

define the set $S \subseteq \{1, 2, \dots, n\} \times \{1, 2, \dots, f+g\}$, and assume the entry $\begin{bmatrix} A_{\text{true}} & B_{\text{true}} \end{bmatrix}_{ij}$ is given for all $(i, j) \in S$. The set of systems compatible with the prior knowledge is given by

$$\Sigma_{pk}(S) := \left\{ (A, B) \mid \begin{bmatrix} A & B \end{bmatrix}_{ij} = \begin{bmatrix} A_{\text{true}} & B_{\text{true}} \end{bmatrix}_{ij} \quad \forall (i, j) \in S \right\}. \quad (6)$$

Subsequently, we define the set of systems compatible with both the data and the prior knowledge as

$$\Sigma := \Sigma_d \cap \Sigma_{pk}(S). \quad (7)$$

Note that the case that all entries of A_{true} and B_{true} are unknown can be captured by setting $S = \emptyset$, which implies $\Sigma = \Sigma_d$. It is clear from (2) and (6) that the system $(A_{\text{true}}, B_{\text{true}})$ belongs to Σ . However, in general, Σ contains other systems because the data may not uniquely determine A_{true} and B_{true} , even if some entries of A_{true} and B_{true} are known.

The goal of this paper is to find a controller that stabilizes the origin of the system $(A_{\text{true}}, B_{\text{true}})$. Since on the basis of the data and the prior knowledge we cannot distinguish between $(A_{\text{true}}, B_{\text{true}})$ and any other system in Σ , we need to find a single controller that stabilizes the origin of all systems in Σ . This motivates the following definition of informative data for stabilization of polynomial systems. In the rest of the paper, we assume that

$$F(0) = 0.$$

Definition 1 The data $(\dot{\mathcal{X}}, \mathcal{X}, \mathcal{U})$ are called *informative for stabilization* if there exist a radially unbounded function $V \in \mathcal{V}$ and a continuous controller $K : \mathbb{R}^n \rightarrow \mathbb{R}^m$ such that $K(0) = 0$ and

$$\frac{\partial V(x)}{\partial x} (AF(x) + BG(x)K(x)) < 0 \quad \forall x \in \mathbb{R}^n \setminus \{0\}, \quad (8)$$

for all $(A, B) \in \Sigma$.

Note that for a controller K satisfying $K(0) = 0$, the origin of the closed-loop system

$$\dot{x} = AF(x) + BG(x)K(x), \quad (9)$$

is an equilibrium point, as $F(0) = 0$. If (8) holds then the origin is globally asymptotically stable for all closed-loop systems obtained by interconnecting any system $(A, B) \in \Sigma$ with the controller $u = K(x)$.

In this paper, we study the following two problems.

Problem 1 (Informativity) *Find conditions under which the data $(\dot{\mathcal{X}}, \mathcal{X}, \mathcal{U})$ are informative for stabilization.*

Problem 2 (Controller design) *Suppose the data $(\dot{\mathcal{X}}, \mathcal{X}, \mathcal{U})$ are informative for stabilization. Find a controller $u = K(x)$ satisfying $K(0) = 0$ and (8).*

3 Connection to previous work

Current approaches for data-driven control of polynomial systems [8, 9] build on the model-based method proposed in [15]. These methods do not incorporate prior knowledge and instead focus on designing a common stabilizing controller for all systems compatible with the data. In these works, the controller is considered to be of the form

$$K(x) = Y(x)PZ(x),$$

where $Y \in \mathbb{R}^{m \times p}[x]$, $P \in \mathbb{S}^p$ is positive definite, and $Z \in \mathbb{R}^p[x]$ is radially unbounded satisfying

$$F(x) = H(x)Z(x), \quad (10)$$

for some $H \in \mathbb{R}^{f \times p}[x]$. The choice of candidate Lyapunov function

$$V(x) = Z^\top(x)PZ(x), \quad (11)$$

then leads to

$$\frac{\partial V}{\partial x}(x)(AF(x) + BG(x)K(x)) = 2Z^\top(x)P\Theta(x)PZ(x),$$

where

$$\Theta(x) := \frac{\partial Z}{\partial x}(x) \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} H(x)P^{-1} \\ G(x)Y(x) \end{bmatrix}.$$

The main idea in this line of work is to find P and $Y(x)$ such that

$$-\Theta(x) - \Theta^\top(x) > 0 \quad \forall x \in \mathbb{R}^n \setminus \{0\}, \quad (12)$$

for all systems (A, B) compatible with the data. In the earlier work [8], $H(x)$ is taken to be equal to the identity matrix, which implies that $Z(x) = F(x)$. In contrast, [9] considers more general $Z(x)$ satisfying (10). This strategy is appealing because it leads to data-based linear matrix inequalities for control design. Unfortunately, however, the method also has some major limitations.

- (1) The matrix $\frac{\partial Z}{\partial x}(x)$ must have full row rank for all $x \in \mathbb{R}^n \setminus \{0\}$.

Indeed, suppose that there exists a nonzero x such that $\frac{\partial Z}{\partial x}(x)$ does not have full row rank. Then $\Theta(x)$ is singular, which implies that (12) does not hold. Note that the full row rank condition can only hold if $p \leq n$, i.e., the number of polynomials in Z is less than or equal to the state-space dimension of the system. This limits the class of Lyapunov functions of the form (11) that can be considered by the