

Throughout we assume the Riemann Hypothesis, in the strong form that the nontrivial zeros are also simple. We also need to assume the following:

Hypothesis D. The level curves $\operatorname{Re}(\eta(s)) = 0$ are differentiable. This is automatic except at isolated points where $\eta'(s) = 0$, so we are really assuming that when $\eta'(s) = 0$, $\arg(\eta(s)) \neq \pm\pi/2$. This prevents the level curves from branching. Hypothesis D is plausible because $\pm\pi/2$ are only two points on the unit circle, while the zeros of $\eta'(s)$ form a countable set.

Here's a summary of the sections of the paper:

- §1 Classification of the zeros of $\zeta(s)$ and $\zeta'(s)$ into different types by means of the level curves, and results on the asymptotics of the types.
- §2 Computation and classification of 10^6 zeros of $\zeta'(s)$ near $T = 10^{10}$.
- §3 Two lemmas.
- §4 A closer look at the type 2 zeros, and the curvature of the level curve.
- §5 A canonical bijection between the complex zeros ρ' of $\zeta'(s)$, and the complex zeros ρ'' of $\zeta''(s)$.
- §6 Adaptation of a theorem of Marden, and the location of ρ'' relative to ρ' .
- §7 Curvature of the level curve at ρ' in terms of all the other zeros $\lambda' \neq \rho'$.
- §8 Appendix: With $p_A(z)$ the characteristic polynomial of a unitary matrix A , we give a classification of the zeros of $p_A(z)$ and $p'_A(z)$ analogous to that in §1.

1. CLASSIFICATION OF ZEROS

Proposition 1. *With the usual indexing $\gamma_1 < \gamma_2 < \dots$ of the imaginary parts of the zeros of $\zeta(s)$, every odd indexed zero lies on a contour $\operatorname{Im}(\eta(s)) < 0$. Every even indexed zero lies on a contour $\operatorname{Im}(\eta(s)) > 0$.*

Proof. This follows from Lemma 2 below, which says that as t increases, the argument of $\eta(1/2 + it)$ decreases by exactly π between