STA 104 Applied Nonparametric Statistics

Chapter 4: One-Way Layout Problems: Nonparametric One-Way Analysis of Variance

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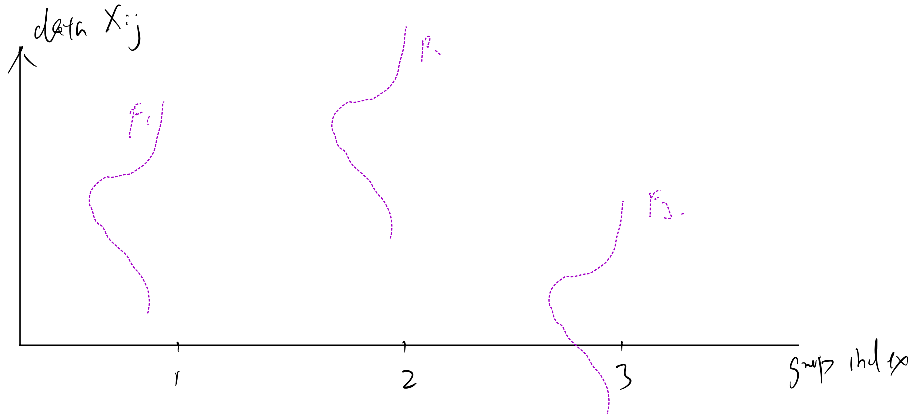
Multiple Comparisons

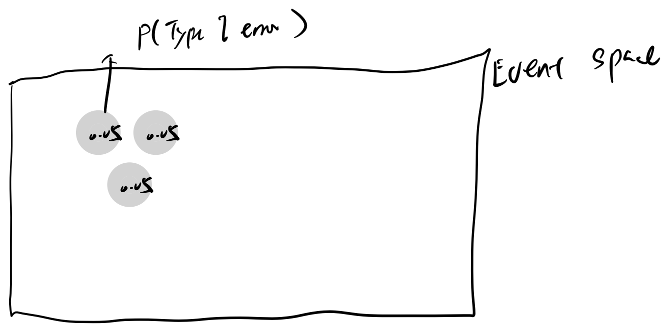
- We have discussed procedures designed to test the null hypothesis of no difference in treatment groups against a variety of alternative hypotheses. Upon rejection of H_0 with one of these test procedures for a given set of data, our conclusions range from the general statement that there are some unspecified differences among the treatment effects (associated with the Kruskal-Wallis test) to the more informative relationships between the treatment effects associated with test procedures designed for the ordered alternatives or the treatments-versus-control setting.
- However, in none of these test procedures are our conclusions specific or pair-specific; that is, the tests are not designed to enable us to reach conclusions about specific pairs of treatment effects, such as which specific treatments are better than the control.

Rationale for Multiple Comparison Procedures.

- To elicit such pairwise specific information, we turn to the class of multiple comparison procedures.
- The aim of applying such procedures goes beyond the point of deciding whether the treatments are equivalent to the (often more important) problem of selecting which, if any, treatments differ from one another. Thus, the user makes $k(k-1)/2$ decisions, one for each pair of treatments.



$$\begin{cases} H_0: \tau_1=\tau_2 & H_1: \tau_1\neq\tau_2 \\ H_0: \tau_1=\tau_3 & H_1: \tau_1\neq\tau_3 \\ H_0: \tau_2=\tau_3 & H_1: \tau_2\neq\tau_3 \end{cases}$$



$P(\text{Type 1 error when 3 tests simultaneously})$

$= P(\text{conclude } \tau_i \neq \tau_j \text{ for some } i,j \mid H_0: \tau_1=\tau_2=\cdots=\tau_3)$

$\approx 0.05 \times 3 = 15\%$

$k=4$? $\binom{4}{2} = 6$ pairwise comparisons

$P(\text{Type 1 error when 4 tests simultaneously})$

$= P(\text{conclude } \tau_i \neq \tau_j \text{ for some } i,j \mid H_0: \tau_1=\tau_2=\cdots=\tau_3)$

$\approx 0.05 \times 6 = 30\%$

$\Rightarrow$ Usual test procedure of p-value $< \alpha = 0.05$ or critical value need to be modified !

Rationale for Multiple Comparison Procedures.

- The multiple comparison procedure is designed so that the Experimentwise Error Rate is controlled to be equal to α; that is, the probability of falsely declaring any pair of treatment effects to be different, when in fact all of the treatment effects are the same, is equal to α.
- The use of an experimentwise error rate represents a very conservative approach to multiple comparisons. We are insisting that the probability of making only correct decisions be $1 - \alpha$ when the hypothesis H_0) of treatment equivalence is true. Thus, we have a high degree of protection when H_0 is true, but we often apply such techniques when we have evidence (rejecting the Kruskal-Wallis test) that H_0 is not true.
- This protection under H_0 also makes it harder for the procedure to judge treatments as differing significantly when in fact H_0 is false, and this difficulty becomes more severe as k increases.

$$P(\underbrace{\text{conclude } \tau_i \neq \tau_j \text{ for some } i,j \mid H_0: \tau_1 = \tau_2 = \cdots = \tau_k}_{\text{experimentwise error rate}}) = \alpha$$

■ Hypothesis testing vs Judge in a court