

- (4) *This is the first time a DTMP is showed for (GE)CMV matrices. Some results in Baire category for certain classes of almost periodic extended CMV matrices were previously obtained by [27, 41]. It is an open question whether it holds for other CMV matrices with, for example, subshift or Sturmian Verblunsky coefficients.*

We base our proof on the recent understanding of Anderson localization for Diophantine frequencies obtained in [20] and techniques developed therein: the Anderson localization for the UAMO in the supercritical setting $\lambda_1 < \lambda_2$ proved in [21] is a full measure result. In [20], an arithmetic version of Anderson localization is proved, albeit for a mosaic model where every other local coin in (2.1) is trivial. However, the proof of [20] works in a straightforward way for UAMO as well, compare also [52].

Theorem 2.4. *Let $\Phi \in \text{DC}(\kappa, \tau)$ and $\lambda_1 < \lambda_2$. Then for each “ Φ -nonresonant” θ , i.e., each θ such that*

$$|\sin 2\pi(\theta + n\Phi)| < \exp(-|n|^{\frac{1}{2\tau}})$$

does not hold for infinitely many n , $W_{\lambda_1, \lambda_2, \Phi, \theta}$ admits Anderson localization.

Proof. In the case $\Phi \in \mathbb{R} \setminus \mathbb{Q}$ and $\lambda_1 < \lambda_2$, according to [21, Theorem 2.9], the Lyapunov exponent characterizing the (typical) decay of generalized eigenfunctions is positive:

$$L_{\lambda_1, \lambda_2, \Phi}(z) \geq \log \left[\frac{\lambda_2(1 + \lambda_1')}{\lambda_1(1 + \lambda_2')} \right] > 0, \quad (2.6)$$

with equality if and only if $z \in \Sigma_{\lambda_1, \lambda_2, \Phi}$. The rest of the proof follows the same outline as the proof of [20, Theorem 6.3]. \square

Remark 2.5. *This result is a full measure result in θ . It is sharp in the sense that it cannot be strengthened to all θ [18].*

We shall also need the following dynamical duality formulation of Aubry-André duality for the UAMO, which can be seen as the reverse statement to [21, Theorem 2.4]. As such, we expect it to be of interest beyond this paper.

Theorem 2.6 (Aubry-André Duality). *Let $\varphi = \varphi^\xi = [\varphi^{\xi, +}, \varphi^{\xi, -}]^\top$, $\xi \in \mathbb{T}$ be a solution to the generalized eigenvalue equation $W_{\lambda_1, \lambda_2, \xi, \Phi}^\# \varphi = z\varphi$ which has the following form*

$$\begin{bmatrix} \varphi_n^{\xi, +} \\ \varphi_n^{\xi, -} \end{bmatrix} = e^{2\pi i n \theta} \begin{bmatrix} \check{\phi}^+(\xi + n\Phi) \\ \check{\phi}^-(\xi + n\Phi) \end{bmatrix} = \frac{1}{\sqrt{2}} e^{2\pi i n \theta} \begin{bmatrix} \check{\psi}^+(\xi + n\Phi) + i\check{\psi}^-(\xi + n\Phi) \\ i\check{\psi}^+(\xi + n\Phi) + \check{\psi}^-(\xi + n\Phi) \end{bmatrix}.$$

Let

$$\begin{bmatrix} \check{\psi}^+ \\ \check{\psi}^- \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} \begin{bmatrix} \check{\phi}^+ \\ \check{\phi}^- \end{bmatrix} \quad (2.7)$$

with n -th Fourier coefficients ψ_n^+ and ψ_n^- , respectively. Then $\psi = [\psi^+, \psi^-]^\top$ solves the eigenvalue equation $W_{\lambda_1, \lambda_2, \Phi, \theta} \psi = z\psi$.

3. PRELIMINARIES

Our proof of Theorem 2.1 utilizes techniques from the theory of one-frequency cocycles of CMV matrices, which we hence review in this section to keep the present treatise as self-contained as possible. We first review the construction of so-called Cantero-Moral-Velázquez matrices (CMV matrices), whose intimate connection with quantum walks on