

conformation tensor, respectively. Here, the operator  $\otimes$  denotes  $(\mathbf{u} \otimes \mathbf{v})_{ij} = u_i v_j$ ,  $(\mathbf{C} \otimes \mathbf{u})_{ijk} = C_{ij} u_k$ . If  $\nabla \cdot \mathbf{u} = 0$ , we have  $\nabla \cdot (\mathbf{u} \otimes \mathbf{u}) = (\mathbf{u} \cdot \nabla) \mathbf{u} + (\nabla \cdot \mathbf{u}) \mathbf{u} = (\mathbf{u} \cdot \nabla) \mathbf{u}$  and  $\nabla \cdot (\mathbf{C} \otimes \mathbf{u}) = (\mathbf{u} \cdot \nabla) \mathbf{C}$ . Therefore,  $\nabla \cdot (\mathbf{u} \otimes \mathbf{u})$  and  $\nabla \cdot (\mathbf{C} \otimes \mathbf{u})$  in (1.1a) and (1.1c) can be replaced by the convective terms  $(\mathbf{u} \cdot \nabla) \mathbf{u}$  and  $(\mathbf{u} \cdot \nabla) \mathbf{C}$ .

The diffusive Peterlin model is discussed in [20], where the global existence of weak solutions for large data is proved. The existence of weak solutions in 3D is established in [4], as well as a conditional weak-strong uniqueness results. On the other hand, various discrete schemes have been proposed for the numerical simulation of the diffusion Peterlin model. The linear and nonlinear schemes of stabilized Lagrange-Galerkin FEM for the Oseen-type Peterlin viscoelastic model are proposed in [22, 23]. Three fully discrete schemes by using Newton's iterative, Picard's iterative and implicit-explicit time-stepping are presented in [16]. [36] introduces a linear decoupled scheme with stabilizing terms, where  $(\mathbf{u}, p)$  and each components of  $\mathbf{C}$  can be computed in parallel. Numerical studies of other viscoelastic models (such as the Oldroyd-B model) also exist [11, 25, 30, 39]. Moreover, some log-conformation formulations [1, 2, 12, 15, 35] are used to guarantee symmetric positive-definiteness of discrete conformation tensor.

When  $\epsilon$  is very small, (1.1c) becomes convection-dominated. However, the numerical schemes used in most existing works [14, 16, 36, 38] are not specifically designed to address the convection-dominated issue. To tackle this problem, [22–24] propose the Lagrange-Galerkin method for the Oseen-type model. By carefully examining the existing results on error estimates, we find that the constant in the error bound often depends on the reciprocal of  $\epsilon$ . As  $\epsilon$  approaches 0, the parabolic equation (1.1c) transitions to a hyperbolic form, which brings difficulty to numerical approximation. To address the convection-dominated issue and to derive an error bound that does not depend on the reciprocal of  $\epsilon$ , we propose a linear HDG scheme for the diffusion Peterlin model and study the error analysis.

The HDG can reduce the computational cost of the DG method while preserving its attractive properties of conservation and stability [8, 18]. The method transfers the computation from the global domain to each element, resulting in a smaller stiffness matrix dimension that facilitates efficient inversion operation [10]. Currently the HDG method has been widely applied to elliptic problem [7, 28], convection-diffusion equation [27], linear elasticity system [29], Stokes equation [6, 9, 31] and Navier-Stokes equations [26, 32]. Especially, for the Navier-Stokes equations, the HDG scheme is mass and momentum conserving, energy stable, and pressure-robust. In addition, the discrete velocity is pointwise divergence-free, and the error estimates are optimal and independent of the viscosity  $\nu$  [17]. The motivation of this work is to employ the HDG method to address the convection-dominated issue of (1.1). In particular, we aim to obtain error estimates that do not depend on  $\epsilon^{-1}$ .

In this work, we design a linear HDG scheme for the diffusion Peterlin problem. We prove the unique existences of the discrete problem. Furthermore, by utilizing the Ritz projections for Stokes and Poisson equations, we establish the error estimates. The numerical experiments are carried out to verify the theoretical convergence rates. The HDG scheme remains stable even for the cases with  $\epsilon = 10^{-3}$  and  $\epsilon = 0$ . A comparison with the ordinary FEM in simulation shows that the HDG scheme performs better in preserving the positive definiteness of the conformation tensor.

The remainder of this paper is organized as follows. Section 2 introduces the fully discrete semi-implicit linearizing HDG scheme which satisfies the mass conservation. Section 3 focuses on stability analysis. The theoretical convergence rate of the error estimates is rigorously proved under proper assumptions in Section 4. We provide several numerical examples in Section 5 to study the experimental convergence rates and illustrate the stability.

## 2. THE LINEAR HDG SCHEME

Let  $\mathcal{T} := \{K\}$  be a regular quasi-uniform triangulation of the domain  $\Omega$ . The boundary of the element  $K$  is denoted by  $\partial K$  and the outward unit normal vector on  $\partial K$  by  $\mathbf{n}$ . Two adjacent cells  $K^+$  and  $K^-$  share an interior facet  $F := \partial K^+ \cap \partial K^-$ . A facet of  $\partial K$  that lies on the boundary  $\partial \Omega$  is called a boundary facet. The interior and boundary facet sets are denoted by  $\mathcal{F}_I$  and  $\mathcal{F}_B$ , respectively.