Computer Science

CSC380: Principles of Data Science

Probability 1

Xinchen Yu

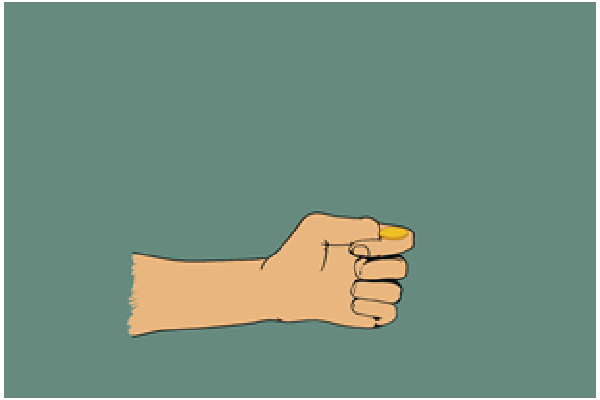
Outline

- What is probability?
- Events
- Calculating probabilities

What is probability?

What is probability?

- Suppose I flip a coin, What is the probability it will come up heads? Most people say 50%, but why?



- “Nolan’s new movie is coming out next weekend! There’s a 100% chance you’re going to love it.”

Interpreting probabilities

Basically two different ways to interpret:

- Objective probability
  - based on logical analysis or long-run frequency of an event. It's derived from known facts, symmetry, or repeated experiments.

- Subjective probability
  - based on personal belief, opinion, or information about how likely an event is, especially when there's uncertainty or limited data.

Objective or Subjective?

r/AskReddit • 2 yr. ago

What statistically improbable thing happened to you?

• 2y ago

When I was a teenager I picked up a hitchhiker and then a few years later the same guy picked me up when I was walking after I ran out of gas. Never saw him before or after those two occasions.

• 2y ago

Got attacked by a robin in the morning, then attacked by a hawk 3 hours later. Weird day.

18K Award Share ...

It is a subjective probability: a belief based on their perception of how rare or meaningful the coincidence is, not a calculation based on statistical data.

Subjective probability

- Probabilities aren't in the world itself; they're in our knowledge/beliefs about the world.

- Can assign a probability to the truth of any statement that I have a degree of belief about.

We will focus on objective probability in this class.

Objective probability

- The probability of an event represents the long run proportion of the time the event occurs under repeated, controlled experimentation.
  - e.g. 000111010011111101000110
- Famous experiments in history on coin tosses

Experimenter	# Tosses	# Heads	Half # Tosses
De Morgan	4092	2048	2046
Buffon	4040	2048	2020
Feller	10000	4979	5000
Pearson	24000	12012	12000

Outcome, Event and Probability

Outcome

- Outcome is a **single** result/observation of a **random experiment**.

- Experiment 1: You flip a coin once.
  - The outcomes are: "Heads" or "Tails”

- Experiment 2: You roll a 6-sided die.
  - The outcomes are: 1, or 2, or 3, or 4, or 5, or 6

- Experiment 3: You tap "shuffle play" on your favorite Spotify playlist with 100 songs and the first song played.
  - The outcomes are: the 1st song in the list, or the 2nd song, ….

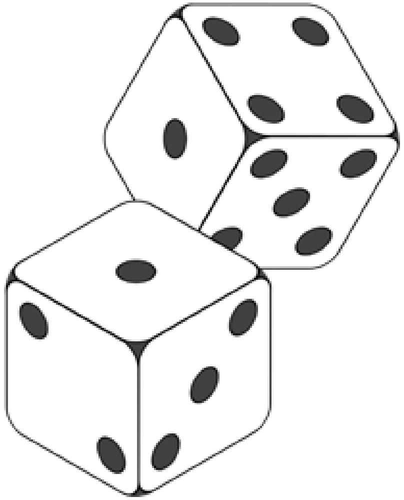
Random Events and Probability

Suppose we roll <u>two fair dice</u>…

Random Events and Probability

Suppose we roll <u>two fair dice</u>…

- What are the possible outcomes?
- What is the *probability* of the following:
  - rolling **even** numbers?
  - having two numbers sum to 6?
  - If one die rolls 1, the second die also rolling 1?



*…this is a **random process**.*

How to formalize all these quantitatively?

The Sample Space

- The set of **all** possible outcomes of a random experiment is called the sample space, written as S.

- In math, the standard notation for a set is to write the individual members in curly braces:
  - S = {Outcome1, Outcome2, . . . , }

- Useful to visualize the sample space with an actual space.

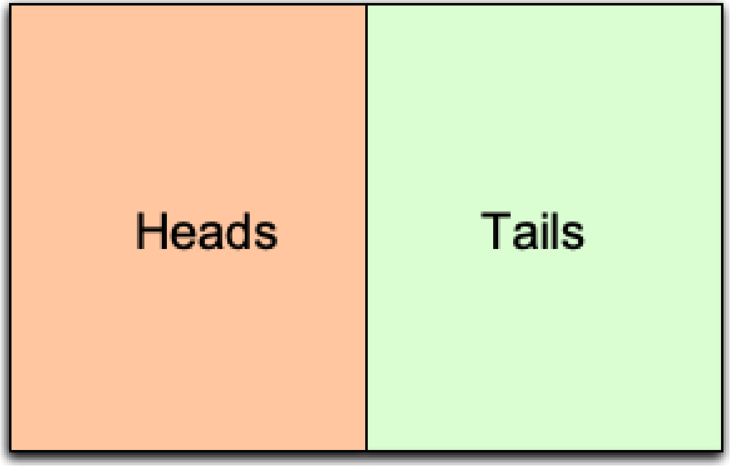
The Sample Space

Figure: Visualization of a Sample Space

Examples of Sample Spaces

What's the sample space for a single coin flip?

- S = {Heads, Tails}



Examples of Sample Spaces

What is the sample space of rolling a die?

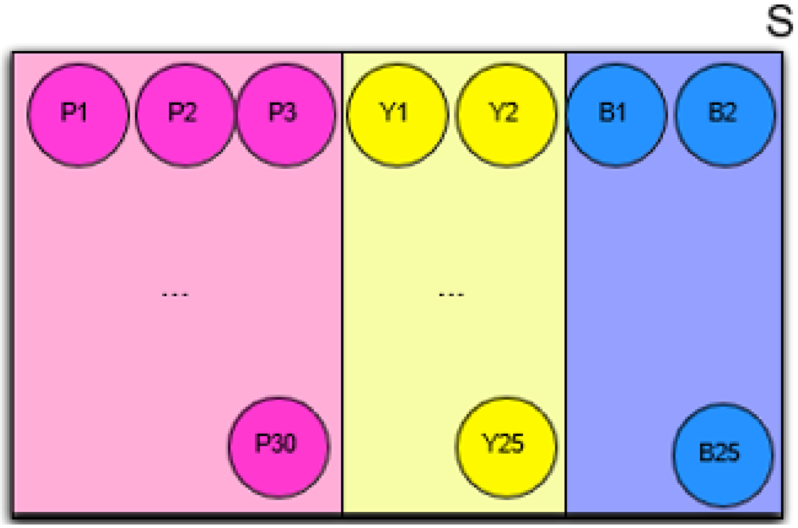
- S = {1, 2, 3, 4, 5, 6}

1	2	3
4	5	6

Examples of Sample Spaces

What is the sample space of drawing a ball out of a box containing 30 pink, 25 yellow, and 25 blue balls?

- S = {P1, P2, …, P30, Y1, …, Y25, B1, …, B25}



Examples of Sample Spaces

What's the sample space for...

- Randomly choosing a student from UA?
  - S = {Aarhus, Amaral, Balkan, . . . , Yao, Zielinski}

- Flipping two different coins?
  - S = {HH, HT, TH, TT}

- Flipping one coin twice?
  - S = {HH, HT, TH, TT}

- Observing the number of earthquakes in San Francisco in a particular year?
  - S = {0, 1, 2, 3, . . . }

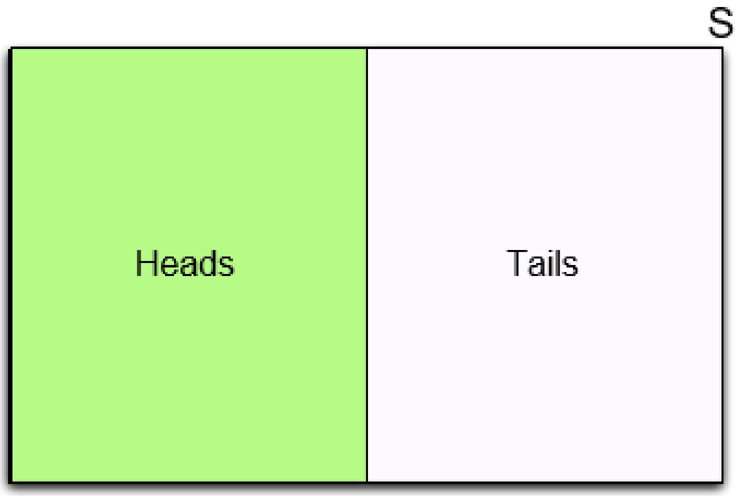
Events

- An event E is a **subset** of the sample space.
- An event E is a **set** of outcomes
  - When we make a particular observation, it is either “in” E or not.
  - Helpful to think about events as propositions (TRUE/FALSE): TRUE when the outcome is among the elements of the event set, and FALSE otherwise.
  - Is 4 in event E = {2, 4, 6}? ✅ YES → the **proposition is TRUE**
  - Is 4 in event F = {1, 3, 5}? ❌ NO → the **proposition is FALSE**

Examples of Events

What's the event set corresponding to the following propositions?

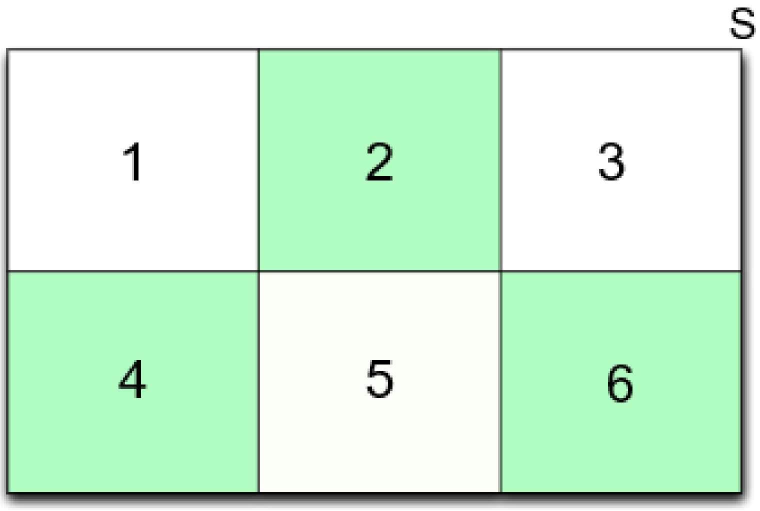
- "The coin comes up heads"
- E = {Heads}



Examples of Events

What's the event set corresponding to the following propositions?

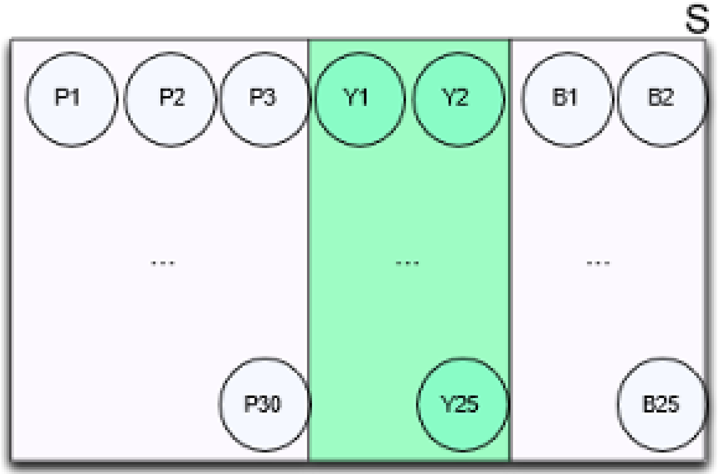
- "The die comes up an even number"
- E = {2, 4, 6}



Examples of Events

What's the event set corresponding to the following propositions?

- "A yellow ball is chosen"
- E = {Y1, Y2, . . . , Y25}

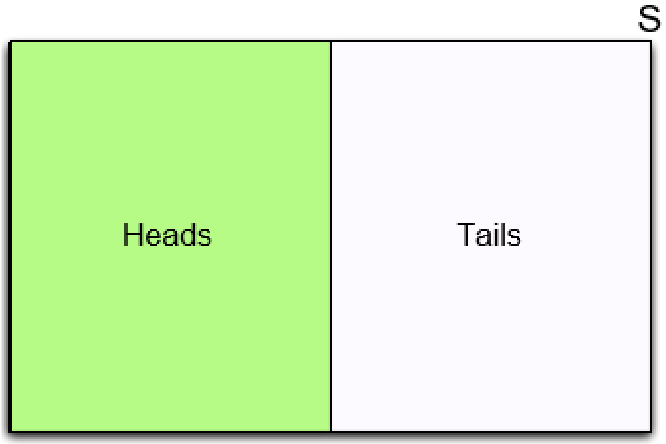


Calculating Probabilities

Calculating probability

- We can think of the probability of an event E as its area, where S (sample space) always has a total area of 1.0

- So, the probability of E is the fraction of S that it takes up.

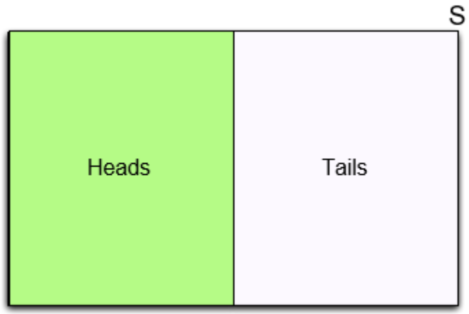


Calculating probability using symmetry

- If we have a sample space for which every outcome is equally likely (has the same area), then we can find event probabilities easily.

- Classic probability model to get the probability of an event E:

$$P(E) = \frac{\#\text{outcomes in } E}{\#\text{outcomes in } S}$$



Probability as Area

What is the probability of

$$P(E) = \frac{\#\text{outcomes in } E}{\#\text{outcomes in } S}$$

- Rolling a fair die and see an even number?
- E = {2,4,6}
- $P(E) = \frac{\#\{2,4,6\}}{\#\{1,2,3,4,5,6\}} = \frac{3}{6} = \frac{1}{2}$

1	2 P(2) = 1/6	3
4 P(4) = 1/6	5	6 P(6) = 1/6

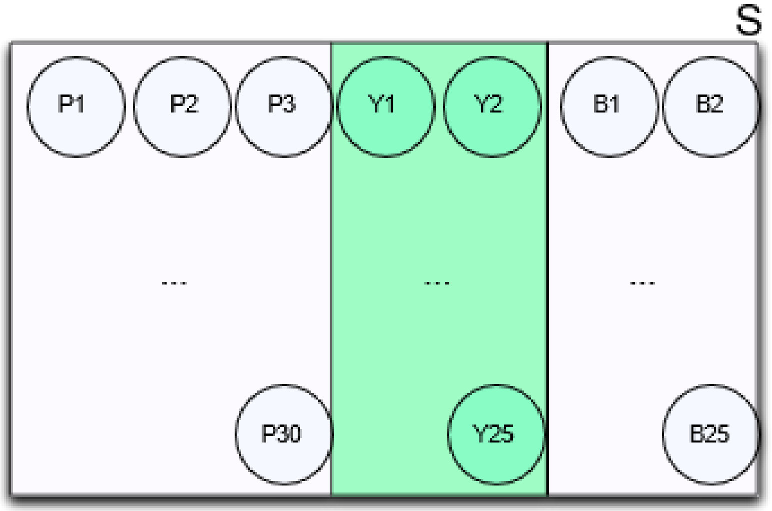
P(S) = 1

P(Even) = P(2) + P(4) + P(6) = 3/6

Probability as Area

What is the probability of

- Selecting a yellow ball?
- E = {Y1, Y2, . . . , Y25}
- $P(E) = \frac{\# E}{\# S} = \frac{25}{30+25+25} = \frac{5}{16}$

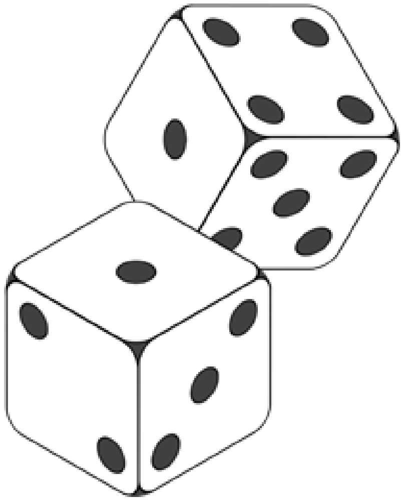


P(Yellow) = P(Y1) + ... + P(Y25) = 25 * (1/80)

Random Events and Probability

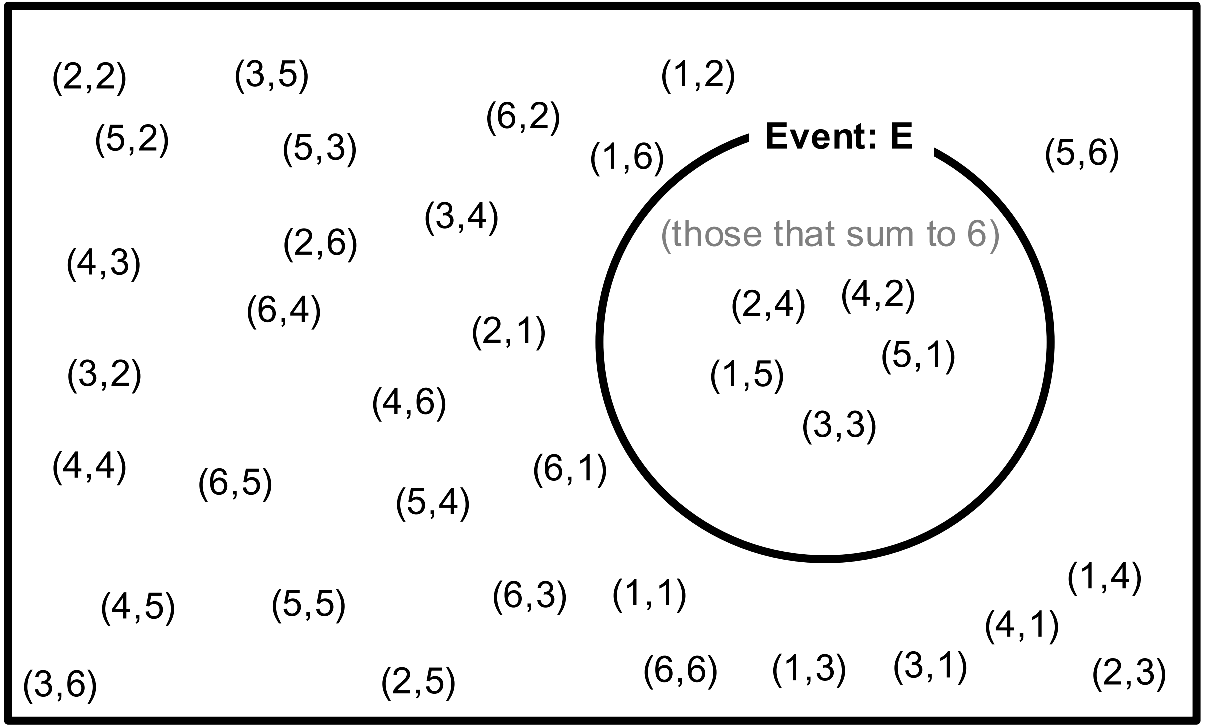
- Suppose we throw two fair dice
  - What is the sample space S (space of all possible outcomes)?

  Event E: the two dice's outcomes sum to 6
  - What is the size of E?
  - What is the probability of E?



Random Events and Probability

What is the probability of having two numbers sum to 6?



$$P(E) = \frac{\#\text{outcomes in } E}{\#\text{outcomes in } S}$$

S = $\{(a, b): a, b \in \{1, \dots, 6\}\}$

Each outcome is equally likely

\# of outcomes that sum to 6:
5

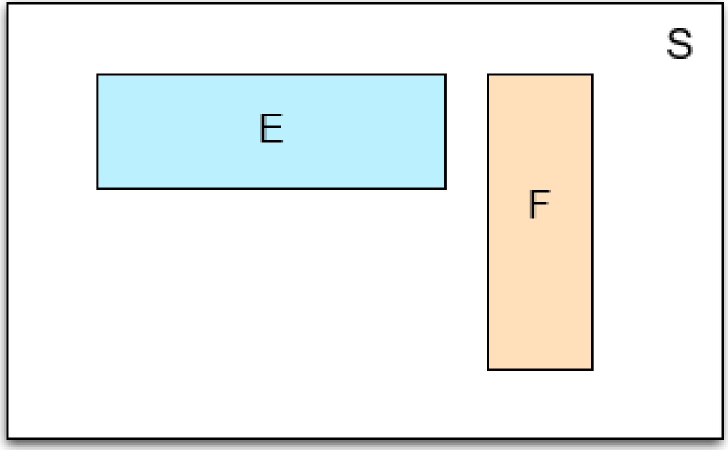
answer:
5/36 = 0.13888…

Disjoint events

- In general, breaking an event into *disjoint events* preserves the total probability
- **Disjoint**: E and F are *disjoint* if they cannot happen simultaneously,
- e.g. E = {2, 4}, F={1, 3, 5}

- In such cases,

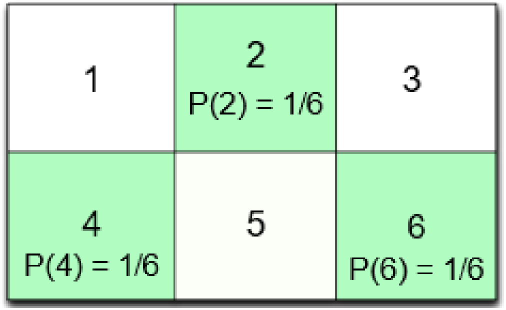
$$P(E \text{ or } F) = P(E) + P(F)$$



Elementary events

- Notice that we can find the total probability of an event by breaking it into pieces and adding up the probabilities of the pieces:

  $$P(\text{Even}) = P(2) + P(4) + P(6) = 3/6$$



- These pieces are called ‘elementary events’: Events that correspond to exactly one outcome
- These elementary events are disjoint events

Partition

- We say that disjoint events $E_1, \ldots, E_n$ form a *partition* of E if any outcome in E lies in exactly one E_i
- e.g.
  - {Fr.} E_1, {Soph.} E_2 form a partition of {Lower division} E
  - {Fr.}, {Soph.} {J.} {Sen.} form a partition of S (sample space)

Probability as Area

For disjoint events E, F:

$$P(E \text{ or } F) = P(E) + P(F)$$

If disjoint events $E_1, \ldots, E_n$ forms a partition of E :

$$P(E) = P(E_1) + P(E_2) + \cdots + P(E_n)$$

If disjoint events $E_1, \ldots, E_n$ forms a partition of S , for event F(law of total probability):

$$P(F) = P(E_1, F) + P(E_2, F) + \cdots + P(E_n, F)$$

Notation: $P(A, B)$ is a shorthand for $P(A \text{ and } B)$

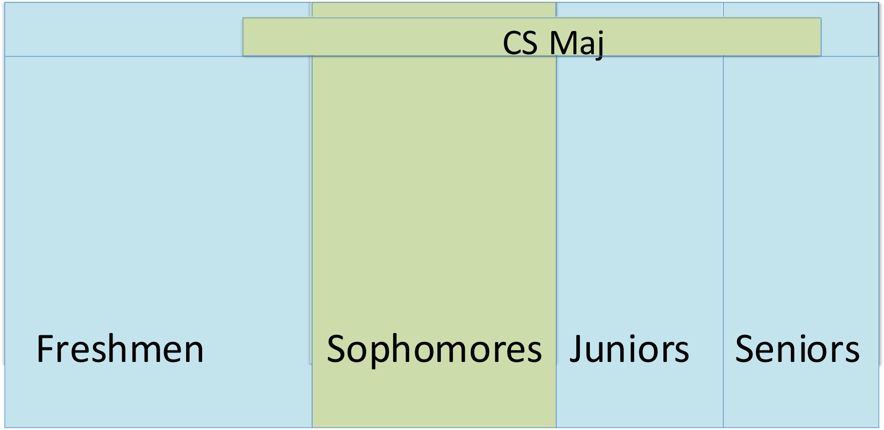
Probability as Area

Examples

- $P(\text{CS}) = P(\text{Fr.}, \text{CS}) + P(\text{Soph.}, \text{CS}) + P(\text{J.}, \text{CS}) + P(\text{Sen.}, \text{CS})$

Notation: $P(A, B)$ is a shorthand for $P(A \text{ and } B)$

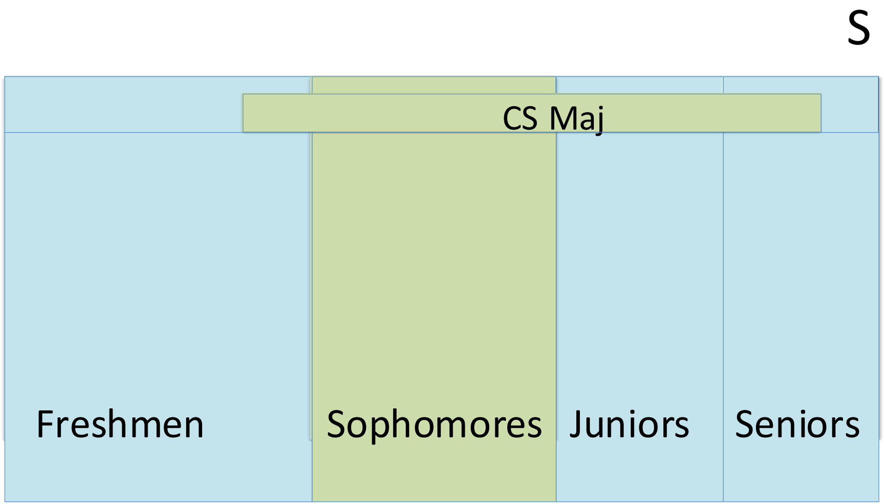
- $P(\text{Soph.}) = P(\text{CS}, \ \text{Soph.}) + P(\text{nonCS}, \ \text{Soph.})_{\text{S}}$



Probability as Area

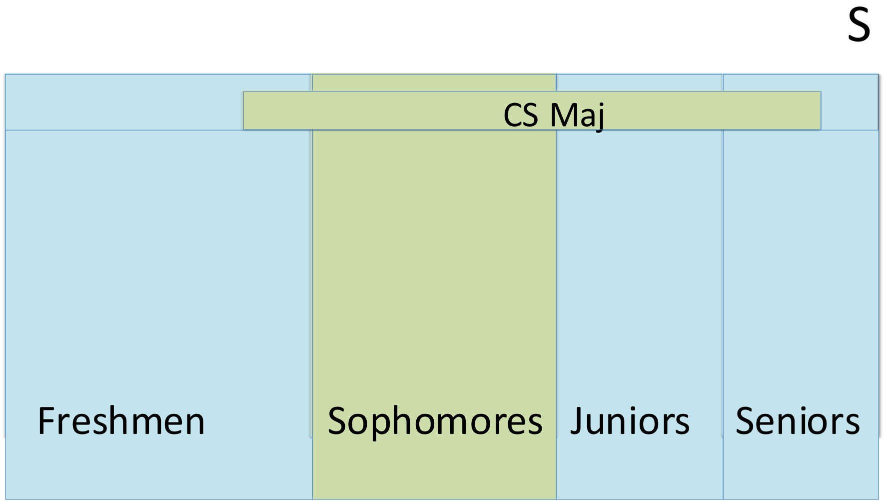
If events E, F are non-disjoint, what is $P(E \text{ or } F)$?

- $P(\text{Soph.} \, or \text{ CS})$?
- Note: events "Sophomore" and "CS Major" may overlap



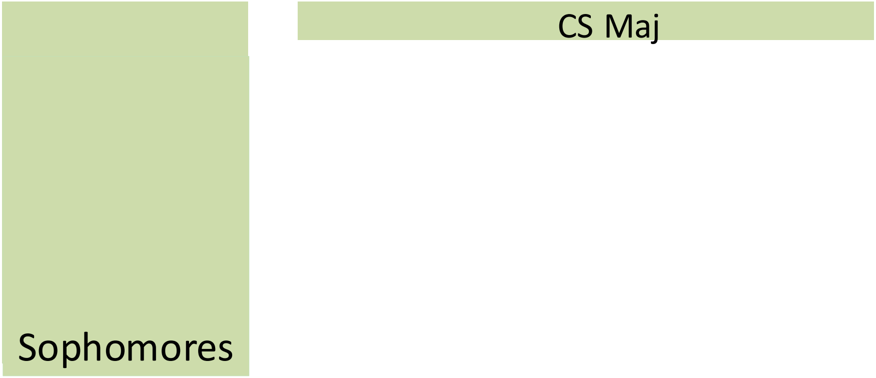
Probability as Area

- E = { Soph OR CS }
- Is P(E) = P(Soph) + P(CS)?
  - No
- Which one is larger?
  - Let's see..

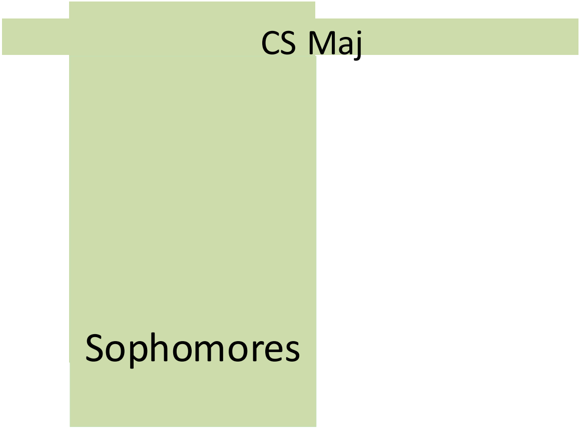


Probability as Area

P(Soph) + P(CS) =



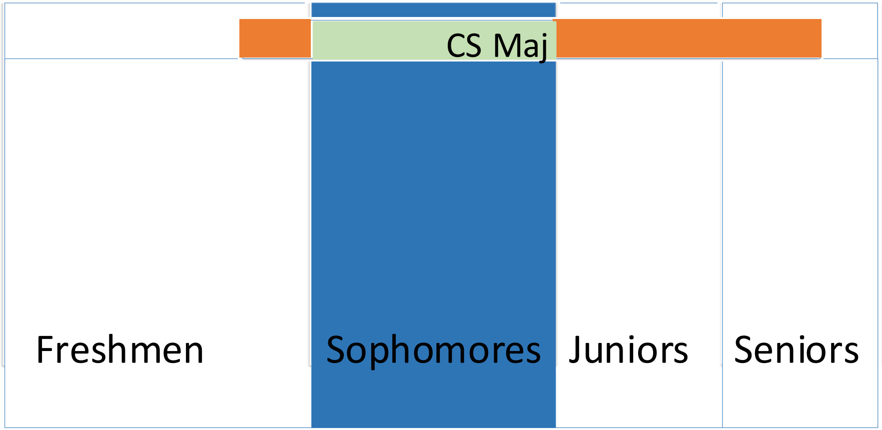
P(Soph or CS) =



Inclusion-Exclusion Principle

- P(Soph) = P(CS , Soph.) + P(Non-CS , Soph.)
- P(CS) = P(Fr. CS) + P(Soph. CS) + P(J. CS) + P(Sen. CS)

- P (Soph or CS) = P(Soph) + P(CS) – P (Soph, CS) Soph. CS is counted twice
S



Inclusion-Exclusion Principle

Inclusion-Exclusion Principle For any events E and F,

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$$

Accounting for overlap
between E and F