Lecture 4: Two-Factor Studies with Equal Sample Sizes

STA 106: Analysis of Variance

Suggested reading: ALSM Chapter 19

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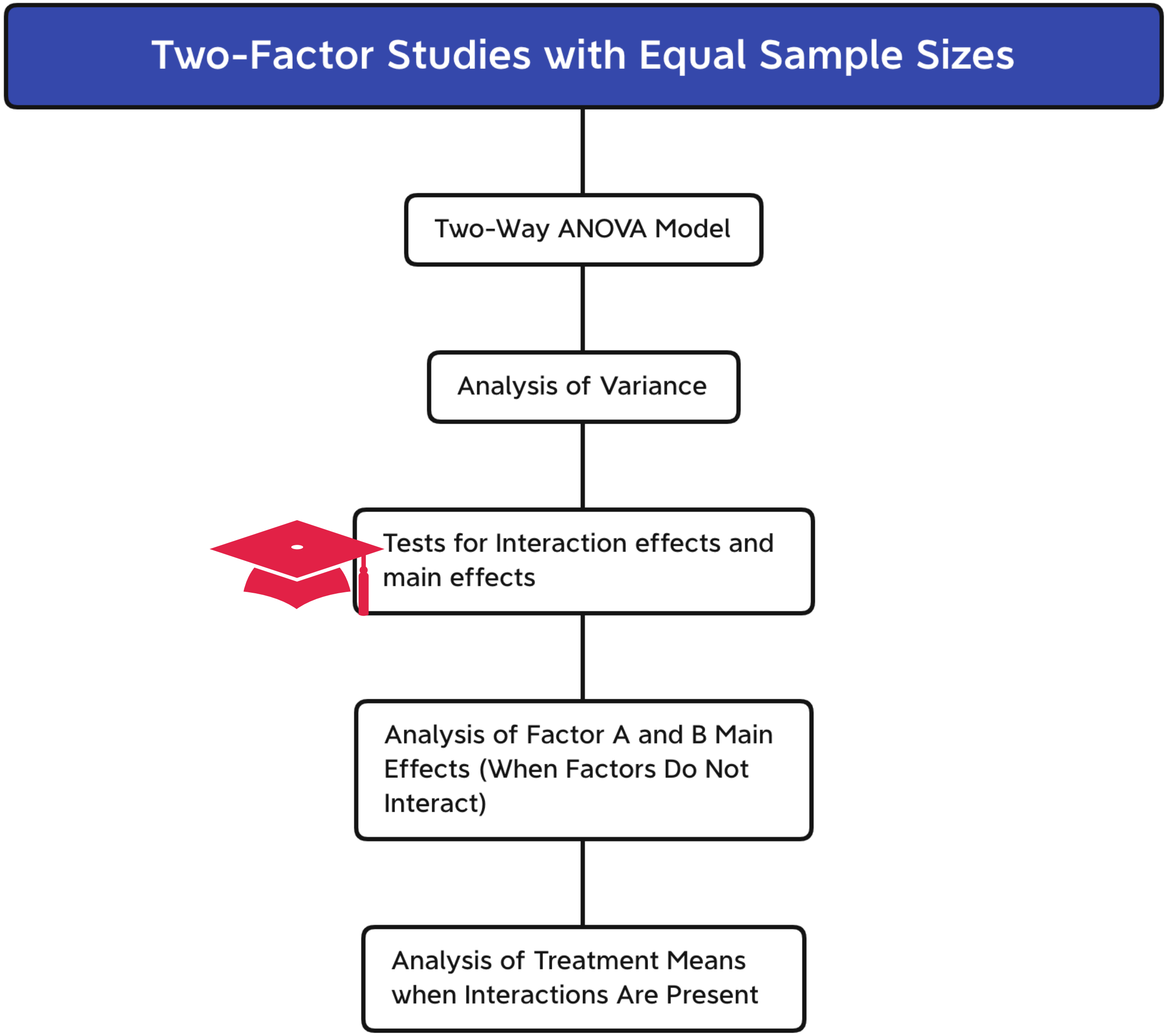

Two-Factor Studies with Equal Sample Sizes
Two-Way ANOVA Model
Analysis of Variance
Tests for Interaction effects and main effects
Analysis of Factor A and B Main Effects (When Factors Do Not Interact)
Analysis of Treatment Means when Interactions Are Present

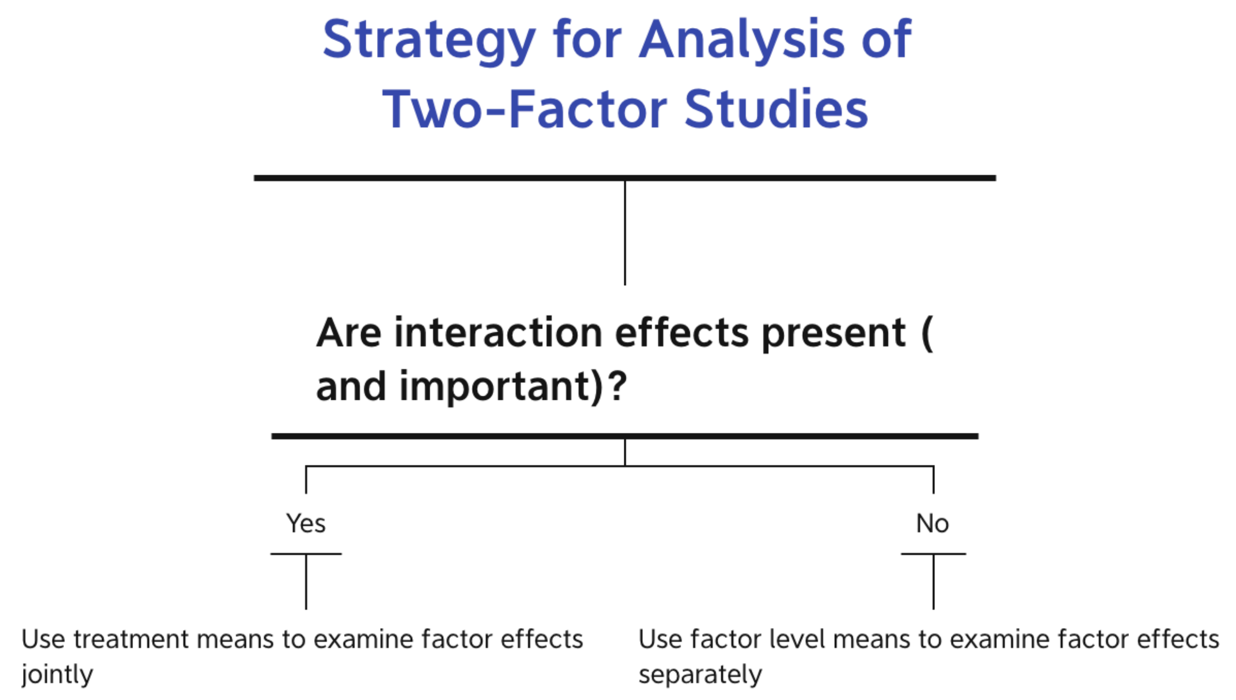
Strategy of Analysis of Two-Factor Studies

Scientific inquiry is guided by the principle:

simple, parsimonious explanations of observed phenomena tend to be the most effective

Additive factor effects : much simpler explanation of factor effects

Interaction effects complicates the explanation



Test for Interactions

To test whether or not the factor level means are the same:

H_0 : all $\gamma_{ij} = 0$

H_a : not all $\gamma_{ij} = 0$

Test statistic: $F^* = \dfrac{MSAB}{MSE}$

Large value of F^* support H_a

Small value, when $F^* \approx 1$ support H_0

—> We reject H_0 for large value of F^*, i.e. $F^* \geq c$

$$F^* = \frac{MSAB}{MSE} = \frac{\frac{\frac{((a-1)(b-1))MSAB}{\sigma^2}}{(a-1)(b-1)}}{\frac{\frac{ab(n-1)MSE}{\sigma^2}}{ab(n-1)}} \sim \frac{\frac{\chi^2_{df=(a-1)(b-1)}}{(a-1)(b-1)}}{\frac{\chi^2_{df=ab(n-1)}}{ab(n-1)}} \sim F((a-1)(b-1), ab(n-1))$$

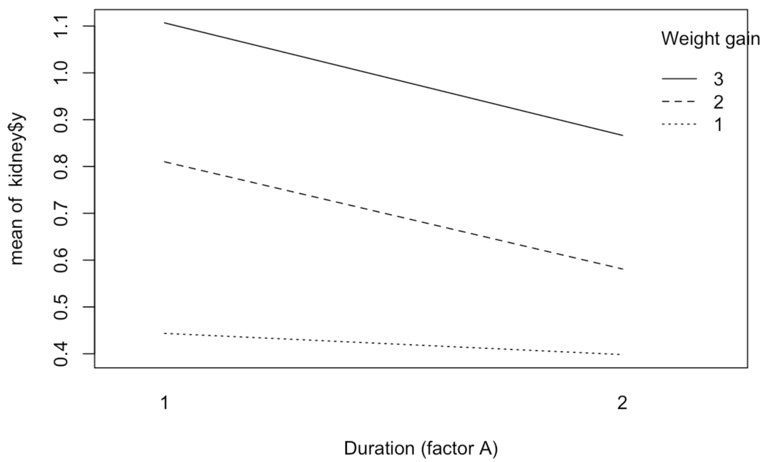
Decision rule:

If $F^* \leq F_{1-\alpha}((a-1)(b-1), ab(n-1))$, then conclude H_0

If $F^* > F_{1-\alpha}((a-1)(b-1), ab(n-1))$, then conclude H_a

Example

Test whether or not interaction effects are present at 0.05 significance level.



H_0 : all $\gamma_{ij} = 0$

H_a : not all $\gamma_{ij} = 0$

Test statistic: $F^* = \dfrac{MSAB}{MSE} = 0.592$

Critical value $F(1-\alpha, 2, 54) = 3.168$

Since the test statistic is less than the critical value, we cannot reject the null hypothesis at 0.05 significance level and conclude that the interaction effect is not statistically significant.

Test for Factor A and Factor B Main Effects

To test whether or not factor A main effects are present:

$$H_0 : \alpha_1 = \ldots = \alpha_a = 0$$

$$H_a : \text{not all } \alpha_i = 0$$

Test statistic: $F^* = \dfrac{MSA}{MSE}$

Decision rule:

If $F^* \leq F_{1-\alpha}(a-1, ab(n-1))$, then conclude H_0

If $F^* > F_{1-\alpha}(a-1, abn-1))$, then conclude H_a

To test whether or not factor B main effects are present:

$$H_0 : \beta_1 = \ldots = \beta_b = 0$$

$$H_a : \text{not all } \beta_j = 0$$

Test statistic: $F^* = \dfrac{MSB}{MSE}$

Decision rule:

If $F^* \leq F_{1-\alpha}(b-1, ab(n-1))$, then conclude H_0

If $F^* > F_{1-\alpha}(b-1, ab(n-1))$, then conclude H_a

Example

Test whether or not main effects for duration and weight gain are present. Use $\alpha = .05$ in each case and state the alternatives, decision rule, and conclusion. Is it meaningful here to test for main factor effects? Explain.

It is meaningful to test main effects, since the interaction effects are not statistically significant, one can examine the effects of two factors separately using main effects only, as one factor's effects no longer depends on the levels of another factor.

To test the significance of Factor A main effect

$$H_0 : \alpha_i = 0, i = 1,2,3 \text{ vs } H_a : \text{ not all } \alpha_i \text{'s are } 0$$

Test statistic: $F^* = \dfrac{MSA}{MSE} = 4.36$

Critical value $F(0.95,2,54) = 4.02$

Since the F test statistics of factor A is larger than the critical value, we reject the null hypothesis at 0.05 significance level and conclude that factor A treatment duration has significant effects on the days hospitalized.

To test the significance of Factor B main effect

$$H_0 : \beta_j = 0, j = 1,2,3 \text{ vs } H_a : \text{ not all } \beta_j' \text{s are } 0$$

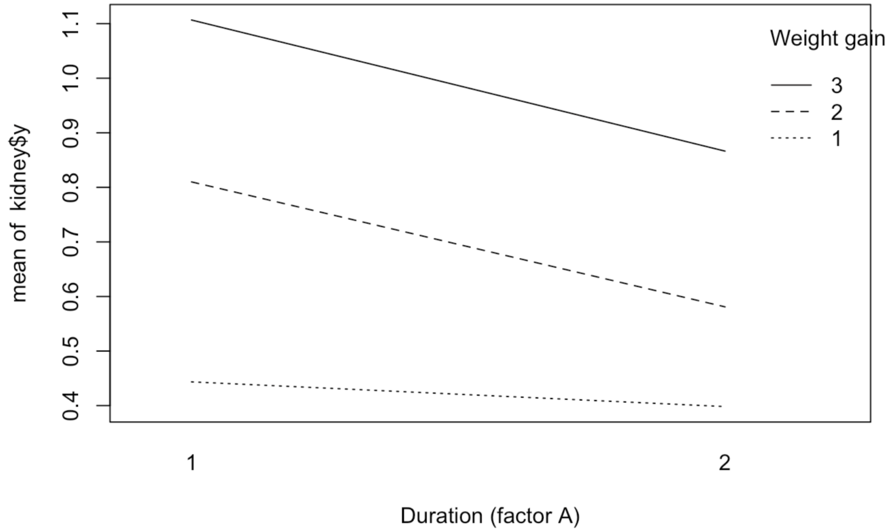
Test statistic: $F^* = \dfrac{MSB}{MSE} = 15.8$

Critical value $F(0.95,2,54) = 4.02$

Since the F test statistic of factor B is larger than the critical value, we reject the null hypothesis at 0.05 significance level and conclude that factor B weight gain as a result of fluid builup has significant effects on the days hosptalized.

Example

Do the results confirm your graphic analysis?



It confirms that the interaction effects we see in the graph might not be systematic and could just due to random chance along, so that the two factors do not interact.

But the factor A and B have significant effects on the response, separately.