



FIGURE 6. Experimentally motivated Plomp-Levelt consonance curve for $f = 440$ Hz, $N = M = 6$.

$$w_c(f, N, A, t) = \sum_{n=1}^N a_n w(nf, t),$$

The next lemma shows that the denominators present in the cosine similarity formula play little role in our considerations under appropriate assumptions on f and N .

Lemma 2. *Let $a_n \in [0, 1]$ for $n = 1, \dots, N$. We have*

$$\left| \int_0^T w_c(f, N, A, t)^2 dt - \frac{NT}{2} \right| \leq \frac{N^2}{2\pi f}$$

Proof. We are looking for a bound on the integral of w_c^2 , that is the following expression

$$\sum_{n=1}^N \sum_{m=1}^N \int_0^T a_n a_m w(nf, t) w(mf, t) dt$$

We obtain the desired bound by looking at the diagonal (i.e. those, where $n = m$) and the non-diagonal entries in the summation, and applying the first and second part of the earlier lemma. For the diagonal entries, we have

$$\frac{NT}{2} - \frac{N}{8\pi f} \leq \sum_{n=1}^N \int_0^T a_n^2 w^2(nf, t) dt \leq \frac{NT}{2} + \frac{N}{8\pi f}$$