Back propogation(Recursive Representation)

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Error of a network can be represented by the formula of each node in the output layer.

$$e = \Sigma_\tau l_\tau(x_\tau)$$

The relationship between derectly connected nodes can be represented as

$$x_r = f_r(\Sigma_\rho w_{r\rho} x_\rho)$$

For the contribution of the differential $\mathrm{d}x_r$ which is the output of a specific node

$$\begin{aligned} \mathrm{d}e &= l_r' \mathrm{d}x_r \\ \mathrm{d}x_p &= f_p' w_{pq} \mathrm{d}x_q \end{aligned} \tag{1}$$

A new symbol β can be introduced to describe the relationship. As a consequence, for the differential $\mathrm{d}x_q$ of a specific node q

$$\mathrm{d}e = \beta_q \mathrm{d}x_q$$

According to the relationship between each $\mathrm{d}x_\rho$ and $\mathrm{d}x_q$ as in eq. (1) as well as the chains from node q to e

$$e \longleftarrow \{x_\rho | x_\rho \leftarrow x_q\} \longleftarrow x_q$$

which means there is a different chain from q to e through different node ρ, there is

$$\begin{aligned} \mathrm{d}e &= \beta_q \mathrm{d}x_q \\ &= \sum_{\rho \in \{\rho | \rho \leftarrow q\}} \beta_\rho f_\rho' w_{\rho q} \mathrm{d}x_q \\ \beta_q &= \sum_{\rho \in \{\rho | \rho \leftarrow q\}} \beta_\rho f_\rho' w_{\rho q} \end{aligned} \tag{2}$$

Further simplification can be deduced by multiplying f_q' to eqn (2)

$$\begin{aligned} \beta_q f_q' &= \sum_{\rho \in \{\rho | \rho \leftarrow q\}} \beta_\rho f_\rho' w_{\rho q} f_q' \\ \delta_q &= \sum_{\rho \in \{\rho | \rho \leftarrow q\}} \delta_\rho w_{\rho q} f_q' \end{aligned} \tag{3}$$

Eqn (3) is the back propogation rule.

For the differential of $\mathrm{d}w_{\mathrm{pq}}$, there is relationship between two node as

$$\mathrm{d}x_p = f_p' \cdot \mathrm{d}w_{pq} \cdot x_q$$

then

$$\begin{aligned} \mathrm{d}e &= \beta_p \mathrm{d}x_p \\ &= \beta_p f_p' \cdot \mathrm{d}w_{pq} \cdot x_q \\ &= \delta_p \cdot \mathrm{d}w_{pq} \cdot x_q \end{aligned}$$

the update of w_{pq} is

$$\begin{aligned} \Delta w_{pq} &= -\eta \delta_p x_q \\ w_{pq} &\leftarrow w_{pq} + \Delta w_{pq} \end{aligned}$$