Lecture 5: Two-Factor Studies with One Case per Treatment

STA 106: Analysis of Variance

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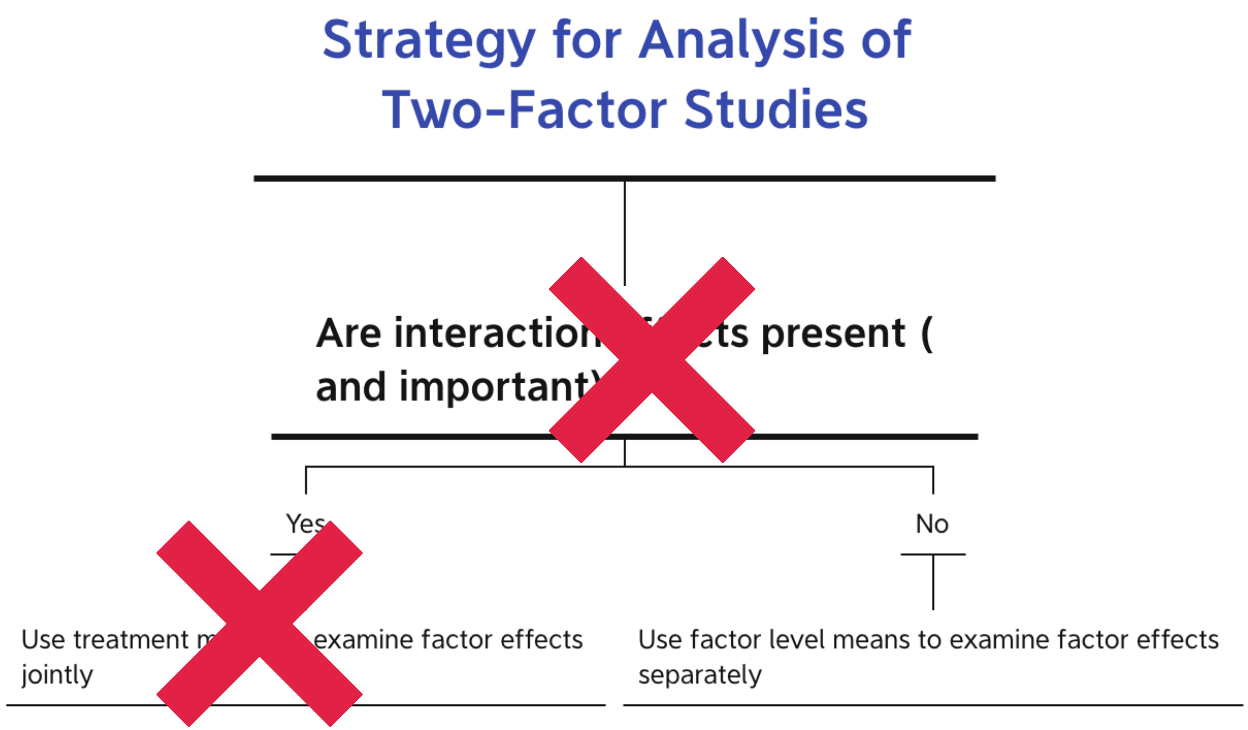
Two-Factor Studies with One Case per Treatment

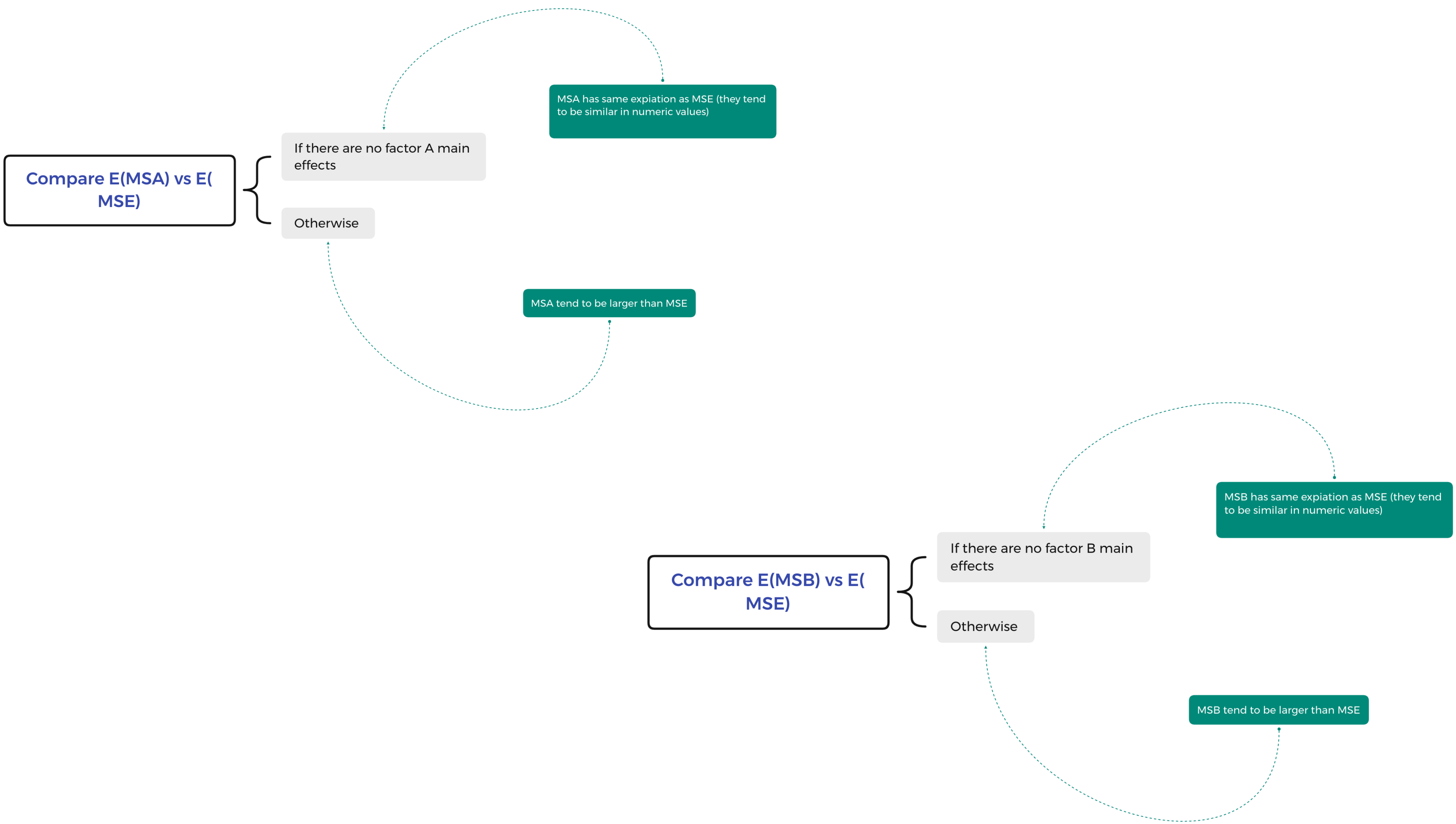
- Two-Way ANOVA Model without Interaction
- Analysis of Variance
- Test for Factor A and B Main Effects
- Multiple Comparison Procedures
- Tukey's test
- Randomized Complete Block Designs

Strategy of Analysis



Inference for two-factor studies with one-case per treatment is the same as general two-factor studies, except that $df(MSE) = (a-1)(b-1)$

Test for Factor A and Factor B Main Effects



They suggest that ratios of Mean Squares provide evidence about the main effects, which will be the basis for F tests

Test for Factor A and Factor B Main Effects

To test whether or not factor A main effects are present:

$$H_0 : \alpha_1 = \ldots = \alpha_a = 0$$

$$H_a : \text{not all } \alpha_i = 0$$

Test statistic: $F^* = \dfrac{MSA}{MSE}$

Decision rule:

If $F^* \leq F_{1-\alpha}(a-1,(a-1)(b-1))$, then conclude H_0

If $F^* > F_{1-\alpha}(a-1,(a-1)(b-1))$, then conclude H_a

To test whether or not factor B main effects are present:

$$H_0 : \beta_1 = \ldots = \beta_b = 0$$

$$H_a : \text{not all } \beta_j = 0$$

Test statistic: $F^* = \dfrac{MSB}{MSE}$

Decision rule:

If $F^* \leq F_{1-\alpha}(b-1,(a-1)(b-1))$, then conclude H_0

If $F^* > F_{1-\alpha}(b-1,(a-1)(b-1))$, then conclude H_a

Example

Conduct separate tests for size and region main effects. In each test, use level of significance $\alpha = .05$ and state the alternatives, decision rule, and conclusion.

To test the significance of Factor A main effect

$$H_0 : \alpha_i = 0, i = 1,2,3 \text{ vs } H_a : \text{ not all } \alpha_i \text{'s are } 0$$

Test statistic: $F^* = \dfrac{MSA}{MSE} = \dfrac{4650}{50} = 93$

Critical value $F(0.95,2,2) = 19$

Since the F test statistics of factor A is larger than the critical value, we reject the null hypothesis at 0.05 significance level and conclude that city size main effects are present.

To test the significance of Factor B main effect

$$H_0 : \beta_j = 0, j = 1,2 \text{ vs } H_a : \text{ not all } \beta_j' \text{s are } 0$$

Test statistic: $F^* = \dfrac{MSB}{MSE} = \dfrac{1350}{50} = 27$

Critical value $F(0.95,1,2) = 18.5$

Since the F test statistic of factor B is larger than the critical value, we reject the null hypothesis at 0.05 significance level and conclude that geographic region main effects are present.