Microeconomics

微观经济学

Yu Gao

27 Oct 2021

Comparative statics: income change

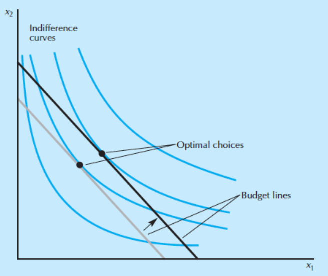


Figure 1: normal goods 正常品: $\frac{\Delta x_1}{\Delta m} > 0$

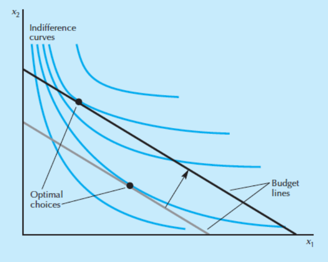


Figure 2: an inferior good 劣等品: $\frac{\Delta x_1}{\Delta m} < 0$

Whether a good is inferior or not depends on the income level that we are examining.

Comparative statics: price change

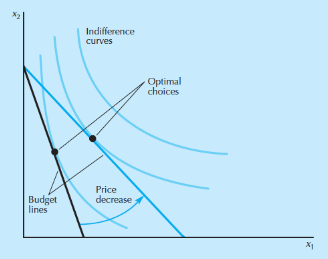


Figure 3: ordinary goods 普通品: $\frac{\Delta x_1}{\Delta p_1} < 0$

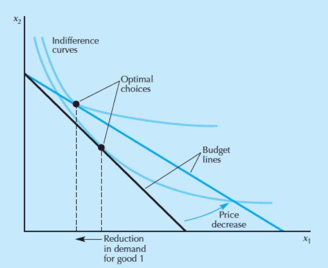


Figure 4: Giffen goods 吉芬品: $\frac{\Delta x_1}{\Delta p_1} > 0$

Even though income remains constant, a change in the price of a good will change purchasing power, and thereby change demand.

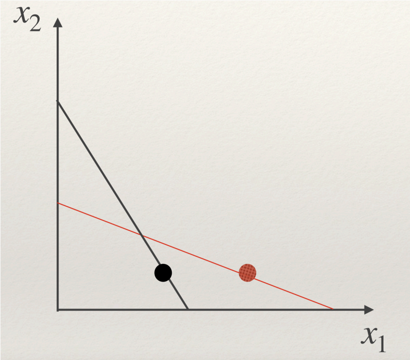
Refresh: Revealed preference

"Revealed preference" $\neq$ "Preference"

- "Revealed preferred" just means that (x_1, x_2) was chosen when (y_1, y_2) was affordable.
- "Preferred" means that the consumer thinks (x_1, x_2) is at least as good as (y_1, y_2).
- **"Revealed preferred" implies "Preference" only if the consumer chooses the best bundle for her when she can afford.**

Refresh: The Weak Axiom of Revealed Preference (WARP)

If the y-bundle is affordable when the x-bundle is purchased, then when the y-bundle is purchased, the x-bundle must not be affordable.



Refresh: The Weak Axiom of Revealed Preference (WARP)

An example that violates WARP: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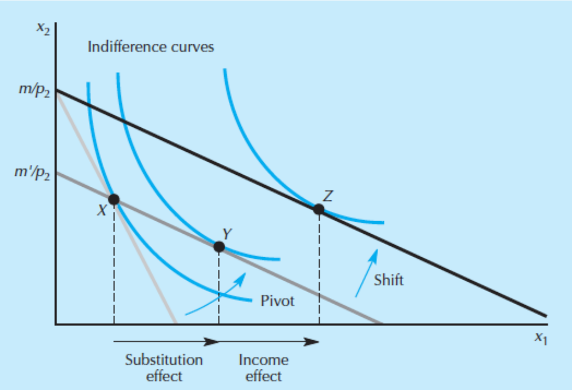
Substitution effect and income effect

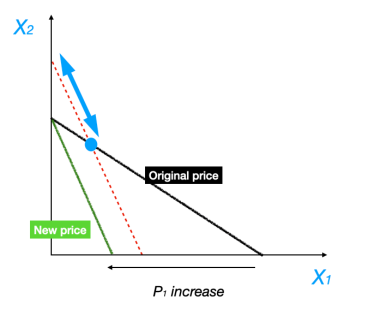
Is the old bundle (x_1, x_2) the optimal purchase at the pivoted budget line?

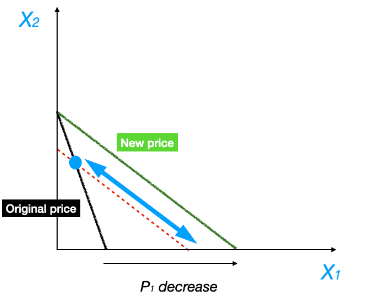


Sign of the total effect

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^n \Rightarrow \frac{\Delta x_1}{\Delta p} = \frac{\Delta x_1^s}{\Delta p} + \frac{\Delta x_1^n}{\Delta p}$$

Substitution effect:





The substitution effect is always negative: the demand and the price change in opposite directions.

The law of demand

Law of demand

If the demand for a good increases when income increases, then the demand for that good must decrease when its price increases.

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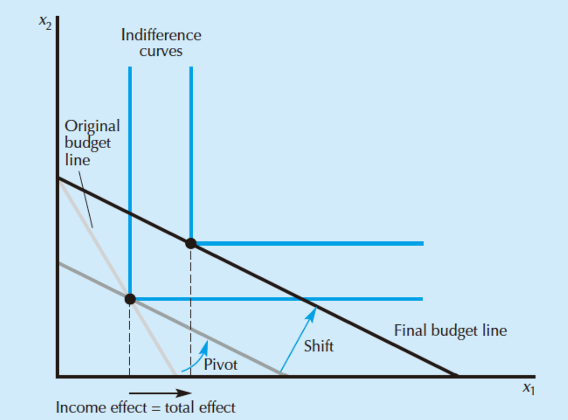
Slutsky equation

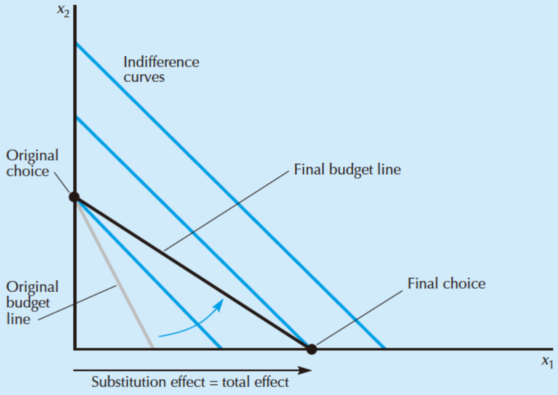
Some examples



Perfect complements. Slutsky decomposition with perfect complements.

The change in demand is due entirely to the **income effect** in this case.

Some examples

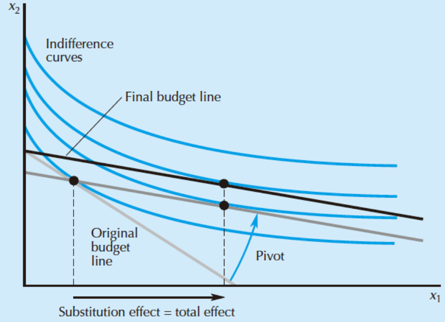


Perfect substitutes. Slutsky decomposition with perfect substitutes.

The entire change in demand is due to the **substitution effect** in this case.

Some examples

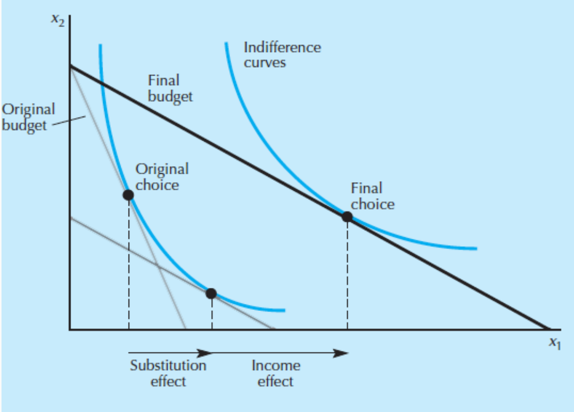
$u(x_1, x_2) = v(x_1) + x_2$



Quasilinear preferences. In the case of quasilinear preferences, the entire change in demand is due to the substitution effect.

The entire change in demand is due to the **substitution effect** in this case.

The Hicksian substitution effect



Choice under uncertainty

Uncertainty

Some examples:

	no rain (s_1)	*some rain* (s_2)	*all rain* (s_3)
x ("ice cream")	400	100	−400
y ("hot dogs")	−400	100	400
0 ("neither")	0	0	0
x + y ("both")	0	200	0

Uncertainty

Some examples:

	price ≥ 2.53 (E_1)	$2.53 >$ *price* ≥ 2.47 (E_2)	$2.47 >$ *price* (E_3)
x	50K	−30K	−30K
y	−30K	−30K	50K
0 (“neither”)	0	0	0
x + y (“both”)	20K	−60K	20K

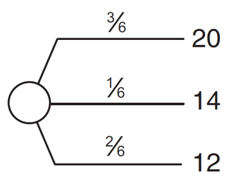
Risk: known probabilities

Decision under risk is a special case of decision under uncertainty.
If we can get probability measure from any source, say, weather forecast, we can turn a uncertainty problem into a risk problem.

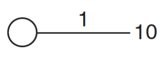
	no rain (s_1) =30%	*some rain* (s_2) =60%	*all rain* (s_3) =10%
x (“ice cream”)	400	100	−400
y (“hot dogs”)	−400	100	400
0 (“neither”)	0	0	0
x + y (“both”)	0	200	0

Risk: known probabilities

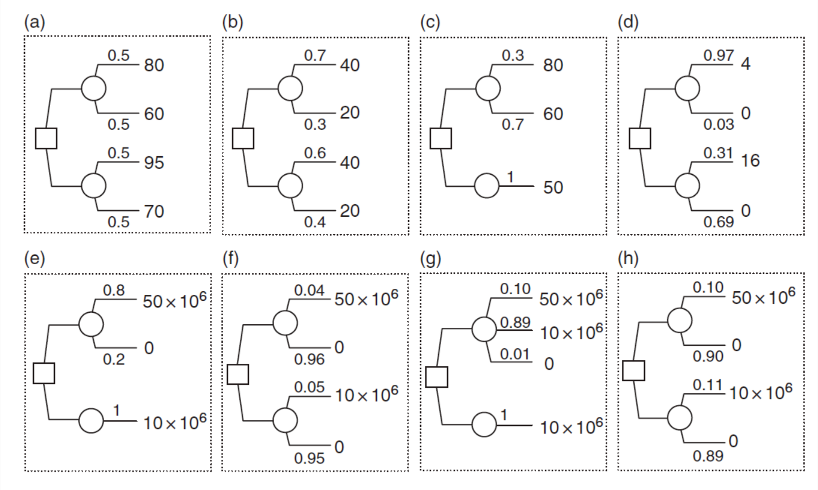
Simple lottery:



Riskless prospect



Risk: known probabilities



Risk: known probabilities

Definition: Expected value (EV) 期望值

Expected value (EV) holds if there exist (subjective) probability P such that preferences maximize the expected value (EV) of prospects defined by

$$E_1x_1...E_nx_n \to P(E_1)x_1 + ... + P(E_n)x_n$$

- "Subjective expected value" means that the probabilities need not have an objective basis but may be subjective.
- Market probabilities in finance are an example of subjective probabilities.

St.Petersburg paradox

Think

A fair coin will be flipped until the first head shows up. If heads shows up at the k^{th} flip, then you receive $\$2^k$. Thus, immediate heads gives only \$2, and after each tails the amount doubles. After 19 tails you are sure to be a millionaire.

Think for yourself how much it would be worth to you to play this game.

St.Petersburg paradox

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Think for yourself how much it would be worth to you to play this game.

If you maximize expected value, then this game is worth more to you than any amount of money: the expected value of the game is

$$\begin{aligned} 1/2 \times 2 + 1/4 \times 4 + 1/8 \times 8 + 1/16 \times 16 + ... + \\ = 1 + 1 + 1 + 1 + ... \\ = \infty \end{aligned}$$

St.Petersburg paradox

- In reality, people pay considerably less to participate in the game, something like \$5.
- It shows that expected value does not hold empirically when large amounts are involved.

Think

Why (almost) no one is willing to pay for ∞?

Expected utility

Definition: Expected utility 期望效用

A utility function U: space of lotteries $\mathbf{P} \to \mathbb{R}$ has an **expected utility form** (or is a von Neumann-Morgenstern utility function) if for each of the N outcomes $(x_1, ..., x_n)$ assigned numbers with u such that for every $\alpha \in \mathbf{P}$, $U(\alpha) = U(\sum_i p_i x_i) = \sum_i p_i u(x_i)$.

- $U(\alpha)$ here is called von Neumann-Morgenstern (vNM) utility function. We not only care about ordering here. We also care about EU being linear in probabilities. Therefore, we can only do affine transformation **正仿射变换** on it: $u(.) = ku(.) + b$, $k > 0$ to ensure the same preference property of α.

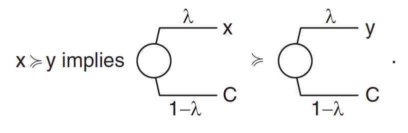
EU and its behavioral foundations: Independence axiom

Independence axiom (IA): 独立性公理

A preference relation over lotteries satisfies the IA if, for any three prospects x, y, and C, and for $\lambda \in (0, 1)$, we have that $x \succeq y$ if and only if the compound lottery $\lambda x + (1 - \lambda) C$ satisfies

$$\lambda x + (1 - \lambda) C \succeq \lambda y + (1 - \lambda) C$$

EU and its behavioral foundations: Independence axiom



Intuition behind IA:

- Consider a decision maker who prefers lottery x to y.
- We can construct a compound lottery: toss a coin, and heads means play x and tails means play C; and construct another compound lottery: toss a coin, and heads means play y and tails means play C.
- IA says that this decision maker must still prefers the first to the second compound lottery.

(first order) Stochastic Dominance （一阶）随机占优

Definition

- Two gambles A FOSD B means that $F_A(x) \leq F_B(x)$ for all x, with strict inequality at some x.
- Gamble A has FOSD over gamble B if for any outcome x, A gives at least as high a probability of receiving at least x as does B, and for some x, A gives a higher probability of receiving at least x.

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State (die result)	1	2	3	4	5	6
Gamble A wins $	1	1	2	2	2	2
Gamble B wins $	1	1	1	2	2	2
Gamble C wins $	3	3	3	1	1	1

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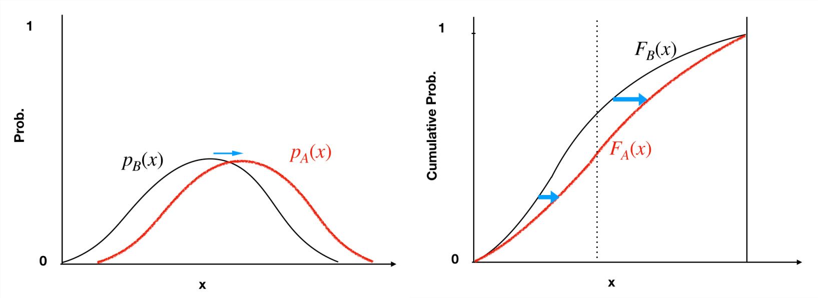
Gamble B wins \$	1	1	1	2	2	2
Gamble C wins \$	3	3	3	1	1	1

A FOSD B, C FOSD B, but no clear order between A and C.

(first order) Stochastic Dominance （一阶）随机占优



(second order) Stochastic Dominance （二阶）随机占优

Another decision criterion when the shape of u(.) is unknown.

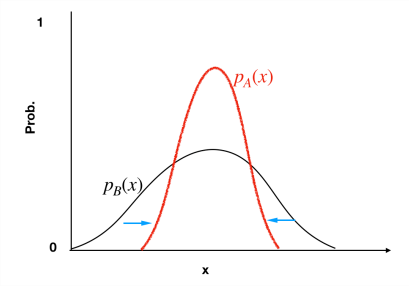
Definition

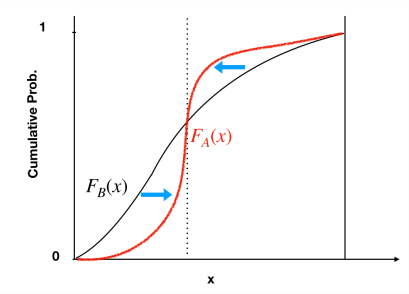
- Two gambles A SOSD B if B is mean-preserving spread **均值保留展开** of A.

For example, obtaining $10 for sure vs. obtaining 0 or $20 with 50% probability.

- **Under EV**, mean-preserving spread should not change your attitudes towards a prospect.
- **Under EU**, aversion to mean-preserving spread is equivalent to *risk aversion*, which is equivalent to *concavity of utility*.

(second order) Stochastic Dominance （二阶）随机占优

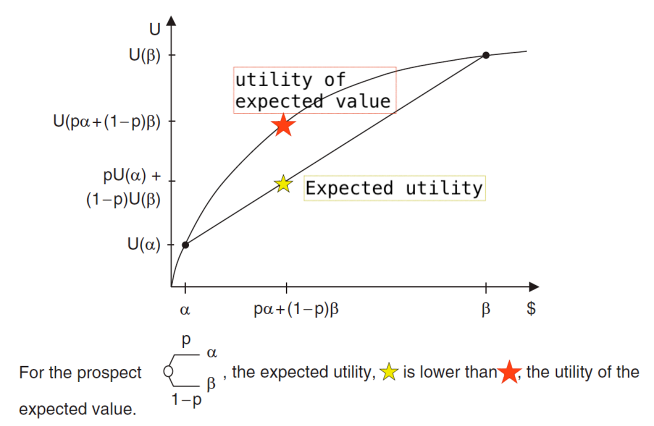




Risk aversion under EU

- Risk aversion $\Leftrightarrow EV(\alpha_p\beta) \succeq \alpha_p\beta \Leftrightarrow$ U is concave
- Risk neutrality $\Leftrightarrow EV(\alpha_p\beta) \sim \alpha_p\beta \Leftrightarrow$ U is linear
- Risk seeking $\Leftrightarrow \alpha_p\beta \succeq EV(\alpha_p\beta) \Leftrightarrow$ U is convex

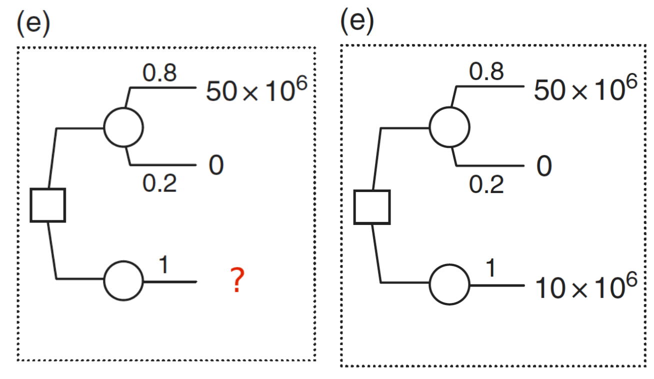
A figure illustration of risk aversion:



Two related concept

1. *Certainty equivalent (CE)* **确定性等值**: is the amount of money for which the individual is indifferent between the lottery and the certain amount:

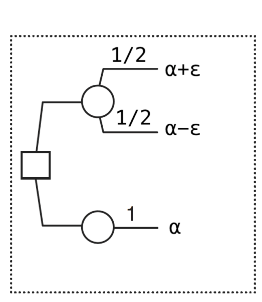
$$\alpha_p\beta \sim CE \Leftrightarrow EU(\alpha_p\beta) = U(CE)$$

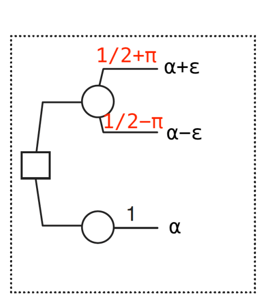


Two related concept

2. *Probability premium* **概率溢价**: for any fixed amount of money α and positive number ε, the *probability premium* demoted by π is the excess in winning probability over fair odds that makes the individual indifferent between the certain outcome x and a lottery between the two outcomes $x+\varepsilon$ and $x-\varepsilon$:

$$U(x) = (\frac{1}{2} + \pi)U(\alpha + \varepsilon) + (\frac{1}{2} - \pi)U(\alpha - \varepsilon)$$





Equivalents statement under EU

Attitude	(1)	Utility Function	(2)	Certainty Equivalent	(3)	Prob. Premium
Risk averse	$\Leftrightarrow$	Concave	$\Leftrightarrow$	CE<EV	$\Leftrightarrow$	$\pi > 0$
Risk neutral	$\Leftrightarrow$	Linear	$\Leftrightarrow$	CE=EV	$\Leftrightarrow$	$\pi = 0$
Risk Loving	$\Leftrightarrow$	Convex	$\Leftrightarrow$	CE>EV	$\Leftrightarrow$	$\pi < 0$

Insurance: Consumer's decision

Assume consumers are risk averse with $U_c^{''} < 0$.

Will not buy insurance

$$W_c = \begin{cases} W_0 & \textit{prob. } 1-p \\ W_0 - L & \textit{prob. } p \end{cases}$$

Buy insurance (premuim I)

$$W_c^{'} = W_0 - I$$

Condition to buy insurance:

$$U_c(W_0 - I) \geq (1-p)U_c(W_0) + pU_c(W_0 - L)$$

[show a figure]

Insurance: Insurance company's decision

Assume insurance companies are risk neutral with $U_I'' = 0$.

<u>Will not sell insurance</u>

$$W_I = 0$$

<u>Sell insurance</u>

$$W_I = \begin{cases} I & \textit{prob. } 1-p \\ I-L & \textit{prob. } p \end{cases}$$

Condition to sell insurance:

$$(1-p)U_I(I) + pU_I(I-L) \geq U_I(0)$$
$$I \geq pL$$

Range of the insurance price

From the discussion above, we know that price has to be in the range

$$I_{min} \leq I \leq I_{max}$$

to make both parites agree on the insurance contract.

Gamble vs. Insurance

How to understand gambling behavior?

Application of EU

Think

- Consider bike insurance. Assume that for the relevant amounts of money which are moderate, your utility is linear. Does EU recommend insuring your bike?
- Consider lotteries. Assume that for the relevant amounts of money which are moderate, your utility is linear. Does EU recommend purchasing a lottery?

Application of EU

Think

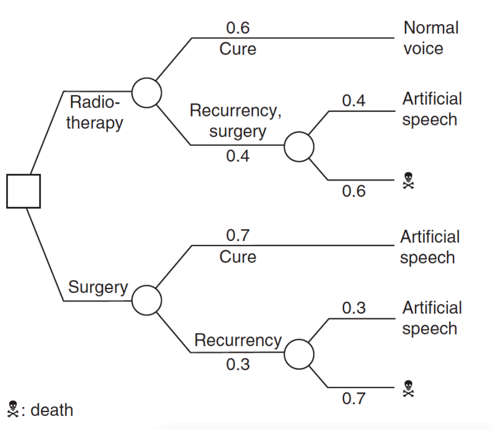
- Consider bike insurance. Assume that for the relevant amounts of money which are moderate, your utility is linear. Does EU recommend insuring your bike?
- Consider lotteries. Assume that for the relevant amounts of money which are moderate, your utility is linear. Does EU recommend purchasing a lottery?

Why do people buy lotteries?
Those who make $13,000 per year or less in the United States, on average, spend about 5% of their gross annual earnings on lottery tickets. An estimated 1/3 of all big ticket lottery winners and others who suddenly come into wealth will file for bankruptcy within 5 years of receiving this cash influx.

EU and its application

EU and its application: a case in medical decision

medical decision: how to make a decision involving health?



EU and its application

Ideally, we want to know the utility for each health state.

- We can normalize the utility of “death” to 0.
- We can normalize the utility of “normal voice” to 1.
- Then how about the utility of “artificial speech”?

EU and its application

Ideally, we want to know the utility for each health state.

- We can normalize the utility of "death" to 0.
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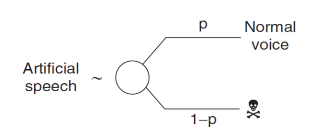

The standard gamble (SG) question: for which p is the gamble equivalent to the certain outcome?

- In this way, the utility of "artificial speech" equals p.

Measuring risk aversion

Two local measures of risk aversion

We need a measure for the degree of risk aversion so that we can compare across individuals and the same individual across different wealth level. To measure risk aversion, we want to measure the concavity of utility functions (assuming that U is twice continuously differentiable).

1. the *Pratt-Arrow measure*, of *absolute measure of risk aversion*

$$-u^{''}(\alpha)/u^{'}(\alpha)$$

2. the *Pratt-Arrow measure*, of *relative measure of risk aversion*

$$\alpha \times -u^{''}(\alpha)/u^{'}(\alpha)$$

   on the domain of positive outcomes $\alpha > 0$.

The concavity of utility can be related to risk aversion only if EU is assumed.

Measure of absolute risk aversion

$$r_A = -\frac{u^{''}(\alpha)}{u^{'}(\alpha)}$$

- $u^{''}(\alpha)$ might be a good measure - A basic measure of a function's "curvature".
- Pros of using $u^{''}(\alpha)$: the *sign* tells us whether agent is risk averse, risk neutral or risk seeking.
- Cons of using $u^{''}(\alpha)$: the *magnitude* is arbitrary. Why? Consider the case of an affine transformation $v(\alpha) = ku(\alpha)$, $v^{''}(\alpha) \neq u^{''}(\alpha)$.

Measure of absolute risk aversion

If we normalize the second derivative by the first derivative, then

$$-\frac{v''(\alpha)}{v'(\alpha)} = -\frac{ku''(\alpha)}{ku'(\alpha)} = -\frac{u''(\alpha)}{u'(\alpha)}$$

- The sign of r_A tells us whether agent is risk averse, risk neutral or risk seeking.
- The magnitude of r_A is independent of affine transformation.
- For two individuals, 1 and 2, person 2 is more risk averse than person 1 at the level of income α if and only if $r_A^1 = -\frac{u_1''(\alpha)}{u_1'(\alpha)} < -\frac{u_2''(\alpha)}{u_2'(\alpha)} = r_A^2$

Measure of relative risk aversion

We have introduced absolute risk aversion r_A.

- $r_A = -\frac{u^{''}(\alpha)}{u^{'}(\alpha)} = -\frac{du^{'}(\alpha)}{d\alpha}\frac{1}{u^{'}(\alpha)}$ tells us the rate at which marginal utility decreases when wealth is increased by *one unit*.
- In other words, r_A is not *unit free*, as it is measured per (euro, dollar, yuan or yen).

Economists often prefer unit-free measurements of sensitivity. To this end, we define r_R as the rate at which marginal utility decreases when wealth is increased by *one percent*.

$$r_R = -\frac{du^{'}(\alpha)/u^{'}(\alpha)}{d\alpha/\alpha} = -\alpha\frac{u^{''}(\alpha)}{u^{'}(\alpha)} = \alpha r_A$$

CARA

The *exponential family*, or the constant absolute risk aversion (CARA) family:

- for $\theta > 0$, $u(\alpha) = 1 - e^{-\theta\alpha}$
- for $\theta = 0$, $u(\alpha) = \alpha$
- for $\theta < 0$, $u(\alpha) = e^{-\theta\alpha} - 1$

$r_A = \theta$ which is independent of wealth level α. The parameter θ is an index of concavity, with linear utility for $\theta = 0$, concave utility for $\theta > 0$, and convex utility for $\theta < 0$.
Constant absolute risk aversion suggests that the agent holds the same amount of dollars in risky assets as wealth increases.

CRRA

The *power family*, or the constant relative risk aversion (CRRA) family: for $\alpha > 0$

- for $\theta > 0$, $u(\alpha) = \alpha^{\theta}$
- for $\theta = 0$, $u(\alpha) = ln(\alpha)$
- for $\theta < 0$, $u(\alpha) = -\alpha^{\theta}$

$r_R = 1 - \theta$, which is independent of α. Therefore, the person with this utility function has constant relative risk aversion.
Constant relative risk aversion suggests that the agent holds the same proportion of dollars in risky assets as wealth increases.

Questions?