Back propogation(Recursive Representation)

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Error of a network can be represented by the formula of each node in the output layer.

$$e = \Sigma_\tau l_\tau(x_\tau)$$

The relationship between derectly connected nodes can be represented as

$$x_r = f_r(\Sigma_\rho w_{r\rho} x_\rho)$$

For the contribution of an increment which is the output of a specific node

$$\begin{aligned} \Delta e &= l'_r \Delta x_r \\ \Delta x_p &= f'_p w_{pq} \Delta x_q \end{aligned} \tag{1}$$

A new symbol β can be introduced to describe the relationship. As a consequence, for specific increment Δx_q of node q

$$\Delta e = \beta_q \Delta x_q$$

According to the relationship between each Δx_ρ and Δx_q as in eq. (1) as well as the chains from node q to e

$$e \longleftarrow \{x_\rho | x_\rho \leftarrow x_q\} \longleftarrow x_q$$

which means there is a different chain from q to e through different node ρ, there is

$$\begin{aligned} \Delta e &= \beta_q \Delta x_q \\ &= \sum_{\rho \in \{\rho | \rho \leftarrow q\}} \beta_\rho f'_\rho w_{\rho q} \Delta x_q \\ \beta_q &= \sum_{\rho \in \{\rho | \rho \leftarrow q\}} \beta_\rho f'_\rho w_{\rho q} \end{aligned} \tag{2}$$

Further simplification can be deduced by multiplying f'_q to eqn (2)

$$\begin{aligned} \beta_q f'_q &= \sum_{\rho \in \{\rho | \rho \leftarrow q\}} \beta_\rho f'_\rho w_{\rho q} f'_q \\ \delta_q &= \sum_{\rho \in \{\rho | \rho \leftarrow q\}} \delta_\rho w_{\rho q} f'_q \end{aligned} \tag{3}$$

Eqn (3) is the back propogation rule.

For the increment of Δw_{pq}, there is relationship between two node as

$$\Delta x_p = f'_p \Delta w_{pq} x_q$$

then

$$\begin{aligned} \Delta e &= \beta_p \Delta x_p \\ &= \beta_p f'_p \Delta w_{pq} x_q \\ &= \delta_p \Delta w_{pq} x_q \end{aligned}$$

the update of w_{pq} is

$$\Delta w_{pq} = -\eta \delta_p x_q$$