Microeconomics

微观经济学

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Refresh: Expected utility (EU) vs. Expected value (EV)

Definition: Expected utility 期望效用

A utility function U: space of lotteries $\mathbf{P} \to \mathbb{R}$ has an **expected utility form** (or is a von Neumann-Morgenstern utility function) if for each of the N outcomes $(x_1, ..., x_n)$ assigned numbers with u such that for every $\alpha \in \mathbf{P}$, $U(\alpha) = U(\sum_i p_i x_i) = \sum_i p_i u(x_i)$.

Refresh: Decision under risk

- Two concepts: certainty equivalence and probability premium

Attitude	(1)	Utility Function	(2)	Certainty Equivalent	(3)	Prob. Premium
Risk averse	$\Leftrightarrow$	Concave	$\Leftrightarrow$	CE<EV	$\Leftrightarrow$	$\pi > 0$
Risk neutral	$\Leftrightarrow$	Linear	$\Leftrightarrow$	CE=EV	$\Leftrightarrow$	$\pi = 0$
Risk loving	$\Leftrightarrow$	Convex	$\Leftrightarrow$	CE>EV	$\Leftrightarrow$	$\pi < 0$

Refresh: Decision under risk

- How to understand insurance and gambles

Table of contents

Application of EU

Think

- Consider bike insurance. Assume that for the relevant amounts of money which are moderate, your utility is linear. Does EU recommend insuring your bike?
- Consider lotteries. Assume that for the relevant amounts of money which are moderate, your utility is linear. Does EU recommend purchasing a lottery?

Application of EU

Think

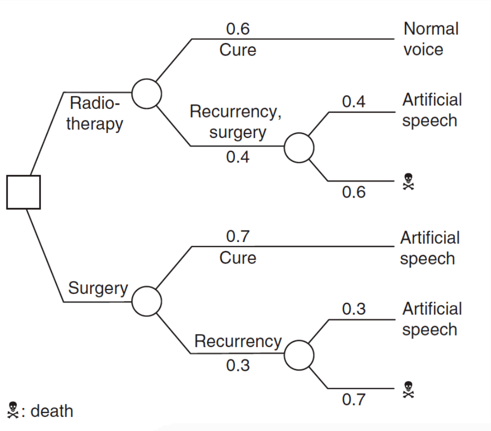
- Consider bike insurance. Assume that for the relevant amounts of money which are moderate, your utility is linear. Does EU recommend insuring your bike?
- Consider lotteries. Assume that for the relevant amounts of money which are moderate, your utility is linear. Does EU recommend purchasing a lottery?

Why do people buy lotteries?
Those who make $13,000 per year or less in the United States, on average, spend about 5% of their gross annual earnings on lottery tickets. An estimated 1/3 of all big ticket lottery winners and others who suddenly come into wealth will file for bankruptcy within 5 years of receiving this cash influx.

EU and its application

EU and its application: a case in medical decision

medical decision: how to make a decision involving health?



EU and its application

Ideally, we want to know the utility for each health state.

- We can normalize the utility of “death” to 0.
- We can normalize the utility of “normal voice” to 1.
- Then how about the utility of “artificial speech”?

EU and its application

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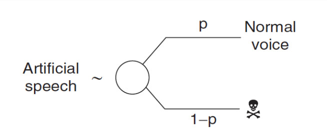


The standard gamble (SG) question: for which p is the gamble equivalent to the certain outcome?

- In this way, the utility of “artificial speech” equals p.

Measuring risk aversion

Measures of risk aversion: Global risk aversion

Consider two persons with preference relations $\succeq_1$ and $\succeq_2$, respectively, over prospects. Assuming that they maximize EU with utilities U_1 and U_2. We say that $\succeq_2$ is *more risk averse* than $\succeq_1$ if

$$\alpha \sim_1 x \Rightarrow \alpha \succeq_2 x$$

for all prospects x and outcomes α.

Theorem

Person 2 is more risk averse than person 1 if and only if $U_2(.) = \phi(U_1(.))$ for a concave transformation ϕ.

Two local measures of risk aversion

We need a measure for the degree of risk aversion so that we can compare across individuals and the same individual across different wealth level.
To measure risk aversion, we want to measure the concavity of utility functions (assuming that U is twice continuously differentiable).

1. the *Pratt-Arrow measure*, of *absolute measure of risk aversion*

$$-u^{''}(\alpha)/u^{'}(\alpha)$$

2. the *Pratt-Arrow measure*, of *relative measure of risk aversion*

$$\alpha \times -u^{''}(\alpha)/u^{'}(\alpha)$$

   on the domain of positive outcomes $\alpha > 0$.

The concavity of utility can be related to risk aversion only if EU is assumed.

Measure of absolute risk aversion

$$r_A = -\frac{u^{''}(\alpha)}{u^{'}(\alpha)}$$

- $u^{''}(\alpha)$ might be a good measure - A basic measure of a function's "curvature".
- Pros of using $u^{''}(\alpha)$: the *sign* tells us whether agent is risk averse, risk neutral or risk seeking.
- Cons of using $u^{''}(\alpha)$: the *magnitude* is arbitrary. Why? Consider the case of an affine transformation $v(\alpha) = ku(\alpha)$, $v^{''}(\alpha) \neq u^{''}(\alpha)$.

Measure of absolute risk aversion

If we normalize the second derivative by the first derivative, then

$$-\frac{v^{''}(\alpha)}{v^{'}(\alpha)} = -\frac{ku^{''}(\alpha)}{ku^{'}(\alpha)} = -\frac{u^{''}(\alpha)}{u^{'}(\alpha)}$$

- The sign of r_A tells us whether agent is risk averse, risk neutral or risk seeking.
- The magnitude of r_A is independent of affine transformation.
- For two individuals, 1 and 2, person 2 is more risk averse than person 1 at the level of income α if and only if $r_A^1 = -\frac{u_1^{''}(\alpha)}{u_1^{'}(\alpha)} < -\frac{u_2^{''}(\alpha)}{u_2^{'}(\alpha)} = r_A^2$

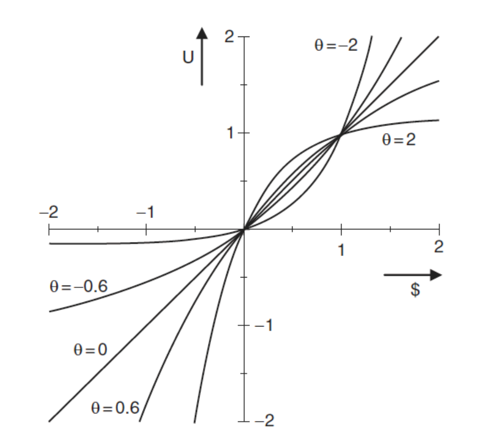
CARA

The *exponential family*, or the constant absolute risk aversion (CARA) family:

- for $\theta > 0$, $u(\alpha) = 1 - e^{-\theta\alpha}$
- for $\theta = 0$, $u(\alpha) = \alpha$
- for $\theta < 0$, $u(\alpha) = e^{-\theta\alpha} - 1$

$r_A = \theta$. The parameter θ is an index of concavity, with linear utility for $\theta = 0$, concave utility for $\theta > 0$, and convex utility for $\theta < 0$. Constant absolute risk aversion suggests that the agent holds the same amount of dollars in risky assets as wealth increases.

CARA



Measure of relative risk aversion

We have introduced absolute risk aversion r_A.

- $r_A = -\frac{u''(\alpha)}{u'(\alpha)} = -\frac{du'(\alpha)}{d\alpha}\frac{1}{u'(\alpha)}$ tells us the rate at which marginal utility decreases when wealth is increased by *one unit*.
- In other words, r_A is not *unit free*, as it is measured per (euro, dollar, yuan or yen).

Economists often prefer unit-free measurements of sensitivity. To this end, we define r_R as the rate at which marginal utility decreases when wealth is increased by *one percent*.

$$r_R = -\frac{du'(\alpha)/u'(\alpha)}{d\alpha/\alpha} = -\alpha\frac{u''(\alpha)}{u'(\alpha)} = \alpha r_A$$

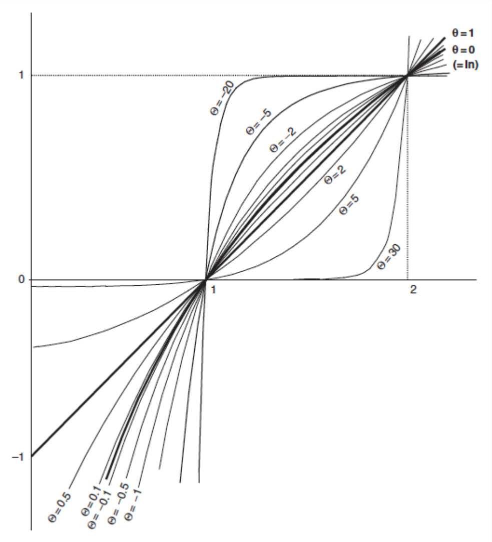
CRRA

The *power family*, or the constant relative risk aversion (CRRA) family: for $\alpha > 0$

- for $\theta > 0$, $u(\alpha) = \alpha^\theta$
- for $\theta = 0$, $u(\alpha) = ln(\alpha)$
- for $\theta < 0$, $u(\alpha) = -\alpha^\theta$

$r_R = 1 - \theta$, which is independent of α. Therefore, the person with this utility function has constant relative risk aversion.
Constant relative risk aversion suggests that the agent holds the same proportion of dollars in risky assets as wealth increases.

CRRA



Behavioral decision theory

Behavioral economics

Behavioral economics modifies and generalizes standard assumptions in psychologically plausible directions to explain the systematic deviations, suggested by introspection, lab experiments and field studies.

The classic theories (like Expected Value and Expected Utility) are more *normative* **规范性** while behavioral theories are *descriptive* **描述性**.

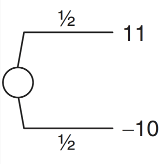
- Behavioral decision theory
  - Risk and Uncertainty
  - Time
  - Social choice
- Behavioral game theory

Behavioral decision theory: risk

Behavioral decision theory: risk

Some puzzles that cannot be explained by classical theories:

1. **Excessive risk aversion** (Rabin paradox)



Behavioral decision theory: risk

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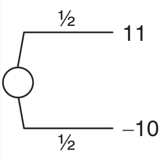


If a person rejects the above lottery, then for this perspn,

$$\frac{1}{2}u(w+11)+\frac{1}{2}u(w-10)<u(w)$$

$$u(w+11)-u(w)<u(w)-u(w-10)$$

$$\frac{u(w+11)-u(w)}{11}<\frac{10}{11}\times\frac{u(w)-u(w-10)}{10}$$

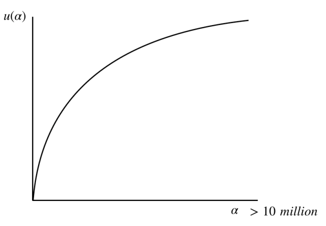
$$u^{'}(w+11)<\frac{10}{11}u^{'}(w-10)$$

Behavioral decision theory: risk

If you reject it, you will refuse a prospect $M_{\frac{1}{2}}(-100)$ even if M is million dollars. Why?

- On each interval [w-10, w+11] of length 21, the average derivative of u over the right part of the interval [w,w+11], is at most $\frac{10}{11}$ times the average derivative of u over the left part [w-10, w].
- Such a fast decline of marginal utility, dropping by a factor of at least $\frac{10}{11}$ over each distance of 21, does not make sense.
- Because such preferences are observed from various people at various wealth levels (Barberis, Huang, & Thaler 2006, AER), it is plausible to assume that a single individual will exhibit this preference not only at the present wealth level w, but also at other wealth levels $w + m$.

Another illustration: say your lifetime wealth is between 3 million (100k per year $\times$ 30 years) to 15 million (500k per year $\times$ 30 years). Let's graph your utility in this range:



Utility curvature should be linear over such a tiny range for any reasonable utility function over wealth.

Some puzzles that cannot be explained by classical theories:

2. **Equity premium puzzle 股权溢价之谜**

Country	Period	Mean real return: Market index (%)	Mean real return: Relatively riskless security (%)	Equity premium (%)
United Kingdom	1900–2005	7.4	1.3	6.1
Japan	1900–2005	9.3	−0.5	9.8
Germany	1900–2005	8.2	−0.9	9.1
France	1900–2005	6.1	−3.2	9.3
Sweden	1900–2005	10.1	2.1	8.0
Australia	1900–2005	9.2	0.7	8.5
India	1991–2004	12.6	1.3	11.3

Sources: Dimson et al. (2002) and Mehra (2007) for India.

Figure 1: Equity premium for selected countries

Investment period	Stocks: Real	Stocks: Nominal	T-bills: Real	T-bills: Nominal
1802–2004	\$655,348	\$10,350,077	\$293	\$4,614
1926–2004	\$238.30	\$2,533.43	\$1.54	\$17.87

Sources: Ibbotson (2005) and Siegel (2002)

Figure 2: Terminal value of \$1 invested in stocks and bonds

2. Equity premium puzzle: an empirical fact that stocks have outperformed bonds over the last century by a surprisingly large margin.

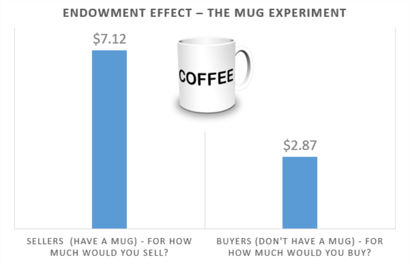
Could this large equity premium be consistent with rational EU maximization?

According to Mehra and Prescott's calculation (1985), people would have to have a coefficient of RRA (relative risk aversion) over 30 to explain the historical pattern of returns.
A person with RRA of 30 would be indifferent between a gamble with \$100,000 and \$50,000 with equal chance and a sure amount of \$51,209.
Few people can be this afraid of risk.

Some puzzles that cannot be explained by classical theories:

3. **Endowment effect 禀赋效应** (Kahneman, Knetsch, and Thaler 1990, 1991)

- Randomly give half of the subjects ("owners") mugs, and half of the subjects ("non-owners") nothing.
- Owners and non-owners are both allowed to examine the mug.



Some puzzles that cannot be explained by classical theories:

4. **Disposition effect 处置效应** (Shefrin & Statman 1985): the tendency of investors to sell shares whose price has increased too early, while keeping assets that have dropped in value for too long.

   Odean (1998) analyzed trading records for 10,000 accounts.
   He defined two ratios:

$$\frac{\textit{Realized Gains}}{\textit{Realized Gains} + \textit{Paper Gains}} = \textit{Proportion of Gains Realized}\,(\textit{PGR})$$

$$\frac{\textit{Realized Losses}}{\textit{Realized Losses} + \textit{Paper Losses}} = \textit{Proportion of Losses Realized}\,(\textit{PLR})$$

	1987–1990	1991–1993	Frequent Traders	Infrequent Traders
Entire year PLR	0.126	0.072	0.079	0.296
Entire year PGR	0.201	0.115	0.119	0.452
Difference in proportions	−0.075	−0.043	−0.040	−0.156
t-statistic	−30	−25	−29	−22
December PLR	0.143	0.110	0.095	0.379
December PGR	0.129	0.097	0.084	0.309
Difference in proportions	0.014	0.013	0.010	0.070
t-statistic	1.9	2.3	2.3	3.5
Jan.–Nov. PLR	0.123	0.069	0.078	0.282
Jan.–Nov. PGR	0.207	0.117	0.123	0.469
Difference in proportions	−0.084	−0.048	−0.045	−0.187
t-statistic	−32	−27	−31	−25

The disposition effect has also been observed in the housing market (Genesove and Mayer, 2001)

Related observation: gamblers stay in Casinos keep losing, and hope for "break even" (Imas, 2017).

Some puzzles that cannot be explained by classical theories:

5. **Unstable risk preference**

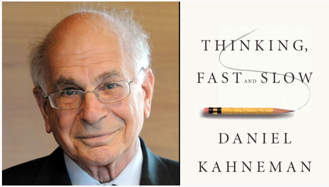
How to explain if the same person buys insurance (risk averse behavior) and lotteries (risk-seeking behavior) at the same time?

An alternative to Expected Utility: Prospect Theory

Behavioral decision theory: risk

Prospect theory (Kahneman and Tversky, 1979, 1992)

Nobel Prize in Economic Sciences, 2002



Origins: Tversky and Kahneman (1981) demonstrated systematic reversals of preference when the same problem is presented in different ways. For example:

Asian Disease Problem

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume the exact scientific estimate of the consequences of the programs are as follows.

1. In a group of 600 people
    - Program A: 200 people will be saved
    - Program B: there is a 1/3 chance that 600 people will be saved, and a 2/3 chance that no people will be saved.

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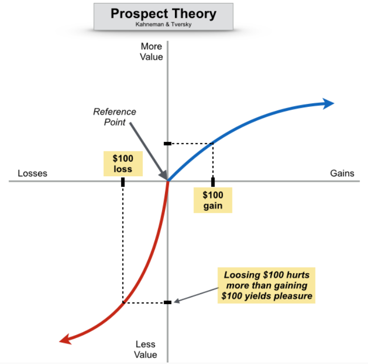
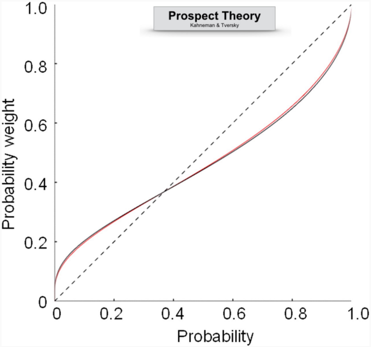
1. In a group of 600 people
   - Program A: 200 people will be saved
   - Program B: there is a 1/3 chance that 600 people will be saved, and a 2/3 chance that no people will be saved.
2. In a group of 600 people
   - Program C: 400 people will die
   - Program D: there is a 1/3 chance that nobody will die, and a 2/3 chance that 600 people will die.

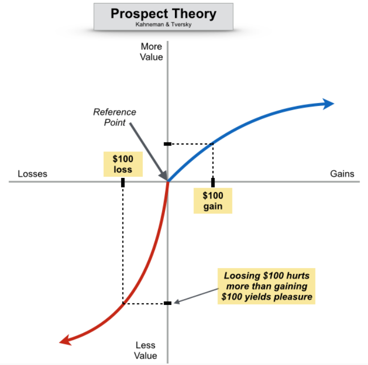
Prospect Theory (Kahneman and Tversky, 1979 ,1992) is a <u>descriptive</u> theory.

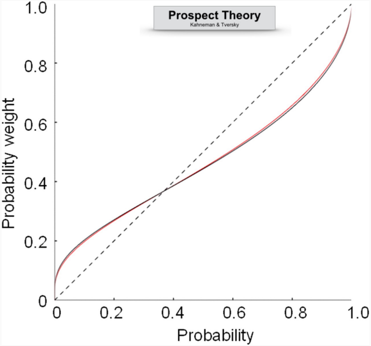




1. Loss aversion (a “kink” around 0) **损失厌恶**: $\lambda u(\alpha) = |u(-\alpha)|$ where $\alpha > 0$ and $\lambda > 1$.
2. Reference dependent **参照依赖**: evaluate outcomes relative to reference point

Prospect Theory (Kahneman and Tversky, 1979 ,1992) is a descriptive theory.





3. Diminishing sensitivity of the value function
4. Inverse s-shape probability weighting: overweighting small probabilities and underweighting middle and large probabilities

Prospect Theory

Discussion
What is “reference point”?

Imagine you get your first salary, your feeling about it may depend on:

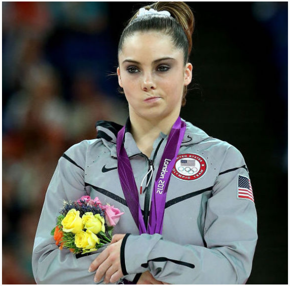
- Some candidates:
  - Status quo 现状
  - Expectation 预期
  - Norms 社会规则
  - Social comparisons 社会比较

  Without a structure, everything can be explained (and this is not good practice!).
- Reference point formation theories

Prospect Theory

Discussion
What is “reference point”?

- Reference point formation theories
  One won a silver medal, and one won a bronze medal. Which is which?



Discussion
What is "reference point"?

- Reference point formation theories
  1. Expectation-based (Kősegi & Rabin 2006, 2007, 2009): the reference point for evaluating an outcome is *recent beliefs* about future outcome.
     Overall utility has two components:
     $$u(c|r) = m(c) + n(c|r)$$
     , where $m(c)$ is "consumption utility" classically stressed in economics, and $n(c|r)$ is "gain-loss utility". Both c and r are K-dimensional vector. We assume both utilities are separable across dimensions, so that $m_c = \sum_k m_k(c_k)$ and $n(c|r) = \sum_k n_k(c_k|r_k)$.
     The person's "gain-loss utility" in dimension k, $n_k(c_k|r_k)$, depends on how consumption utility in that dimension compares to what the person could have achieved with the reference consumption level:
     $$n_k(c_k|r_k) = \mu(m_k(c_k) - m_k(r_k))$$
     .

Discussion
What is “reference point”?

- Reference point formation theories
    2. “natural reference points...”

Examples in professional sports:

- Marathon runners (Allen et al., 2016)

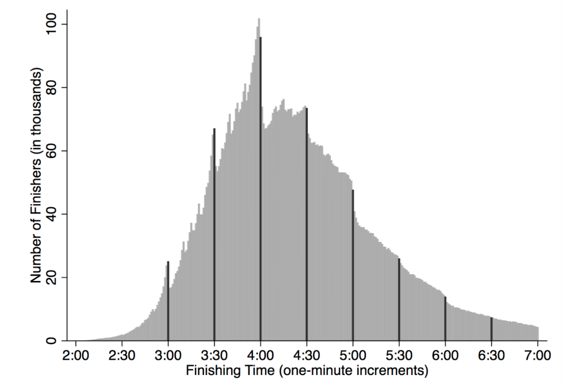


Figure 3: Distribution of marathon finishing times (n=9,662,885)

The dark bars highlight the density in the minute bin just prior to each 30 minute threshold.

Examples in professional sports:

- Marathon runners (Allen et al., 2016)

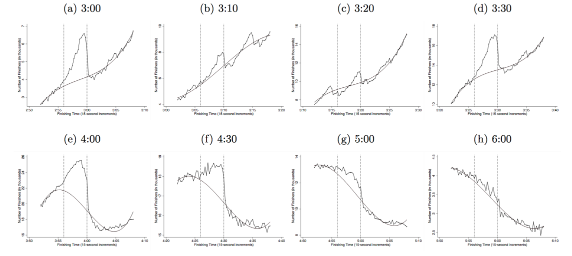


Figure 4: Distribution of the number of finishers around round number reference point and the fitted conterfactural distribution

The vertial axis shows the number of finishers in each 15 second bin.

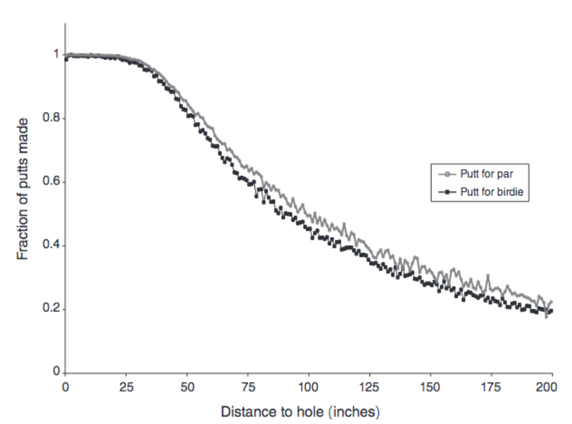
Critics of the decision bias literature believe that biases are likely to be extinguished by competition, large stakes, and experience. Examples in professional sports:

- Professional golf player ("Is Tiger Woods Loss Averse? Persistent Bias in the Face of Experience, Competition, and High Stakes", Pope and Schweitzer, 2011)

About golf:

- During the PGA tour **职业联赛**, there are a collection of 40-50 tournamwents. In each tournament, golfers attempt to minimize the total number of shots they take across 72 holes.
- For historical reasons, each hole is assigned a value or "par". On PGA Tour courses, each hole has a par value equal to 3, 4 or 5.
- Par **标准杆**: the typical number of shots professional golfers take to complete a hole.
- Birdie: complete a hole with one shot less than par.
- Bogey: complete a hole with one shot more than the par.

- Data include 2.5 million putts with laser measurements of initial and final ball placement (x, y, z coordinates).



This figure depicts the fraction of successful par and birdie putts by distance to the hole (in inches). The sample included 2,525,161 putts attempted in the PGA Tour between 2004 and 2008.

Prospect Theory

Discussion
What is “reference point”?

- Reference point formation theories
  3. Security-based: the minimum health state that can reach for sure (van Osch et al., 2004, 2006)

$$? \text{ \textit{years of life}} \sim \begin{cases} 50\%, \text{\textit{the rest of your life expectancy}} \\ 50\%, 1 \text{ \textit{week}} \end{cases}$$

Think
In Chinese tradition, some families tend not to celebrite birthdays for the old.

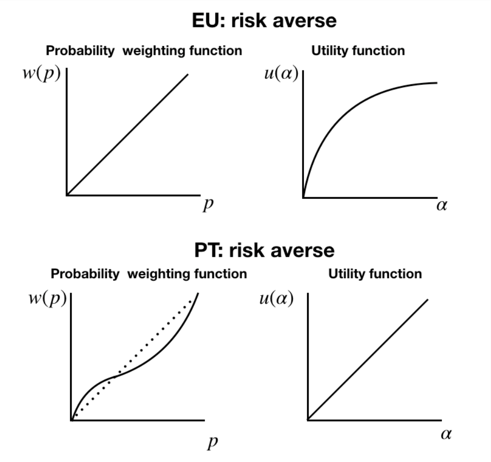
Discussion
How to understand risk aversion under the framework of Prospect Theory.

Under EU, probability weighting has to be linear. The curvature of utility is a measure of risk attitudes.

But wait a minute!

Under PT, probability weighting is nonlinear. A person can be risk seeking or averse with linear utility function.

Discussion
How to understand risk aversion under the framework of Prospect Theory.



Prospect Theory

Discussion
How to understand diminishing sensitivity (if it is not risk averse anymore)?

Under EU, the curvature of the utility function is interpreted as risk attitudes. A more natural explanation of the curvature is diminishing sensitivity: people's sensitivity to changes in an outcome is smaller for outcome levels that are further away from the reference point.

- For example, a change from getting \$0 to getting \$10 feels greater than a change from getting \$1,000 to getting \$1,010.

Reconcile theories with realities

Behavioral decision theory

How to explain **excessive risk aversion** (Rabin paradox)?

Behavioral decision theory

How to explain **excessive risk aversion** (Rabin paradox)?
Loss aversion + reference dependence
If the person is loss averse, meaning that the utility of gaining 11 is smaller than the absolute value of utility of losing 10.
And the person changes the reference point to 0.

It also explains that many consumers buy extended warranties and other very expensive small-scale insurance (say, Apple Care).

When should we buy insurance? When you cannot bear the possible loss.

How to explain **Equity premium puzzle**?

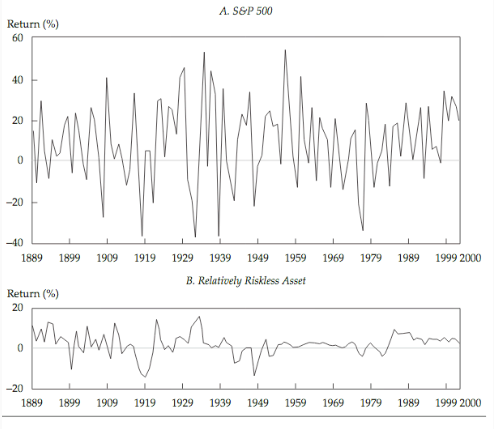


Figure 5: Real Return on S&P 500 and Relatively Risk-less Asset, 1889–2000

Behavioral decision theory

How to explain **Equity premium puzzle**?
It is caused by loss aversion and frequent evaluations.
loss aversion + **mental accounting** = **myopic loss aversion**

- The term "mental accounting" was introduced by Thaler's landmark article "Mental accounting and consumer choice"(1985).
- "Mental accounting is the set of cognitive operations used by individuals and households to code, categorize and evaluate financial activities".
- Examples include attitudes towards salaries vs. windfalls, gamblers keeping money that they have won during the day in a separate pocket from their "own" money (house money effect), use of credit cards, etc.

Behavioral decision theory

How to explain **endowment effect**?
A combination of reference dependence and loss aversion.

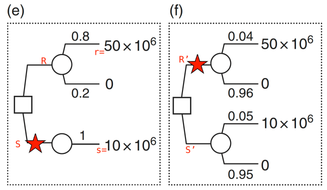
- Owner's reference point: having one mug.
- Non-owner's reference point: having zero mugs.
- Thus, selling entails a loss of the mug, while buying entails a gain of the mug.beginenumerate
- Since people are more sensitive to losses than they are to same-sized gains, the sellers "value" the mug more.

What about money?

- If there is a difference, spending money is a loss for non-owners, and getting money is only a gain for owners.
- This reinforces the endowment effect.

Behavioral decision theory

How to explain **Allais paradox**?



Assume u(0)=0,
Under EU, $S \succeq R \Leftrightarrow u(s) \geq 0.8u(r) \Leftrightarrow$
$0.05u(s) \geq 0.05 \times 0.8u(r) \Leftrightarrow S' \succeq R'$
Under PT, we can find a probability weighting function w(p) that satisfies both $S \succeq R \Leftrightarrow w(1)u(s) \geq w(0.8)u(r)$ and
$w(0.05)u(s) \leq w(0.04)u(r) \Leftrightarrow R' \succeq S'$

Behavioral decision theory

How to explain **unstable risk preferences** (buying lotteries and insurance at the same time)?
Overweighting of extreme events (winning a lottery, losing a bike, etc.)

Questions?