

To derive the asymptotic properties of the MCLEs, we use results obtained in [8] and [9]. In [8] we considered sums of square increments of an isotropic fractional Brownian field based on the edges of the Delaunay triangles and provided asymptotic results using Malliavin calculus (see Theorem 1 in [8]). In [9] we considered sums of square increments of the pointwise maximum of two independent isotropic fractional Brownian fields and showed that the asymptotic behaviors of the sums depend on the local time at the level 0 of the difference between the two fractional Brownian fields and no more on the Gaussian limits of the sums of square increments of each isotropic fractional Brownian field (see Theorem 2 in [9]). In this paper we generalize this result to the max-stable Brown-Resnick random field which is built as the pointwise maximum of an infinite number of isotropic fractional Brownian fields (see Theorem 3). Using approximations of the pairwise and triplewise CL objective functions, we then derive the asymptotic properties of the MCLEs (see Theorem 6). The rates of convergence of the estimators as well as their limit distributions are not standard and are specific to the structure of the max-stable process. It is important to identify them in order to avoid using the Gaussian distributions typically obtained with data from multiple time observations.

It is noteworthy that we only consider isotropic fractional Brownian fields with Hurst index in $(0, 1/2)$ as in [8] and [9]. This is not a very restrictive constraint since almost all empirical studies that use the spatial Brown-Resnick random field obtain estimates in this interval (see e.g. [12, 13, 19, 20]).

Our paper is organized as follows. In Section 2 we present the family of stationary Brown-Resnick random fields introduced in [27], then we review some established concepts related to the Delaunay triangulation, and we end with the definition of the local time between two independent and identically distributed fractional Brownian random fields. In section 3, we first study the asymptotic distributions of the “normalized” increments of the logarithm of the Brown-Resnick random field based on pairs and triples of sites as the distances between sites tend to zero. We then provide asymptotic results for squared increment sums of the max-stable Brown-Resnick random field. In Section 4, we introduce the randomized sampling scheme and define the CL estimators of the scale and Hurst parameters. The asymptotic properties of the MCLEs for these parameters are then given. The proofs and some intermediate results are deferred to Section 5 and Section 6.

2 Preliminaries

2.1 Max-stable Brown-Resnick random fields

In this paper we are concerned with the class of max-stable random fields known as Brown-Resnick random fields introduced in [27]. This class of random fields is based on Gaussian random fields with linear stationary increments. Recall that a random process $(W(x))_{x \in \mathbf{R}^d}$ is said to have linear stationary increments if the law of $(W(x+x_0) - W(x_0))_{x \in \mathbf{R}^d}$ does not depend on the choice of $x_0 \in \mathbf{R}^d$. A prominent example is the isotropic fractional Brownian field where $W(0) = 0$ a.s. and semi-variogram given by

$$\gamma(x) = \frac{\text{var}(W(x))}{2} = \frac{\sigma^2 \|x\|^\alpha}{2} \quad (2.1)$$

for some $\alpha \in (0, 2)$ and $\sigma^2 > 0$, where $\|x\|$ is the Euclidean norm of x . The parameter σ is called the scale parameter while α is called the range parameter ($H = \alpha/2$ is also known as the Hurst parameter and relates to the Hölder continuity exponent of W). It is noteworthy that W is a self-similar random