

after a fixed choice of parametrization of boundary of \mathcal{D} by \mathbb{R} . (In the case the end of \mathcal{D} is negative, the above is meant to be analogous.)

A Hamiltonian connection \mathcal{A} on $[0, 1] \times M$ uniquely corresponds to a choice of a smooth function $H : [0, 1] \times M \rightarrow \mathbb{R}$, normalized to have mean zero at each moment. For the holonomy path of \mathcal{A} over $[0, 1]$ is a path $\phi_{\mathcal{A}} : [0, 1] \rightarrow \text{Ham}(M, \omega)$, generated by a Hamiltonian $H : [0, 1] \times M \rightarrow \mathbb{R}$, and this uniquely determines the connection. Conversely, H uniquely determines a Hamiltonian connection with holonomy path generated by H . We can say that H **generates** \mathcal{A} .

Lemma 2.26. *Let p and $\mathcal{L}_p \subset \partial\mathcal{D} \times M$ be as in definition above with $L^\pm(\tilde{p}) = \rho$, where \tilde{p} is some lift of p to $\text{Ham}(M, \omega)$, that is $p(t) = \tilde{p}(t)(p(0))$. Let \mathcal{A}_0 be a Hamiltonian connection on $[0, 1] \times M$, generated by a Hamiltonian $H : [0, 1] \times M \rightarrow \mathbb{R}$ with L^\pm length κ , constant for t near $0, 1$. Then there is a Hamiltonian connection $\tilde{\mathcal{A}}_0^p$ on $\mathcal{D} \times M$, preserving \mathcal{L}_p , compatible with respect to \mathcal{A}_0 , and satisfying*

$$\text{area}(\tilde{\mathcal{A}}_0^p) \leq \kappa + \rho.$$

The construction is natural in the sense that $(\tilde{p}, \mathcal{A}_0) \mapsto \tilde{\mathcal{A}}_0^p$ can be made into a smooth map (of Frechet manifolds).

Proof. Let $q : [0, 1] \rightarrow \text{Ham}(M, \omega)$ be the holonomy path of \mathcal{A}_0 , $q(0) = \text{id}$, generated by H . Let $\tilde{p} \cdot q$ be the usual path concatenation in diagrammatic order, and H' be its generating Hamiltonian.

Define a coupling form Ω' on $D^2 \times M$:

$$\Omega' = \omega - d(\eta(\text{rad}) \cdot H' d\theta),$$

for (rad, θ) the modified angular coordinates on D^2 , $\theta \in [0, 1]$, $0 \leq \text{rad} \leq 1$, and $\eta : [0, 1] \rightarrow [0, 1]$ is a smooth function satisfying

$$0 \leq \eta'(\text{rad}),$$

and

$$(2.27) \quad \eta(\text{rad}) = \begin{cases} 1 & \text{if } 1 - \delta \leq \text{rad} \leq 1, \\ \text{rad}^2 & \text{if } \text{rad} \leq 1 - 2\delta, \end{cases}$$

for a small $\delta > 0$. By an elementary calculation

$$\text{area}(\mathcal{A}') = L^+(p \cdot q) = L^+(p) + L^+(q),$$

where \mathcal{A}' is the connection induced by Ω' . Set

$$\text{arc} = \{(1, \theta) \in D^2 \mid 0 \leq \theta \leq 1/2\}.$$

Let arc^c denote the complement of arc in ∂D^2 . Fix a smooth embedding $i : D^2 \hookrightarrow \mathcal{D}$ such that the following is satisfied (see Figure 1):

- The image of the embedding contains $\partial\mathcal{D} - \text{end}$, where end is the image of the distinguished (say positive) strip end chart

$$[0, 1] \times (0, \infty) \rightarrow \mathcal{D}.$$

- $i(\text{arc}) \subset \text{end}^c$,
- $i(\text{arc}^c) \subset \text{end}$.

Next fix a deformation retraction ret of \mathcal{D} onto $i(D^2)$, so that in the strip end chart above, for $r \geq 1$ ret is the composition $i \circ \text{param} \circ \text{pr}$, where

$$\text{pr} : [0, 1] \times (0, \infty) \rightarrow [0, 1]$$

the projection and where

$$\text{param} : [0, 1] \rightarrow \text{arc}^c \subset D^2$$