

which converges in $\mathcal{O}_{\mathcal{M}_{\text{LT},\infty}}^{L\text{-la}}(U)$. Later we will show $D_i(s)$ actually lies in the smaller subspace $\mathcal{O}_{\mathcal{M}_{\text{LT},\infty}}^{L\text{-la},\mathbf{m}^0=0}(U)$. Using $E_{d,i}(x_i - x_{i,n}) = 1$, a direct computation shows $E_{d,i}D_i(s) = 0$ for all $s \in \mathcal{O}_{\mathcal{M}_{\text{LT},\infty}}^{L\text{-la},\mathbf{m}^0=0}(U)$, and

$$s = \sum_{l=0}^{\infty} a_{i,l}(x_i - x_{i,n})^l$$

with $a_{i,l} = D_i\left(\frac{(E_{d,i})^l \cdot s}{l!}\right)$. Here we note the coefficients $a_{i,l}$ are determined by s . From the construction we know the coefficients $a_{i,l}$ are killed by $E_{d,i}$ for all l . For $j \neq i$, we further express the coefficients $a_{i,l}$ along $E_{d,j}$, namely

$$a_{i,l} = \sum_{k=0}^{\infty} b_{i,l,j,k}(x_j - x_{j,n})^k$$

with $b_{i,l,j,k} = D_j\left(\frac{(E_{d,i})^l \cdot a_{i,l}}{k!}\right)$. As $[E_{d,i}, E_{d,j}] = 0$ for $i, j = 0, 1, \dots, d-1$, we see $b_{i,l,j,k}$ is killed by $E_{d,i}$ and $E_{d,j}$. After we expand s along each derivation $E_{d,i}$ for $i = 0, 1, \dots, d-1$, we arrive at an expression

$$s = \sum_{i_0=0}^{\infty} \sum_{i_1=0}^{\infty} \cdots \sum_{i_{d-1}=0}^{\infty} c_{i_0 \dots i_{d-1}}(x_0 - x_{0,n})^{i_0} \cdots (x_{d-1} - x_{d-1,n})^{i_{d-1}}$$

such that $E_{d,i}c_{i_0 \dots i_{d-1}} = 0$ for all $i = 0, 1, \dots, d-1$. As \mathbf{m}^0 acts trivially on $\mathcal{O}_{\mathcal{F}\ell}$, we know \mathbf{m}^0 acts trivially on $x_i - x_{i,n}$ for any $i = 0, \dots, d-1$. Therefore,

$$0 = \sum_{i_0, \dots, i_{d-1}} (\mathbf{m}^0 c_{i_0 \dots i_{d-1}})(x_0 - x_{0,n})^{i_0} \cdots (x_{d-1} - x_{d-1,n})^{i_{d-1}}$$

This implies $\mathbf{m}^0 c_{i_0 \dots i_{d-1}} = 0$. Indeed, the coefficients $c_{i_0 \dots i_{d-1}}$ are determined by s . Combined this with the fact that $\mathcal{O}_{\mathcal{M}_{\text{LT},\infty}}^{L\text{-la}}$ is killed by $\pi^{-1}\mathbf{n}^0$, we see \mathbf{p}^0 kills $c_{i_0 \dots i_{d-1}}$. Hence there is an induced action of $\bar{\mathbf{n}}^0 = \mathbf{g}^0/\mathbf{p}^0$ on $c_{i_0 \dots i_{d-1}}$. On U , $\bar{\mathbf{n}}^0$ is generated by $E_{d,i}$ for $i = 0, 1, \dots, d-1$. Indeed, on the open locus $V_o = \{[z_0 : z_1 : \dots : z_{d-1} : 1]\} \subset \mathbb{P}^d$, the matrix $Z = \begin{pmatrix} I_{d \times d} & 0 \\ (z_0, z_1, \dots, z_{d-1}) & 1 \end{pmatrix}$ is a lifting of the point $z = [z_0 : z_1 : \dots : z_{d-1} : 1]$ as $oZ = z$ where $o = [0 : 0 : \dots : 0 : 1]$. Then $Z^{-1}E_{d,i}Z = E_{d,i}$ for $i = 0, 1, \dots, d$ generates $\bar{\mathbf{n}}^0$ on V_o . From the construction we know $c_{i_0 \dots i_{d-1}}$ is killed by $E_{d,i}$ for all $i = 0, 1, \dots, d-1$, hence it is killed by $\bar{\mathbf{n}}^0$. This shows that \mathbf{g}^0 acts trivially on $c_{i_0 \dots i_{d-1}}$, which implies $c_{i_0 \dots i_{d-1}} \in \mathcal{O}_{\mathcal{M}_{\text{LT},\infty}}^{\text{sm}}(U)$. \square

Corollary 0.3.2. For any $U \in \mathfrak{B}_{\text{LT}}$, the image of $\mathcal{O}_{\mathcal{M}_{\text{LT},\infty}}^{\text{sm}}(U) \otimes_C \pi^{-1}\mathcal{O}_{\mathcal{F}\ell}(U)$ inside $\mathcal{O}_{\mathcal{M}_{\text{LT},\infty}}^{L\text{-la},\mathbf{m}^0=0}(U)$ is dense.

Proof. This follows directly from Theorem 0.3.1. \square

To simplify the notation, we denote \mathcal{O}_{LT} by the sheaf $\mathcal{O}_{\mathcal{M}_{\text{LT},\infty}}^{L\text{-la},\mathbf{m}^0=0}$ on $\mathcal{M}_{\text{LT},\infty}$. Put $\mathcal{O}_{\text{LT}}^{\text{sm}} := \mathcal{O}_{\mathcal{M}_{\text{LT},\infty}}^{G\text{-sm}}$. Let $\Omega_{\mathcal{M}_{\text{LT},n}}^k$ be the sheaf of k -differential forms on $\mathcal{M}_{\text{LT},n}$ for $k = 0, 1, \dots, d$ and put $\Omega_{\text{LT}}^{k,\text{sm}} := \varinjlim_n \pi_n^{-1}\Omega_{\mathcal{M}_{\text{LT},n}}^k$ with $\pi_n : \mathcal{M}_{\text{LT},\infty} \rightarrow \mathcal{M}_{\text{LT},n}$. Clearly $\mathcal{O}_{\text{LT}}^{\text{sm}} = \Omega_{\text{LT}}^{0,\text{sm}}$ and $\Omega_{\text{LT}}^{k,\text{sm}} = \wedge_{\Omega_{\text{LT}}^{\text{sm}}}^k \Omega_{\text{LT}}^{1,\text{sm}}$.

Proposition 0.3.3. There exists a differential operator $d : \mathcal{O}_{\text{LT}} \rightarrow \mathcal{O}_{\text{LT}} \otimes_{\mathcal{O}_{\text{LT}}^{\text{sm}}} \Omega_{\text{LT}}^{1,\text{sm}}$, such that

- (i) d is given by the usual derivation on $\mathcal{O}_{\text{LT}}^{\text{sm}}$,
- (ii) d is $\pi^{-1}\mathcal{O}_{\mathcal{F}\ell}$ -linear.

Moreover, d is uniquely determined by these two properties up to constants.

Proof. Define $d|_{\mathcal{O}_{\text{LT}}^{\text{sm}}}$ as the differential map on finite levels $\mathcal{O}_{\text{LT}}^{\text{sm}} \rightarrow \Omega_{\text{LT}}^{1,\text{sm}}$, and define $d|_{\pi^{-1}\mathcal{O}_{\mathcal{F}\ell}}$ to be the zero map. By Theorem 0.3.1, for any $U \in \mathfrak{B}_{\text{LT}}$ such that $z_d \neq 0$ on U , we may write any section $s \in \mathcal{O}_{\text{LT}}(U)$ of the form for some sufficiently large n :

$$s = \sum_{i_0=0}^{\infty} \sum_{i_1=0}^{\infty} \cdots \sum_{i_{d-1}=0}^{\infty} c_{i_0 \dots i_{d-1}}(x_0 - x_{0,n})^{i_0} \cdots (x_{d-1} - x_{d-1,n})^{i_{d-1}}$$