

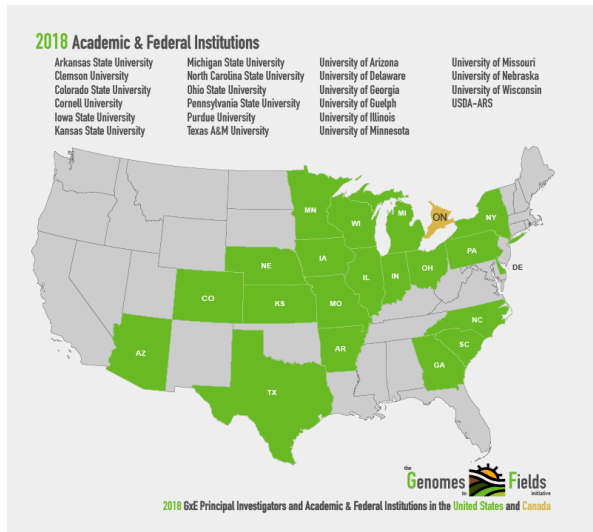
A Hierarchical Spatial Finlay-Wilkinson Model for Multi-Environment Trial Analysis

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The Genomes To Fields (G2F) Initiative



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Multi-Environment Field Trial Analysis for G2F Data

- Initially, we focus on a subset of 24 **environments**.
- We have **yield recorded for 10,971 field plots** with known **spatial locations**.
- A total of **1,105 hybrids** are planted in approximately 10 plots on average.
- Hybrid **genotypes at $\sim 1\text{M}$ genomic locations** are available.
- To characterize environments, weather stations provide **time-indexed measurements for 10 weather variables**, and several **soil variables** are available.

Finlay-Wilkinson (FW) Model

- FW model:

$$y_{ijk} = \mu + g_i + h_j + b_i h_j + e_{ijk},$$

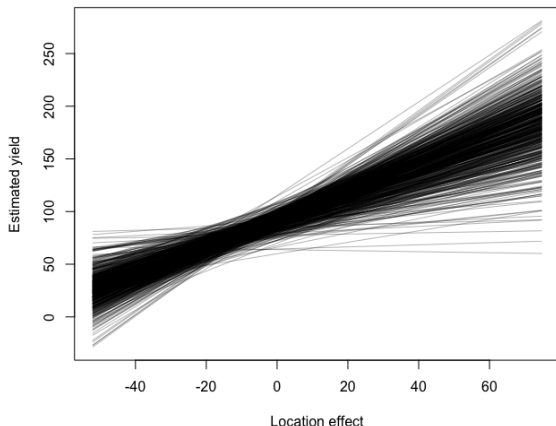
- A model for characterizing genotype-by-environment interaction.
- For genotype i :

$$y_{ijk} = (\mu + g_i) + (b_i + 1)h_j + e_{ijk},$$

FW model becomes a linear model with intercept $\mu + g_i$ and slope $b_i + 1$.

Finlay-Wilkinson (FW) Model

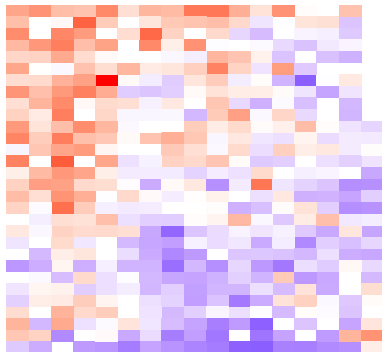
This plot gives one example of the fitted FW model (using G2F data). Each line represents the linear relationship between grain yield and location effect for one genotype.



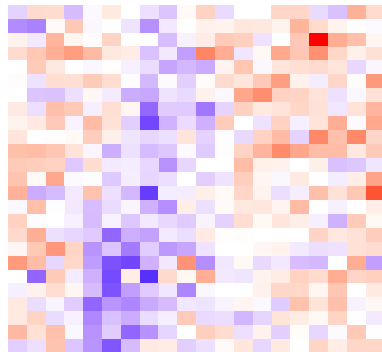
Residuals of FW Model for Two Fields

Problem: the residuals are **highly spatially correlated**.

MOH1



WIH2



Hierarchical Spatial Finlay-Wilkinson (SFW) Model

- Data model:

$$[y_{ijk} | \mu, \mathbf{g}, \mathbf{b}, \mathbf{h}, \phi] \stackrel{\text{indep}}{\sim} \mathcal{N}(\mu + g_i + h_j + b_i h_j + \phi_{ijk}, \sigma_e^2),$$

- Prior distributions for genotype, slope, and field effects:

$$[\mathbf{g}] \sim \mathcal{N}(\mathbf{0}, \mathbf{A}\sigma_g^2); \quad [\mathbf{b}] \sim \mathcal{N}(\mathbf{0}, \mathbf{A}\sigma_b^2);$$

$$[\mathbf{h} | \gamma] \sim \mathcal{N}(\gamma_1 \mathbf{Z}_1 + \cdots + \gamma_I \mathbf{Z}_I + \cdots + \gamma_L \mathbf{Z}_L, \mathbf{I}\sigma_h^2)$$

\mathbf{A} is a known matrix describing the correlation structure of \mathbf{g} and \mathbf{b} , \mathbf{Z}_l is the l th environment-specific covariate.

Intrinsic Autoregression Model for Spatial Effects

- A popular model for fertility adjustment in agricultural field trials is the **first order intrinsic autoregression** (Besag and Higdon, 1999; Dutta and Mondal, 2015).
- First order Intrinsic Autoregressive prior:

$$[\psi_j | \theta_j, \sigma_j^2] \propto |\sigma_j^{-2} \mathbf{W}_j|_+^{1/2} \exp \left(-\frac{1}{2} \sigma_j^{-2} \psi_j \mathbf{W}_j \psi_j \right)$$

where

$$\psi_j \mathbf{W}_j \psi_j = \theta_j \sum \sum (\psi_{u,v} - \psi_{u-1,v})^2 + \bar{\theta}_j \sum \sum (\psi_{u,v} - \psi_{u,v-1})^2$$

- The distribution of ψ_j is **invariant** to the addition of arbitrary constant.

Intrinsic Autoregression Model for Spatial Effects

- The intrinsic model has an indeterminate overall level.
- The overall levels of the field specific spatial effects are completely confounded with the location effects.
- Not directly applicable for yield prediction.
- **A hard constraint:** set the average of the spatial effects to zero.

Projected Intrinsic Autoregression (PIAR) Prior

- The Gaussian projected intrinsic autoregression (PIAR) on the $r_j \times c_j$ regular array is then defined as:

$$\phi_j = \mathbf{B}_j \varphi_j, \quad \varphi_j \sim \mathcal{N}(\mathbf{0}, \mathbf{D}_j^{-1}),$$

- \mathbf{B}_j is an $r_j c_j \times (r_j c_j - 1)$ matrix.
- \mathbf{D}_j is an $(r_j c_j - 1) \times (r_j c_j - 1)$ diagonal matrix.

Projected Intrinsic Autoregression (PIAR) Prior

- Then the spectral decomposition of \mathbf{W}_j is given by:

$$(\mathbf{N}_{r_j} \otimes \mathbf{N}_{c_j}) \mathbf{W}_j (\mathbf{N}_{r_j}^T \otimes \mathbf{N}_{c_j}^T) = \theta_j \mathbf{\Lambda}_{r_j} \otimes \mathbf{I}_{c_j} + \bar{\theta}_j \mathbf{I}_{r_j} \otimes \mathbf{\Lambda}_{c_j}.$$

- $\mathbf{\Lambda}_k$ denote the $k \times k$ diagonal matrix whose u th diagonal entry is $4 \sin^2\{\pi(u-1)/(2k)\}$.
- \mathbf{N}_k denotes the $k \times k$ orthogonal matrix whose (u, v) th entry is $1/\sqrt{k}$ if $u = 1, \forall v$, and $(2/k)^{1/2} \cos\{\pi(u-1)(v-1/2)/k\}$ otherwise.
- \mathbf{B}_j^T denotes the $(r_j c_j - 1) \times r_j c_j$ matrix consisting of last $r_j c_j - 1$ rows of $\mathbf{N}_{r_j} \otimes \mathbf{N}_{c_j}$.
- \mathbf{D}_j denotes the diagonal matrix consisting of the nonzero elements of $\theta_j \mathbf{\Lambda}_{r_j} \otimes \mathbf{I}_{c_j} + \bar{\theta}_j \mathbf{I}_{r_j} \otimes \mathbf{\Lambda}_{c_j}$.

Matrix Free Computation

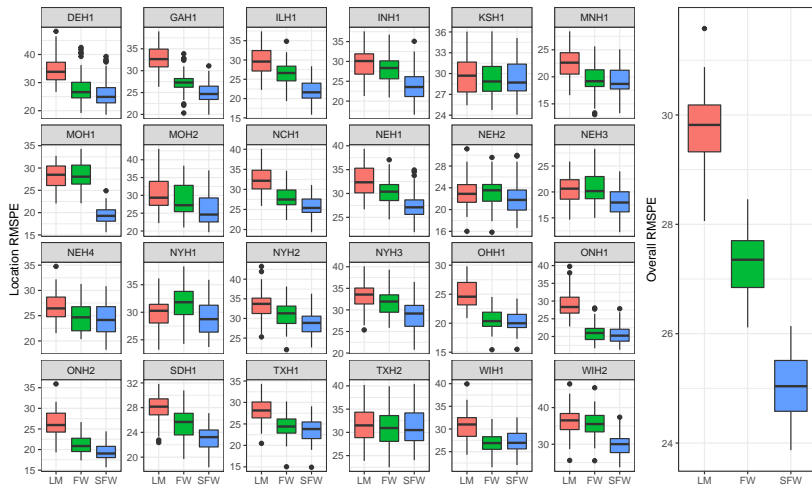
- The covariance matrix of the Gaussian PIAR is a **dense** singular matrix.
- The computation load for generating ϕ_j from PIAR using knowledge of multivariate statistics is $\mathcal{O}((r_j c_j)^3/3)$.
- Assume **small number of missing plots** (denote $r_j c_j - N_j$ as the number of missing plots).
- Thus matrix-vector multiplications with \mathbf{B}_j and \mathbf{B}_j^T can also be performed using these **discrete cosine transformations (DCT)**.
- The computation load of our proposed algorithm is $\mathcal{O}(r_j c_j + (r_j c_j - N_j) r_j c_j \log r_j c_j + (r_j c_j - N_j)^3/3)$.

Within-fields prediction

- Predict yield for plots in a field using information from other plots in the same field.
- Implement **posterior predictive distributions**.
- Important to account for the spatial correlation between plots.
- Kinship information plays a decisive role for an accurate prediction.
- Mainly used for **model evaluation**.

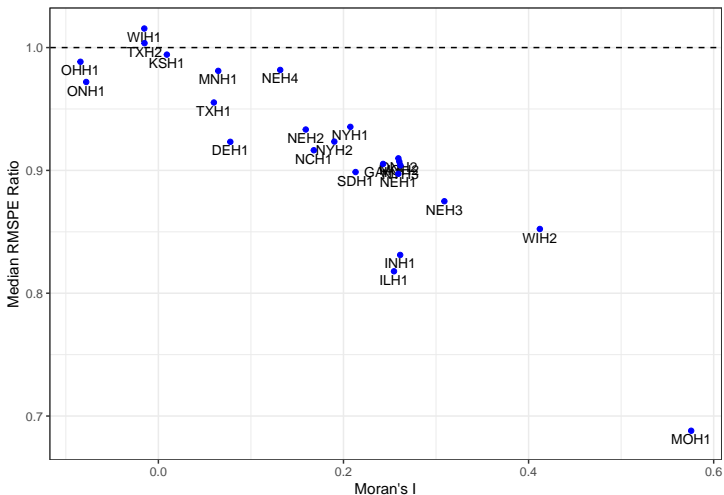
Model Evaluation via Prediction

Reduced error for yield prediction.



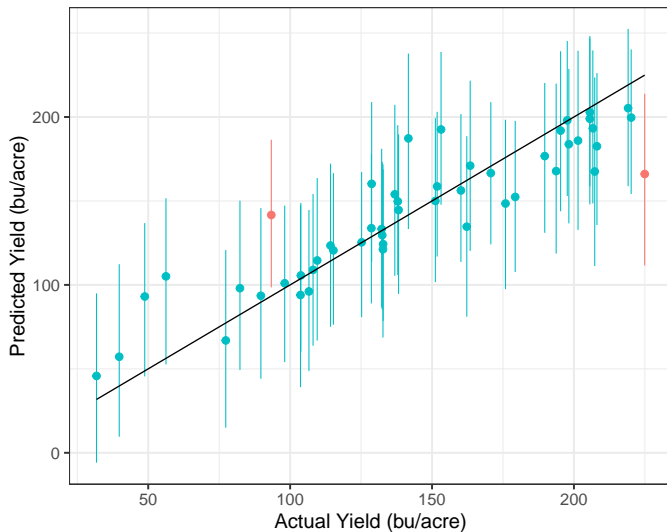
Model Evaluation via Prediction

Level of spatial correlation vs performance of SFW model.



Prediction Intervals

50 plot yield prediction intervals (95% credible level).

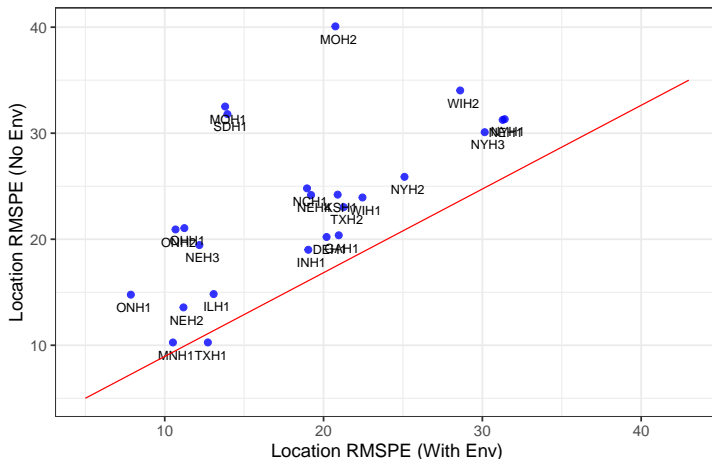


Predict in New Environments

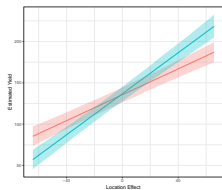
- Need to learn how field effects depend on the weather and soil variables.
- The better we explain the environment effects using environmental covariates, the higher prediction accuracy we will gain.
- Collecting more elaborate field and weather variables will lead to a more accurate and robust prediction.

Predict in New Environments

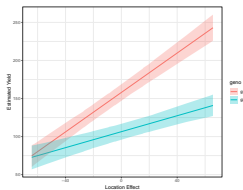
Location-wise RMSPEs computed using temperature and rainfall data (x-axis), versus the location-wise RMSPEs computed not using any environment information (y-axis).



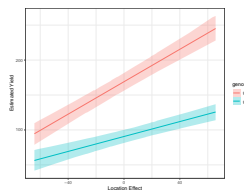
Assessing Uncertainty about FW Regression Lines



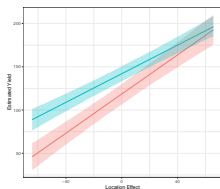
(a)



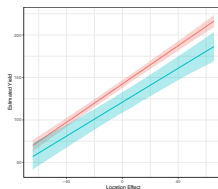
(b)



(c)



(d)



(e)

Figure: Estimated Yield vs Location Effect for pairs of genotypes

Selected References

- Besag, J. and Higdon, D. (1999). Bayesian analysis of agricultural field experiments. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 61(4):691–746.
- Dutta, S. and Mondal, D. (2015). An h-likelihood method for spatial mixed linear models based on intrinsic auto-regressions. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 77(3):699–726.

Acknowledgements

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Any opinions, findings, conclusions, or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the U.S. Department of Agriculture.

Thank You!