2020年 清华大学 理论物理+原子分子物理优秀大学生暑期学校
2020.08.17--2020.08.20

Stochastic Thermodynamics and Fluctuation Theorem

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Physics School, PKU

Outline

- Thermodynamics in the19th and the 21st century
- Stochastic thermodynamics
- Fluctuation Theorems
- Maxwell's demon
- Summary

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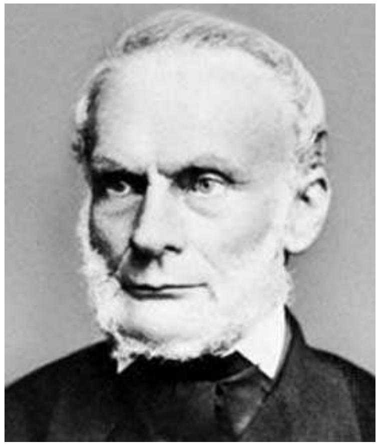
- Perspective

$1820 \simeq 1850$	classical thermodynamics	$dW = dU + dQ$ $dS \geq 0$
$\simeq 1900$	eq stat phys	$p_i = \exp[-(E_i - F)/k_B T]$
$1930 \simeq 1960$	non-eq: linear response	Onsager Green-Kubo, FDT
≥ 1993	non-eq: beyond linear response stochastic thermodynamics	Fluctuation theorem Jarzynski relation

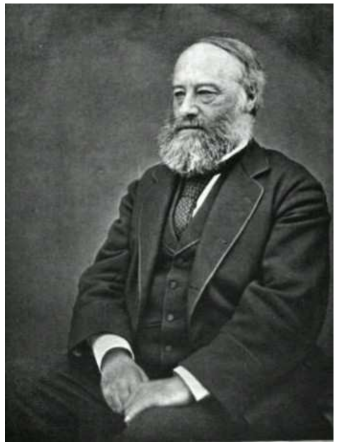
Background: Thermodynamics of the 19th century

Large system: many degrees of freedom,
many particles,
average values,
vanishingly small fluctuation,
near equilibrium process,

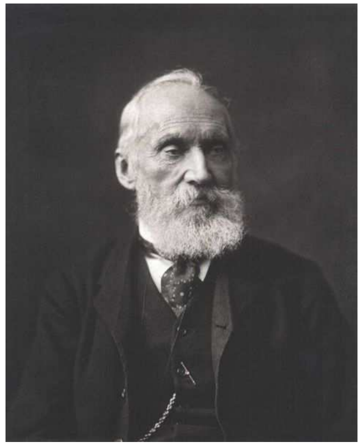
Sadi Carnot



Rdolf Clausius



James Joule



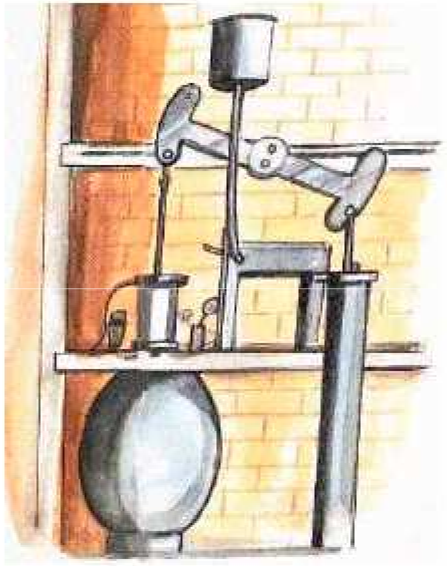
William Thomson

Laws of thermodynamics

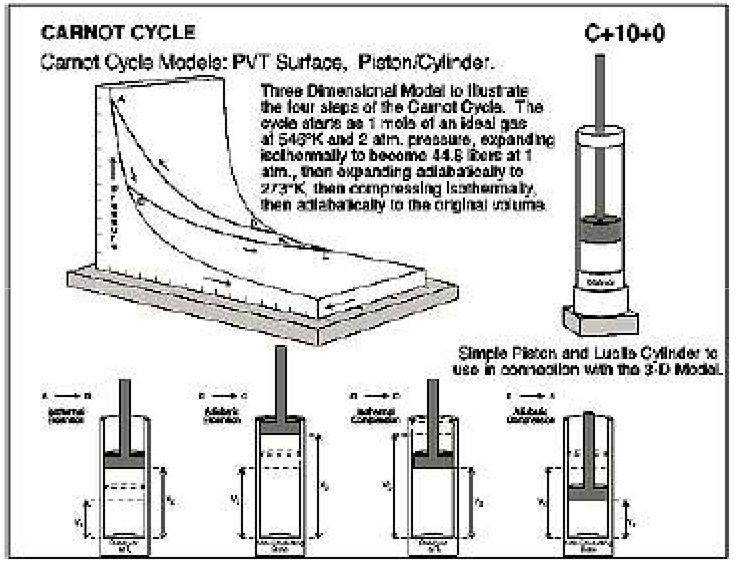
- Zeroth law of thermodynamics: If two systems are in thermal equilibrium separately, with a third system, they must be in thermal equilibrium with each other. This law helps define the notion of temperature.
- First law of thermodynamics: When energy passes, as work, as heat, or with matter, into or out from a system, its internal energy changes in accord with the law of conservation of energy. Equivalently, perpetual motion machines of the first kind are impossible.
- Second law of thermodynamics: The entropy of any isolated system never decreases. Such systems spontaneously evolve towards thermodynamic equilibrium — the state of maximum entropy of the system. Equivalently, perpetual motion machines of the second kind are impossible.
- Third law of thermodynamics: The entropy of a system approaches a constant value as the temperature approaches absolute zero.[2] With the exception of glasses the entropy of a system at absolute zero is typically close to zero, and is equal to the log of the multiplicity of the quantum ground state.

Thermodynamic cycles and Carnot engine

steam engine

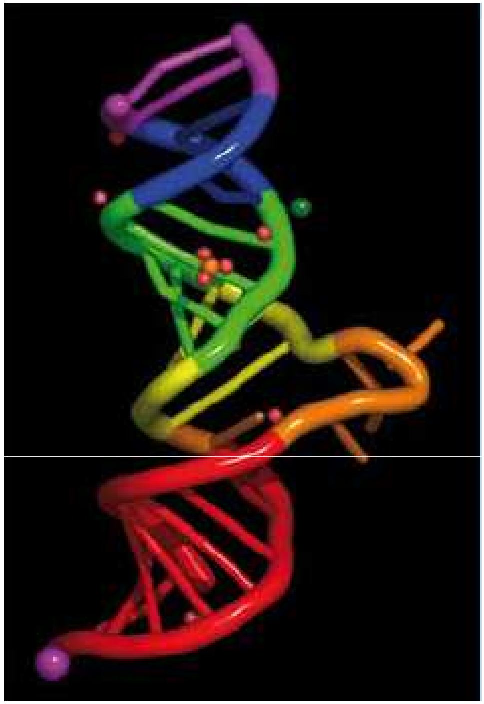


Carnot cycle



textbook thermodynamics

Thermodynamics of the 21th century



Mark Haw, Physics World, **20**, No 11, 25, 2007.

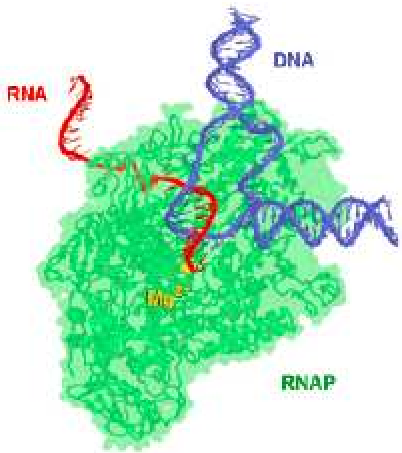
Small system: few degrees of freedom,
few particles,
prominent fluctuations,
quantum effects,
non-equilibrium process,

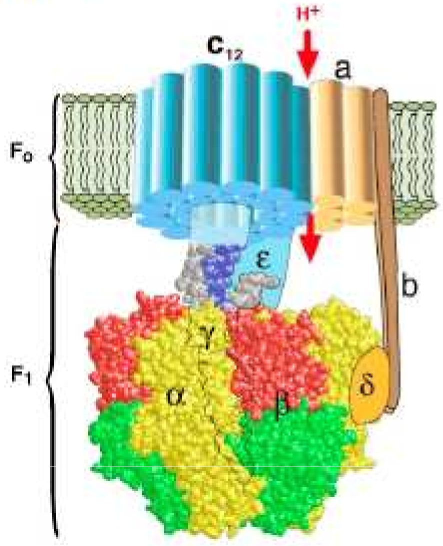
Magnetic domains in ferromagnets: less than 300nm
Quantum dots: less than 100nm
Biological molecular machines: range from 2 to 100nm

Stochastic molecular machines based on fluctuations

Molecular machines

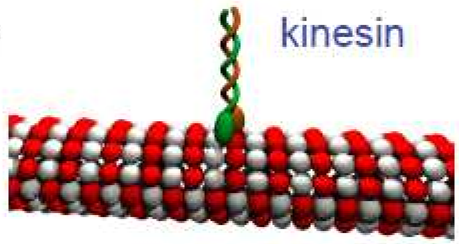
RNA polymerase





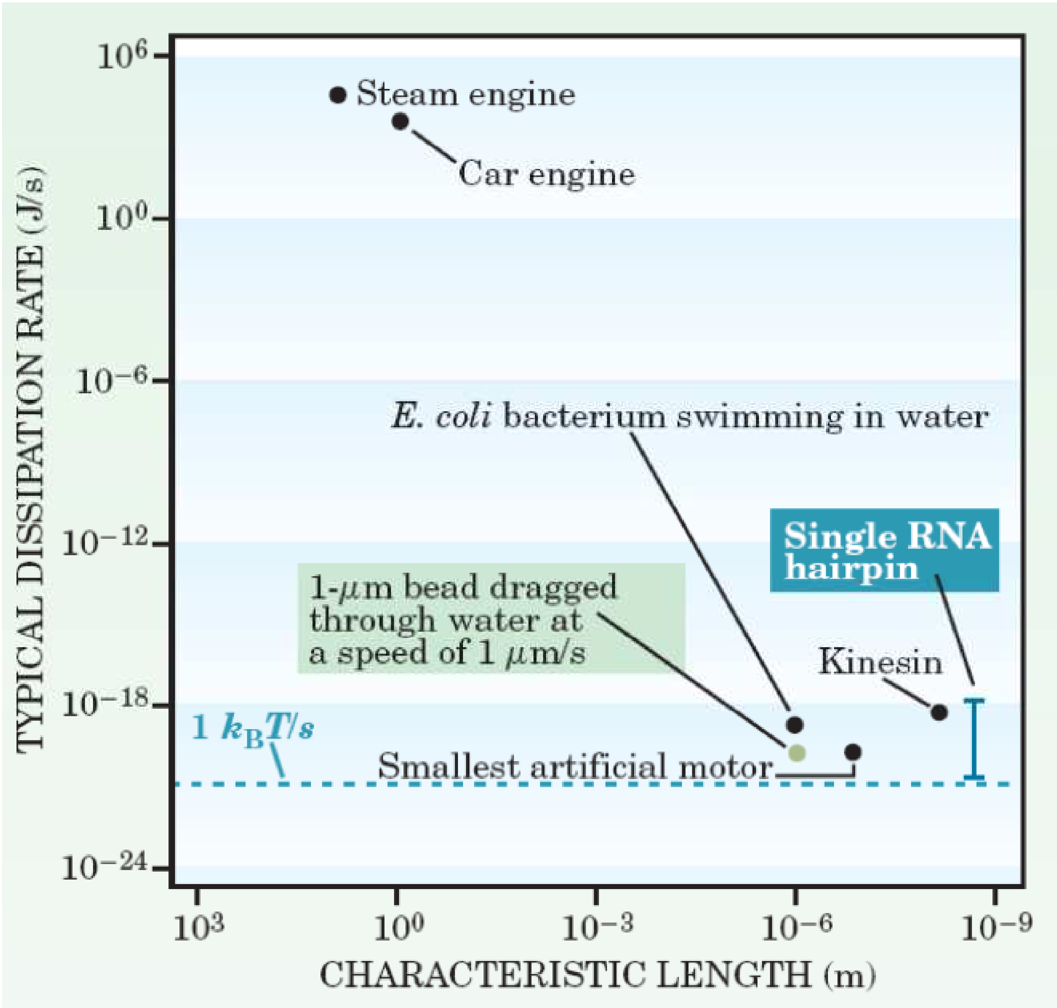
H. Wang and G. Oster (1998). Nature 396:279-282.

ATP synthase



kinesin

What are the underlying thermodynamics?
How is chemical energy converted to mechanical motion?



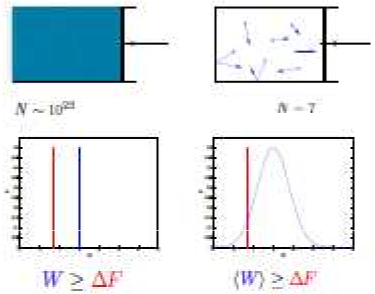
C. Bustamante, J. Liphardt, and F. Ritort, Physics Today, **58**, no 7, page 43-48, 2005.

Thermodynamics of small systems are important and interesting!

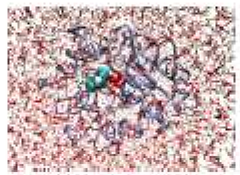
Features of small system thermodynamics

$$W \simeq \Delta F \simeq k_B T, \; \Delta S \simeq k_B$$

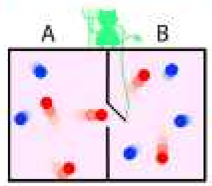
1. Fluctuating thermodynamic quantities, in particular in non-equilibrium processes.



2. Strong coupling to the reservoir(s).



3. Information acquired in measurements becomes thermodynamically relevant.



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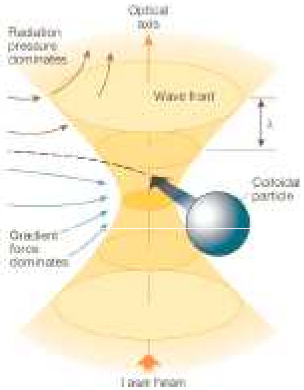
Quantitative analysis: Langevin dynamics

- select relevant degrees of freedom
- subsume the rest into a heat bath
- model the interaction with the bath by friction and noise (FDT)



works nicely if timescales separate,
Here: overdamped version

$$\dot{x} = -V'(x,\lambda) + \sqrt{2/\beta}\,\xi(t) \qquad \langle \xi(t)\xi(t') \rangle = \delta(t-t')$$

Continuous stochastic processes

Stochastic differential equation:

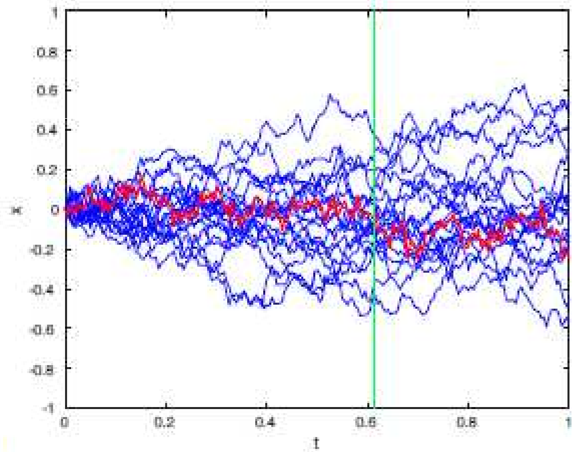
$$\dot{\mathrm{x}} = f(\mathrm{x}, t) + \sqrt{\frac{2}{\beta}}\,\boldsymbol{\xi}(t) \qquad \langle \xi_i(t)\xi_j(t')\rangle = \delta_{ij}\delta(t-t')$$

Fokker-Planck equation:

$$\partial_t P(\mathrm{x}, t) = -\nabla \left(\mathbf{f}(\mathrm{x}, t) P(\mathrm{x}, t) - \frac{1}{\beta} \nabla P(\mathrm{x}, t) \right)$$

Path measure in function space:

$$P_T[\mathrm{x}(\cdot)] = \mathcal{N}_T[\mathrm{x}(\cdot)] \exp\left(-\frac{\beta}{4} \int_0^T dt \left(\dot{\mathrm{x}} - \mathbf{f}(\mathrm{x}, t) \right)^2 \right)$$



Stochastic thermodynamics

Let $\mathbf{f}(\mathbf{x}, t) = -\nabla V(\mathbf{x}, \lambda(t))$ with some protocol $\lambda(t)$ (driven system).

Change of energy of the system:

$$dU = dV = \frac{\partial V}{\partial \lambda} d\lambda + \frac{\partial V}{\partial x} dx = dW + dQ$$

First Law of thermodynamics for a single fluctuating trajectory.
(Sekimoto, 1994)

Work and heat become stochastic variables $W[x(\cdot)], Q[x(\cdot)]$.

What are their distributions?

Langevin equation

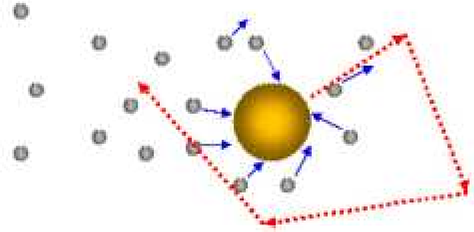
Observation time $\tau \gg \tau_{H_2O}$

$$m\ddot{x} = f(x,t) + \boxed{[-\gamma\dot{x} + \xi(t)]}$$

effect from the fast water molecules

$$\langle \xi(t)\xi(\tau)\rangle = 2\gamma k_B T \delta(t-\tau)$$

relating fluctuating force to friction



Observation time $\tau \gg m/\gamma$

$$0 = f(x,t) + [-\gamma\dot{x} + \xi(t)]$$

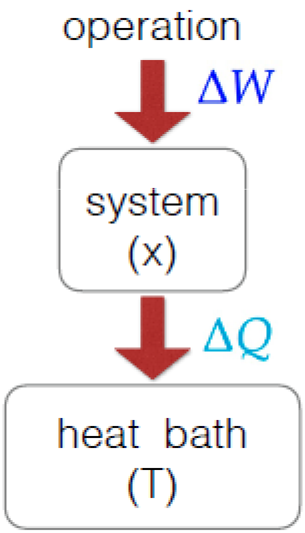
Over-damped Langevin equation

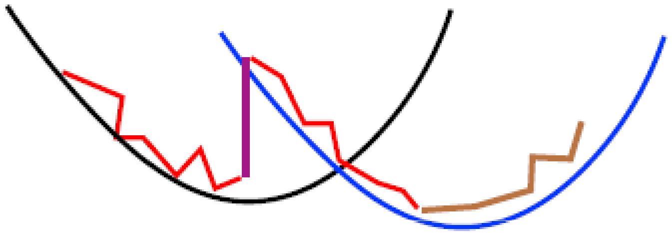
discretization

$$x' = x + \delta t f(x,t) + \sqrt{2D\delta t}\eta$$

$$P(\eta) \sim \exp\left(-\frac{\eta^2}{2}\right) \qquad D = \frac{k_B T}{\gamma}$$

Energy transduction along a stochastic process





potential energy —> heat

work —> potential energy

heat —> potential energy

heat —> work?

First Law: energy balance for a trajectory

$$m\ddot{x} = -\frac{\partial U(x,\lambda)}{\partial x} + [-\gamma\dot{x} + \xi(t)]$$

for a small step dx

$$d[\frac{1}{2}m\dot{x}^2 + U(x,\lambda)] = \frac{\partial U(x,\lambda)}{\partial \lambda}d\lambda + [-\gamma\dot{x} + \xi(t)]dx$$

$$dE \quad = \quad \Delta W \quad + \quad \Delta Q$$

change of internal energy	work by external operation	heat production in the medium

K. Sekimoto, *Progr. Theor. Exp. Phys.*, 130:17–27, 1998

Transformation of probability

$$W[\mathrm{x}(\cdot)] = \int_0^T dt\, \frac{\partial V}{\partial \lambda}(\mathrm{x}(t), \lambda(t))\, \dot{\lambda}(t)$$

$$Q[\mathrm{x}(\cdot)] = \int_0^T dt\, \nabla V(\mathrm{x}(t), \lambda(t)) \cdot \dot{\mathrm{x}}(t)$$

$$P_T[\mathrm{x}(\cdot)] = \mathcal{N}_T[\mathrm{x}(\cdot)]\, \exp\left(-\frac{\beta}{4}\int_0^T dt\, \left(\dot{\mathrm{x}} + \nabla V(\mathrm{x}, \lambda)\right)^2\right)$$

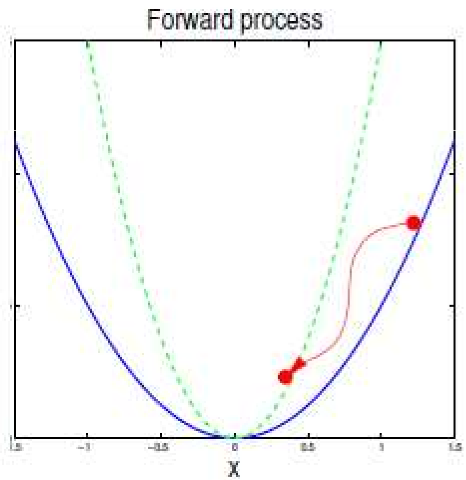
$$P(W) := \int_{(\mathbf{x}_0,0)}^{(\mathbf{x}_T,T)} \mathcal{D}\mathrm{x}(\cdot) P_T[\mathrm{x}(\cdot)]\, \delta(W - W[\mathrm{x}(\cdot)])$$

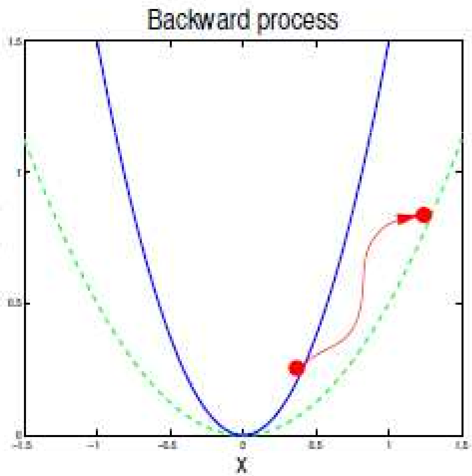
$$P(Q) := \int_{(\mathbf{x}_0,0)}^{(\mathbf{x}_T,T)} \mathcal{D}\mathrm{x}(\cdot) P_T[\mathrm{x}(\cdot)]\, \delta(Q - Q[\mathrm{x}(\cdot)])$$

What to do with it?

Time inversion

Reverse process: $\bar{\lambda}(t) := \lambda(T-t)$, mirror trajectory: $\bar{x}(t) := x(T-t)$





The detailed fluctuation theorem

$$\frac{P_T[\mathbf{x}(\cdot)]}{\bar{P}_T[\bar{\mathbf{x}}(\cdot)]} = \frac{\mathcal{N}_T[\mathbf{x}(\cdot)]\, \exp\left(-\frac{\beta}{4}\int_0^T dt\, \left(\dot{\mathbf{x}} + \nabla V(\mathbf{x},\lambda)\right)^2\right)}{\mathcal{N}_T[\bar{\mathbf{x}}(\cdot)]\, \exp\left(-\frac{\beta}{4}\int_0^T dt\, \left(\dot{\bar{\mathbf{x}}} + \nabla V(\bar{\mathbf{x}},\bar{\lambda})\right)^2\right)}$$

$$= \frac{\mathcal{N}_T[\mathbf{x}(\cdot)]\, \exp\left(-\frac{\beta}{4}\int_0^T dt\, \left(\dot{\mathbf{x}} + \nabla V(\mathbf{x},\lambda)\right)^2\right)}{\mathcal{N}_T[\mathbf{x}(\cdot)]\, \exp\left(-\frac{\beta}{4}\int_0^T dt\, \left(-\dot{\mathbf{x}} + \nabla V(\mathbf{x},\lambda)\right)^2\right)}$$

$$= \exp\left(-\beta \int_0^T dt\, \dot{\mathbf{x}} \cdot \nabla V(\mathbf{x},\lambda)\right) = e^{-\beta \Delta Q}$$

Exact for arbitrarily large deviations from equilibrium!

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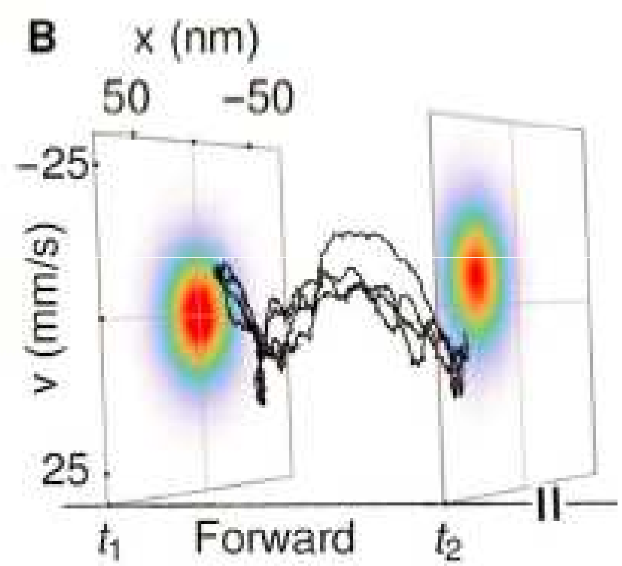
Path integral over all classical trajectories

$$\left\langle e^{-\beta W[x(\tau)]} \right\rangle \equiv \int dx_0 dx_t D[x(\tau)] \frac{e^{-\beta H_0(x_0)}}{z_0} P[x(\tau)\,|\,x_0] e^{-\beta W[x(\tau)]}$$

$$= \frac{z_1}{z_0} \int dx_0 dx_t D[x(\tau)] \frac{e^{-\beta H_1(x_t)}}{z_1} P[x(\tau)\,|\,x_0] e^{-\beta Q[x(\tau)]}$$

$$= \frac{z_1}{z_0} \int dx_0 dx_t D[x(\tau)] \frac{e^{-\beta H_1(\tilde{x}_0)}}{z_1} \tilde{P}[\tilde{x}(\tau)\,|\,\tilde{x}_0]$$

$$= \frac{z_1}{z_0} = e^{-\beta \Delta F} \quad \Longrightarrow \quad \left\langle W[x(\tau)] \right\rangle \geq \Delta F$$

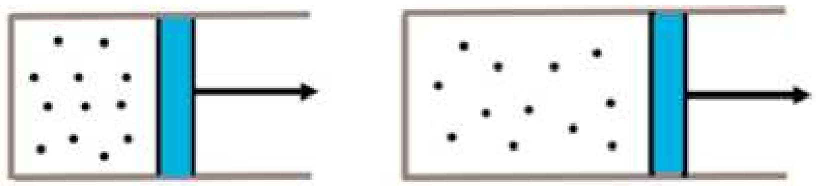


C. Jarzynski, Phys. Rev. Lett 78, 2690 (1997)

- Jarzynski equality (1997): $\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$

equilibrium information from non-equilibrium processes.

Jarzynski equality: relation between nonequilibrium work and equilibrium free energy difference



The second law of theromdynamics （equality for quasistatic process only）

$$\langle W \rangle \geq \Delta F$$

J. Gibbs, (1876)

Jarzynski equality （always equality, no matter slow or fast）

$$\left\langle e^{-\beta W} \right\rangle = e^{-\beta \Delta F}$$

C. Jarzynski, PRL, 78, 2690 (1997)

（cited for 4437 times）

50 years PRL moving physics forward

2019 Lars Onsager Prize Recipient

Christopher Jarzynski
University of Maryland, College Park

Citation:

"For seminal contributions to non-equilibrium thermodynamics and statistical mechanics that have had remarkable impact on experimental research in single-molecule and biological physics, engendering whole new fields of theoretical, numerical, and laboratory research, as well as for groundbreaking work on the thermodynamics of small systems."

Past Recipients

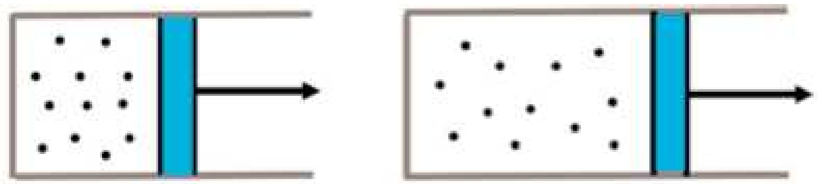
2018: Subir Sachdev
2017: Natan Andrei
Paul B. Wiegmann
2016: Giorgio Parisi
Marc Mezard
Riccardo Zecchina
2015: Franz Wegner

2003: Pierre Claude Hohenberg
2002: Anatoly Larkin
2001: Bertrand I. Halperin
2000: David James Thouless
John Michael Kosterlitz
1999: Chen Ning Yang
1998: Leo P. Kadanoff
1997: Robert H. Kraichnan
1995: Michael E. Fisher

Timeline of the second law

- Maximum Work Principle (1876)

$$\langle W \rangle \geq \Delta F$$



- Fluctuation-Dissipation relation (1950)

$$\langle W \rangle - \Delta F = \frac{1}{2}\beta\sigma_W^2$$

- Jarzynski equality (1997)

$$\left\langle e^{-\beta W} \right\rangle = e^{-\beta \Delta F}$$

$$\Downarrow \quad \left\langle e^{x} \right\rangle \geq e^{\langle x \rangle}$$

$$\langle W \rangle \geq \Delta F$$

- Hummer-Szabo relation (2001)

$$\left\langle \delta(\tilde{\Gamma} - \Gamma_\tau) e^{-\beta W} \right\rangle = \frac{e^{-\beta U_\tau(\tilde{\Gamma})}}{Z_0}$$

- Crooks Fluctuation Theorem (1998)

$$\frac{P_R(-W)}{P_F(W)} = e^{-\beta(W - \Delta F)}$$

- Differential Fluctuation Theorem (2008)

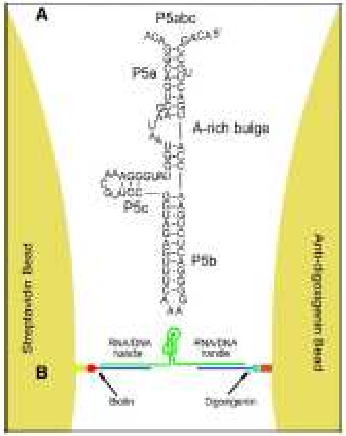
$$P_F(W, \Gamma_0 \to \Gamma_\tau) e^{-\beta(W - \Delta F)} = P_R(-W, \Gamma_\tau^* \to \Gamma_0^*)$$

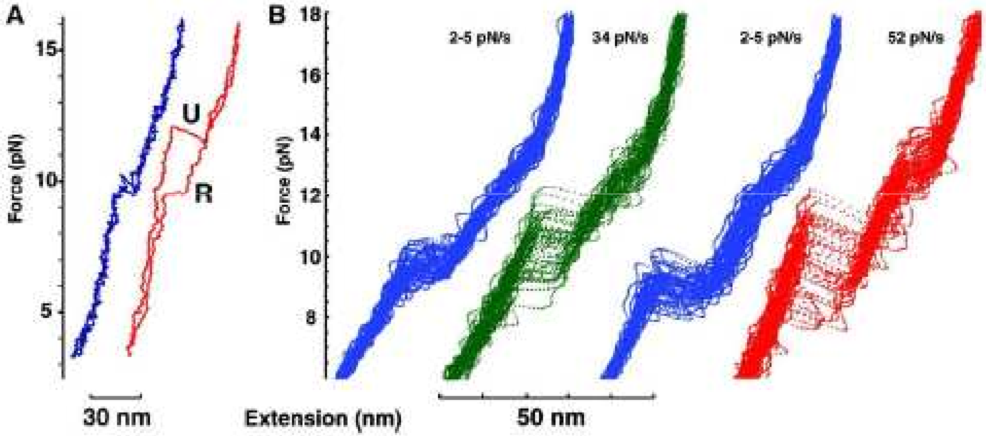
Time reversed: $\overline{\lambda}_t = \lambda_t$

Experimental test of Jarzynski's equality

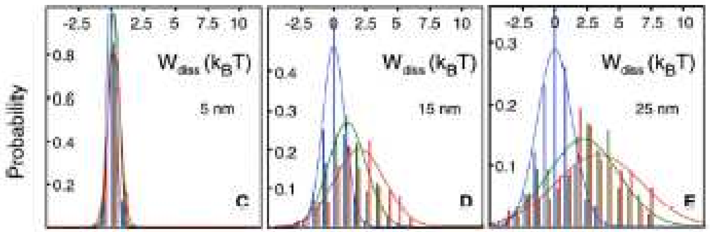
- Nano-world Experiment: Stretching RNA

[Liphardt et al, Science **296** 1832, 2002.]



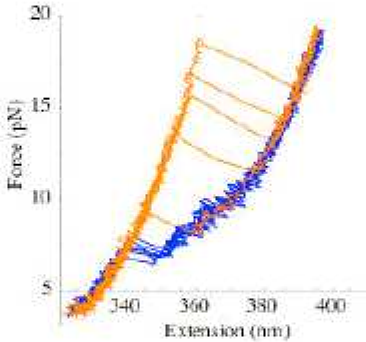


– distributions of W_{diss}:

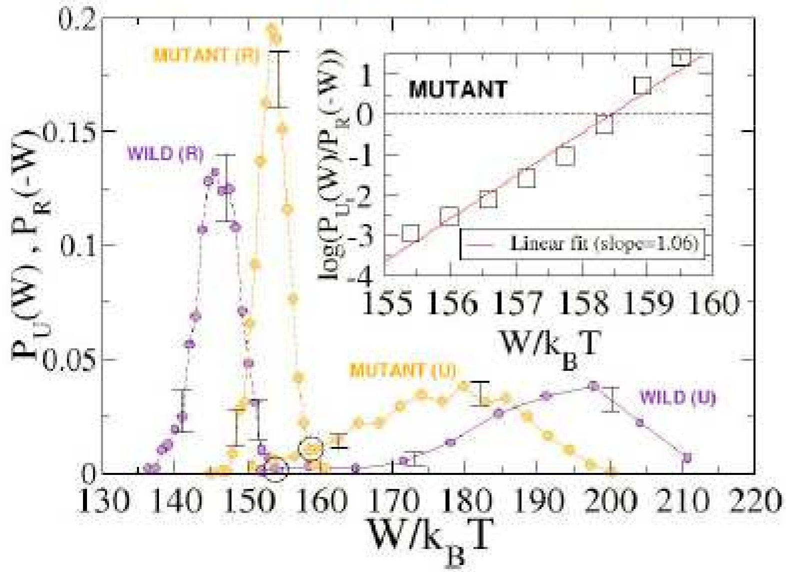


Unfolding & refolding of ribosomal RNA

$$\frac{\rho_{unfold}(+W)}{\rho_{refold}(-W)} = \exp\left[\beta(W - \Delta F)\right]$$



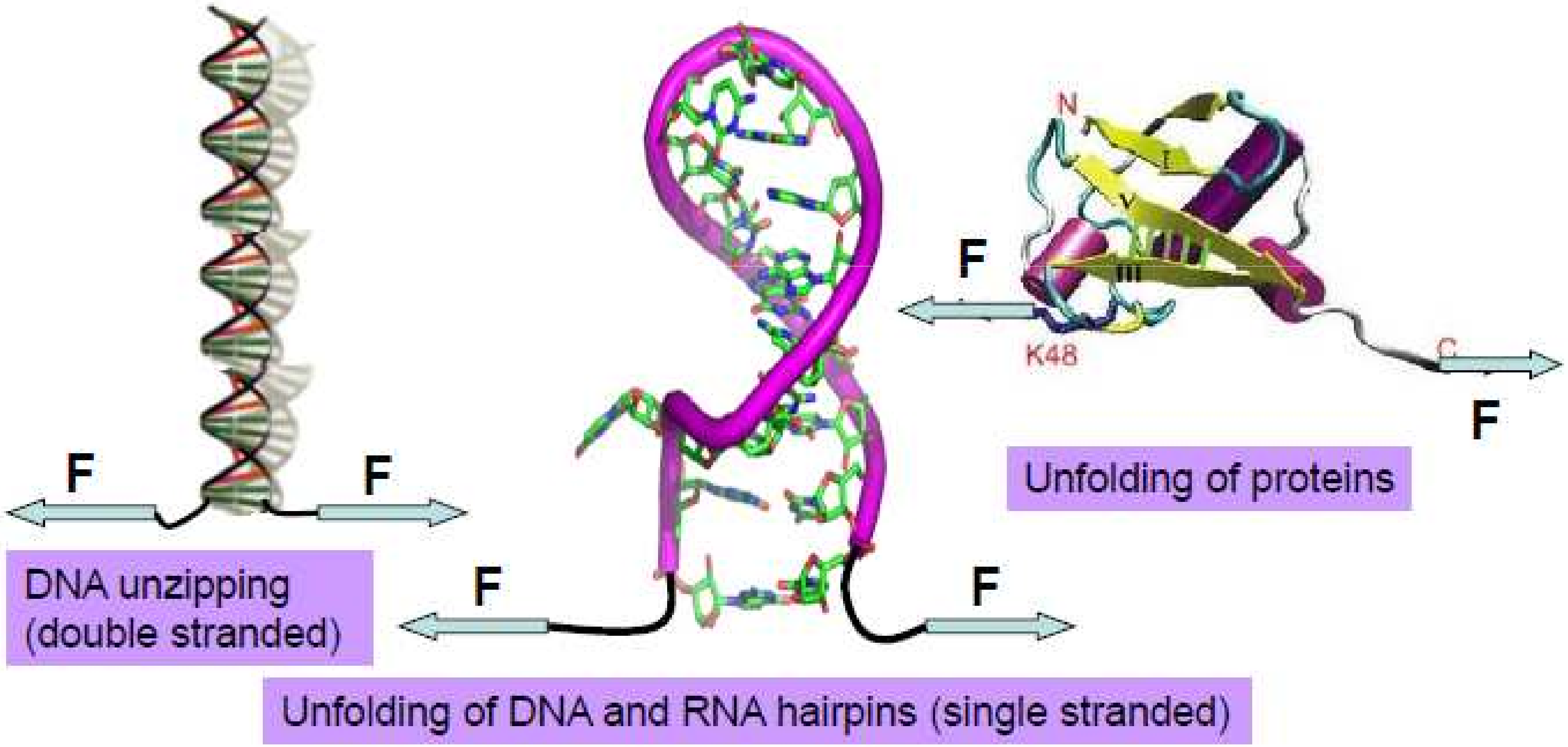
Collin *et al*, *Nature* **437**, 231 (2005)



Molecular unzipping

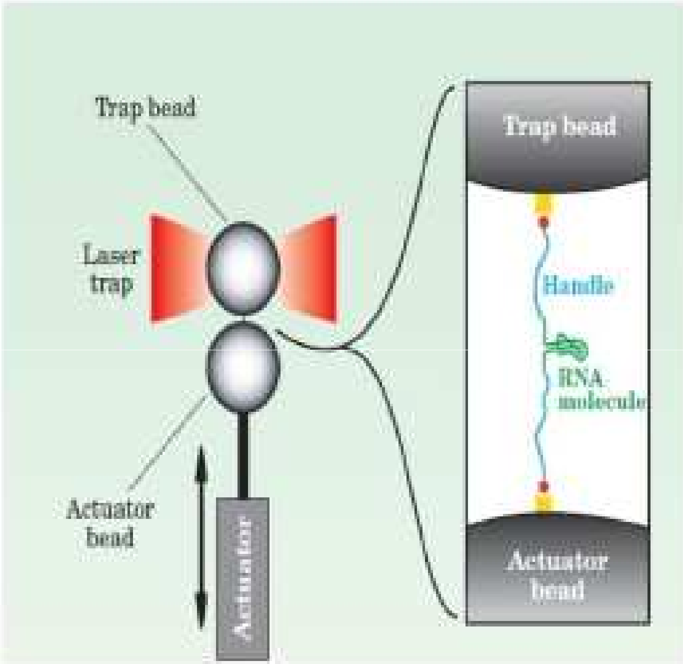
KTH/CSC

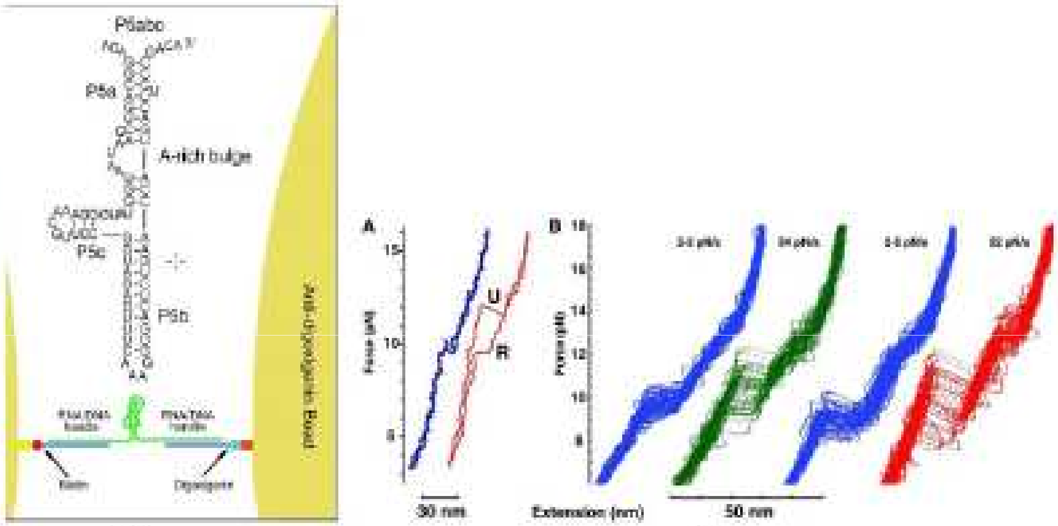
B. Essevaz-Roulet, U. Bockelmann , F. Heslot F (1997) *Proc Natl Acad Sci* USA 94:11935-11940
M. Rief, H. Clausen-Schaumann, H.E. Gaub (1999) *Nat Struct Biol* 6:346-349
C. Danilowicz et al. (2003) *Proc Natl Acad Sci* USA 100:1694-1699



F. Ritort, J. Phys. (Cond. Matter) **18** R531 (2006))

Non-equilibrium work can be measured in small systems and determines ΔF...





(a) (b)

...or at least $\Delta\Delta F$, the change in free energy when the RNA changes

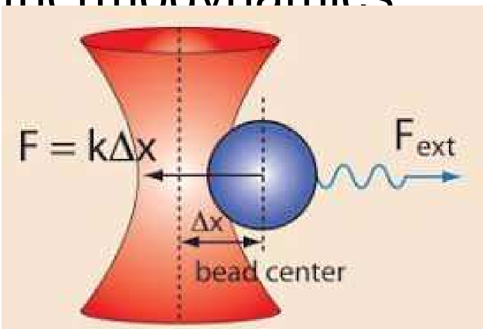
Different pulling speeds give different force-elongation curves

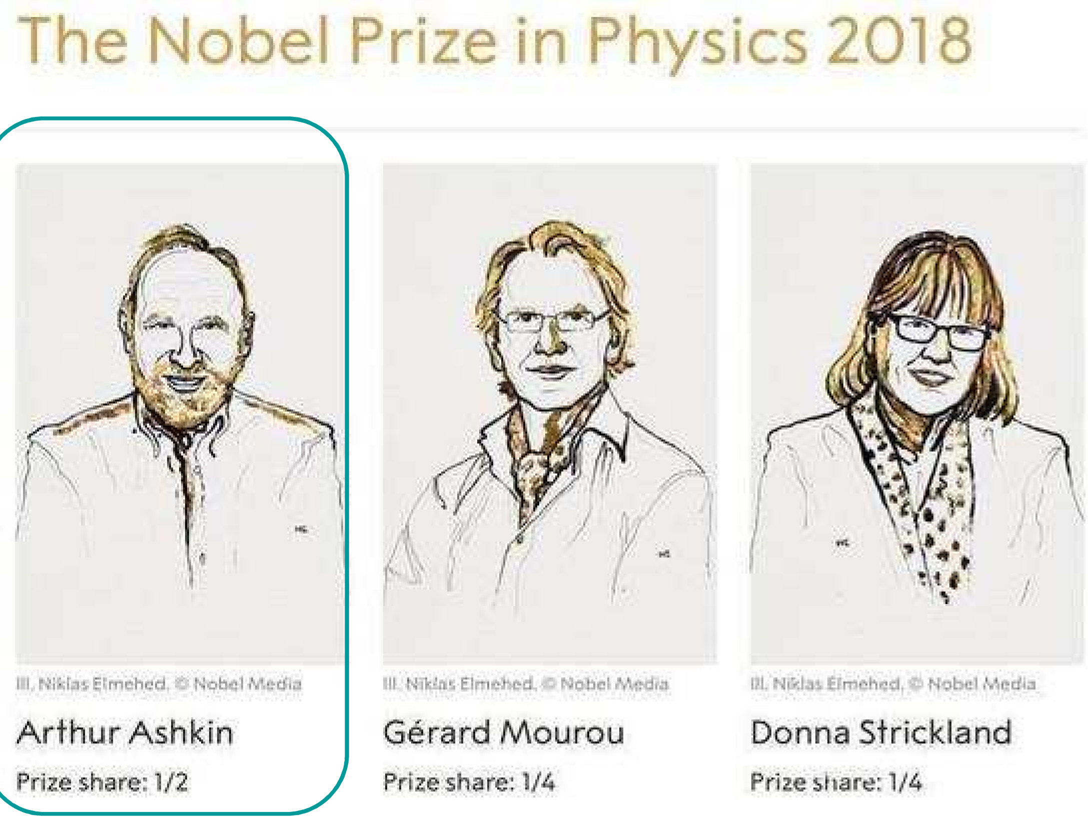
Theory: Jarzynski, Crooks, Evans, and others

Experiments: Bustamante, Ritort, and others

Arthur Ashkin
“Optical tweezer”

Key application:
Study of
Stochastic
thermodynamics





KUNGL.
VETENSKAPS-
AKADEMIEN
THE ROYAL SWEDISH ACADEMY OF SCIENCES

RNA molecules under mechanical force [36]. Force-extension curves for an RNA molecule were subsequently used for the first experimental test [37] of Jarzynski’s equality in stochastic thermodynamics, which relates nonequilibrium work distributions to equilibrium free energy differences [38].

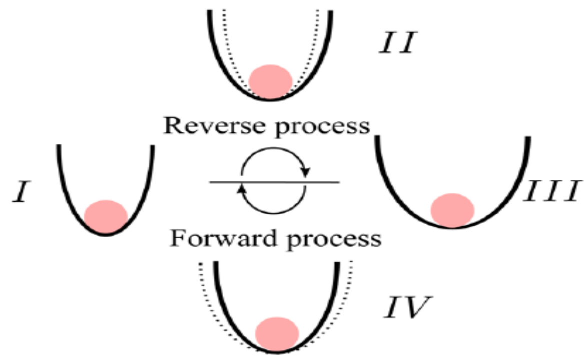
Application of Stochastic Thermodynamics

QUICK STUDY

A single-atom heat engine

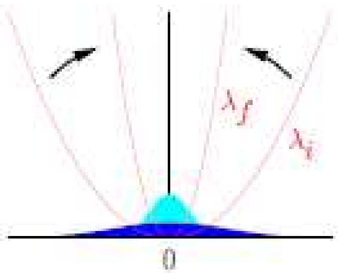
Eric Lutz

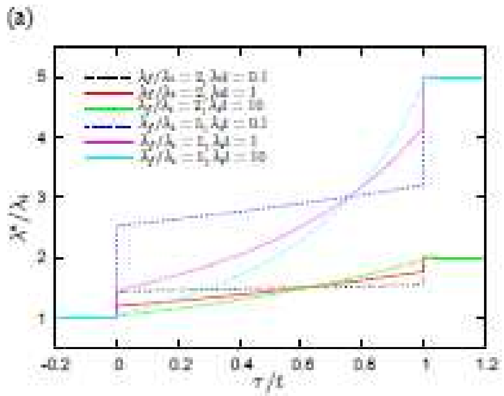
The power of an engine scales with the number of particles that make up its working fluid, a generalization that has proven true down to a single atom.



Optimal Protocol in a Finite-Time Process

$$V(x, \lambda) = \lambda(\tau)x^2/2$$





– jumps are generic

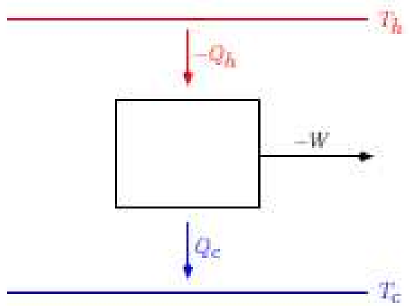
– should help to improve convergence of $\langle \exp(-W) \rangle$

– generalization: underdamped dynamics $\Rightarrow$ delta-peaks

[A. Gomez-Marin, T.Schmiedl , U.S., J. Chem. Phys., 129 : 024114, 2008]

Carnot Efficiency at the Maximum Power

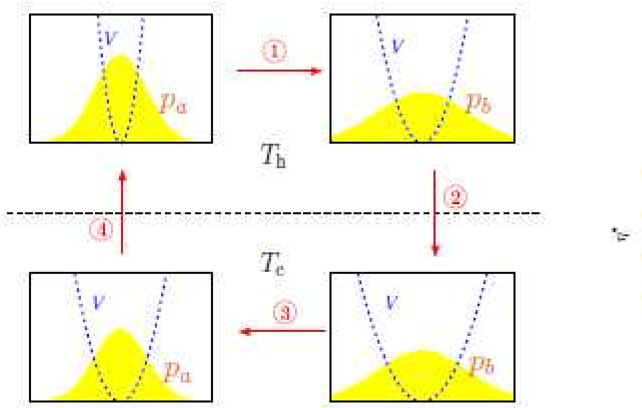
– Carnot (1824)



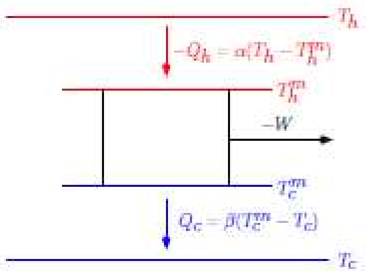
– $\eta_c \equiv 1 - T_c/T_h$
but zero power

Brownian heat engine at maximal power

[T. Schmiedl and U.S., EPL **81**, 20003, (2008)]



– Curzon-Ahlborn (1975)

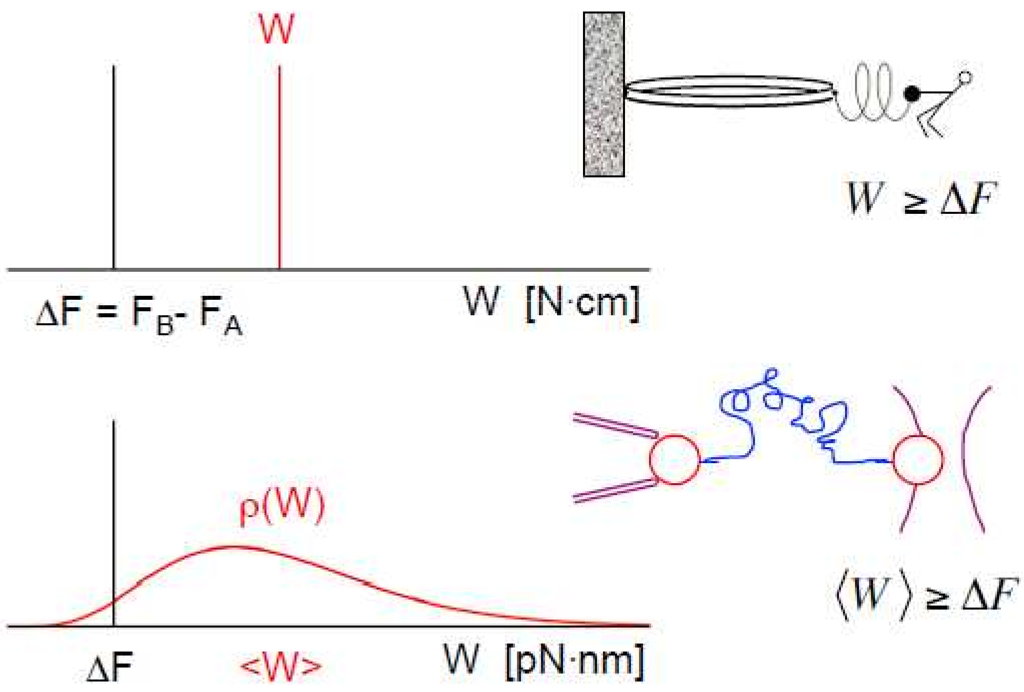


- – efficiency at maximum power
  $\eta_{ca} \equiv 1 - \sqrt{T_c/T_h}$
- – recent claims for universality(?)
- – what about fluctuations?

– Curzon-Ahlborn neither universal nor a bound

“Violation” of the Second Law at the Nanoscale

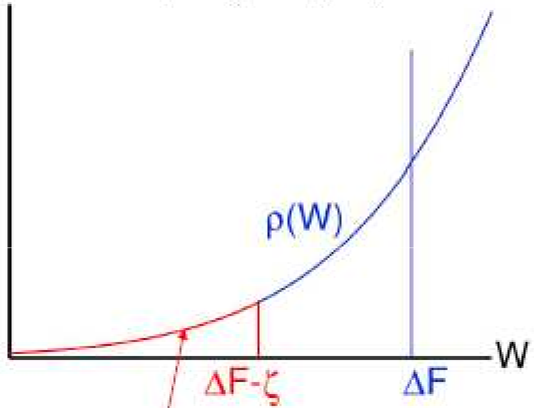
Clausius inequality, *macro* & *micro*



Irreversibility in microscopic systems

$$\left\{\begin{array}{l}\left\langle e^{-\beta W}\right\rangle = e^{-\beta\Delta F} \\ \text{Jensen's inequality} \quad \left\langle e^{x}\right\rangle \geq e^{\langle x\rangle}\end{array}\right\} \longrightarrow \langle W\rangle \geq \Delta F$$

(Clausius inequality)



What is the probability that the 2nd law will be "violated" by at least ζ units of energy?

$$P[W < \Delta F - \zeta] = \int_{-\infty}^{\Delta F-\zeta} dW\, \rho(W) \leq \int_{-\infty}^{\Delta F-\zeta} dW\, \rho(W)\, e^{\beta(\Delta F-\zeta-W)}$$

$$\leq e^{\beta(\Delta F-\zeta)} \int_{-\infty}^{+\infty} dW\, \rho(W)\, e^{-\beta W} = \exp(-\zeta/kT)$$

The probability of observing the "violating" the second law decay exponentially with energy scale of the system, and becomes unobservable for any systems whose energy scale is a few times of KT.

quantum Jarzynski equality

Jarzynski,
Phys. Rev. Lett. 78
(1997) 2690

$$\Delta F = -k_B T \, ln\langle e^{-W/k_BT}\rangle$$

free energy difference

$$\langle e^{-W/k_BT}\rangle = \int dW e^{-W/k_BT} \, P(W)$$

average exponented work

$$P(W) = \sum_{m,n} \delta[W - (E_m^\tau - E_n^0)] \, P_{m,n}^\tau \, P_n^0$$

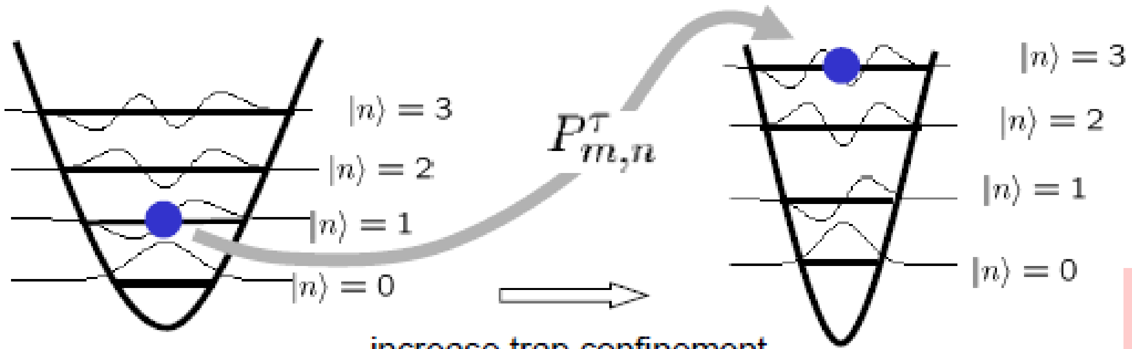
quantum work probabilty

Energy difference

Transition probabilities

Thermal occupation

no expectation value,
but correlation function



increase trap confinement
non-adaibatically

P. Talkner et al.,
Phys. Rev. E 75 R
(2007) 050102

Non-equilibrium phonon States in a Paul trap

Proposed exp. Scheme:

1) Start with thermal state n=0… ~ 10
2) Determine E^0
3) Act (non-adiabatically) on trap potential
4) Determine E^t

quantum work probabilty

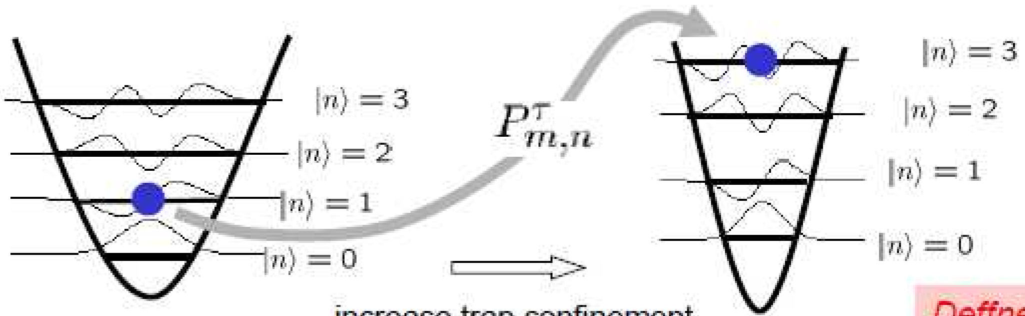
$$P(W) = \sum_{m,n} \delta[W - (E_m^\tau - E_n^0)]\; P_{m,n}^\tau\; P_n^0$$

Energy difference

Transition probabilities

Thermal occupation

no expectation value, but correlation function



increase trap confinement non-adaibatically

*Huber et al., PRL **101**, 070403 (2008)*

Deffner, Lutz, Phys. Rev. E 77, 021128 (2008)

Proof of Jarzynski equality in quantum systems

$$\left\langle e^{-\beta W} \right\rangle = \sum_{m,n} \frac{1}{Z_A} e^{-\beta E_n^A} \left| \left\langle E_m^B \middle| U \middle| E_n^A \right\rangle \right|^2 e^{-\beta (E_m^B - E_n^A)}$$

$$= \sum_{m,n} \frac{1}{Z_A} e^{-\beta E_m^B} \left| \left\langle E_m^B \middle| U \middle| E_n^A \right\rangle \right|^2$$

$$= \sum_{m} \frac{1}{Z_A} e^{-\beta E_m^B} \sum_{n} \left| \left\langle E_m^B \middle| U \middle| E_n^A \right\rangle \right|^2$$

=1, Unitary evolution

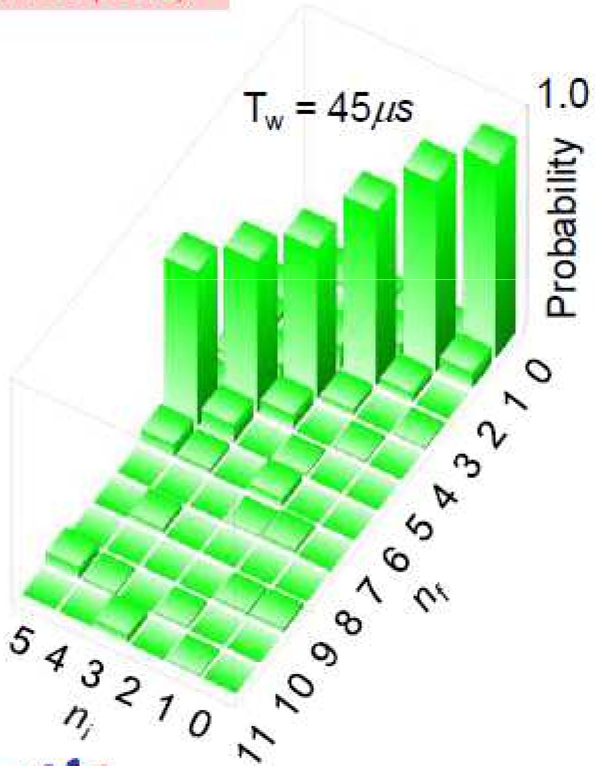
$$= \sum_{m} \frac{1}{Z_A} e^{-\beta E_m^B}$$

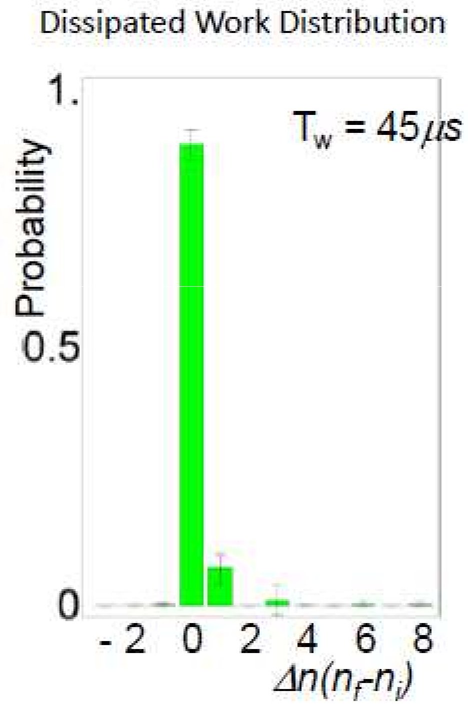
$$= \frac{Z_B}{Z_A} = e^{-\beta \Delta F}$$

Experiment test of quantum Jarzynski equality

Final State Measurements – Intermediate Work

An et al., Nat. Phys.11, 193 (2015)



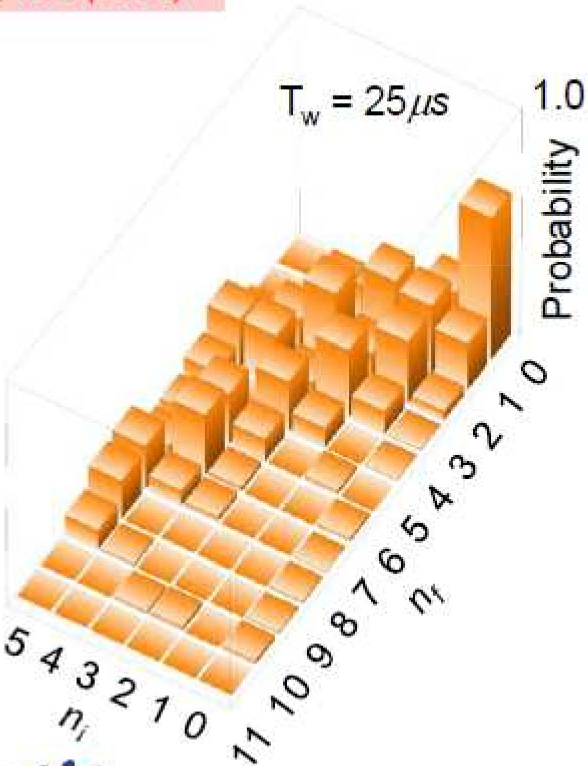


$$\left\langle e^{-\beta W-\beta\Delta F}\right\rangle = 0.989 \pm 0.072$$

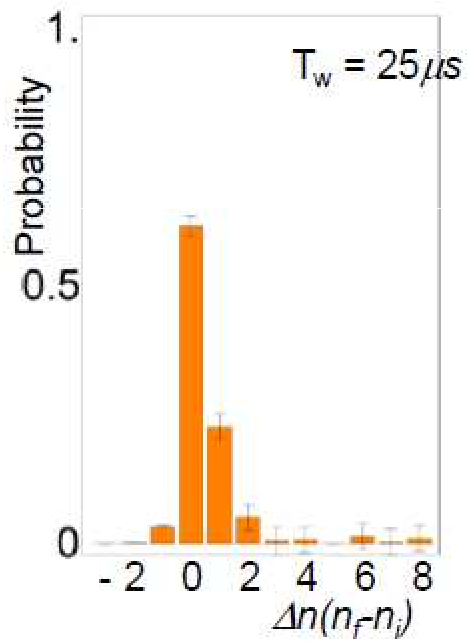
Experiment test of quantum Jarzynski equality

Final State Measurements – Intermediate Work

An et al., Nat. Phys.11, 193 (2015)

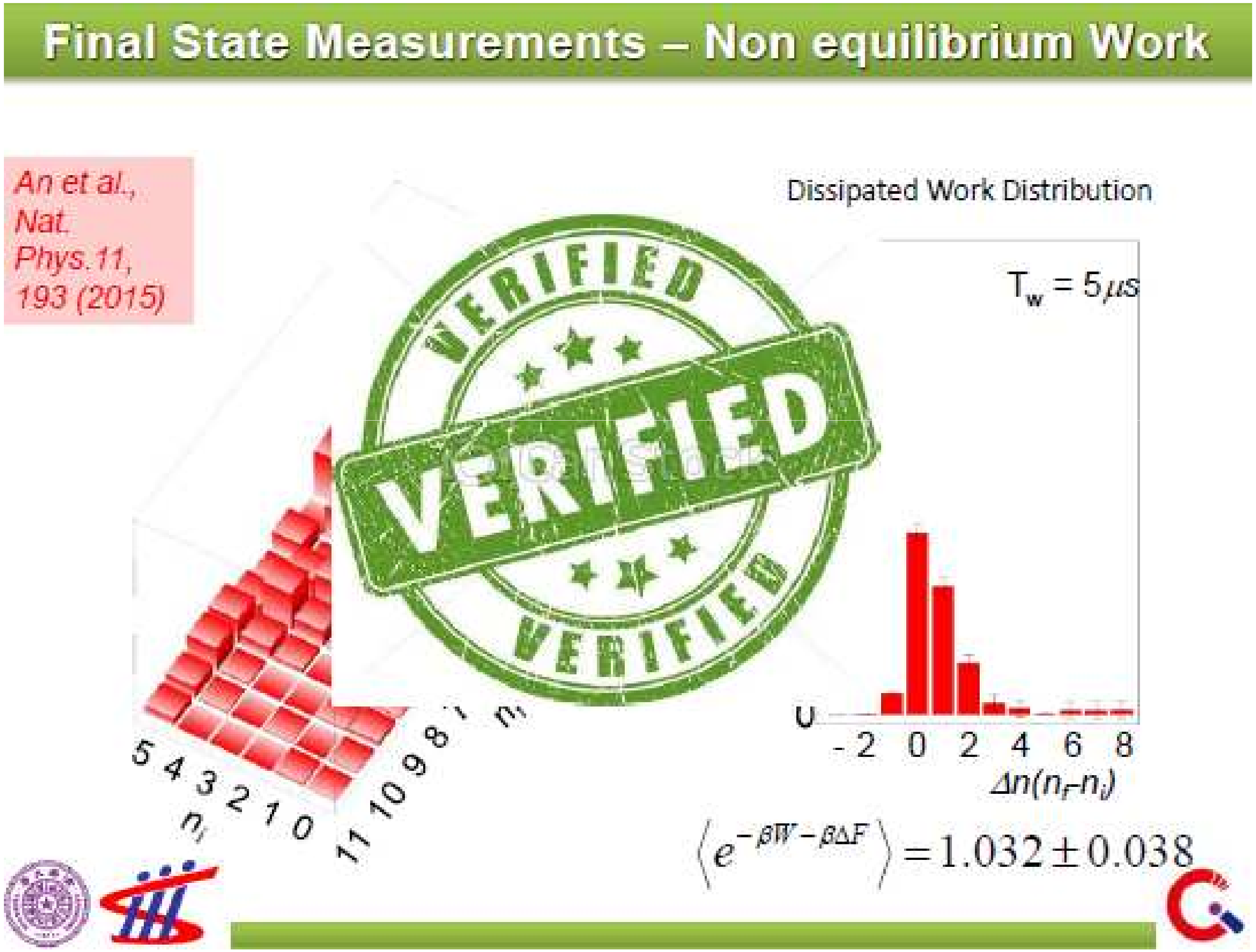


Dissipated Work Distribution

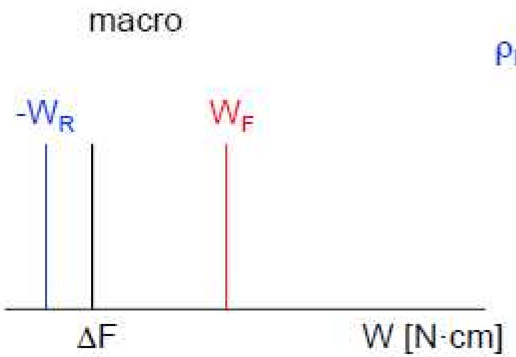


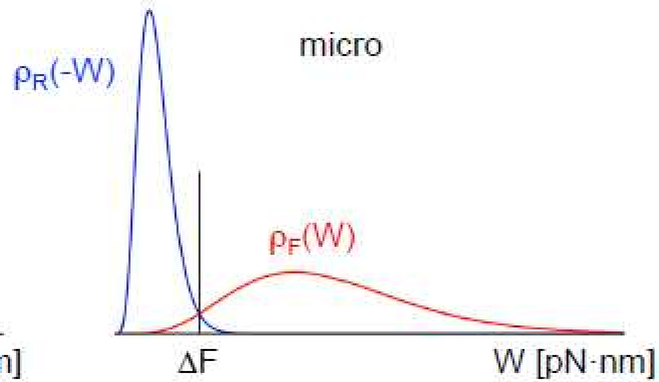
$$\left\langle e^{-\beta W - \beta \Delta F} \right\rangle = 0.995 \pm 0.045$$

Experiment test of quantum Jarzynski equality



Summary





$$\langle W \rangle \geq \Delta F \qquad\qquad \left\langle e^{-\beta W} \right\rangle = e^{-\beta \Delta F}$$

$$-\langle W \rangle_R \leq \Delta F \leq \langle W \rangle_F \qquad\qquad \frac{\rho_F(+W)}{\rho_R(-W)} = \exp\left[\beta(W - \Delta F)\right]$$

$$\left\langle \delta\left(x - x_t\right) \right\rangle \neq p^{eq}\left(x, \lambda_t\right) \qquad\qquad \left\langle \delta\left(x - x_t\right) \exp\left(-\beta w_t\right) \right\rangle \propto p^{eq}\left(x, \lambda_t\right)$$

Summary of Part I

- A MICROSCOPIC theory of nonequilibrium thermodynamics of small systems is being established (delayed by one century!!!)

- The theory of stochastic processes provides the mathematical foundation

- The second law of thermodynamics is sharpened, which has many applications in bio physics and chemical physics

- Quantum extension of the stochastic thermodynamics is not completed yet

Outline

- Thermodynamics in the19th and the 21st century
- Stochastic thermodynamics
- Fluctuation Theorems
- Maxwell's demon
- Summary

Introduction to information

Is there any relation between information and heat?

Or is there any relation between the computer and the refrigerator?

What is information?

What is one bit of information?

- Information is related to probability, once you know the probability distribution, you know the information amount from one measurement

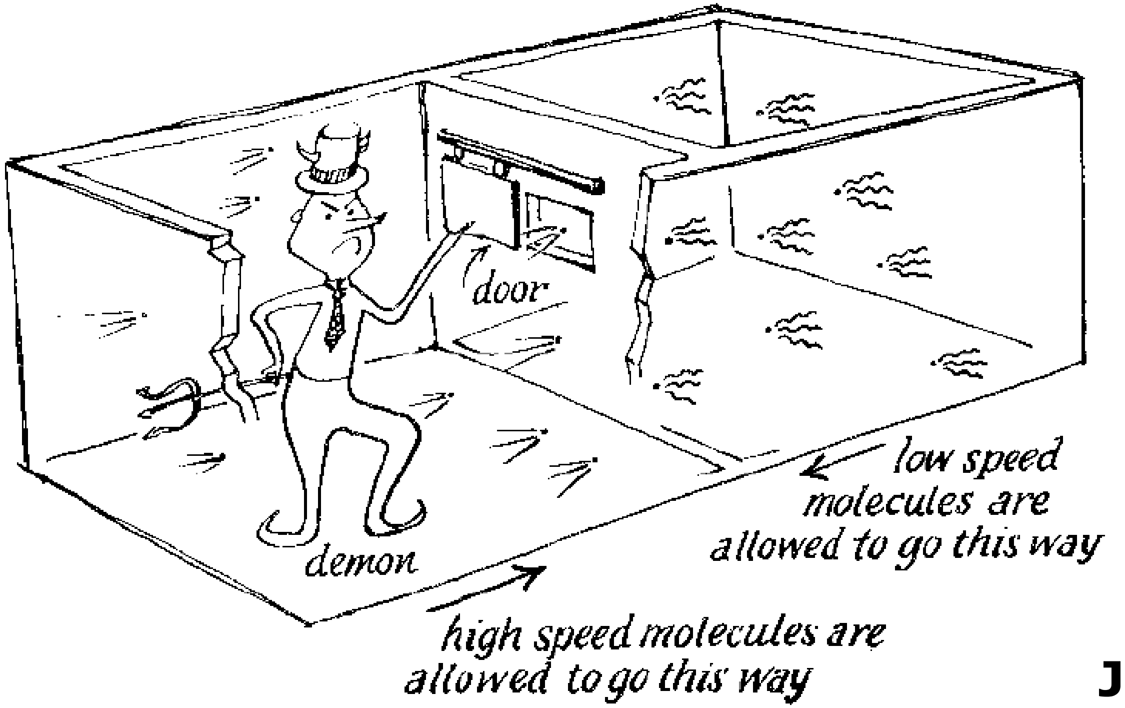
- Obtaining information $\Longleftrightarrow$ Reducing uncertainty

- Shannon Information amount: $I = -\sum_i p_i \log_2 p_i$

$$I = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2} = 1 \text{ (bit)}$$

$$p_{up} = \frac{1}{2} \qquad p_{down} = \frac{1}{2}$$

Maxwell's demon thought experiment



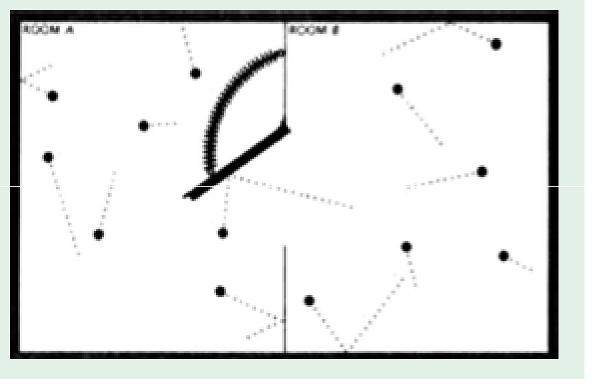
James Clerk Maxwell

1831 - 1879

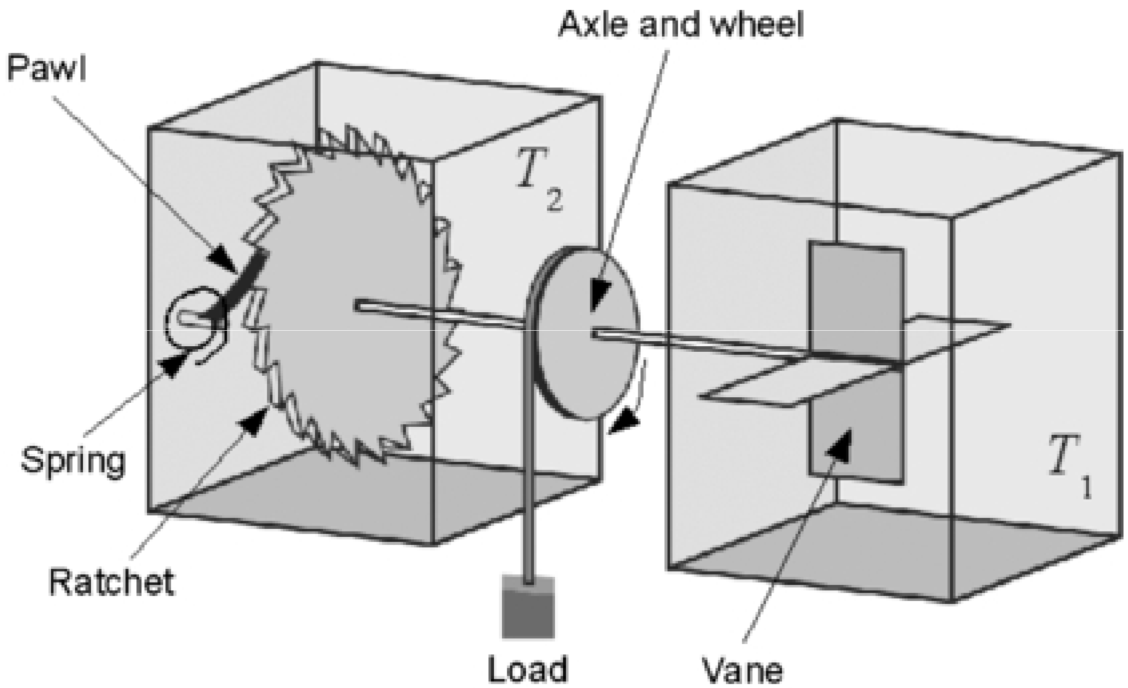
Theory of Heat (Longmans, London, 1871)

A perpetual motion machine?

Smoluchowski trapped door (1912)



Feynman’s Ratchet (1963)

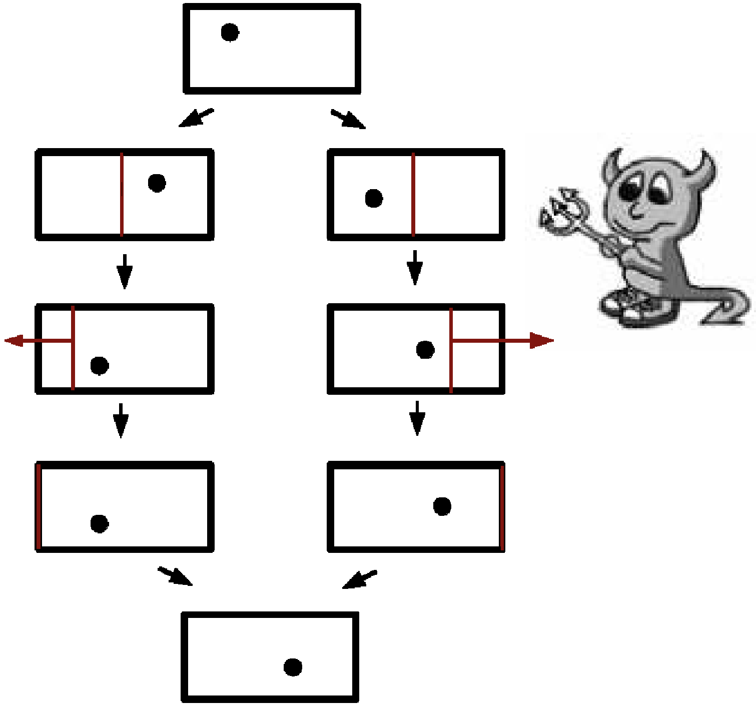


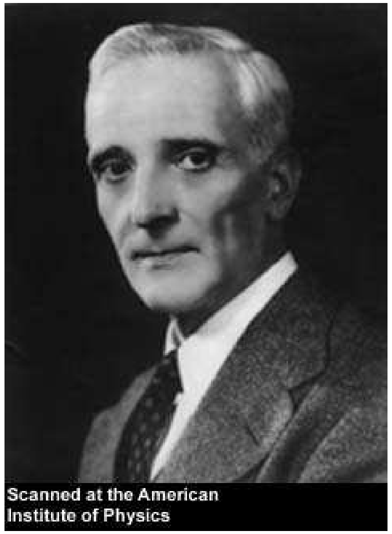
C. Jarzynski, et al PRE, 59, 6448 (1999); Z. C. Tu, J. Phys. A, 41, 312003 (2008)

Sekimoto introduced the concept of trajectory work/heat in 1997 when he studied Feynman’s ratchet, and thus initiated the whole field of stochastic thermodynamics

Szilard's engine

(establish connection between information and entropy)





Leo Szilard
1898-1964

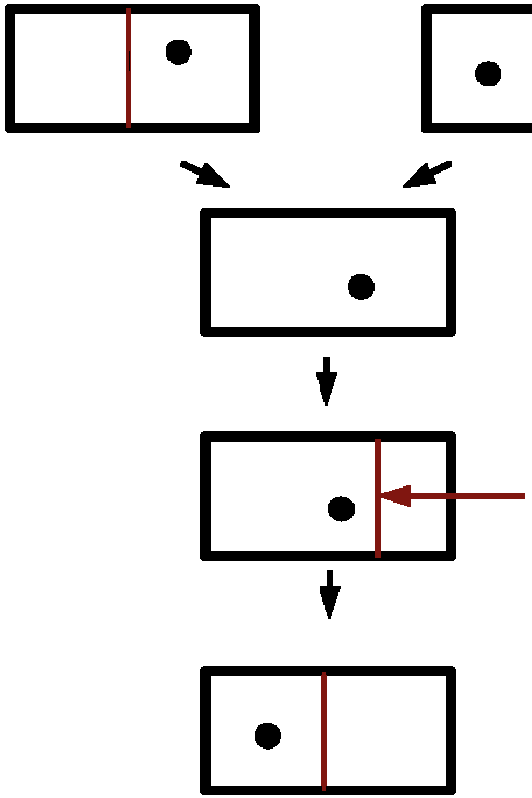
Szilard's Single Molecule
Engine (1929)

Leon Brillouin
1889-1969

Leon Brillouin: Measurement process will cost energy and leads to entropy increase

$$...100101101 \Rightarrow 0000000...$$

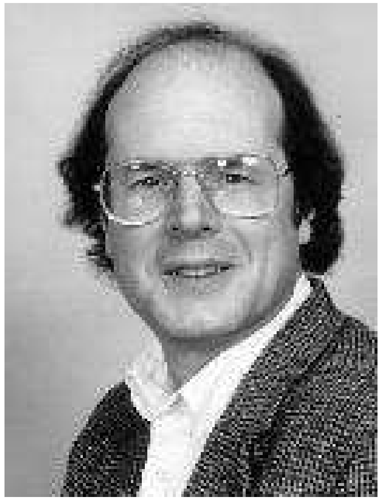
Rolf Landauer
1927-1999



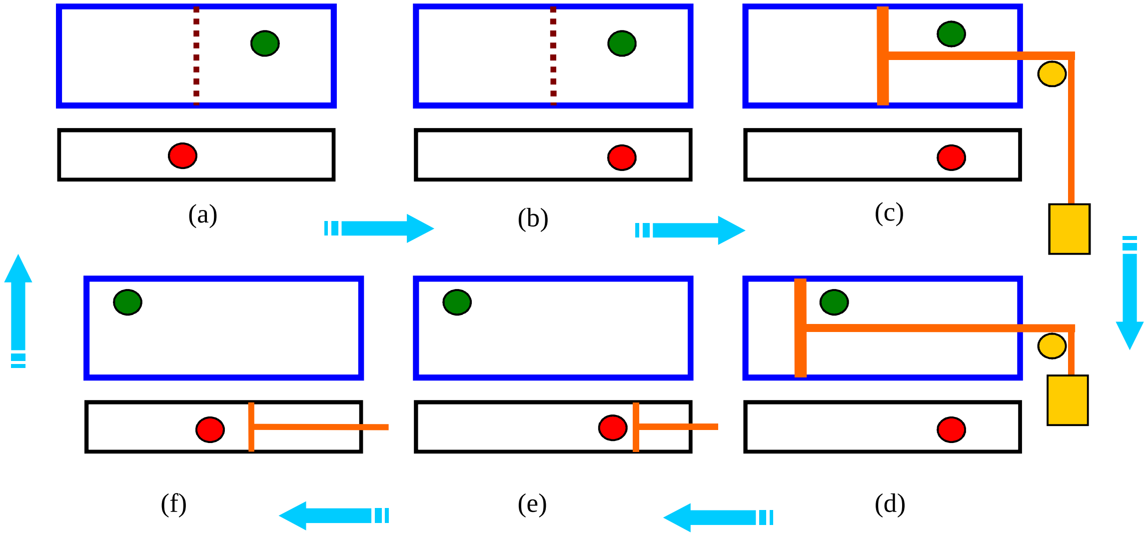
Landauer's principle (1961): Each bit of lost information will lead to an release of amount of KTln 2 heat

$$W = \int_{V}^{V/2} P dV = \int_{V}^{V/2} \frac{KT}{V} dV = -KT \ln 2$$

There is a lower bound of heat a computer must dissipate to process a given amount of information.

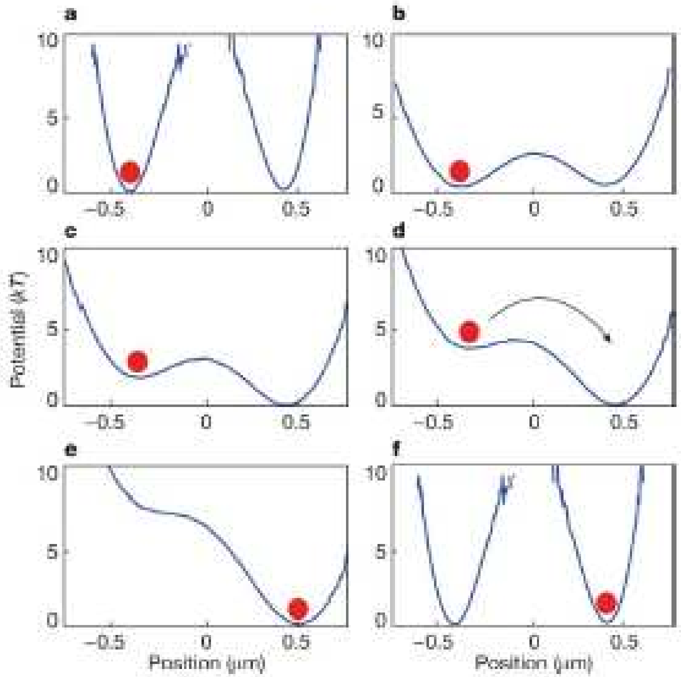


Charles H. Bennett
1943-

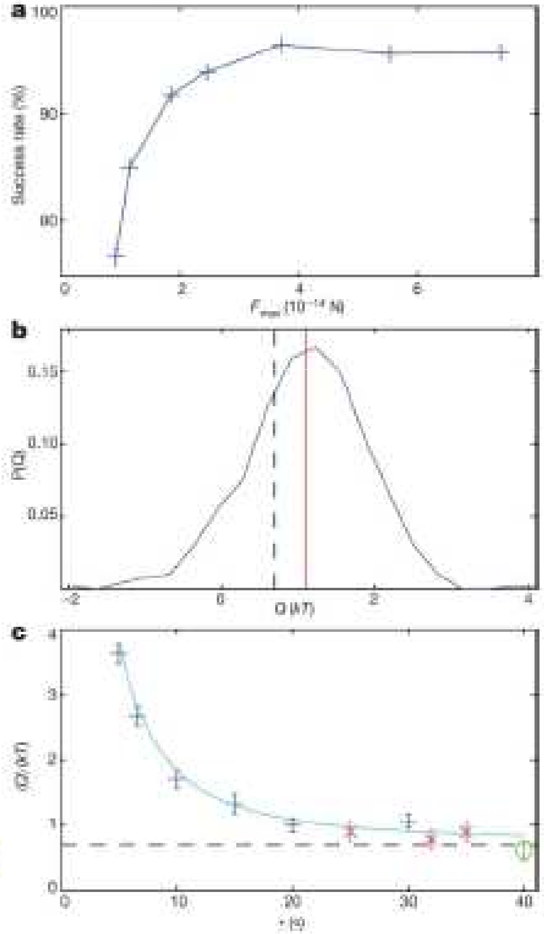


“The erasure of the memory of the demon compensates the entropy decreases and thus save the second law.”

So moving a particle between wells in a measurable fashion is now feasible



Bérut et al. *Nature* (2012) **483**:187–189



Fluctuating released heat

Released heat *vs* time duration & Landauer

…there are also recent concrete proposals for Maxwell demons and Szilard engines…

…which have been realized in single-electron traps

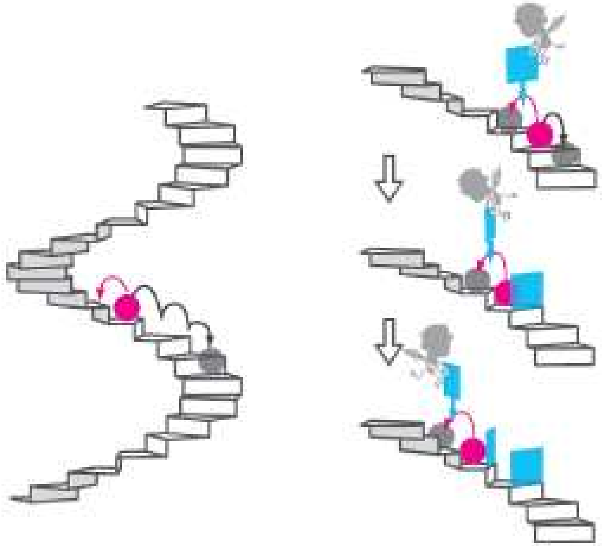
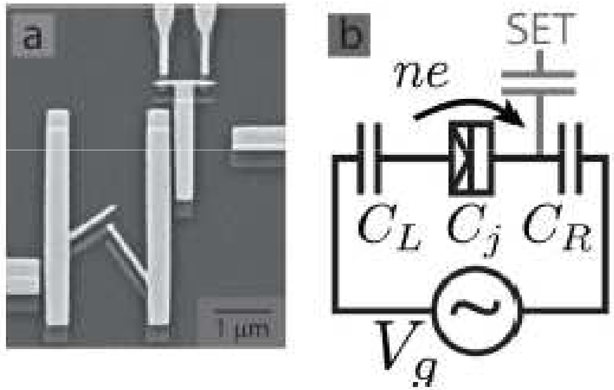


Figure 1 | Schematic illustration of the experiment. a, A microscopic particle on a spiral-staircase-like potential with a step height comparable to k_BT. The particle stochastically jumps between steps owing to thermal fluctuations. As the downward jumps along the gradient are more frequent than the upward ones, the particle falls down the stairs, on average. **b**, Feedback control. When an upward jump is observed, a block is placed behind the particle to prevent downward jumps. By repeating this cycle, the particle is expected to climb up the stairs without direct energy injection.

Toyabe *et al.*, *Nature Physics* 2010

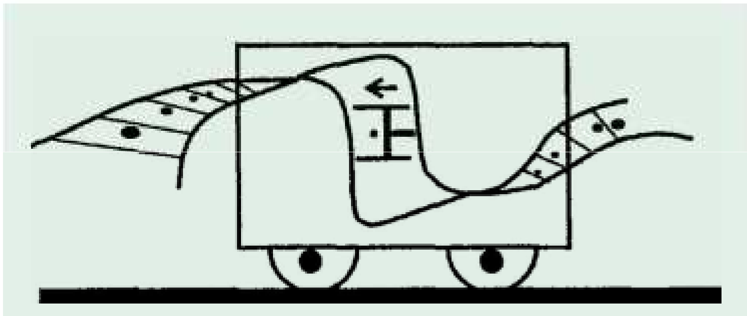


Jukka Pekola group, Aalto University

Saira *et al Phys Rev Lett* **109**, 180601 (2012)
Averin *et al Phys Rev B* **84**, 245448 (2011)
Koski *et al Phys Rev Lett* **113**, 030601 (2014)
Koski *et al*, *PNAS* **111**, 13786 (2014)

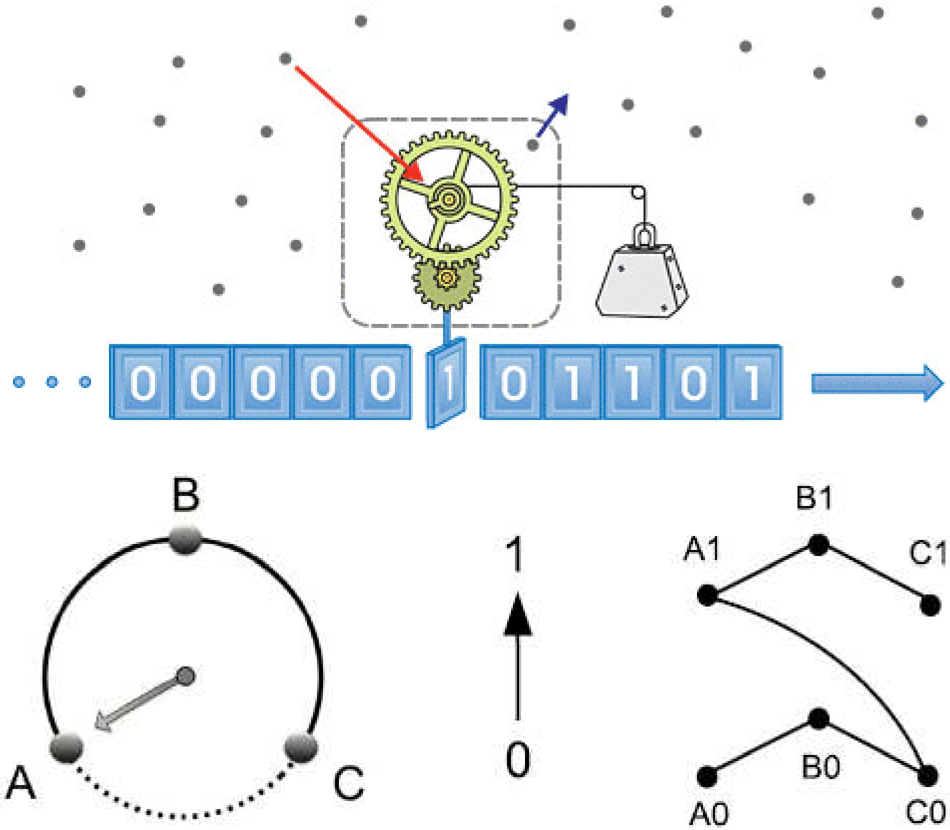
Can one use information as “energy” to drive a car?

In principle yes, but how?



R. Feynman, Feynman Lecture on Computation

An autonomous information engine proposal



Two competing
Driving forces

•Information
(entropy force)

•Mechanical
work

Information Engine
/Eraser

D. Mandal, C. Jarzynski, PNAS, 109, 11641, (2012)

Summary of Part II

•Maxwell’s demon paradox was proposed by Maxwell in 1871

•Smoluchowski proposed a mechanical demon in 1912; Similarly in 1963 by Feynman. Both did not violate the second law of thermodynamics

•Szilard simplified Maxwell’s model to a single-molecular engine in 1929

•Brillouin proposed a tentative “solution” in 1949, but was later invalidated

•Landauer proposed the Landauer’s principle in 1961

•Bennett exorcised Maxwell’s demon by using Landauer’s principle in 1982

•Sekimoto introduced the concept of trajectory work/heat when studying Feynman’s ratchet in 1997

•Landuaer’s principle was first experimented tested by Cilliberto’s group in 2012

•Jarzynski proposed a model of autonomous information engine in 2012

Thank you!

Some materials from the internet. Acknowledgements to all authors!