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1 Introduction

In the present paper, we are interested in the phase transition at the critical inverse temperature of the XY model on the three-dimensional square lattice.

To formulate the problem we start by considering a finite subset of a three-dimensional square lattice

$$\mathcal{L} = [-L, L]^3 \cap \mathbb{Z}^3. \quad (1.1)$$

On this lattice the Hamiltonian is defined as

$$H_{L,\nu} := - \sum_{k,l \in \mathcal{L}} J_{k,l} S_k \cdot S_l \quad (1.2)$$

and $\nu = +$ or 0 . Here, depending on the location of the site k , $S_k \in \mathbb{S}^1$ satisfies different conditions: the lattice is decomposed into two parts,

$$\mathcal{L} = \mathcal{L}^\circ \cup \partial\mathcal{L}, \quad (1.3)$$

with \mathcal{L}° being the interior, and $\partial\mathcal{L} = \{(z_1, z_2, z_3) \mid \max\{|z_1|, |z_2|, |z_3|\} = L\}$ the boundary; for $k \in \mathcal{L}^\circ$, the only requirement for S_k is that $|S_k| = 1$; when $k \in \partial\mathcal{L}$ i.e. it is on the boundary, we consider two boundary conditions: when $\nu = 0$, we use the free boundary condition, specifically

$$S_k = 0, \text{ if } k \in \partial\mathcal{L}; \quad (1.4)$$

when $\nu = +$, the plus boundary condition,

$$S_k = (1, 0)^T, \text{ if } k \in \partial\mathcal{L}. \quad (1.5)$$