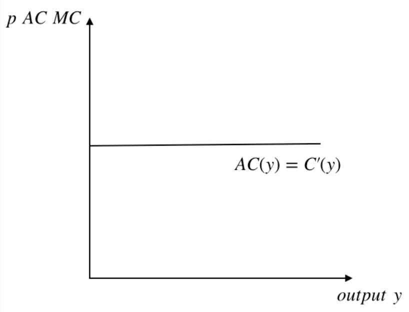




- Because of DRS, both $C'(q)$ and $AC(q)$ are increasing.

## The geometry of cost: Case 2

A constant returns to scale technology:

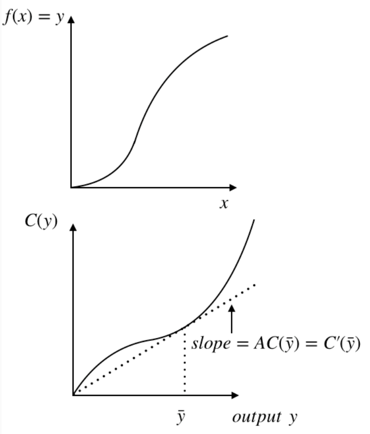


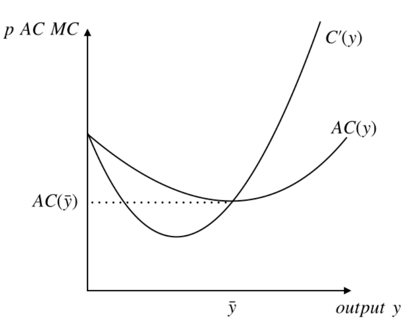




- Because of CRS, $C'(q) = AC(q)$.

# The geometry of cost: Case 3

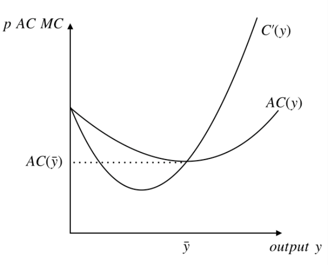
First increasing then decreasing returns to scale technology:



# The geometry of cost: Case 3

Remark:



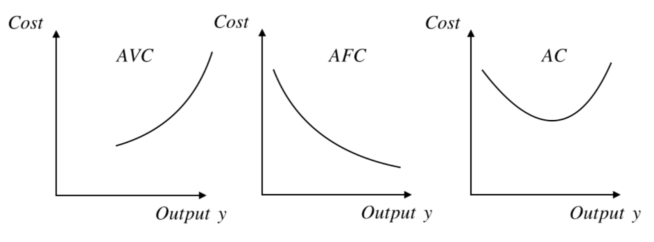

1. AC=MC at $y = 0$, i.e., AC(0)=MC(0).
2. When MC<AC, the AC curve decreases, and when MC>AC, the AC curve increases.
   - Intuition: using the example of grades.
   - If the new exam score raises your average grade, it must be that such new grade is better than your average grade thus far.
   - If, in contrast, the new exam grade score lowers your average grade, it must be that such new grade is lower than your average grade thus far.
3. In other words, AC and MC curves cross (AC=MC) at exactly the minimum of the AC curve.

# Case 4: Variable cost and fixed cost

Think about the last situation:

- At first, when we increase output, AC is decreasing.
- Only when we approach some capacity level of output, our AC starts to increase.

An important source of nonconvex cost is fixed setup costs.




Average cost (AC) = Average variable cost (AVC) + Average fixed cost (AFC)

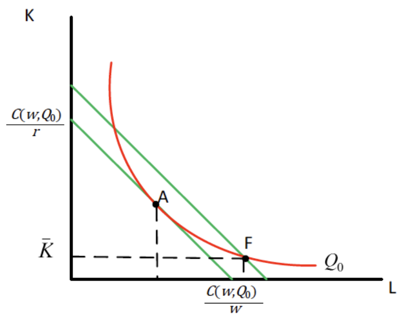
# Short run vs. Long run

The key difference between short run and long run is if there is any prior input commitments (inputs are adjustable or not).

- In the short run, the firm does not have the flexibility of all input choice (e.g., labor is flexible, but not capital).
- In the long run, the firm can modity the amounts of all inputs.

## Short run vs. Long run

- In the short-run
  - capital is fixed at $\bar{K}$
  - the firm cannot equate MRTS with the ratio of input prices
- In the long-run
  - firm can choose input vection A, which is a cost-minimizing input combination.



# Short run vs. Long run

Let $\mathbf{x}_f$ be the vector of fixed factors, $\mathbf{x}_v$ be the vector of variable factors.

- Short-run cost function (STC):

$$c(\mathbf{w}, y, \mathbf{x}_f) = \mathbf{w}_v \mathbf{x}_v(\mathbf{w}, y, \mathbf{x}_f) + \mathbf{w}_f \mathbf{x}_f$$

We can omit $\mathbf{w}$ because factor prices are taken as given,

$$c(y, \mathbf{x}_f) = \mathbf{w}_v \mathbf{x}_v(y, \mathbf{x}_f) + \mathbf{w}_f \mathbf{x}_f$$

- Long-run cost function (LTC):

$$c(\mathbf{w}, y) = \mathbf{w}_v \mathbf{x}_v(\mathbf{w}, y) + \mathbf{w}_f \mathbf{x}_f(\mathbf{w}, y)$$

Omit $\mathbf{w}$,

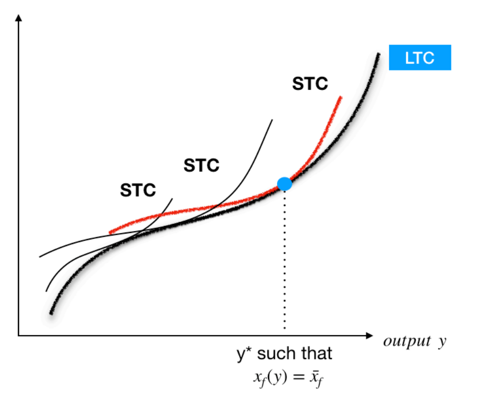
$$c(y) = \mathbf{w}_v \mathbf{x}_v(y) + \mathbf{w}_f \mathbf{x}_f(y)$$

# Short run vs. Long run: Total cost

LTC is the lower envelope of the family of short-run functions STC generated by letting $x_f$ take all possible values.

$$STC = c(y, \mathbf{x}_f) = \mathbf{w}_v \mathbf{x}_v(y, \mathbf{x}_f) + \mathbf{w}_f \mathbf{x}_f$$

$$LTC = c(y) = \mathbf{w}_v \mathbf{x}_v(y) + \mathbf{w}_f \mathbf{x}_f(y)$$

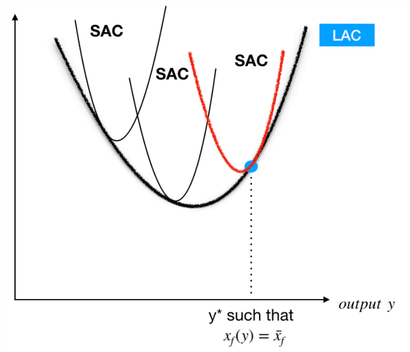


# Short run vs. Long run: Average cost

LAC is the lower envelope of the family of short-run functions SAC generated by letting $x_f$ take all possible values.

$$SAC = \frac{c(y, \mathbf{x}_f)}{y} = \frac{\mathbf{w}_v \mathbf{x}_v(y, \mathbf{x}_f) + \mathbf{w}_f \mathbf{x}_f}{y}$$

$$LAC = \frac{c(y)}{y} = \frac{\mathbf{w}_v \mathbf{x}_v(y) + \mathbf{w}_f \mathbf{x}_f(y)}{y}$$



# Aggregation in Production

# Is there a "representative producer"?

Consider $J$ firms with production levels $y_1, y_2, ..., y_J$. Define the aggregate supply correspondence as the sum of the individual supply correspondences

$$y(p, w) = \sum_{j=1}^{J} y_j(p, w)$$

for $j = 1, 2, ..., J$.

# Is there a "representative producer"?

There exists a representative producer:

- Producing an aggregate supply $y^*(p, w)$ that exactly coincides with the sum $\sum_{j=1}^{J} y_j(p)$; and
- Obtaining aggregate profits $\pi^*(p, w)$ that eactly coincide with the sum $\sum_{j=1}^{J} \pi_j(p, w)$.

Intuition: The aggregate profit obtained by each firm maximizing its profits separately (taken prices as given) is the same as that which would be obtained if all firms were to coordinate their actions (i.e., $y_j'$s) in a joint PMP.

# Is there a "representative producer"?

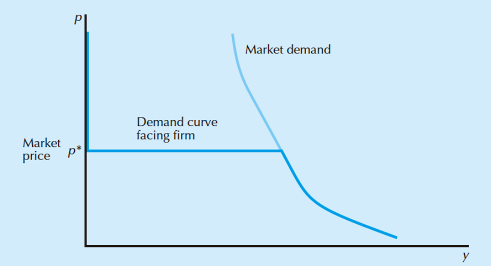
- It is a "decentralization" result: to find the solution of the joint PMP for given prices $p, w$, it is enough to "let each individual firm maximize its own profits" and add the solutions of their individual PMPs.
- Key: price taking assumptions.
  - The results does not hold if firms have market power.

# Market

# Competitive market 竞争市场

# Competitive market

A competitive firm takes the market price of output as being given.




- If sell above market price, no demand.
- If sell below the market price, not wise, because customers are willing to buy at higher.
- So it must sell at the market price.

# Firm's supply

Shutdown vs. Exit

- **Shutdown**: A short-run decision not to produce anything because of market conditions
- **Exit**: A long-run decision to leave the market
- A key difference:
  - If shut down in the short-run, must still pay fixed cost.

Example: a restaurant that shuts down during holidays.

# A firm's short run decision to shut down

The firm has two concerns:

1. The cost faced by a firm in the short run:

$$c(y) = c_v(y) + c_f$$

   The firm will produce when the revenue of producing exceeds the variable cost:

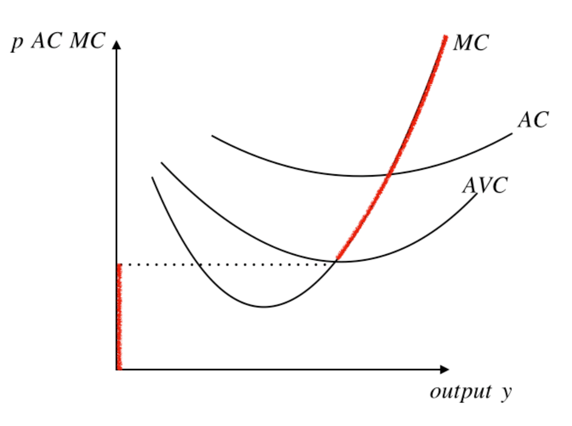
$$py - c_v(y) \geq 0$$

$$p \geq \frac{c_v(y)}{y} = AVC$$

2. Remember from PMP and CMP we know that price equals marginal cost:

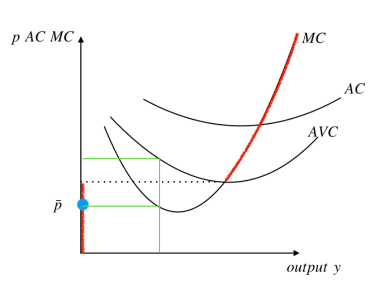
$$p = \frac{\partial c(w, y^*)}{\partial y} = MC$$

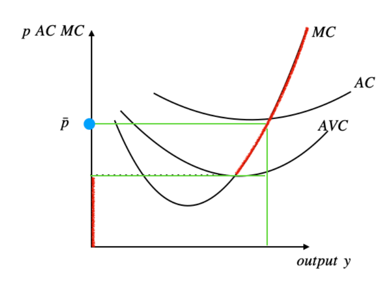
# A firm's short run supply curve




$$y = \begin{cases} 0 & if\ \bar{p} < minAVC(y) \\ MC^{-1}(p) & if\ \bar{p} \geq minAVC(y) \end{cases}$$

# Identify loss and gain



# A firm's long run decision to exit

In the long-run, fixed cost should be taken into consideration. The firm's concern becomes:

$$py - c_(y) \geq 0$$

$$p \geq \frac{c(y)}{y} = ATC$$

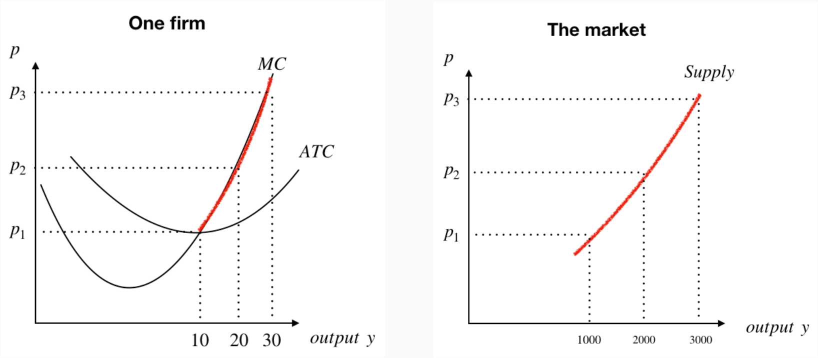
- The firm will exit if $p < ATC$
- New firm will enter if $p > ATC$

# The industry supply function

The **industry supply function** is simply the sum of the individual firm supply function:

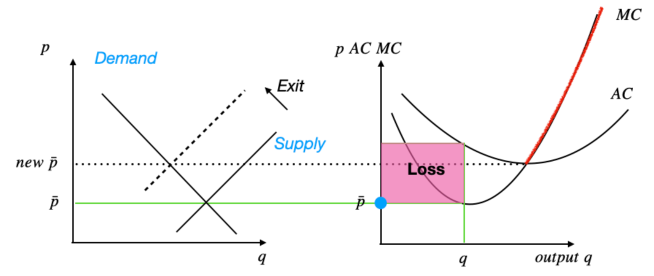
$$Y(p) = \sum_i y_i(p)$$

Intuition: consider 100 identical firms (in the short run)



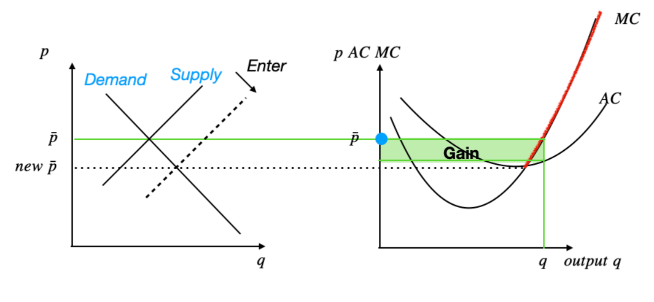
# Long-run supply curve

Long-run equilibrium: the process of entry or exit is complete – remaining firms earn zero economic profits.



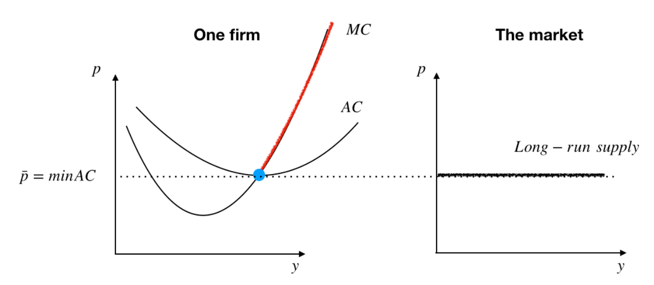
# Long-run supply curve

Long-run equilibrium: the process of entry or exit is complete – remaining firms earn zero economic profits.



## Long-run supply curve

The long-run market (industry) supply is horizontal at $p = minimumATC$.



**Questions?**