

Many examples satisfying (H) are given in Section 2.2, including normal and Cauchy random variables.

Moreover, a second contribution is that, as we show in Theorem 4.2, the averaged exact controllability does not hold in the L^2 set up, which is a surprising result because one would expect that such property would be inherited from the exact controllability properties of the Schrödinger equation.

Finally, a third contribution, as we show in Theorem 4.4, is that the preimage of 0 under $\xi \mapsto y(T, \cdot; \xi; y_0; u)$ is finite for all $y_0 \in L^2(G) \setminus \{0\}$ and $u \in L^2((0, T) \times G_0)$. In particular, the event: “the system (1.1) is simultaneous null controllable” is negligible for absolutely continuous random variables, that is,

$$\mathbb{P}[\omega : y(T, \cdot; \alpha(\omega); y_0; u) = 0] = 0.$$

A brief overview of the related literature. Regarding the control of averaged properties, the problem and its abstract formulation were first presented in the work [30], where finite-dimensional systems were considered. The pioneering study on averaged controllability in the context of partial differential equations (PDEs) was introduced in [23]. In this study, the authors addressed the problem of controlling the average of PDEs and examined the properties of the transport, heat, and Schrödinger equations in specific scenarios. For the Schrödinger equation, they analyzed the equation when the diffusivity follows uniform, exponential, normal, Laplace, chi-square, and Cauchy probability laws. In [31] the continuous average of the heat equation was considered. Further contributions, such as [17] and [13] investigated perturbations of probability density functions modeled by Dirac masses. In [8] and [4] the controllability of the heat equation with random diffusivity was studied. In the second paper, it was discovered that some probability density functions caused fractional dynamics for the heat equation. Moreover, we would like to point out that there are many known results for lower-order random terms, as shown in the survey [22] and the book [21], and some of them are recent results for nonlinear parabolic stochastic equations such as in [10] and [11].

Regarding the controllability of the Schrödinger equation, there are many classical papers such as [14], [24], [27], [7], and [2], involving geometric control conditions. As for simultaneous control, we would like to highlight [26], where there is a bilinear control, and [18], where cascade-like systems are studied. Also, for stochastic equations with random lower-order terms, we may highlight, for example, [20]. Finally, as mentioned above, for random diffusivity, it was studied in [23] for specific probability laws.

Our contribution to the literature is to obtain the averaged controllability of the Schrödinger equation with random diffusivity in a more general setting and to determine how much we can generalize the known results.

Structure of this paper. In Section 2, we introduce several key concepts along with their characterization, and analyze the main assumption of the paper. Section 3 is devoted to proving the main result on the averaged null controllability of the system (1.1). In Section 4, we prove the lack of exact average for general absolutely continuous random variables and of simultaneous controllability. In Section 5, we present a numerical method to validate our theoretical result in the case of Cauchy and normal distributions. Finally, Section 6 is dedicated to the conclusion and outlines several potential directions for future research.

Notation: Throughout this paper, we adopt some standard notations and usual abbreviations in