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0. Executive Summary

0.1 Data Set

This Project analyzes 5 years of monthly closing price data from the end of October 2014 through the end of October 2019.

0.2 Mutual Fund Descriptions

a. S&P 500 index: vfinx

   The fund employs an indexing investment approach designed to track the performance of the Standard & Poor's 500 Index, a widely recognized benchmark of U.S. stock market performance that is dominated by the stocks of large U.S. companies. European stock index: veurx

b. Emerging markets fund: veiex

   The fund employs an indexing investment approach by investing all, or substantially all, of its assets in the common stocks included in the FTSE Developed Europe All Cap Index. The index is a market-capitalization-weighted index.

c. Long-term bond fund: vbltx

   This index measures the performance of a wide spectrum of public, investment-grade, taxable, fixed income securities in the United States-including government, corporate, and international dollar-denominated bonds, as well as

mortgage-backed and asset-backed securities-all with maturities of more than 1 year.

d. Short-term bond fund: vbisx

   This index includes all medium and larger issues of U.S. government, investment-grade corporate, and investment-grade international dollar-denominated bonds that have maturities between 1 and 5 years and are publicly issued.

e. Pacific stock index: vpacx

   The fund employs an indexing investment approach by investing all, or substantially all, of its assets in the common stocks included in the FTSE Developed Asia Pacific All Cap Index. The FTSE Developed Asia Pacific All Cap Index is a market-capitalization weighted index.

0.3 Main Findings

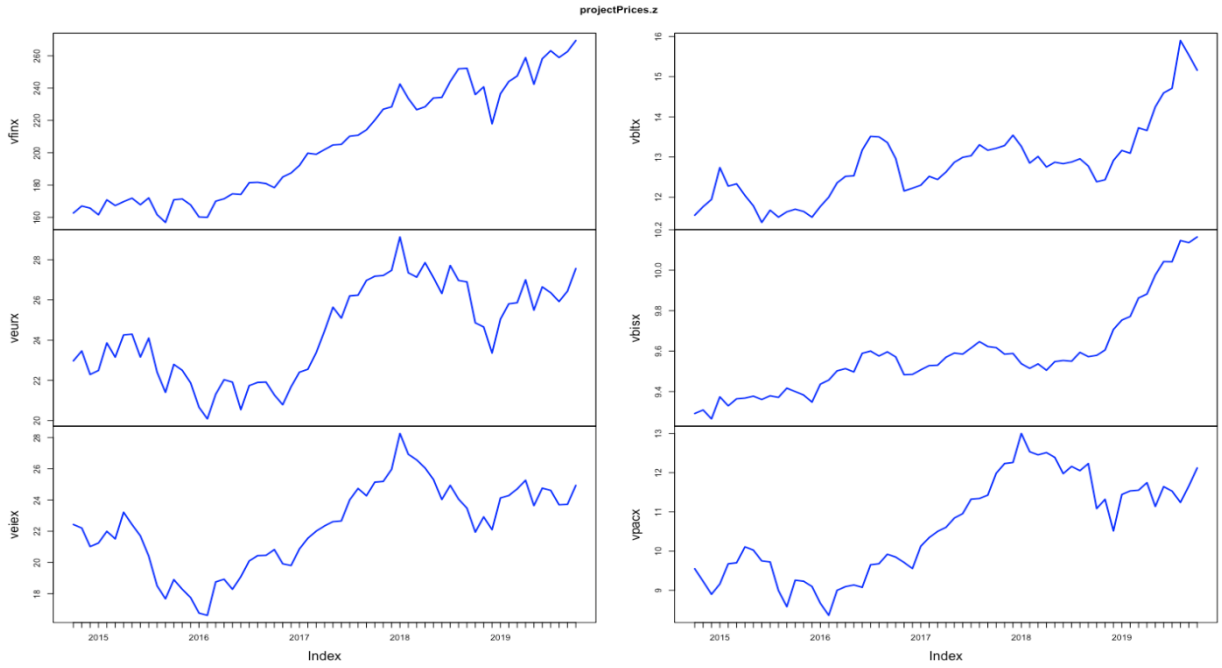
- The prices and continuously compounded returns of all six mutual funds experienced drop with different extents from 2018. After the end of 2018, all six mutual funds revived and showed a similar increasing trend.
- None of the indexes has a perfect normal distribution.
- S&P 500 index has the highest average return and Short-term bond fund has the lowest average return.

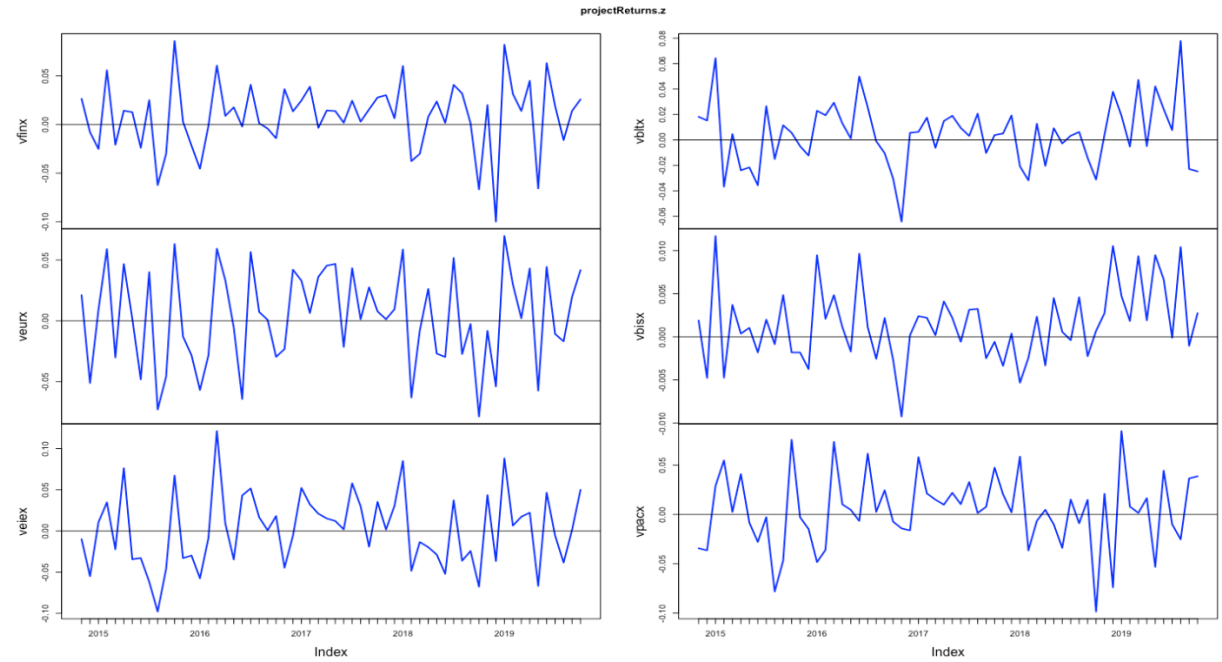
- The tradeoff between risk and volatility didn't perform perfectly in the six assets, but the asset with higher expected return has a lower volatility.
- Long-term bond fund has the highest Sharpe ratio and Emerging markets fund has the lowest Sharpe ratio.
- The correlations between VBLTX and VFINX, VBISX and VFINX, VBLTX and VEURX, VBISX, and VEURX, VBLTX and VPACX, VBISX and VPACS are negative. Other pairs are positively correlated.
- VPACX and VEIEX are most highly correlated, but VEURX and VBLTX are least correlated. However, it's a wise strategy to use them for the purpose of reducing risks.
- VEIEX has the highest Value-at-Risk losses, while VBISX has the least Value-at-Risk losses.
- Global minimum Variance Portfolio has less Value-at-Risk losses than any of the individual asset. The Sharpe Ratio in the Global minimum Variance Portfolio not allowing for short sales is smaller than that in the Global minimum Variance portfolio allowing short sales.
- The expected return of the tangency portfolio not allowing for short sales is lower than the expected return of the tangency portfolio allowing for short sales. The volatility of returns in the tangency portfolio not allowing for short sales is larger than the in the the tangency portfolio allowing for short sales. Sharpe Ratio in the tangency portfolio not allowing for short sales is smaller than that in the tangency portfolio with short sales.

- If only containing the risky assets without safe asset such as T-Bills, the short sales are also not allowed, there is a limit to the target expected return when allocating the assets using efficient portfolio.

1. Return calculations and Sample Statistics

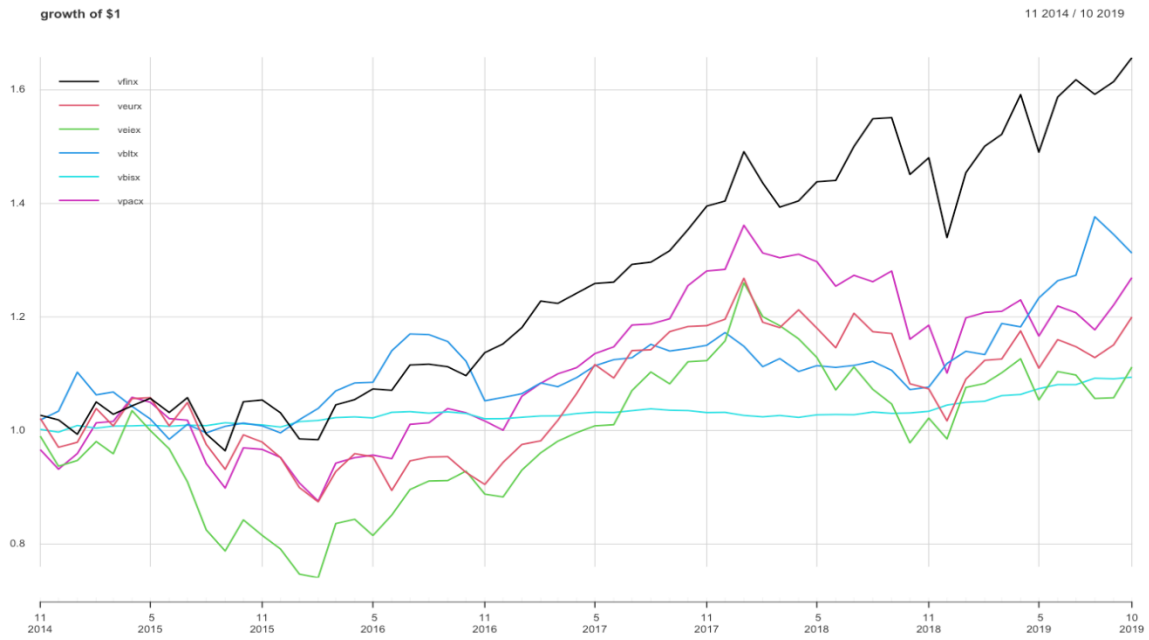
1.1 Time Plots of Monthly Prices+and Returns





The monthly prices and continuously compounded returns of the six mutual funds in the plots above seem to move together. In general, they all show an increasing trend over 2015 to 2019 although there are some fluctuations between the months of years. The prices and continuously compounded returns of all six mutual funds experienced drop with different extents from 2018. The decreasing trend of European stock index (veurs), Emerging markets fund (veiex) and Pacific stock index (vpacx) are most severe. The rest three assets had a relatively smoothie decreases. The reason for this unusual decreasing prices and returns may be due to the trade war and interest rising since the end of 2018. Fortunately, after the end of 2018, all six mutual funds revived and showed a similar increasing trend.

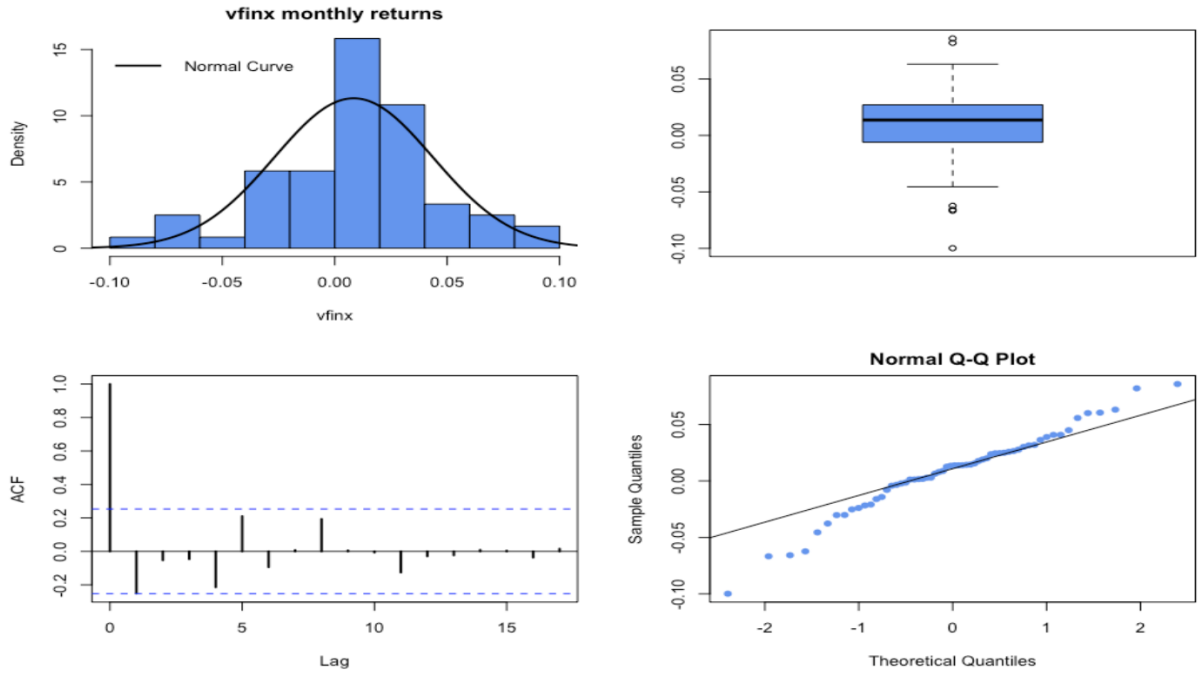
In addition, Long-term bond fund (vbltx) and Short-term bond fund (vbisx) move closely together. Both of them show great increasing trend but the prices and continuously compound returns of both mutual funds experienced a slight and short-time drop from the midyear of 2016 to the start of 2017.



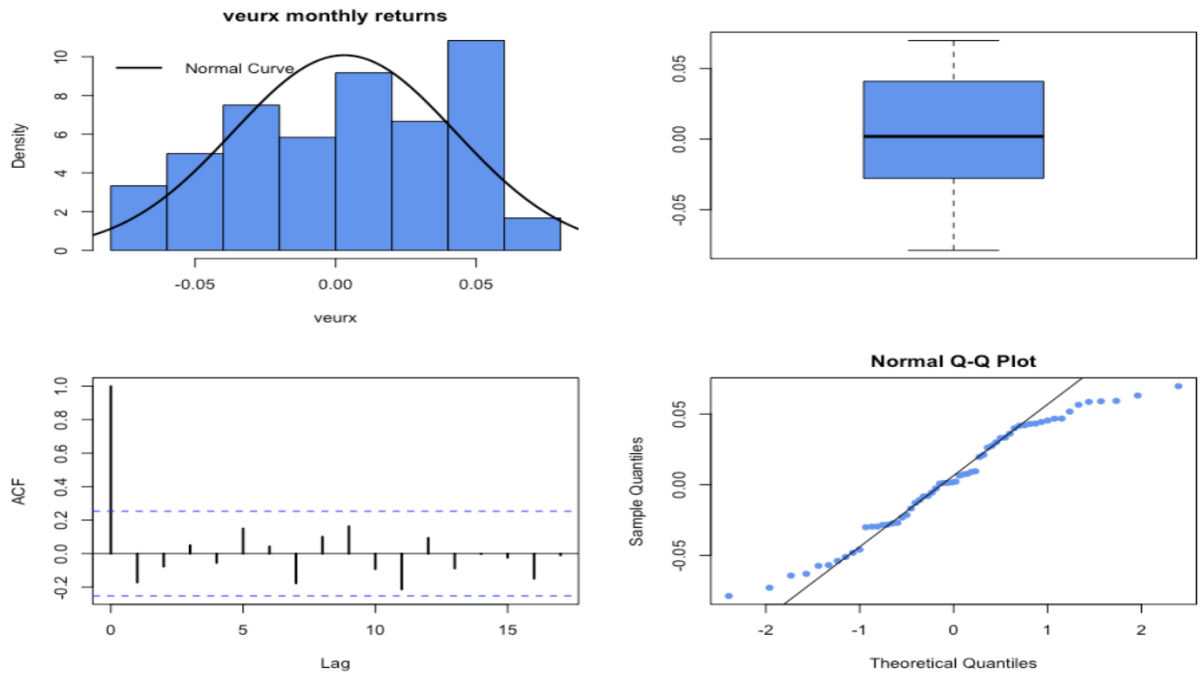
The equity curve above shows the trend of the growth of $1 in each of the six mutual funds over the five year period. S&P 500 index (vfinx) gives the highest future value with a growth over $1.6 for each $1. This is not surprised because S&P 500 include the best stable and wealthier stocks and the pay-off investments are very wise.

1.2 Normality of Monthly Continuously Compounded Returns

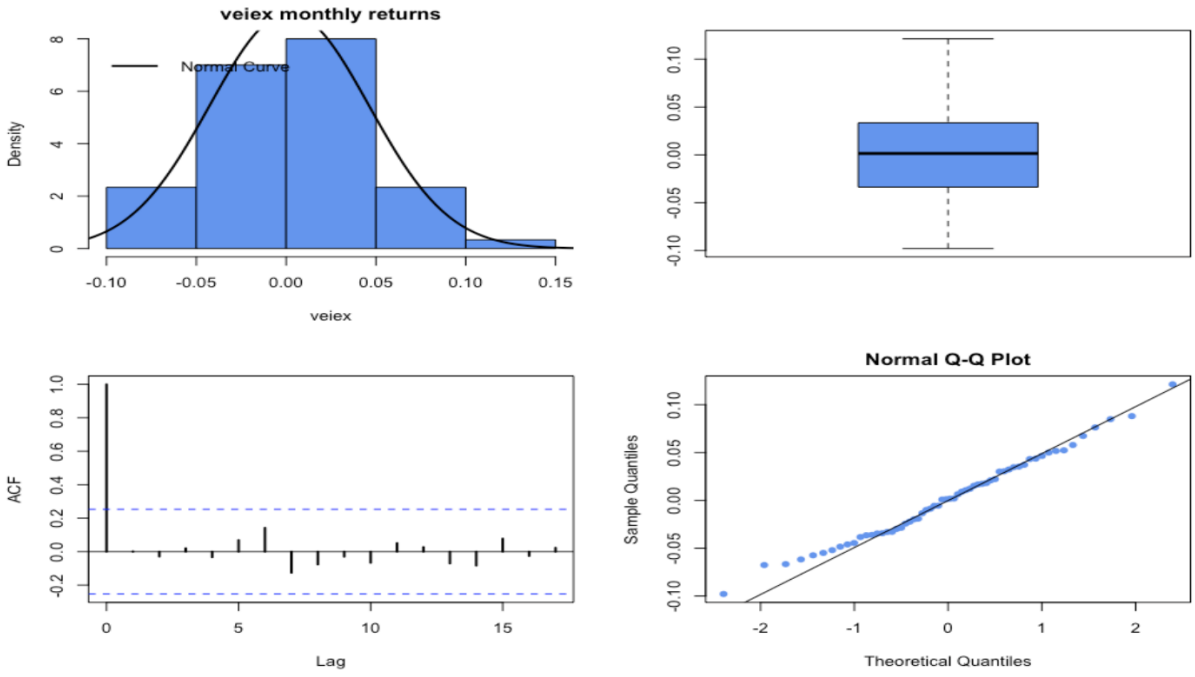
Here are four-panel diagnostic plots containing histograms, boxplots, Q-Q-plots, and SACFs for each return series.



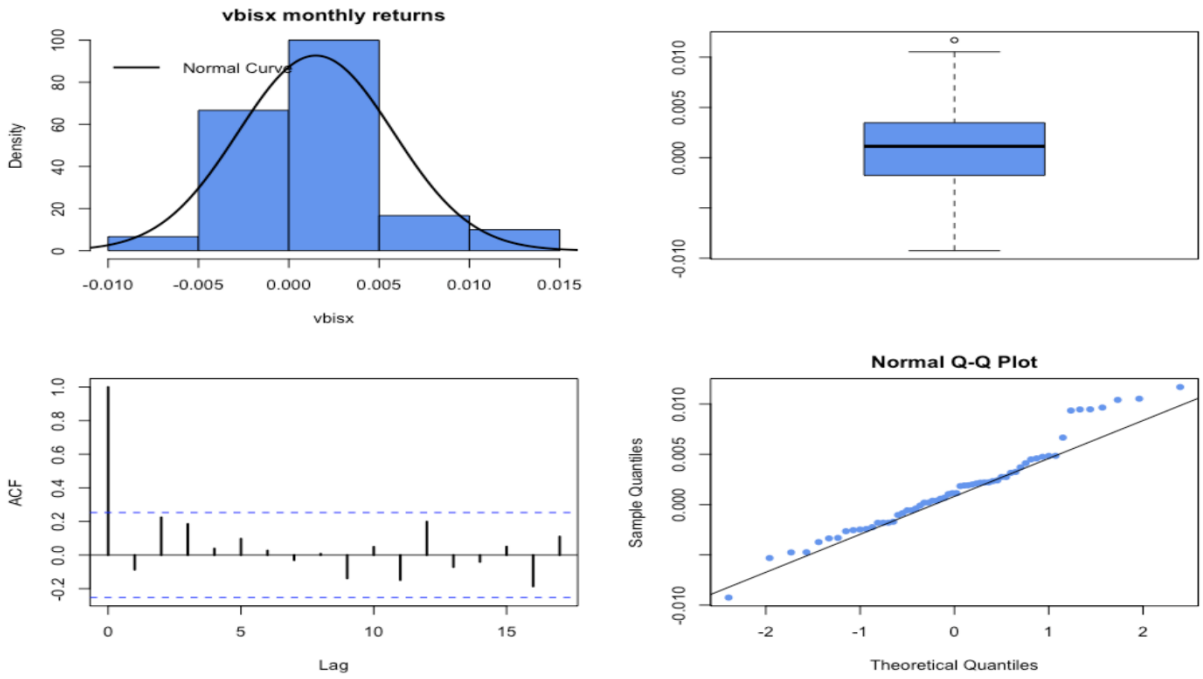
The histogram of VFINX monthly returns is not a normal distribution, which is left skewed. The box-plot looks very normal. The QQ plot is not precisely linear with many outliers at the extremes of lines. The SACF plot shows no significant autocorrelation for lags less than 15. All of the sample autocorrelations are small (close to zero) and show no distinct pattern over time. Hence, there does not appear to be any linear time dependence in the returns.



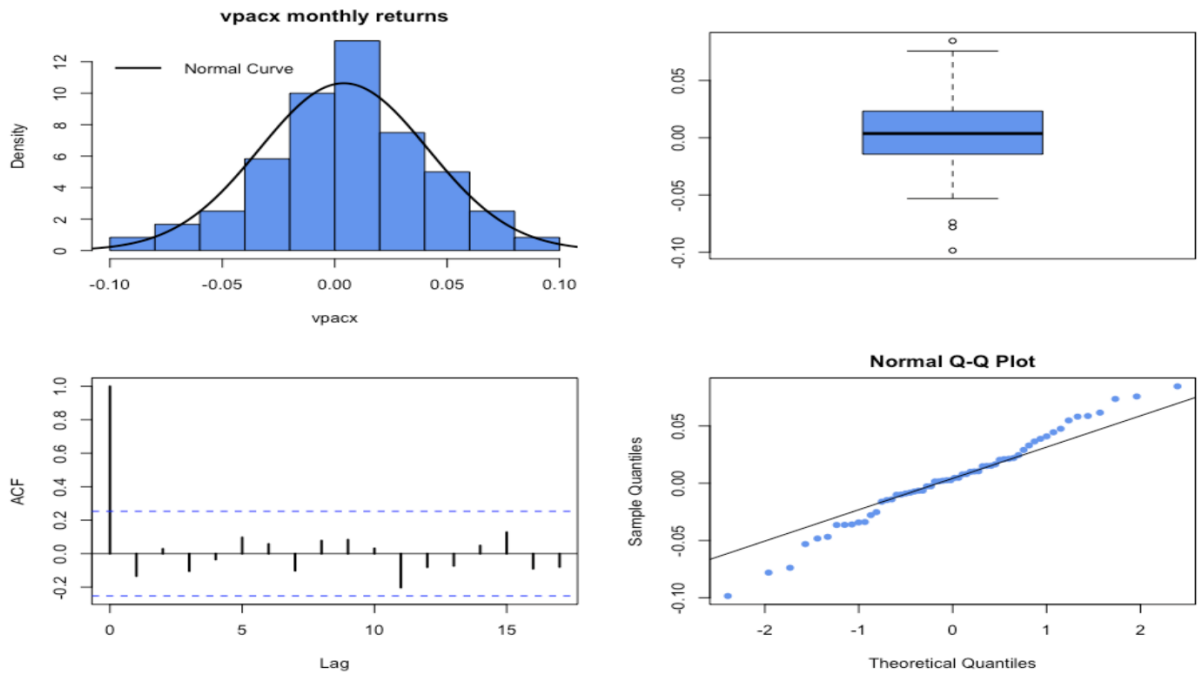
The histogram of VEURX monthly returns is not a normal distribution since the highest value is on the far rightside. The box-plot looks roughly normal. The QQ plot shows imprecise linear with many outliers. The SACF plot shows a significant autocorrelation for lags less than 15. All of the sample autocorrelations are between (-0.2, 0.2) and show no distinct pattern over time. Hence, there does not appear to be any linear time dependence in the returns.



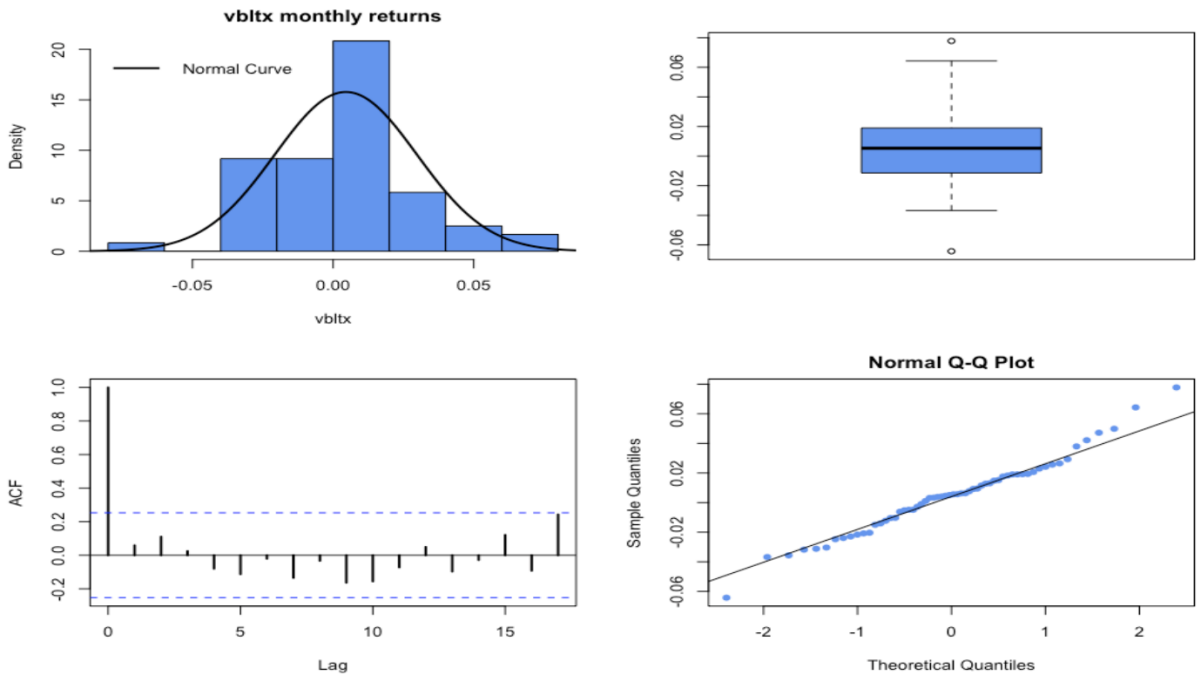
The histogram of VEIEX monthly returns is not a normal distribution, whic is right skewed. The box-plot looks normal. The QQ plot shows more precise linearity than the above two with less outliers. The SACF plot shows a significant autocorrelation for lags less than 15. All of the sample autocorrelations are small around zero and show no distinct pattern over time. Hence, there does not appear to be any linear time dependence in the returns.



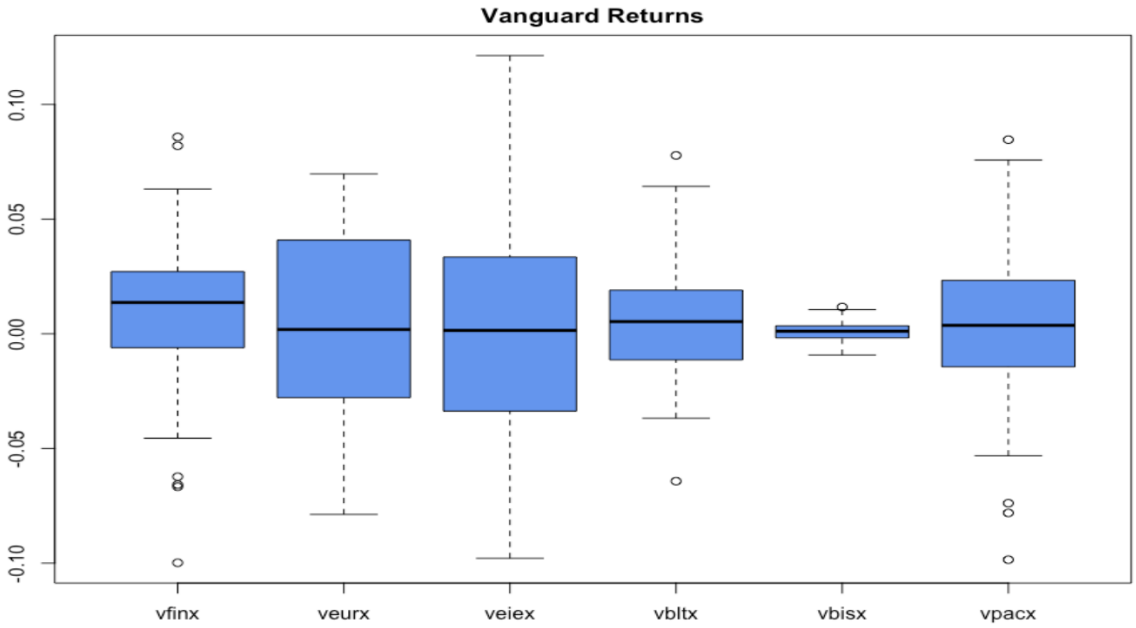
The histogram of VBISX monthly returns is not a normal distribution which is right skewed. The box-plot looks roughly normal. The QQ plot shows imprecise linearity with large outliers. All of the sample autocorrelations in the SACF plot are between (-0.3, 0.3) are not very close to 0. There may be weak linear time dependence in the returns.



The histogram of VPACX monthly returns looks roughly bell-shape which indicates a normal distribution. The box-plot looks normal. More than half of dots in the QQ plot are not on the straight line. The SACF plot shows a significant autocorrelation for lags less than 15 and shows no distinct pattern over time. All of the sample autocorrelations in the SACF plot are between (-0.3, 0.3) are not very close to 0. There may be weak linear time dependence in the returns.



The histogram of VBLTX monthly returns looks roughly bell-shape which indicates a normal distribution. The box-plot looks normal. Almost all dots in the QQ plot are on the straight line with several outliers. The SACF plot shows a significant autocorrelation for lags less than 15. All of the sample autocorrelations in the SACF plot are between (-0.3, 0.3) are not very close to 0. There may be weak linear time dependence in the returns.



This is a boxplot showing the distributions of all of the assets in one graph. All six boxplots look normal. The range of VEURX is the largest, and the range of VBISX are the smallest.

1.3 Univariate descriptive statistics

	vfinx	veurx	veiex	vbltx	vbisx	vpacx
Mean	0.00841	0.00304	0.00176	0.00453	0.00150	0.00397
Std Dev	0.03525	0.03959	0.04447	0.02529	0.00431	0.03751
Skewness	-0.52061	-0.22656	0.26473	0.20838	0.42018	-0.23519
Excess Kurtosis	0.87697	-0.96999	-0.24699	0.80207	0.19338	0.23588
1% Quantile	-0.08028	-0.07532	-0.08004	-0.04805	-0.00693	-0.08646
5% Quantile	-0.06249	-0.06315	-0.06200	-0.03200	-0.00474	-0.05419

The table above contains the univariate descriptive statistics (mean, variance, standard deviation, skewness, kurtosis, quantiles) for each return series. VFINX has the

highest average return (0.00841) and VBISX has the lowest average return (0.0015). The volality of VEIEX is the biggest with a standard deviation equal to 0.4447, and the volatility of VBLTX is the smallest with a standard deviation equal to 0.02529.

In addition, the absolute value of excess kurtosis of VBISX is the smallest, which is the closest to zero. Conversely, VEURX is the furthest away from zero. Therefore, VBISX looks most normally distributed and VEURX looks least normally distributed. All six assets have fat tails. Spacifically, VFINX, VEURX, VPACX have negative skewness and the other three assets have positive skewness.

1.4 Monthly Sharpe's Ratio

	SharpeRatios <dbl>	SE <dbl>
vfinx	0.2269	0.141
veurx	0.0662	0.135
veiex	0.0303	0.130
vbltx	0.1627	0.124
vbisx	0.2507	0.131
vpacx	0.0948	0.134

The table above shows Sharpe's slope/ratio for each asset and the estimated standard errors calculated using the bootstrap. The monthly risk-free rate equal to 0.0004167 per month (which corresponds to a continuously compounded annual rate of 0.5%). According to the table, VBLSX has the highest slope of 0.2507 and VEIEX has the lowest slope of 0.0303. The size of estimated standard errors of the six assets are similar to the values of Sharpe's Ratios. Therefore, the Sharpe slopes are estimated not precisely.

1.5 95% Confidence Intervals for the estimates of the mean and standard deviation

	muhat.vals <dbl>	se.muhat <dbl>	mu.lower.95 <dbl>	mu.upper.95 <dbl>
vfinx	0.00841	0.004550	-0.000504	0.01733
veurx	0.00304	0.005111	-0.006980	0.01305
veiex	0.00176	0.005741	-0.009486	0.01302
vbltx	0.00453	0.003265	-0.001866	0.01093
vbisx	0.00150	0.000556	0.000407	0.00259
vpacx	0.00397	0.004843	-0.005517	0.01346

	sd.vals <dbl>	se.sigmahat <dbl>	sigma.lower.95 <dbl>	sigma.upper.95 <dbl>
vfinx	0.03525	0.003217	0.02894	0.04155
veurx	0.03959	0.003614	0.03250	0.04667
veiex	0.04447	0.004059	0.03651	0.05242
vbltx	0.02529	0.002308	0.02076	0.02981
vbisx	0.00431	0.000393	0.00354	0.00508
vpacx	0.03751	0.003424	0.03080	0.04422

The top table above shows the value of average returns, the estimated standard errors, and the 95% confidence intervals for the estimates of average returns. The estimated standard errors of average returns in VEURX, VEIEX, and VPACX is bigger than the value of mean, which indicate imprecise emtimation. For the rest three assets, the size of standard errors is bigger than one third of size of mean. Therefore, the estimates of the mean in all six assets are not precise. In addition, except VBISX, the confidence intervals of all other five assets contain both positive and negative values, which approves the imprecise estimation.

The bottom table above shows the value of standard deviation, the estimated standard errors, and the 95% confidence intervals for the estimates of standard deviation. For each asset, the size of estimated standard errors of standard deviation is approximately 1/10 of the value of standard deviation, which indicate precise emtimation.

The confidence intervals of all six assets contain only positive values, which approve the precise estimation.

Therefore, we can conclude that the estimated standard deviations are more precise than the the estimated means.

1.6 Annual Sharpe Ratios

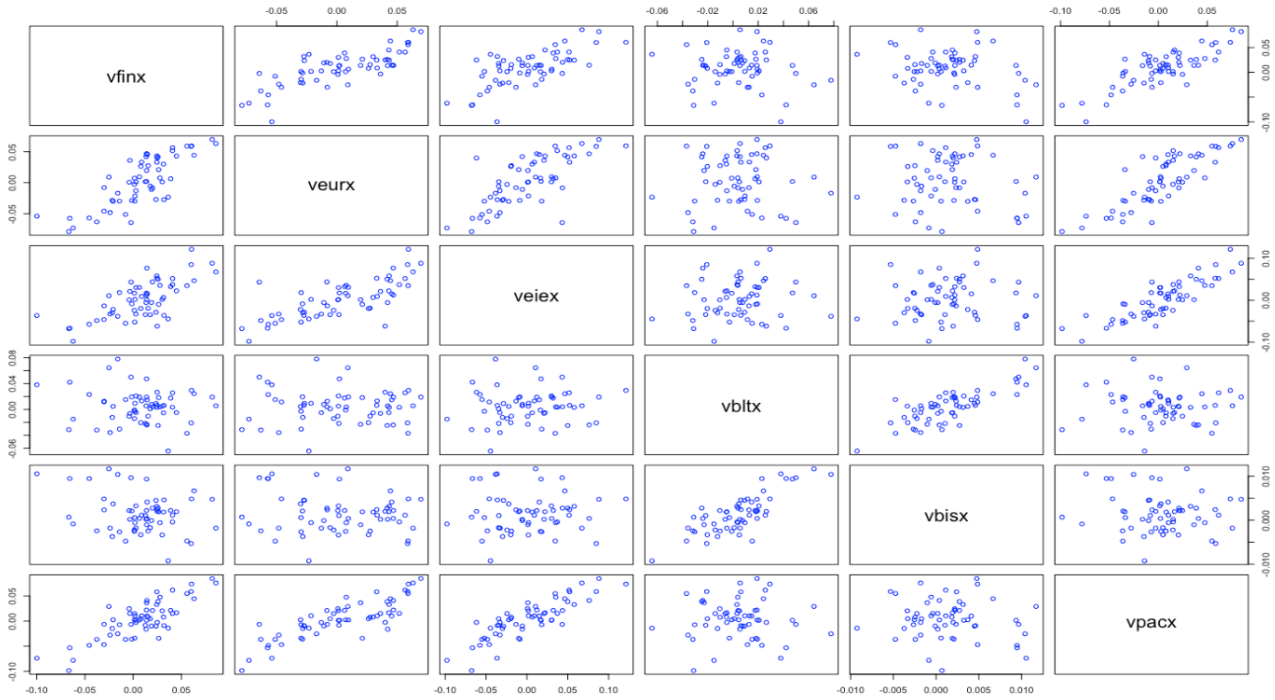
	annual_SharpeRatios <dbl>
vfinx	0.786
veurx	0.229
veiex	0.105
vbltx	0.564
vbisx	0.869
vpacx	0.329

The table above shows the annual Sharpe Ratio of six assets. In this case, we use the square-root-of-time rule to convert the monthly sample means by multiplying by 12 and convert SDs into annual estimates by multiplying by the square root of 12. For all six assets, the annual Sharpe Ratio is much higher than the monthly Sharpe Ratio. The asset rankings with annual Sharpe Ratios are same as with the monthly Sharpe ratios. (i.e. VBISX is still the asset with highest Sharpe Ratio and VEIEX is the asset with lowest Sharpe Ratio.)

	r_fiveYears <dbl>
vfinx	1.62
veurx	1.20
veiex	1.11
vbltx	1.30
vbisx	1.09
vpacx	1.26

Assuming we get the average annual return every year for 5 years, the table above shows how much $1 would grow to after 5 years.

1.7 Pair-wise scatterplots and linear association between the asset returns

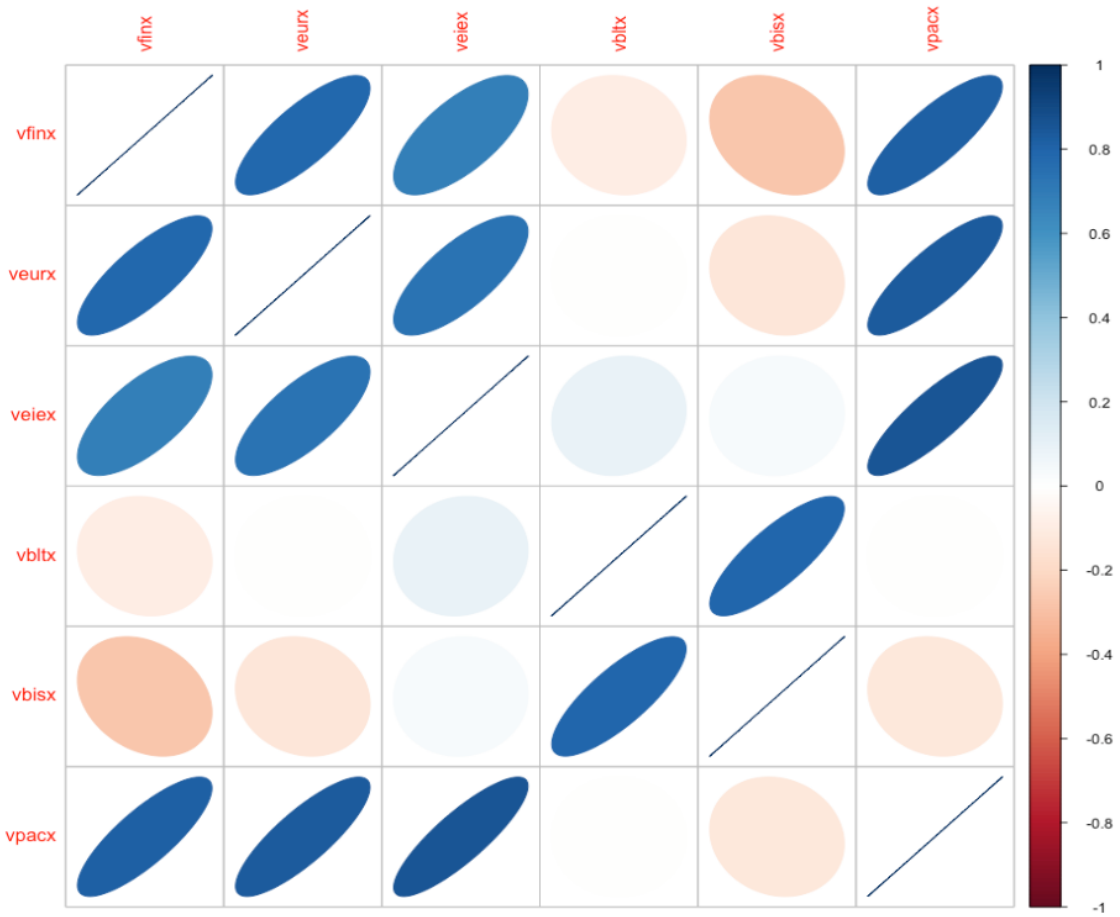


	vfinx	veurx	veiex	vbltx	vbisx	vpacx
vfinx	1.24e-03	1.10e-03	1.08e-03	-8.36e-05	-4.11e-05	1.08e-03
veurx	1.10e-03	1.57e-03	1.30e-03	-6.76e-07	-2.25e-05	1.23e-03
veiex	1.08e-03	1.30e-03	1.98e-03	1.08e-04	7.57e-06	1.42e-03
vbltx	-8.36e-05	-6.76e-07	1.08e-04	6.39e-04	8.69e-05	-5.72e-06
vbisx	-4.11e-05	-2.25e-05	7.57e-06	8.69e-05	1.85e-05	-1.99e-05
vpacx	1.08e-03	1.23e-03	1.42e-03	-5.72e-06	-1.99e-05	1.41e-03

The table above is the sample covariance matrix of the returns on six assets Compute and the pair-wise scatterplots shows the relationship between assets. We can find that the correlations between VBLTX and VFINX, VBISX and VFINX, VBLTX and VEURX, VBISX, and VEURX, VBLTX and VPACX, VBISX and VPACS are negative. The corresponding twelve plots (two scatterplots for each pair) for the six correlations

shows negative linear associations. The dots are on or around a downward straight line. For other asset pairs, the correlations are positive, and the dots in the scatterplots follow upward straight lines.

1.8 Plot of correlation matrix of the returns



	vfinx	veurx	veiex	vbltx	vbisx	vpacx
vfinx	1.0000	0.788149	0.6867	-0.093841	-0.2710	0.81547
veurx	0.7881	1.000000	0.7372	-0.000675	-0.1321	0.83009
veiex	0.6867	0.737189	1.0000	0.095865	0.0396	0.85022
vbltx	-0.0938	-0.000675	0.0959	1.000000	0.7982	-0.00603
vbisx	-0.2710	-0.132068	0.0396	0.798225	1.0000	-0.12338
vpacx	0.8155	0.830092	0.8502	-0.006029	-0.1234	1.00000

The plot and table above show the correlation matrix of the returns between the six assets. VPACX and VEIEX are most highly correlated with an absolute value of correlation as 0.85022. Also on the plot, the darkest blue at the sixth column of the third row and the third column of the last row) represents the correlation between VPACX and VEIEX. VEURX and VBLTX are least correlated with an absolute value of correlation less than 0.001 (0.000675). Also on the plot, the color at the position of this pair is nearly white. Another obvious weakly correlated relationship is VPACX and VBLTX pair, which is the second least correlated. For the assets which are weakly correlated, it's a good strategy to combine a portfolio with them in order to reduce risk because when one asset dropped suddenly, other assets which have weak correlation with it would not be influenced.

2. Value-at-Risk Calculations

2.1 Value-at-risk of the $100,000 investment over a one-month investment horizon

	VaR.normal.01 <dbl>	VaR.normal.05 <dbl>
vfinx	-7094	-4835
veurx	-8520	-6019
veiex	-9668	-6889
vbltx	-5285	-3638
vbisx	-848	-557
vpacx	-7991	-5609

The first column of the table above shows the 1% value-at-risk of the $100,000 investment over a one-month investment horizon of the six assets. With 1% probability, VEIEX produces the largest loss ($9668) and VBISX produces the lowest loss ($848).

The second column of the table above shows the 5% value-at-risk of the $100,000 investment over a one-month investment horizon of the six assets. With 5% probability, VEIEX produces the largest loss ($6889) and VBISX produces the lowest loss ($557).

	VaR.normal.01 <dbl>	SE <dbl>	lower_bound <dbl>	upper_bound <dbl>
vfinx	-7094	1029.0	-9202	-5168
veurx	-8520	819.0	-10183	-6974
veiex	-9668	856.0	-11441	-8084
vbltx	-5285	625.0	-6567	-4116
vbisx	-848	97.6	-1056	-673
vpacx	-7991	965.0	-9998	-6217

The table above shows the estimated standard errors (2nd column) and 95% confidence intervals (3rd and 4th column) for 1% VaR estimates (1st column). For each asset, the size of estimated standard errors is approximately 1/10 of the value of 1% VaR estimates. The CIs all contain negative values. Therefore, the 1% VaR estimates are precise.

	VaR.normal.05 <dbl>	SE <dbl>	lower_bound <dbl>	upper_bound <dbl>
vfinx	-4835	846.0	-6587	-3269
veurx	-6019	681.0	-7408	-4739
veiex	-6889	706.0	-8355	-5589
vbltx	-3638	503.0	-4672	-2701
vbisx	-557	73.4	-710	-423
vpacx	-5609	770.0	-7151	-4131

The table above shows the estimated standard errors (2nd column) and 95% confidence intervals (3rd and 4th column) for 5% VaR estimates (1st column). For each asset, the size of estimated standard errors is far less than the value of 1% VaR estimates. The CIs all contain negative values. Therefore, the 5% VaR estimates are precise.

2.2 Value-at-risk of the $100,000 investment over a one-year investment horizon

	VaR.normal.01.annual <dbl>	VaR.normal.05.annual <dbl>
vfinx	-16728	-9503
veurx	-24616	-17231
veiex	-28621	-20720
vbltx	-13877	-8579
vbisx	-1660	-656
vpacx	-22477	-15299

The first column of the table above shows the 1% value-at-risk of the $100,000 investment over a one-year investment horizon of the six assets. With 1% probability, VEIEX produces the largest loss ($28621) and VBISX produces the lowest loss ($1660).

The second column of the table above shows the 5% value-at-risk of the $100,000 investment over a one-year investment horizon of the six assets. With 5% probability, VEIEX produces the largest loss ($20720) and VBISX produces the lowest loss ($656).

2.3 Value-at-risk of the $100,000 investment over a one-month investment horizon based on historical simulation

	VaR.empirical.01 <dbl>	VaR.empirical.05 <dbl>	VaR.normal.01 <dbl>	VaR.normal.05 <dbl>
vfinx	-7714	-6058	-7094	-4835
veurx	-7256	-6120	-8520	-6019
veiex	-7692	-6011	-9668	-6889
vbltx	-4691	-3149	-5285	-3638
vbisx	-691	-473	-848	-557
vpacx	-8283	-5275	-7991	-5609

The first column: 1% value-at-risk of the $100,000 investment over a one-month investment horizon of the six assets estimated using empirical quantiles.
The second column: 5% value-at-risk of the $100,000 investment over a one-month investment horizon of the six assets estimated using empirical quantiles.

The third column: 1% value-at-risk of the $100,000 investment over a one-month investment horizon of the six assets estimated using normal distribution.

The fourth column: 5% value-at-risk of the $100,000 investment over a one-month investment horizon of the six assets estimated using normal distribution.

(Column3&4 are used for comparision)

Using the empirical 1% quantiles, VPCX produces the largest loss ($8283) and VBISX produces the lowest loss ($691).

Using the empirical 5% quantiles, VEURX produces the largest loss ($6120) and VBISX produces the lowest loss ($473).

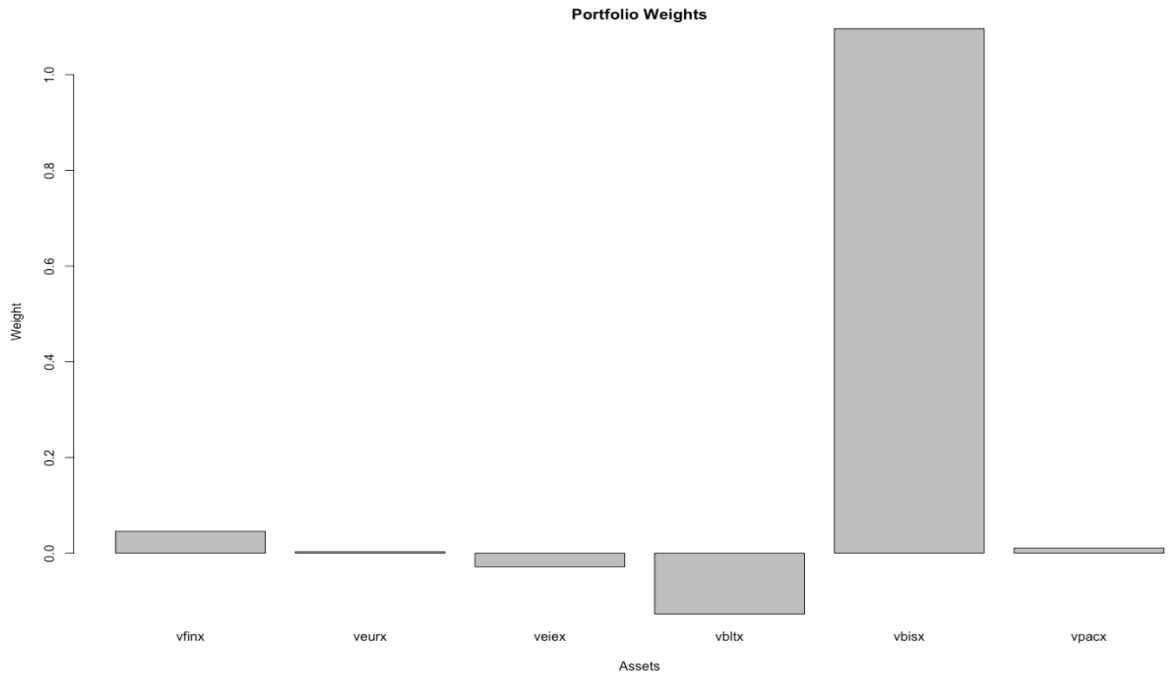
Comparing the first and last columns, the values of losses calculated using two different methods are similar, but the rankings of losses change. For example, the largest 1% VaR loss estimated using normal distribution is VEIEX, but that estimated using 1% empirical quantile is VPCX.

3. Portfolio Theory

3.1 Global Minimum Variance Portfolio With Short Sales

```
Portfolio expected return:     0.00145
Portfolio standard deviation:  0.00262
Portfolio Sharpe Ratio:        0.395
```

	gmin.weights.mat <dbl>
vfinx	0.04591
veurx	0.00285
veiex	-0.02845
vbltx	-0.12733
vbisx	1.09613
vpacx	0.01088



The tables and plot above show the summary of estimations using global minimum variance portfolio.

Based on the firts table, the expected return of this portfolio is 0.00145, the standard deviation of this portfolio is 0.00262, and the Sharpe Ratio is 0.395.

The second table and plot shows the weights of six assets in the global minimum variance portfolio. VBISX has the highest wights in this portfolio. There are two negative weights (VEIEX: -0.02845%; VBLTX: -0.12733). Therefore, the global minimum variance portfolio is calculated allowing short sales, which is not replicable in mutual funds.

3.2 Annual Sharpe Ratio In Global Minimum Variance Portfolio With Short Sales

	annual_SharpeRatios <dbl>
vfinx	0.786
veurx	0.229
veiex	0.105
vbltx	0.564
vbisx	0.869
vpacx	0.329

Estimates <chr>	Monthly <dbl>	Annual <dbl>
Expected return	0.00145092	0.0174110
Variance	0.00000687	0.0000824
Standard Deviation	0.00262087	0.0090790
Sharp Ratio	0.39462219	1.3670114
VaR 1%	-463.53579209	-370.2903837
VaR 5%	-285.59409534	248.0553067

6 rows

The first table above is from 1.6, which is the annual Sharpe Ratios of individual assets. The second above is the summary of estimations in global minimum variance portfolio. Comparing these two tables, the annual Sharp Ratio of this portfolio (1.367) is higher than any of individual asset's Sharp Ratio.

3.3 Value-at-risk Over A One-Month Investment Horizon In Global Minimum Variance Portfolio With Short Sales

	VaR.normal.01 <dbl>	VaR.normal.05 <dbl>
vfinx	-7094	-4835
veurx	-8520	-6019
veiex	-9668	-6889
vbltx	-5285	-3638
vbisx	-848	-557
vpacx	-7991	-5609

Estimates <chr>	Monthly <dbl>	Annual <dbl>
Expected return	0.00145092	0.0174110
Variance	0.00000687	0.0000824
Standard Deviation	0.00262087	0.0090790
Sharp Ratio	0.39462219	1.3670114
VaR 1%	-463.53579209	-370.2903837
VaR 5%	-285.59409534	248.0553067

6 rows

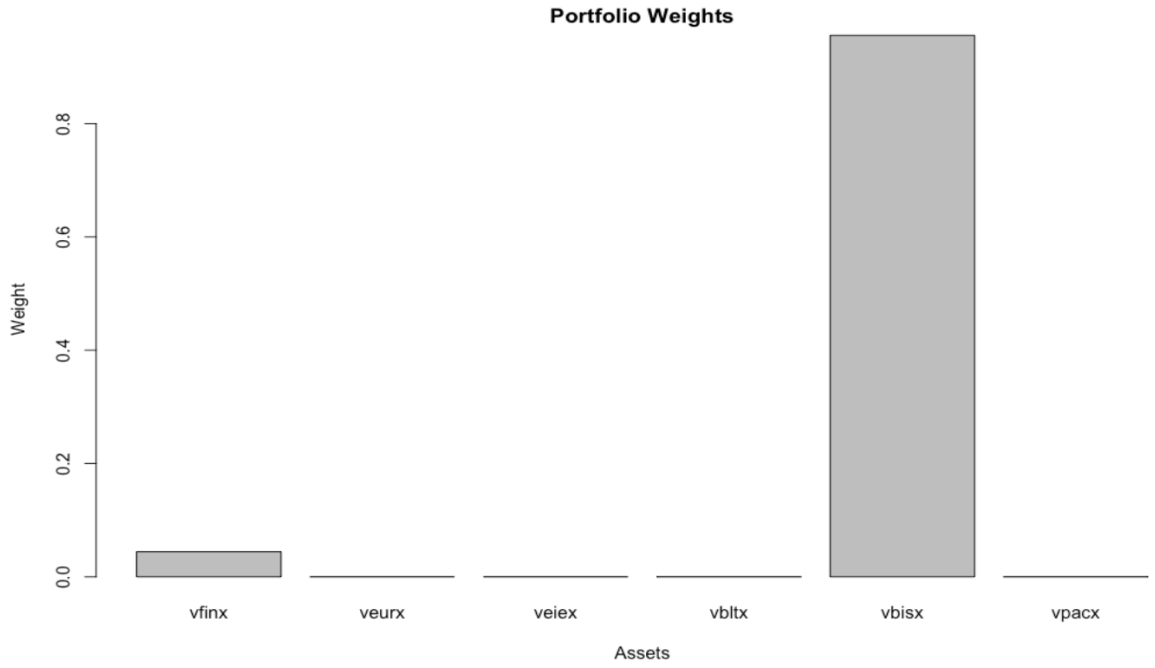
The initial investment to the global minimum variance portfolio is $100,000.

The first table above is from 2.1, which contains the 1 and 5% % value-at-risk of the $100,000 investment over a one-month investment horizon of the six individual assets.The second above is the summary of estimations in global minimum variance portfolio.

With 1% probability, the loss produced by global minimum variance portfolios is approximately $463.53. With 5% probability, the loss produced by global minimum variance portfolios is approximately $283.59. Both are much less than the losses of individual asset. Therefore, it's great strategy to use portfolio in order to reduce risk.

3.4 Wights Of Assets In Global Minimum Variance Portfolio With No Short Sales

```
Portfolio weights:
 vfinx  veurx  veiex  vbltx  vbisx  vpacx
0.0444 0.0000 0.0000 0.0000 0.9556 0.0000
```



The table and plot above show the weights of assets in global minimum variance portfolio with no short sales. VBISX has the highest wights in this portfolio. There are no negative weights this time, since we add the restrictions of no short sales. Therefore, this portfolio is replicable in mutual funds.

3.5 Compare Sharpe Ratio And VaR With Global Minimum Variance Portfolio (With Short Sales)

Table 3.5.1: global minimum variance portfolio with short sales

Estimates <chr>	Monthly <dbl>	Annual <dbl>
Expected return	0.00145092	0.0174110
Variance	0.00000687	0.0000824
Standard Deviation	0.00262087	0.0090790
Sharp Ratio	0.39462219	1.3670114
VaR 1%	-463.53579209	-370.2903837
VaR 5%	-285.59409534	248.0553067

6 rows

Table 3.5.2: global minimum variance portfolio with no short sales

Estimates <chr>	Monthly <dbl>	Annual <dbl>
Expected return	0.0018034	0.021641
Variance	0.0000159	0.000191
Standard Deviation	0.0039859	0.013807
Sharp Ratio	0.3479157	1.205215
VaR 1%	-744.1247567	-1042.519971
VaR 5%	-474.1459985	-106.968840

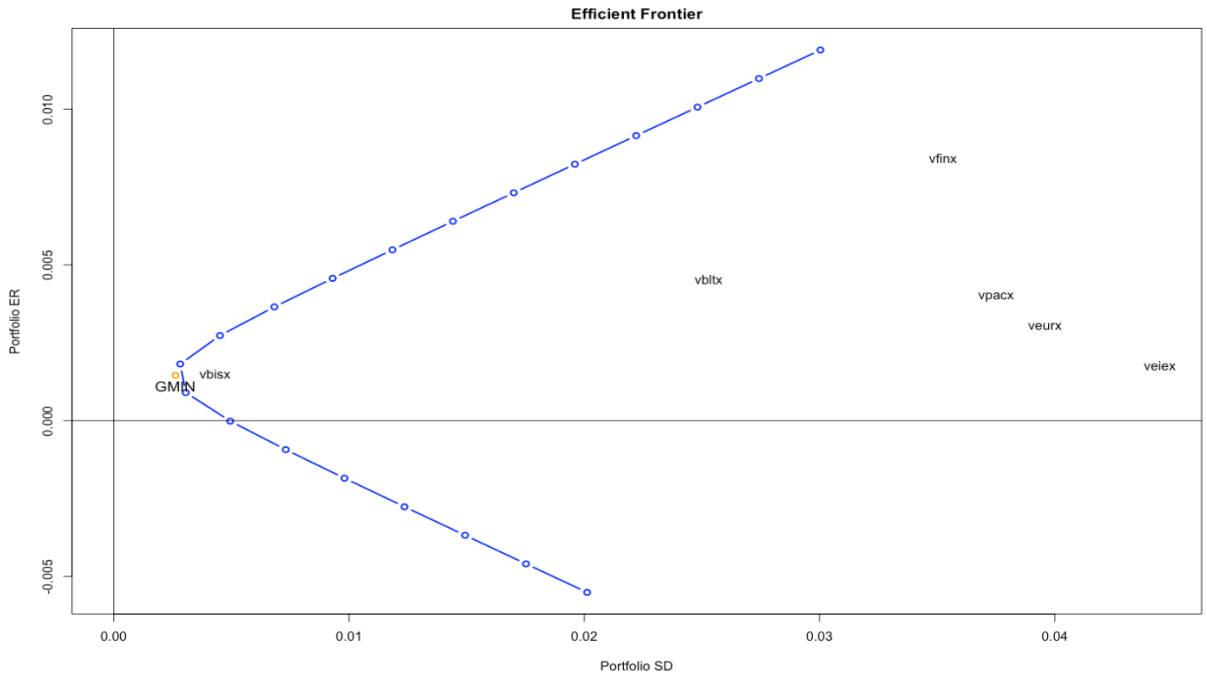
Comparing the two tables above, the annual Sharpe Ratio in the portfolio not allowing short sales is about 1.21 (see Table 3.5.2), which is smaller than that in the portfolio allowing short sales (1.37, see Table 3.5.1).

The initial investment to the global minimum variance portfolio with no short sale is also $100,000.

Based on Table 3.5.2, with 1% probability, the monthly loss produced by global minimum variance portfolio with no short sales is approximately $744.12, which is larger than the loss in the portfolio with short sales (463.54, see Table 3.5.1). With 5% probability, the loss produced by global minimum variance portfolio with no short sale is approximately $474.15, which is smaller also larger than the loss in the portfolio with short sales (285.59). Therefore, the 1% and 5% value-at-risk of the $100,000 investment

over a one-month investment horizon are larger in global minimum variance portfolio with no short sales than the portfolio allowing short sales.

3.6 Markowitz Bullets, Locations Of Six Asssets, And Global Minimum (Short Sale Allowed)

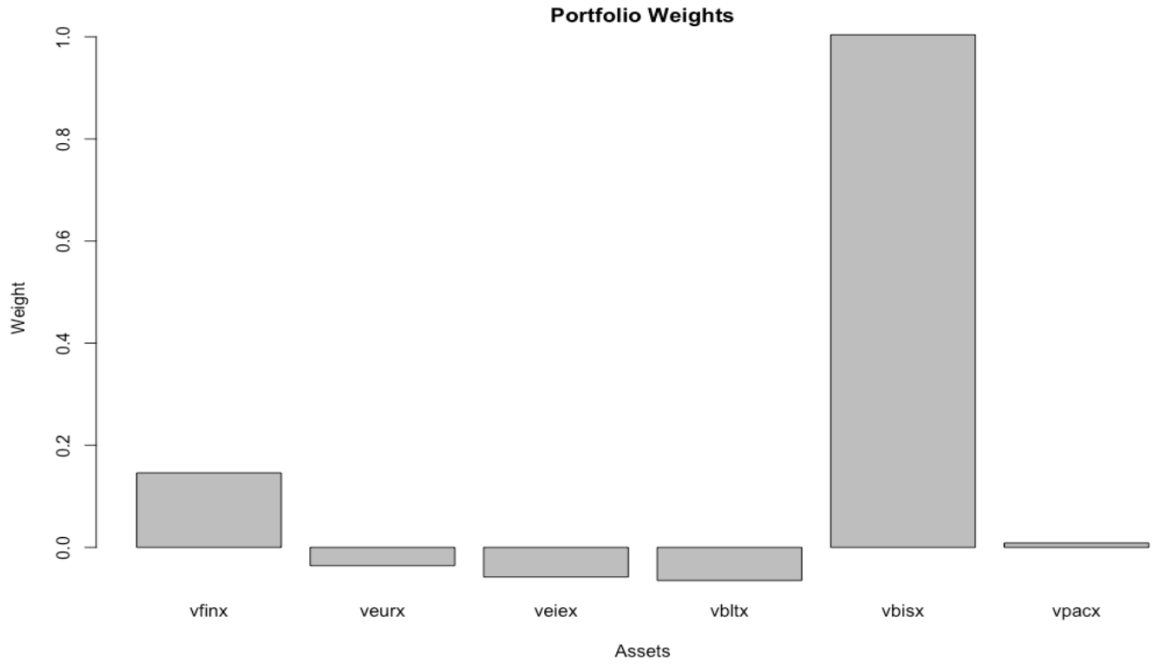


The Markowitz bullet is constructed using global minimum variance portfolio and the efficient minimum variance portfolio with a target return equal to the maximum of the average returns for the six assets. The orange point on the plot is the location of global minimum which is calculate in 3.1. The locations of six assets are calculated using the Markowitz algorithm.

3.7 Tangency Portfolio Allowing For Short Sales

The tangency portfolio is calculated using a monthly risk-free rate equal to 0.0004167 per month (which corresponds to an annual rate of 0.5%).

```
Portfolio weights:
  vfinx   veurx   veiex   vbltx   vbisx   vpacx
 0.1460 -0.0357 -0.0580 -0.0648  1.0039  0.0086
```



The plot above shows the weights of the six assets in the tangency portfolio. There are three assets with negative wights: VEURX (-0.0357), VEIEX (-0.0580), and VBLTX (-0.0648).

Estimates <chr>	Monthly <dbl>	Annual <dbl>
Expected return	0.0022602	0.027122
Variance	0.0000122	0.000147
Standard Deviation	0.0034991	0.012121
Sharp Ratio	0.5268580	1.825090

The table above shows the monthly and annual expected return, variance and standard deviation of the tangency portfolio.

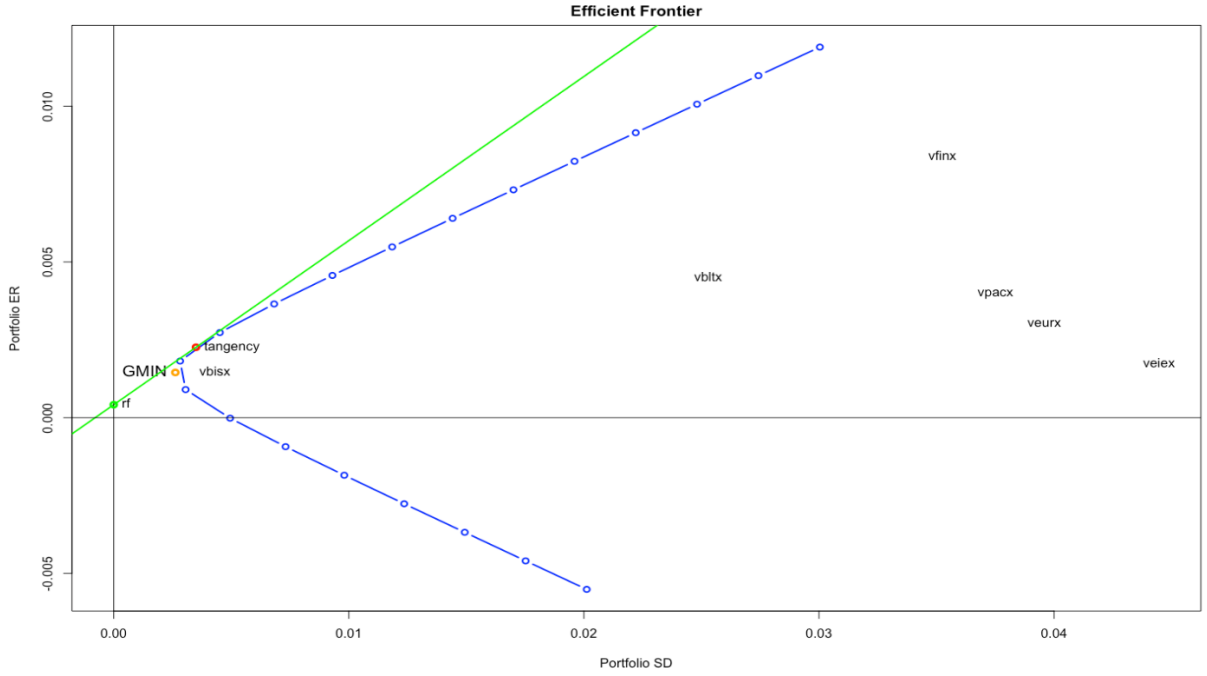
Table 3.7.1: Monthly Sharpe Ratio of six individual assets (from 1.4)

	SharpeRatios <dbl>	SE <dbl>
vfinx	0.2269	0.141
veurx	0.0662	0.135
veiex	0.0303	0.130
vbltx	0.1627	0.124
vbisx	0.2507	0.131
vpacx	0.0948	0.134

Table 3.7.2: Annual Sharpe Ratio of six individual assets (from 1.6)

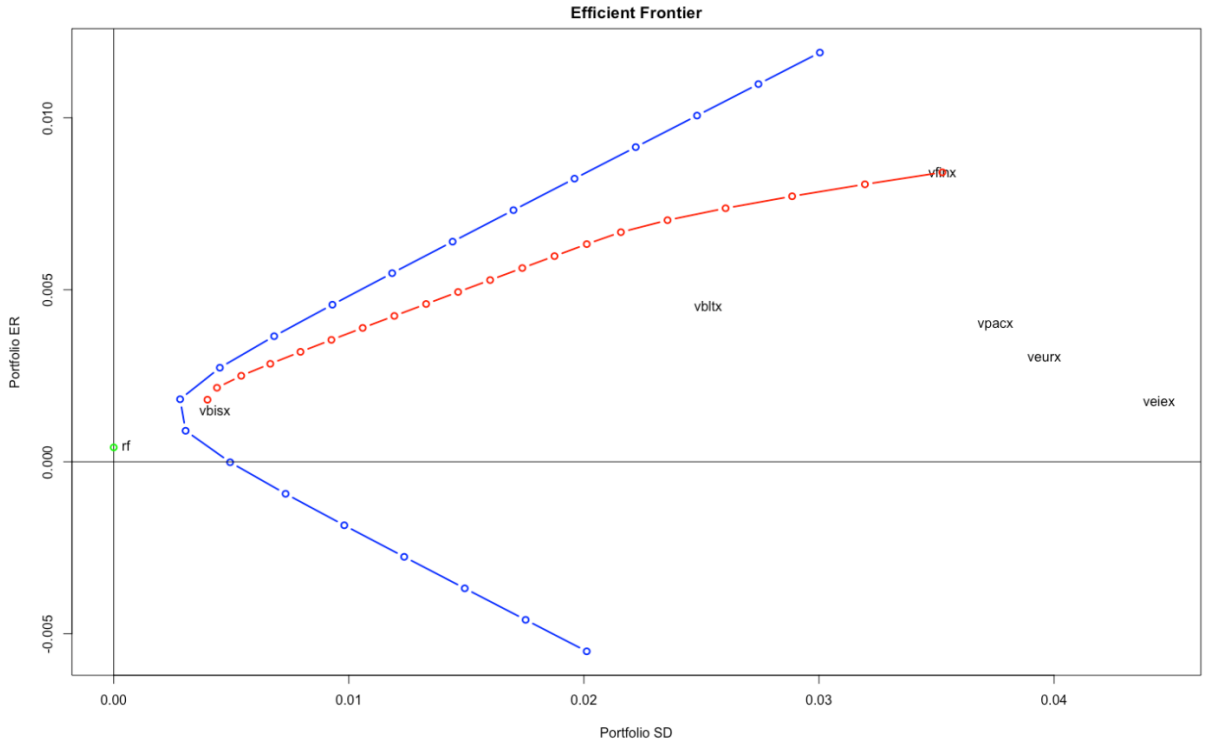
	annual_SharpeRatios <dbl>
vfinx	0.786
veurx	0.229
veiex	0.105
vbltx	0.564
vbisx	0.869
vpacx	0.329

The monthly Sharpe ratio in tangency portfolio is approximately 0.53, which is much higher than the monthly sharpe ratio of any individual asset. The annual Sharpe ratio in tangency portfolio is about 1.83, which is also higher than the maximum of annnual Sharpe ratio among the six individual assets.



Based on the plot in 3.6, the location of tangency portfolio (the red point) and the tangent line (green line) are added.

3.8 Efficient Portfolio Frontier Not Allowing For Short sales



The blue Markowitz bullet represents the efficient portfolio frontier with short sales. The red Markowitz bullet represents the efficient portfolio frontier not allowing short sales. Based on the graph, the efficient portfolio frontier not allowing short sales limites the portfolios. In specific, the highest expected return invested in the efficient portfolio frontier not allowing short sales cannot be higher than the efficient portfolio frontier with short sales. Similarly, the lowest expected return invested in the efficient portfolio frontier not allowing short sales cannot be lower than the efficient portfolio frontier with short sales.

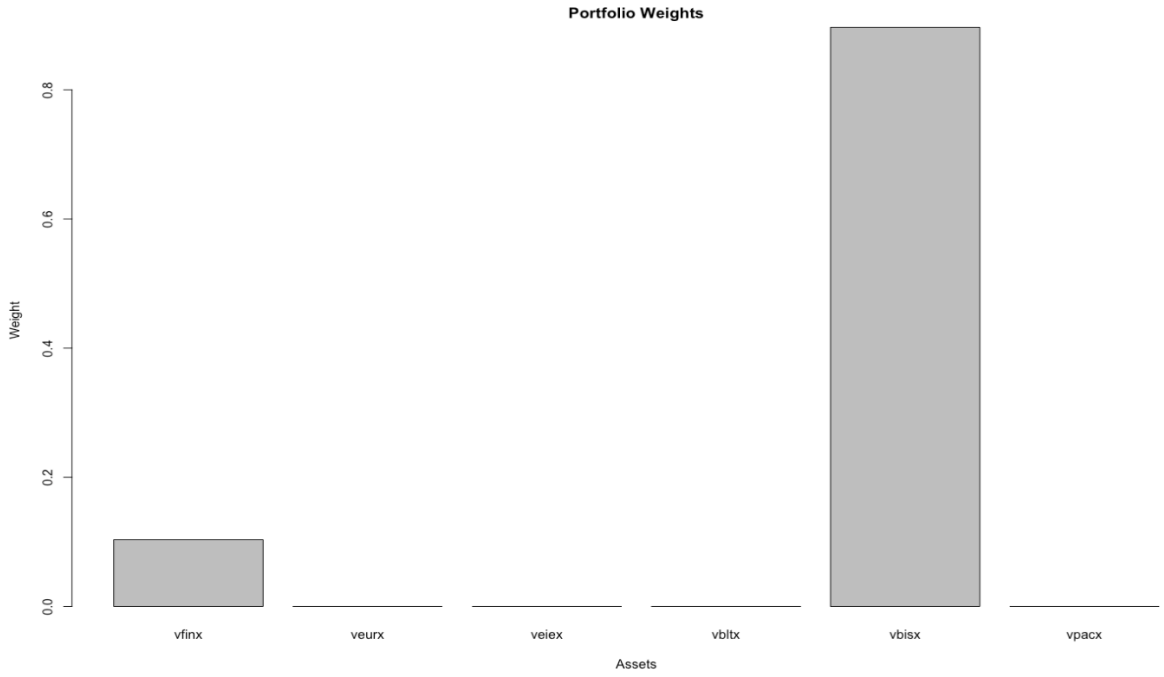
Consider a portfolio with a target volatility of 2% per month. According to the plot, the approximate cost in the expected return of investing in a no short sale efficient

portfolio versus a short sale efficient portfolio is the difference vertically between the blue and red bullet point when portfolio SD is equal to 0.02, which is about 0.002.

3.9 Tangency Portfolio Not Allowing For Short Sales

This tangency portfolio is calculated using a monthly risk-free rate equal to 0.0004167 per month (which corresponds to an annual rate of 0.5%).

```
Portfolio weights:
vfinx veurx veiex vbltx vbisx vpacx
0.103 0.000 0.000 0.000 0.897 0.000
```



The table and plot above show the weights of assets in tangency portfolio not allowing for short sales. VBISX has the highest wights in this portfolio. There are no negative weights this time, since we add the restrictions of no short sales. Therefore, this portfolio is replicable in mutual funds.

Table 3.9.1: Tangency portfolio with short sales

Estimates <chr>	Monthly <dbl>	Annual <dbl>
Expected return	0.0022602	0.027122
Variance	0.0000122	0.000147
Standard Deviation	0.0034991	0.012121
Sharp Ratio	0.5268580	1.825090

Table 3.9.2: Tangency portfolio not allowing for short sales

Estimates <chr>	Monthly <dbl>	Annual <dbl>
Expected return	0.0022117	0.026540
Variance	0.0000206	0.000247
Standard Deviation	0.0045348	0.015709
Sharp Ratio	0.3958306	1.371197

Comparing the two tables above, among both monthly and annual data, the expected return of the tangency portfolio not allowing for short sales is lower than the expected return of the tangency portfolio allowing for short sales. Also, the volatility of returns in the tangency portfolio not allowing for short sales is larger than the in the the tangency portfolio allowing for short sales.

In addition, the monthly Sharpe Ratio in the tangency portfolio not allowing for short sales is about 0.396 (see Table 3.5.2), which is smaller than that in the portfolio allowing short sales (0.527, see Table 3.5.1). The annual Sharpe Ratio in the tangency portfolio not allowing for short sales is about 1.37 (see Table 3.5.2), which is smaller than that in the portfolio allowing short sales (1.83, see Table 3.5.1).

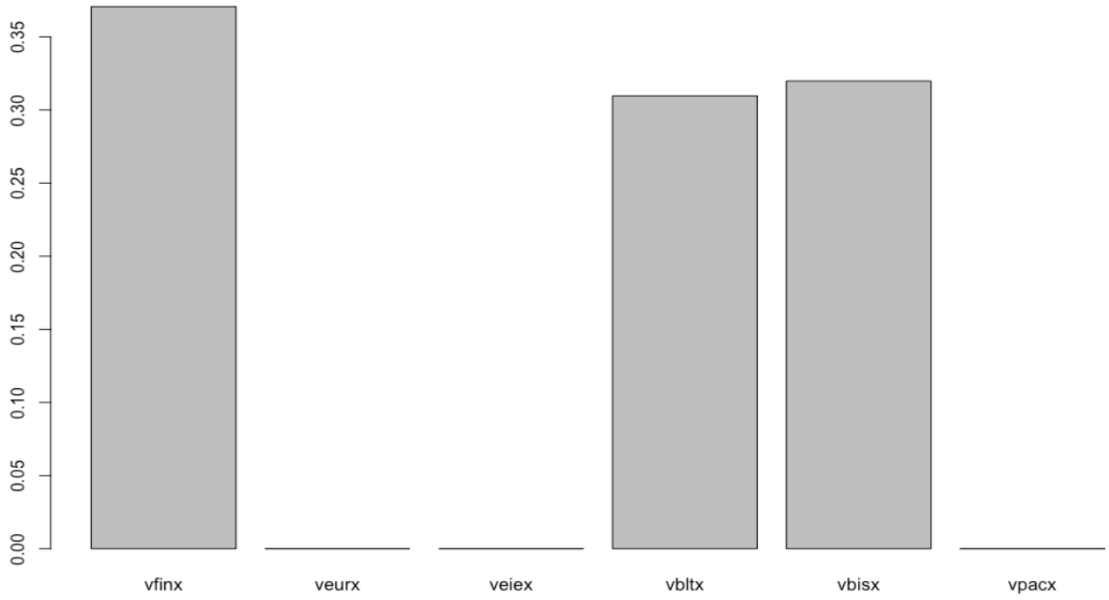
4. Asset Allocation

4.1 Asset Allocation with target expected return of 6% per year

The efficient portfolio below achieves a target expected return of 6% per year (which corresponds 0.5% per month) using only the six risky assets with no short sales:

Table 4.1 The weights of assets invested in this portfolio

```
Portfolio weights:
vfinx veurx veiex vbltx vbisx vpacx
0.371 0.000 0.000 0.310 0.320 0.000
```



4.2 Statistics of Asset Allocation with target expected return of 6% per year

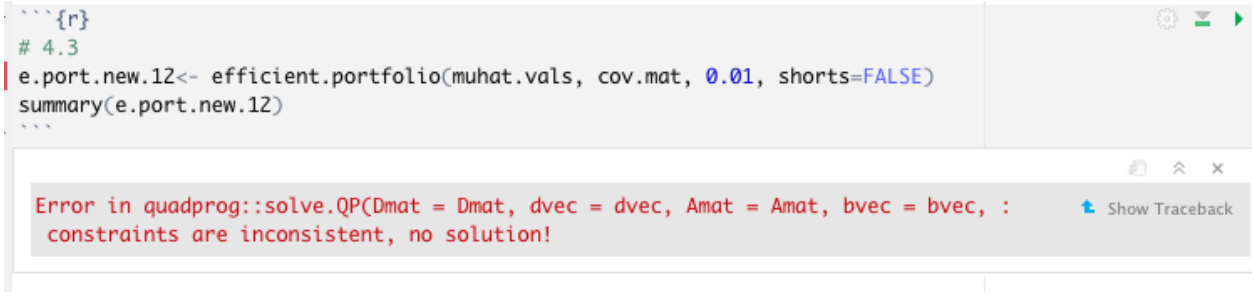
Table 4.2: Summary of statistics

Estimates <chr>	Monthly <dbl>	Annual <dbl>
Expected return	0.005000	0.06000
Variance	0.000222	0.00267
Standard Deviation	0.014903	0.05162
Sharp Ratio	0.307552	1.06539
VaR 1%	-2923.287253	-5832.56628
VaR 5%	-1932.348130	-2460.64034

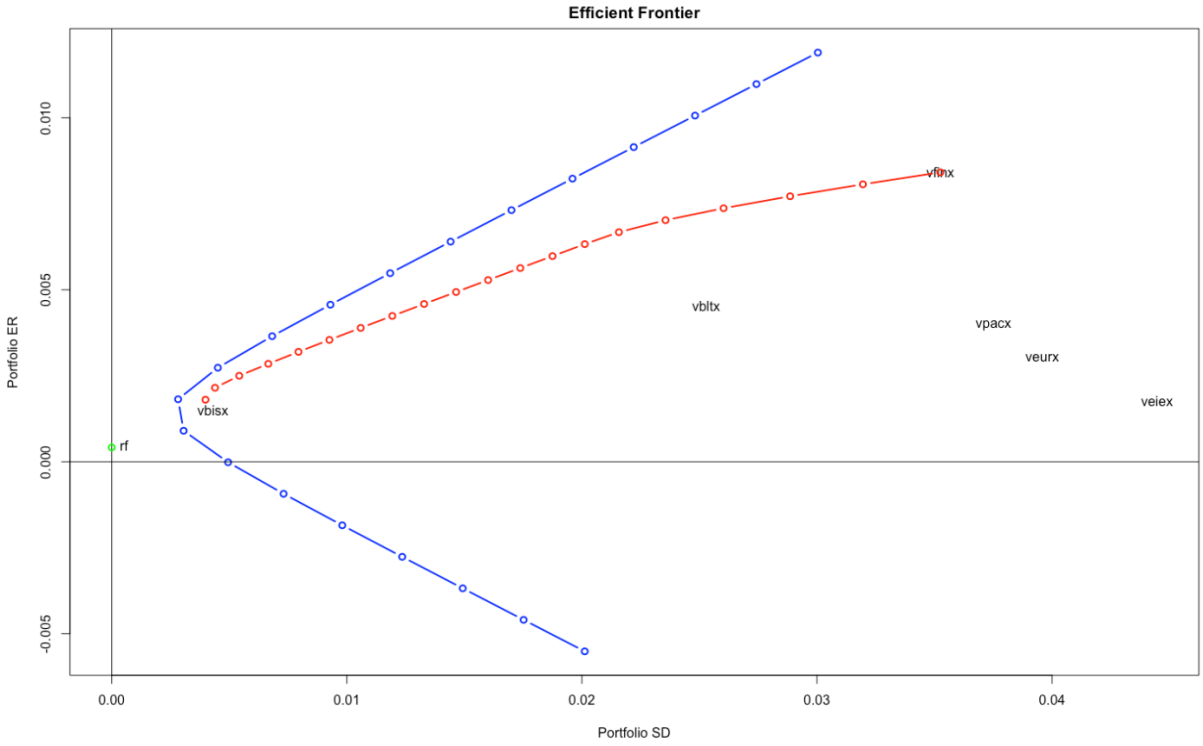
The monthly standard deviation on this efficient portfolio is 0.000222.

Based on an initial $100,000 investment, with 1% probability, this efficient portfolio produces a loss approximately $2923.29 over a one-month investment horizon . With 5% probability, this efficient portfolio produces a loss approximately $1932.35 over a one-month investment horizon. In addition, over an annual investment horizon, with 1% probability, this efficient portfolio produces a loss approximately $5832.57. With 5% probability, this efficient portfolio produces a loss approximately $2460.64.

4.3 Asset Allocation with target expected return of 12% per year



In this case, if adding other riskless asset (such as T-Bills) and the short sales are both not allowed, there is NO solutions for generating an efficient portfolio achieving target expected return of 12% per year.



The plot in 3.8 can approved this. On the red bullet point, the highest monthly expected return in the efficient portfolio not allowing for short sales is obviously less than 0.01.

Citation

Yahoo! finance site: https://finance.yahoo.com/