



FIGURE 3. Base  $b = 1 + 2i$  with all possible one missing digit.

of Hausdorff the metric. With one missing digit,  $C_{b,D}$  is  $1/N^{1/2}(b)$ -close to  $C_{b,D_b}$ . Notice that  $C_{b,D}$  is self-similar whose contraction ratio is  $1/N^{1/2}(b)$ . From here, we see that the thickness of  $C_{b,D}$  is at least  $c_d N^{1/2}(b)$  for some constant  $c_d > 0$ .  $\square$

The thickness estimate holds for all  $C_{b,D,j}$  because of the self-similarity. However, we cannot directly use Theorem 11 to obtain intersections between different missing digit sets without checking the conditions (2,3).

Consider different integers  $b_1, \dots, b_k \in \mathbb{Z}[i]$  with large enough norms so that Theorem 11 applies. Consider the logarithmic map

$$\log : a \in \mathbb{Z}[i] \rightarrow (\log |a|, \arg a) \in (0, \infty) \times \mathbb{T}.$$

The first component is the standard log for positive numbers. The second component is the argument of  $a$  as a complex number.<sup>3</sup> For simplicity, let us first look at the two integers  $b_1, b_2$ . We claim that there are infinitely many positive rational integer pairs  $(l_1, l_2)$  such that  $b_1^{l_1}/b_2^{l_2}$  is close to one. In fact, consider the set

$$P = \{b_1^{l_1}/b_2^{l_2} : (l_1, l_2) \in \mathbb{Z}_{>0}^2\} \subset \mathbb{C}.$$

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<sup>3</sup>In general, such a logarithmic map can be defined in each Galois embedding individually.