Topic 5: Hypothesis Testing

Optional Reading: Chapter 9

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- **Test about a population mean**
- **Test about a population proportion**
- **Test about the difference between two population means**
- **Test about the difference between two population proportions**

Hypothesis Testing a Population Proportion p

$p = 80\%$?

$p < 80\%$?

$p > 80\%$?

Summary: Hypothesis Testing about a Population Proportion

Step 1: null hypothesis
$H_0 : p = p_0$

Step 2: Alternative hypothesis

One-sided
$H_a : p > p_0$ or $H_a : p < p_0$

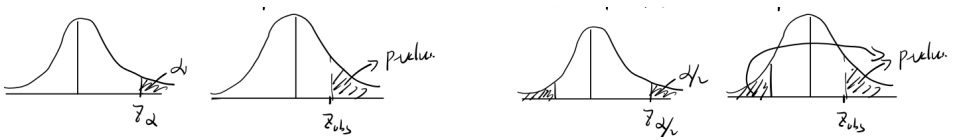
Two-sided
$H_a : p \neq p_0$

Step 3: calculate test statistic

$$Z_{obs} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Step 4: critical value or p-value approach

Critical value | P-value | Critical value | P-value



Step 5: Decision and Conclusion

Reject H_0 if:

$Z_{obs} \geq Z_\alpha$ | p-value $\leq \alpha$ | $|Z_{obs}| \geq Z_\alpha$ | p-value $\leq \alpha$

Overall 20% of American adults participate in fitness activities regularly.
However, does these fitness activities change as people getting older?

Survey:
n=100 adults over 40 years old
X=15 participate fitness activities regularly

Does the data indicate that participation rate for adults over 40 years old is less than 20%?
Use significance level 0.05.

Step 1 : null , alternative hypotheses

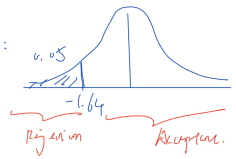
$H_0: p = 0.2$ vs $H_a: p < 0.2$

Step 2 : test statistic

$$Z_{obs} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{\frac{15}{100} - 0.2}{\sqrt{\frac{0.2 \times 0.8}{100}}} = -1.25$$

Step 3 :

critical value approach :



p-value approach : p-value $= P(Z \leq Z_{obs})$

$= P(Z \leq -1.25)$

$= 0.1056$

Step 4: conclusion and decision:

Critical value approach:

Since the observed test statistic falls in the acceptance region,
We do not reject H_0 and conclude that, the participation rate for adults over 40 is not significantly less than 20%.

P-value approach:

Since p-value =0.1056>0.05,
We do not reject H_0 and conclude that, the participation rate for adults over 40 is not significantly less than 20%.

Note: Observe some difference does not mean there is real difference, due to randomness and chance we could still see difference!

Early Detection of Breast Cancer Of those women who are diagnosed to have early-stage breast cancer, one-third eventually die of the disease. Suppose a community public health department instituted a screening program to provide for the early detection of breast cancer and to increase the survival rate p of those diagnosed to have the disease. A random sample of 200 women was selected from among those who were periodically screened by the program and who were diagnosed to have the disease. Let x represent the number of those in the sample who survive the disease.

1. If you wish to determine whether the community screening program has been effective, state the alternative hypothesis that should be tested.
2. State the null hypothesis.
3. If 164 women in the sample of 200 survive the disease, can you conclude that the community screening program was effective? Test using $\alpha = .05$ and explain the practical conclusions from your test.

Step 1 : null , alternative hypotheses

$H_0 : p = \frac{1}{3}$ vs $H_a : p > \frac{1}{3}$

Step 2 : test statistic

$$z_{obs} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{\frac{164}{200} - \frac{1}{3}}{\sqrt{\frac{\frac{1}{3}\cdot\frac{2}{3}}{200}}} = 4.6$$

Step 3 :

critical value approach :

$z_{0.05} = 1.64$

Acceptance region rejection region

p-value approach : p-value $= P(z \geq z_{obs})$

$= P(z \geq 4.6)$

≈ 0

Step 4: conclusion and decision:

Critical value approach:

Since the observed test statistic falls in the rejection region,
We reject H_0 and conclude that, the community screening program has been effective at reducing mortality due to breast cancer.

P-value approach:

Since p-value =0+ < 0.05,
We reject H_0 and conclude that, the community screening program has been effective at reducing mortality due to breast cancer.