

around 2000 due to the studies of quantum computers. As well known, Grover's algorithm can find targets much faster than the corresponding classical algorithm, and it can be considered as a quantum walk on a graph [4]. For database search in quantum computers, quantum walks have been focused on in quantum informations [5].

The major study in quantum walks is their finding probabilities and they have been investigated numerically or analytically. Analytic researches have resulted in long-time limit theorems in mathematics. The limit theorems are useful to understand how the quantum walkers distribute in space after they have repeated their unitary processes a lot of times. The first limit theorem was proved in 2002 and it stated a long-time limit distribution which was different from the limit distributions of classical random walks [6]. Since then, many limit theorems have been demonstrated and they told us interesting behavior of quantum walks [7].

In 2015, a remarkable behavior was discovered [8]. A quantum walk held a gap in distribution and the walker was never observed around the launching location. After the paper had been published, two types of quantum walk were reported and they also had a gap in distribution. One of them was a 3-state quantum walk in 2006 and its probability distribution showed a gap and localization [9]. The other was a 2-state walk in 2018 and its model was complicated [10]. Motivated by the quantum walk in 2018, we try to seek a simpler model which makes a probability distribution with a gap. We define a quantum walk in the next section and numerically observe its probability distribution. Since the numerical experiments let us expect that the quantum walk distributes with a gap, we will work on limit distributions in Sect. 3 so that the quantum walk is proved to hold a gap in distribution.

2 Definition of a quantum walk

The quantum walker with two coin states $|0\rangle$ and $|1\rangle$ is supposed to locate at points, whose set is represented by $\mathbb{Z} = \{0, \pm 1, \pm 2, \dots\}$, in superposition. Its system is described on a tensor Hilbert space $\mathcal{H}_p \otimes \mathcal{H}_c$. The Hilbert space \mathcal{H}_p represents the locations and it is spanned by the orthogonal normalized basis $\{|x\rangle : x \in \mathbb{Z}\}$. Also, the Hilbert space \mathcal{H}_c represents the coin states and it is spanned by the orthogonal normalized basis $\{|0\rangle, |1\rangle\}$. Let us define

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad (1)$$

for the Hilbert space \mathcal{H}_c in this paper. Given two unitary operations, the coin states and the position states of the walker are operated with them. More precisely, the system of quantum walk at time t ($= 0, 1, 2, \dots$), represented by $|\Psi_t\rangle \in \mathcal{H}_p \otimes \mathcal{H}_c$, gets a new system at time $t + 1$, represented by $|\Psi_{t+1}\rangle \in \mathcal{H}_p \otimes \mathcal{H}_c$, with unitary operations U_1 and U_2 ,

$$|\Psi_{t+1}\rangle = U_2 U_1 |\Psi_t\rangle, \quad (2)$$