

First- and Half-order Schemes for Regime Switching Stochastic Differential Equation with Non-differentiable Drift Coefficient

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Abstract

An explicit first-order drift-randomized Milstein scheme for a regime switching stochastic differential equation is proposed and its bi-stability and rate of strong convergence are investigated for a non-differentiable drift coefficient. Precisely, drift is Lipschitz continuous while diffusion along with its derivative is Lipschitz continuous. Further, we explore the significance of evaluating Brownian trajectories at every switching time of the underlying Markov chain in achieving the convergence rate 1.0 of the proposed scheme. In this context, possible variants of the scheme, namely modified randomized and reduced randomized schemes, are considered and their convergence rates are shown to be 1/2. Numerical experiments are performed to illustrate the convergence rates of these schemes along with their corresponding non-randomized versions. Further, it is illustrated that the half-order non-randomized reduced and modified schemes outperforms the classical Euler scheme.

Keywords: Randomized Milstein scheme, SDEs with Markovian switching, Bi-stability, First- and Half-order schemes, Rate of Convergence.

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1. Introduction

Let $(\tilde{\Omega}, \tilde{\mathcal{F}}, \tilde{\mathbb{P}})$ be a complete probability space. Consider a \tilde{d} -dimensional standard Brownian motion $B := \{B(t)\}_{t \geq 0}$ with natural filtration $\tilde{\mathbb{F}}^B := \{\tilde{\mathcal{F}}_t^B\}_{t \geq 0}$. Further, assume that $r := \{r(t)\}_{t \geq 0}$ is a Markov chain with finite state space $S := \{1, \dots, m'\}$ and generator $\mathcal{Q} := (\mathbf{q}_{j_0 k_0})_{j_0, k_0 \in S}$ where $\mathbf{q}_{j_0 k_0} \geq 0$ for $j_0 \neq k_0$ and $\mathbf{q}_{j_0 j_0} = - \sum_{k_0 \neq j_0} \mathbf{q}_{j_0 k_0}$ which implies that the transition probability matrix of r is given by,

$$\tilde{\mathbb{P}}(r(t + \Delta) = k_0 | r(t) = j_0) = \begin{cases} \mathbf{q}_{j_0 k_0} \Delta + o(\Delta), & k_0 \neq j_0, \\ 1 + \mathbf{q}_{j_0 j_0} \Delta + o(\Delta), & k_0 = j_0, \end{cases}$$

for any $t \geq 0$, $j_0, k_0 \in S$ and $\Delta > 0$. The natural filtration of r is denoted by $\tilde{\mathbb{F}}^r := \{\tilde{\mathcal{F}}_t^r\}_{t \geq 0}$. Also, consider an $\tilde{\mathcal{F}}_0^B$ -measurable random variable X_0 and assume that r , B and X_0 are independent. Define $\tilde{\mathbb{F}} := \{\tilde{\mathcal{F}}_t\}_{t \geq 0}$ where $\tilde{\mathcal{F}}_t := \tilde{\mathcal{F}}_t^B \vee \tilde{\mathcal{F}}_t^r$ for $t \geq 0$. For a fixed $T > 0$, let $b : [0, T] \times \mathbb{R}^d \times S \mapsto \mathbb{R}^d$ and $\sigma : [0, T] \times \mathbb{R}^d \times S \mapsto \mathbb{R}^{d \times \tilde{d}}$ be Borel measurable functions.

Consider the following d -dimensional regime switching stochastic differential equation, also referred to as stochastic differential equation with Markovian switching (SDEwMS),

$$X(t) = X_0 + \int_0^t b(s, X(s), r(s)) ds + \sum_{\ell=1}^{\tilde{d}} \int_0^t \sigma_\ell(s, X(s), r(s)) dB_\ell(s) \quad (1)$$

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