MLP Report

Name: Xiangyun Ding (丁相允)

Student ID: 2016011361

Overview

In this homework, I implemented multilayer perceptron with the original code structure. All codes are in directory `codes`.

In addition to the normal parts to complete, I split Softmax and CrossEntropyLoss and wrote an extra Softmax layer. I will explain it in the following part.

Details

Changes to files:

- `layers.py`: Add details to class Relu, Sigmoid and Linear. Add class Softmax layer.
- `loss.py`: Add details to EuclideanLoss. Change SoftmaxCrossEntropyLoss to CrossEntropyLoss.
- `utils.py`: Add functions to record run time.
- `run_mlp.py`: Change the network to test more frequently. Add codes to draw loss-acc figures.

Since the Euclidean loss function is: $E_k(n) = \frac{1}{2}(t_k(n) - y_k(n))^2$, and the $t_k(n)$ we have is either 0 or 1, it should be a good idea to normalize all $y_k(n)$ to $(0, 1)$. I noticed that the Softmax function happens to do this normalization, so I decided to add a Softmax layer before the EuclideanLoss or CrossEntropyLoss layer.

The SoftmaxCrossEntropyLoss function has a gradient with a simple format. Since I split out the Softmax layer, we have to re-calculate its gradient. In the forward propagation, given input $\mathbf{x} = [x_1, x_2, \ldots, x_n]$, the output of Softmax layer is $\mathbf{y} = [y_1, y_2, \ldots, y_n]$, where we have:

$$y_i = \frac{e^{x_i}}{\sum_{j=1}^{n} e^{x_j}}$$

In the backward propagation, we've known $\frac{\partial E}{\partial y_i}$, and we want to calculate $\frac{\partial E}{\partial x_i}$. So we should use the chain rule:

$$\frac{\partial E}{\partial x_i} = \sum_{j=1}^{n} \frac{\partial E}{\partial y_j} \times \frac{\partial y_j}{\partial x_i}$$

If $j \neq i$, then we have:

$$\frac{\partial y_j}{\partial x_i} = \frac{\partial \frac{e^{x_j}}{\sum_{j=1}^{n} e^{x_j}}}{\partial x_i} = e^{x_i} \times -\frac{e^{x^j}}{(\sum_{j=1}^{n} e^{x_j})^2} = -y_i \times y_j$$

If $j = i$, then we have:

$$\frac{\partial y_j}{\partial x_i} = \frac{\partial \frac{e^{x_i}}{\sum_{j=1}^{n} e^{x_j}}}{\partial x_i} = \frac{(\sum_{j=1}^{n} e^{x_j}) \times e^{x_i} - e^{x_i} \times e^{x_i}}{(\sum_{j=1}^{n} e^{x_j})^2} = y_i \times (1 - y_i)$$

So we can simply use the result of $j \neq i$, then add y_i to the answer. Finally we have:

$$\frac{\partial E}{\partial x_i} = \sum_{j=1}^{n} \frac{\partial E}{\partial y_j} \times -y_i \times y_j + \frac{\partial E}{\partial y_i} \times y_i$$

This is the forward and backward propagation equation for Softmax layer.

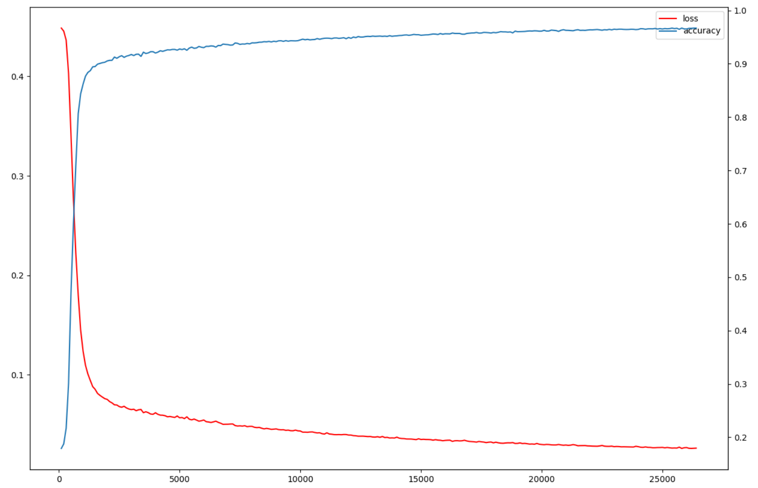
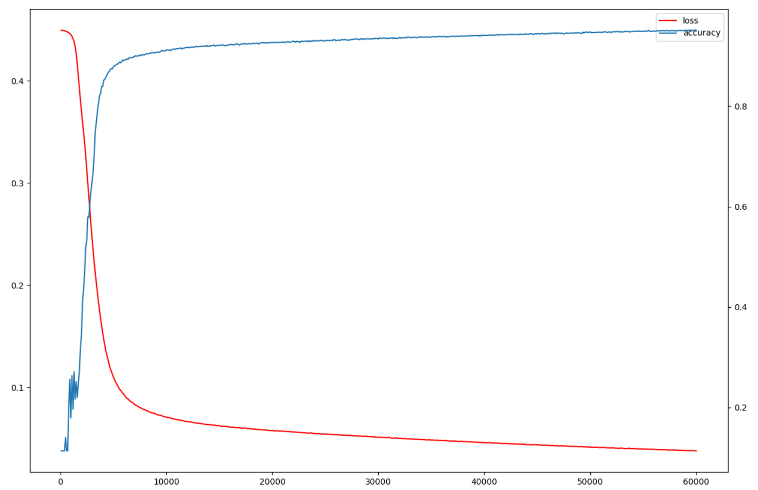
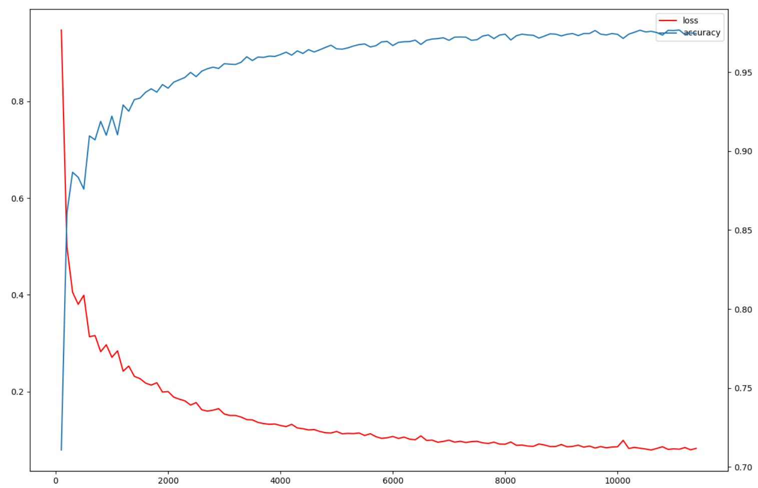
Experiments

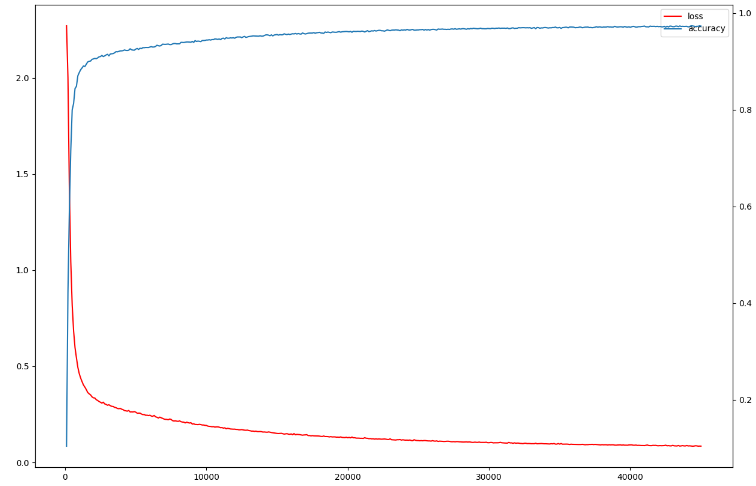
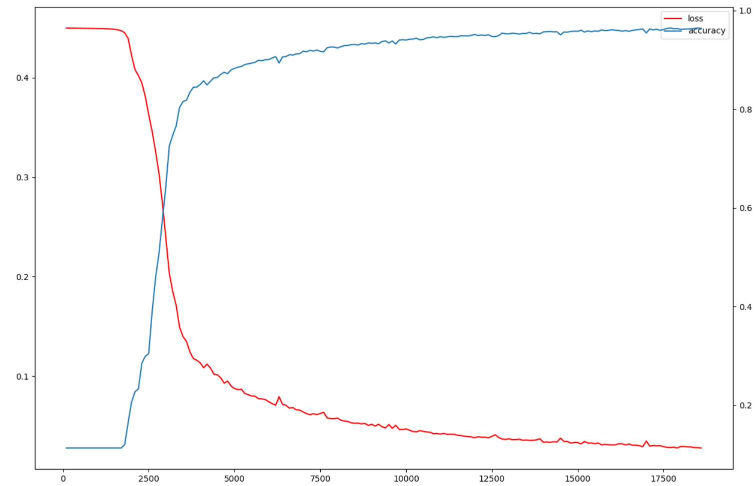
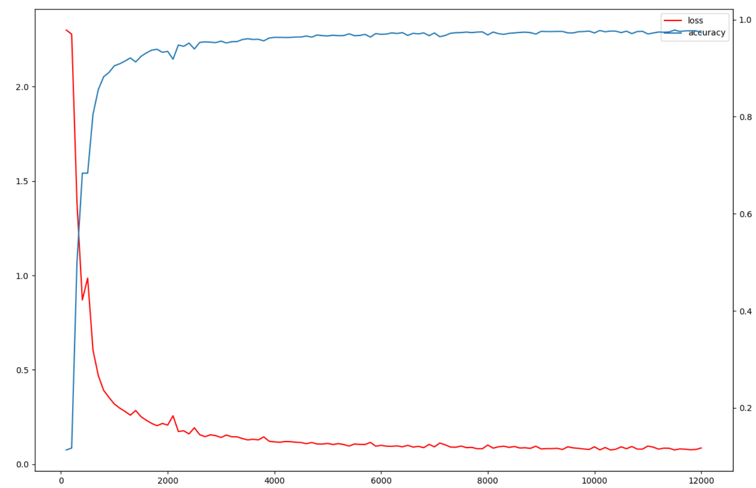
Basic Results:

ID	Network Structure	Loss Function	Learning Rate	Train Time(s)	Test Set Accuracy(%)
0	Linear(784×128)->Relu->Linear(128×10)->Softmax	EuclideanLoss	0.1	159	96.7
1	Linear(784×128)->Sigmoid->Linear(128×10)->Softmax	EuclideanLoss	0.1	431	95.1
2	Linear(784×128)->Relu->Linear(128×10)->Softmax	CrossEntropyLoss	0.1	84.3	97.6
3	Linear(784×128)->Sigmoid->Linear(128×10)->Softmax	CrossEntropyLoss	0.1	326	97.4
4	Linear(784×128)->Relu->Linear(128×128)->Relu->Linear(128×10)->Softmax	EuclideanLoss	0.1	162	96.5
5	Linear(784×128)->Relu->Linear(128×128)->Relu->Linear(128×10)->Softmax	CrossEntropyLoss	0.1	101	98.1

Loss-Accuracy Figures:

ID	Figure

ID	Figure
0	
1	
2	

ID	Figure
3	
4	
5	

For higher resolution images, please visit directory `images`.

It can be summarized from table and figures above that:

- Given same network structure, CrossEntropyLoss is better than EuclideanLoss as loss function.
- Given same network structure, Relu is better than Sigmoid as activate function.
- Comparing different network structures, the network with more hidden layers tends to comsume more time to train, but also have higher test accuracy.

Summary

During the homework period, I met the problem that the loss became 'NAN' while training. I debug it for quite a time and finally found that I made a mistake in the simplist Relu layer. This also reminds me that, those best deep learning frameworks, such as Tensorflow, are build from small units of codes. How difficult it is to avoid bugs in the huge system.

I've learned a lot through this homework. Thank Mr.Huang and the TAs.