A Hierarchical Spatial Finlay-Wilkinson Model for Analysis of Multi-Environment Field Trials

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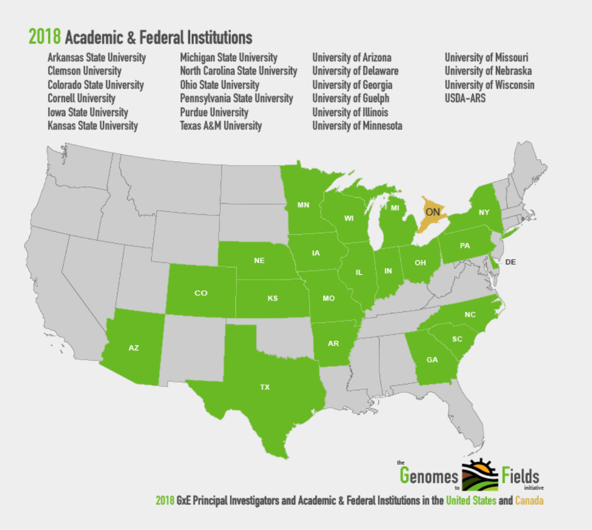
The Genomes to Fields (G2F) Initiative



Multi-Environment Field Trial Analysis for G2F Data

- Initially, we focus on a subset of 24 **environments**.
- We have **yield recorded for 10,971 field plots** with known **spatial locations**.
- A total of **1,105 hybrids** are planted in approximately 10 plots on average.
- Hybrid **genotypes at ∼1M genomic locations** are available.
- To characterize environments, weather stations provide **time-indexed measurements for 10 weather variables**, and several **soil variables** are available.

Finlay-Wilkinson (FW) Model

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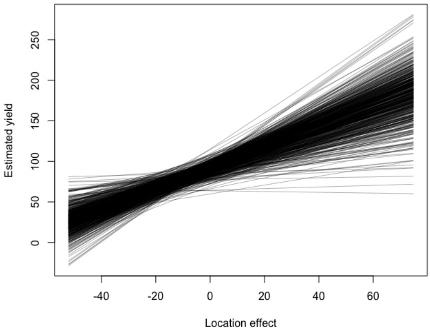
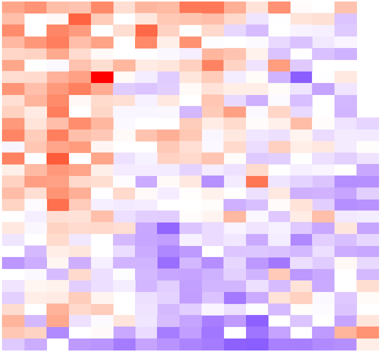


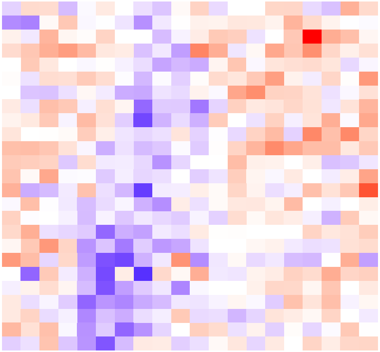
Residuals of FW Model for Two Fields

Problem: the residuals are **highly spatially correlated**.

MOH1



WIH2



Hierarchical Spatial Finlay-Wilkinson (SFW) Model

- Data model:

$$[y_{ijk}|\mu, \mathbf{g}, \mathbf{b}, \mathbf{h}, \phi] \overset{indep}{\sim} \mathcal{N}(\mu + g_i + h_j + b_i h_j + \phi_{ijk}, \sigma_e^2),$$

- Prior distributions for genotype, slope, and field effects:

$$[\mathbf{g}] \sim \mathcal{N}(\mathbf{0}, \mathbf{A}\sigma_g^2); \quad [\mathbf{b}] \sim \mathcal{N}(\mathbf{0}, \mathbf{A}\sigma_b^2);$$

$$[\mathbf{h}|\gamma] \sim \mathrm{N}(\gamma_1 \mathbf{Z}_1 + \cdots + \gamma_l \mathbf{Z}_l + \cdots + \gamma_L \mathbf{Z}_L, \mathbf{I}\sigma_h^2).$$

- **A** is the kinship matrix describing the correlation structure of **g** and **b** (R package/software: rrBLUP, Tassel 5).

- $\mathbf{Z}_l$ is the lth standardized environmental covariate.

Intrinsic Autoregression Model for Spatial Effects

- A popular model for fertility adjustment in agricultural field trials is the **first order intrinsic autoregression** (Besag and Higdon, 1999; Dutta and Mondal, 2015).

- First order Intrinsic Autoregressive prior:

$$[\psi_j|\theta_j, \sigma_j^2] \propto |\sigma_j^{-2}\mathbf{W}_j|_+^{1/2} \exp\left(-\tfrac{1}{2}\sigma_j^{-2}\boldsymbol{\psi}_j\mathbf{W}_j\boldsymbol{\psi}_j\right)$$

  where

$$\boldsymbol{\psi}_j\mathbf{W}_j\boldsymbol{\psi}_j = \theta_j \sum\sum(\psi_{u,v} - \psi_{u-1,v})^2 + \bar{\theta}_j \sum\sum(\psi_{u,v} - \psi_{u,v-1})^2$$

- The distribution of $\boldsymbol{\psi}_j$ is **invariant** to the addition of arbitrary constant.

Intrinsic Autoregression Model for Spatial Effects

Recall:

- $[y_{ijk}|\mu, \mathbf{g}, \mathbf{b}, \mathbf{h}, \phi] \overset{indep}{\sim} \mathcal{N}(\mu + g_i + h_j + b_i h_j + \phi_{ijk}, \sigma_e^2),$

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Problem:

- The intrinsic spatial prior has an **indeterminate overall level**.
- The overall levels of **spatial effects** are **confounded with** the **location effects**.
- Estimation of **b** is biased.
- Hierarchical structure of **h** is not applicable.

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Solution:

- **A hard constraint**: set the average of the spatial effects to zero.

Projected Intrinsic Autoregression (PIAR) Prior

- The Gaussian projected intrinsic autoregression (PIAR) on the $r_j \times c_j$ regular array is then defined as:

$$\phi_j = \mathbf{B}_j \varphi_j, \qquad \varphi_j \sim \mathcal{N}(\mathbf{0}, \mathbf{D}_j^{-1}),$$

- $\mathbf{B}_j$ is an $r_j c_j \times (r_j c_j - 1)$ matrix.
- $\mathbf{D}_j$ is an $(r_j c_j - 1) \times (r_j c_j - 1)$ diagonal matrix.
- We can show: $\varphi_j = \mathbf{B}_j^{\mathrm{T}} \phi_j$.

Matrix Free Computation

- The covariance matrix of the Gaussian PIAR is a **dense** singular matrix.
- The computation load for generating ϕ_j from PIAR using knowledge of multivariate statistics is $\mathcal{O}((r_j c_j)^3/3)$.
- Assume **small number of missing plots** (denote $r_j c_j - N_j$ as the number of missing plots).
- Thus matrix-vector multiplications with $\mathbf{B}_j$ and $\mathbf{B}_j^{\mathrm{T}}$ can also be performed using these **discrete cosine transformations (DCT)**.
- The computation load of our proposed algorithm is $\mathcal{O}(r_j c_j + (r_j c_j - N_j) r_j c_j \log r_j c_j + (r_j c_j - N_j)^3/3)$.

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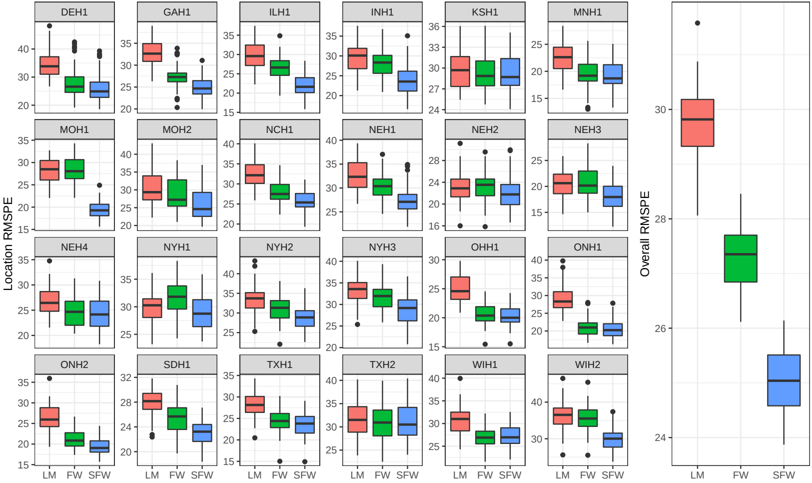
Predict in new environments:

- By learning how environment effects depend on the weather and soil variables.

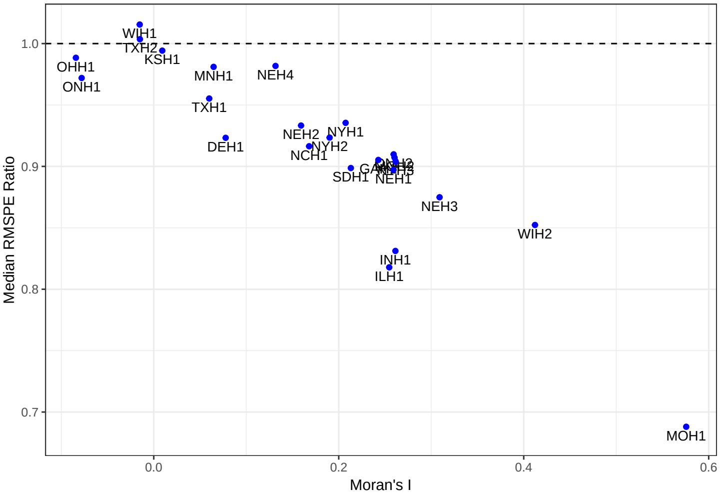
Model Evaluation via Within-Field Prediction

Reduced error for yield prediction.



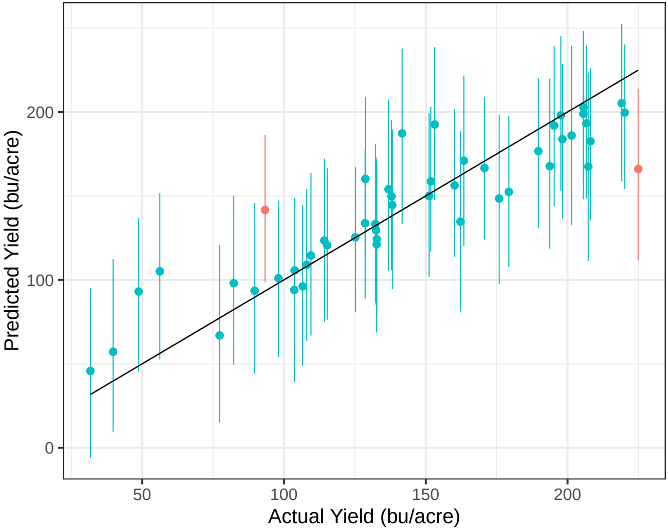
Model Evaluation via Within-Field Prediction

Level of spatial correlation vs performance of SFW model.



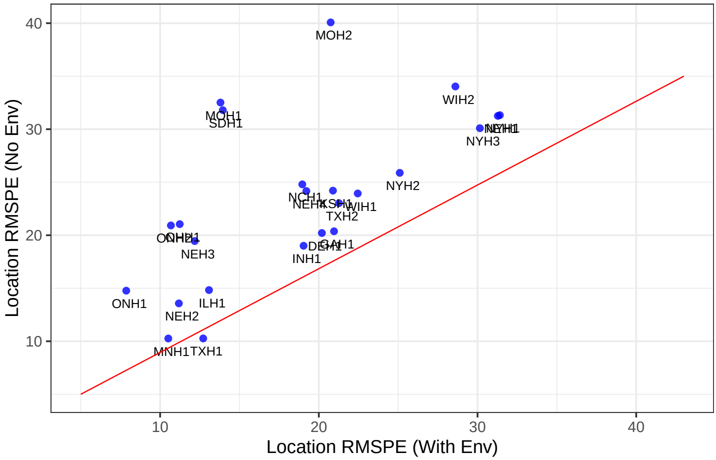
Prediction Intervals

50 plot yield prediction intervals (95% credible level).



Predict in New Environments

Location-wise RMSPEs computed using temperature and rainfall data (x-axis), versus the location-wise RMSPEs computed not using any environment information (y-axis).



Selected References

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Acknowledgements

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Any opinions, findings, conclusions, or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the U.S. Department of Agriculture.

Thank You!

Construction of $\mathbf{B}_j$ and $\mathbf{D}_j$

- Then the spectral decomposition of $\mathbf{W}_j$ is given by:

$$(\mathbf{N}_{r_j} \otimes \mathbf{N}_{c_j})\mathbf{W}_j(\mathbf{N}_{r_j}^{\mathrm{T}} \otimes \mathbf{N}_{c_j}^{\mathrm{T}}) = \theta_j \boldsymbol{\Lambda}_{r_j} \otimes \mathbf{I}_{c_j} + \bar{\theta}_j \mathbf{I}_{r_j} \otimes \boldsymbol{\Lambda}_{c_j}.$$

- $\boldsymbol{\Lambda}_k$ denote the $k \times k$ diagonal matrix whose uth diagonal entry is $4\sin^2\{\pi(u-1)/(2k)\}$.
- $\mathbf{N}_k$ denotes the $k \times k$ orthogonal matrix whose (u, v)th entry is $1/\sqrt{k}$ if $u = 1$, $\forall v$, and $(2/k)^{1/2}\cos\{\pi(u-1)(v-1/2)/k\}$ otherwise.
- $\mathbf{B}_j^{\mathrm{T}}$ denotes the $(r_j c_j - 1) \times r_j c_j$ matrix consisting of last $r_j c_j - 1$ rows of $\mathbf{N}_{r_j} \otimes \mathbf{N}_{c_j}$.
- $\mathbf{D}_j$ denotes the diagonal matrix consisting of the nonzero elements of $\theta_j \boldsymbol{\Lambda}_{r_j} \otimes \mathbf{I}_{c_j} + \bar{\theta}_j \mathbf{I}_{r_j} \otimes \boldsymbol{\Lambda}_{c_j}$.

Assessing Uncertainty about FW Regression Lines

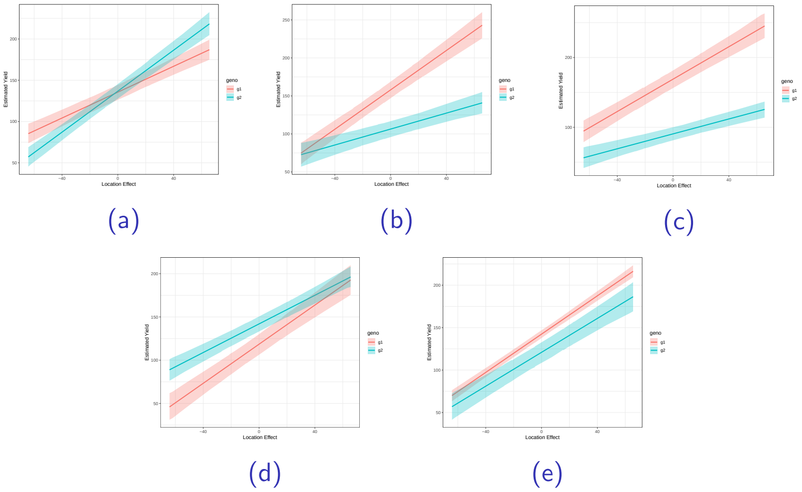

Figure: Estimated Yield vs Location Effect for pairs of genotypes