

Finally, we define the nonlinear mapping $F^M : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ as

$$F^M(U) := (f_a^M(U) + \varepsilon^{-1}Q^M(U), f_b^M(U) - \varepsilon^{-1}Q^M(U), f_v^M(U)).$$

Note that the functions f_ν^M, ψ^M, ϕ^M are globally Lipschitz and bounded. Therefore, F^M is globally Lipschitz, bounded and

$$F^M(U) := (f_a(U) + \varepsilon^{-1}Q(U), f_b(U) - \varepsilon^{-1}Q(U), f_v(U)), \quad \text{if } |U| \leq M.$$

Given $p \in (1, +\infty)$, we consider the operator A_p on $X_p := (L^p(\Omega))^3$ defined by

$$\begin{cases} D(A_p) = D_p^3 & (\text{see (2.1)}) \\ A_p U = (d_a \Delta u_a, d_b \Delta u_b, d_v \Delta v) & \text{for } U \in D(A_p), \end{cases} \quad (\text{A.1})$$

and the abstract initial value problem

$$U'(t) = A_p U(t) + F^M(U(t)), \quad t > 0, \quad U(0) = U^{\text{in}} := (u_a^{\text{in}}, u_b^{\text{in}}, v^{\text{in}}). \quad (\text{A.2})$$

We will solve (A.1), (A.2) and then we will get rid of the truncation. The main ingredient is that $A_p : D(A_p) \subset X_p \rightarrow X_p$ is a sectorial operator ([21], Theorem 3.1.3). Hence it generates in X_p an analytic semigroup denoted $(e^{tA_p})_{t \geq 0}$ ([21], Chapter 2). Moreover, A_p is closed so that $D(A_p)$, endowed with the graph norm, is a Banach space. $D(A_p)$ being also dense in X_p , the semigroup is strongly continuous, i.e. $\lim_{t \rightarrow 0} e^{tA_p} U = U$, for all $U \in X_p$. Furthermore, there exists $K_p > 0$ and $\omega_p \in \mathbb{R}$ such that (see [21], Proposition 2.1.1)

$$\|e^{tA}\|_{L(X_p)} \leq K_p e^{\omega_p t}, \quad \forall t \geq 0. \quad (\text{A.3})$$

First step: well-posedness of (A.1), (A.2). Let $\|\cdot\|_p$ denote the usual norm in X_p . We start proving that (A.1), (A.2) has a unique mild solution, i.e. a unique function $U \in C([0, \infty), X_p)$ such that

$$U(t) = e^{tA} U^{\text{in}} + \int_0^t e^{(t-s)A} F^M(U(s)) ds, \quad \forall t \geq 0. \quad (\text{A.4})$$

It is easily seen that F^M maps X_p into X_p and

$$\|F^M(U)\|_p \leq \|F^M(0)\|_p + L_M \|U\|_p = L_M \|U\|_p, \quad \forall U \in X_p. \quad (\text{A.5})$$

Therefore, (A.4) makes sense since, by assumption (2.3), $U^{\text{in}} \in X_p$ and, for all $U \in C([0, \infty), X_p)$ and all $t > 0$, $F^M(U(\cdot)) \in L^1((0, t); X_p)$. Moreover, the Lipschitz property of F^M together with (A.3) and Gronwall's Lemma gives us the uniqueness of (A.4). The same ingredients give us the continuous dependence of U with respect to U^{in} . Therefore, it remains to prove the existence of U and that U belongs to $C^1([0, \infty); X_p) \cap C([0, \infty); D_p(A))$ for all $p \in (1, +\infty)$.

Let $\theta > 0$ be such that $\omega_p + \theta > 0$. The existence is proved using the contraction mapping principle in the space

$$E := \{U \in C([0, \infty), X_p) : \|U\|_E = \sup_{t \geq 0} e^{-(\omega_p + \theta)t} \|U(t)\|_p < \infty\},$$

that is a Banach space when endowed with the norm $\|U\|_E$. Hence, given $U \in E$, we set

$$\Phi(U)(t) = e^{tA} U^{\text{in}} + \int_0^t e^{(t-s)A} F^M(U(s)) ds, \quad \forall t \geq 0.$$