Cho phương trình $8x^2 - 8x + m^2 + 1 = 0$.
Định m để phương trình có hai nghiệm thỏa $x_1^4 - x_2^4 = x_1^3 - x_2^3$.

$\Delta' = 4^2 - 8(m^2 + 1) = 8 - 8m^2$

Để phương trình có hai nghiệm thì

$\Delta' \geq 0 \Leftrightarrow 8 - 8m^2 \geq 0 \Leftrightarrow m^2 \leq 1 \Leftrightarrow -1 \leq m \leq 1$

Khi đó,

$* \ S = x_1 + x_2 = \dfrac{-(-8)}{2.8} = \dfrac{1}{2}$

$* \ P = x_1 x_2 = \dfrac{m^2 + 1}{8}$

Ta có:

$$x_1^4 - x_2^4 = x_1^3 - x_2^3$$
$$\Leftrightarrow (x_1^2 + x_2^2)(x_1^2 - x_2^2) = (x_1 - x_2)(x_1^2 + x_2^2 - x_1 x_2)$$
$$\Leftrightarrow (S^2 - 2P)(x_1 - x_2)(x_1 + x_2) = (x_1 - x_2)(S^2 - 3P)$$
$$\Leftrightarrow (S^3 - 2SP)(x_1 - x_2) - (S^2 - 3P)(x_1 - x_2) = 0$$
$$\Leftrightarrow (S^3 - S^2 - 2SP + 3P)(x_1 - x_2) = 0$$
$$\Leftrightarrow \left[\begin{array}{l} S^3 - S^2 - 2SP + 3P = 0 \\ x_1 - x_2 = 0 \end{array}\right.$$
$$\Leftrightarrow \left[\begin{array}{l} S^3 - S^2 + 2P = 0 \\ x_1 = x_2 \end{array}\right.$$
$$\Leftrightarrow \left[\begin{array}{l} \left(\dfrac{1}{2}\right)^3 - \left(\dfrac{1}{2}\right)^2 + 2\left(\dfrac{m^2 + 1}{8}\right) = 0 \\ \text{Phương trình có nghiệm kép} \end{array}\right.$$
$$\Leftrightarrow \left[\begin{array}{l} 2m^2 + 1 = 0 \\ \Delta' = 0 \end{array}\right.$$
$$\Leftrightarrow \left[\begin{array}{l} m \in \varnothing \\ 8 - 8m^2 = 0 \end{array}\right.$$
$$\Leftrightarrow \left[\begin{array}{l} m \in \varnothing \\ m = 1 \\ m = -1 \end{array}\right.$$
$$\Leftrightarrow \left[\begin{array}{l} m = 1 \\ m = -1 \end{array}\right.$$

Vậy $m \in \{-1; 1\}$ thỏa ycbt.

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