

Therefore,

$$\begin{aligned}\mathcal{I}_E(z) &= \frac{1}{(2\pi)^3} \int_{\mathbb{S}^2} \int_0^{+\infty} \mathcal{F}[J](k\hat{x}) e^{ik\hat{x}\cdot z} k^2 dk ds_{\hat{x}} \\ &= \mathcal{F}^{-1} \mathcal{F}[J] \\ &= J(z), \quad z \in \mathbb{R}^3.\end{aligned}$$

The proof is complete.  $\square$

Note that, different from the acoustic and elastic source scattering problems, there exists non-radiating sources for the electromagnetic waves. Precisely, the electromagnetic far field patterns may vanish for the sources  $J$  satisfying  $\operatorname{div} J \neq 0$ . We refer to [7, 17] for more details on the non-radiating electromagnetic sources. Physically,  $\rho := \frac{1}{i\omega} \operatorname{div} J$  is the charge density. We define

$$\mathcal{I}_H(z) := \frac{\sqrt{\mu}}{2\pi^2} \int_{\mathbb{S}^2} \int_0^{+\infty} H_{\infty}(\hat{x}, k) e^{ik\hat{x}\cdot z} k^2 dk ds_{\hat{x}}, \quad z \in \mathbb{R}^3, \quad (3.18)$$

$$\mathcal{I}_{\rho}(z) := \frac{1}{(2\pi)^3} \int_{\mathbb{S}^2} \int_0^{+\infty} (\omega \mathcal{F}[\rho](k\hat{x}) - 4\pi i \sqrt{\varepsilon} E_{\infty}(\hat{x}, k)) e^{ik\hat{x}\cdot z} k dk ds_{\hat{x}}, \quad z \in \mathbb{R}^3. \quad (3.19)$$

Following the arguments in the proof of Theorem 3.1, we obtain the following theorem. To avoid repetition, we omit the proof.

**Theorem 3.2.** *For  $J \in (H^1(\mathbb{R}^3))^3$ , we have*

$$\mathcal{I}_H = \operatorname{curl} J.$$

*If we know the charge density  $\rho \in L^2(\mathbb{R}^3)$  in advance, we have*

$$\mathcal{I}_{\rho} = J.$$

We finally remark that, without any a priori information on the charge density, we can always reconstruct  $\operatorname{curl} J$  from the magnetic far field patterns.

## 4 Stability estimates for the discrete indicators

Definitely, the theorems in the previous two sections show that the indicators  $\mathcal{I}_f, \mathcal{I}_p, \mathcal{I}_s, \mathcal{I}_E$  and  $\mathcal{I}_H$  can be used to determine the full or partial information of unknown sources. In practice, the far field patterns are taken for finitely many observation directions and frequencies. Therefore, we have to consider the indicators in the form of a finite sum. In this section, we derive the stability analyses for such indicators.

We begin with the elastic source reconstructions in  $\mathbb{R}^2$ . The observation direction set is defined by

$$\Theta_L := \left\{ \left( \cos \frac{2l\pi}{L}, \sin \frac{2l\pi}{L} \right)^T \mid l = 0, 1, \dots, L-1 \right\}.$$

We take the circular frequencies

$$\omega_m = m\Delta\omega, \quad m = 1, 2, \dots, \Lambda.$$