Assignment 2 description and solutions CTA200H

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1 Question 1

1.1 Methods

In order to plot the complex number z on a 2D plane, I separated z and c(the fixed point $x+iy$) into Real and Imaginary parts corresponding to x and y (axis) components, respectively. I found the expression $Z_{n+1} = [(ReZ_n)^2 - (ImZ_n)^2 + x] + 2i[(ReZ_n)(ImZ_n) + y]$ for the sequence $Z_{n+1} = (Z_n)^2 + c, Z_0 = 0$ as n increases. To test the divergence, I selected some c points with iteration steps varying from $10 to 50$ and computed the $|z|^2$for each step. The iteration terminated if $|z|^2$ is larger than 2, which indicated the sequence is diverging under the initial choice of c. I created a n-dim array to partition the domain of c into a $Matrix_{n\times n}$ so that I can collect the diverging c points and marked them with colours to distinguish from the bounded ones. I attempted to demonstrate the rate of divergence for the diverging c points according to the relative difference between their $|z|^2$ value (i.e. the magnitude of the entries in $Matrix_{n\times n} = np.abs(z)$).

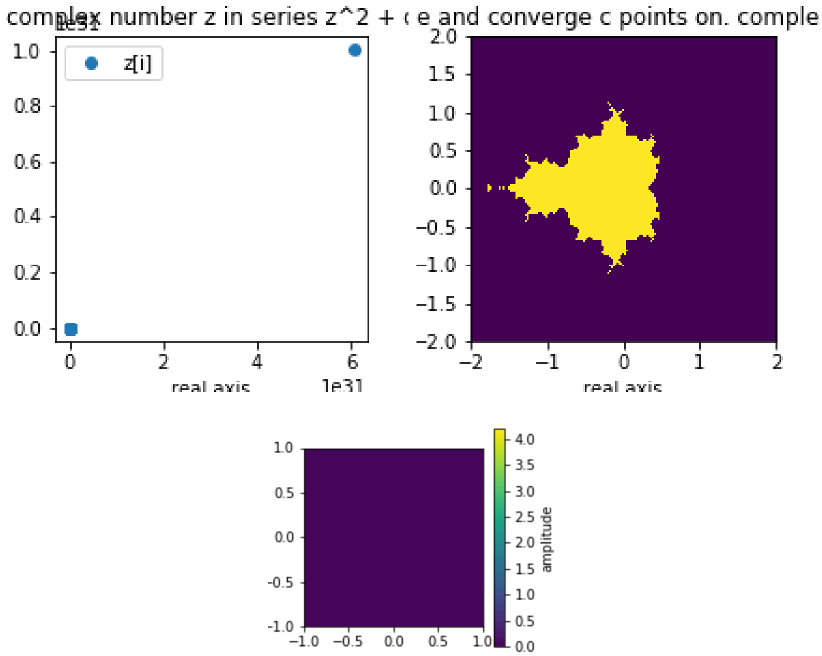

Figure 1: a) The set of c points confined within a 4 by 4 square are plotted on the complex plane for $c = 1 + 1i$. b) The shaded region represents the c points that generate diverging sequence $z_{i+1} = (z_i)^2 + c$. The yellow region shows the c points that yield bounded sequences. c)The rate of divergence with color bar generated from n-dim matrix entries. The color resembles the $3rd$ axis apart from x and y axis.

2 Discussion

In Fig. 1a, since $c = 1 + 1i$ yields diverging sequence from the $8th$ iteration step, the blue dots are far away from each other as expected. In Fig. 1b, the bounded c points are bounded by an open ball of radius 2 centered at the origin. The n-dim array of $|z|^2$ contains different entries, unlike the homogeneous color as shown in Fig. 1c.

3 Question 2

3.1 Methods

I implemented the initial condition: $N = 1000, S(t)+I(t)+R(t) = N for any t \in [0, 200]$ to solve the ODE by using the $Scipy.odeint$ package. The parameters: $\beta and \gamma$ determine the slope of the first derivative of S, I, R. I expect S to decrease while I increases in the beginning followed by the delayed increase in R. To take into account of the death rate arising from the infected population, I added $D(t) * I(t)$ to dI/dt and subtracted the death term from dR/dt to satisfy the initial condition.

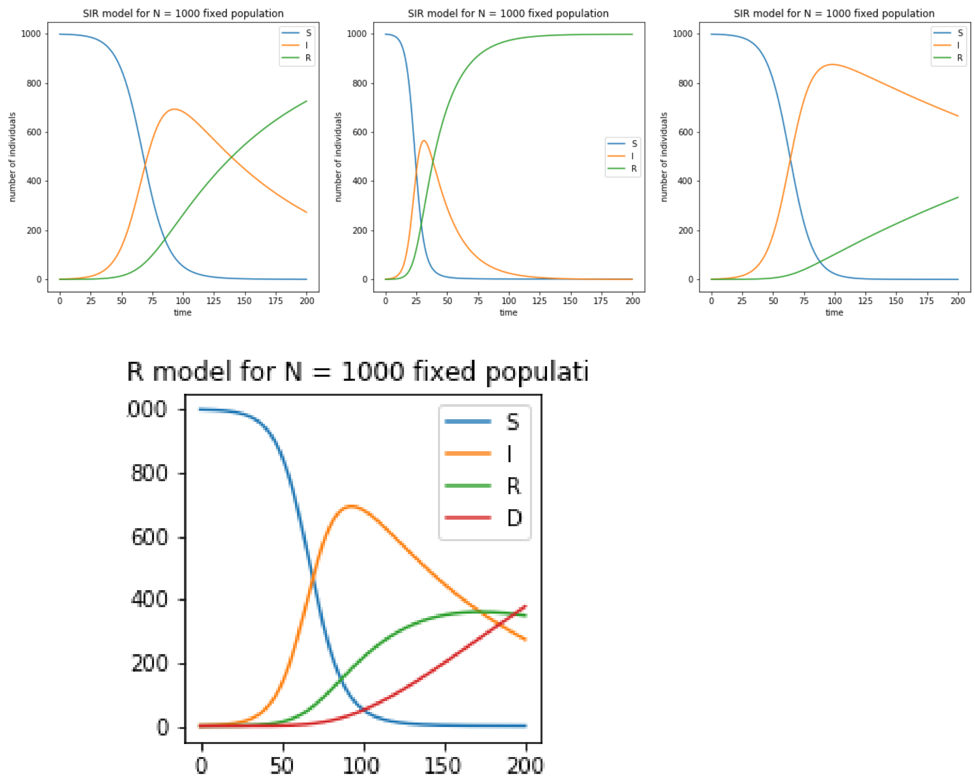

Figure 2: a) The S(t), I(t), R(t) curves plotted for $t = 200 days$, representing the number of vulnerable(not yet affected) individuals, infected individuals and recovered individuals, respectively. The first graph has parameters $\beta = 9 \times 10^6, \gamma = 0.01$, second one with $\beta = 3 \times 10^6, \gamma = 0.05$ and the last one with $\beta = 9 \times 10^6, \gamma = 0.003$. The initial condition is $S(0) = 999, I(0) = 1 and R(0) = 0$ b)Same as a) with a death term included $\beta = 3 \times 10^6, \gamma = 0.005, death2 = 0.01$.

4 Discussion

Among the 3 figures in Fig. 2a, the one the middle best represents the disease spreading model with a reasonable infected curve and a flattened out recovered curve. Unlike the first and the third plot, the decaying tails of $I(t)$ and $R(t)$ expanded beyond the 200 days time interval which is too slow for most common diseases. In Fig. 2b, the death rate exceeds the recovered rate near $t = 180 days$. Vulnerable but not infected individuals vanished after $t = 75 days$, they become either recovered or dead as shown in this new model.