HARVARD
School of Engineering and Applied Sciences

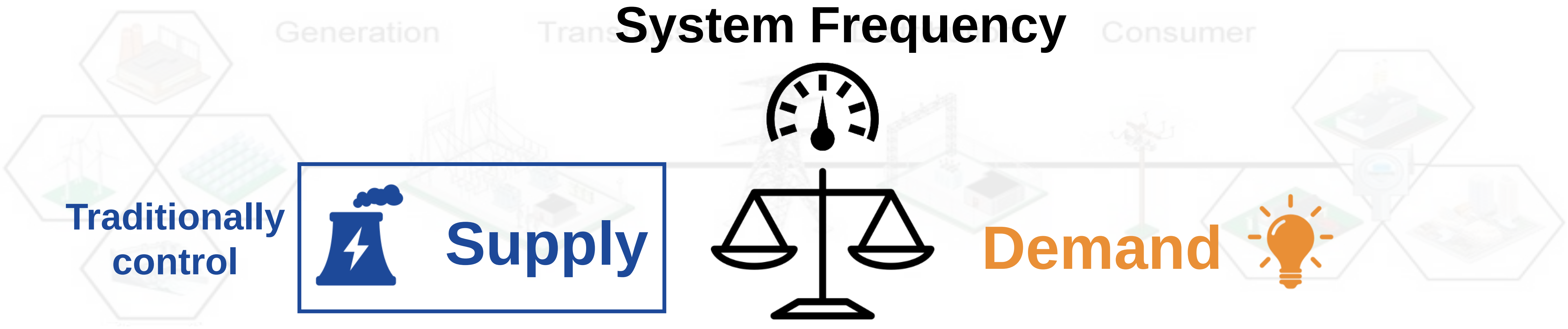
Distributed Automatic Load-Frequency Control with Optimality in Power Systems

Xin Chen[1], Changhong Zhao[2], Na Li[1]

1 **School of Engineering and Applied Sciences, Harvard University**
2 **National Renewable Energy Laboratory, USA**

2018-8-22

Frequency Regulation

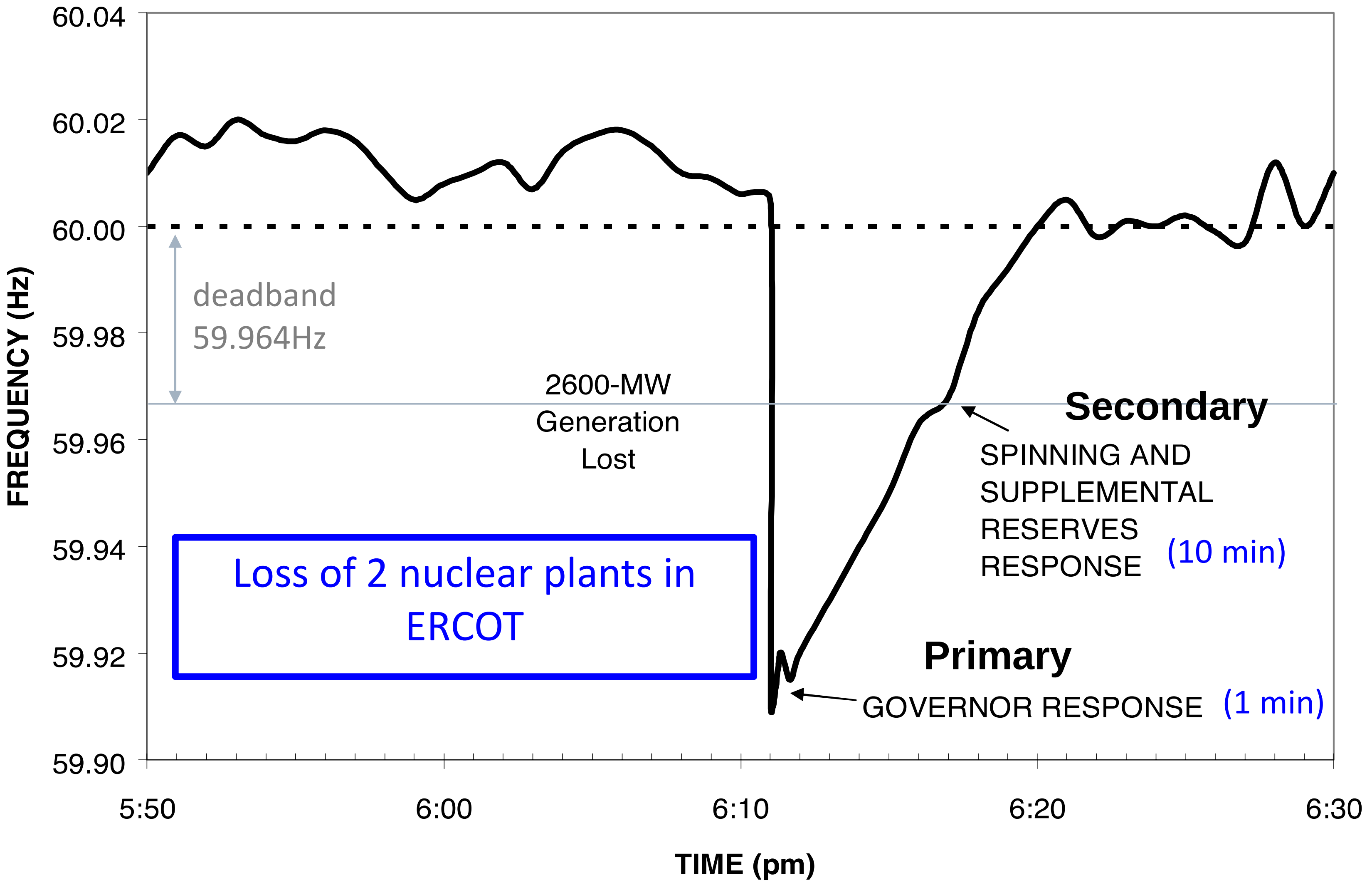


After a power disturbance:

Primary Freq. Control	Secondary Freq. Control	Economic Dispatch

sec → min → 5 min → hour → day

Frequency Regulation



Kirby 2003 [ORNL/TM-2003/19]

System Frequency

Traditionally control

Supply

uncertainty **fluctuation**

Large-scale renewable generation

Demand

flexibility **controllability**

controllable load demands

Problem

Fast controllable generators are insufficient and expensive

Solution

Control Load for frequency regulation

Literature: load-frequency control

- **Centralized Control** [M. Aldeen and H. Trinh 1994] [H. Shayeghi, H. Shayanfar 2006]

  Heavy computational and communication burden.

- **Distributed Control**

  - **Centralized optimization problem with decomposition solution.**

    [X. Zhang and A. Papa 2015] [C. Wu and T. Chang 2016] [S. Abhinav, I. Schizas 2017]

    Parameter selection and convergence issues.

  - **Local PI load controller.** [M. Andreasson, D. V. Dimarogonas 2014]

    Without regard to network constraints and load control limits.

  - **Reverse-engineering method.**

    [E. Mallada, C. Zhao 2017] [N. Li, C. Zhao 2016] [C. Zhao, U. Topcu 2014]

    Need accurate information of power imbalance and communication among boundary buses.

Our Work

Develop $\boxed{\textbf{Distributed Load Controller}}$ for real-time frequency regulation.

- No control center
- Local measurement and communication

Optimal Control (steady state)

1- Restore frequency to its nominal value.

2- Respect the operation constraints.

3- Each area absorbs its own power change.

4- Economic efficiency.

Optimal Load Control Model

$$
\begin{aligned}
\text{Obj.}\quad & \min_{d,\theta} \sum_{i\in\mathcal{N}} C_i\left(d_i\right) \\
\text{s.t.}\quad & d_i = P_i^{in} - \sum_{j:ij\in\mathcal{E}_{in}} B_{ij}\left(\theta_i-\theta_j\right) \\
& \qquad + \sum_{k:ki\in\mathcal{E}_{in}} B_{ki}\left(\theta_k-\theta_j\right) && \forall i\in\mathcal{N} \\
& \underline{d}_i \le d_i \le \overline{d}_i && \forall i\in\mathcal{N} \\
& \underline{P}_{ij} \le B_{ij}\left(\theta_i-\theta_j\right) \le \overline{P}_{ij} && \forall ij\in\mathcal{E}
\end{aligned}
$$

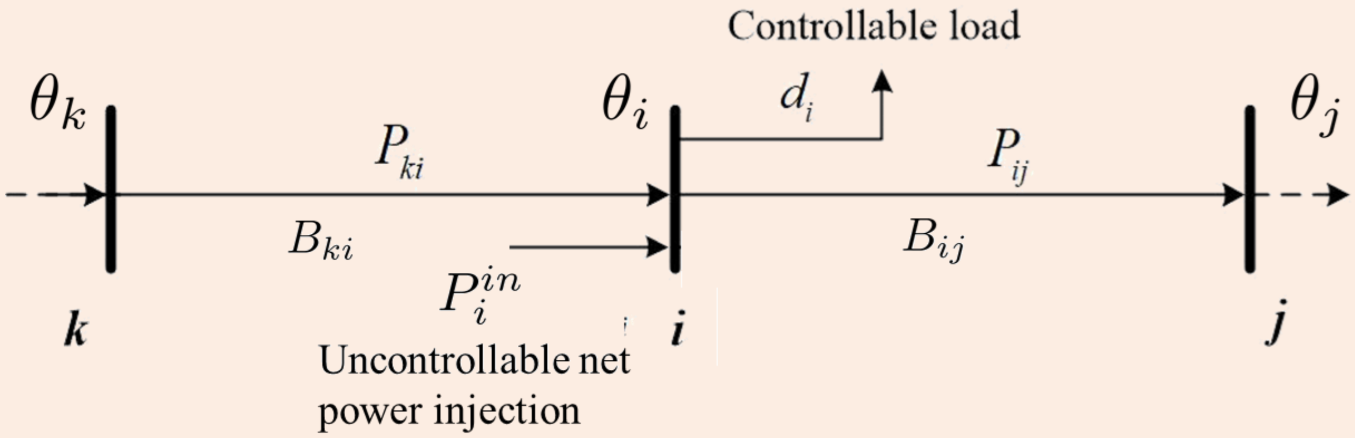
Direct Current Power Flow

$$P_{ij} = B_{ij}\left(\theta_i - \theta_j\right)$$

power flow on line *ij*

line constant

phase angle



Optimal Control (steady state)

1- Restore frequency to its nominal value.

2- Respect the operation constraints.

3- Each area absorbs its own power change.

4- Economic efficiency.

Optimal Load Control Model

$$\text{Obj.} \quad \min_{d,\theta} \sum_{i \in \mathcal{N}} C_i\left(d_i\right) \longrightarrow \text{cost function}$$

$$\text{s.t.} \quad d_i = P_i^{in} - \sum_{j:ij \in \mathcal{E}_{in}} B_{ij}\left(\theta_i - \theta_j\right) + \sum_{k:ki \in \mathcal{E}_{in}} B_{ki}\left(\theta_k - \theta_j\right) \quad \forall i \in \mathcal{N}$$

$$\underline{d}_i \leq d_i \leq \overline{d}_i \quad \forall i \in \mathcal{N}$$

$$\underline{P}_{ij} \leq B_{ij}\left(\theta_i - \theta_j\right) \leq \overline{P}_{ij} \quad \forall ij \in \mathcal{E}$$

System Dynamics

Distributed Load Controller

$(\omega_i^\star = 0, d_i^\star, P_{ij}^\star)$ **Steady State** $=$ (d_i^*, P_{ij}^*) **Optimal Solution** $+$ $\omega_i^* = 0$

Optimal Control (steady state)

1- Restore frequency to its nominal value.

2- Respect the operation constraints.

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Optimal Load Control Model

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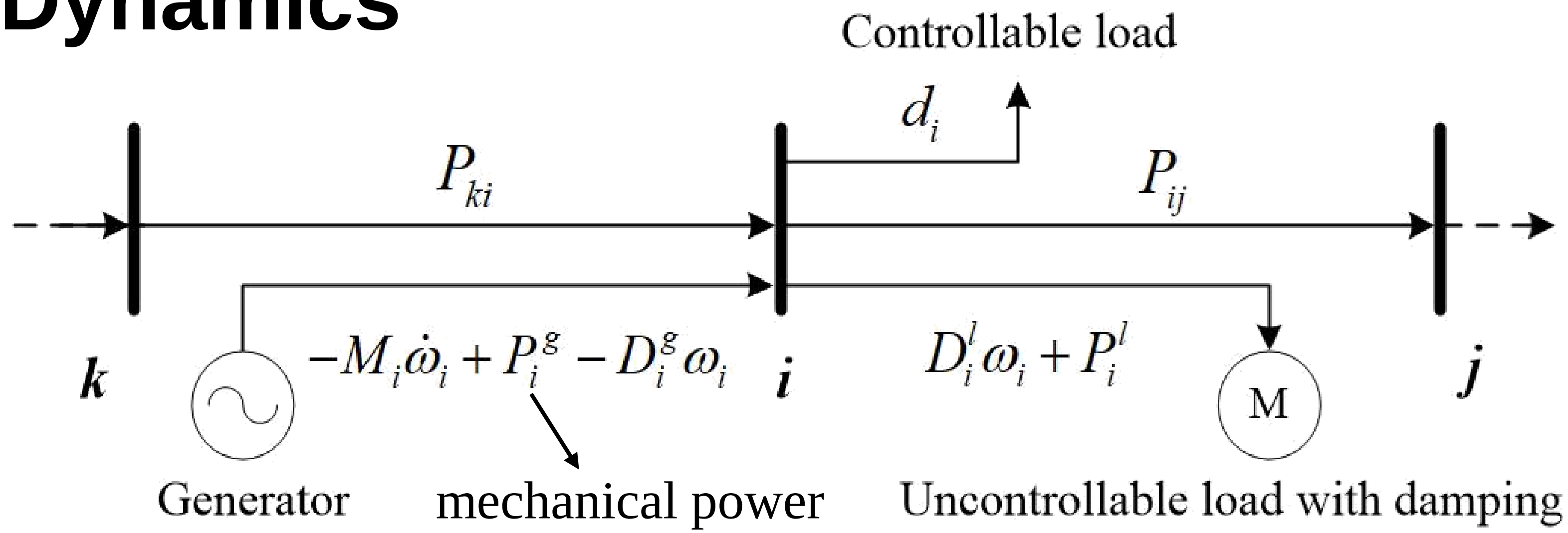
& \underline{P}_{ij} \le B_{ij}\left(\theta_i - \theta_j\right) \le \overline{P}_{ij} && \forall ij \in \mathcal{E}
\end{aligned}
$$

System Dynamics



Bus *i* : $$M_i\dot{\omega}_i = -\left(D_i\omega_i + d_i - P_i^{in} + \sum_{j:ij\in\mathcal{E}} P_{ij} - \sum_{k:ki\in\mathcal{E}} P_{ki}\right)$$ ***Swing Dynamics***

generator inertia

frequency

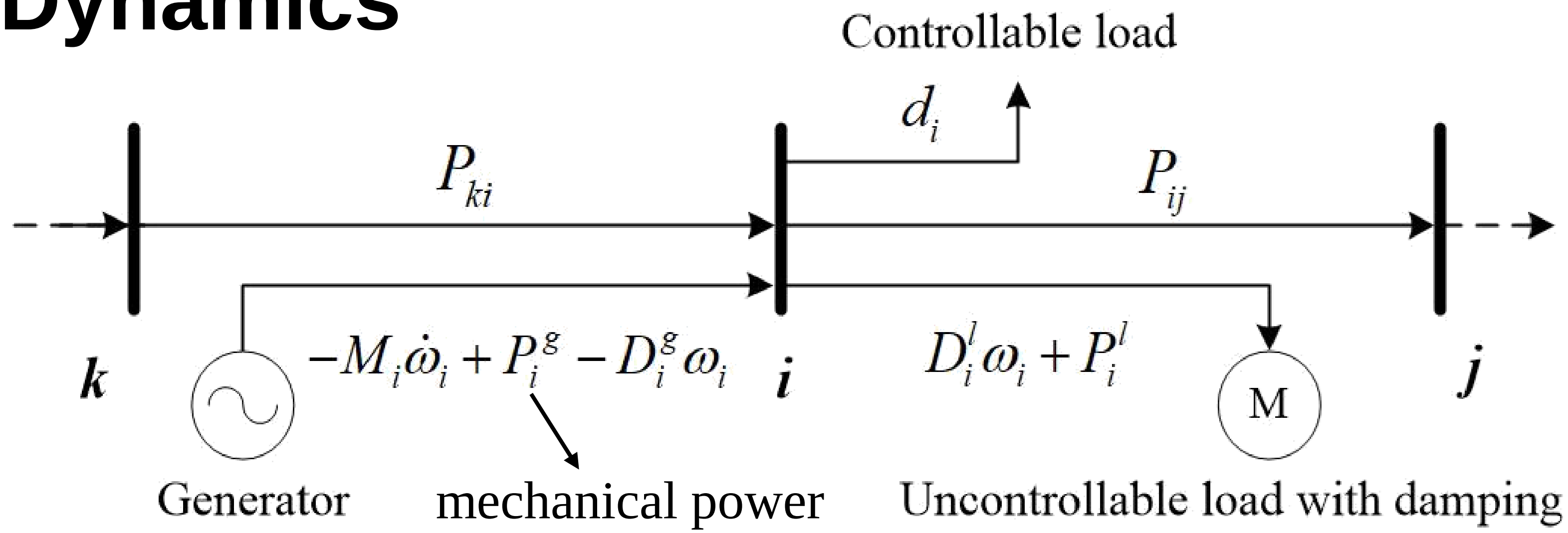
damping coefficient

controllable load

$P_i^{in} = P_i^g - P_i^l$
uncontrollable power injection

power flow on connected lines

System Dynamics



Generator bus:

$$M_i\dot{\omega}_i = -\left(D_i\omega_i + d_i - P_i^{in} + \sum_{j:ij\in\mathcal{E}} P_{ij} - \sum_{k:ki\in\mathcal{E}} P_{ki}\right) \quad \forall i \in \mathcal{G}$$

Load bus:

$$0 = D_i\omega_i + d_i - P_i^{in} + \sum_{j:ij\in\mathcal{E}} P_{ij} - \sum_{k:ki\in\mathcal{E}} P_{ki} \quad \forall i \in \mathcal{L}$$

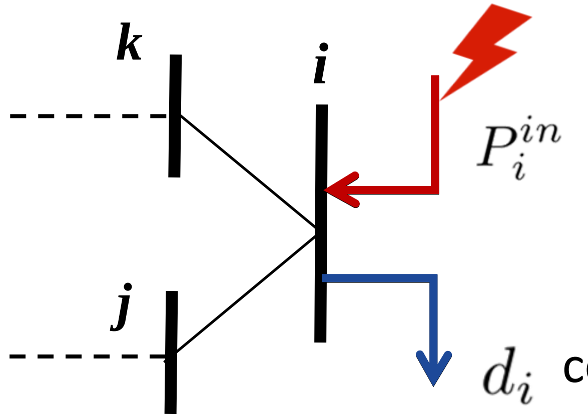
Swing Dynamics

Line ij:

$$\dot{P}_{ij} = B_{ij}\left(\omega_i - \omega_j\right) \quad \forall ij \in \mathcal{E}$$

Power Flow Dynamics

each bus



P_i^{in} uncontrollable power injection

d_i controllable load

Distributed Load Controller

$d_i = f(\omega_i, P_{ij}, P_{ki}, \cdots)$

Optimal steady state

- All the variables denote the deviation from their nominal values, e.g. $\omega_i \longrightarrow 0$.

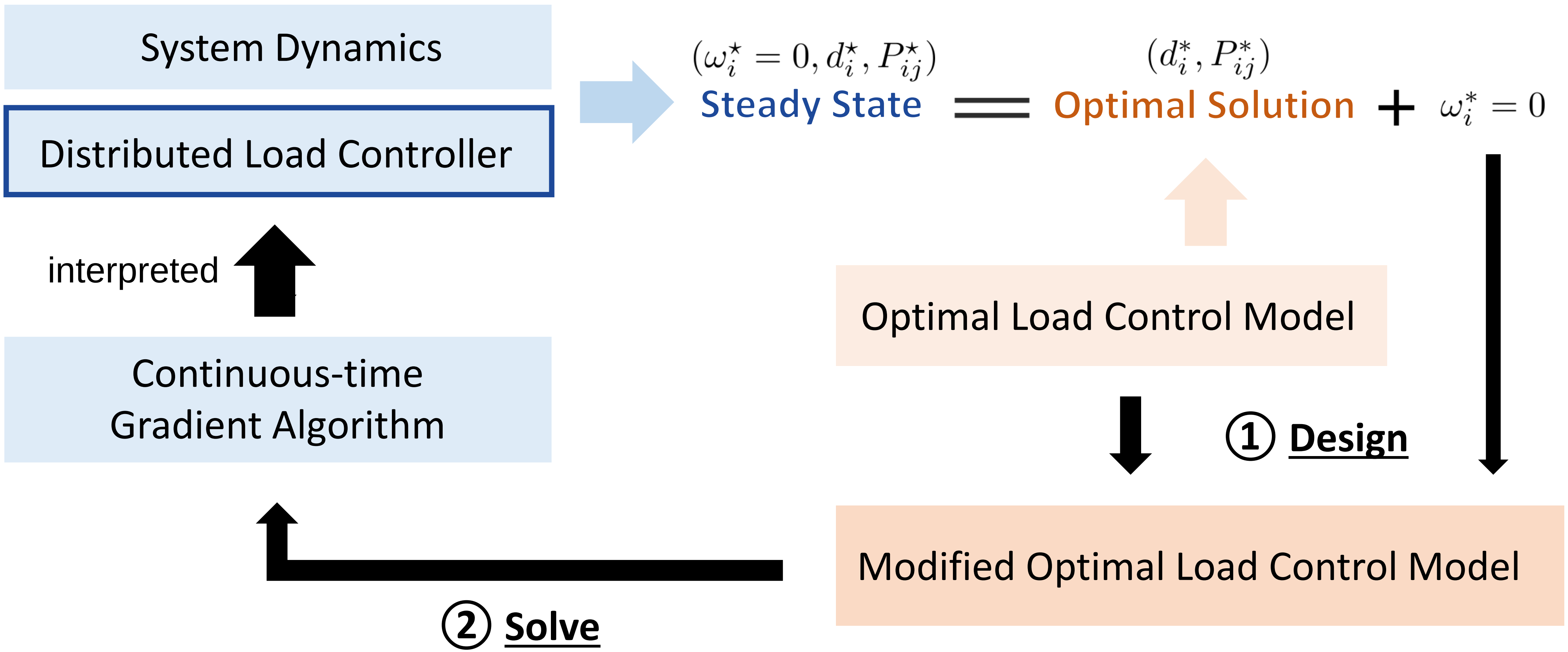
Generator bus: $M_i \dot{\omega}_i = -\left(D_i \omega_i + d_i - P_i^{in} + \sum_{j:ij \in \mathcal{E}} P_{ij} - \sum_{k:ki \in \mathcal{E}} P_{ki} \right) \quad \forall i \in \mathcal{G}$

Load bus: $0 = D_i \omega_i + d_i - P_i^{in} + \sum_{j:ij \in \mathcal{E}} P_{ij} - \sum_{k:ki \in \mathcal{E}} P_{ki} \quad \forall i \in \mathcal{L}$

Line *ij*: $\dot{P}_{ij} = B_{ij} (\omega_i - \omega_j) \quad \forall ij \in \mathcal{E}$

System Dynamics

Our Tool: Reverse-Engineering Method [1][2]



[1] E. Mallada, C. Zhao and S. Low, ``Optimal load-side control for frequency regulation in smart grids," 2017.
[2] N. Li, C. Zhao, and L. Chen, ``Connecting automatic generation control and economic dispatch from an optimization view," 2016.

① Design

Optimal Load Control (OLC) Model

$$
\begin{aligned}
\text{Obj.} \quad & \min_{d,\theta} \quad \sum_{i\in\mathcal{N}} C_i\left(d_i\right) \\
\text{s.t.} \quad & d_i = P_i^{in} - \sum_{j:ij\in\mathcal{E}_{in}} B_{ij}\left(\theta_i-\theta_j\right) \\
& \qquad + \sum_{k:ki\in\mathcal{E}_{in}} B_{ki}\left(\theta_k-\theta_j\right) && \forall i\in\mathcal{N} \\
& \underline{d}_i \le d_i \le \overline{d}_i && \forall i\in\mathcal{N} \\
& \underline{P}_{ij} \le B_{ij}\left(\theta_i-\theta_j\right) \le \overline{P}_{ij} && \forall ij\in\mathcal{E}
\end{aligned}
$$

Modified OLC Model

for algorithm design

to make $\omega_i^* = 0$ system frequency

$$
\begin{aligned}
\text{Obj.} \quad & \min_{d,\omega,P,\psi} \quad \sum_{i\in\mathcal{N}} C_i\left(d_i\right) + \sum_{i\in\mathcal{N}} \frac{1}{2} D_i w_i^2 \\
\text{s.t.} \quad & d_i = P_i^{in} - D_i\omega_i - \sum_{j:ij\in\mathcal{E}} P_{ij} + \sum_{k:ki\in\mathcal{E}} P_{ki} && \forall i\in\mathcal{N} \\
& \underline{d}_i \le d_i \le \overline{d}_i && \forall i\in\mathcal{N} \\
& d_i = P_i^{in} - \sum_{j:ij\in\mathcal{E}_{in}} B_{ij}\left(\psi_i-\psi_j\right) \\
& \qquad + \sum_{k:ki\in\mathcal{E}_{in}} B_{ki}\left(\psi_k-\psi_i\right) && \forall i\in\mathcal{N} \\
& \underline{P}_{ij} \le B_{ij}\left(\psi_i-\psi_j\right) \le \overline{P}_{ij} && \forall ij\in\mathcal{E}
\end{aligned}
$$

virtual phase angle

to guarantee the network constraints in steady state

① Design

Lemma 1

For the **optimal solution** of Modified OLC

- $\omega_i^* = 0$ for all $i \in \mathcal{N}$.
- d^* is optimal for cost and satisfies the constraints.

Optimal Control

- Restore nominal frequency.
- Economic efficiency.
- Respect operational constraints.
- Each area absorbs its own disturbance.

Modified OLC Model

$$
\begin{aligned}
\text{Obj.}\quad & \min_{d,\omega,P,\psi} \quad \sum_{i\in\mathcal{N}} C_i\left(d_i\right) + \sum_{i\in\mathcal{N}} \frac{1}{2} D_i w_i^2 \\
\text{s.t.}\quad & d_i = P_i^{in} - D_i\omega_i - \sum_{j:ij\in\mathcal{E}} P_{ij} + \sum_{k:ki\in\mathcal{E}} P_{ki} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \forall i \in \mathcal{N} \\
& \underline{d}_i \le d_i \le \overline{d}_i \qquad\qquad\qquad\qquad\qquad \forall i \in \mathcal{N} \\
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& \underline{P}_{ij} \le B_{ij}\left(\psi_i - \psi_j\right) \le \overline{P}_{ij} \qquad\qquad \forall ij \in \mathcal{E}
\end{aligned}
$$

② Solution

Lagrangian function

Modified OLC $\longrightarrow$ $\underbrace{\min_{d,\omega,P,\psi}}_{\text{primal variables}} \underbrace{\max_{\lambda_{\mathcal{G}},\lambda_{\mathcal{L}},\mu,\sigma,\gamma}}_{\text{dual variables}} L(\omega, d, P, \psi, \lambda_{\mathcal{G}}, \lambda_{\mathcal{L}}, \mu, \sigma, \gamma)$

partial primal-dual gradient algorithm

to find saddle point

- ***Step 1***: min L on ω and max L on $\lambda_{\mathcal{L}} := (\lambda_i)_{i\in\mathcal{L}}$ by taking

$$\frac{\partial L}{\partial \omega_i} = 0 \quad \text{and} \quad \frac{\partial L}{\partial \lambda_{\mathcal{L}}} = 0$$

- ***Step 2***: use the primal-dual algorithm for the rest variables as

$$\dot{x} = \epsilon_x \cdot \frac{\partial L}{\partial x} \quad \text{with} \quad x := (d, P, \psi, \lambda_{\mathcal{G}}, \mu, \sigma, \gamma)$$

② Solution

(done by **Cyber System**)

Distributed Load Controller

$$\frac{\partial L}{\partial \omega_i} = 0$$

$$\frac{\partial L}{\partial \lambda_{\mathcal{L}}} = 0$$

$$\dot{x} = \epsilon_x \cdot \frac{\partial L}{\partial x}$$

=

$$\dot{d}_i = f\left(d_i, \omega_i, \{P_{ij}\}_{j\to i}, \{\mu_j\}_{j\to i}, \{\psi_j\}_{j\to i}\right)$$

$$\dot{w}_i = \epsilon_{w_i}\left(P_i^{in} - d_i - D_i w_i - \sum_{j:ij\in\mathcal{E}} P_{ij} + \sum_{k:ki\in\mathcal{E}} P_{ki}\right) \quad \forall i \in \mathcal{G}$$

$$0 = d_i - P_i^{in} + D_i w_i + \sum_{j:ij\in\mathcal{E}} P_{ij} - \sum_{k:ki\in\mathcal{E}} P_{ki} \quad \forall i \in \mathcal{L}$$

$$\dot{P}_{ij} = \epsilon_{P_{ij}}(w_i - w_j) \qquad \forall ij \in \mathcal{E}$$

Network Dynamics

(implemented by **Physical System itself**)

② Solution

(done by **Cyber System**)

Distributed Load Controller

Partial

Primal-Dual

Gradient

Algorithm

=

$$\dot{d}_i = f\left(d_i, \omega_i, \{P_{ij}\}_{j\to i}, \{\mu_j\}_{j\to i}, \{\psi_j\}_{j\to i}\right)$$

$$\dot{w}_i = \epsilon_{w_i}\left(P_i^{in} - d_i - D_i w_i - \sum_{j:ij\in\mathcal{E}} P_{ij} + \sum_{k:ki\in\mathcal{E}} P_{ki}\right) \quad \forall i \in \mathcal{G}$$

$$0 = d_i - P_i^{in} + D_i w_i + \sum_{j:ij\in\mathcal{E}} P_{ij} - \sum_{k:ki\in\mathcal{E}} P_{ki} \quad \forall i \in \mathcal{L}$$

$$\dot{P}_{ij} = \epsilon_{P_{ij}}\left(w_i - w_j\right) \qquad \forall ij \in \mathcal{E}$$

Network Dynamics

(implemented by **Physical System itself**)

Save considerable computation

② Solution

(done by **Cyber System**)

Distributed Load Controller

Partial

Primal-Dual

Gradient

Algorithm

$$\dot{d}_i = f\left(\underbrace{d_i, \omega_i, \{P_{ij}\}_{j\to i}}_{\text{Local measurement of frequency and power flow}}, \underbrace{\{\mu_j\}_{j\to i}, \{\psi_j\}_{j\to i}}_{\text{Local communication with neighbor buses}}\right)$$

Local measurement of frequency and power flow

Local communication with neighbor buses

Fully distributed control

Detailed Distributed Controller

$$\dot{d}_i = f\left(d_i, \omega_i, \{P_{ij}\}_{j\to i}, \{\mu_j\}_{j\to i}, \{\psi_j\}_{j\to i}\right)$$

$$\dot{d}_i = \epsilon_{d_i}\left(-C_i'\left(d_i\right) + \omega_i + \mu_i - \gamma_i^+ + \gamma_i^-\right)$$

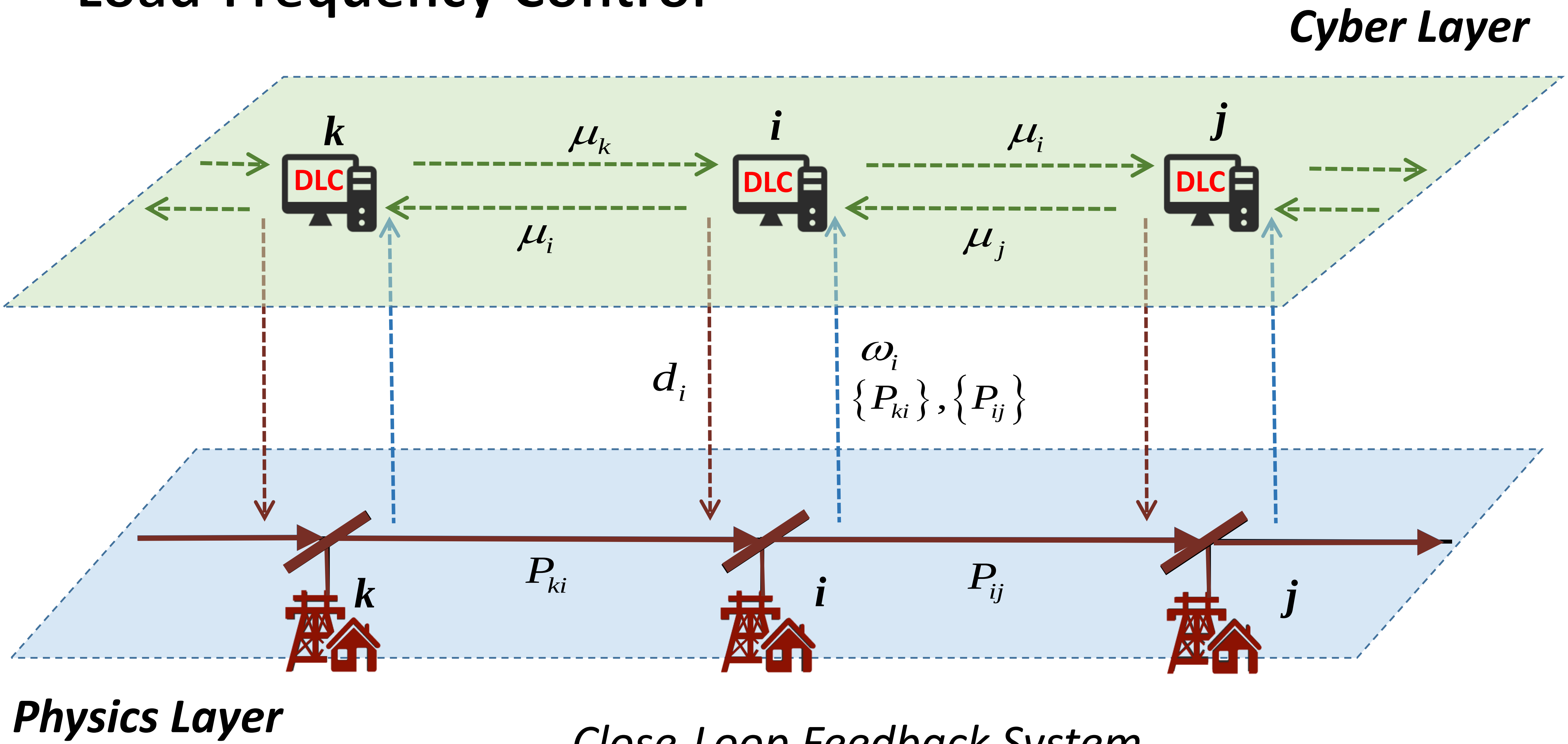
$$\dot{\psi}_i = \epsilon_{\psi_i}\left[\sum_{j:ij\in\mathcal{E}_{in}}\left(\mu_i - \mu_j - \sigma_{ij}^+ + \sigma_{ij}^-\right)B_{ij} + \sum_{k:ki\in\mathcal{E}_{in}}\left(\mu_i - \mu_k + \sigma_{ki}^+ - \sigma_{ki}^-\right)B_{ki}\right]$$

$$\dot{\mu}_i = \epsilon_{\mu_i}\left(P_i^{in} - d_i - \sum_{j:ij\in\mathcal{E}_{in}} B_{ij}\left(\psi_i - \psi_j\right) + \sum_{k:ki\in\mathcal{E}_{in}} B_{ki}\left(\psi_k - \psi_i\right)\right)$$

$$\dot{\gamma}_i^+ = \epsilon_{\gamma_i^+}\left[d_i - \overline{d}_i\right]_{\gamma_i^+}^+, \qquad \dot{\gamma}_i^- = \epsilon_{\gamma_i^-}\left[-d_i + \underline{d}_i\right]_{\gamma_i^-}^+$$

$$\dot{\sigma}_{ij}^+ = \epsilon_{\sigma_{ij}^+}\left[B_{ij}\left(\psi_i - \psi_j\right) - \overline{P}_{ij}\right]_{\sigma_{ij}^+}^+, \quad \dot{\sigma}_{ij}^- = \epsilon_{\sigma_{ij}^-}\left[-B_{ij}\left(\psi_i - \psi_j\right) + \underline{P}_{ij}\right]_{\sigma_{ij}^-}^+$$

Load-Frequency Control



Close-Loop Feedback System

Convergence

Theorem 1. (informal) Suppose that the cost function is strictly convex and the OLC problem is feasible. Then the proposed load control algorithm together with the power system dynamics asympotically converges to an optimal solution of M-OLC problem.

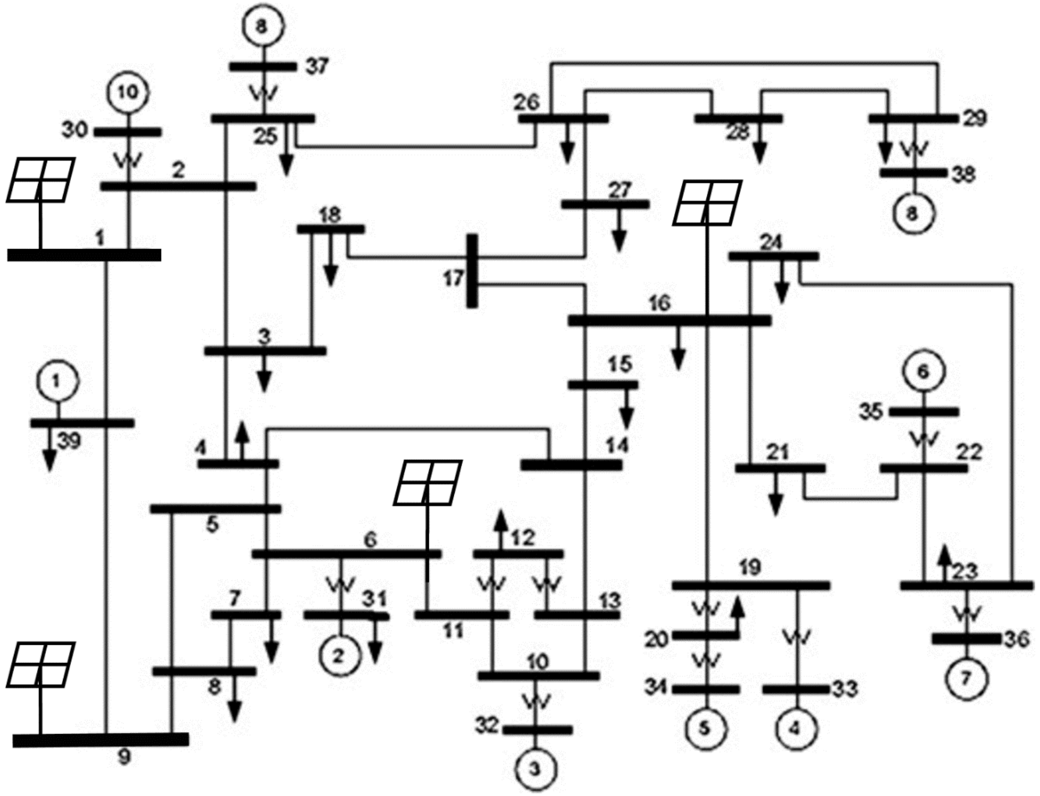
Robustness: Inaccurate damping

Theorem 2. (informal) Under some conditions on the cost function, when the inaccuracy in the damping coefficient D_i is not large, the proposed load control algorithm can still converge to an optimal solution of M-OLC problem.

Simulations

- Simulations are run on **Power System Toolbox** [1]
  - ✓ AC power flow model.
  - ✓ Classic two-axis sub-transient generator model.
  - ✓ IEEE Type DC1 excitation system model.
  - ✓ Dynamic period is 0.01s.

- 39-bus new England system**.**
  - ✓ Bus 1-29: load buses. Bus 30-39: generator buses.
  - ✓ 4 PV (photovoltaic) units for continuous power change simulation.
  - ✓ Cost for load buses 1-5: 1 p.u., the others are 5 p.u.



The 39-bus New England Power Network.

[1] K. W. Cheung, J. Chow, and G. Rogers, Power System Toolbox ver. 3.0., Rensselaer Polytechnic Institute and Cherry Tree Scientific Software, 2009.

Case 1: Step Power Change

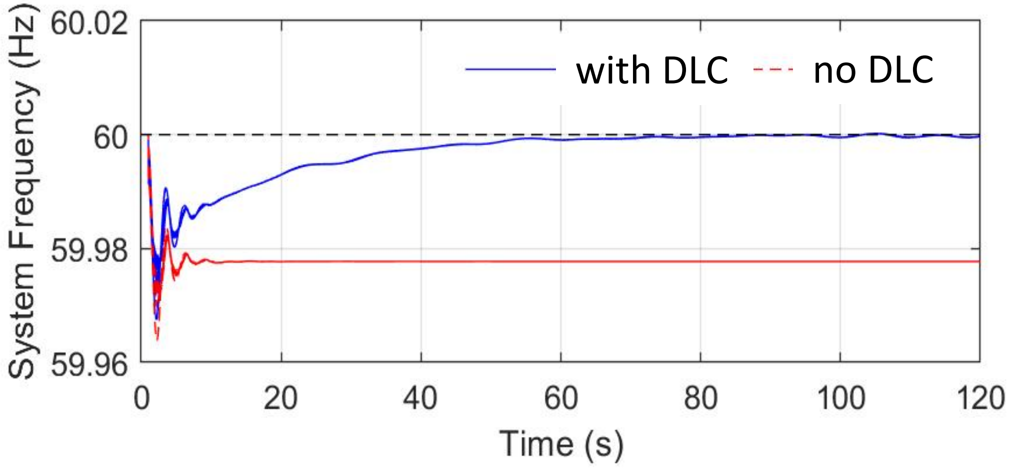


Fig. 1 Frequency dynamics under step power change.

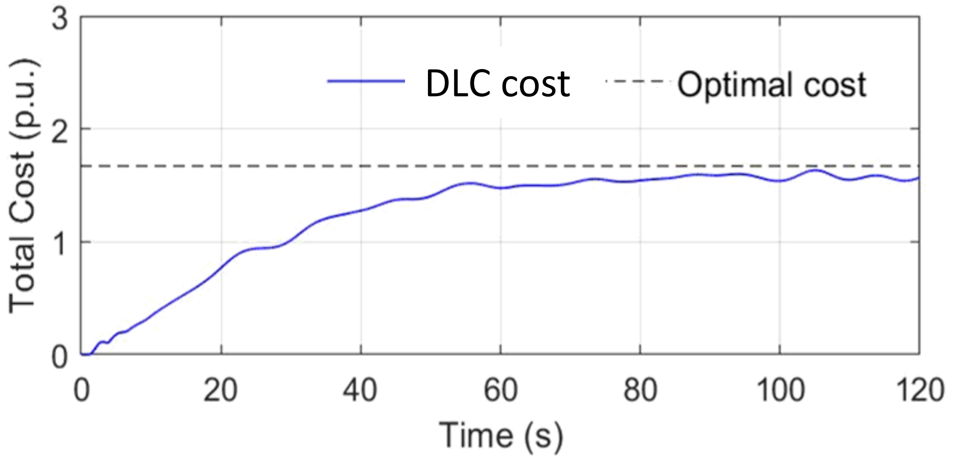


Fig.2 Total DLC cost in Case 1.

Case 2: Continuous Power Change

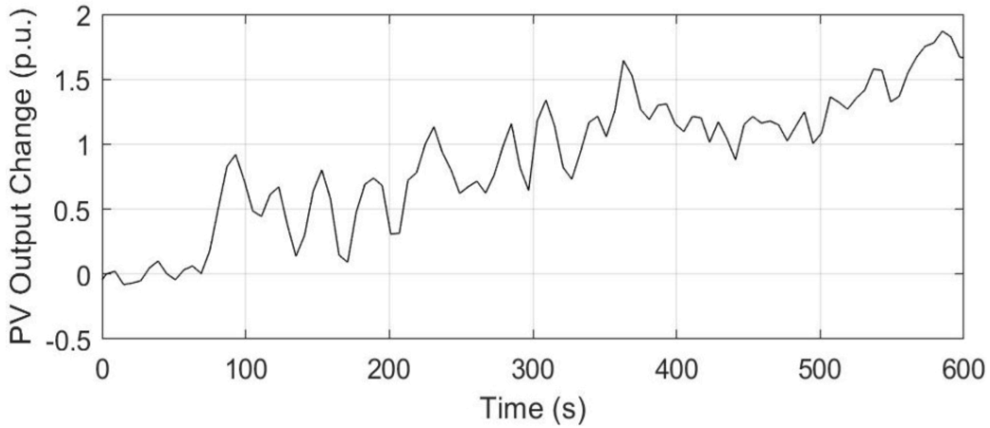


Fig.3 Time-varying PV outputs.

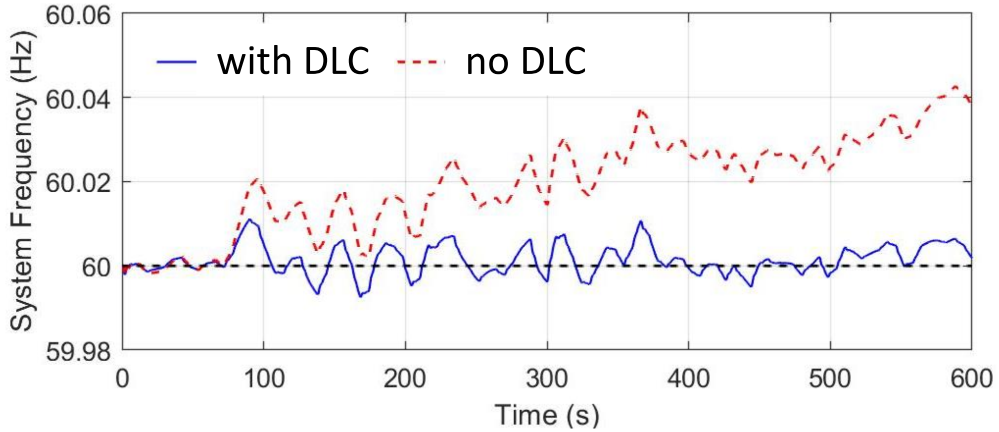


Fig.4 Frequency dynamics in Case 2.

Case 3: Inaccurate Damping

- Under step power change, let the used damping coefficient $\tilde{D}_i$ be k-times of the accurate D_i .

$$\tilde{D}_i = k \cdot D_i$$

Increase k from 0.01 to 30

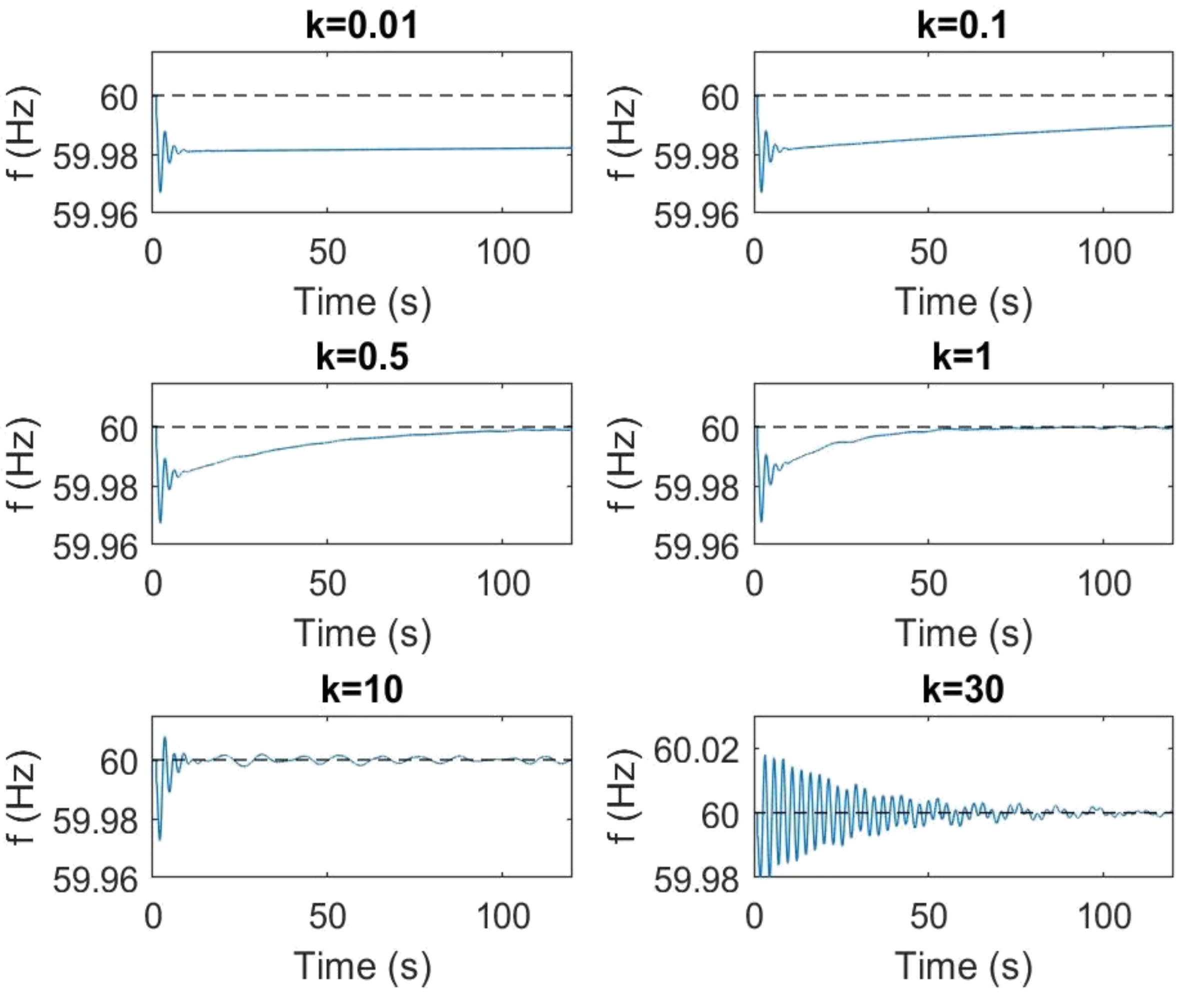


Fig. 5 The frequency dynamics under inaccurate damping coefficients

Conclusion

- A distributed load control method is developed for secondary frequency regulation:

1- The information of aggregate power imbalance is not required.

2- Only require local measurement and communication.

3- Satisfy critical operational constraints and achieve economic optimality.

4- Globally asymptotical stability.

Thank you!