

Remark 2.10. In Proposition 2.8 and all of our computation examples, there is a symmetry between the x_i and y_i coordinates of $X_{\Delta_n}(\mathbb{N}^{2n})$ for all i . Antoine de Saint-Germain has kindly pointed out to the second author that the $x_i \mapsto y_i$ direction of this symmetry has the following names in different contexts:

- the Auslander–Reiten translate in finite-dimensional representation theory;
- the Fomin–Zelevinsky twist in Lie theory (cf. [dSG23, §5.6];
- the Donaldson–Thomas transformation in Calabi–Yau theory;
- the maximal green sequence in combinatorics.

2.4. Effective bounds. As before, let $A_{\Delta_n} = (a_{i,j})$ be a Cartan matrix for Dynkin type Δ_n . Denote its inverse by $A_{\Delta_n}^{-1} = (a_{i,j}^{-1})$. Define the following associated values:

$$b_{i,\Delta_n} := \prod_{j=1}^n 2^{a_{i,j}^{-1}}$$

$$c_{i,\Delta_n} := \prod_{j=1}^n \left(1 + 2^{\sum_{k \neq i} a_{j,k}}\right)^{a_{i,j}^{-1}}.$$

Muller [Mul23, Proposition 2.3 and Example 3.1] gives bounds on frieze entries in terms of these values.

LEMMA 2.11. [Mul23, Proposition 2.3 and Example 3.1] *Let F be a positive integral Δ_n -frieze. Then there is the following upper bound on the product of entries in its i -th row: $\prod_{j=1}^{P_{\Delta_n}} F_{i,j} \leq b_{i,\Delta_n}^{P_{\Delta_n}}$. Furthermore, if all entries in F are at least 2, then $\prod_{j=1}^{P_{\Delta_n}} F_{i,j} \leq c_{i,\Delta_n}^{P_{\Delta_n}}$.*

Remark 2.12. There is a misprint in [Mul23, Example 3.1]: the bound $(\frac{151875}{16384})^{16} \approx 2^{51}$ on the eighth row of an E_8 -frieze should be $c_{8,E_8}^{16} = (\frac{177347025604248046875}{144115188075855872})^{16} \approx 2^{164}$.

An immediate consequence of Lemma 2.11 and Proposition 2.3 is that if the frieze F corresponds to the point $(x_1, \dots, x_n; y_1, \dots, y_n) \in X_{\Delta_n}(\mathbb{N}^{2n})$, then $x_i \leq b_{i,\Delta_n}^{P_{\Delta_n}}$. Furthermore, if all entries in F are at least 2, then $x_i \leq c_{i,\Delta_n}^{P_{\Delta_n}}$.

The Diophantine model of friezes allows us to find a minimal element in each $\mathbb{Z}/P_{\Delta_n}\mathbb{Z}$ -orbit on which we can reduce existing bounds by a power of $\frac{1}{P_{\Delta_n}}$.

PROPOSITION 2.13. *Let F be a positive integral Δ_n -frieze and fix an $i \in \{1, \dots, n\}$. There is a Δ_n -frieze F' corresponding to $(x_1, \dots, x_n; y_1, \dots, y_n) \in X_{\Delta_n}(\mathbb{N}^{2n})$ such that F is a horizontal translation of F' and $x_i \leq b_{i,\Delta_n}$. Furthermore, if all entries in F are at least 2, then $x_i \leq c_{i,\Delta_n}$.*

Proof. Each Δ_n -frieze F is a horizontal translation of a frieze F' in its $\mathbb{Z}/P_{\Delta_n}\mathbb{Z}$ -orbit such that $F'_{i,1} \leq F'_{i,j}$ for all $1 \leq j \leq P_{\Delta_n}$. By Lemma 2.11, there is a bound on $\prod_{j=1}^{P_{\Delta_n}} F'_{i,j} \leq b_{i,\Delta_n}^{P_{\Delta_n}}$. But $x_i = F'_{i,1}$ is the smallest factor in the product of P_{Δ_n} -many terms, hence $x_i \leq b_{i,\Delta_n}$. If the entries of F are all at least 2, then the entries of the translation F' are also all at least 2. Hence the same argument gives the bound in terms of c_{i,Δ_n} instead of b_{i,Δ_n} . \square