Lecture 5: Two-Factor Studies with One Case per Treatment

STA 106: Analysis of Variance

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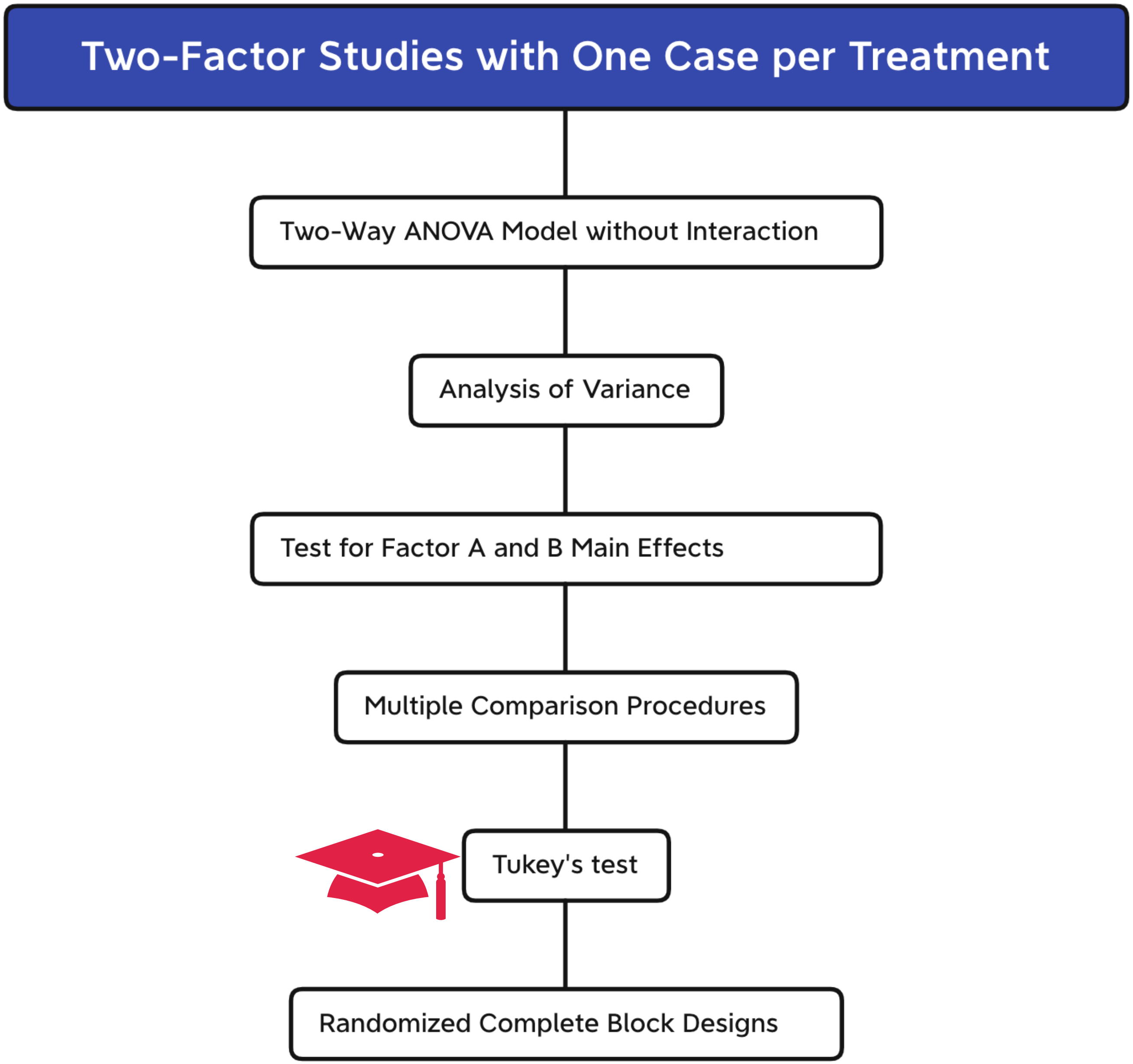

Two-Factor Studies with One Case per Treatment
Two-Way ANOVA Model without Interaction
Analysis of Variance
Test for Factor A and B Main Effects
Multiple Comparison Procedures
Tukey's test
Randomized Complete Block Designs

How do we know “No Interaction” assumption is correct or wrong?

Tukey Test for Additivity (Tukey one degree of freedom test)

Problem: can’t allow arbitrary forms of interaction because of limited data

Tukey’s idea: allow some restricted form of interaction

ij th interaction effect is proportional to the product of the main effects

$$\gamma_{ij} = D\alpha_i\beta_j$$

Motivation:

if in fact, the interaction effect γ_{ij} depends on main effects α_i, β_j in a relatively simple way, then the Turkey’s assumption can be shown to be accurate.

How do we know “No Interaction” assumption is correct or wrong?

Two-way ANOVA model with Turkey’s interaction:

$$Y_{ij} = \mu_{..} + \alpha_i + \beta_j + D\alpha_i\beta_j + \varepsilon_{ij}$$

Least squares estimates :

$$\hat{\mu}_{..} = \frac{\sum_i \sum_j \hat{\mu}_{ij}}{ab} = \frac{\sum_i \sum_j Y_{ij}}{ab} = \bar{Y}_{..}$$

$$\hat{\alpha}_i = \hat{\mu}_{i.} - \hat{\mu}_{..} = \frac{\sum_j Y_{ij}}{b} - \bar{Y}_{..} = \bar{Y}_{i.} - \bar{Y}_{..}$$

$$\hat{\beta}_j = \hat{\mu}_{.j} - \hat{\mu}_{..} = \frac{\sum_i Y_{ij}}{a} - \bar{Y}_{..} = \bar{Y}_{.j} - \bar{Y}_{..}$$

$$\hat{D} = \frac{\sum_i \sum_j \hat{\alpha}_i \hat{\beta}_j Y_{ij}}{\sum_i \hat{\alpha}_i^2 \sum_j \hat{\beta}_j^2} = \frac{\sum_i \sum_j (\bar{Y}_{i.} - \bar{Y}_{..})(\bar{Y}_{.j} - \bar{Y}_{..})Y_{ij}}{\sum_i (\bar{Y}_{i.} - \bar{Y}_{..})^2 \sum_j (\bar{Y}_{i.} - \bar{Y}_{..})^2}$$

How do we know "No Interaction" assumption is correct or wrong?

$$Y_{ij} - \bar{Y}_{..} = \left(Y_{ij} - \hat{Y}_{ij}\right) + \hat{\alpha}_i + \hat{\beta}_j + \hat{D}\hat{\alpha}_i\hat{\beta}_j$$

Total Deviation

Deviation due to extraneous factors

A main effect

B main effect

AB interaction effect

$$\sum_i \sum_j \left(\bar{Y}_{ij} - \bar{Y}_{..}\right)^2 = \sum_i \sum_j e_{ij}^2 + \sum_i \hat{\alpha}_i + \sum_j \hat{\beta}_j + \sum_i \sum_j \left(\hat{D}\hat{\alpha}_i\hat{\beta}_j\right)^2$$

SSTO

SSE

SSA

SSB

SSAB

$df(SSE) = ab - a - b$

$df(SSAB) = 1$

$$\boxed{\text{SSTO} = \text{SSA + SSB + SSAB+ SSE}}$$

How do we know "No Interaction" assumption is correct or wrong?

Tukey Test for Additivity (Tukey one degree of freedom test)

$H_0 : D = 0$ no interaction present $\quad\quad$ $H_a : D \neq 0$ interaction is present

Test statistic: $F^* = \dfrac{MSAB}{MSE}$

Large value of F^* support H_a

Small value, when $F^* \approx 1$ support H_0

—> We reject H_0 for large value of F^*, i.e. $F^* \geq c$

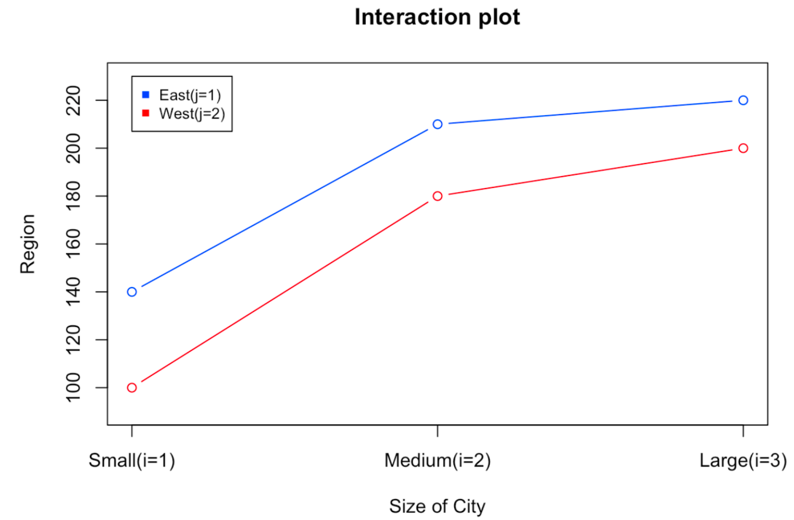
Decision rule:

If $F^* \leq F_{1-\alpha}(1, ab - a - b)$, then conclude H_0

If $F^* > F_{1-\alpha}(1, ab - a - b)$, then conclude H_a

Example

Conduct the Tukey test for additivity; use $\alpha = .1$. State the alternatives, decision rule, and conclusion. If the additive model is not appropriate, what might you do?



$H_0 : D = 0$ no interaction present $\qquad H_a : D \neq 0$ interaction is present

Test statistic: $F^* = \dfrac{MSAB}{MSE}$

$$MSAB = \sum_i \sum_j \left(\hat{D}\hat{\alpha}_i\hat{\beta}_j\right)^2$$

$$MSE = \sum_i \sum_j \left(Y_{ij} - \hat{Y}_{ij}\right)^2$$

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## [1] 6.75
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Code

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## [1] 39.86346
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For $\alpha = .10$, we require $F(.90; 1, 1) = 39.9$. Since $F^* = 6.8 \leq 39.9$, we conclude that region and size of city do not interact. Use of the no-interaction model for the data therefore appears to be reasonable.

How do we know “No Interaction” assumption is correct or wrong?

Tukey Test for Additivity (Tukey one degree of freedom test)

- Effective in detecting the interactions that are “simple” and approximately in the form $D\alpha_i\beta_j$
- Remedial actions are needed if interaction effects are present by Tukey’s test

  Transformation of Y to remove interaction effects

  $\sqrt{Y}$, $logY$, Box-Cox transformation Y^{λ}

  If no such transformation can remove the interaction, then be cautious about the reliability of model result