STA 104 Applied Nonparametric Statistics

Chapter 3: Two-Sample Methods

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In this chapter the data consist of two random samples, a sample from the control population and an independent sample from the treatment population.

On the basis of these samples, we wish to investigate the presence of a treatment effect that results in a shift of location.

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Why Ranks? Scoring Systems

Ranks can be thought of as scores that are used in place of the original observations in nonparametric methods. This leads us to consider coming up with other scores to be used in the same way. The key is to figure out a reasonable way to generate scores.

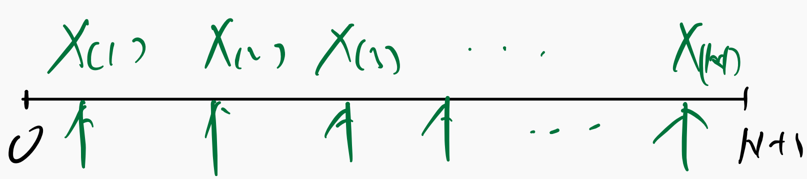
First, consider a particular way in which we might think of ranks. Suppose we take a random sample from a uniform probability distribution on the interval $[0, N+1]$; that is, the population distribution is

$$f(w) = \frac{1}{N+1}, 0 \leq w \leq N+1$$

Let $W_{(1)} < W_{(2)} < \cdots < W_{(\overset{N}{\cancel{M}})}$ denote the order statistics of this random sample, where $W_{(1)}$ is the smallest observation, $W_{(2)}$ is the next smallest, and so on. It can be shown that the ranks are just the expected values of the $W_{(i)}$'s; that is, $E\left(W_{(i)}\right) = i$.

N samples:

$E[X_{(i)}] = i$



In this vein, we consider scoring systems as looking at the expected order of data, we can generalize based on distributions other than the uniform, which leads to more general scoring systems.

So why ignore the exact measurements we get from the original data?

In some sense we lose partial information coded in the exact numbers of the data, but by replacing in with its relative order, i.e. ranks, so in face value we lost something,

but the bonus is that, we can get procedures that are more generally applicable for many types of data, no distributional assumptions are necessary.

- For instance, two reviewers with similar tastes may rate products based on different numerical systems, as long as their general tastes are the same, we get exact same results from using either one of their ratings.