Room 2-317

Ultrafast Optics

→ Briefing on course: English Course.

Homework, exams, text books.

→ Course outlines:

*. fundamentals: How to describe pulsed laser field.

*. Pulse propagation: linear / nonlinear effects.

*. Femto-second optics: components, sources. measurement techniques; controlling / manipulation; Applications.

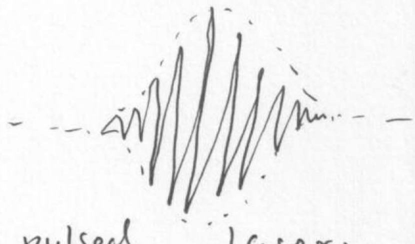
Introduction:

Let's look at E-field of laser

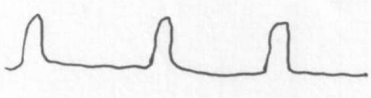
Time scale: optical wave

optical period: $T = \frac{\lambda}{c} = \frac{600\,nm}{c} \approx 2fs$
$= 2 \times 10^{-15}\ s$

pulsed E-field:



The ways to describe pulsed laser:

repetition rate:  Hz

pulse width: $\sim 10^{-9} \sim 10^{-15} \rightarrow 10^{-18}$ s

average power: W.

center wavelength: nm.

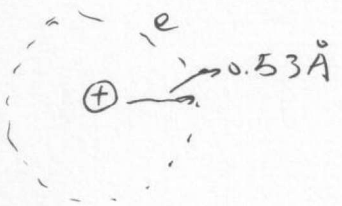
why pulsed laser:

* peak power: eg. 100 fs , 1 mJ . focus on 10 × 10 um spot.
(Intensity)

$$I_{peak} = \frac{1\,mJ / 100\,fs}{10\,um \times 10\,um} \sim 10^{20}\ W/m^2 \sim 10^{16}\ W/cm^2$$

$$I = \frac{1}{2}\varepsilon_0 E^2 c \rightarrow E_{opt-peak} \sim 2.7\ GV/cm$$

meanwhile: for H - Atom



$$\sim E_m = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \sim 5\,GV/cm$$

*. time ~~ten~~ ruler: ps: 10^{-12} s. fs: 10^{-15} s

Mathmatically: ~~CW~~ Continous Wave (CW) laser: $\sim A\cos(\omega t - \phi)$

~~pulsed~~ or. $A e^{-j(\omega t - \phi)}$

pulse: $A(t) \cdot e^{-j[\omega t - \phi(t)]}$

~~Introd~~

Now we dicuss how to handle above description in ~~via~~ math. before we carry on, some review on mathmatical methods.

*. Fourier Transform.

function : $f(t)$

$$\begin{cases} F(\omega) = F.T.\{f(t)\} = \int_{-\infty}^{\infty} f(t) \cdot e^{j\omega t} \cdot dt \\ f(t) = I.F.T\{F(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{-j\omega t} \cdot d\omega \end{cases}$$

where $\omega = 2\pi\nu$

more symmetric way:

$$\begin{cases} F(\nu) = F.T.\{f(t)\} = \int_{-\infty}^{\infty} f(t) \cdot e^{j2\pi\nu t} \cdot dt \\ f(t) = I.F.T.\{F(\nu)\} = \int_{-\infty}^{\infty} F(\nu) \cdot e^{-j2\pi\nu t} \cdot d\nu \end{cases}$$

Important theorems:

* Scaling Theorem:

$$h(t) = f(at)$$
$$\Downarrow$$
$$H(\omega) = \frac{1}{a} F\left(\frac{\omega}{a}\right)$$

*. Time-delay Theorem:

$$h(t) = f(t-\tau)$$
$$\Downarrow$$
$$H(\omega) = F(\omega) \cdot e^{+j\omega\tau}$$

*. Frequency-offset Theorem:

$$h(t) = f(t) \cdot e^{-j\omega_0 t}$$
$$\Downarrow$$
$$H(\omega) = F(\omega - \omega_0)$$

*. Convolution Theorem: used a lot in signal/imaging processing

$$h(t) = f(t) * g(t)$$
$$\Downarrow$$
$$H(\omega) = F(\omega) \cdot G(\omega)$$

Special cases of F.T.

① $\delta(t) \Longleftrightarrow 1$; ② Gaussian: $e^{-\pi\nu^2} \longleftrightarrow e^{-\pi t^2}$

③ $F.T.\left\{\frac{\partial}{\partial t} f(t)\right\} = -j\omega F(\omega)$ → partial integration theorem

recall: $\int u(x) \cdot v'(x) \cdot dx = u(x) \cdot v(x) - \int u'(x) \cdot v(x) \cdot dx$

④ spatial domain F.T. and Matlab.

{ Now let's look at pulsed E-field]

$$A(t) \cdot e^{-j\{\omega t - \phi(t)\}}$$

ω ↓ carrier frequency

example: ~~t~~ measurable field $x(t)$ in real number.

$t=0$, $x(t)$, $\Rightarrow Re\{A(0) \cdot e^{j\phi(0)}\} = 0.7$

But: problem: how to determine $A(t)$. and $\phi(t)$?

need the help of particular mathmatical ~~app~~ ~~meothd~~ method.

$\Rightarrow$ Start with real field $x(t)$

$$\begin{cases} \tilde{x}(\nu) = F.T\{x(t)\} \\ x(t) = I.F.T\{\tilde{x}(\nu)\} \end{cases}$$

$\because$ $x(t)$ is real

$\therefore$ $\tilde{x}(-\nu) = \tilde{x}^*(\nu)$

means: negtive frequency does not ~~xx~~ contain any information.

proof: $$\tilde{x}(-\nu) = \int_{-\infty}^{\infty} x(t) \cdot e^{j2\pi(-\nu) \cdot t} \cdot dt$$

$$= \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi\nu t} \cdot dt$$

$$= \left[\int_{-\infty}^{\infty} x(t) \cdot e^{j2\pi\nu t} \cdot dt\right]^*$$

$$= \tilde{x}^*(\nu)$$

Now we define:

$$\tilde{X}^{+}(\nu) = \begin{cases} \hat{X}(\nu) & \nu \geq 0 \\ 0 & < 0 \end{cases}$$

in time domain:

$$\overset{+}{X}(t) = \int_{-\infty}^{\infty} \tilde{X}^{+}(\nu) \cdot e^{-j2\pi\nu t} \cdot d\nu = \int_{0}^{+\infty} \tilde{X}(\nu) \cdot e^{-j2\pi\nu t} \cdot d\nu$$

$\downarrow$

Complex Analytical Signal of $X(t)$, why useful?

$$\Rightarrow X(t) = \int \hat{X}(\nu) \cdot e^{-j2\pi\nu t} \, d\nu$$

$$= \int_{0}^{+\infty} \hat{X}(\nu) \cdot e^{-j2\pi\nu t} \cdot d\nu + \int_{-\infty}^{0} \tilde{X}(\nu) \cdot e^{-j2\pi\nu t} \cdot d\nu$$

$$\int_{-\infty}^{0} \tilde{X}(\nu) \cdot e^{-j2\pi\nu t} \cdot d\nu = \int_{0}^{+\infty} \tilde{X}(-\nu') \cdot e^{j2\pi\nu' t} \, d\nu' = \int_{0}^{+\infty} \tilde{X}^{*}(\nu) \, e^{j2\pi\nu t} \cdot d\nu$$

$$= X^{+}(t) + \left(X^{+}(t) \right)^{*}$$

$\therefore$ given $\overset{+}{X}(t)$, we can get $X(t)$

$$\Rightarrow X(t) = 2\mathrm{Re}\left\{ X^{+}(t) \right\}$$

assume know $\tilde{X}(\nu)$: $a(\nu) \cdot e^{j\phi(\nu)}$ → complex field in frequency domain

$$X(t) = \int_{0}^{\infty} a(\nu) \cdot e^{-j[2\pi\nu t - \phi(\nu)]} \cdot d\nu + \text{c.c.}$$

$$= 2\int_{0}^{\infty} a(\nu) \cdot \cos[2\pi\nu t - \phi(\nu)] \cdot d\nu \rightarrow \text{real number}$$

where the Complex Analytical Sigal → complex number

$$X^{+}(t) = \int_{0}^{\infty} a(\nu) \cdot e^{-j[2\pi\nu t - \phi(\nu)]} \cdot d\nu$$

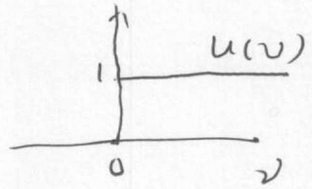
Comment: $x^+(t)$ is a complete description of E-field: Both phase and amplitude

how to obtain $x^+(t)$ give $x(t)$

$$x(t) \to \tilde{x}(\nu) \to \tilde{x}^+(\nu) \to x^+(t)$$

① $x(t) \xrightarrow{F.T.} \tilde{x}(\nu)$

② Step function: $u(\nu)$



③
$$x^+(t) = I.F.T\left[\tilde{x}^+(\nu)\right]$$
$$= I.F.T\left[\tilde{x}(\nu)\cdot u(\nu)\right]$$
$$= x(t) \otimes \underbrace{I.F.T\left[u(\nu)\right]}_{\frac{1}{2}\delta(t) + \frac{1}{j2\pi t}}$$

$$\Rightarrow x^+(t) = x(t) \otimes \left[\frac{1}{2}\delta(t) + \frac{1}{2j\pi t}\right]$$
$$= \frac{1}{2}x(t) - j\, x(t) \otimes \frac{1}{2\pi t}$$

define new $y(t) = -x(t) \otimes \frac{1}{\pi t} = \frac{-1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{(t-\tau)}\cdot d\tau$

$$\Rightarrow \begin{cases} y(t) = x(t) \otimes \frac{1}{\pi t} \to \text{Hilbert Transform.} \\ x(t) = \frac{-1}{\pi}\int_{-\infty}^{\infty}\frac{y(\tau)}{t-\tau}\cdot d\tau \end{cases}$$

Comments: ① Hilbert

② $x^+(t) = \frac{1}{2}\left[x(t) + j\, y(t)\right]$

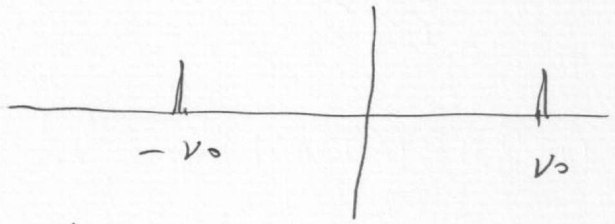
One example: $x(t) = \cos(\omega_0 t) = \cos(2\pi\nu_0 t)$

We can use two method:

method ①: $x(t) \to \tilde{x}(\nu) \to \tilde{x}^+(\nu) \to x^+(t)$

$$\tilde{x}(\nu) = \int_{-\infty}^{\infty} \frac{1}{2}\left[e^{j2\pi\nu_0 t} + e^{-j2\pi\nu_0 t}\right] \cdot e^{j2\pi\nu t} \cdot dt$$

$$= \frac{1}{2}\left[\delta(\nu-\nu_0) + \delta(\nu+\nu_0)\right]$$



② $\tilde{x}^+(\nu) = \frac{1}{2}\delta(\nu-\nu_0)$

③ $x^+(t) = \frac{1}{2} e^{-j2\pi\nu_0 t}$

method ② : More general one: Hilbert Transform

$$x(t) = \cos(2\pi\nu_0 t) \Rightarrow y(t)\ ?$$

$$y(t) = \frac{-1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} \cdot d\tau \qquad \text{let } 2\pi\nu_0 = 1$$

$$= \frac{-1}{\pi}\int_{-\infty}^{\infty} \frac{\cos(\tau)}{t-\tau} \cdot d\tau$$

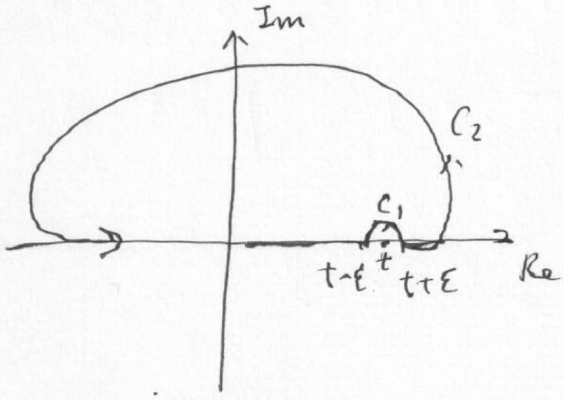
$$= \frac{-1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{j\tau} + e^{-j\tau}}{t-\tau} \cdot d\tau$$

$$= \frac{-1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{j\tau}}{t-\tau} \cdot d\tau + c.c.$$

Construct auxiliary function: $f(z) = \dfrac{e^{jz}}{t-z}$

where $z = z_r + jz_i$

$$\Rightarrow e^{jz} \propto e^{-z_i}$$



$f(z)$: analytical upper high plane

if $f(z)$ is analytical

$\oint_C f(z)\cdot dz = 0 \Rightarrow$ theorem.

$$\underset{①}{\int_{-\infty}^{t-\varepsilon} \frac{e^{j\tau}}{t-\tau}\cdot d\tau} + \underset{②}{\int_{t+\varepsilon}^{\infty} \frac{e^{j\tau}}{t-\tau}\cdot d\tau} + \underset{③}{\int_{C_1} \frac{e^{jz}}{t-z}\cdot dz}$$

$$+ \underset{④}{\int_{C_2} \frac{e^{jz}}{t-z}\cdot dz} = 0$$

④ disappears: since $e^{jz} \propto e^{-z_i} \Rightarrow 0$

$$\therefore \lim_{\varepsilon\to 0}\left[\int_{-\infty}^{t-\varepsilon} \frac{e^{j\tau}}{t-\tau}\cdot d\tau + \int_{t+\varepsilon}^{+\infty} \frac{e^{j\tau}}{t-\tau}\cdot d\tau\right]$$

$$= -\int_{C_1} \frac{e^{jz}}{t-z}\cdot dz \Rightarrow$$

Let: $z - t = \varepsilon\cdot e^{j\theta}$

$$\int_{C_1} \frac{e^{jz}}{z-t}\cdot dz = \int_{C_1} \frac{e^{jz}}{\varepsilon\cdot e^{j\theta}}\cdot dz$$

[knowing: $dz = \varepsilon\cdot e^{j\theta}\cdot j\cdot d\theta$ if $\varepsilon\to 0$, $z=t$]

$$= \int_{C_1} \frac{e^{jt}}{\varepsilon\cdot e^{j\theta}}\cdot \varepsilon\cdot e^{j\theta}\cdot j\cdot d\theta = \int_{C_1} e^{jt}\cdot j\cdot d\theta$$

$$= \int_{\pi}^{0} e^{jt}\, j \cdot d\theta = -j\pi e^{jt}$$

$$\therefore y(t) = \frac{-1}{2\pi}\left[-j\pi e^{jt} + j\pi e^{-jt}\right] = -\sin t$$

$$\therefore x^{+}(t) = \frac{1}{2}\left[x(t) + j\,y(t)\right] = \frac{1}{2}\left[\cos t - j\sin t\right]$$

recall we ~~set~~ set $2\pi\nu_0 = 1$

Well. time to review:

properties of Complex Analytical Signal. $x^+(t)$

$$\begin{cases} x(t) = 2\,\mathrm{Re}\left[x^+(t)\right] \\ x^+(t) = \frac{1}{2}\left[x(t) + j\,y(t)\right] \end{cases}$$

$$|x^+(t)|^2 = \frac{1}{4}\left[x^2(t) + y^2(t)\right]$$

Energy: $\int |x^+(t)|^2 \cdot dt = \frac{1}{4}\int x^2(t)\, dt + \frac{1}{4}\int y^2(t)\, dt$

$$\int |x^+(t)|^2 \cdot dt = \int |\tilde{X}^+(\nu)|^2 \cdot d\nu$$

$$= \frac{1}{2}\int |X(\nu)|^2 \cdot d\nu$$

$$= \frac{1}{2}\int |x(t)|^2 \cdot dt$$

$$\therefore \frac{1}{4}\int x^2(t)\cdot dt + \frac{1}{4}\int y^2(t)\cdot dt = \frac{1}{2}\int |x(t)|^2 \cdot dt$$

$$\Rightarrow \frac{1}{4}\int y^2(t)\cdot dt = \frac{1}{4}\int x^2(t)\cdot dt$$

$$\Rightarrow \boxed{\int x^2(t)\, dt = \int y^2(t)\cdot dt = 2\int |x^+(t)|^2 \cdot dt}$$

knowing from Hilbert Transform: $\int x(t)\, y(t) = 0$. orthognal

$$\Rightarrow \int x^{+2}(t)\cdot dt = \int x^+(t)\cdot x^{-*}(t)\cdot dt = \int \tilde{X}^+(\nu)\cdot \hat{X}^{-*}(\nu)\, d\nu = 0 \Rightarrow \text{No overlap}$$