CSC380: Principles of Data Science

Data Analysis, Collection, and Visualization 2

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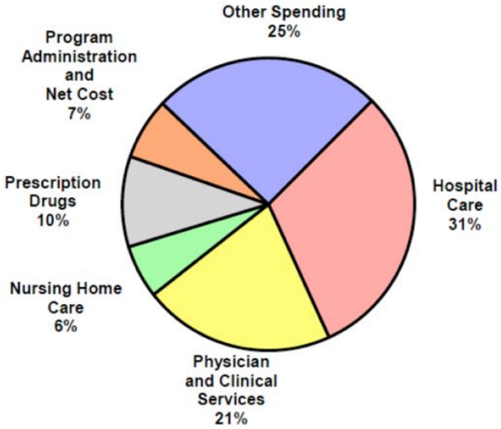
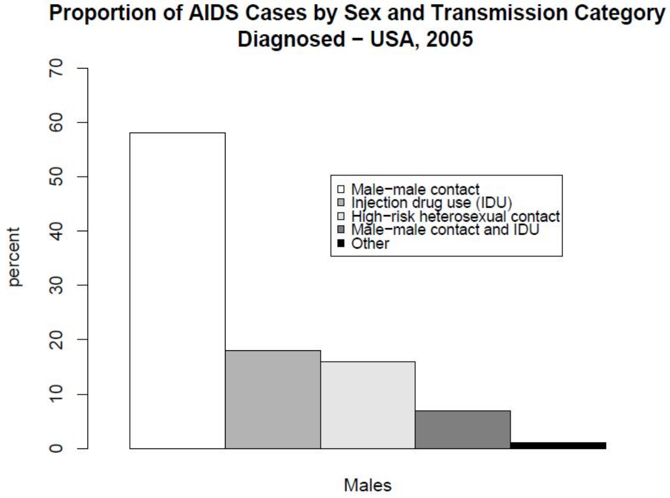
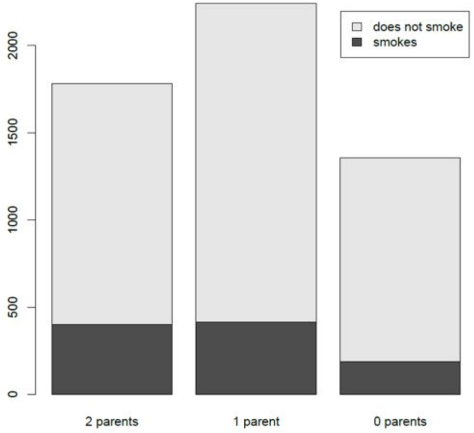
Acknowledgement: Built on Jason Pacheco, Kwang-Sung Jun, Chicheng Zhang’s slides

Visualizing Categorical Variables



	student smokes	student does not smoke	total
2 parents smoke	400	1380	1780
1 parent smokes	416	1823	2239
0 parents smoke	188	1168	1356
total	1004	4371	5375





Two-Way Table

Also called <u>contingency table</u> or <u>cross tabulation table</u>…

Frequency

	student smokes	student does not smoke	total
2 parents smoke	400	1380	1780
1 parent smokes	416	1823	2239
0 parents smoke	188	1168	1356
total	1004	4371	5375

Two-Way Table

Also called <u>contingency table</u> or <u>cross tabulation table</u>…

Relative Frequency

	student smokes	student does not smoke	total
2 parents smoke	7.4%	25.7%	33.1%
1 parent smokes	7.7%	33.9%	41.7%
0 parents smoke	3.5%	21.8%	25.2%
total	18.7%	81.3%	100%

Column Variable

Marginal Distribution Of Row Variable

Row Variable

Marginal Distribution Of Column Variable

Joint Distribution

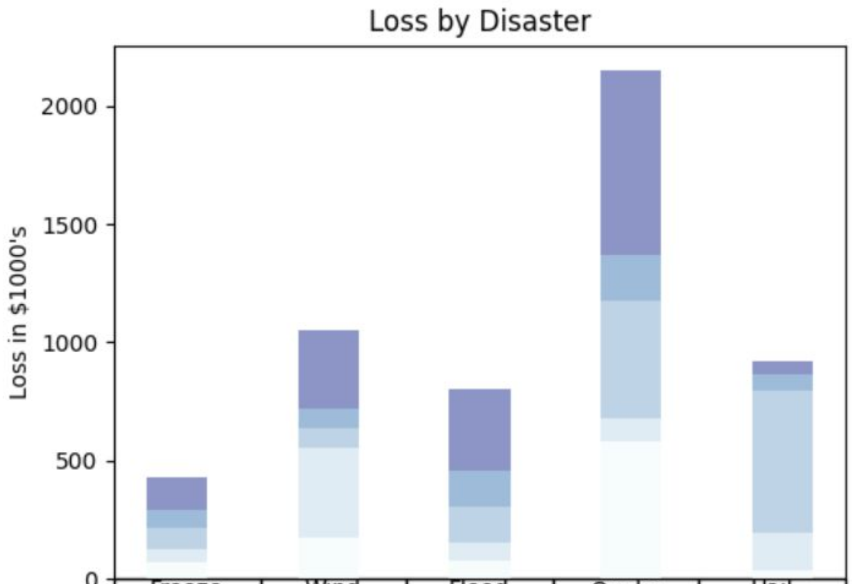
Q: how do you compute the conditional probability P(student smokes | 2 parents smoke)?

Two-Way Table


```python
data = [[ 66386, 174296,  75131, 577908,  32015],
        [ 58230, 381139,  78045,  99308, 160454],
        [ 89135,  80552, 152558, 497981, 603535],
        [ 78415,  81858, 150656, 193263,  69638],
        [139361, 331509, 343164, 781380,  52269]]

columns = ('Freeze', 'Wind', 'Flood', 'Quake', 'Hail')
rows = ['%d year' % x for x in (100, 50, 20, 10, 5)]
colors = plt.cm.BuPu(np.linspace(0, 0.5, len(rows)))
```

```python
the_table = plt.table(cellText=cell_text,
                      rowLabels=rows,
                      rowColours=colors,
                      colLabels=columns,
                      loc='bottom')
```

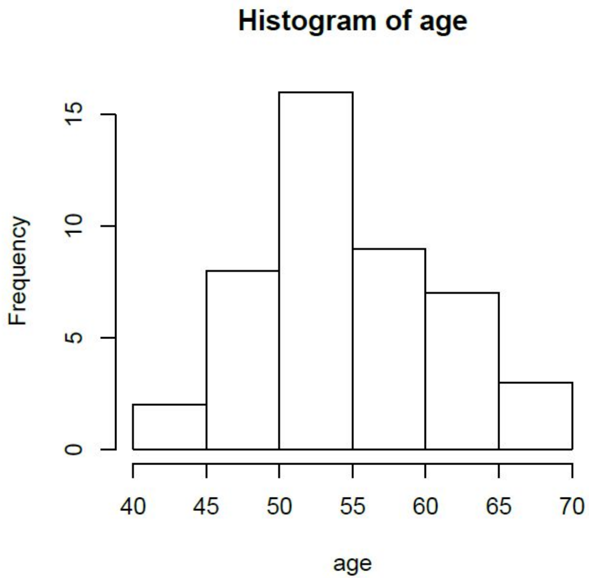
Adding stacked bars requires more steps, full code here: [*https://matplotlib.org/stable/gallery/misc/table_demo.html*](https://matplotlib.org/stable/gallery/misc/table_demo.html)

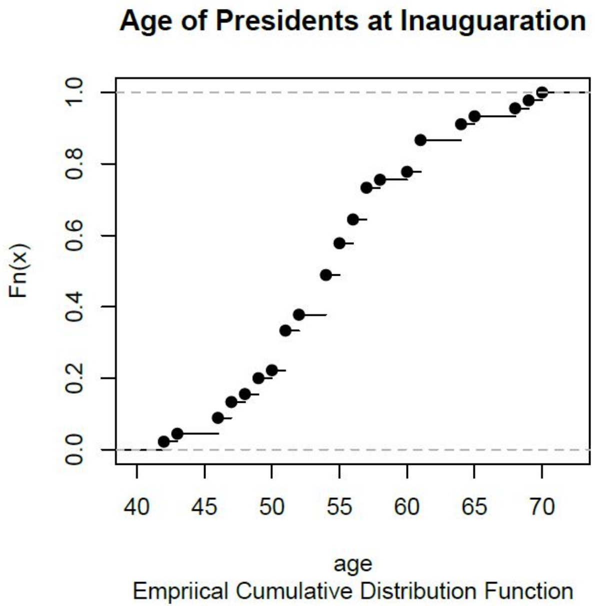


	Freeze	Wind	Flood	Quake	Hail
100 year	431.5	1049.4	799.6	2149.8	917.9
50 year	292.2	717.8	456.4	1368.5	865.6
20 year	213.8	636.0	305.7	1175.2	796.0
10 year	124.6	555.4	153.2	677.2	192.5
5 year	66.4	174.3	75.1	577.9	32.0

Histogram

Empirical approximation of (quantitative) data generating distribution



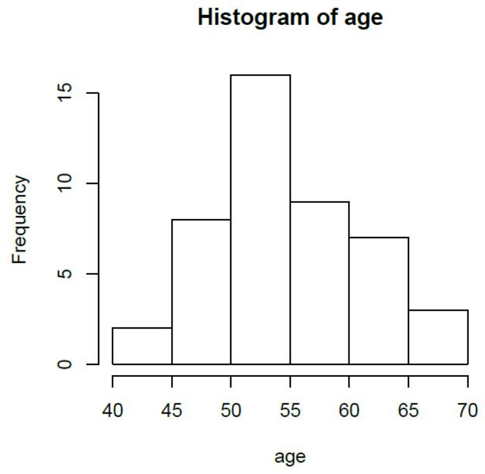


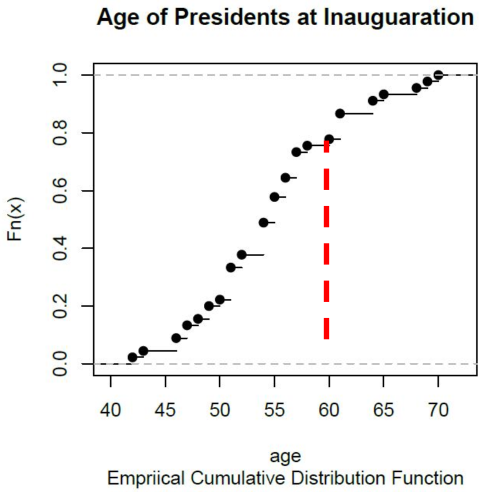
Empirical CDF for each x gives P(X<x),

$$F_n(x) = \frac{1}{n}\#(\text{observations less than or equal to x})$$

Quantile / Percentile

Question Is 60yrs old for a US president? Why or why not?





Empirical CDF for each x gives P(X<x),

$$F_n(x) = \frac{1}{n}\#(\text{observations less than or equal to x})$$

Compute probability of being <60,

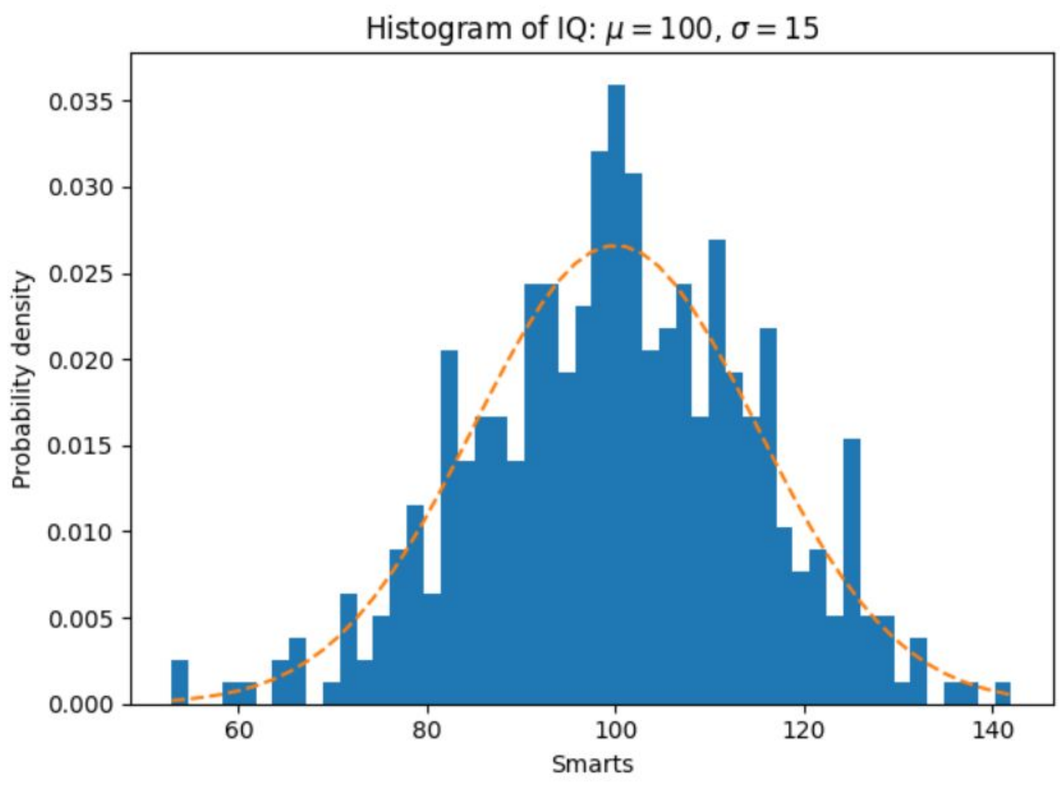
$$F_n(60) \approx 0.8$$

0.8 Quantile or 80th Percentile → About 80% of presidents younger than 60

Histogram

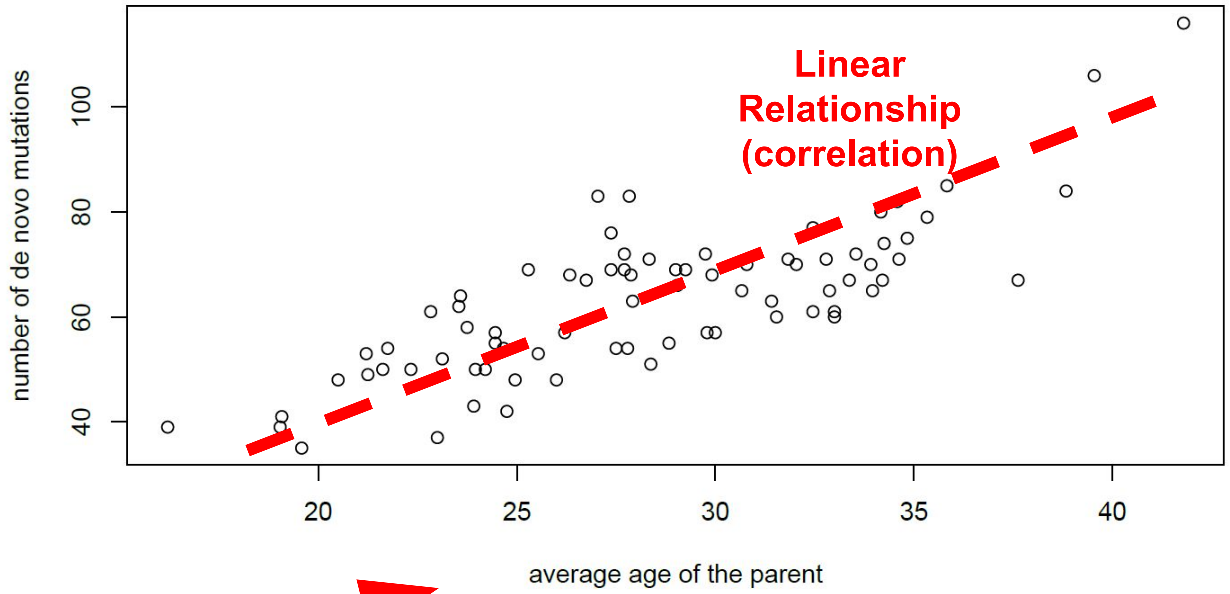

```python
import numpy as np
import matplotlib.pyplot as plt

np.random.seed(19680801)

# example data
mu = 100  # mean of distribution
sigma = 15  # standard deviation of distribution
x = mu + sigma * np.random.randn(437)

num_bins = 50

fig, ax = plt.subplots()

# the histogram of the data
n, bins, patches = ax.hist(x, num_bins, density=True)

# add a 'best fit' line
y = ((1 / (np.sqrt(2 * np.pi) * sigma)) *
     np.exp(-0.5 * (1 / sigma * (bins - mu))**2))
ax.plot(bins, y, '--')
ax.set_xlabel('Smarts')
ax.set_ylabel('Probability density')
ax.set_title(r'Histogram of IQ: $\mu=100$, $\sigma=15$')

# Tweak spacing to prevent clipping of ylabel
fig.tight_layout()
plt.show()
```

Standard normal dist



Scatterplot

Compares relationship between two quantitative variables…

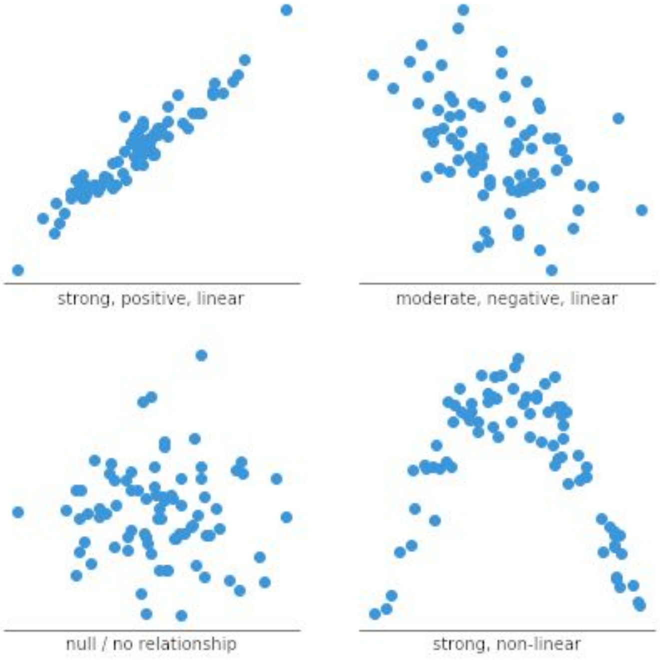


Target (or response) Variable

Explanatory Variable
Also: predictor, descriptor, input

Scatterplot

Compares relationship between two quantitative variables…

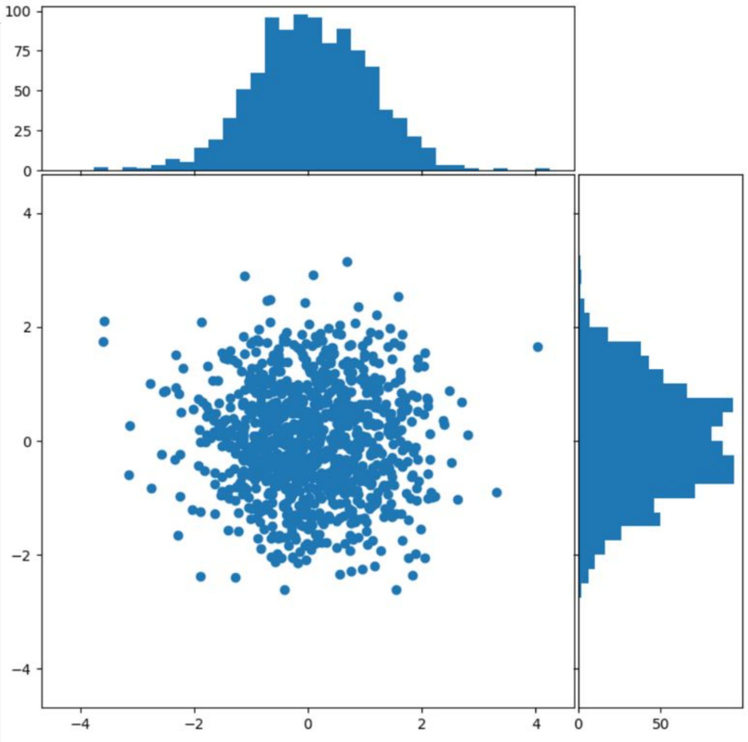


Relationship can also be:

- Nonlinear (e.g. "curvy")
- Clustered or grouped

Scatterplot + Histogram


```
import numpy as np
import matplotlib.pyplot as plt

# Fixing random state for reproducibility
np.random.seed(19680801)

# some random data
x = np.random.randn(1000)
y = np.random.randn(1000)


def scatter_hist(x, y, ax, ax_histx, ax_histy):
    # no labels
    ax_histx.tick_params(axis="x", labelbottom=False)
    ax_histy.tick_params(axis="y", labelleft=False)

    # the scatter plot:
    ax.scatter(x, y)

    # now determine nice limits by hand:
    binwidth = 0.25
    xymax = max(np.max(np.abs(x)), np.max(np.abs(y)))
    lim = (int(xymax/binwidth) + 1) * binwidth

    bins = np.arange(-lim, lim + binwidth, binwidth)
    ax_histx.hist(x, bins=bins)
    ax_histy.hist(y, bins=bins, orientation='horizontal')
```



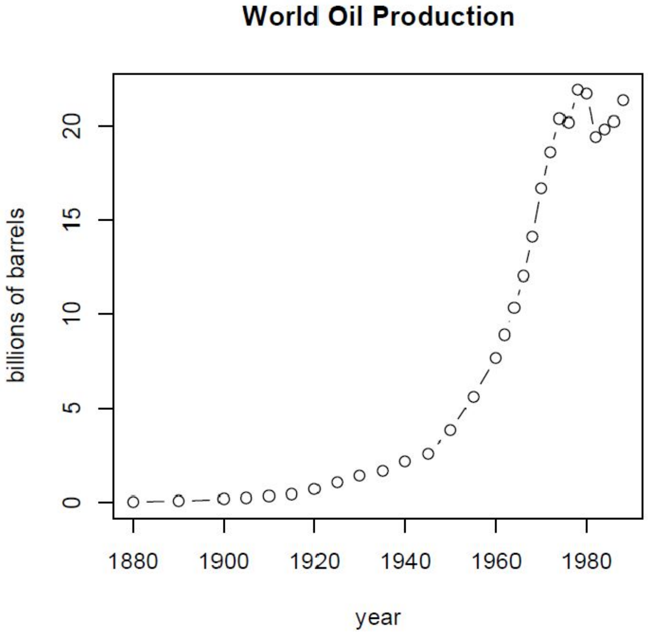
Full Code:
https://matplotlib.org/stable/gallery/lines_bars_and_markers/scatter_hist.html

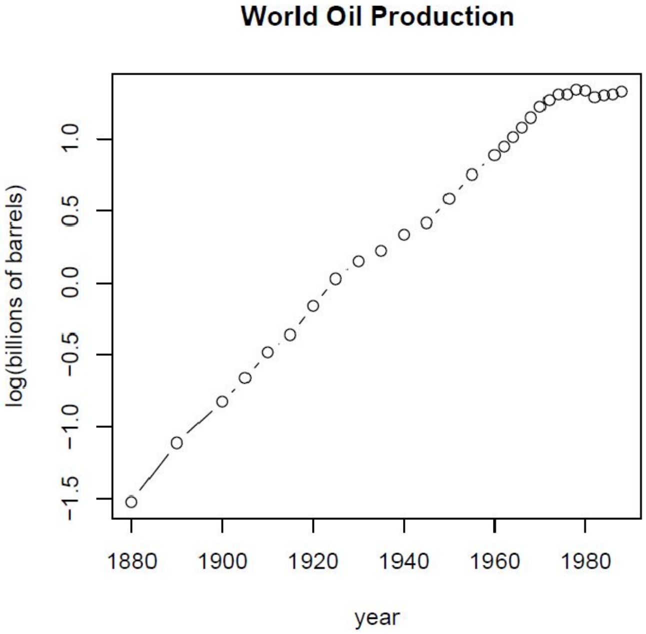
Logarithm Scale

Changing limits and base of y-scale highlights different aspects…

if y = e^x, then log(y) = x
if y = b^x, then log(y) = log(b)*x

=> becomes linear in x

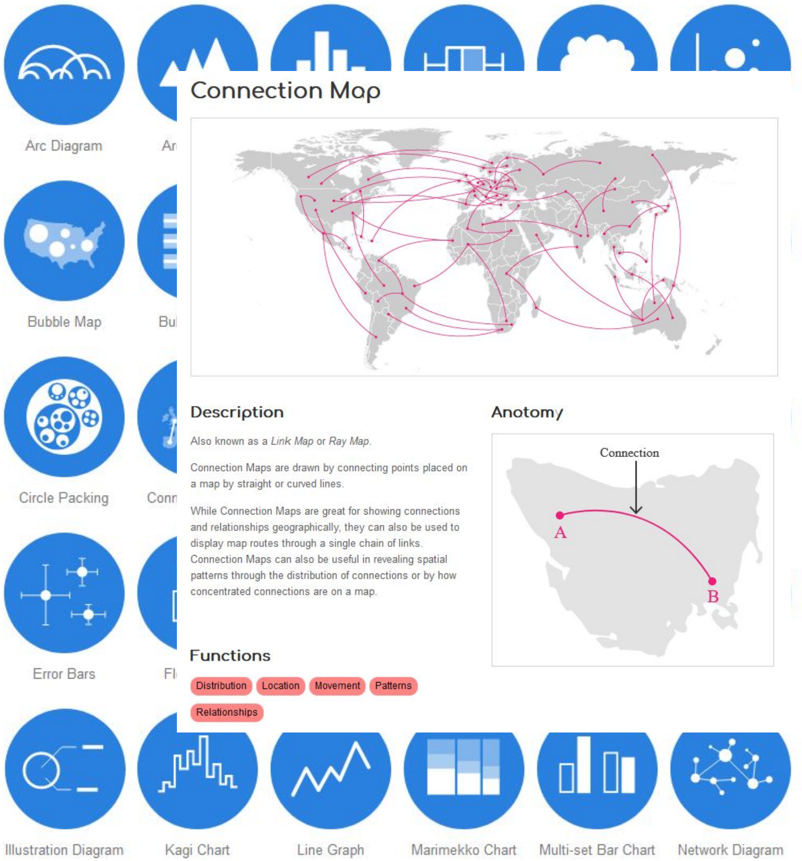




…log-scale emphasizes relative changes in smaller quantities

More Visualization Resources

datavizcatalogue.com



matplotlib

matplotlib.org

scikit learn

scikit-learn.org

Outline

- Data Visualization
- Data Summarization
- Data Collection and Sampling

Data Summarization

- Raw data are hard to interpret

- Visualizations summarize important aspects of the data

- The *empirical distribution* estimates the distribution on data, but can be hard to interpret

- **Summary statistics** characterize aspects of the data distribution like:
  - Location / center
  - Scale / spread

Measuring Location

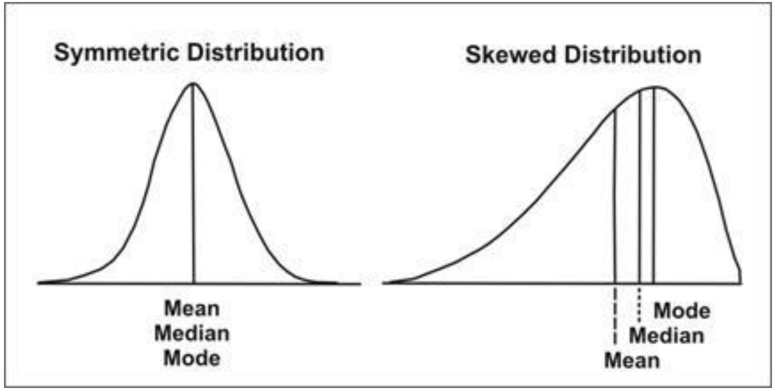
Three common measures of the distribution location…

Mean Average (expected value) of the data distribution

Median Midpoint – 50% of the probability is below and 50% above

Mode Value of highest probability (mass or density)

E.g., [1,2,3] vs [0,10,11] compute mean and median



…align with symmetric distributions, but diverge with asymmetry

Median

For data $x_1, x_2, \ldots, x_N$ sort the data,

$$x_{(1)}, x_{(2)}, \ldots, x_{(n)}$$

- Notation $x_{(i)}$ means the i-th *lowest* value, e.g. $x_{(i-1)} \leq x_{(i)} \leq x_{(i+1)}$
- $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ are called *order statistics*

not summary info, but rather a transformation

If n is **odd** then find the middle datapoint,

$$\mathrm{median}(x_1, \ldots, x_n) = x_{((n+1)/2)}$$

If n is **even** then average between both middle datapoints,

$$\mathrm{median}(x_1, \ldots, x_n) = \frac{1}{2}\left(x_{(n/2)} + x_{(n/2+1)}\right)$$

Median

What is the median of the following data?

1, 2, 3, 4, 5, 6, 8, 9 **4.5**

What is the median of the following data?

1, 2, 3, 4, 5, 6, 8, 100 **4.5**

Median is *robust* to outliers

Sample Mean

Empirical estimate of the true mean of the data distribution,

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

Alternative definition: if the value x occurs n(x) times in the data then,

$$\bar{x} = \frac{1}{N} \underbrace{\sum_{x}}_{\text{for the unique values of } \{x_1, \dots, x_N\}} x n(x) = \sum_{x} x p(x) \quad \text{where} \quad \underbrace{p(x) = \frac{n(x)}{N}}_{\text{Empirical Distribution}}$$

Recall

- Law of Large Numbers says $\bar{x}$ goes to mean E[X]
- Central Limit Theorem says $\bar{x}$ has Normal distribution, asymptotically.

Sample Mean

Example 2.1. *For the data set* $\{1,2,2,2,3,3,4,4,4,5\}$, *we have* $n = 10$ *and the sum*

$$\begin{aligned} 1+2+2+2+3+3+4+4+4+5 &= 1n(1)+2n(2)+3n(3)+4n(4)+5n(5) \\ &= 1(1)+2(3)+3(2)+4(3)+5(1) = 30 \end{aligned}$$

Thus, $\bar{x} = 30/10 = 3$.

Sample Mean

↓
(bacterium)

Example 2.2. *For the data on the length in microns of wild type* Bacillus subtilis *data, we have*

length x	frequency $n(x)$	proportion $p(x)$	product $xp(x)$
1.5	18	0.090	0.135
2.0	71	0.355	0.710
2.5	48	0.240	0.600
3.0	37	0.185	0.555
3.5	16	0.080	0.280
4.0	6	0.030	0.120
4.5	4	0.020	0.090
sum	200	1	2.490

So the sample mean $\bar{x} = 2.49$.

Sample Mean

For any real-valued function h(x) we can compute the mean as,

$$\overline{h(x)} = \frac{1}{N}\sum_{i=1}^{N} h(x_i)$$

Note $\overline{h(x)} \neq h(\bar{x})$ in general.

Example Compute the average of the square of values,

{ 1, 2, 3, 4, 5, 5, 6 }

$$\overline{x^2} = \frac{1}{7}(1 + 2^2 + 3^3 + 4^2 + 2(5^2) + 6^2) \approx 16.57$$

$(\bar{x})^2 \approx 13.80$

Weighted Mean

In some cases we may weigh data differently,

$$\sum_{i=1}^{N} w_i x_i \quad \text{where} \quad \sum_{i=1}^{N} w_i = 1 \qquad 0 \leq w_i \text{ for } i = 1, \ldots, N$$

For example, grades in a class:

$$\text{Grade} = 0.2 \cdot x_{\text{midterm}} + 0.2 \cdot x_{\text{final}} + 0.6 \cdot x_{\text{homework}}$$

Grading Breakdown (example)

- Homework: 60%
- Midterm: 20%
- Final: 20%

Measuring Spread

We have seen estimates of spread via the sample variance,

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

Biased

$$s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

Unbiased

But you might be interested in more detailed information about the spread.

For example, fraction of people with heights <= 5 feet

Measuring Spread

Quartile divide data into 4 equally-sized bins,

- **1st Quartile** : Lowest 25% of data
- **2nd Quartile** : Median (lowest 50% of data)
- **3rd Quartile** : 75% of data is below 3rd quartile
- **4th Quartile** : All the data… not useful

Compute using `np.quantile()` :

```
x = np.random.rand(10) * 100
q = np.quantile(x, (0.25, 0.5, 0.75))
np.set_printoptions(precision=1)
print( "X: " , x )
print( "Q: " , q )
```

```
X:  [90.7 73.9 31.7  2.8 56.3 95.7 15.6 75.8  4.1 19.5]
Q:  [16.6 44.  75.3]
```

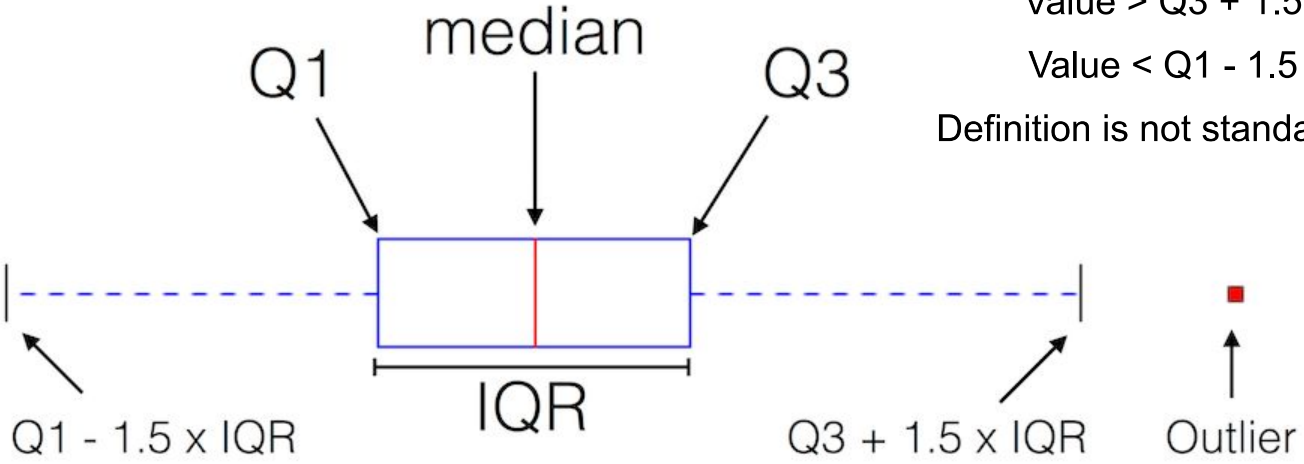
Box Plot

Outliers Are atypical data,

Value > Q3 + 1.5 x IQR

Value < Q1 - 1.5 x IQR

Definition is not standard.



Interquartile-Range (IQR) Measures interval containing 50% of data

$$IQR = Q3 - Q1$$

Region of *typical* data

Box Plot

48 52 57 61 64 72 76 77 81 85 88

Median

48 52 57 61 64 72 76 77 81 85 88

$\underbrace{48\quad 52\quad 57\quad 61\quad 64\quad 72}_{\text{First half}}\quad \underbrace{72\quad 76\quad 77\quad 81\quad 85\quad 88}_{\text{Second half}}$

Q1 Q3

$\underbrace{48\quad 52\quad 57\quad 61\quad 64\quad 72}_{\text{First half}}\quad \underbrace{72\quad 76\quad 77\quad 81\quad 85\quad 88}_{\text{Second half}}$

$Q1 = \frac{57+61}{2} = 59$

$Q3 = \frac{77+81}{2} = 79$

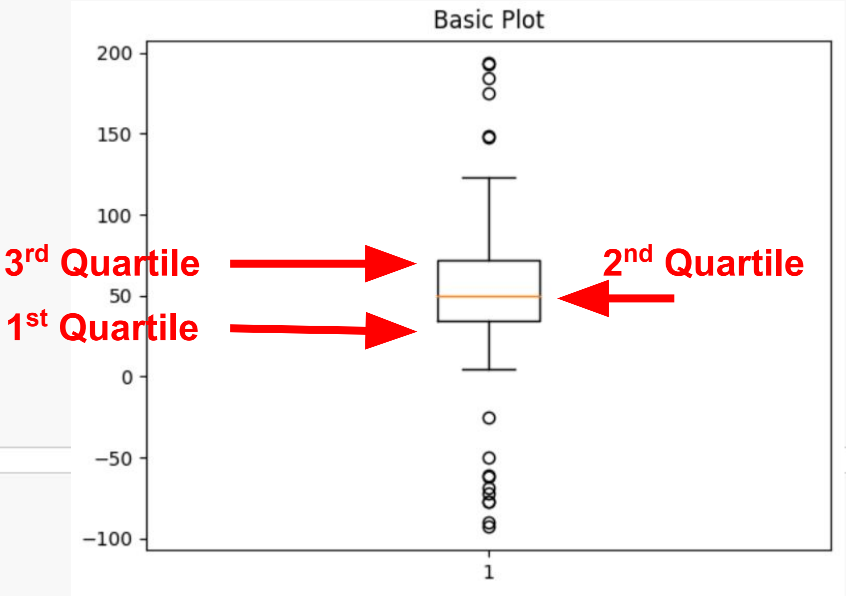
$IQR = Q3 - Q1$

$IQR = 79 - 59 = 20$

Box Plot

```
import numpy as np
import matplotlib.pyplot as plt

# Fixing random state for reproducibility
np.random.seed(19680801)

# fake up some data
spread = np.random.rand(50) * 100
center = np.ones(25) * 50
flier_high = np.random.rand(10) * 100 + 100
flier_low = np.random.rand(10) * -100
data = np.concatenate((spread, center, flier_high, flier_low))
```

```
fig1, ax1 = plt.subplots()
ax1.set_title('Basic Plot')
ax1.boxplot(data)
```



SciPy

Python-based ecosystem for math, science and engineering.

As usual, install with Anaconda:

```
> conda install scipy
```

Or with PyPI:

```
> pip install scipy
```

SciPy includes some libraries that directly works with:

SciPy Statistics

To compute summary stats (e.g., **mode**):

```
>>> a = np.array([[6, 8, 3, 0],
...               [3, 2, 1, 7],
...               [8, 1, 8, 4],
...               [5, 3, 0, 5],
...               [4, 7, 5, 9]])
>>> from scipy import stats
>>> stats.mode(a)
ModeResult(mode=array([[3, 1, 0, 0]]), count=array([[1, 1, 1, 1]]))
```

numpy has mean, but not mode.

- **numpy** provides popular numerical functions.
- **scipy** provides more serious & specialized functions.

kind of stupid example; tie breaking leads to choose the smallest value

Compute the mode of the whole array set `axis=None`:

```
>>> stats.mode(a, axis=None)
ModeResult(mode=array([3]), count=array([3]))
```

SciPy Statistics

SciPy is a large library, so we import it in bits and pieces…

```
>>> from scipy import stats
```

Access the object norm and call its function mean(): stats.norm.mean()

In some cases, you will import only the functions that you need:

```
>>> from scipy.stats import norm
```

SciPy Statistics

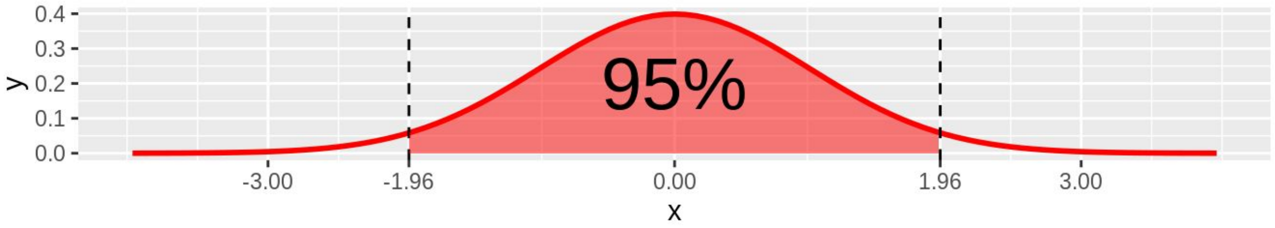
In some cases, you will import only the functions that you need:

```
>>> from scipy.stats import norm
```

contains information about the standard normal distribution

```
>>> norm.mean(), norm.std(), norm.var()
(0.0, 1.0, 1.0)
>>> norm.stats(moments="mv")
(array(0.0), array(1.0))
```

norm.ppf(0.975) returns 0.975-quantile, which is ≈ 1.96



<u>(Fact 2)</u> If $Z \sim \mathcal{N}(0,1)$,

$$P(Z \in [-z, z]\,) = 1 - 2\big(1 - \Phi(z)\big)$$

where $\Phi(z) := \mathrm{P}(\mathrm{Z} \le z)$ is the CDF of Z.

z = 1.96: RHS $\approx$ .95,   95% confident

z = 2.58: RHS $\approx$ .99,

SciPy Statistics

SciPy

Other useful summary statistics:

`moment`(a[, moment, axis, nan_policy])	Calculate the nth moment about the mean for a sample.
`trim_mean`(a, proportiontocut[, axis])	Return mean of array after trimming distribution from both tails.
`iqr`(x[, axis, rng, scale, nan_policy, …])	Compute the interquartile range of the data along the specified axis.
`bootstrap`(data, statistic, *[, vectorized, …])	Compute a two-sided bootstrap confidence interval of a statistic.

do not use this for your homework

. . .

Anscomb's Quartet : The Data

We'll see the risk of looking at the **statistics** only, not the **actual data**.

Four distinct datasets of X and Y…

I		II		III		IV	
x	y	x	y	x	y	x	y
10.0	8.04	10.0	9.14	10.0	7.46	8.0	6.58
8.0	6.95	8.0	8.14	8.0	6.77	8.0	5.76
13.0	7.58	13.0	8.74	13.0	12.74	8.0	7.71
9.0	8.81	9.0	8.77	9.0	7.11	8.0	8.84
11.0	8.33	11.0	9.26	11.0	7.81	8.0	8.47
14.0	9.96	14.0	8.10	14.0	8.84	8.0	7.04
6.0	7.24	6.0	6.13	6.0	6.08	8.0	5.25
4.0	4.26	4.0	3.10	4.0	5.39	19.0	12.50
12.0	10.84	12.0	9.13	12.0	8.15	8.0	5.56
7.0	4.82	7.0	7.26	7.0	6.42	8.0	7.91
5.0	5.68	5.0	4.74	5.0	5.73	8.0	6.89

[Source: https://www.geeksforgeeks.org/anscombes-quartet/]

Anscomb’s Quartet : Summary Statistics


```python
# Import the csv file
df = pd.read_csv("anscombe.csv")

# Convert pandas dataframe into pandas series
list1 = df['x1']
list2 = df['y1']

# Calculating mean for x1
print('%.1f' % statistics.mean(list1))

# Calculating standard deviation for x1
print('%.2f' % statistics.stdev(list1))

# Calculating mean for y1
print('%.1f' % statistics.mean(list2))

# Calculating standard deviation for y1
print('%.2f' % statistics.stdev(list2))

# Calculating pearson correlation
corr, _ = pearsonr(list1, list2)
print('%.3f' % corr)

# Similarly calculate for the other 3 samples

# This code is contributed by Amiya Rout
```

Summary statistics, e.g. Dataset 1:

Mean X1: 9.0

STDEV X1: 3.32

Mean Y1: 7.5

STDEV Y1: 2.03

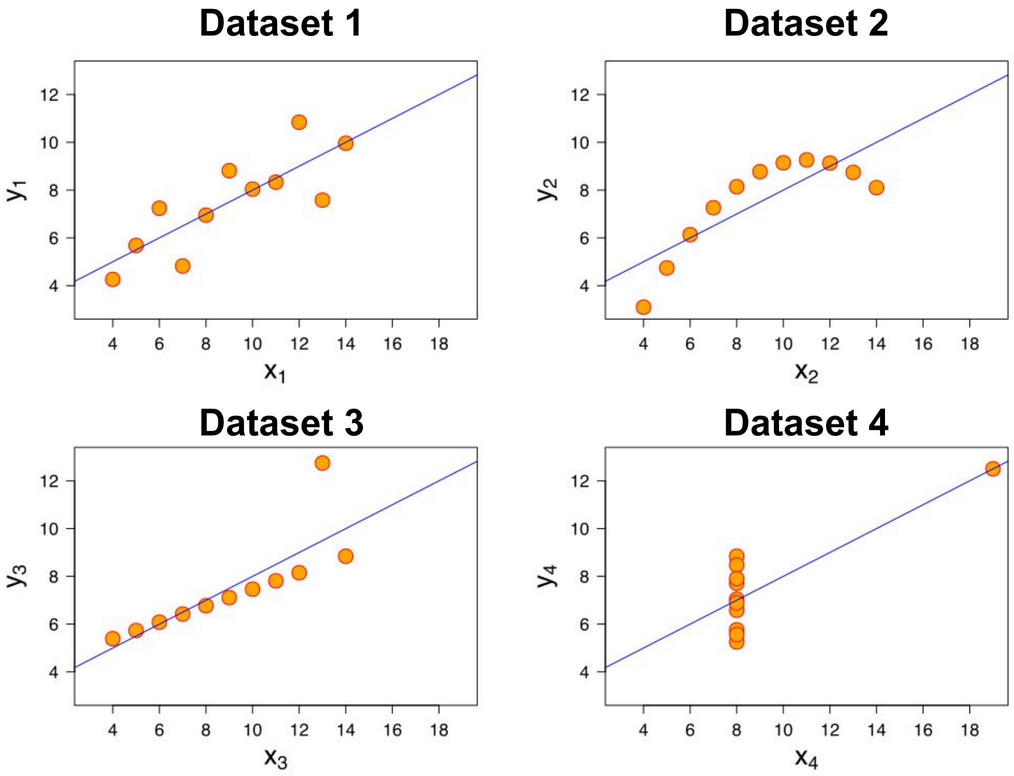
Correlation: 0.816

Actually, **all datasets have the same statistics…**

Question What can we conclude about these data? Are they the same?

[Source: https://www.geeksforgeeks.org/anscombes-quartet/]

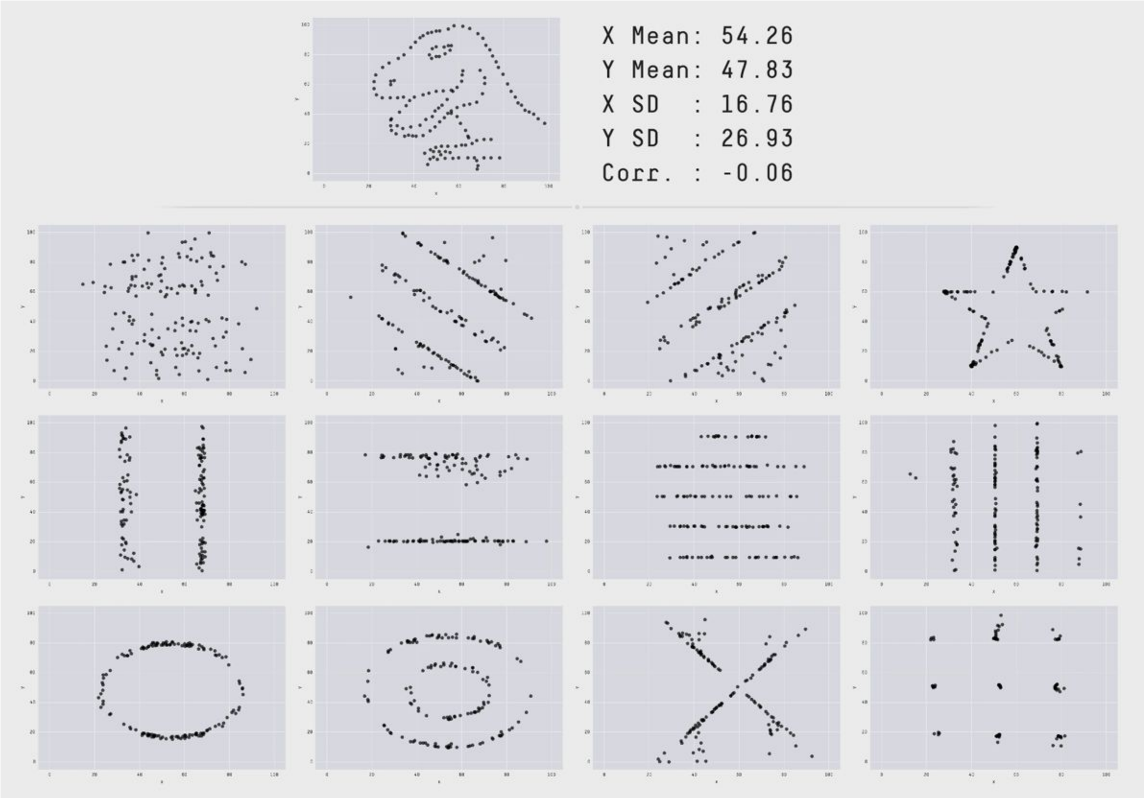
Anscomb’s Quartet : Visualization



Visualizing data clearly indicates that these are *very different* datasets…

…this highlights the **importance of visualizing data**

Datasaurus



13 datasets that all have the same summary statistics, but look very different in simple visualizations

Can be very difficult to see differences in high dimensions, however

[Source: Alberto Cairo]