计算视觉与模式识别

透视缩减

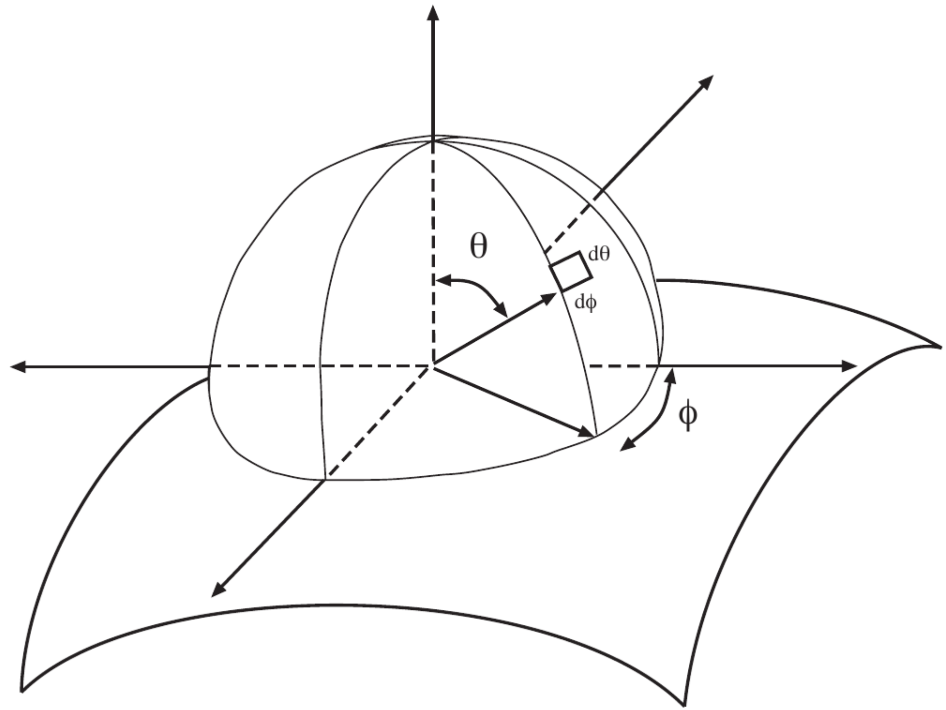

Figure 2.1. A point on a surface sees the world along a hemisphere of directions centered at the point; the surface normal is used to orient the hemisphere, to obtain the θ, ϕ coordinate system that we use consistently from now on to describe angular coordinates on this hemisphere. Usually in radiation problems we compute the brightness of the surface by summing effects due to all incoming directions, so that the fact we have given no clear way to determine the direction in which $\phi = 0$ is not a problem.

立体角

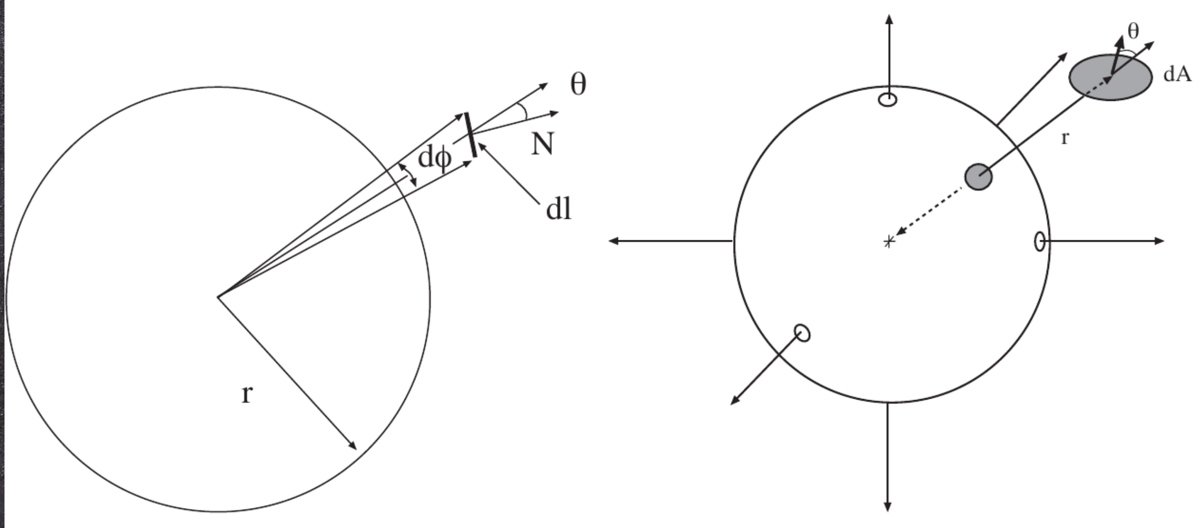

Figure 2.2. Top: The angle subtended by a curve segment at a particular point is obtained by projecting the curve onto the unit circle whose center is at that point, and then measuring the length of the projection. For a small segment, the angle is $(1/r)dl\cos\theta$. **Bottom:** A sphere, illustrating the concept of solid angle. The small circles surrounding the coordinate axes are to help you see the drawing as a 3D surface. An infinitesimal patch of surface is projected onto the unit sphere centered at the relevant point; the resulting area is the solid angle of the patch. In this case, the patch is small, so that the angle is $(1/r^2)dA\cos\theta$.

$$
\begin{aligned}
\mathrm{d}\varphi &= \frac{\mathrm{d}l\cos\theta}{r} \\
\mathrm{d}\omega &= \frac{\mathrm{d}A\cos\theta}{r^2}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{d}\omega &= \frac{r\sin\theta\mathrm{d}\varphi\cdot r\mathrm{d}\theta}{r^2} \\
&= \sin\theta\mathrm{d}\theta\mathrm{d}\varphi
\end{aligned}
$$

辐射率

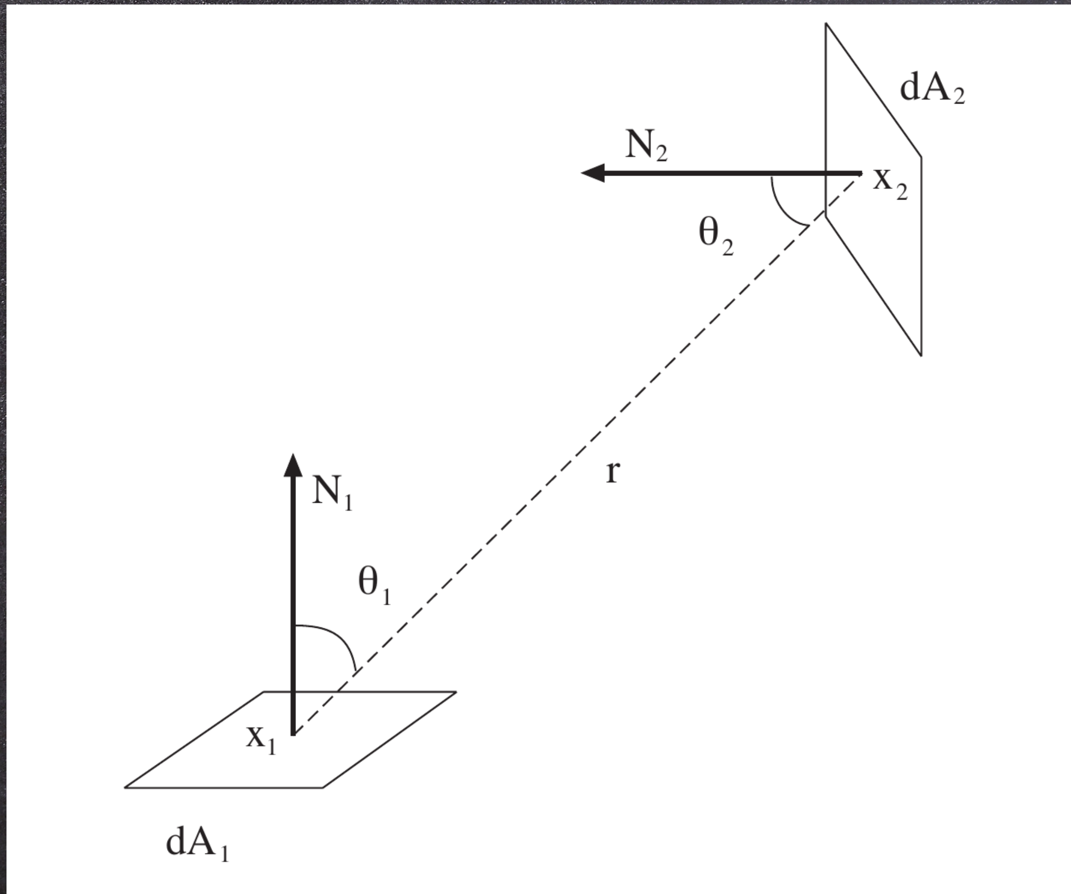

Figure 2.3. Light intensity is best measured in radiance, because radiance does not go down along straight line paths in a vacuum (or, for reasonable distances, in clear air). This is shown by an energy conservation argument in the text, where one computes the energy transferred from a patch dA_1 to a patch dA_2

$$L(x_1, \theta, \varphi)(\cos\theta_1 \mathrm{d}A_1)(d\omega)(dt)$$

$$\begin{aligned}
\mathrm{d}^3 E_{1\to 2} &= L(x_1, x_1 \to x_2)\cos\theta_1 \mathrm{d}\omega_{2(1)} \mathrm{d}A_1 \mathrm{d}t \\
&= L(x_1, x_1 \to x_2)\frac{\cos\theta_1 \cos\theta_2 \mathrm{d}A_2 \mathrm{d}A_1 \mathrm{d}t}{r^2} \\
\mathrm{d}\omega_{2(1)} &= \frac{\cos\theta_2 \mathrm{d}A_2}{r^2} \\
\mathrm{d}^3 E_{1\to 2} &= L(x_1, x_1 \to x_2)\cos\theta_2 \mathrm{d}\omega_{1(2)} \mathrm{d}A_2 \mathrm{d}t \\
&= L(x_1, x_1 \to x_2)\frac{\cos\theta_2 \cos\theta_1 \mathrm{d}A_1 \mathrm{d}A_2 \mathrm{d}t}{r^2} \\
\mathrm{d}\omega_{1(2)} &= \frac{\cos\theta_1 \mathrm{d}A_1}{r^2}
\end{aligned}$$

辐照度

$$L_i(x, \theta_i, \varphi_i)\cos\theta_i \mathrm{d}\omega$$

双向反射分布函数

$$\rho_{\mathrm{bd}}(\theta_o, \varphi_o, \theta_i, \varphi_i) = \frac{\mathrm{d}L_0(x, \theta_o, \varphi_o)}{L_i(x, \theta_i, \varphi_i)\cos\theta_i \mathrm{d}\omega}$$

设

$$
\begin{aligned}
L_i(\boldsymbol{x}, \theta_i, \varphi_i) &= \int_\Omega \frac{1}{\cos\theta} \cos\theta \mathrm{d}\omega \\
&= \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \sin\theta \mathrm{d}\theta \mathrm{d}\varphi \\
&= 2\pi
\end{aligned}
$$

得

$$
L(\boldsymbol{x}, \theta_o, \varphi_o) = \int_\Omega \rho_{\mathrm{bd}}(\theta_o, \varphi_o, \theta_i, \varphi_i) L_i(\boldsymbol{x}, \theta_i, \varphi_i) \cos\theta_i \mathrm{d}\omega
$$

$$
\begin{aligned}
2\pi &\geqslant \int_{\Omega_o} L_o(\boldsymbol{x}, \theta_o, \varphi_o)\cos\theta_o \mathrm{d}\omega_o \\
&= \int_{\Omega_o}\int_{\Omega_i} \rho_{\mathrm{bd}}(\theta_o, \varphi_o, \theta_i, \varphi_i) L_i(\boldsymbol{x}, \theta_i, \varphi_i)\cos\theta_i \mathrm{d}\omega_i \cos\theta_o \mathrm{d}\omega_o \\
&= \int_{\Omega_o}\int_{\Omega_i} \rho_{\mathrm{bd}}(\theta_o, \varphi_o, \theta_i, \varphi_i) \mathrm{d}\omega_i \cos\theta_o \mathrm{d}\omega_o \\
&= \int_{\Omega_o}\int_{\Omega_i} \rho_{\mathrm{bd}}(\theta_o, \varphi_o, \theta_i, \varphi_i) \sin\theta_i \mathrm{d}\theta_i \mathrm{d}\varphi_i \cos\theta_o \sin\theta_o \mathrm{d}\theta_o \mathrm{d}\varphi_o
\end{aligned}
$$

辐射度

$$B(\boldsymbol{x}) = \int_\Omega L(\boldsymbol{x},\theta,\varphi)\cos\theta \mathrm{d}\omega$$

设

$$L_o(\boldsymbol{x},\theta_o,\varphi_o) = L_o(\boldsymbol{x})$$

$$\begin{aligned}
B(\boldsymbol{x}) &= \int_\Omega L_o(\boldsymbol{x})\cos\theta \mathrm{d}\omega \\
&= L_o(\boldsymbol{x}) \int_o^{\frac{\pi}{2}} \int_o^{2\pi} \cos\theta\sin\theta \mathrm{d}\varphi \mathrm{d}\theta \\
&= \pi L_o(\boldsymbol{x})
\end{aligned}$$

方向性半球反射

$$
\begin{aligned}
\rho_{\mathrm{dh}}(\theta_i, \varphi_i) &= \frac{\int_\Omega L_o(\boldsymbol{x}, \theta_o, \varphi_o)\cos\theta_o \mathrm{d}\omega_o}{L_i(\boldsymbol{x}, \theta_i, \varphi_i)\cos\theta_i \mathrm{d}\omega_i} \\
&= \int_\Omega \frac{L_o(\boldsymbol{x}, \theta_o, \varphi_o)}{L_i(\boldsymbol{x}, \theta_i, \varphi_i)\cos\theta_i}\cos\theta_o \mathrm{d}\omega_o \\
&= \int_\Omega \rho_{\mathrm{bd}}(\theta_o, \varphi_o, \theta_i, \varphi_i)\cos\theta_o \mathrm{d}\omega_o
\end{aligned}
$$

漫反射

$$
\begin{aligned}
\rho_{\mathrm{bd}}(\theta_o, \varphi_o, \theta_i, \varphi_i) &= \rho \\
\rho_d &= \int_\Omega \rho_{\mathrm{bd}}(\theta_o, \varphi_O, \theta_i, \varphi_i)\cos\theta_o \mathrm{d}\omega_o \\
&= \int_\Omega \rho\cos\theta_o \mathrm{d}\omega_o \\
&= \rho \int_o^{\frac{\pi}{2}} \int_o^{2\pi} \cos\theta_o \sin\theta_o \mathrm{d}\theta_o \mathrm{d}\varphi_o \\
&= \pi\rho \\
\rho_{\mathrm{brdf}} &= \frac{\rho_d}{\pi}
\end{aligned}
$$

镜面反射

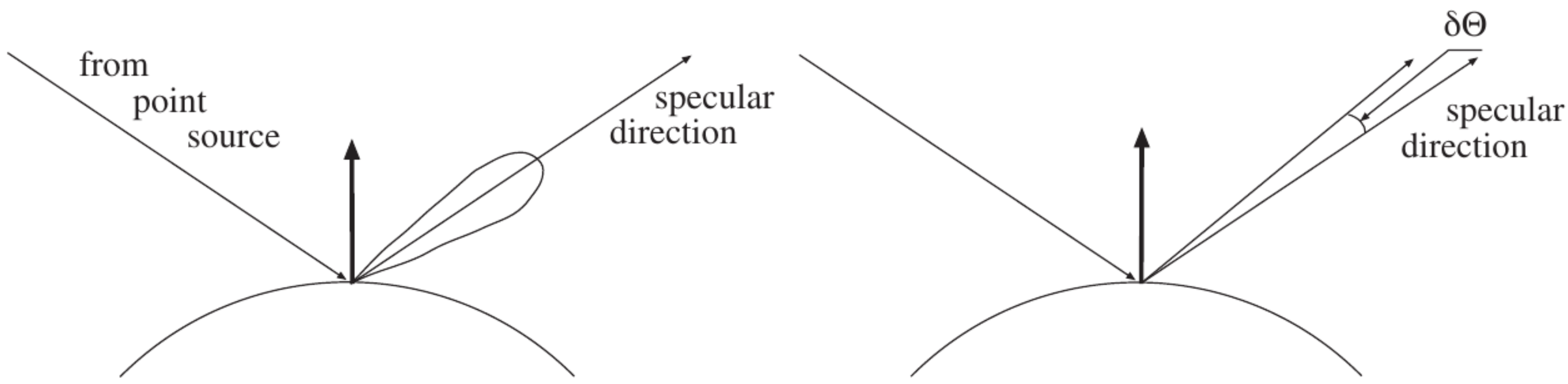

Figure 2.4. Specular surfaces commonly reflect light into a lobe of directions around the specular direction, where the intensity of the reflection depends on the direction, as shown on the left. Phong's model is used to describe the shape of this lobe, in terms of the offset angle from the specular direction.

漫反射与镜面反射

$$\begin{aligned} k(\delta\theta) &= \cos^n(\delta\theta) \\ &= \cos^n(\theta_s - \theta_o) \end{aligned}$$

$$\begin{aligned} L(\boldsymbol{x}, \theta_o, \varphi_o) &= \rho_d(\boldsymbol{x}) \int_\Omega L(\boldsymbol{x}, \theta_i, \varphi_i) \cos\theta_i \mathrm{d}\omega + \rho_s(\boldsymbol{x}) L(\boldsymbol{x}, \theta_s, \varphi_s) k(\delta\theta) \\ &= \rho_d(\boldsymbol{x}) \int_\Omega L(\boldsymbol{x}, \theta_i, \varphi_i) \cos\theta_i \mathrm{d}\omega + \rho_s(\boldsymbol{x}) L(\boldsymbol{x}, \theta_s, \varphi_s) \cos^n(\theta_s - \theta_o) \end{aligned}$$