STA13 Homework 3

October 16, 2021

1. Underage drinking Data collected by the Substance Abuse and Mental Health Services Administration (SAMSHA) suggests that 69.7% of 18-20 year olds consumed alcoholic beverages in any given year.

   1. Suppose a random sample of ten $18-20$ year olds is taken. Is the use of the binomial distribution appropriate for calculating the probability that exactly six consumed alcoholic beverages? Explain.

   2. Calculate the probability that exactly 6 out of 10 randomly sampled 18- 20 year olds consumed an alcoholic drink.

   3. What is the probability that exactly four out of ten $18-20$ year olds have not consumed an alcoholic beverage?

   4. What is the probability that at most 2 out of 5 randomly sampled 18-20 year olds have consumed alcoholic beverages?

   5. What is the probability that at least 1 out of 5 randomly sampled 18-20 year olds have consumed alcoholic beverages?

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2. Arachnophobia. A Gallup Poll found that 7% of teenagers (ages 13 to 17) suffer from arachnophobia and are extremely afraid of spiders. At a summer camp there are 10 teenagers sleeping in each tent. Assume that these 10 teenagers are independent of each other.

   1. Calculate the probability that at least one of them suffers from arachnophobia.

   2. Calculate the probability that exactly 2 of them suffer from arachnophobia.

   3. Calculate the probability that at most 1 of them suffers from arachnophobia.

   4. If the camp counselor wants to make sure no more than 1 teenager in each tent is afraid of spiders, does it seem reasonable for him to randomly assign teenagers to tents?

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3. Male children. While it is often assumed that the probabilities of having a boy or a girl are the same, the actual probability of having a boy is slightly higher at 0.51. Suppose a couple plans to have 3 kids.

1. Use the binomial model to calculate the probability that two of them will be boys.
2. Write out all possible orderings of 3 children, 2 of whom are boys. Use these scenarios to calculate the same probability from part (a) but using the addition rule for disjoint outcomes. Confirm that your answers from parts (a) and (b) match.
3. If we wanted to calculate the probability that a couple who plans to have 8 kids will have 3 boys, briefly describe why the approach from part (b) would be more tedious than the approach from part (1).

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4. Area under the curve What percent of a standard normal distribution $N(\mu = 0, \sigma = 1)$ is found in each region? Be sure to draw a graph.

1. $Z < -1.35$
2. $Z > 1.48$
3. $-0.4 < Z < 1.5$
4. $|Z| > 2$

5.CAPM. The Capital Asset Pricing Model (CAPM) is a financial model that assumes returns on a portfolio are normally distributed. Suppose a portfolio has an average annual return of 14.7% (i.e. an average gain of 14.7%) with a standard deviation of 33%. A return of 0% means the value of the portfolio doesn't change, a negative return means that the portfolio loses money, and a positive return means that the portfolio gains money.

1. What percent of years does this portfolio lose money, i.e. have a return less than 0% ?
2. What is the cutoff for the highest 15% of annual returns with this portfolio?