STA 104 Applied Nonparametric Statistics

Chapter 5: Two-Way Layout Problems: Nonparametric Two-Way Analysis of Variance

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Two-Sided All-Treatments Multiple Comparisons for General Alternative in a Randomized Complete Block Design

After rejection of

$$H_0 : \underbrace{\tau_1 = \ldots = \tau_k}_{F_{i1},\ldots,F_{ik} \text{ within block } i \text{ are the same; that is, } F_{i1} \equiv F_{i2} \equiv \cdots \equiv F_{ik} \equiv F_i, \text{ for each fixed } i=1,\ldots,n}$$

$$H_1 : \underbrace{\tau_1 \ldots \tau_k \text{ not all equal}}_{\text{at least two of the treatment effects are not equal}}$$

with the Friedman test based on within block ranks, it is important to reach conclusions about all $\binom{k}{2} = k(k-1)/2$ pairs of treatment effects and these conclusions are naturally two-sided.

Setting

The data consist of $N = \sum_{i=1}^{n} \sum_{j=1}^{k} 1 = nk$ observations, with 1 observations from the combination of the i th block with the j th treatment (i.e., the (i, j) th cell), for $i = 1, \ldots, n$ and $j = 1, \ldots, k$

	Treatments			
Blocks	1	2	...	k
1	X_{11}	X_{12}	...	X_{1k}
2	X_{21}	X_{22}	...	X_{2k}
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	X_{n1}	X_{n2}	...	X_{nk}

Hypothesis

$$\left\{\begin{array}{ll} H_0 : \tau_1 = \tau_2 & H_1 : \tau_1 \neq \tau_2 \\ H_0 : \tau_1 = \tau_3 & H_1 : \tau_1 \neq \tau_3 \\ \ldots & \\ H_0 : \tau_{k-1} = \tau_k & H_1 : \tau_{k-1} \neq \tau_k \end{array}\right\}$$

$\frac{k(k-1)}{2}$ simultaneous tests/multiple comparisons

Optional :

To test each : $H_0: \tau_u = \tau_v$ $H_1: \tau_u \neq \tau_v$

$1 \leq u < v \leq K$

test statistic

$\boxed{|R_u - R_v|}$ tend to be small when null $\tau_u = \tau_v$ is true, and tend to be large when alternative is $\tau_u \neq \tau_v$ is true

$\Rightarrow$ So we use absolute difference in within block rank sums as the test statistics

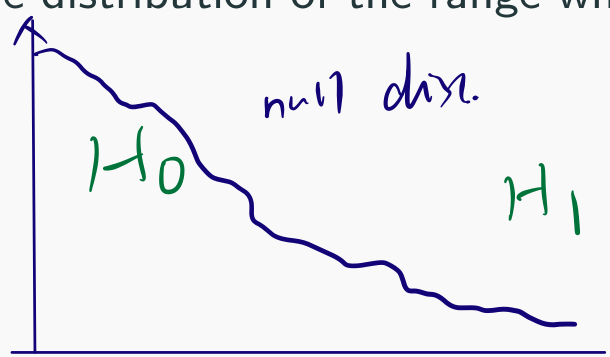
$\Rightarrow$ $(R_1, \ldots, R_k)$ as n tends to infinity, an asymptotic multivariate normal distribution $\cancel{(Z_1, \ldots, Z_k)}$

$\Rightarrow$

$$\max_{1 \leq u < v \leq k} |R_u - R_v| \sim range(Z_1 \ldots Z_k) \times \sqrt{\frac{nk(k+1)}{12}}$$

$\Rightarrow$ To get the null distribution for the simultaneous tests, it is equivalent to know the null distribution of $\max_{1 \leq u < v \leq k} |R_u - R_v|$

$\Rightarrow$ It is then equivalent to the distribution of the range when we draw k independent $N(0, 1)$



Procedure

For each pair of treatments (u, v), for $1 \leq u < v \leq k$,

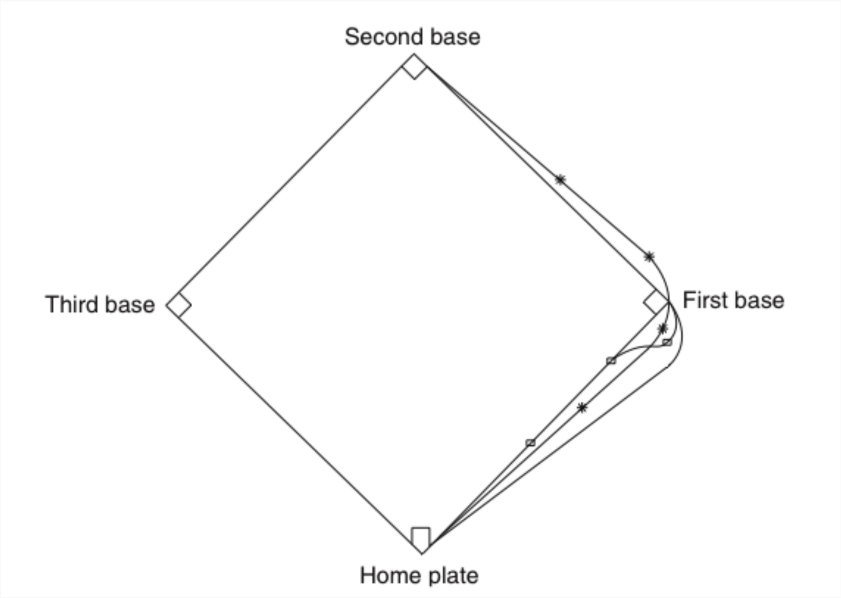
Decide $\tau_u \neq \tau_v$ if $|R_u - R_v| \geq \boxed{q_\alpha \left[\frac{nk(k+1)}{12}\right]^{1/2}}$ critical value

otherwise decide $\tau_u = \tau_v$

- q_α is the upper α quantile of the range of k normal variates.

Example:Rounding First Base

We have had found that there is strong evidence to conclude that the three methods of running to first base are not equivalent with respect to time to reach second base. Here we want to determine which of the three running methods differ in median times to second base.



$H_0: \tau_1 = \tau_2 \quad H_1: \tau_1 \neq \tau_2$

$H_0: \tau_1 = \tau_3 \quad H_1: \tau_1 \neq \tau_3$

$H_0: \tau_2 = \tau_3 \quad H_1: \tau_2 \neq \tau$

Simultaneous testing

Experimentwise error rate 5%

	Methods		
Players	Round out	Narrow angle	Wide Angle
1	5.40(1)	5.50(2)	5.55(3)
2	5.85(3)	5.70(1)	5.75(2)
3	5.20(1)	5.60(3)	5.50(2)
4	5.55(3)	5.50(2)	5.40(1)
5	5.90(3)	5.85(2)	5.70(1)
6	5.45(1)	5.55(2)	5.60(3)
7	5.45(2)	5.50(3)	5.35(1)
8	5.25(3)	5.15(2)	5.00(1)
9	5.85(3)	5.80(2)	5.70(1)
10	5.25(3)	5.20(2)	5.10(1)
11	5.65(3)	5.55(2)	5.45(1)
12	5.60(3)	5.35(1)	5.45(2)
13	5.05(3)	5.00(2)	4.95(1)
14	5.45(1)	5.55(3)	5.50(2)
15	5.45(1)	5.50(2)	5.55(3)
16	5.50(3)	5.45(2)	5.25(1)
17	5.65(3)	5.60(2)	5.40(1)
18	5.70(3)	5.65(2)	5.55(1)
	$R_1 = 43$	$R_2 = 37$	$R_3 = 28$

```
> library(NSM3)
> cRangeNor(0.05,k=3)
[1] 3.315
```

$q_{0.05}$

critical value

Decide $\tau_u \neq \tau_v$ if $|R_u - R_v| \geq \boxed{(3.315)\left[\frac{18(3)(4)}{12}\right]^{1/2}} = 14.06435.$

Using the treatments sums of within-runners ranks given, we find that

$$
\begin{aligned}
|R_2 - R_1| &= 6 < 14.06 \Rightarrow \text{ decide } \tau_2 = \tau_1 \\
|R_3 - R_1| &= 15 \geq 14.06 \Rightarrow \text{ decide } \tau_3 \neq \tau_1 \\
|R_3 - R_2| &= 9 < 14.06 \Rightarrow \text{ decide } \tau_3 = \tau_2
\end{aligned}
$$

Thus, at an approximate experimentwise error rate of .05, we have reached the conclusion that only the round out (treatment 1) and wide angle (treatment 3) running methods yield significantly different median times to second base.