

the crack tip for $\beta = 0$. The greater the positive β is, the higher the reduction is. Similar to Fig. 4, all curves show slightly wavy patterns due to the impact of the density-dependent moduli.

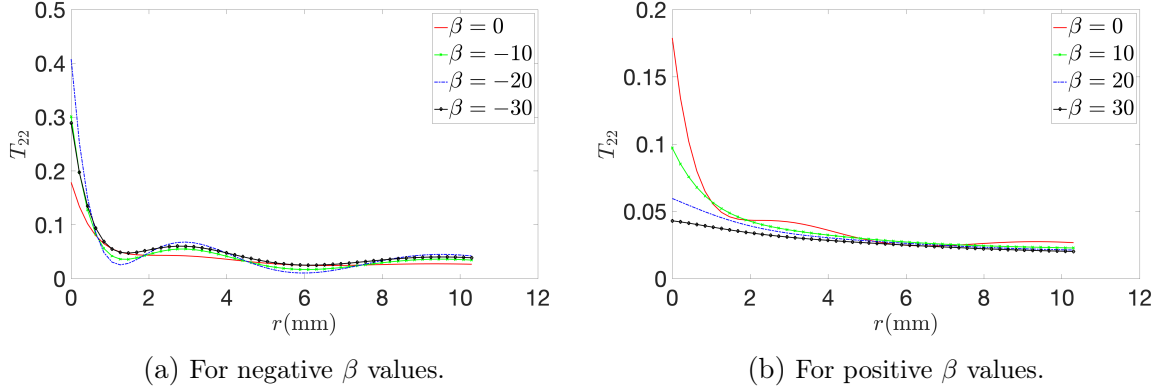


Figure 6: T_{22} vs. r (T_{22} unit: 10^4 Pa). In the first panel, at $r = 0$ (the crack-tip), negative β does not reduce T_{22} compared with T_{22} at $\beta = 0$. Generally, a more negative value does not necessarily generate greater T_{22} at the crack tip. In the second panel for $\beta > 0$, positive β generates smaller T_{22} value compared with T_{22} at $\beta = 0$. Greater β creates smaller T_{22} at $r = 0$. Slightly wavy patterns also appear in all the curves due to the effect of the density-dependent moduli.

5.3 SIF

For the current problem, we do not have an asymptotic or analytical solution. Therefore, an explicit description of crack-tip SIF is not available. However, one can utilize the crack-tip SIF defined for the linear elastic fracture mechanics model and use the finite element solution for the nonlinear model to glean some vital physical insight into SIF.

The SIF K_I in the context of linear elasticity defined as

$$K_I = \lim_{r \rightarrow 0^+} \sqrt{2\pi r} T_{22}. \quad (33)$$

It is challenging to analytically compute K_I for complicated models such as the ones in 3-D that we are investigating in this paper. We compute K_I numerically for illustrative purposes to understand its value as a function of the nonlinear modeling parameter β . The function $\sqrt{2\pi r} T_{22}$ vs. r is shown in Fig. 7. For $\beta = 0, -10, -20, -30$, the K_I values are, respectively, 0.0110, 0.0185, 0.0251, 0.0178; For $\beta = 0, 10, 20, 30$, the K_I values are, respectively, 0.0110, 0.0060, 0.0037, 0.0026 (Unit: $10^4 \text{mm}^{1/2} \text{Pa}$). All the K_I values are very close, implying an identical theoretical K_I value for all β is highly likely. Such numerical approximation for K_I shows that the nonlinear model with density-dependent moduli is designed appropriately under the common crack criteria used for the linear-elasticity fracture model. Also, in the two panels of Fig. 7, the curve shapes for $\beta < 0$ and $\beta > 0$ are slightly distinct. The $\beta < 0$ curves show a greater curvature, while those for $\beta > 0$ show flatter behaviors. Such difference is caused by the sign of β , disclosing how negative and positive β values impact the SIF differently.