Computer Science

# CSC380: Principles of Data Science

## Predictive Modeling and Classification 3

Xinchen Yu

# Midterm score curving

$$new = score + \sqrt{1 - score}$$

| origin_scores | new_scores |
|---|---|
| 38 | 45.87400787401181 |
| 39 | 46.810249675906654 |
| 40 | 47.74596669241483 |
| 41 | 48.681145747868605 |
| 42 | 49.61577310586391 |
| 43 | 50.54983443527075 |
| 44 | 51.48331477354788 |
| 45 | 52.41619848709566 |
| 46 | 53.348469228349536 |
| 47 | 54.28010988928052 |
| 48 | 55.211102550927976 |
| 49 | 56.14142842854285 |
| 50 | 57.071067811865476 |
| 51 | 58.0 |
| 52 | 58.92820323027551 |
| 53 | 59.855654600401046 |
| 54 | 60.782329983125265 |
| 55 | 61.70820393249937 |
| 56 | 62.633249580710796 |

| origin_scores | new_scores |
|---|---|
| 63 | 69.08276253029823 |
| 64 | 70.0 |
| 65 | 70.91607978309962 |
| 66 | 71.8309518948453 |
| 67 | 72.74456264653803 |
| 68 | 73.65685424949238 |
| 69 | 74.56776436283002 |
| 70 | 75.47722557505166 |
| 71 | 76.3851648071345 |
| 72 | 77.29150262212919 |
| 73 | 78.19615242270663 |
| 74 | 79.09901951359278 |
| 75 | 80.0 |
| 76 | 80.89897948556636 |
| 77 | 81.79583152331271 |
| 78 | 82.69041575982342 |
| 79 | 83.58257569495584 |
| 80 | 84.47213595499957 |
| 81 | 85.35889894354068 |

| origin_scores | new_scores |
|---|---|
| 82 | 86.24264068711929 |
| 83 | 87.12310562561765 |
| 84 | 88.0 |
| 85 | 88.87298334620742 |
| 86 | 89.74165738677394 |
| 87 | 90.605551275464 |
| 88 | 91.46410161513775 |
| 89 | 92.3166247903554 |
| 90 | 93.16227766016839 |
| 91 | 94.0 |
| 92 | 94.82842712474618 |
| 93 | 95.64575131106459 |
| 94 | 96.44948974278317 |
| 95 | 97.23606797749979 |
| 96 | 98.0 |
| 97 | 98.73205080756888 |
| 98 | 99.41421356237309 |
| 99 | 100.0 |
| 100 | 100.0 |

# Estimating current scores

- Assignments: 36%
- Midterm: 20%
- Project: 14%
- Final Exam: 20%
- Participation: 10%

- 7 homeworks, drop the lowest one.
  - Each 6 points
  - e.g., 6 * 120/130 = 5.54 -- if a homework total points are 130 and you got 120

# Model Evaluation

# Confusion Matrix



Suppose our classifier distinguishes between cats and non-cats.

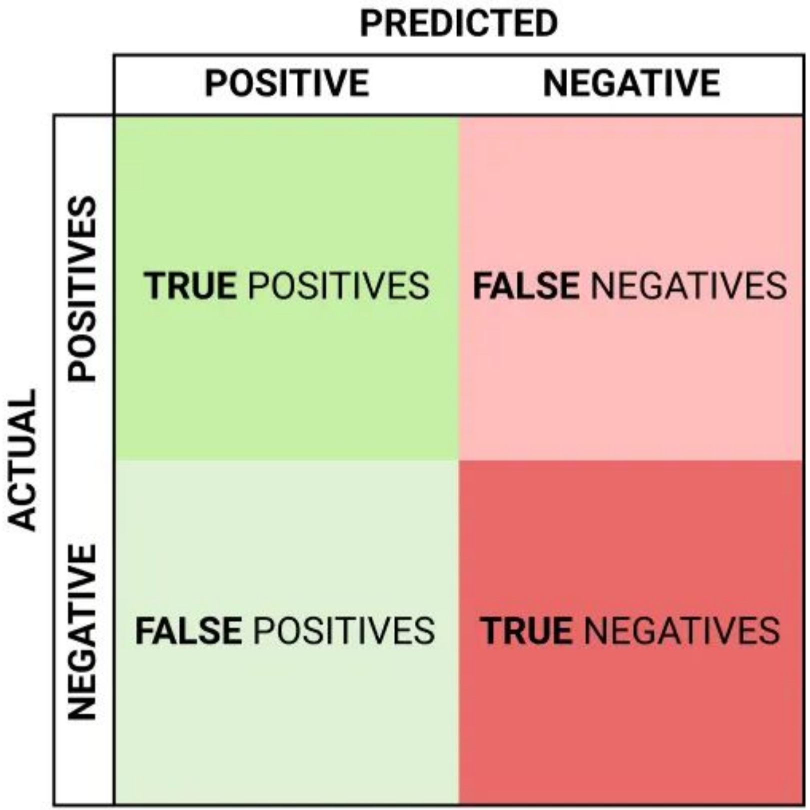
We can make the following table called **confusion matrix**:

| Predicted class / Actual class | Cat | Non-cat |
|---|---|---|
| **Cat** | **6 true positives** | 2 false negatives |
| **Non-cat** | 1 false positive | **3 true negatives** |

It tells us if classifier is biased towards certain mistakes (False Positives, False Neg.)

Good for investigating opportunities to improve the classifier.

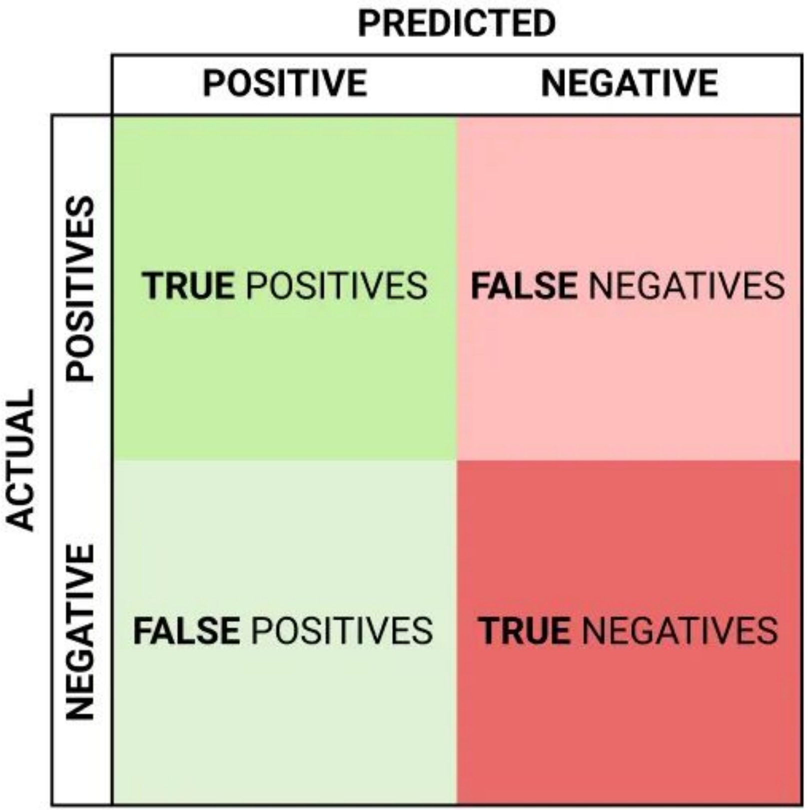
# Evaluating Classifiers - Precision




**Precision**: dividing the true positives by anything that was predicted as a positive.

$$\frac{\text{TRUE POSITIVES}}{\text{TRUE POSITIVES} + \text{FALSE POSITIVES}}$$

# Evaluating Classifiers - Recall




**Recall** (or True Positive Rate): dividing the true positives by anything that should have been predicted as positive.

$$\frac{\text{TRUE POSITIVES}}{\text{TRUE POSITIVES} + \text{FALSE NEGATIVES}}$$

# Evaluating Classifiers

F1 score symmetrically represents both precision and recall in one metric.

$$F_1 = \frac{2}{\mathrm{recall}^{-1} + \mathrm{precision}^{-1}} = 2 \cdot \frac{\mathrm{precision} \cdot \mathrm{recall}}{\mathrm{precision} + \mathrm{recall}} = \frac{\mathrm{tp}}{\mathrm{tp} + \frac{1}{2}(\mathrm{fp} + \mathrm{fn})}$$

- This is the *harmonic mean* of precision and recall
  - harmonic_mean(x,y)

$$\frac{1}{\frac{1}{2}(\frac{1}{x} + \frac{1}{y})}$$

- Gives equal importance to precision and recall – F1 may not be best when you care about one more than the other (e.g., in medical tests we care about recall)

# Evaluation in Scikit-Learn



## Evaluation functions live in `metrics`

| | |
|---|---|
| `metrics.confusion_matrix`(y_true, y_pred, *) | Compute confusion matrix to evaluate the accuracy of a classification. |
| `metrics.dcg_score`(y_true, y_score, *[, k, ...]) | Compute Discounted Cumulative Gain. |
| `metrics.det_curve`(y_true, y_score[, ...]) | Compute error rates for different probability thresholds. |
| `metrics.f1_score`(y_true, y_pred, *[, ...]) | Compute the F1 score, also known as balanced F-score or F-measure. |
| `metrics.fbeta_score`(y_true, y_pred, *, beta) | Compute the F-beta score. |
| `metrics.hamming_loss`(y_true, y_pred, *[, ...]) | Compute the average Hamming loss. |
| `metrics.hinge_loss`(y_true, pred_decision, *) | Average hinge loss (non-regularized). |
| `metrics.jaccard_score`(y_true, y_pred, *[, ...]) | Jaccard similarity coefficient score. |
| `metrics.log_loss`(y_true, y_pred, *[, eps, ...]) | Log loss, aka logistic loss or cross-entropy loss. |
| `metrics.matthews_corrcoef`(y_true, y_pred, *) | Compute the Matthews correlation coefficient (MCC). |
| `metrics.multilabel_confusion_matrix`(y_true, ...) | Compute a confusion matrix for each class or sample. |
| `metrics.ndcg_score`(y_true, y_score, *[, k, ...]) | Compute Normalized Discounted Cumulative Gain. |
| `metrics.precision_recall_curve`(y_true, ...) | Compute precision-recall pairs for different probability thresholds. |
| `metrics.precision_recall_fscore_support`(...) | Compute precision, recall, F-measure and support for each class. |
| `metrics.precision_score`(y_true, y_pred, *[, ...]) | Compute the precision. |
| `metrics.recall_score`(y_true, y_pred, *[, ...]) | Compute the recall. |

# Naïve Bayes in Scikit-learn



*Scikit-learn has separate classes each feature type*

`sklearn.naive_bayes.GaussianNB`

Real-valued features

`sklearn.naive_bayes.MultinomialNB`

Discrete K-valued feature counts (e.g. multiple die rolls)

`sklearn.naive_bayes.BernoulliNB`

Binary features (e.g. coinflip)

`sklearn.naive_bayes.CategoricalNB`

Discrete K-valued features (e.g. single die roll)

For large training data that don't fit in memory use Scikit-learn's out-of-core learning

https://scikit-learn.org/stable/modules/naive_bayes.html

# Naïve Bayes

# Naïve Bayes Overview



**<u>Heads Up</u>** This section will return to some math as we go in depth. But, much of it is that you already know with a new application (Naïve Bayes Classification) – **ask questions if you are lost**

- Introduction to Naïve Bayes Classifier
- Maximum Likelihood Estimation

# Math Prep

- N RVs conditionally independent, given Z, if and only if:

$$p(X_1, \ldots, X_N \mid Z) = \prod_{i=1}^{N} p(X_i \mid Z)$$

Probability 3, page 25

- Bayes' rule

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

Probability 2, page 24

- Maximum likelihood estimation

Statistics 2

- Bernoulli and Gaussian distribution

Probability 3, 4
Statistics 2

# Intuition

**Training Data:**

| Person | height (feet) |
|---|---|
| male | 6 |
| male | 5.92 (5'11") |
| male | 5.58 (5'7") |
| male | 5.92 (5'11") |
| female | 5 |
| female | 5.5 (5'6") |
| female | 5.42 (5'5") |
| female | 5.75 (5'9") |

Y X

Task: observe feature $x_n$ , predict label $y_n \in (0,1)$

Choose the class with higher probability:

$$P(Y_n = 1 | X_n = x_n) \quad \text{vs} \quad P(Y_n = 0 | X_n = x_n)$$

$$P(Y_n = 1 | X_n = x_n) = \frac{P(Y_n = 1, X_n = x_n)}{P(X_n = x_n)}$$

$$P(Y_n = 0 | X_n = x_n) = \frac{P(Y_n = 0, X_n = x_n)}{P(X_n = x_n)}$$

How to learn a joint probability model for $P(Y, X)$ ?

# Probabilistic Approach to ML

**Training Data:**

| Person | height (feet) | weight (lbs) | foot size(inches) |
|---|---|---|---|
| male | 6 | 180 | 12 |
| male | 5.92 (5'11") | 190 | 11 |
| male | 5.58 (5'7") | 170 | 12 |
| male | 5.92 (5'11") | 165 | 10 |
| female | 5 | 100 | 6 |
| female | 5.5 (5'6") | 150 | 8 |
| female | 5.42 (5'5") | 130 | 7 |
| female | 5.75 (5'9") | 150 | 9 |

**Features**

**Task:** Observe features $x_1, \ldots, x_D$ and predict class label $y \in \{1, \ldots, C\}$

**Model:** Assume that the feature $x$ and its label $y$ follows certain type of distribution $\mathcal{D}$ with parameter $\theta$.

$$(x, y) \overset{\text{i.i.d.}}{\sim} \mathcal{D}_\theta$$

**Training Algorithm**: Estimate $\theta$ e.g., MLE $\hat{\theta}$

**To classify**: Compute

$$\hat{y} = \arg \max_{c \in \{1,\ldots,C\}} p(y = c \mid x; \hat{\theta})$$

what comes after semicolon is the parameter of the distribution

# Naïve Bayes is a Specific Probabilistic Approach



**Training Data:**

| Person | height (feet) | weight (lbs) | foot size(inches) |
|---|---|---|---|
| male | 6 | 180 | 12 |
| male | 5.92 (5'11") | 190 | 11 |
| male | 5.58 (5'7") | 170 | 12 |
| male | 5.92 (5'11") | 165 | 10 |
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| female | 5.75 (5'9") | 150 | 9 |

**Features**

**Task:** Observe features $x_1, \ldots, x_D$ and predict class label $y \in \{1, \ldots, C\}$

**Naïve Bayes Model:** Treat features as *conditionally independent* given class label,

$$p(x, y) = p(y)p(x|y) = p(y)\prod_{d=1}^{D} p(x_d \mid y)$$

build individual models for these

**To classify a given instance $x$:** Bayes rule!

$$p(y = c \mid x) = \frac{p(y = c)p(x \mid y = c)}{p(x)}$$

# Key concept in Naïve Bayes

$$p(x,y) = p(y)p(x|y) = p(y)\prod_{d=1}^{D} p(x_d \mid y)$$

**Class prior distribution** **Class conditional distribution**

Given one data point, it has 4 features (input), and the label is 0 (output)

$$p(x_1, x_2, x_3, x_4, y = 0) = p(y = 0) \cdot p(x_1, x_2, x_3, x_4 | y = 0)$$

$$= p(y = 0) \cdot p(x_1 | y = 0) \cdot p(x_2 | y = 0) \cdot p(x_3 | y = 0) \cdot p(x_4 | y = 0)$$

# Class prior distribution



$$p(x,y) = p(y)p(x|y) = p(y)\prod_{d=1}^{D} p(x_d \mid y)$$

**Class prior distribution** **Class conditional distribution**

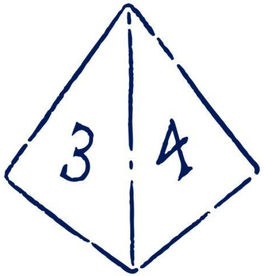
For the class prior distribution, take categorical distribution.

$$y \sim \text{Categorical}(\pi), \qquad \pi \in \mathbb{R}^C, \pi_c \geq 0, \sum_c \pi_c = 1$$

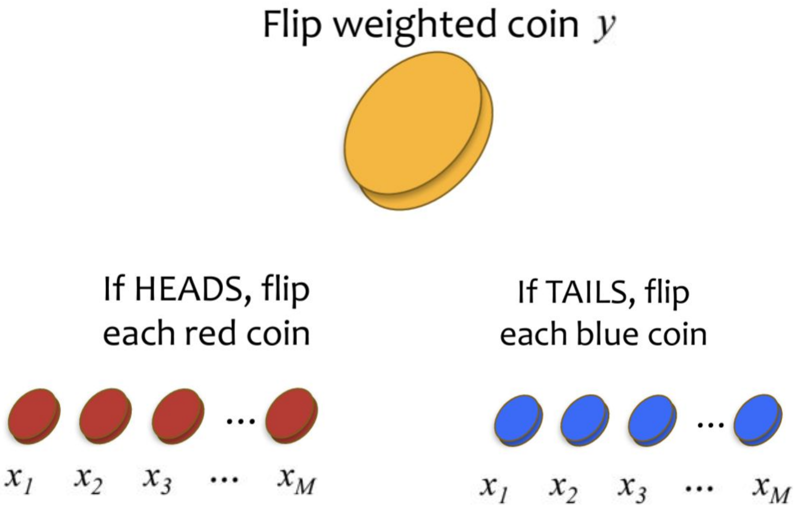
=> $p(y = c) = \pi_c$

Example: biased 4-sided die Y, given:

- P(Y=1) = 0.2, P(Y=2) = 0.3, P(Y=3) = 0.1, P(Y=4) = 0.4



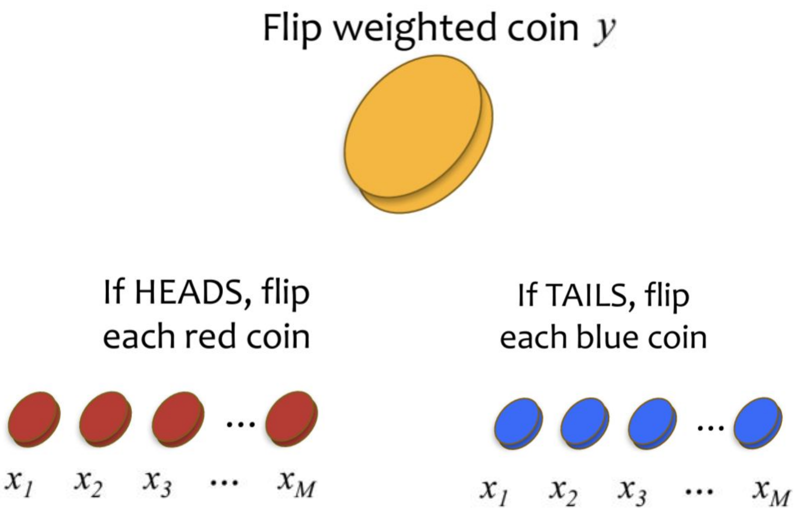
# Class prior distribution




| $y$ | $x_1$ | $x_2$ | $x_3$ | … | $x_M$ |
|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | … | 1 |
| 1 | 0 | 1 | 0 | … | 1 |
| 1 | 1 | 1 | 1 | … | 1 |
| 0 | 0 | 0 | 1 | … | 1 |
| 0 | 1 | 0 | 1 | … | 0 |
| 1 | 1 | 0 | 1 | … | 0 |

$p(y)$

# Class conditional distribution

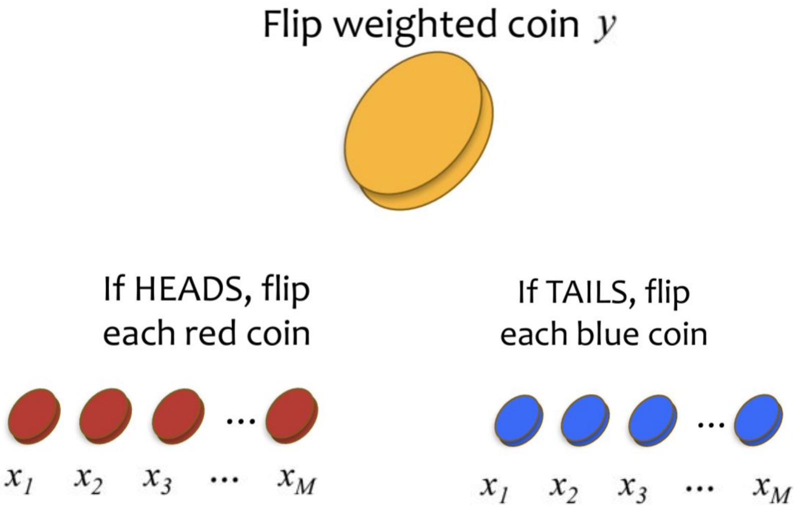



Each red / blue coin biases can be different.

| $y$ | $x_1$ | $x_2$ | $x_3$ | … | $x_M$ |
|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | … | 1 |
| 1 | 0 | 1 | 0 | … | 1 |
| 1 | 1 | 1 | 1 | … | 1 |
| 0 | 0 | 0 | 1 | … | 1 |
| 0 | 1 | 0 | 1 | … | 0 |
| 1 | 1 | 0 | 1 | … | 0 |

$p(x_1|y = 1)$

# Class conditional distribution

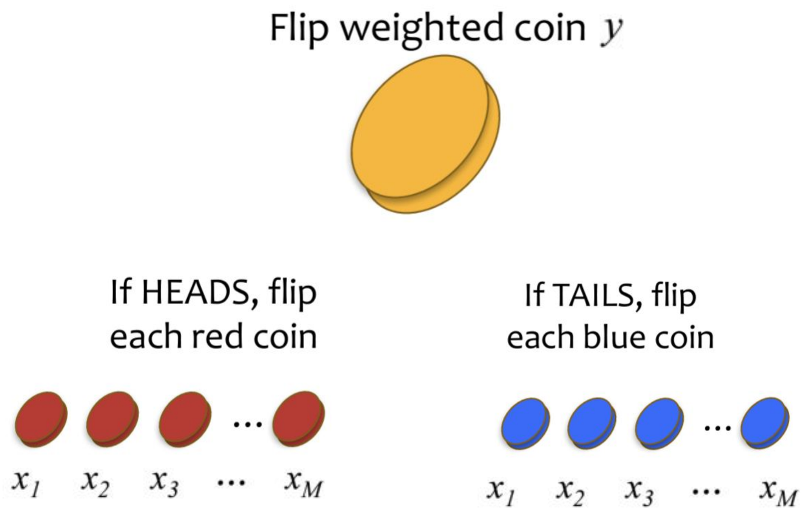



Each red / blue coin biases can be different.

| $y$ | $x_1$ | $x_2$ | $x_3$ | ... | $x_M$ |
|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | ... | 1 |
| 1 | 0 | 1 | 0 | ... | 1 |
| 1 | 1 | 1 | 1 | ... | 1 |
| 0 | 0 | 0 | 1 | ... | 1 |
| 0 | 1 | 0 | 1 | ... | 0 |
| 1 | 1 | 0 | 1 | ... | 0 |

$p(x_2|y = 1)$

# Class conditional distribution




Each red / blue coin biases can be different.

| $y$ | $x_1$ | $x_2$ | $x_3$ | … | $x_M$ |
|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | … | 1 |
| 1 | 0 | 1 | 0 | … | 1 |
| 1 | 1 | 1 | 1 | … | 1 |
| 0 | 0 | 0 | 1 | … | 1 |
| 0 | 1 | 0 | 1 | … | 0 |
| 1 | 1 | 0 | 1 | … | 0 |

$p(x_1 | y = 0)$

# Class conditional distribution



**Simplifying Assumption:** "*Class conditional*" distribution assumes features are conditionally independent given class

$$p(x \mid y) = \prod_{d=1}^{D} p(x_d \mid y)$$

- "Naïve" as in general features are likely to be dependent.
- Every feature can have a different class-conditional distribution

$P(x_1|y)$ can be different from $P(x_2|y)$

# Class conditional distribution

**Features are typically not independent!**

**Example 1** If a recent news article contains word “Donald” it is much more likely to contain the word “Trump”.


**Example 2** If flower <u>petal *width*</u> is very large then <u>petal *length*</u> is also likely to be high.


Source: <u>Matt Gormley</u>

# Class conditional distribution



**Simplifying Assumption:** "*Class conditional*" distribution assumes features are conditionally independent given class

$$p(x \mid y) = \prod_{d=1}^{D} p(x_d \mid y)$$

- "Naïve" as in general features are likely to be dependent.
- Every feature can have a different class-conditional distribution

Doesn't capture correlation among features. But why would it be a good idea?

- Easy computation: For C classes and D features only $O(CD)$ parameters
- Prevents overfitting
- Simplicity

# Class conditional distribution



For real-valued features we can use Normal distribution:

$$p(x \mid y = c) = \prod_{d=1}^{D} \mathcal{N}(x_d \mid \underbrace{\mu_{cd}, \sigma_{cd}^2}_{\text{Parameters of featured for class } c})$$

quiz candidate
Q: how many parameters?
2cd

For binary features $x_d \in \{0,1\}$ can use Bernoulli distributions:

$$p(x \mid y = c) = \prod_{d=1}^{D} \text{Bernoulli}(x_d \mid \theta_{cd})$$

“Coin bias” for d[th] feature and class $c$

quiz candidate
Q: how many parameters?
cd

- K-valued discrete features: use Categorical.
- Can mix-and-match, e.g. some discrete, some continuous features

$$p(x \mid y = c) = \prod_{d=1}^{D'} \text{Bernoulli}(x_d \mid \theta_{cd}) \prod_{d=D'+1}^{D} \mathcal{N}(x_d \mid \mu_{cd}, \sigma_{cd}^2)$$

# Naïve Bayes Model : Maximum Likelihood



Fitting the model requires learning all parameters…

$$p(x, y = c) = p(y = c; \pi) \prod_{d=1}^{D} p(x_d \mid \theta_{cd})$$

**Class Prior Parameters** **Likelihood Parameters**

Given training data $\mathcal{D} = \{(x^{(i)}, y^{(i)})\}_{i=1}^{N}$ maximize the likelihood function,

$$\theta^{\text{MLE}} = \arg\max_{\pi, \theta} \log p(\mathcal{D}; \pi, \theta)$$

# Naïve Bayes Model : Maximum Likelihood



$$\theta^{\text{MLE}} = \arg\max_{\pi,\theta} \log p(\mathcal{D}; \pi, \theta) \qquad \left( \mathcal{D} \coloneqq \{(x^{(i)}, y^{(i)})\}_{i=1}^{m} \right)$$

**Since data are iid**

$$= \arg\max_{\pi,\theta} \log \prod_{i=1}^{m} p(x^{(i)}, y^{(i)}; \pi, \theta)$$

**log(ab) = log a + log b**

$$= \arg\max_{\pi,\theta} \sum_{i=1}^{m} \log p(x^{(i)}, y^{(i)}; \pi, \theta)$$

**Conditional probability +**
**Naïve Bayes assumption**

$$= \arg\max_{\pi,\theta} \sum_{i=1}^{m} \log p(y^{(i)}; \pi) + \sum_{i=1}^{m} \sum_{d=1}^{D} \log p\left(x_d^{(i)} \middle| y^{(i)}; \theta_{y^{(i)}d}\right)$$

$\theta_{cd}$: parameter for feature d for class c

*Find zero-gradient if concave, or gradient-based optimization otherwise*

# Example: Naïve Bayes with Bernoulli Features

**Analogy:**

| $y$ | $x_1$ | $x_2$ | … | | | $x_D$ |
|---|---|---|---|---|---|---|
| 5 | 0 | 1 | 1 | 0 | 1 | 0 |
| 2 | 1 | 0 | 0 | 0 | 0 | 0 |
| 3 | 1 | 1 | 1 | 1 | 0 | 0 |
| … | | | … | | | |
| 4 | 0 | 0 | 1 | 1 | 0 | 1 |

# Example: Naïve Bayes with Bernoulli Features 

Let each feature follow a Bernoulli distribution then the model is…

$$y \sim \text{Categorical}(\pi) \qquad x_d \mid y = c \sim \text{Bernoulli}(\theta_{cd})$$

The Naïve Bayes joint distribution is then:

$$p(\mathcal{D}; \pi, \theta) = \prod_{i=1}^{m} \left( p(y^{(i)}; \pi) \prod_{d} p\left(x_d^{(i)}; \theta_{y^{(i)}d}\right) \right)$$

$$= \prod_{i=1}^{m} \left( \prod_{c} \left( \pi_c^{\mathbf{I}\{y_i=c\}} \prod_{j} p(x_{ij} | \theta_{jc})^{\mathbf{I}\{y_i=c\}} \right) \right)$$

*Write down log-likelihood and optimize…*

# Bernoulli Naïve Bayes MLE



Let $m_c := \sum_{i=1}^{m} \mathbf{I}\{y^{(i)} = c\}$ be number of training examples in class c then,

$$\sum_{i=1}^{m} \log p(\mathcal{D}; \pi, \theta) = \sum_{c=1}^{C} m_c \log \pi_c + \sum_{c=1}^{C} \sum_{i:y^{(i)}=c} \sum_{d=1}^{D} \log p\left(x_d^{(i)}; \theta_{cd}\right)$$

Log-likelihood function is concave in all parameters so…

1. Take derivatives with respect to $\pi$ and $\theta$ separately.
2. Set derivatives to zero and solve

$$\hat{\pi}_c = \frac{m_c}{m}$$

**Fraction of training examples from class c**

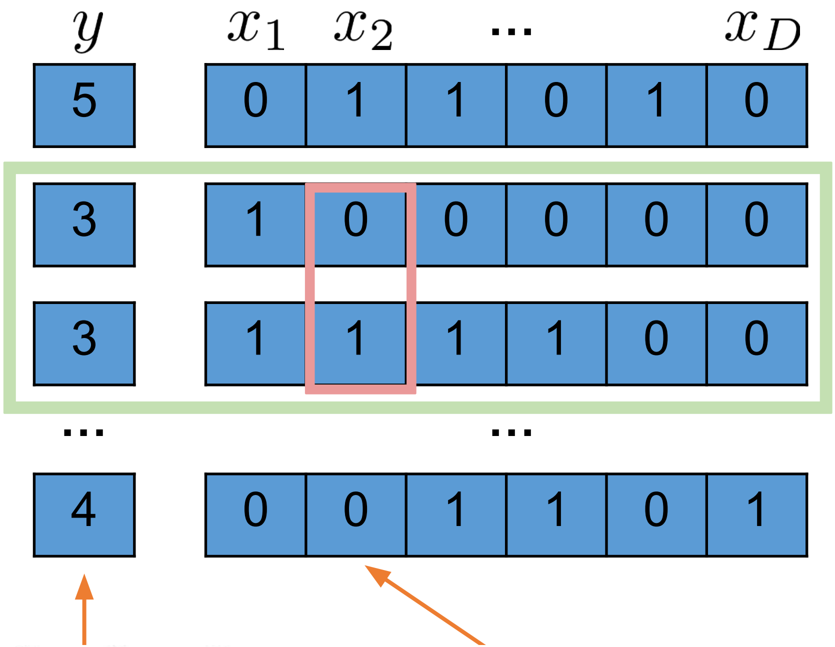
$$\hat{\theta}_{cd} = \frac{m_{cd}}{m_c}$$

**Number of "heads" in training set from class c**

$$m_{cd} = \sum_{i=1}^{m} I\{y^{(i)} = c, x_d^{(i)} = 1\}$$

# Example: Naïve Bayes with Bernoulli Features

**Analogy:**




$$\hat{\pi}_c = \frac{m_c}{m}$$

$$\hat{\theta}_{cd} = \frac{m_{cd}}{m_c}$$

$$m_{cd} = \sum_{i=1}^{m} I\{y^{(i)} = c, x_d^{(i)} = 1\}$$

$$\hat{\pi}_3 = \frac{\#\, number\ of\ y = 3}{\#\, total\ data}$$

$$\hat{\theta}_{3,2} = \frac{\#\, number\ of\ y = 3\ and\ x_2 = 1}{\#\, number\ of\ y = 3}$$

# Bernoulli Naïve Bayes MLE



$\hat{\pi}_c = \frac{m_c}{m}$ **Fraction of training examples from class c**

$\hat{\theta}_{cd} = \frac{m_{cd}}{m_c}$ **Number of "heads" in training set from class c**

What if there are *no* examples of class c in the training set?

$\hat{\pi}_c = 0$ **Model will never learn to guess class c**

What if all data points $i$ in class c has $x_d^{(i)} = 0$ in the training set?

$\hat{\theta}_{cd} = 0$

**Model will assign 0 likelihood for test data with $x_d = 1$ for class c (i.e., $p(x|y = c)$ ).**

What does it imply on $p(y = c|x)$ ? 0!

Training data needs to see *<u>every possible outcome</u> for each feature*

**Any ideas how we can fix this problem?**

# Fixing Bernoulli MLE



We could add a small constant to prevent zero probabilities…

$\hat{\pi}_c \propto m_c + \alpha$ $\qquad$ $\hat{\theta}_{cj} \propto m_{cj} + \beta$ $\qquad$ $\alpha, \beta > 0$

**Pseudocounts**
**add-$\alpha$ Smoothing**
**Laplace smoothing**
**….**

**typical choice: set $\boldsymbol{\alpha = \beta = 1}$**

Another smoothing method:

$$\hat{P}(w_i|c) = \frac{count(w_i, c) + 1}{\sum_{w \in V} (count(w, c) + 1)} = \frac{count(w_i, c) + 1}{\left(\sum_{w \in V} count(w, c)\right) + |V|}$$

**Word count in category c**

**Vocabulary size in whole corpus**

# Naïve Bayes in Sentiment Classification

| | Cat | Documents |
|---|---|---|
| Training | - | just plain boring |
| | - | entirely predictable and lacks energy |
| | - | no surprises and very few laughs |
| | + | very powerful |
| | + | the most fun film of the summer |
| Test | ? | predictable with no fun |

$$\boxed{\hat{\pi}_c = \frac{m_c}{m}} \qquad P(-) = \frac{3}{5} \qquad P(+) = \frac{2}{5}$$

# Naïve Bayes in Sentiment Classification

$$\hat{P}(w_i|c) = \frac{count(w_i,c)}{\sum_{w\in V} count(w,c)}$$

smoothing

$$\frac{count(w_i,c)+1}{\left(\sum_{w\in V} count(w,c)\right)+|V|}$$

| | Cat | Documents |
|---|---|---|
| Training | - | just plain boring |
| | - | entirely predictable and lacks energy |
| | - | no surprises and very few laughs |
| | + | very powerful |
| | + | the most fun film of the summer |
| Test | ? | predictable with no fun |

20 = 14 + 9 - 3

3: the, and, very (duplicate)

$$P(\text{“predictable”}|-) = \frac{1+1}{14+20} \qquad P(\text{“predictable”}|+) = \frac{0+1}{9+20}$$

$$P(\text{“no”}|-) = \frac{1+1}{14+20} \qquad P(\text{“no”}|+) = \frac{0+1}{9+20}$$

$$P(\text{“fun”}|-) = \frac{0+1}{14+20} \qquad P(\text{“fun”}|+) = \frac{1+1}{9+20}$$

# Naïve Bayes in Sentiment Classification

| | Cat | Documents |
|---|---|---|
| Training | - | just plain boring |
| | - | entirely predictable and lacks energy |
| | - | no surprises and very few laughs |
| | + | very powerful |
| | + | the most fun film of the summer |
| Test | ? | predictable with no fun |

Ignore unknown words

$$P(-)P(S|-) = \frac{3}{5} \times \frac{2 \times 2 \times 1}{34^3} = 6.1 \times 10^{-5}$$

$$P(+)P(S|+) = \frac{2}{5} \times \frac{1 \times 1 \times 2}{29^3} = 3.2 \times 10^{-5}$$

The model thus predicts the class *negative* for the test sentence.