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A Lane Departure Warning System based on a Linear-Parabolic Lane Model

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Abstract— **In this paper, we propose a new lane departure warning system based on a linear-parabolic lane boundary model. A linear function is used to fit the near vision field, and a quadratic function fits the far field. The linear part of the model provides robust information about the orientation of the vehicle with respect to both lane boundaries, while the parabolic part is flexible enough to fit curved parts of the road. The orientation of both lane boundaries is then computed and used to anticipate lane crossings.**

I. INTRODUCTION

Nowadays, an important social and economic problem is traffic safety. In 1999, about 800,000 people died globally in road related accidents, causing losses of around US$ 518 billion [1]. The annual report of brazilian national traffic department (DENATRAN) of 2001 [2] shows a total of 300,000 accidents, with 20,000 fatalities only in Brazil. A considerable fraction of these accidents is due to driver fatigue and/or inattention. In many cases, the driver falls asleep (mostly bus and truck drivers), making the vehicle to leave its designated lane and possibly causing and accident.

It is expected that machine vision systems can be used to improve safety on the roads, decreasing the number of accidents. However, the contribution is not so high at the moment, because machine vision systems have not achieved enough reliability so far. In this paper, we propose a new lane departure warning system based on a linear-parabolic lane boundary model. The orientations of both left and right lane boundaries are used to compute a lane departure measure.

II. RELATED WORK

The first step to develop a lane departure warning system is a robust detection of lane boundaries. In fact, several road boundary detection systems have been developed in the past years. However, the number of lane departure systems reported in the literature is much smaller. A plausible explanation for this fact is that existing lane detection techniques are not reliable enough for orientation estimation. Next, some of these techniques are briefly described.

Beucher and his colleagues [3], [4] worked on road segmentation and obstacle detection based on watersheds. Their technique consists of applying a temporal filter for noise reduction (and connection of ground markings), followed by edge detection and watershed segmentation. Such methods demand a relatively high computational cost, and the resulting road boundaries are typically jagged (due to the watershed transform).

Another class of lane detection methods [5], [6] rely on top-view (birds eye) images computed from images acquired by the camera. These methods are reliable in obtaining lane orientation in world coordinates, but require online computation of the top-view images (and camera calibration).

Deformable road models have been widely used for lane detection [7], [8], [9], [10], [11], [12]. These techniques attempt to determine mathematical models to fit road boundaries. In general, simpler models (e.g. linear) do not provide an accurate fit, but they are more robust with respect to image artifacts. On the other hand, more complex models (such as parabolic and splines) are more flexible, but also more sensitive to noise.

Apostoloff and Zelinsky [13] proposed a lane tracking system based on particle filtering and multiple cues. In fact, this method does not track explicitly the lanes, but it computes parameters such as lateral offset and yaw of the vehicle with respect to the center of the road. Although the method appears to be robust under a variety of conditions (shadows, different lighting conditions, etc.), it cannot be used to estimate curvature or detect if the vehicle is approaching a curved part of the road.

Lee [14] proposed a lane departure detection system that estimates lane orientation through an edge distribution function (EDF), and identifies changes in the travelling direction of a vehicle. However, the EDF may fail in curved roads with dashed lane markings. A modification of this technique [15] includes a boundary pixel extractor to improve its robustness. However, curved lanes may still cause problems, because a linear model (computed using the Hough Transform) is used for fitting lane boundaries.

In this paper, we propose a linear-parabolic model for lane boundaries. A linear function is used in the near field (locally, the road is assumed to be straight), and a quadratic function is utilized in the far field (such that incoming curves can be efficiently detected). This model combines the robustness of the linear model with the flexibility of the parabolic model, showing good detection results in the presence of noise, shadows and different illumination conditions. The orientation of both lane boundaries in the near field is used to provide a measure of deviation from the center of the lane.

An overview of the lane departure warning system in presented in the next Section. Our approach for lane de-

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tection is described in Section IV. Our lane departure warning system is described in Section V, followed by some experimental results. Finally, the concluding remarks are presented in the final Section.

III. Overview of Our Technique

Let us consider a camera attached to the windshield of the vehicle, with its optical axis aligned with the central axis of the vehicle. If the vehicle is travelling at the center of the lane, we should expect symmetry in the orientations of left and right lane boundaries ($\theta_l + \theta_r = 0$), as depicted in Fig. 1.

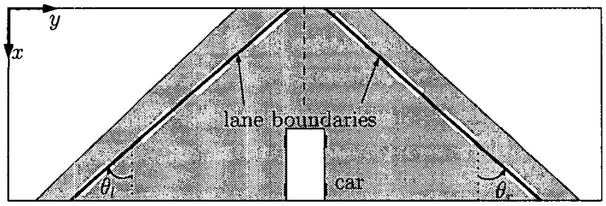

Fig. 1. Orientation of lane boundaries.

A simple and efficient measure for deviation with respect to the center of the lane is given by $\alpha = |\theta_l + \theta_r|$. If α exceeds a certain threshold T, the vehicle has a tendency for lane departure.

Our technique is based on a robust lane detection algorithm, that is used to compute both lane orientations θ_l and θ_r. The symmetry measure α is computed at every frame, and a lane departure alarm is triggered when $\alpha > T$.

IV. Lane Detection

Let us consider a coordinate system matching image coordinates, and a threshold x_m that separates the near and far vision fields, as shown in Fig. 2 (the choice for x_m depends on the size and the quality of the acquired images). Our model is a combination of a linear function in the near field, and a parabolic function in the far field, as described next.

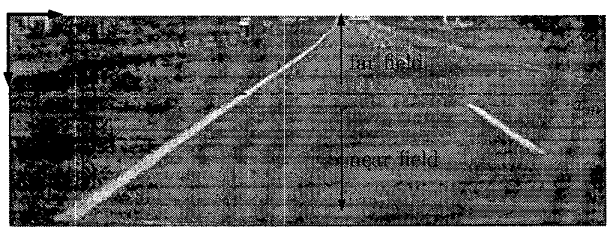

Fig. 2. Our coordinate system and the definition of the near and far fields.

A. The Proposed Lane Boundary Model

Let us consider the following model $f(x)$ for lane boundary:

$$f(x) = \begin{cases} a + bx, & \text{if } x > x_m \\ c + dx + ex^2, & \text{if } x \leq x_m \end{cases}, \tag{1}$$

where x_m represents the border between near and far fields. If we impose continuity and differentiability conditions on the function f, we must have $f(x_m^+) = f(x_m^-)$ and $f'(x_m^+) = f'(x_m^-)$. These conditions imply that:

$$\begin{cases} a + bx_m = c + dx_m + ex_m^2 \\ b = d + 2ex_m \end{cases}. \tag{2}$$

Solving this system for variables c and e, and replacing the result back into equation (2) leads to:

$$f(x) = \begin{cases} a + bx, & \text{if } x > x_m \\ \frac{2a + x_m(b-d)}{2} + dx + \frac{(b-d)}{2x_m}x^2, & \text{if } x \leq x_m \end{cases}. \tag{3}$$

Equation (3) indicates that our lane boundary model is characterized by only three coefficients (a, b and d). To determine these parameters, we use a weighted least squares method, fitting the proposed model to the images acquired by the camera. This procedure is applied independently for each lane boundary, and is described next.

B. Fitting the Linear-Parabolic Model

Let us assume that both (right and left) lane boundaries were detected in the prior frame of the video sequence. In the current frame, it is expected that lane boundaries will be constrained to a neighborhood of the prior detection. This neighborhood will be called lane boundary region of interest (LBROI), and will be the search space for boundaries in the current frame. The size of the LBROI depends on the resolution of the camera and the speed of the vehicle, and in this work they will be "thick" curves obtained by dilating lane boundary models w pixels in the y direction. Fig. 3(a) depicts the LBROIs for a certain frame of a video sequence.

The edge image $|\nabla I(x,y)|$ of the current frame is computed within the LBROI. Although most of the edges will be related to the lane boundary, some edges related to noise, road texture or other structures will also appear. To remove these undesired edges, we apply an adaptive threshold based on the mean magnitude M_{mean} of the edges. More specifically, we remove all the edges with magnitudes smaller than $0.5M_{\text{mean}}$. Let $g(x,y)$ denote the thresholded edge image:

$$g(x,y) = \begin{cases} |\nabla I(x,y)|, & \text{if } |\nabla I(x,y)| \geq 0.5M_{\text{mean}} \\ 0, & \text{otherwise} \end{cases}. \tag{4}$$

It should be noticed that this adaptive threshold is not affected by varying illumination conditions, and does not require any *a priori* information about color and/or contrast between the road and lane markings. For example, Fig. 3(b) shows the thresholded edge image $g(x,y)$ for the frame subsequent to the one illustrated in Fig. 3(a).

Let (x_{n_i}, y_{n_i}), for $i = 1, ..., m$, denote the m coordinates of the non-zero pixels of the thresholded edge image $g(x,y)$ belonging to the near field, and $M_{n_i} = g(x_{n_i}, y_{n_i})$ the respective magnitudes. Analogously, let (x_{f_j}, y_{f_j}) and $M_{f_j} = g(x_{f_j}, y_{f_j})$, for $j = 1, ..., n$, represent the same characteristics for the n edge pixels in the far field.

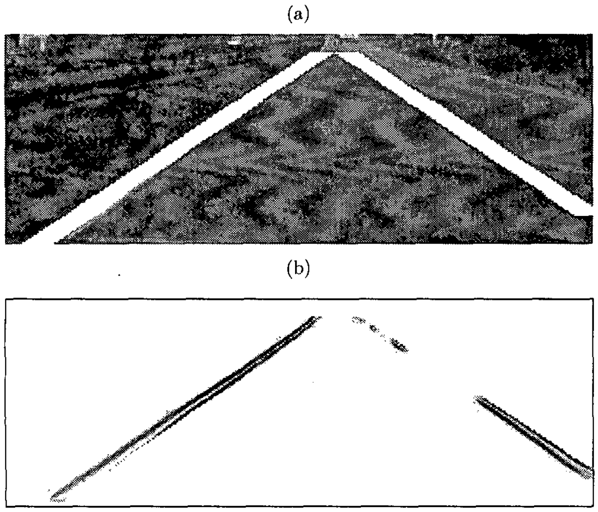

Fig. 3. (a) LBROIs superimposed to a frame in our video sequence. (b) Thresholded edge image corresponding to the LBROIs in (a) for the subsequent frame (darker pixels relate to larger magnitudes).

Fitting the lane model (3) to the edge data results in a linear system with 3 unknowns and $n+m$ equations:

$$\begin{cases} a+bx_{n_i}=y_{n_i}, & \text{for } i=1,2,...,m \\ \frac{2a+x_m(b-d)}{2}+dx_{f_j}+\frac{(b-d)}{2x_m}x_{f_j}^2=y_{f_j}, & \text{for } j=1,2,...,n \end{cases}. \tag{5}$$

Typically, $(n+m)$ will be much greater than 3, and this system will not admit an exact solution. However, we can find an approximated solution such that a specific error measure is minimized. Assuming that edges related to lane boundaries usually have larger magnitudes than edges related to other irrelevant structures (such as noise, road texture, etc.), we propose a quadratic error weighted by the respective edge magnitudes:

$$E=\sum_{i=1}^{m} M_{n_i}\left[y_{n_i}-f(x_{n_i})\right]^2+\sum_{j=1}^{n} M_{f_j}\left[y_{f_j}-f(x_{f_j})\right]^2. \tag{6}$$

This error is minimized when the following 3×3 linear system is solved:

$$\mathbf{A}^T\mathbf{W}\mathbf{A}\mathbf{c}=\mathbf{A}^T\mathbf{W}\mathbf{b}, \tag{7}$$

where

$$\mathbf{A}=\begin{bmatrix} 1 & x_{n_1} & 0 \\ \vdots & \vdots & \vdots \\ 1 & x_{n_m} & 0 \\ 1 & \frac{1}{2x_m}\left(x_{f_1}^2+x_m^2\right) & -\frac{1}{2x_m}\left(x_{f_1}-x_m\right)^2 \\ \vdots & \vdots & \vdots \\ 1 & \frac{1}{2x_m}\left(x_{f_n}^2+x_m^2\right) & -\frac{1}{2x_m}\left(x_{f_n}-x_m\right)^2 \end{bmatrix},$$

$$\mathbf{W}=\operatorname{diag}\left(M_{n_1},\ldots,M_{n_m},M_{f_1},\ldots,M_{f_n}\right),$$

$$\mathbf{c}=[a,b,d]^T \text{ and } \mathbf{b}=\left[y_{n_1},\ldots,y_{n_m},y_{f_1},\ldots,y_{f_n}\right]^T.$$

It should be noticed that $\mathbf{A}^T\mathbf{W}\mathbf{A}$ is a symmetric matrix. Hence, only a triangular portion (upper or lower) of the matrix must be computed, reducing the computation burden.

Fig. 4 shows the lane boundaries detected by fitting the proposed model to the edge magnitudes illustrated in Fig. 3(b). Due to the minimization process, the model is fitted at center of the lane markings (thus, we do not have to deal with inner and outer boundaries, as done in [15]). Also, it should be noticed that a straight portion of the road is depicted. Consequently, the parabolic part of our model is approximately linear.

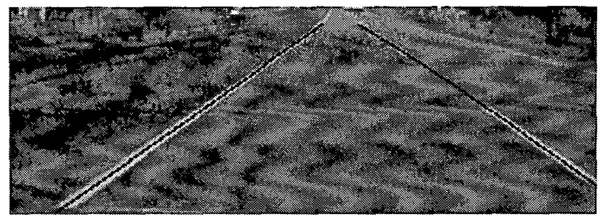

Fig. 4. Detected lane boundaries.

Similarly to the procedure applied in the initial segmentation, the LBROI is obtained by dilating the fitted function in the y direction, w pixels to the right and w pixels to the left.

In the subsequent frame, the edge image will be computed only in a small region delimited by the LBROI obtained in the current frame. The procedure described in Section IV-B is then repeated for the remaining frames.

V. Lane Departure Detection

As a result of the lane detection technique described in the previous section, we have an explicit linear-parabolic model $f(x)$ for each lane boundary. Let $f_l(x)$ and $f_r(x)$ denote the left and right lane boundaries, respectively. The corresponding orientations $\theta_l(x)$ and $\theta_r(x)$ are given by:

$$\theta_l(x)=\tan^{-1}\left(f_l'(x)\right), \quad \theta_r(x)=\tan^{-1}\left(f_r'(x)\right). \tag{8}$$

Equation (8) provides both lane boundary orientations at any given position x. In this work, we want to estimate these orientations in the near field. Since $f_l(x)$ and $f_r(x)$ are both linear functions for $x\geq x_m$, $\theta_l(x)=\theta_l$ and $\theta_r(x)=\theta_r$ are actually constant values within the near field. To reduce the influence of noise, the orientations θ_l and θ_r are averaged in five consecutive frames.

We compute the symmetry measure $\alpha=|\theta_l+\theta_r|$, and then decide for lane departure if $\alpha>T$, where T is a fixed threshold determined experimentally (we used $T=10^\circ$ in this work).

VI. Experimental Results

Our preliminary results indicate the robustness of the lane detection technique, and the validity of our lane departure system. For example, Fig. 5(a) shows the orientations θ_l and θ_r for a video sequence containing 1151 frame acquired at

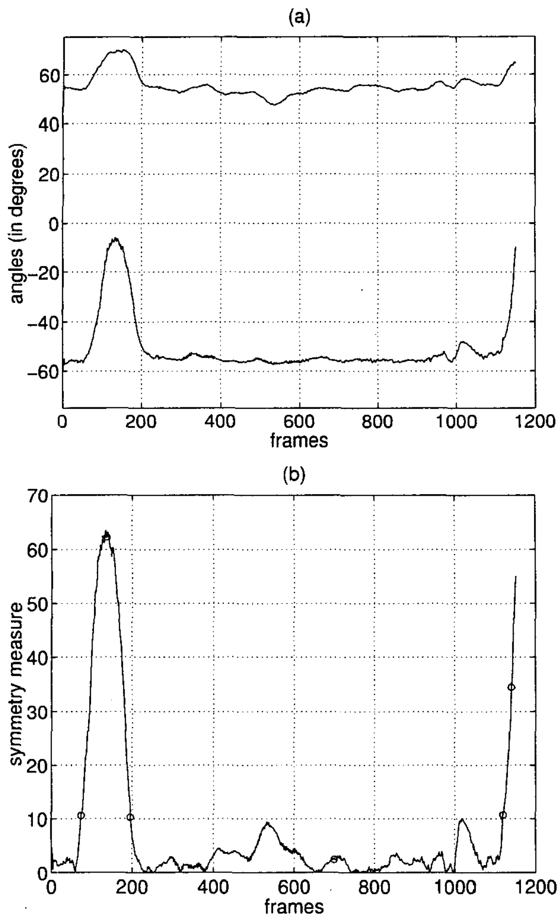

Fig. 5. (a) Orientations θ_r and θ_l for the first video sequence. (b) Corresponding symmetry measure α.

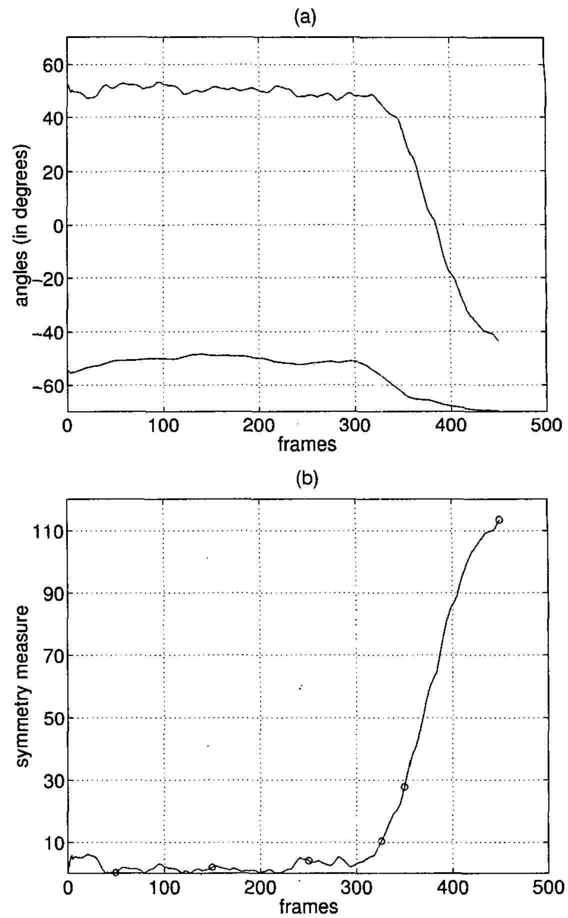

Fig. 7. (a) Orientations θ_r and θ_l for the second video sequence. (b) Corresponding symmetry measure α.

15 frames per second. Fig. 5(b) shows the corresponding symmetry measure α. According to our symmetry metric, lane departure occurred from frames 74 to 196 (with a maximum deviation at frame 137), and again from frame 1120 on. In the other frames, no lane departure was identified. Fig. 6 shows some of these frames where lane departure was detected (and also frame 700, where the vehicle was travelling within its designated lane). It can be noticed that our method correctly detected lane departure: during frames 74 to 196, the vehicle was overtaking a motorcycle; from frame 1120 to 1151, the vehicle was overtaking another car. In the remaining frames, the vehicle was driving in its lane.

Another video sequence obtained with a different camera and acquisition rate (30 frames per second) is shown in Figures 7 and 8, depicting a vehicle changing from the left to the right lane. Fig. 7(a) shows the orientations θ_l and θ_r, and Fig. 7(b) shows the corresponding symmetry measure α, indicating that lane departure occurs from frame 326 on. The frames marked with circles in Fig. 7(b) are shown in Fig. 8.

VII. Concluding Remarks

In this paper, we proposed a new lane departure warning system based on a linear-parabolic lane boundary model. Our preliminary results indicate that the proposed model provides an accurate fit to lane boundaries, and can be used to obtain robust information about their orientation. Also, these orientations are used to produce a symmetry measure, that correctly indicates tendencies of lane departure.

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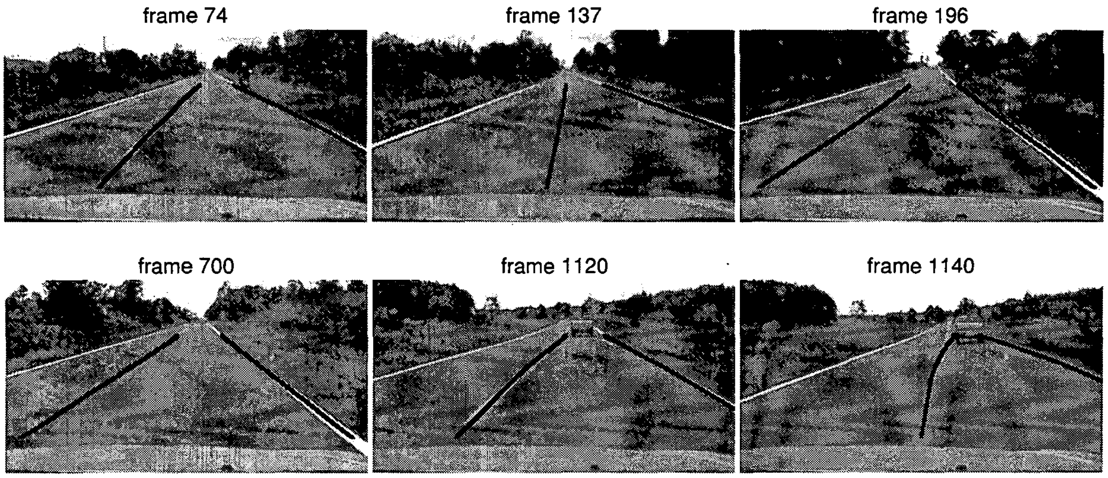

Fig. 6. Frames of the first video sequence.

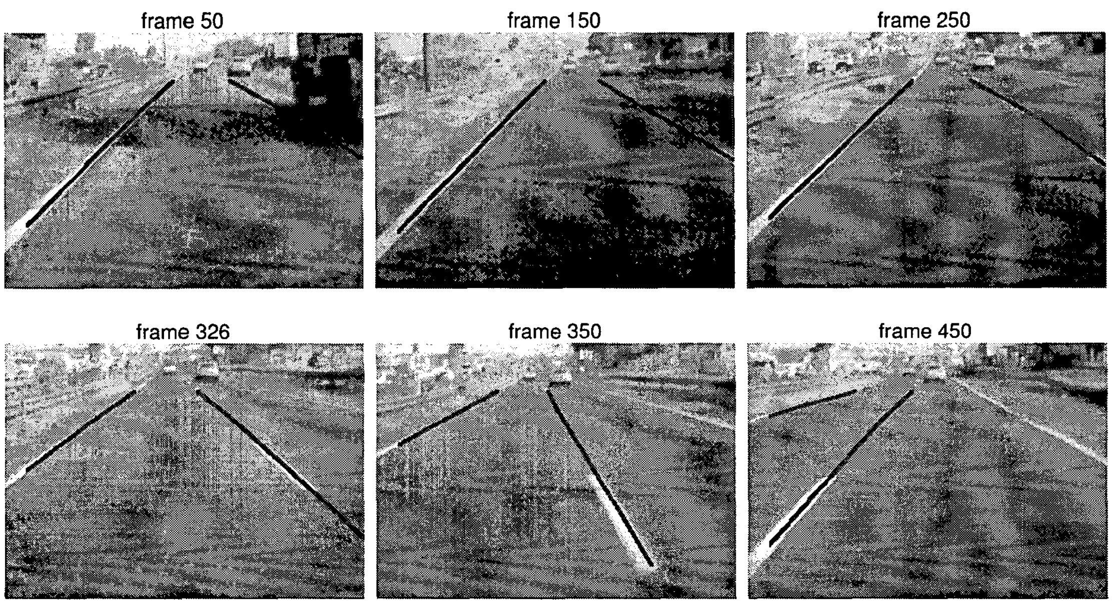

Fig. 8. Frames of the second video sequence.

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