Adaboost

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问题描述

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\begin{aligned}
t_n &= \{-1, 1\} & \text{标签} \\
y_m(x) &= \begin{cases} 1 & , x > 0 \\ -1 & , x < 0 \end{cases} & \text{弱分类器} \\
F_i(x) &= \frac{1}{2}\sum_{m=1}^{i} \alpha_m y_m(x) & \text{弱分类器组合} \\
E &= \sum_{n=1}^{N} e^{-t_n F_i(x_n)} & \text{损失函数}
\end{aligned}
$$

得

$$
\begin{aligned}
E &= \sum_{n=1}^{N} e^{-\frac{1}{2}\sum_{m=1}^{i} t_n \alpha_m y_m(x)} \\
&= \sum_{n=1}^{N} e^{-\frac{1}{2}\sum_{m=1}^{i-1} t_n \alpha_m y_m(x) - \frac{1}{2}\alpha_i t_n y_i(x_n)} \\
&= \sum_{n=1}^{N} e^{-\frac{1}{2}\sum_{m=1}^{i-1} t_n \alpha_m y_m(x)} e^{-\frac{1}{2}\alpha_i t_n y_i(x_n)} \\
&= \sum_{n=1}^{N} w_n^{(i)} e^{-\frac{1}{2}\alpha_i t_n y_i(x_n)} \\
w_n^{(i)} &= e^{-\frac{1}{2}\sum_{m=1}^{i-1} t_n \alpha_m y_m(x_n)}
\end{aligned}
$$

与$t_n y_i(x_n)$有关的化简方法

$$
\begin{aligned}
T &= \{n | t_n y_i(x_n) = 1\} \\
F &= \{n | t_n y_i(x_n) = -1\} \\
e^{-\frac{1}{2}\alpha_i t_n y_i(x_n)} &= \frac{1 + t_n y_i(x_n)}{2} e^{-\frac{1}{2}\alpha_i} + \frac{1 - t_n y_i(x_n)}{2} e^{\frac{1}{2}\alpha_i} \\
&= \frac{e^{-\frac{1}{2}\alpha_i} + e^{\frac{1}{2}\alpha_i}}{2} + \frac{e^{-\frac{1}{2}\alpha_i} - e^{\frac{1}{2}\alpha i}}{2} t_n y_i(x_n)
\end{aligned}
$$

求解$y_i(\cdot)$，将E看作$y_i(\cdot)$的函数

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\begin{aligned}
E(y_i) &= \text{const} \cdot \sum_{n=1}^{N} w_n^{(i)} t_n y_i(x_n) + \text{const} \\
y_i &= \arg\min_{y_i} \sum_{n \in F} w_n^{(i)}
\end{aligned}
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求解α

$$
\begin{aligned}
\frac{\partial E}{\partial \alpha_i} &= -\frac{1}{2} \sum_{n=1}^{N} w_n^{(i)} e^{-\frac{1}{2}\alpha_i t_n y_i(x_n)} t_n y_i(x_n) \\
0 &= \sum_{n=1}^{N} w_n^{(i)} e^{-\frac{1}{2}\alpha_i t_n y_i(x_n)} t_n y_i(x_n) \\
0 &= \sum_{n \in T} w_n^{(i)} e^{-\frac{1}{2}\alpha_i} - \sum_{n \in F} w_n^{(i)} e^{\frac{1}{2}\alpha_i} \\
\sum_{n \in F} w_n^{(i)} e^{\frac{1}{2}\alpha_i} &= \sum_{n \in T} w_n^{(i)} e^{-\frac{1}{2}\alpha_i} \\
\sum_{n \in F} w_n^{(i)} e^{\alpha_i} &= \sum_{n \in T} w_n^{(i)} \\
e^{\alpha_i} &= \frac{\sum_{n \in T} w_n^{(i)}}{\sum_{n \in F} w_n^{(i)}} \\
&= \frac{\sum_{n=1}^{N} w_n^{(i)} - \sum_{n \in F} w_n^{(i)}}{\sum_{n \in F} w_n^{(i)}}
\end{aligned}
$$

求解$w_n^{(i)}$

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\begin{aligned}
w_n^{(i)} &= e^{-\frac{1}{2}\sum_{m=1}^{i-1} t_n \alpha_m y_m(x_n)} \\
w_n^{(i+1)} &= w_n^{(i)} e^{-\frac{1}{2} t_n \alpha_i y_i(x_n)}
\end{aligned}
$$