

by $U(X)$ in **Alt**. In fact, one may verify that the semi-direct product $B \ltimes_{\varphi} U(X)$ with multiplication

$$(b, x) \cdot (b', x') = (bb', xx' + \varphi(b)x' + x\varphi(b'))$$

is an alternative algebra. We thus have a natural isomorphism

$$\text{SplExt}(-, U(X)) \cong \text{Hom}_{\mathbf{Alt}}(-, U(X))$$

and we can state the following.

Theorem 3.2. *The category \mathbf{Alt}_1 is action representable with the actor of a unitary alternative algebra X being isomorphic to X itself.* \square

4. POISSON ALGEBRAS

The aim of this section is to prove that the categories \mathbf{Pois}_1 of unitary Poisson algebras and \mathbf{CPois}_1 of unitary commutative Poisson algebras over a field \mathbb{F} with $\text{char}(\mathbb{F}) \neq 2$ are action representable.

We recall that a Poisson algebra is a vector space X over \mathbb{F} equipped with two bilinear multiplications

$$\cdot : X \times X \rightarrow X \quad \text{and} \quad [-, -] : X \times X \rightarrow X$$

such that (X, \cdot) is an associative algebra, $(X, [-, -])$ is a Lie algebra and the *Poisson identity* holds:

$$[x, yz] = [x, y]z + y[x, z], \quad \forall x, y, z \in X.$$

A Poisson algebra is said to be *commutative* (resp. *unitary*) if the underlying associative algebra is commutative (resp. unitary).

It was proved in [6] that for any Poisson algebra X there exists a natural monomorphism of functors

$$\tau : \text{SplExt}(-, X) \hookrightarrow \text{Hom}_{\mathbf{Alg}^2}(\tilde{U}(-), [X]),$$

where \mathbf{Alg}^2 denotes the category of algebras with two non-necessarily associative bilinear operations, $\tilde{U} : \mathbf{Pois} \rightarrow \mathbf{Alg}^2$ is the forgetful functor and

$[X] = \{f = (f * -, - * f, [f, -]) \in \text{Bim}(X) \times \text{Der}(X) \mid \dots$
 $\dots \mid f * [x, y] = [f * x, y] - [f, y]x, [x, y] * f = [x * f, y] - x[f, y], [f, xy] = [f, x]y + x[f, y]\}$
 is the universal strict general actor of X , which is endowed with the bilinear multiplications

$$f \cdot g = (f * (f' * -), (- * f) * f'), f * [f', -] + [f, -] * f')$$

and

$$[f, g] = (f * [f', -] - [f', f * -], [f', -] * f - [f', - * f], [f, [f', -]] - [f', [f, -]]).$$

Furthermore, a morphism $\varphi = (\varphi_1, \varphi_2, \varphi_3) : \tilde{U}(B) \rightarrow [X]$ belongs to $\text{Im}(\tau_B)$ if and only if $(\varphi_1, \varphi_2) : (B, \cdot) \rightarrow \text{Bim}(X)$ is an acting morphism in **Assoc**.

If X has trivial annihilator or $(X^2, \cdot) = (X, \cdot)$, then eq. (3.1) holds in $\text{Bim}(X)$ and we have a natural isomorphism

$$\text{SplExt}(-, X) \cong \text{Hom}_{\mathbf{Alg}^2}(\tilde{U}(-), [X]).$$

This happens, for instance, when X is a unitary Poisson algebra. In this case, it is possible to prove that $[X]$ is a Poisson algebra. Indeed, by the results in Section 3, the universal strict general actor $[X]$ may be described as the subalgebra of all pairs $(\alpha, [f, -]) \in X \times \text{Der}(X)$ such that

$$\begin{aligned} \alpha[x, y] &= [\alpha x, y] - [f, y]x, \\ [x, y]\alpha &= [x\alpha, y] - x[f, y] \end{aligned}$$