

# A Hierarchical Spatial Finlay-Wilkinson Model for Analysis of Multi-Environment Field Trials

Xingche Guo, Somak Dutta, Dan Nettleton

Dept. of Statistics, Iowa State University

Joint Statistical Meetings, 2019

# The Genomes to Fields (G2F) Initiative



copyright: <https://www.genomes2fields.org>

# Multi-Environment Field Trial Analysis for G2F Data

- Initially, we focus on a subset of 24 **environments**.
- We have **yield recorded for 10,971 field plots** with known **spatial locations**.
- A total of **1,105 hybrids** are planted in approximately 10 plots on average.
- Hybrid **genotypes at  $\sim 1\text{M}$  genomic locations** are available.
- To characterize environments, weather stations provide **time-indexed measurements for 10 weather variables**, and several **soil variables** are available.

# Finlay-Wilkinson (FW) Model

- Finlay-Wilkinson (FW) model (Finlay and Wilkinson, 1963):

$$y_{ijk} = \mu + g_i + h_j + b_i h_j + e_{ijk},$$

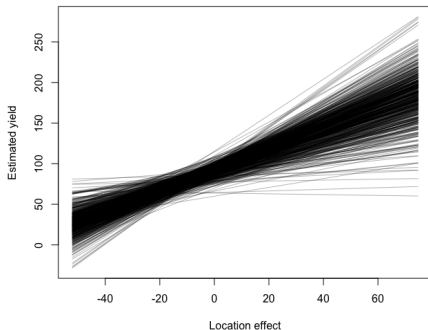
- For genotype  $i$ , FW model becomes a **linear model** with intercept  $\mu + g_i$  and slope  $b_i + 1$ .

# Finlay-Wilkinson (FW) Model

- Finlay-Wilkinson (FW) model (Finlay and Wilkinson, 1963):

$$y_{ijk} = \mu + g_i + h_j + b_i h_j + e_{ijk},$$

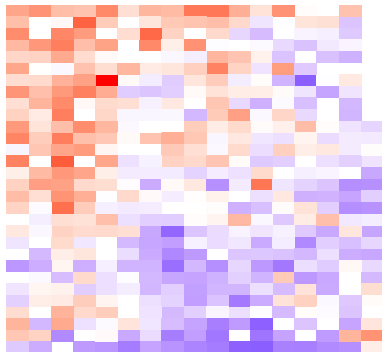
- For genotype  $i$ , FW model becomes a **linear model** with intercept  $\mu + g_i$  and slope  $b_i + 1$ .



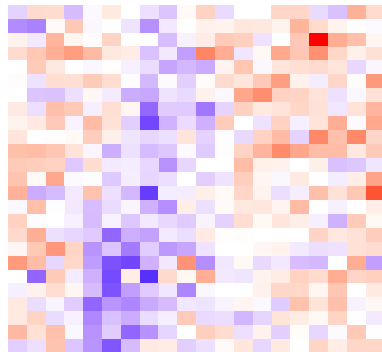
# Residuals of FW Model for Two Fields

**Problem:** the residuals are **highly spatially correlated**.

MOH1



WIH2



# Hierarchical Spatial Finlay-Wilkinson (SFW) Model

- Data model:

$$[y_{ijk} | \mu, \mathbf{g}, \mathbf{b}, \mathbf{h}, \phi] \stackrel{\text{indep}}{\sim} \mathcal{N}(\mu + g_i + h_j + b_i h_j + \phi_{ijk}, \sigma_e^2),$$

- Prior distributions for genotype, slope, and field effects:

$$[\mathbf{g}] \sim \mathcal{N}(\mathbf{0}, \mathbf{A}\sigma_g^2); \quad [\mathbf{b}] \sim \mathcal{N}(\mathbf{0}, \mathbf{A}\sigma_b^2);$$

$$[\mathbf{h} | \gamma] \sim \mathcal{N}(\gamma_1 \mathbf{Z}_1 + \cdots + \gamma_I \mathbf{Z}_I + \cdots + \gamma_L \mathbf{Z}_L, \mathbf{I}\sigma_h^2).$$

- $\mathbf{A}$  is the kinship matrix describing the correlation structure of  $\mathbf{g}$  and  $\mathbf{b}$  (R package/software: rrBLUP, Tassel 5).
- $\mathbf{Z}_l$  is the  $l$ th standardized environmental covariate.

# Intrinsic Autoregression Model for Spatial Effects

- A popular model for fertility adjustment in agricultural field trials is the **first order intrinsic autoregression** (Besag and Higdon, 1999; Dutta and Mondal, 2015).
- First order Intrinsic Autoregressive prior:

$$[\psi_j | \theta_j, \sigma_j^2] \propto |\sigma_j^{-2} \mathbf{W}_j|_+^{1/2} \exp \left( -\frac{1}{2} \sigma_j^{-2} \psi_j \mathbf{W}_j \psi_j \right)$$

where

$$\psi_j \mathbf{W}_j \psi_j = \theta_j \sum \sum (\psi_{u,v} - \psi_{u-1,v})^2 + \bar{\theta}_j \sum \sum (\psi_{u,v} - \psi_{u,v-1})^2$$

- The distribution of  $\psi_j$  is **invariant** to the addition of arbitrary constant.



# Intrinsic Autoregression Model for Spatial Effects

Recall:

- $[y_{ijk} | \mu, \mathbf{g}, \mathbf{b}, \mathbf{h}, \phi] \stackrel{\text{indep}}{\sim} \mathcal{N}(\mu + g_i + h_j + b_i h_j + \phi_{ijk}, \sigma_e^2),$

# Intrinsic Autoregression Model for Spatial Effects

## Recall:

$$\bullet [y_{ijk} | \mu, \mathbf{g}, \mathbf{b}, \mathbf{h}, \phi] \stackrel{\text{indep}}{\sim} \mathcal{N}(\mu + g_i + h_j + b_i h_j + \phi_{ijk}, \sigma_e^2),$$

## Problem:

- The intrinsic spatial prior has an **indeterminate overall level**.
- The overall levels of **spatial effects** are **confounded with the location effects**.
- Estimation of **b** is biased.
- Hierarchical structure of **h** is not applicable.

# Intrinsic Autoregression Model for Spatial Effects

## Recall:

$$\bullet [y_{ijk} | \mu, \mathbf{g}, \mathbf{b}, \mathbf{h}, \phi] \stackrel{\text{indep}}{\sim} \mathcal{N}(\mu + g_i + h_j + b_i h_j + \phi_{ijk}, \sigma_e^2),$$

## Problem:

- The intrinsic spatial prior has an **indeterminate overall level**.
- The overall levels of **spatial effects** are **confounded with the location effects**.
- Estimation of **b** is biased.
- Hierarchical structure of **h** is not applicable.

## Solution:

- **A hard constraint:** set the average of the spatial effects to zero.

# Projected Intrinsic Autoregression (PIAR) Prior

- The Gaussian projected intrinsic autoregression (PIAR) on the  $r_j \times c_j$  regular array is then defined as:

$$\phi_j = \mathbf{B}_j \varphi_j, \quad \varphi_j \sim \mathcal{N}(\mathbf{0}, \mathbf{D}_j^{-1}),$$

- $\mathbf{B}_j$  is an  $r_j c_j \times (r_j c_j - 1)$  matrix.
- $\mathbf{D}_j$  is an  $(r_j c_j - 1) \times (r_j c_j - 1)$  diagonal matrix.

# Matrix Free Computation

- The covariance matrix of the Gaussian PIAR is a **dense** singular matrix.
- The computation load for generating  $\phi_j$  from PIAR using knowledge of multivariate statistics is  $\mathcal{O}((r_j c_j)^3/3)$ .
- Assume **small number of missing plots** (denote  $r_j c_j - N_j$  as the number of missing plots).
- Thus matrix-vector multiplications with  $\mathbf{B}_j$  and  $\mathbf{B}_j^T$  can also be performed using these **discrete cosine transformations (DCT)**.
- The computation load of our proposed algorithm is  $\mathcal{O}(r_j c_j + (r_j c_j - N_j) r_j c_j \log r_j c_j + (r_j c_j - N_j)^3/3)$ .

# Prediction

- Implement **posterior predictive distributions**.
- Easy to obtain predictive credible intervals.

# Prediction

- Implement **posterior predictive distributions**.
- Easy to obtain predictive credible intervals.

## Within-field prediction:

- Important to account for the spatial correlation between plots.
- Kinship information plays a decisive role for an accurate prediction.
- Mainly used for **model evaluation**.

# Prediction

- Implement **posterior predictive distributions**.
- Easy to obtain predictive credible intervals.

## Within-field prediction:

- Important to account for the spatial correlation between plots.
- Kinship information plays a decisive role for an accurate prediction.
- Mainly used for **model evaluation**.

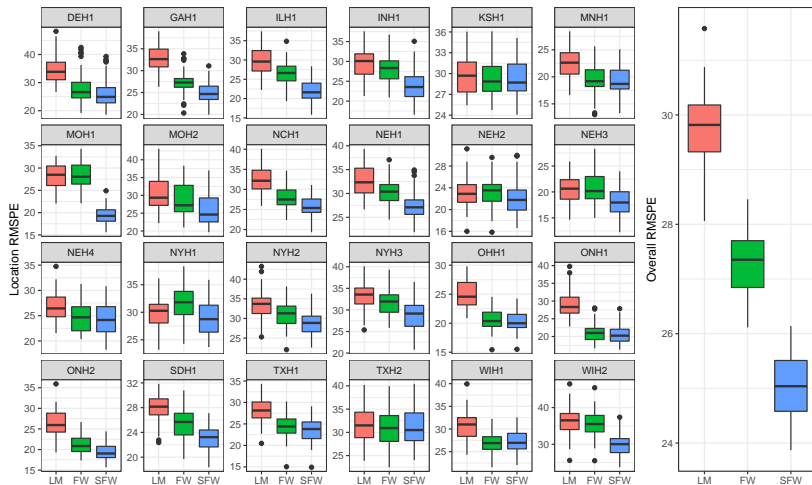
## Predict in new environments:

- By learning how environment effects depend on the weather and soil variables.



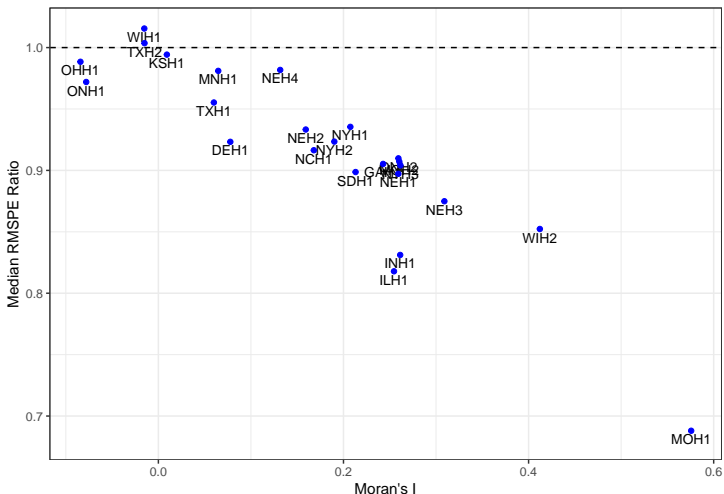
# Model Evaluation via Within-Field Prediction

Reduced error for yield prediction.



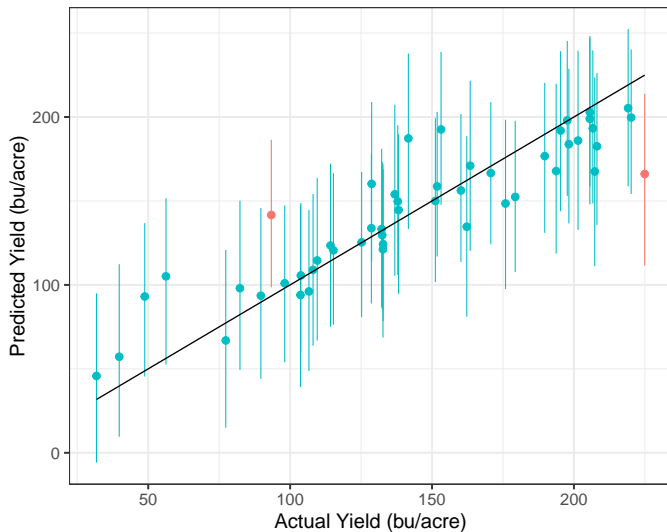
# Model Evaluation via Within-Field Prediction

Level of spatial correlation vs performance of SFW model.



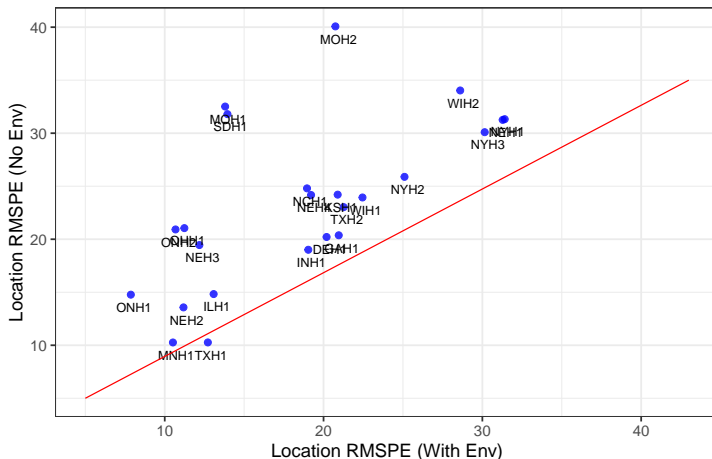
# Prediction Intervals

50 plot yield prediction intervals (95% credible level).



# Predict in New Environments

Location-wise RMSPEs computed using temperature and rainfall data (x-axis), versus the location-wise RMSPEs computed not using any environment information (y-axis).



# Selected References

- Besag, J. and Higdon, D. (1999). Bayesian analysis of agricultural field experiments. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 61(4):691–746.
- Dutta, S. and Mondal, D. (2015). An h-likelihood method for spatial mixed linear models based on intrinsic auto-regressions. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 77(3):699–726.
- Finlay, K. and Wilkinson, G. (1963). The analysis of adaptation in a plant-breeding programme. Australian Journal of Agricultural Research, 14(6):742–754.

# Acknowledgements

The authors acknowledge financial support of Iowa State University Plant Sciences Institute Scholars Program, the Baker Center for Bioinformatics and Biological Statistics, and the Iowa Agriculture and Home Economics Experiment Station, Ames, Iowa, Project No. IOW03617, which is supported by USDA/NIFA and State of Iowa funds.

Any opinions, findings, conclusions, or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the U.S. Department of Agriculture.

# Thank You!

# Construction of $\mathbf{B}_j$ and $\mathbf{D}_j$

- Then the spectral decomposition of  $\mathbf{W}_j$  is given by:

$$(\mathbf{N}_{r_j} \otimes \mathbf{N}_{c_j}) \mathbf{W}_j (\mathbf{N}_{r_j}^T \otimes \mathbf{N}_{c_j}^T) = \theta_j \mathbf{\Lambda}_{r_j} \otimes \mathbf{I}_{c_j} + \bar{\theta}_j \mathbf{I}_{r_j} \otimes \mathbf{\Lambda}_{c_j}.$$

- $\mathbf{\Lambda}_k$  denote the  $k \times k$  diagonal matrix whose  $u$ th diagonal entry is  $4 \sin^2\{\pi(u-1)/(2k)\}$ .
- $\mathbf{N}_k$  denotes the  $k \times k$  orthogonal matrix whose  $(u, v)$ th entry is  $1/\sqrt{k}$  if  $u = 1, \forall v$ , and  $(2/k)^{1/2} \cos\{\pi(u-1)(v-1/2)/k\}$  otherwise.
- $\mathbf{B}_j^T$  denotes the  $(r_j c_j - 1) \times r_j c_j$  matrix consisting of last  $r_j c_j - 1$  rows of  $\mathbf{N}_{r_j} \otimes \mathbf{N}_{c_j}$ .
- $\mathbf{D}_j$  denotes the diagonal matrix consisting of the nonzero elements of  $\theta_j \mathbf{\Lambda}_{r_j} \otimes \mathbf{I}_{c_j} + \bar{\theta}_j \mathbf{I}_{r_j} \otimes \mathbf{\Lambda}_{c_j}$ .



# Assessing Uncertainty about FW Regression Lines

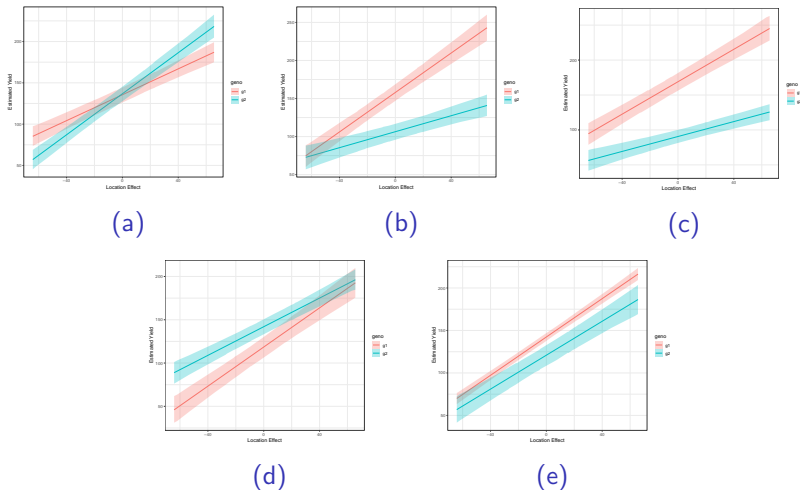


Figure: Estimated Yield vs Location Effect for pairs of genotypes