



Figure 12: The cycle  $C$  created by two distinct paths from  $v'$  to  $x'$

to it in  $C$ . Let  $v_{i_l}y$  be one such chord; then we show that  $y$  lies in  $P(v'_1, x'_1)$  in the next claim.

**Claim 6.6.**  $y$  lies in  $P(v'_1, x'_1)$ .

*Proof.* If  $y$  does not lie in  $P(v'_1, x'_1)$ , then  $y$  must lie in the path between  $v_{i_l}$  and  $x'_1$  (or the path between  $v_{i_l}$  and  $v'_1$ ). If it lies on the path between  $v_{i_l}$  and  $x'_1$  (resp. between  $v_{i_l}$  and  $v'_1$ ), then we have a path from  $v_{i_l}$  to  $x'$  (resp. between  $v_{i_l}$  and  $v'$ ) which has length smaller than  $d(v_{i_l}, x')$  (resp.  $d(v_{i_l}, v')$ ), which is a contradiction. Hence,  $y$  lies in  $P(v'_1, x'_1)$  (refer to Figure 12b).  $\square$

Note that  $d(x'_1, y) \geq d(x'_1, v_{i_l})$  and  $d(v'_1, y) \geq d(v'_1, v_{i_l})$ . If not, then there exists a shorter or equal length path from  $v'$  to  $v_{i_l}$  (from  $x'$  to  $v_{i_l}$ ) that bypasses the edge  $v_{i_l}v$  (resp.  $v_{i_l}x$ ), which is a contradiction to the fact that the pair  $v_{i_l}, v'$  (resp.  $v_{i_l}, x'$ ) monitors the edge  $v_{i_l}v$  (resp.  $v_{i_l}x$ ). Hence  $d_P(x'_1, v'_1) = d(x'_1, y) + d(y, v'_1) \geq d(v'_1, v_{i_l}) + d(x'_1, v_{i_l})$ , implying  $d_P(x'_1, v'_1) = d(v'_1, v_{i_l}) + d(x'_1, v_{i_l})$ . This implies that  $d(x'_1, y) = d(x'_1, v_{i_l})$  and  $d(v'_1, y) = d(v'_1, v_{i_l})$ , which implies that the vertex  $y$  is unique in  $P(v'_1, x'_1)$ . This fact together with Claim 6.6 implies that in  $C$ ,  $v_{i_l}$  is incident to exactly one chord.

Now note that if  $v$  and  $y$  are not adjacent, then the induced 2-path  $yv_{i_l}v$  is part of a chordless cycle of length at least 4, which is a contradiction. Hence  $vy \in E(G_{k+1})$ . Similarly, it can be shown that  $xy \in E(G_{k+1})$ . Hence  $vv_{i_l}x$  is part of a 4-cycle  $vv_{i_l}xyv$ , which leads to a contradiction. Hence  $v'$  and  $x'$  monitor  $vv_{i_l}$ .

Hence, until now, we showed that all edges in  $E_l$  can be monitored by the vertices of  $Man(G_k) \setminus \{v_{i_l}\}$ . Hence, combining this fact with Claim 6.1 and 6.3, we can conclude that  $Man(G_{k+1})$  forms an optimal MEG set of  $G_{k+1}$ . Hence,  $meg(G) = |Man(G)|$  for every strongly chordal graph  $G$ .  $\square$

## 7. Conclusion and future aspects

In this paper, we solved the complexity status of the MIN-MEG problem for some well-known graph classes. Next, it will be interesting to address the following questions: