

for some $0 < \omega_0 < \tilde{\omega}$ and sufficiently small $\epsilon \in \mathbb{R}$. The spectral structure in combination with resolvent bounds proven uniformly in the parameter imply

$$\|\mathbf{S}_\epsilon(\tau)(\mathbf{I} - \mathbf{P}_\epsilon)\mathbf{u}\|_{s,k} \lesssim e^{-\omega_0 \tau} \|(\mathbf{I} - \mathbf{P}_\epsilon)\mathbf{u}\|_{s,k},$$

and $\mathbf{P}_\epsilon \mathbf{S}_\epsilon(\tau) = e^\tau \mathbf{P}_\epsilon$ for \mathbf{P}_ϵ denoting the spectral projection onto the eigenspace spanned by \mathbf{g}_ϵ , see [30], Theorem A.1. For the nonlinearity $\widehat{\mathbf{N}}_\epsilon$, we establish Lipschitz bounds by imposing again the above assumptions (1.19) on the Sobolev exponents and apply the estimates of Appendix A. With these results at hand we construct in Section 4.2 global, exponentially decaying strong $\mathcal{H}_r^{s,k}$ -solutions of Eq. (1.20). For this, we use the standard approach and first suppress the exponential growth induced by the symmetry eigenvalue $\lambda = 1$ by a correction with values in $\text{ran} \mathbf{P}_\epsilon$. In a second step we account for this using the T -dependence of the initial condition to determine the suitable blowup time T_ϵ . In Proposition 4.10 we upgrade the constructed strong solutions to classical ones. By defining

$$v_\epsilon^T(t, x) := \frac{1}{T-t} \psi_\epsilon \left(\frac{x}{T-t} \right), \quad \psi_\epsilon(\xi) := |\xi|^{-1} f_\epsilon(|\xi|) \quad (1.21)$$

for $x \in \mathbb{R}^n$ and $t \in [0, T)$ we get the following result.

Theorem 1.6. *Let $n \geq 5$ and choose $\epsilon^* > 0$ as in Theorem 1.2. For $\epsilon \in \mathbb{R}$, $|\epsilon| \leq \epsilon^*$, let v_ϵ^T be defined as in Eq. (1.21) and let $(s, k) \in \mathbb{R} \times \mathbb{N}$ satisfy*

$$\frac{n}{2} - 1 < s \leq \frac{n}{2} - 1 + \frac{1}{2n-2}, \quad k > n. \quad (1.22)$$

Then there exist $\omega > 0$ and $0 < \bar{\epsilon} \leq \epsilon^$ such that for every $\epsilon \in \mathbb{R}$, $|\epsilon| \leq \bar{\epsilon}$ there are $\delta > 0$ and $M > 1$ such that the following holds: For any pair of radial, real-valued functions $\varphi_0, \varphi_1 \in \mathcal{S}(\mathbb{R}^n)$ satisfying*

$$\|(\varphi_0, \varphi_1)\|_{\dot{H}^s \cap \dot{H}^k(\mathbb{R}^n) \times \dot{H}^{s-1} \cap \dot{H}^{k-1}(\mathbb{R}^n)} < \frac{\delta}{M}, \quad (1.23)$$

there exists $T = T_\epsilon \in [1 - \delta, 1 + \delta]$ and a unique radial solution $v \in C^\infty([0, T) \times \mathbb{R}^n)$ to Eq. (1.14) with

$$v(0, \cdot) = v_\epsilon^1(0, \cdot) + \varphi_0, \quad \partial_t v(0, \cdot) = \partial_t v_\epsilon^1(0, \cdot) + \varphi_1.$$

Moreover, v blows up at $(T, 0)$ and can be decomposed as

$$v(t, x) = v_\epsilon^T(t, x) + \frac{1}{T-t} \varphi \left(\log \left(\frac{T}{T-t} \right), \frac{x}{T-t} \right)$$

for all $(t, x) \in [0, T) \times \mathbb{R}^n$, where $\varphi \in C^\infty([0, T) \times \mathbb{R}^n)$ is radially symmetric and satisfies

$$\|\varphi(-\log(T-t) + \log T, \cdot)\|_{\dot{H}^r(\mathbb{R}^n)} \lesssim \delta(T-t)^\omega \quad (1.24)$$

for all $r \in [s, k]$. Furthermore,

$$\|(\partial_0 + \Lambda + 1)\varphi(-\log(T-t) + \log T, \cdot)\|_{\dot{H}^{r-1}(\mathbb{R}^n)} \lesssim \delta(T-t)^\omega. \quad (1.25)$$

Finally, we rephrase the results of Theorem 1.6 in terms of normal coordinates using the equivalence of norms of corotational maps and their radial profiles, see [23], to obtain Theorem 1.3.