

where $\varphi(z)$ is described by the capacitary formula induced by F_0 (see (3.13)). It is worth noting that a similar analysis in the case of non local functionals of convolution type has been performed in a recent paper by R. Alicandro, M.S. Gelli and C. Leone [4]. From a technical point of view we mainly adopt the strategy exploited in [4], which in turn is inspired by [5]. The idea is to use a separation-of-scales argument, formalized in [5] and then deal with the non locality of our functionals to estimate the contribution near the perforations. A crucial role is played by a discrete version of the Gagliardo-Nirenberg-Sobolev inequality, which is derived from the corresponding non local variant proved in [4]. This inequality allows us to show the convergence of minimum problems on unbounded domains, defining the approximating capacitary densities, to the limit energy density defined in (3.13).

The paper is organized as follows. In Section 2 we introduce some notation. In Section 3 we present the setting of the problem and state the main result of the paper. In Section 4 we recall some preliminary results. In Section 5 we state and prove the discrete version of the Gagliardo-Nirenberg-Sobolev inequality and some other results which are instrumental for the proof of the main theorem, which is the core of Section 6.

2 Notation

In what follows $d, m \in \mathbb{N}$ will be two fixed natural numbers denoting the dimension of the reference and the target spaces of the functions we consider, respectively. The set of vectors $\{e_1, \dots, e_d\}$ will denote the standard orthonormal basis in \mathbb{R}^d . Given $t \in \mathbb{R}$, $[t]$ denotes the integer part of t ; for $\alpha \in \mathbb{Z}^d$, $r > 0$, $Q(\alpha, r) = \alpha + (-r/2, r/2)^d$ (if $\alpha = 0$, simply Q_r) is the open cube in \mathbb{R}^d of center α and side length r . We denote by \mathbb{S}^{d-1} the unit sphere in \mathbb{R}^d . If A is a subset of \mathbb{R}^d then $\text{dist}(x, A) = \inf\{|y - x| : y \in A\}$; $\mathcal{A}^{\text{reg}}(A)$ is the family of open subsets with Lipschitz boundary. We use standard notation for Γ -convergence [10]. Unless otherwise stated, C will always denote a generic strictly positive constant that may change from line to line.

3 Setting of the problem and the main result

We fix $p \in (1, d)$ and we let $\Omega \subset \mathbb{R}^d$ be a bounded open set with Lipschitz boundary. For fixed $\varepsilon > 0$, we denote by Ω_ε the lattice $\Omega_\varepsilon := \varepsilon\mathbb{Z}^d \cap \Omega$ and later for a given infinitesimal sequence ε_j we will use the notation $\Omega_j := \Omega_{\varepsilon_j}$. We denote by $\mathcal{A}_\varepsilon(\Omega; \mathbb{R}^m)$ the set of functions

$$\mathcal{A}_\varepsilon(\Omega; \mathbb{R}^m) := \{u : \Omega_\varepsilon \rightarrow \mathbb{R}^m\}.$$

We will identify the functions in $\mathcal{A}_\varepsilon(\Omega; \mathbb{R}^m)$ by their piecewise constant interpolation on the cells of the lattice $\varepsilon\mathbb{Z}^d$ that is

$$\mathcal{A}_\varepsilon(\Omega; \mathbb{R}^m) = \{u : \mathbb{R}^d \rightarrow \mathbb{R}^m : u \text{ constant on } \alpha + [0, \varepsilon)^d \text{ for any } \alpha \in \Omega_\varepsilon\}.$$

Given $\xi \in \mathbb{Z}^d$ and $E \subset \Omega$ we define

$$E_\varepsilon(\xi) := \{\alpha \in E \mid \alpha + \varepsilon\xi \in E\} \cap \varepsilon\mathbb{Z}^d. \quad (3.1)$$

Given a function $v \in \mathcal{A}_\varepsilon(\Omega; \mathbb{R}^m)$, we denote by $D_\varepsilon^\xi v$ the different quotient along the direction ξ ; i.e. for $\alpha \in \Omega_\varepsilon(\xi)$

$$D_\varepsilon^\xi v(\alpha) := \frac{v(\alpha + \varepsilon\xi) - v(\alpha)}{\varepsilon|\xi|} \quad (3.2)$$