

is the origin of Y). Then $D^b(\text{Unip}(X)) \simeq D^b(\text{Coh}_{\{\hat{0}\}}(Y))$ and, by Corollary 2.3 and Proposition 3.6, there is no smooth projective variety M such that $D^b(\text{Unip}(X))$ is equivalent to $D^b(\text{Coh}(M))$.

In the positive-dimensional case the characterization of point like objects in $D_Z^b(\text{Coh}(X))$ is possible under a positivity assumption on the restriction of the canonical bundle to the support.

Proposition 3.8. *Suppose $\omega_X|_Z$ or $\omega_X^{-1}|_Z$ is ample. Then the point like objects of $D_Z^b(\text{Coh}(X))$ are the objects isomorphic to $\iota_Z^! \mathbb{C}(p)[r]$ where $p \in Z$ is a closed point and $r \in \mathbb{Z}$.*

Proof. Let P be a point like object in $D_Z^b(\text{Coh}(X))$ and denote by \mathcal{H}^j the cohomology sheaves of $\iota_Z P$. Note that the \mathcal{H}^j 's are coherent sheaves on X supported in Z . Since $S_Z(P) \simeq P[\dim X]$, we have the following isomorphisms:

$$\begin{aligned} \iota_Z^! (\iota_Z P \otimes \omega_X) &\simeq P \\ \iota_Z \iota_Z^! (\iota_Z P \otimes \omega_X) &\simeq \iota_Z P \\ \iota_Z \iota_Z^! \iota_Z P \otimes \omega_X &\simeq \iota_Z P \\ \iota_Z P \otimes \omega_X &\simeq \iota_Z P. \end{aligned}$$

By taking cohomology we have

$$(3) \quad \mathcal{H}^j \otimes \omega_X \simeq \mathcal{H}^j \quad \text{for all } j.$$

Now we show that \mathcal{H}^j is supported in dimension zero for any j . Suppose $\omega_X|_Z$ is ample, the other case being similar. Let $k > 0$ be an integer such that $N := \omega_X^{\otimes k}|_Z$ is very ample and let $i: Z \hookrightarrow X$ be the inclusion map (here Z is equipped with the reduced induced subscheme structure). Then the Hilbert polynomial of $P_{i^* \mathcal{H}^j}(m) = \chi(i^* \mathcal{H}^j \otimes N^{\otimes m})$ has degree equal to

$$s_j := \dim \text{Supp}(i^* \mathcal{H}^j) = \dim \text{Supp}(\mathcal{H}^j)$$

([Ser55, p. 276, Proposition 6]). Moreover, by tensoring (3) with positive powers of ω_X and by restricting the isomorphisms to Z , we find

$$i^* \mathcal{H}^j \otimes N \simeq i^* \mathcal{H}^j \quad \text{for all } j.$$

Therefore $P_{i^* \mathcal{H}^j}(m) = P_{i^* \mathcal{H}^j \otimes N}(m) = P_{i^* \mathcal{H}^j}(m+1)$ for all $m \in \mathbb{Z}$ which is impossible if $\deg P_{i^* \mathcal{H}^j} > 0$. Hence $s_j = 0$ for all j and $\iota_Z P \simeq \mathbb{C}(p)[r]$ for some closed point $p \in Z$ and $r \in \mathbb{Z}$ by Lemma 3.4. It follows that $P \simeq \iota_Z^! \mathbb{C}(p)[r]$.

□

Example 3.9. We construct further instances of equivalences between derived categories with support extending the equivalences (1). Denote by

$$R: \text{Aut}^0(X) \times \text{Pic}^0(X) \rightarrow \text{Aut}^0(Y) \times \text{Pic}^0(Y)$$

the Rouquier isomorphism induced by an equivalence $F: D^b(\text{Coh}(X)) \rightarrow D^b(\text{Coh}(Y))$ (cf. [Rou11, Théorème 4.18] or [Huy06, Proposition 9.45], and [PS11, p.531, footnote (1)]). By following [Lom14, Proposition 3.1], if $\alpha \in \text{Pic}^0(X)$ is a topologically trivial line bundle such that $H^0(X, \omega_X^{\otimes k_0} \otimes \alpha) \neq 0$ for some $k_0 \in \mathbb{Z}$, then $R(\text{id}_X, \alpha) = (\text{id}_Y, \beta)$ for some $\beta \in \text{Pic}^0(Y)$ and moreover there are isomorphisms $R_k: H^0(X, \omega_X^{\otimes k} \otimes \alpha) \rightarrow H^0(Y, \omega_Y^{\otimes k} \otimes \beta)$ for all $k \in \mathbb{Z}$. As in [Tod06], one can prove that if $E \in |\omega_X^{\otimes k} \otimes \alpha|$ and $E' = R_k(E) \in |\omega_Y^{\otimes k} \otimes \beta|$ is the corresponding divisor, then F restricts to an equivalence of triangulated categories $D_{\text{Supp}(E)}^b(\text{Coh}(X)) \simeq D_{\text{Supp}(E')}^b(\text{Coh}(Y))$.