计算视觉与模式识别

线性滤波器

线性滤波与卷积

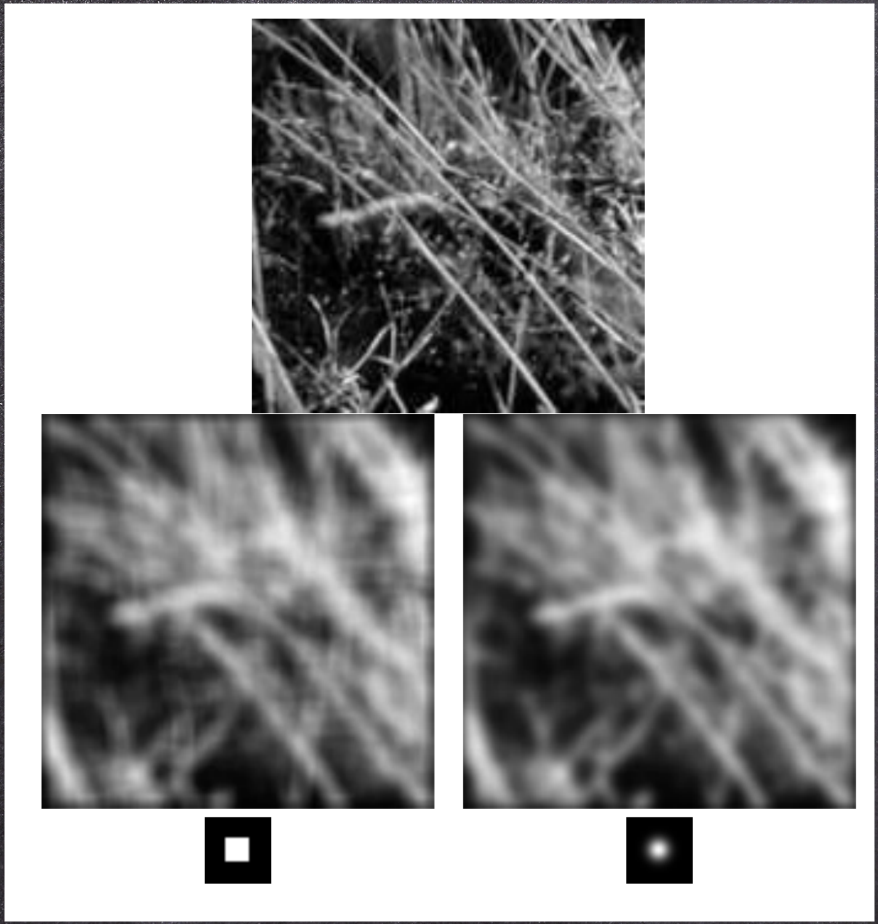
平均值：

$$\mathcal{R}_{ij} = \frac{1}{(2k+1)^2} \sum_{u=i-k}^{u=i+k} \sum_{v=j-k}^{v=j+k} \mathcal{F}_{uv}$$

卷积：

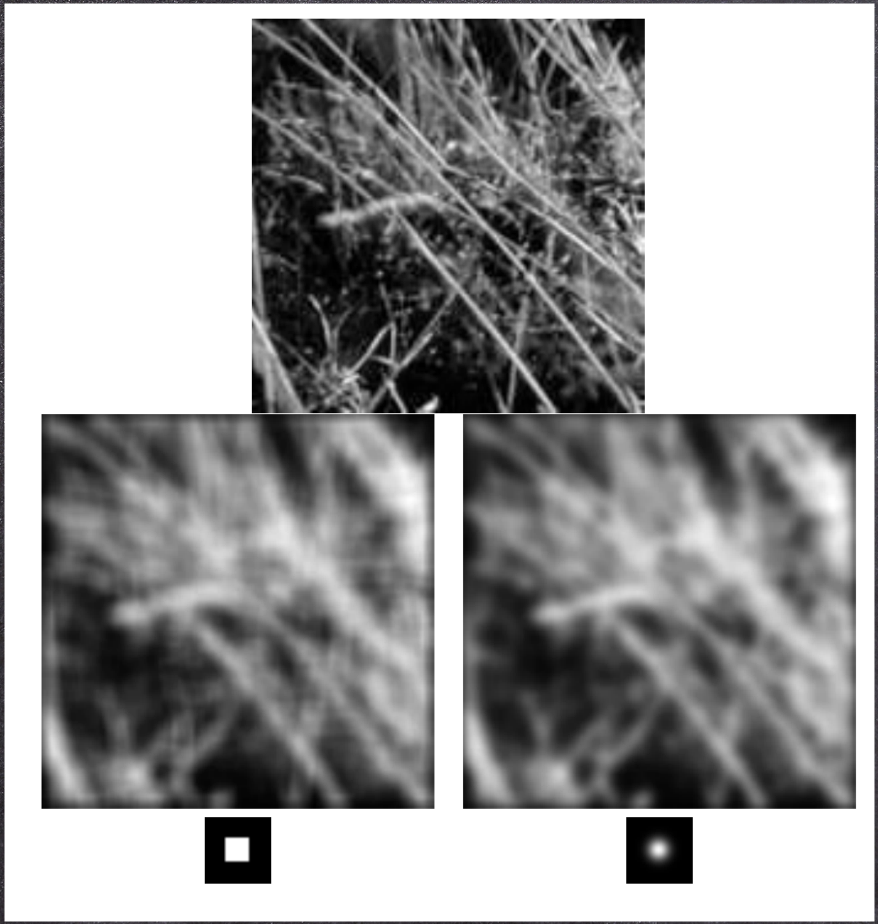
$$\mathcal{R}_{ij} = \sum_{u,v} H_{i-u,j-v} F_{u,v}$$

均值滤波示例



- Figure 8.1.

均值滤波示例



- Figure 8.1.

Although a uniform local average may seem to give a good blurring model, it generates effects that are not usually seen in defocussing a lens. The images above compare the effects of a uniform local average with weighted average. The image at the top shows a view of grass. On the left in the second row, the result of blurring this image using a uniform local model and on the right, the result of blurring this image using a set of Gaussian weights. The degree of blurring in each case is about the same, but the uniform average produces a set of narrow vertical and horizontal bars — an effect often known as ringing. The bottom row shows the weights used to blur the image, themselves rendered as an image; bright points represent large values and dark points represent small values (in this example the smallest values are zero).

高斯核

对称高斯核：

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$

离散形式，$2k+1\times 2k+1$矩阵：

$$H_{ij} = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{((i-k-1)^2+(j-k-1)^2)}{2\sigma^2}\right)$$

高斯核

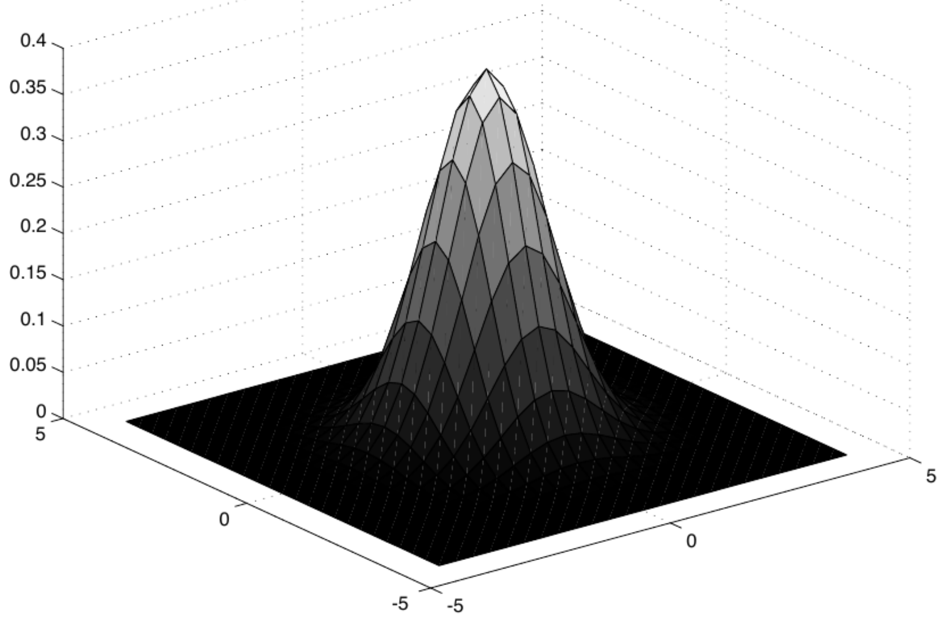
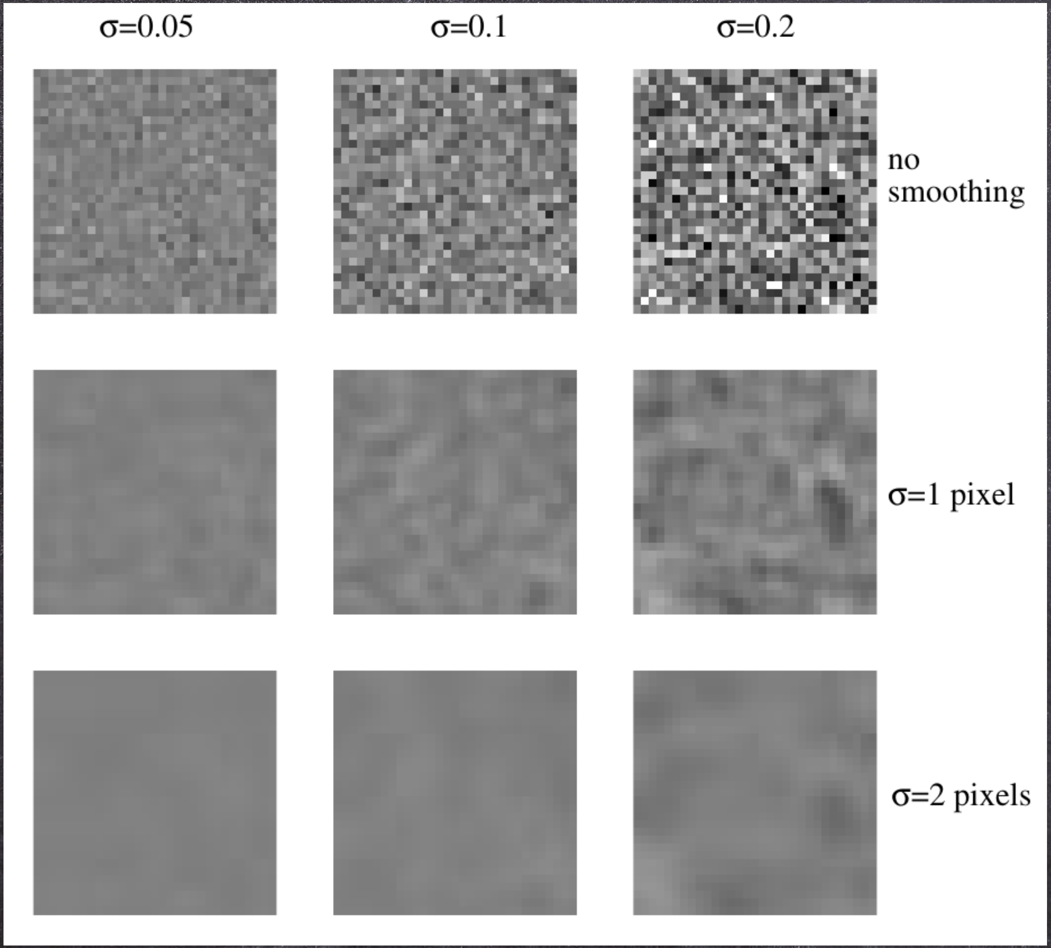


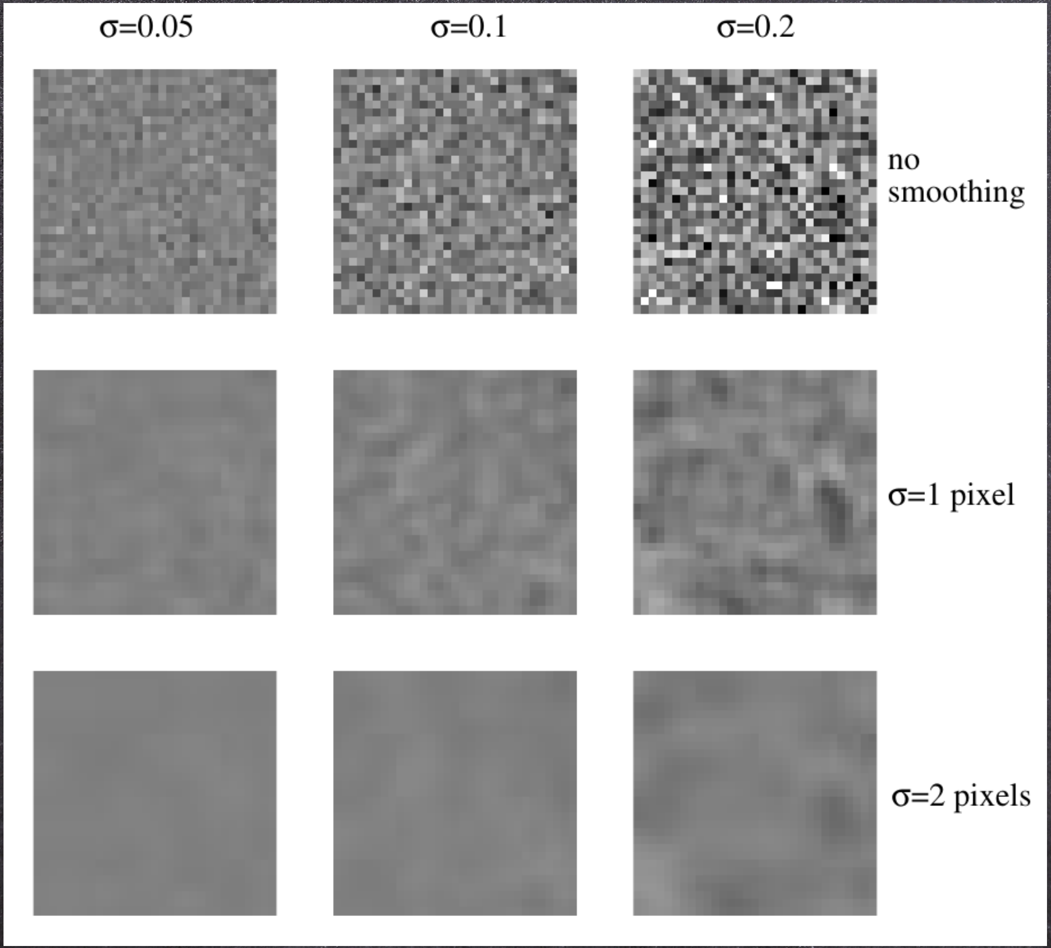
Figure 8.2. The symmetric Gaussian kernel in 2D. This view shows a kernel scaled so that its sum is equal to one; this scaling is quite often omitted. The kernel shown has $\sigma = 1$. Convolution with this kernel forms a weighted average which stresses the point at the center of the convolution window, and incorporates little contribution from those at the boundary. Notice how the Gaussian is qualitatively similar to our description of the point spread function of image blur; it is circularly symmetric, has strongest response in the center, and dies away near the boundaries.

高斯噪声与高斯滤波器



- FIGURE 4.3:

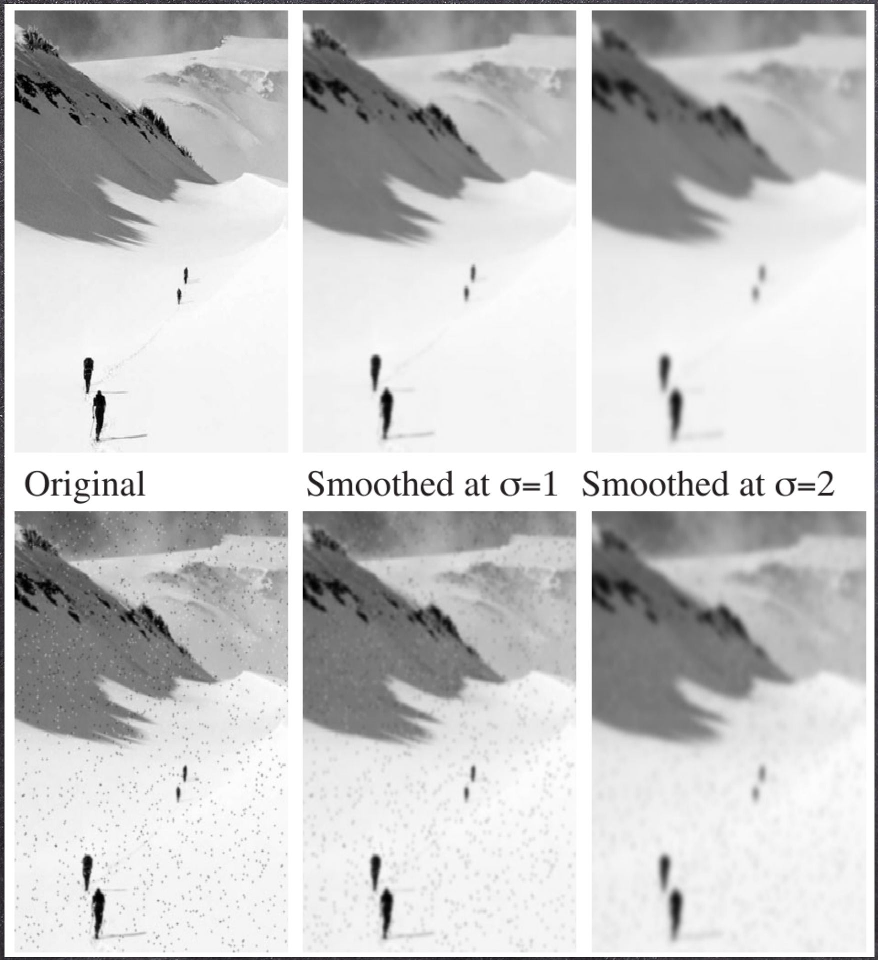
高斯噪声与高斯滤波器



- FIGURE 4.3:

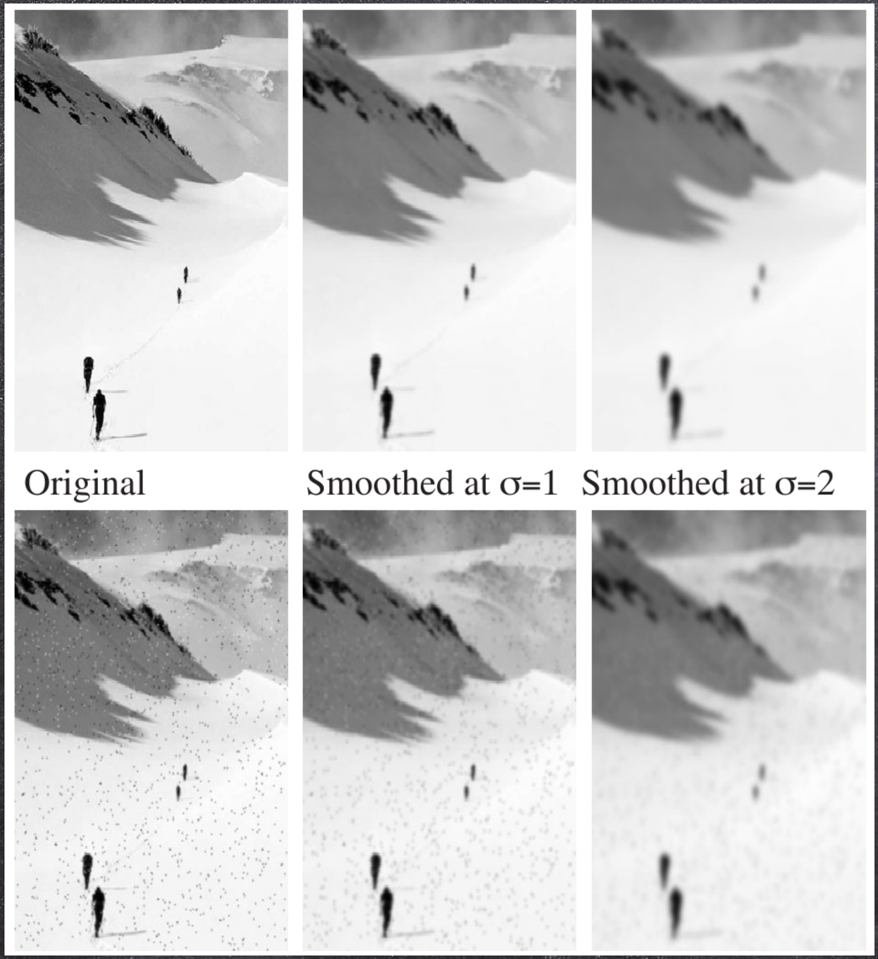
The top row shows images of a constant mid-gray level corrupted by additive Gaussian noise. In this noise model, each pixel has a zero-mean normal random variable added to it. The range of pixel values is from zero to one, so that the standard deviation of the noise in the first column is about $1/20$ of full range. The center row shows the effect of smoothing the corresponding image in the top row with a Gaussian filter of σ one pixel. Notice the annoying overloading of notation here; there is Gaussian noise and Gaussian filters, and both have σ's. One uses context to keep these two straight, although this is not always as helpful as it could be, because Gaussian filters are particularly good at suppressing Gaussian noise. This is because the noise values at each pixel are independent, meaning that the expected value of their average is going to be the noise mean. The bottom row shows the effect of smoothing the corresponding image in the top row with a Gaussian filter of σ two pixels.

椒盐噪声与高斯滤波



- Figure 8.3.

椒盐噪声与高斯滤波



- Figure 8.3.

In salt-and-pepper noise, we choose pixels uniformly at random, and uniformly at random make them either black or white. Gaussian smoothing is particularly effective at suppressing the effects of salt-and-pepper noise. The top row shows an image, and versions smoothed by a symmetric Gaussian with σ two pixels and four pixels. The images in the second row are obtained by corrupting the images in the top row by this noise model and then smoothing the result. Notice that, as the smoothing increases, detail is lost, but the effects of the noise diminish, too — the smoothed versions of the noisy images look very much like the smoothed version of the noise-free images.

导数与有限差分

偏导数

$$\frac{\partial f}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x, y)}{\varepsilon}$$

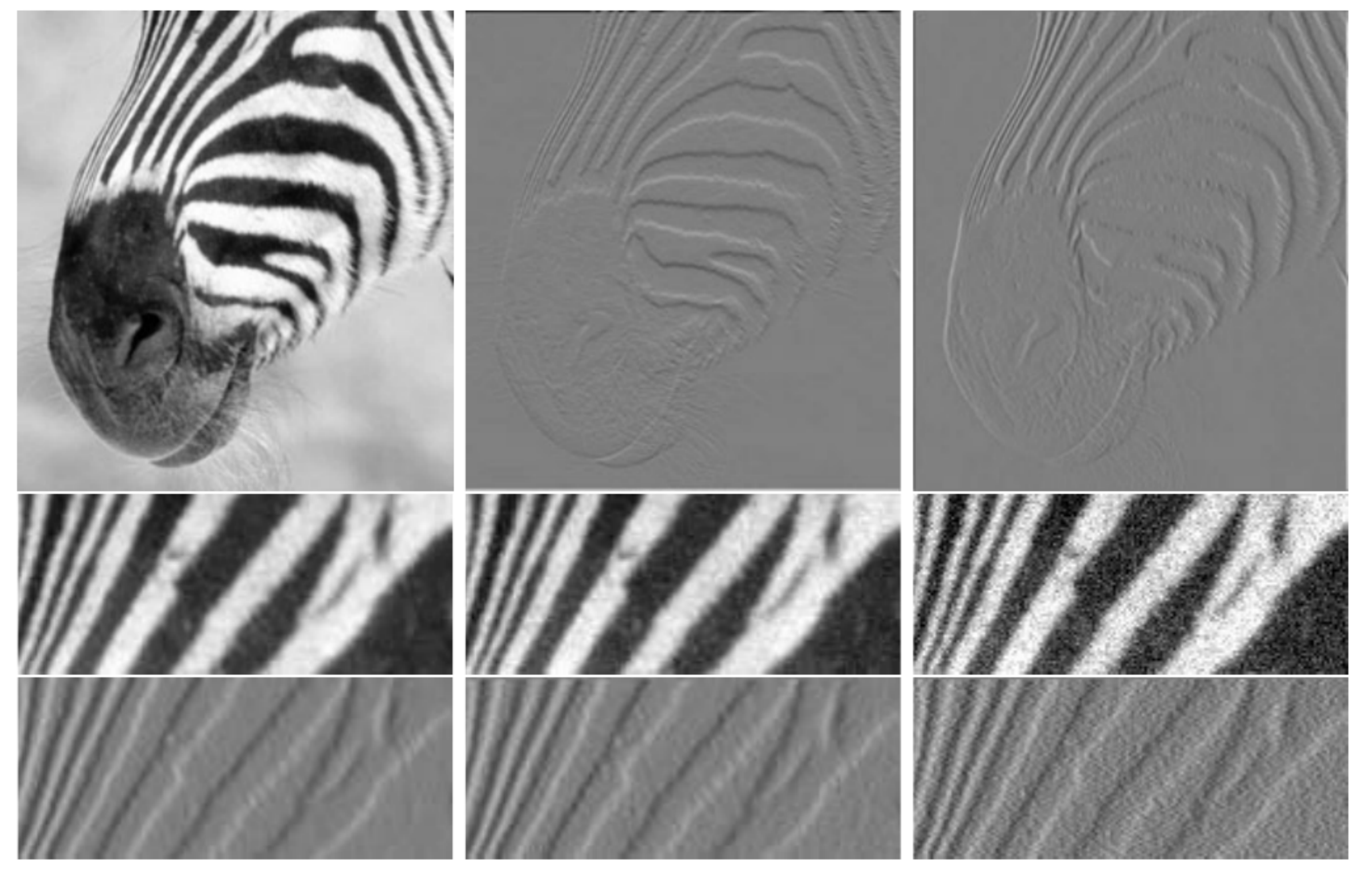
有限差分

$$\frac{\partial h}{\partial x} \approx h_{i+1,j} - h_{i-1,j}$$

卷积核

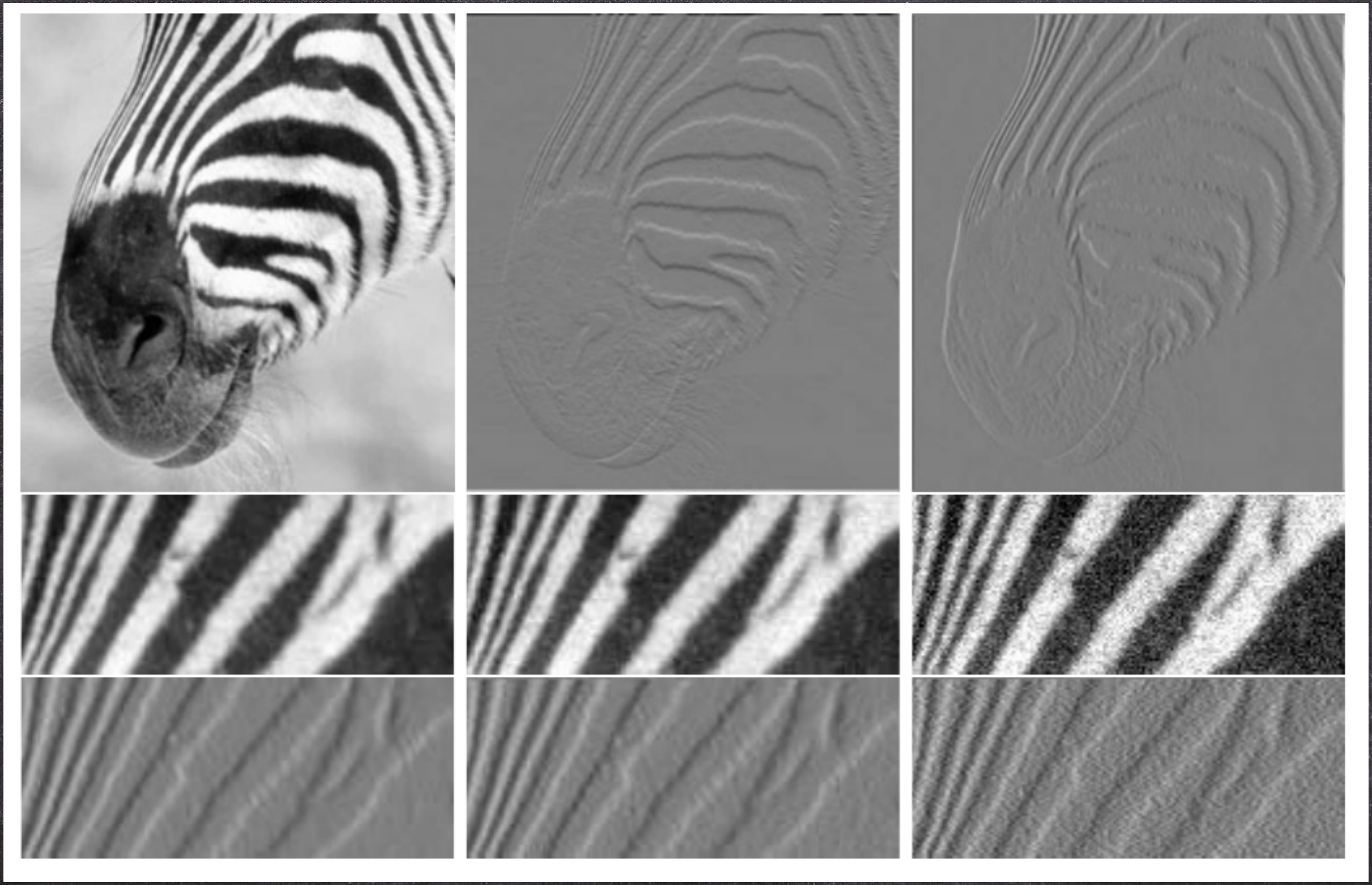
$$\mathcal{H} = \begin{Bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{Bmatrix}$$

差分图像



- FIGURE 4.4:

差分图像



- FIGURE 4.4:

The top row shows estimates of derivatives obtained by finite differences. The image at the left shows a detail from a picture of a zebra. The center image shows the partial derivative in the y-direction—which responds strongly to horizontal stripes and weakly to vertical stripes—and the right image shows the partial derivative in the x-direction—which responds strongly to vertical stripes and weakly to horizontal stripes. However, finite differences respond strongly to noise. The image at center left shows a detail from a picture of a zebra; the next image in the row is obtained by adding a random number with zero mean and normal distribution ($\sigma = 0.03$; the darkest value in the image is 0, and the lightest 1) to each pixel; and the third image is obtained by adding a random number with zero mean and normal distribution ($\sigma = 0.09$) to each pixel. The bottom row shows the partial derivative in the x-direction of the image at the head of the row. Notice how strongly the differentiation process emphasizes image noise; the derivative figures look increasingly grainy. In the derivative figures, a mid-gray level is a zero value, a dark gray level is a negative value, and a light gray level is a positive value.

平移不变线性系统

线性

$$
\begin{aligned}
R(f+g) &= R(f)+R(g) \\
R(kf) &= kR(f)
\end{aligned}
$$

平移不变

$$
\begin{aligned}
R(f(x)) &= g(x) \\
R(f(x-y)) &= g(x-y)
\end{aligned}
$$

离散卷积

$$
\begin{aligned}
\boldsymbol{e}_0 &= \ldots .0,0,0,1,0,0,0.\ldots \\
\boldsymbol{f} &= \sum_i f_i \mathrm{Shift}(\boldsymbol{e}_0, i) \\
R(\mathrm{Shift}(\boldsymbol{f}, k)) &= \mathrm{Shift}(R(\boldsymbol{f}), k) \\
R(k\boldsymbol{f}) &= kR(\boldsymbol{f})
\end{aligned}
$$

$$
\begin{aligned}
R(\boldsymbol{f}) &= R\left(\sum_i f_i \mathrm{Shift}(\boldsymbol{e}_0, i)\right) \\
&= \sum_i R(f_i \mathrm{Shift}(\boldsymbol{e}_0, i)) \\
&= \sum_i f_i R(\mathrm{Shift}(\boldsymbol{e}_0, i)) \\
&= \sum_i f_i \mathrm{Shift}(R(\boldsymbol{e}_0), i)
\end{aligned}
$$

离散卷积（续）

$$
\begin{aligned}
\boldsymbol{g} &= R(\boldsymbol{e}_0) \\
R(\boldsymbol{f}) &= \sum_i f_i \mathrm{Shift}(\boldsymbol{g}, i) \\
&= \boldsymbol{g} * \boldsymbol{f} \\
R_j &= \sum_i g_{j-i} f_i
\end{aligned}
$$

二维卷积

$$
\begin{aligned}
\mathcal{E}_{00} &= \begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix} \\
R_{ij} &= \sum_{u,v} G_{i-u,j-v} F_{uv} \\
\mathcal{R} &= \mathcal{G} ** \mathcal{H}
\end{aligned}
$$

连续卷积

$$
\begin{aligned}
R(kf) &= kR(f) \\
\mathrm{Shift}(f, c) &= f(u-c) \\
R(\mathrm{Shift}(f, c)) &= \mathrm{Shift}(R(f), c) \\
\mathrm{box}_\varepsilon(x) &= \begin{cases} 0 & \mathrm{abs}(x) > \frac{\varepsilon}{2} \\ 1 & \mathrm{abs}(x) < \frac{\varepsilon}{2} \end{cases} \\
R\left(\sum_i f_i \mathrm{Shift}(\mathrm{box}_\varepsilon, x_i)\right) &= \sum_i R(f_i \mathrm{Shift}(\mathrm{box}_\varepsilon, x_i)) \\
&= \sum_i f_i R(\mathrm{Shift}(\mathrm{box}_\varepsilon, x_i)) \\
&= \sum_i f_i \mathrm{Shift}\left(R\left(\frac{\mathrm{box}_\varepsilon}{\varepsilon}\varepsilon\right), x_i\right) \\
&= \sum_i f_i \mathrm{Shift}\left(R\left(\frac{\mathrm{box}_\varepsilon}{\varepsilon}\right), x_i\right)\varepsilon
\end{aligned}
$$

连续卷积（续）

δ函数

$$
\begin{aligned}
d_\varepsilon(x) &= \frac{\mathrm{box}_\varepsilon(x)}{\varepsilon} \\
\delta(x) &= \lim_{\varepsilon\to 0} d_\varepsilon(x)
\end{aligned}
$$

卷积

$$
\begin{aligned}
R(f) &= \int \{R(\delta(u-x'))\} f(x')\mathrm{d}x' \\
&= \int g(u-x') f(x')\mathrm{d}x' \\
&= (g*f) \\
(g*f) &= (h*g) \\
(f*(g*h)) &= ((f*g)*h)
\end{aligned}
$$

二维卷积

$$
\begin{aligned}
d_\varepsilon(x,y) &= \frac{\mathrm{box}_{\varepsilon^2}(x,y)}{\varepsilon^2} \\
R(h) &= \iint g(x-x', y-y')h(x',y')\mathrm{d}x\mathrm{d}y \\
&= (g**h) \\
(g**h) &= (h**g)
\end{aligned}
$$

傅里叶变换

$$\begin{aligned}\mathcal{F}(g(x,y)) &= \iint g(x,y)e^{-i2\pi(ux+vy)}\mathrm{d}x\mathrm{d}y \\ e^{-i2\pi(ux+vy)} &= \cos(2\pi(ux+vy))+i\sin(2\pi(ux+vy))\end{aligned}$$

线性

$$\begin{aligned}\mathcal{F}(g+h) &= \mathcal{F}(g)+\mathcal{F}(h) \\ \mathcal{F}(kg(x,y)) &= k\mathcal{F}(g(x,y))\end{aligned}$$

反变换

$$g(x,y) = \iint \mathcal{F}(g)e^{i2\pi(ux+vy)}\mathrm{d}u\mathrm{d}v$$

傅里叶基实部示例

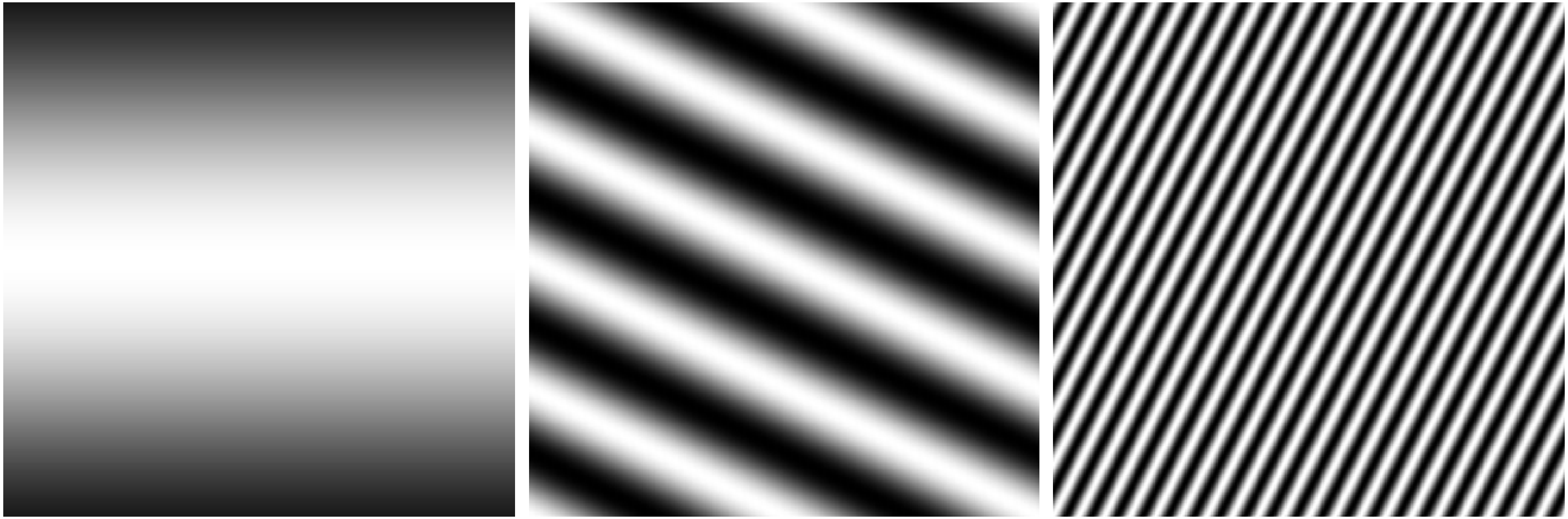


Figure 8.6. The real component of Fourier basis elements, shown as intensity images. The brightest point has value one, and the darkest point has value zero. The domain is $[-1,1]x[-1,1]$, with the origin at the center of the image. On the left, $(u,v) = (0, .4)$; in the center, $(u,v) = (1,2)$ and on the right $(u,v) = (10,-5)$. These are sinusoids of various frequencies and orientations, described in the text.

幅值与相位

$$
\begin{aligned}
\mathcal{F}(g(x,y)) &= \iint g(x,y)\cos(2\pi ux+vy)\mathrm{d}x\mathrm{d}y \\
&\quad +i\iint g(x,y)\sin(2\pi ux+vy)\mathrm{d}x\mathrm{d}y \\
&= \Re(\mathcal{F}(g))+i*\Im(\mathcal{F}(g)) \\
&= \mathcal{F}_R(g)+i*\mathcal{F}_I(g)
\end{aligned}
$$

傅里叶变换示例

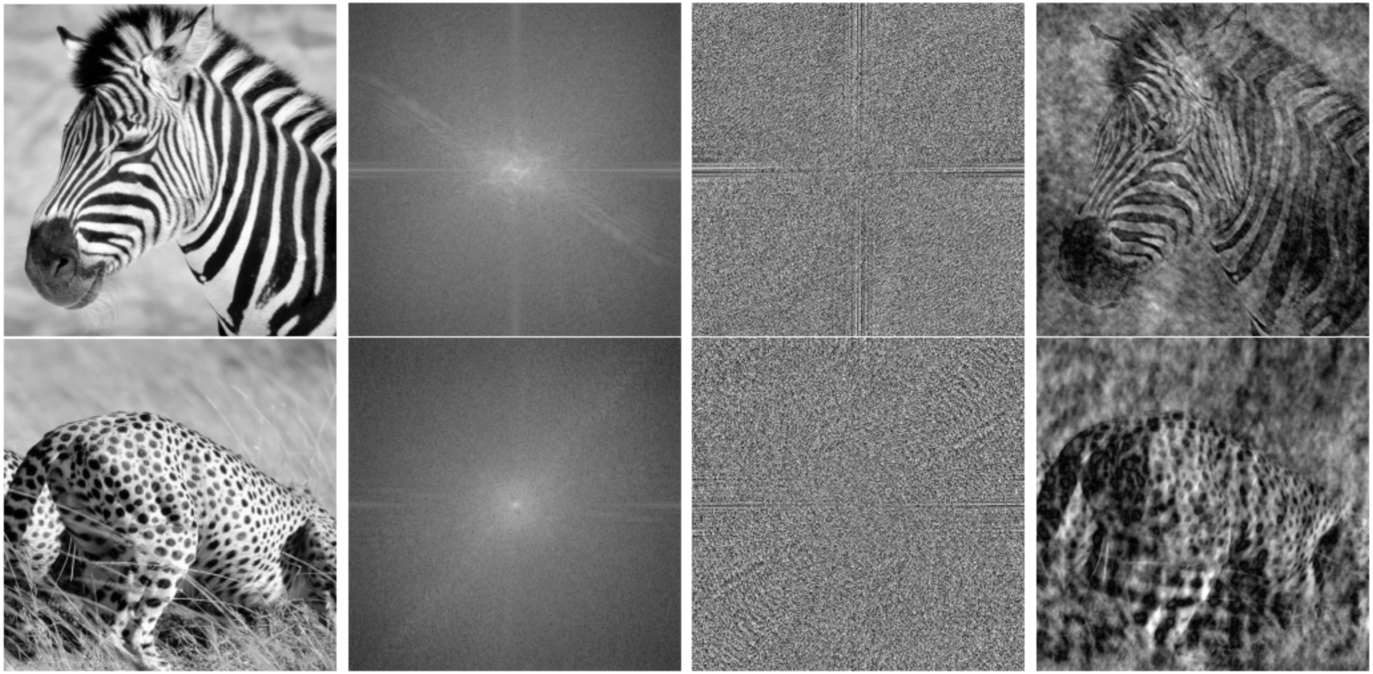


Figure 8.7. The second image in each row shows the log of the magnitude spectrum for the first image in the row; the third image shows the phase spectrum, scaled so that $-\pi$ is dark and π is light. The final images are obtained by swapping the magnitude spectra. While this swap leads to substantial image noise, it doesn't substantially affect the interpretation of the image, suggesting that the phase spectrum is more important for perception than the magnitude spectrum.