GAMMT: Generative Ambiguity Modeling Using Multiple Transformers

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Abstract

We introduce a new model based on sets of probabilities for sequential data. We name the model **GAMMT**, which stands for **G**enerative **A**mbiguity **M**odels using **M**ultiple **T**ransformers. We suppose that data generating process of a sequence is ambiguous and determined by a set of probabilities rather than one as in the conventional model. We use multiple parallel transformers connected by a selection mechanism to approximate ambiguous probabilities. The GAMMT allows for ambiguity modeling in a generative way and multiple representations of the input tokens and the input sequence. This work explores the combination of attention mechanism and ambiguity by deep neural networks. We expect that this framework will facilitate new research into machine learning, improving our understanding of the *attention-ambiguity mechanism*.

1 Introduction

Risk and ambiguity are two different uncertainty modeling. Under risk, we can assess the probability for different outcomes, whereas under ambiguity, the probabilistic structure of the random outcomes is unknown, i.e. there is uncertainty about probability.

Standard approaches in machine learning take data as sampling from a deterministic probability. In this work, we suppose that data generating process of a sequence, such as a sentence of words, is ambiguous and determined by a set of probabilities rather than one. We then introduce a class of *generative ambiguity models using multiple transformers* (GAMMT for short).

In order to illustrate risk and ambiguity, suppose we have three urns with identical balls except for color: one urn has 30 red, 30 black and 30 yellow balls; the second urn has 30 red balls and 60 black or yellow balls, but we do not know the exact number of black or yellow balls; the third urn has 90 balls, 30 in red but the number of black or yellow balls are uniformly distributed in this magic urn. When people face these three urns, they have different uncertainty. The first urn is described by one probability, i.e. P(red) = P(black) = P(yellow) = 1/3, which is a deterministic urn.

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The second urn is described by P(red) = 1/3, P(black) + P(yellow) = 2/3, $0 \le P(black) \le 2/3$. The probabilities are not objectively known for people. People should have multiple prior beliefs when facing this ambiguous situation. The source of ambiguity is because of the missing information that is relevant and could be known but is unknown. The third urn is described by P(red) = 1/3, P(black) + P(yellow) = 2/3, but $P(black) = n_b/90$ with $n_b \sim \mathcal{U}(\{0, 1, \dots, 60\})$ is a uniform random variable. The urn is characterized by a set of probabilities, which is ambiguous in nature. When people face this uniform urn, it is like that they are playing with 61 latent deterministic urns at the same time. If they pick a ball from this uniform urn, it is the same as they randomly choose a latent deterministic urn then pick one ball from it. Actually, the last two urns represent two sources of model uncertainty: *subjective ignorance* and *objective randomness*.

The above urns are also called Ellsberg's urns, given in Ellsberg (1961) [9]. The Ellsberg Paradox has inspired a long line of theoretical and empirical researches on ambiguity in economics and finance. We refer the readers interested to see the review literature on ambiguity e.g. Camerer and Weber (1992) [4] and Ilut and Schneider (2022) [10] and related references therein in this field.

In this work, Our GAMMT model is constructed by using multiple parallel transformers connected with a selection mechanism. The transformer is an attention mechanism in deep neural networks, proposed in Vaswani et al. (2017) [19], which has become a powerful building block in machine learning. It gets its success in the field of nature language process (NLP). For example, its successive models BERT (Devlin et al., 2018 [6]), OpenAI GPT (Radford et al., 2018 [15]), GPT-2 (Radford et al., 2019 [16]), GPT-3 (Brown et al., 2020 [2]), Google T5 (Raffel et al., 2019 [17]) and many others are came up and show their effectiveness in many NLP tasks. The attention-based neural networks Transformers have moved from NLP to computer vision (e.g. Vision Transformers by Dosovitskiy et al., 2020 [7]) and structural biology (e.g. AlphaFold 2 by Jumper et al., 2021 [11]) in achieving state of the art results.

Our models utilize the effectiveness of Transformers for modeling probabilities. We use multiple parallel transformers to model a set of probabilities in sequential data like token sequences of language. The parallel transformers are like a distributed network of brain regions to some extent. Each transformers receive an input sequence and output a deterministic probability. They are connected together through some selection mechanism, then work together as a system to response to ambiguity. Our models are two-fold. Firstly, the models are generative and they can characterize ambiguity in the data. Secondly, the last hidden layer of each parallel transformer gives an embedding of each token and of the input sequence. Each embedding is an ambiguous representation. Then multiple transformers give multiple representations of the underlying input tokens and the input sequence.

A number of works have introduced the concept of ambiguity into the field of machine learning, such as Ek et al. (2008) [8], Patel et al. (2019) [13], Yang et al. (2021) [20], Buisson et al. (2022) [3] and many others. Ambiguity ubiquitously arises in vision, natural language, general learning and decision making. Visual relationships in an image are often semantically ambiguous. As Yang et al. [20] suggested, the semantic ambiguity can be classified into three types: synonymy ambiguity, hyponymy ambiguity, and multi-view ambiguity. For natural language, perceptual vagueness appears at different levels of granularity, e.g. words, sentences, paragraphs and documents. Ambiguity of image or language maybe have its source in an external world, or due to the feelings,

perception, experiences or incomplete knowledge of people into the generation process. In terms of representation, an image, a word or a sentence, is usually modeled as a fixed feature vector in a deterministic way in the machine learning field. However, such fixed representation is not sufficient to characterize the ambiguity we mentioned above. Ek et al. [8] proposed to use two or more complementary representations of an underlying phenomenon and demonstrated its use for multi-modal regression on a benchmark human pose estimation dataset. Yang et al. [20] utilized a probability distribution to represent each union region of an image by Gaussian embedding. Each union region is parameterized by a mean and variance. The mean vector acts like the normal feature vector as in the conventional model and the variance measures the feature uncertainty.

In the field of neuroscience, cognitive neuroscientists have examined brain regions activations and the neuronal correlates in humans on decision making under ambiguity through neuroimaging devices, such as functional magnetic resonance imaging (fMRI), see e.g. Levy et al. (2010) [12], Bach et al. (2011) [1], Chumbley et al. (2012) [5], Taya (2012) [18] and references therein. Chumbley et al. [5] found that the hippocampus expresses clear ambiguity-dependent responses, which suggest candidate neuronal systems may be involved in resolving ambiguity. Taya [18] reviewed a number of neuroimaging studies on decision making under ambiguity, have shown that a distributed network of brain regions regarding cognitive control and reward processing is closely related to decision making in the face of ambiguity.

To our knowledge, we are the first to explore the combination of attention mechanism and ambiguity by deep neural networks. We introduce the framework of our models without providing experimental validation in this work. We expect that this framework will facilitate new research into the field of machine learning, improving our understanding of how the *attention-ambiguity mechanism* works.

2 Framework

Standard approaches in machine learning for sequential modeling such as language models are to factorize the joint probabilities P(x) of a sequence $x = (s_1, s_2, \dots, s_n) \in V^*$ in a space of sequences $V^* = \bigcup_{\ell=0}^{\infty} V^{\ell}$ (V is a vocabulary of tokens) as the product of conditional probabilities (e.g. GPT [15], GPT-2 [16], GPT-3 [2])

$$P(x) = \prod_{t=1}^{n} P(s_t | s_1, s_2, \dots, s_{t-1})$$
(2.1)

or to factorize the conditional probabilities P(x|z) of a pair of sequences $x = (s_1, s_2, \dots, s_n) \in V^*$ and $z \in V^*$ as the product of following conditionals (e.g. the original Transformer [19])

$$P(x|z) = \prod_{t=1}^{n} P(s_t|s_1, s_2, \dots, s_{t-1}, z).$$
 (2.2)

In this paper, we propose a novel approach to tackle sequential modeling problems. Since ambiguity appears in many situations, the randomness of sequences is not always characterized by

only one probability distribution but perhaps a set of probabilities instead. It is like that we have a number of (different) dice in a magic box rather than one (see Figure 1). Every time we randomly pick and roll one of them, or roll all of them altogether. Then an event happens, or decision makes based on the rolled dice. The data generating process is then determined by the set of probabilities

$$\mathcal{P}(x) = \{ P(x) | P \in \mathcal{M} \}, \tag{2.3}$$

where \mathcal{M} is a set of basic probability measures for generating the data process. The size of the set is the degree of model ambiguity, or called model uncertainty.

The objective of a sequence modeling is to maximize the following quantity

$$\prod_{t=1}^{n} S\left(\left\{P_{\theta_{j}}(s_{t}|s_{1}, s_{2}, \cdots, s_{t-1})\right\}_{j=1}^{M}\right)$$
(2.4)

if we assume that $\mathcal{M} = \{P_{\theta_1}, P_{\theta_2}, \cdots, P_{\theta_M}\}$ is finite, where S is a selection mechanism function on probabilities. The conditional probabilities P_{θ_j} are modeled using deep neural networks with parameters θ_j , such as self-attention architectures like the multi-layer Transformer decoders for language models. The aim of the selection S is to choose a mechanism to tackle the kind of ambiguity in the data. It could be a random choice of probabilities (e.g. subject to a uniform random variable), or the maximum of all possible conditionals. Note that if $\mathcal{M} = \{P\}$ is a singleton set, the method degenerates to the standard approach.



Figure 1: A set of dice

3 Architecture

In this section, we propose a class of deep neural networks, the self-attention Transformer decoders (see e.g. [2, 6, 14, 15, 16, 17, 19]), to approximate a set of probabilities which characterize ambiguity of the data generating process. The architecture is as following, see Figure 2.

There are M parallel Transformer decoders in the architecture. Each Transformer decoder is composed of a stack of N identical layers, in each layer there are a masked multi-head self-attention sublayer with h parallel attention heads, and a feed-forward network followed by layer normalization. Then a linear transformation and softmax function are applied to transform the decoder output to next-token probabilities. All the M Transformer decoders are connected by a selection mechanism simulating the type of ambiguity.

The model described here can be generalised to the encoder-decoder architectures with Transformers to approximate a set of conditional probabilities $\mathcal{P}(x|z) = \{P(x|z) | P \in \mathcal{M}\}$ where x and z are a pair of input or output token sequences as the original Transformer [19].

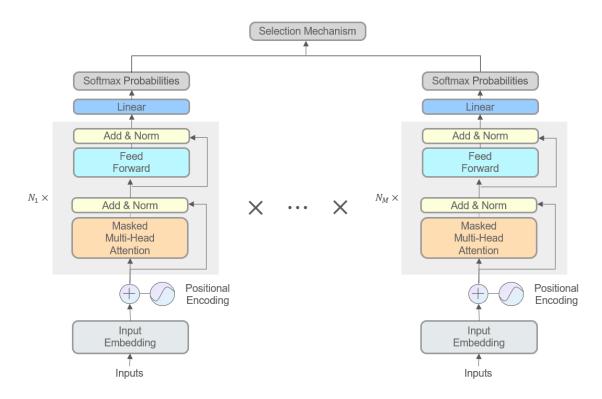


Figure 2: Model architecture - The Transformer decoders to approximate a set of probabilities

4 Algorithms

The section presents the algorithms for the model architecture (see Algorithm 1), model training and inference (see Algorithm 2 and 3). We omit the details of the initial token and positional embedding and Transformer decoders in the following pseudocode. We refer to Phuong and Hutter (2022) [14] for an overview of algorithms about transformers and the related transformer-based models.

Algorithm 1 shows the model architecture of GAMMT in a way of forward pass. The input is a sequence of tokens x with length ℓ_x and the output is probabilities of next token matrices. V is a vocabulary of tokens and N_V is the size of the vocabulary V. The operations Embedding(x) and TransformerDecoder (m,\cdot) are almost the same as Algorithms 1, 2, and 10 in the work [14]. Algorithm 2 is for the model training. The M parallel Transformer decoders are trained together through the given selection mechanism. We get all the trained parameters after $N_{epoches}$ epochs. Algorithm 3 is for the inference of our model. When a prompt is feed to the trained model, it will output a continuation of the prompt which is sampled from our ambiguity machine GAMMT. The sampling methods depend on the given selection mechanism when training the model.

```
Algorithm 1: \mathcal{P} \leftarrow \text{DTransformers}(x|\theta)
  /* Model architecture - the Transformer decoders, forward pass
  /* Input: a sequence of token IDs with length \ell_x
  /* Output: probabilities of next token
  Input: x \in V^*
  Output: P_{\theta_m} \in [0,1]^{N_V \times \ell_x}, m \in [M] := \{1,2,\cdots,M\} and the Selection S
  Hyperparameters: The number of Transformer decoders M, the Transformer decoders
                           Layers N_i, heads H_i, input embedding dimension d_e, hidden layer
                           dimension d_{mlp}, maximum length of input \ell_{max}, selection S
  Parameters: \theta = (\theta_m)_{m \in [M]} includes all token and positional embedding matrices, all
                   Transformers' parameters
1 for m = 1, 2, \dots, M do
      X_m \leftarrow \text{TransformerDecoder}(m, \text{Embedding}(x))
      P_{\theta_m} = \operatorname{softmax}(W_o^{(m)} X_m)
4 end
s return (P_{\theta_m})_{m\in[M]}, S=S(P_{\theta_1},\cdots,P_{\theta_M})
```

Algorithm 3: $y \leftarrow \text{Inference}(x, \hat{\theta})$

```
/* Model inference - generate a sequence based on the trained model and a prompt
    /* Input: the trained parameters and a prompt
                                                                                                                                */
    /* Output: a continuation of the prompt sampled from the trained model
                                                                                                                                */
   Input: \hat{\theta} = (\hat{\theta}_m)_{m \in [M]}, x \in V^*
    Output: y \in V^*
   Hyperparameters: temprature \tau > 0
 1 \ell_x \leftarrow \text{length}(x)
 y \leftarrow \emptyset
 3 while y \neq eos\_token do
         (P_{\theta_m})_{m \in [M]}, -\leftarrow \text{DTransformers}(x|\hat{\theta})
         if S \sim \mathcal{R}([M]) then
              u \leftarrow \text{sample a random variable } \mathcal{R} \text{ on } [M]
              p \leftarrow P_{\hat{\theta}_u}[:, \ell_x]
 7
              sample a token y from the probability q \propto p^{1/\tau}
              x \leftarrow [x, y]
            \ell_x \leftarrow \ell_x + 1
10
         end
11
12
         else if S = \max then
              for m = 1, 2, \dots, M do
13
                   p_m \leftarrow P_{\hat{\theta}_m}[:, \ell_x]
14
                   sample a token y_m from the probability q_m \propto p_m^{1/\tau}
15
16
              y \leftarrow y_{m^*} with m^* = \operatorname{argmax} \{q_m(y_m), m \in [M]\}
17
              x \leftarrow [x, y]
18
              \ell_x \leftarrow \ell_x + 1
19
         end
21 end
22 return y = x
```

5 Conclusion

In this paper, we introduce a new model framework GAMMT for sequential data in order to model ambiguity in the data distribution. We use multiple parallel transformers connected by a selection mechanism to approximate a set of probabilities. The GAMMT allows for ambiguity modeling in a generative way and multiple representations of the input tokens and the input sequence. The model described in Section 3 and 4 can be generalised to the encoder-decoder architectures with Transformers to approximate a set of conditional probabilities $\mathcal{P}(x|z) = \{P(x|z) | P \in \mathcal{M}\}$ where x and z are a pair of input or output token sequences.

Our work is the first to explore the combination of attention mechanism and ambiguity by deep neural networks. For further research, we plan to provide experimental results to validate the effectiveness of our models based on the attention-ambiguity mechanism. In addition, we plan to extend the GAMMT framework to vision, audio, reinforcement learning and other domains in the future works. We expect that this new framework will help enable new research into a broad machine learning field.

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