Stat 580 (Spring 2017): Homework 2

Due, in class, February 9, 2017

Please submit your homework with codes (hard copy) in class and upload the corresponding codes to the Blackboard. Problems marked with * will be graded in detail and they are worth 50% of the total score. Remaining problems, worth the remaining 50% of the total score, will be given full mark if reasonable amount of work is shown.

1. * Write a C program that prints a table of binomial probability mass function

$$f(x) = \binom{n}{x} p^x (1-p)^{(n-x)},$$

similar to this one:

```
x\p   0.0100 0.0644 0.1189 0.1733 0.2278 0.2822 0.3367 0.3911 0.4456 0.5000

0     0.9510 0.7167 0.5311 0.3861 0.2746 0.1905 0.1284 0.0837 0.0524 0.0312
1     0.0480 0.2469 0.3583 0.4047 0.4050 0.3746 0.3259 0.2688 0.2105 0.1562
2     0.0010 0.0340 0.0967 0.1697 0.2389 0.2945 0.3308 0.3453 0.3384 0.3125
3     0.0000 0.0023 0.0130 0.0356 0.0705 0.1158 0.1679 0.2218 0.2719 0.3125
4     0.0000 0.0001 0.0009 0.0037 0.0104 0.0228 0.0426 0.0712 0.1093 0.1562
5     0.0000 0.0000 0.0000 0.0002 0.0006 0.0018 0.0043 0.0092 0.0176 0.0312
```

You are only allowed to use the C standard IO (stdio.h) and math (math.h) libraries. Use the following symbolic constants to define the behavior of the table:

```
#define P0 0.01  /* lower limit of the probability (p) */
#define P1 0.5   /* upper limit of the probability (p) */
#define PLEN 10  /* number of columns */
#define N 5      /* number of experiments (n) */
```

2. * Use the inverse transform method to sample from the density

$$f(x) \propto 1/x, \quad 1 < x < 10.$$

   (a) Derive the algorithm.
   (b) Write a C program that draws and prints one observation from this density.
   (c) Write an R program to draw a sample of 5000 observations. Plot the estimated density of the sample.

3. Consider the following probability density function:

$$f(x) \propto q(x) = \frac{e^{-x}}{1+x^2}, \quad x > 0.$$

   Use rejection sampling to sample from $f(x)$ with the following envelope density functions:

$$g_1(x) = e^{-x}, \quad g_2(x) = \frac{2}{\pi(1+x^2)}, \quad x > 0.$$

   (a) For each density function ($f(x)$, $g_1(x)$ and $g_2(x)$), write an R program to draw a sample of 5000 random observations and plot the estimated density function for $0 < x < 5$.
   (b) Comments on the speeds of sampling and the results using $g_1(x)$ and $g_2(x)$.

4. Design a rejection algorithm to sample from the following density on the upper right quarter of the unit disc:

$$f(x, y) \propto x^\alpha y, \quad x > 0, \quad y > 0, \quad x^2 + y^2 \leq 1.$$

   You don't need to run simulations for this problem. Just describe your algorithm in detail. Note that $\alpha > -1$. (Finding an algorithm that targets for $\alpha \geq 0$ is simple. However, it is tricky to get an algorithm that works for any $\alpha > -1$.)