<u>ELEC ENG 2CJ4 Circuits and Systems Lab 5</u>

<u>Instructor: Dr. Elamien</u>

<u>Kun Xing, 400460968, xing8</u>

<u>Section L01</u>

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Introduction

Active filters are fundamental components in analog circuits design, playing a crucial role in applications such as signal conditioning, communication systems, and control electronics. In this experiment, we investigate the behaviour and performance of a Butterworth Low-Pass Filter using an LM358P op-amp, resistors, and capacitors. The primary objective is to analyze the filter's frequency response, derive the theoretical transfer function, and verify its cutoff frequency both analytically and experimentally. This experiment reinforces key concepts in analog signal processing, such as frequency-domain behaviour, gain attenuation, and the practical implementation of filters using op-amps. The comparison between analytical and measured results will provide insights into real-world component tolerances and filter accuracy, while also highlighting the effectiveness in achieving reliable low-pass filtering.

Consider the shown Butterworth Low-Pass Filter. Take $R_1 = R_1 = 10k\Omega$ and $C_1 = C_2 = 100nF$.

a) Derive an expression for the transfer function of the filter.

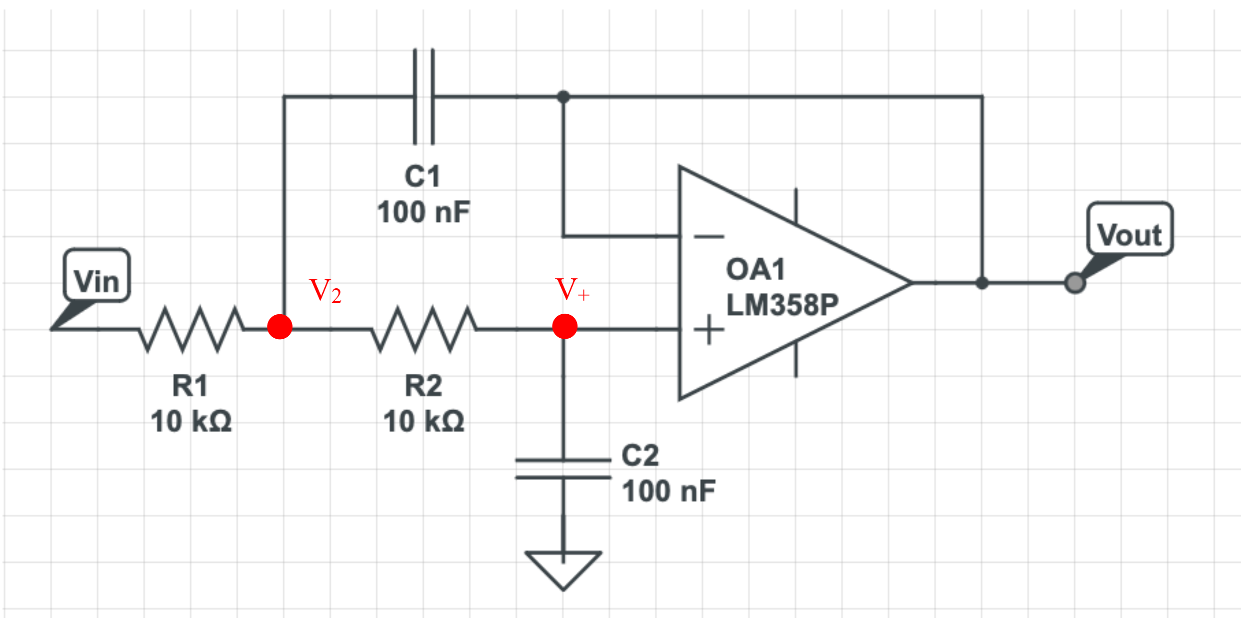

Figure 1: Circuit Schematic

$I_+ = I_- = 0$ mA

$V_+ = V_- = V_{out}$

$$Z_1 = Z_2 = \frac{1}{s(100n)} = \frac{1}{s}10^7$$

Nodal Analysis at V_+:

$\frac{V_+}{Z_2} + \frac{V_+ - V_2}{R_2} = 0$, where V_+ = Vout from ideal op-amps

$\frac{V_{out}}{Z_2} + \frac{V_{out} - V_2}{R2} = 0$

$V_{out}\left(\frac{1}{Z_2} + \frac{1}{R_2}\right) = \frac{V_2}{R_2}$

$V_{out}\left(\frac{R_2}{Z_2} + 1\right) = V_2$

Nodal Analysis at V_2:

$\frac{V_2 - V_{in}}{R_1} + \frac{V_2 - V_{out}}{Z_1} + \frac{V_2 - V_+}{R2} = 0$, where $V_+ = V_{out}$ from ideal op-amps

$\frac{V_2 - V_{in}}{R_1} + \frac{V_2 - V_{out}}{Z_1} + \frac{V_2 - V_{out}}{R_2} = 0$

$V_2\left(\frac{1}{R_1} + \frac{1}{Z_1} + \frac{1}{R_2}\right) - V_{out}\left(\frac{1}{Z_1} + \frac{1}{R_2}\right) = \frac{V_{in}}{R_1}$

$V_{out}\left(\frac{R_2}{Z_2} + 1\right)\left(\frac{1}{R_1} + \frac{1}{Z_1} + \frac{1}{R_2}\right) - V_{out}\left(\frac{1}{Z_1} + \frac{1}{R_2}\right) = \frac{V_{in}}{R_1}$

$V_{out}\left(\frac{R_2}{Z_2 R_1} + \frac{R_2}{Z_2 Z_1} + \frac{R_2}{Z_2 R_2} + \frac{1}{R_1} + \frac{1}{Z_1} + \frac{1}{R_2} - \frac{1}{Z_1} - \frac{1}{R_2}\right) \times R_1 = V_{in}$

$V_{out}\left(\frac{R_2}{Z_2} + \frac{R_2 R_1}{Z_2 Z_1} + \frac{R_1}{Z_2} + 1\right) = V_{in}$

$H(s) = \frac{V_{out}}{V_{in}} = \frac{1}{\frac{R_2}{Z_2} + \frac{R_2 R_1}{Z_2 Z_1} + \frac{R_1}{Z_2} + 1}$ where R_1 = R2 = 10kΩ and $Z_1 = Z_2 = \frac{1}{s(100n)} = \frac{1}{s}10^7$

$H(s) = \frac{1}{(1\times10^{-6})s^2 + (2\times10^{-3})s + 1}$

b) **Evaluate the filter transfer function abs(V_o/V_i) using the transfer function derived in part (a) for the frequencies in the table.**

$H(jw) = \frac{1}{-(1\times10^{-6})w^2 + (2\times10^{-3})jw + 1}$

$|H(jw)| = \frac{1}{\sqrt{(1-10^{-6}w^2)^2 + (2\times10^{-3}w)^2}}$ where $w = 2\pi f$

$|H(jw)| = \frac{1}{\sqrt{\left(1-10^{-6}(4\pi^2 f^2)\right)^2 + (10^{-3}(4\pi f))^2}}$

Frequency	$abs(V_o/V_i)$ (analytical)
50 Hz	0.91
100 Hz	0.72
200 Hz	0.39
500 Hz	0.092
1 kHz	0.025
1.1 kHz	0.021
1.2 kHz	0.017
1.3 kHz	0.015
1.4 kHz	0.013
1.5 kHz	0.011
1.6 kHz	0.0098
1.7 kHz	0.0087
1.8 kHz	0.0078
1.9 kHz	0.0070
2 kHz	0.0063
5 kHz	0.0010

c) Measure the transfer function using the AD2 board and fill the corresponding components of the table below. Use a sine wave with an amplitude of 2V and offset of 0V ($V_{CC} = \pm 5V$).

```
octave:1> R1 = 10000;
R2 = 10000;
C1 = 100e-9;
C2 = 100e-9;
num = [1];
den = [R1*R2*C1*C2 R1*C1+R2*C1+R2*C2 1];
H = tf(num,den);
p = bodeoptions('cstprefs');
p.FreqUnits = 'Hz';
bode(H,p)
grid on
Fc = 1/(2*pi*sqrt(R1*R2*C1*C2));
```

Figure 2: MATLAB Code

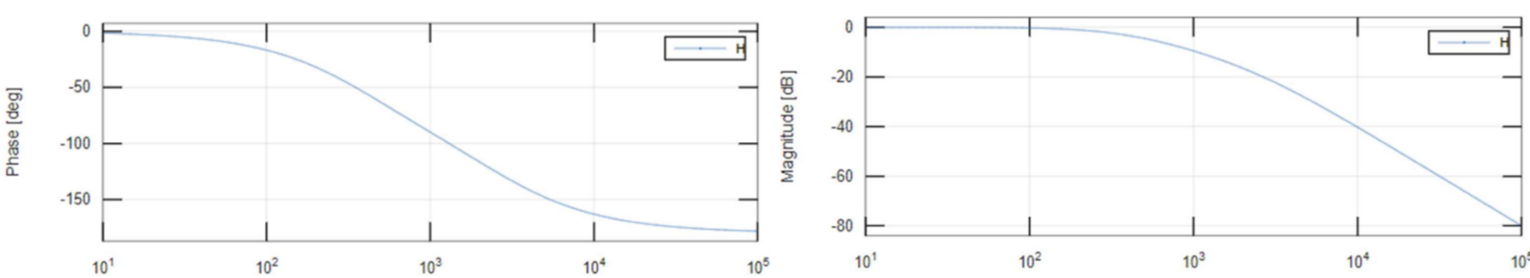

Figure 3: Bode Plots

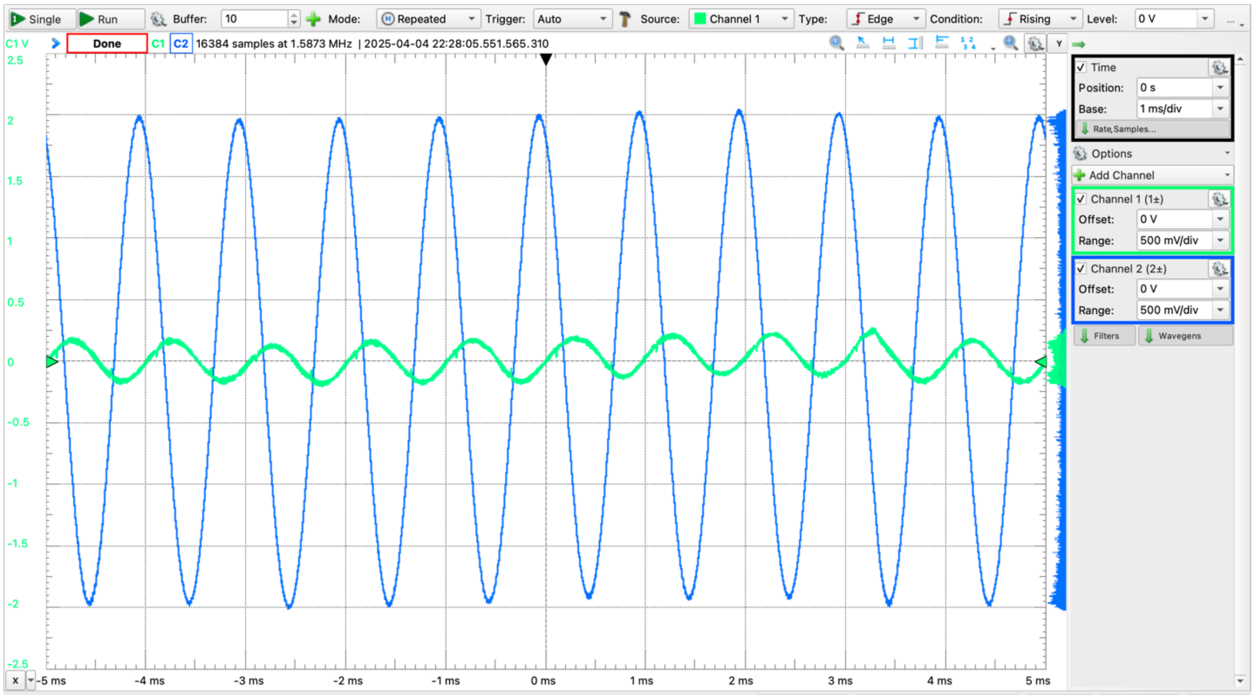

Figure 4: AD3 Simulation Results Graph

Frequency	abs(V_o/V_i) (analytical)	abs(V_o/V_i) (measured)
50 Hz	0.91	0.872
100 Hz	0.72	0.769
200 Hz	0.39	0.426
500 Hz	0.092	0.104
1 kHz	0.025	0.0391
1.1 kHz	0.021	0.0342
1.2 kHz	0.017	0.0283
1.3 kHz	0.015	0.0279
1.4 kHz	0.013	0.0245
1.5 kHz	0.011	0.0238
1.6 kHz	0.0098	0.0214
1.7 kHz	0.0087	0.0202
1.8 kHz	0.0078	0.0209
1.9 kHz	0.0070	0.0215
2 kHz	0.0063	0.0207
5 kHz	0.0010	0.0201

d) What is the cut-off frequency of this filter?

From transfer function of a 2nd order low pass filter general equation:

$H(s) = \frac{H_{DC}w_c^2}{s^2+2\zeta w_c s+w_c^2}$ and comparing with the H(s) function from the question we get that:

$H(s) = \frac{1}{(1\times10^{-6})s^2+(2\times10^{-3})s+1} \rightarrow w_c^2 = \frac{1}{10^{-6}} \rightarrow w_c = 1000$

Since w = 2πf, $w_c = 2\pi f_c$

$$f_c = \frac{w_c}{2\pi} = 159.15\ Hz$$

e) How do the theoretical and measured results compare? Comment on your results.

Frequency	abs(V_o/V_i) Percent Error
50 Hz	4.18%
100 Hz	6.81%
200 Hz	9.23%
500 Hz	13.04%
1 kHz	56.4%
1.1 kHz	62.86%
1.2 kHz	66.47%
1.3 kHz	86.00%
1.4 kHz	88.46%
1.5 kHz	116.36%
1.6 kHz	118.36%
1.7 kHz	132.18%
1.8 kHz	167.95%
1.9 kHz	207.14%
2 kHz	228.57%
5 kHz	1909.99%

The theoretical and measured results are generally good and similar at lower frequencies (less than 500 Hz), but very noticeable discrepancies arise as the frequency values increase. At lower frequency values, the percent error remains relatively low (4-13%), indicating that the circuit performs as expected and closely follow the theoretical predictions. However, as these frequency values increase, the percent error increases significantly. This suggests that the filters real world behaviour is not of that of the ideal model. This may be due to many factors, including the roll-off of the filter not being as sharp as predicted by the second order filter, or the increased impact of noise and non-ideal behaviour of the op-amp at higher frequencies. Additionally, the AD3 and circuit itself may have limitations for higher frequency signals. The Bode plot further supports this by showing a clear transition in the filter's behaviour around the cutoff frequency, approximately 1.5 kHz (10000 rad/s). Beyond this point, the gain drops more rapidly, as expected from a low-pass filter, but the measured gain values deviate more from the ideal curve. These results suggests that while the theoretical model provides a strong foundation, real-world limitations like bandwidth constraints, parasitic effects, and resolution must be considered for higher frequencies within filters.