Homework 1

STAT 547, Fall 2019

You are encouraged to discuss the homework questions with classmates or the instructor, but you must write and submit your individual copy. Please write down the name of the persons with whom you discussed the homework, and submit your homework in a pdf file through Canvas.

1. Exercise 1.1 (b) & (c), Kokoszka and Riemherr (2017).
2. Exercise 1.4, Kokoszka and Riemherr (2017). The datasets in the book can be found here: `http://www.personal.psu.edu/mlr36/Documents/KRBook_DataSets.zip`
3. For this and the next question, consider a multivariate random variable $X \in \mathbb{R}^d$, $d \geq 2$. Find the projection directions v_k, $k = 1, \ldots, d$ for the principal component analysis obtained in the following stepwise fashion:

$$v_1 = \underset{\|l_1\|=1}{\arg\max}\ \mathrm{Var}(l_1^\intercal X)$$

$$v_k = \underset{\substack{\|l_k\|=1 \\ l_k^\intercal l_j=0, \\ j=1,\ldots,k-1}}{\arg\max}\ \mathrm{Var}(l_k^\intercal X), \quad k = 2, \ldots, d.$$

4. Let $\xi_k = v_k^\intercal (X - \mu)$, $k = 1, \ldots, d$, where $\mu = E(X)$. Then
   (a) $E(\xi_k) = 0$
   (b) $\mathrm{Var}(\xi_k) = \lambda_k$
   (c) $\mathrm{Cov}(\xi_j, \xi_k) = \lambda_k \delta_{jk}$, where $\delta_{jk} = 1$ if $j = k$ and 0 otherwise.
   (d) $\mathrm{Corr}(X_j, \xi_k) = \sqrt{\lambda_k} v_{jk} / \sqrt{\sigma_{jj}}$, where X_j and v_{jk} are the jth entry of X and v_j, respectively, and σ_{jj} is the jth diagonal entry of $\Sigma = \mathrm{Cov}(X)$.
5. Exercise 10.1, Kokoszka and Reimherr 2017
6. Exercise 10.3, Kokoszka and Reimherr 2017
7. Let $X(t)$, $t \in [0, 1]$ be a stochastic process for which the sample paths lie in $L^2([0, 1])$. Show that the solution to the following problem minimizing the residual variance coincides with the projection directions in the functional principal component analysis:

$$\min E \|X - \sum_{k=1}^{K} \langle X, e_k \rangle e_k\|^2.$$

   The minimum is taken over orthonormal functions $e_1, \ldots, e_K$, $K \geq 1$.