STA 104 Applied Nonparametric Statistics

Chapter 5: Two-Way Layout Problems: Nonparametric Two-Way Analysis of Variance

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Two-Sided All-Treatments Multiple Comparisons for General Alternative in a Randomized Block Design with Equal Number of Replications Per treatment-Block Combination

After rejection of

$$H_0 : \underbrace{\tau_1 = \ldots = \tau_k}_{F_{i1},\ldots,F_{ik} \text{ within block } i \text{ are the same; that is, } F_{i1}\equiv F_{i2}\equiv\cdots\equiv F_{ik}\equiv F_i, \text{ for each fixed } i=1,\ldots,n}$$

$$H_1 : \underbrace{\tau_1 \ldots \tau_k \text{ not all equal}}_{\text{at least two of the treatment effects are not equal}}$$

with Mack-Skillings test, it is important to reach conclusions about all $\binom{k}{2} = k(k-1)/2$ pairs of treatment effects and these conclusions are naturally two-sided.

Hypothesis

$$\left\{\begin{array}{ll} H_0 : \tau_1 = \tau_2 & H_1 : \tau_1 \neq \tau_2 \\ H_0 : \tau_1 = \tau_3 & H_1 : \tau_1 \neq \tau_3 \\ \ldots & \\ H_0 : \tau_{k-1} = \tau_k & H_1 : \tau_{k-1} \neq \tau_k \end{array}\right\}$$

$\frac{k(k-1)}{2}$ simultaneous tests/multiple comparisons

Motivation

Optional:

$R_1 \dots R_k$

$\Rightarrow$ $\cancel{S_1, \dots, S_k}$: the treatment sums of cellwise averages of within-blocks ranks

$\Rightarrow$ $|R_u - R_v|$ tend to be small when null $\tau_u = \tau_v$ is true, and tend to be large when alternative $\tau_u \neq \tau_v$ is true

$\Rightarrow$ So we use absolute difference in within block rank sums as the test statistics

$\Rightarrow$ $(R_1, \dots, R_k)$ as N tends to infinity, an asymptotic multivariate normal distribution $(Z_1 \dots Z_k)$

$\Rightarrow$

$$\max_{1 \le u < v \le k} |R_u - R_v| \sim range(Z_1 \dots Z_k) \times \sqrt{\frac{k(N+n)}{12}}$$

$\Rightarrow$ To get the null distribution for the simultaneous tests, it is equivalent to know the null distribution of $\max_{1 \le u < v \le k} |R_u - R_v|$

$\Rightarrow$ It is then equivalent to the distribution of the range when we draw k independent $N(0, 1)$

Procedure

For each pair of treatments (u, v), for $1 \leq u < v \leq k$,

Decide $\tau_u \neq \tau_v$ if $|S_u - S_v| \geq \boxed{[k(N+n)/12]^{1/2} q_\alpha}$ critical value

otherwise decide $\tau_u = \tau_v$

- q_α is the upper α quantile of the range of k normal variates.

Example:Determination of Niacin in Bran Flakes

We have found rather strong evidence that the studied process for assessing niacin content in bran flakes does not produce consistent results across a variety of laboratories.

To determine which of the laboratories differ in median detected niacin content in the bran flakes.

$H_0: \tau_1 = \tau_2$ $\quad H_1: \tau_1 \neq \tau_2$

Laboratory	Amount of niacin enrichment (milligrams per 100 g bran flakes)		
	0	4	8
1	7.58(3)	11.63(7)	15.00(2)
	7.87(8)	11.87(11)	15.92(9)
	7.71(6)	11.40(3)	15.58(4)
2	7.95(9)	12.20(12)	16.60(12)
	8.27(12)	11.6(8)	16.40(11)
	8.05(10)	11.80(10)	15.90(7)
3	7.60(4)	11.04(2)	15.87(6)
	7.30(1)	11.45(5)	15.91(8)
	7.82(7)	11.49(4)	16.28(10)
4	8.03(11)	11.50(6)	15.10(3)
	7.35(2)	10.10(1)	14.80(1)
	7.66(5)	11.70(9)	15.70(5)

```
> library(NSM3)
> cRangeNor(0.05,k=4)
[1] 3.634
```

critical value

Decide $\tau_u \neq \tau_v$ if $|S_u - S_v| \geq \boxed{[4(36+3)/12]^{1/2}(3.634)} = 13.1$

$$
\begin{aligned}
|S_2 - S_1| &= 12.83 < 13.1 \Rightarrow \text{ decide } \tau_2 = \tau_1, \\
|S_3 - S_1| &= 1.84 < 13.1 \Rightarrow \text{ decide } \tau_3 = \tau_1, \\
|S_4 - S_1| &= 3.67 < 13.1 \Rightarrow \text{ decide } \tau_4 = \tau_1, \\
|S_3 - S_2| &= 14.67 > 13.1 \Rightarrow \text{ decide } \tau_3 \neq \tau_2, \\
|S_4 - S_2| &= 16.5 > 13.1 \Rightarrow \text{ decide } \tau_4 \neq \tau_2, \\
|S_4 - S_3| &= 1.83 < 13.1 \Rightarrow \text{ decide } \tau_4 = \tau_3.
\end{aligned}
$$

These numbers may not right ......

Thus, at an approximate experimentwise error rate of .05, we see that Laboratory 2 yielded significantly different median detected niacin content than either Laboratory 3 or Laboratory 4.

These multiple comparison decisions help to focus the rationale for the original rejection of the Mack-Skillings test, as it now seems reasonable to question the reliability of Laboratory 2 in conducting this niacin content process.