

for the Dirichlet form associated to sticky Brownian motion. Denoting $a_0 = \frac{1}{2+\omega L}$ and $b_0 = \omega a_0$, for all $s > 0$ we have

$$\begin{aligned} \int f^2 d\mu &= a_0(f(0)^2 + f(L)^2) + b_0 \int_0^L f^2 dx, \\ &\leq a_0(f(0)^2 + f(L)^2) + b_0 s \int_0^L (f')^2 dx + b_0 \beta(s) \left(\int_0^L |f| dx \right)^2, \\ &\leq b_0 s \int_0^L (f')^2 dx + b_0 \max(b_0^{-2}, a_0^{-1}) \beta(s) \left(\int |f| d\mu \right)^2. \end{aligned}$$

Therefore the sticky Brownian satisfies a super Poincaré inequality. Then by [Wan00, Th. 5.1], it has an empty essential spectrum. Now, by [BGL14, Th. A.6.4], the resolvent is compact and thus the generator has discrete spectrum. \square

Corollary 19. *Choosing $T = m^{-1/2}$, the transition semigroup of the RTP process is exponentially contractive in T -average with rate*

$$\nu = \Omega \left(\frac{\omega}{1 + (\omega L)^2} \right).$$

Note that the relaxation time corresponding to this decay rate is of the same order as the mixing time obtained in [GHM24]. It reveals the existence of two regimes controlled by the parameter ωL . In the ballistic regime $\omega L \ll 1$, velocity flips are rare, leading to a fast exploration of the position space \mathcal{S} and a comparatively slow exploration of the velocity space \mathcal{V} . This results in the scaling $\nu \propto \omega$. On the contrary, in the diffusive regime $\omega L \gg 1$, the high frequency of velocity flips makes the exploration of \mathcal{V} faster than the exploration of \mathcal{S} . This leads to the scaling $\nu \propto \omega^{-1} L^{-2}$.

Proof. We begin by verifying Assumption (A). Recall that $\text{Dom}(\mathcal{L}_{C^0})$ is a core of \mathcal{L} by Theorem 7. For all $f \in \text{Dom}(\mathcal{L}_{C^0})$ we have $\hat{\mathcal{L}}_v(f \circ \pi) = 0$ hence $\hat{\mathcal{L}}_{\text{tr}}$ is a lift of \mathcal{L} by Remark 8. Furthermore, for $f \in \text{Dom}(\mathcal{L}_{C^0})$ one has

$$\hat{\mathcal{L}}_{\text{tr}}^*(f \circ \pi)(x, v) = -v 1_{\{0 < x < L\}} f'(x) = -\hat{\mathcal{L}}_{\text{tr}}(f \circ \pi)(x, v).$$

A straightforward computation yields

$$\int_{\mathcal{V}} \hat{\mathcal{L}}_v f(x, v) d\kappa_x(v) = 0 \text{ for all } x \in \mathcal{S} \text{ and } f \in \text{Dom}(\hat{\mathcal{L}}).$$

Finally, we prove $\|f - \Pi_v f\|_{L^2(\hat{\mu})}^2 \leq \frac{1}{m_v} \mathcal{E}_v(f)$ with $m_v = 2$. Define the matrices

$$S = \begin{pmatrix} 1/4 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix}, \quad \mathcal{Q} = \begin{pmatrix} -2 & 2 & 0 \\ 1 & -2 & 1 \\ 0 & 2 & -2 \end{pmatrix},$$

as well as the scalar product $\langle x, y \rangle_S = x^\top S y$ and let Π be the orthogonal projection on the kernel of \mathcal{Q} with respect to $\langle \cdot, \cdot \rangle_S$. The matrix \mathcal{Q} is symmetric w.r.t. the scalar