Data Structures
Homework #4

Due: December 3, 2024

Reminder:
1. Please write all the hand-written solutions on **A4-sized papers** for submission.
2. **Mark you calendar**: The Final Exam is scheduled in the evening from 18:00-21:00 on **January 6 (Mon), 2025**.

1. There is a simple, but inefficient, algorithm, called *bubble-sort*, for sorting a sequence S of n comparable elements. This algorithm scans the sequence $n-1$ times, where, in each scan, the algorithm compares the current element with the next one and swaps them if they are out of order. Give a pseudo-code description of bubble-sort that is as efficient as possible assuming S is implemented with a doubly linked list. What is the running time of this algorithm (using the Big-Oh notation)?
   hint: Avoid index-based operations.

2. Suppose that we are given a binary search tree T with distinct keys. We would like to find out the kth smallest and largest keys in T. The value of k is of course smaller than the size of T. Please provide efficient approaches to find these two keys and analyze your methods.

3. Suppose two binary trees, T_1 and T_2, hold entries satisfying the heap-order property. Describe a method for combining T_1 and T_2 into a tree T whose internal nodes hold the union of the entries in T_1 and T_2 and also satisfy the heap-order property. Your algorithm should run in time $O(h_1 + h_2)$ where h_1 and h_2 are the respective heights of T_1 and T_2.

4. Two binary trees are ***isomorphic*** if one of them can be reshaped into the other by swapping the left children and right children at some nodes and vice versa. For example, consider the two isomorphic binary trees T and T' in Figure 1. Trees T and T' are isomorphic because tree T' can be reshaped into T by swapping the children of the root r and node b, respectively. Of course, a binary tree is isomorphic to itself.

   (a) Given a binary tree T, how many binary trees are isomorphic to T, including T itself. Why?

   (b) We define the ***distance*** between two isomorphic binary trees to be the total number of swaps for reshaping one to the other. Please give an approach to find the distance of two given isomorphic binary trees T and T'. Write your approach first and then give the pseudo-code and the time complexity of the approach.

5. **Programming (Distance of Two Isomorphic Binary Trees)**
   Recall problem 4. In this problem, please implement the approach you proposed for finding the distance of two isomorphic binary trees. You need to first build the given binary trees. Two text files, `inFileA.txt` and `inFileB.txt`, are provided for the given trees and each of which contains the information of the input binary tree. The input text file format is as follow.

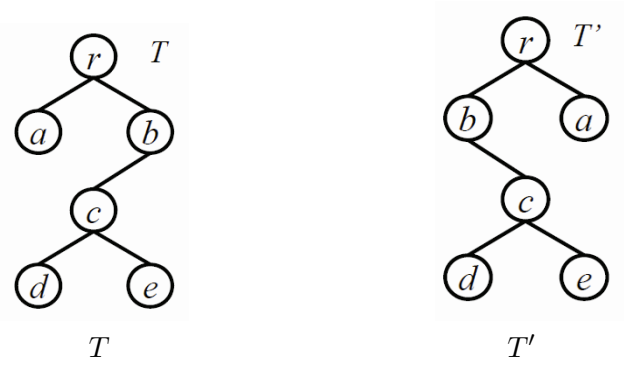

Figure 1: Two isomorphic binary trees, T and T'

```
r a b
b c -
c d e
```

In the text file, each line indicates a relation between parent and children. For example, the first line says that root r has two children: left child a and right child b. The second line indicates that node b has only left child c and no right child as denoted by -

We have provided the function `readLines()` for reading lines from the text file and you then need `re` package to parse each line for building the binary tree using ***linked structure***. For this, you need to implement the `node` and `binary tree` classes. The program performs the *preorder* traversals of the binary trees for showing the content of the trees. Then, please provide the function `deriveTheDistance()` to derive the distance of these two given isomorphic binary trees. The template file `HW4.ipynb` with output example and sample input text files for binary trees will be provided for you.

About submitting this homework

1. **Honest Policy**: We encourage students to discuss their work with the peers. However, each student should write the program or the problem solutions on her/his own. Those who copy others' work will get 0 on the homework grade.

2. For problem 1, 2, 3 and 4, Please

   (1) write all of your solutions on the **papers of size A4**,

   (2) leave you name and student ID on the first page, and

   (3) hand in your solutions for problem 1, 2, 3 and 4 to me **in class**.

3. For problem 5,

   (1) please finish the problem right after the problem description in the `HW4.ipynb` file provided on the **i-school(Plus)** (`https://istudy.ntut.edu.tw/learn/index.php`) platform; and

(2) please upload the completed `.ipynb` file with the filename as `HW4_studentID.ipynb` to **i-school(Plus)**

4. **Late work** is not acceptable. Remember, the **deadline** is the midnight of **December 3, 2024**.