计算视觉与模式识别

对极几何

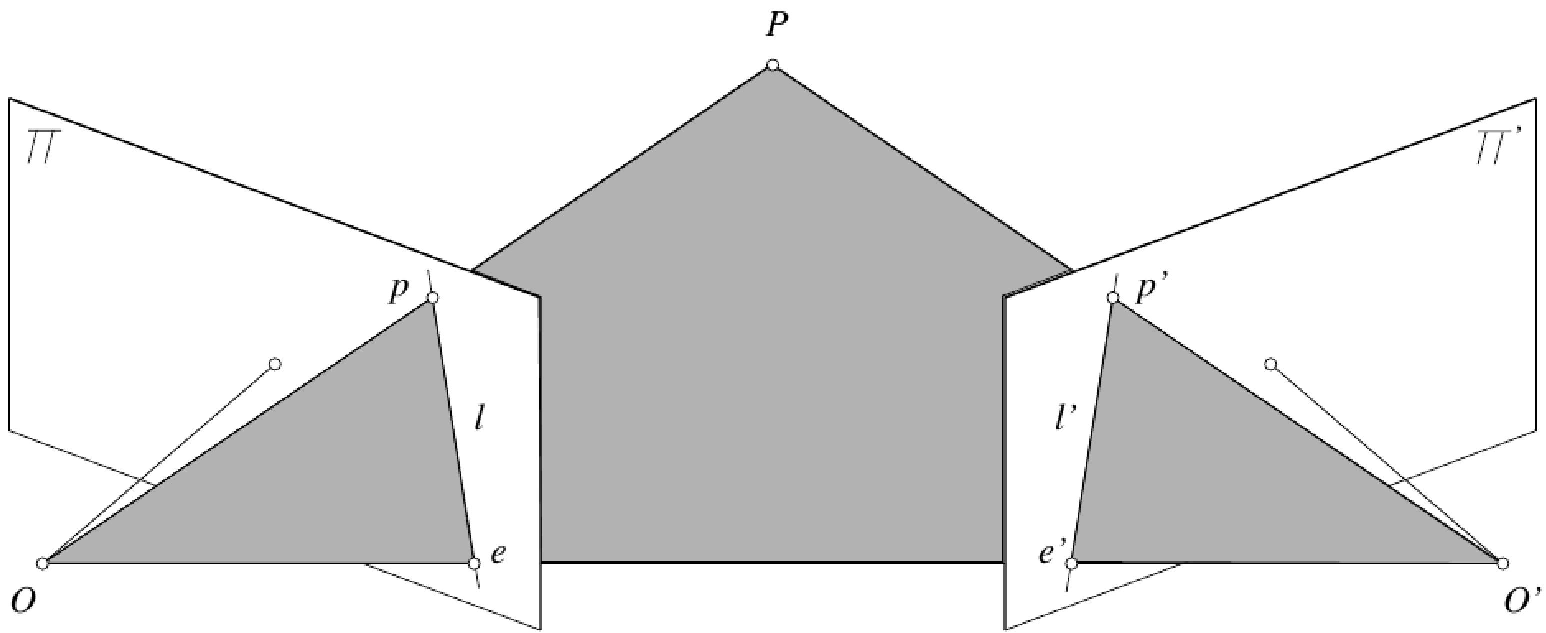

Figure 12.1. Epipolar geometry: the point P, the optical centers O and O' of the two cameras, and the two images p and p' of P all lie in the same plane.

对极约束

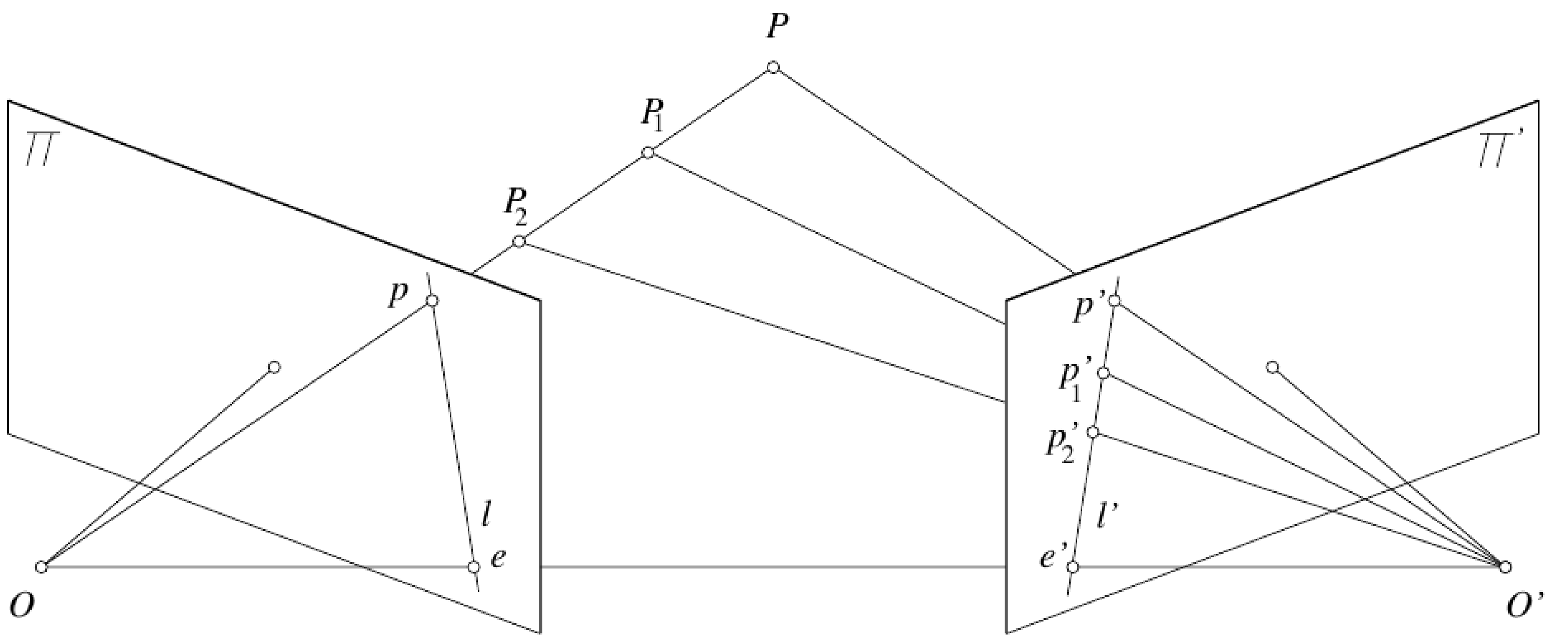

Figure 12.2. Epipolar constraint: given a calibrated stereo rig, the set of possible matches for the point p is constrained to lie on the associated epipolar line l'.

本质矩阵

$$
\begin{aligned}
\overrightarrow{\mathrm{Op}}\cdot[\overrightarrow{\mathrm{OO}'}\times\overrightarrow{O'p'}] &= 0\\
\boldsymbol{p}\cdot[\boldsymbol{t}\times(\mathcal{R}\boldsymbol{p}')] &= 0\\
\mathcal{M} &= \begin{pmatrix} I & \mathbf{0} \end{pmatrix}\\
\mathcal{M}' &= \begin{pmatrix} \mathcal{R}^T & -\mathcal{R}\boldsymbol{t} \end{pmatrix}\\
\boldsymbol{p} &= \begin{pmatrix} u & v & 1 \end{pmatrix}^T\\
\boldsymbol{p}' &= \begin{pmatrix} u' & v' & 1 \end{pmatrix}^T
\end{aligned}
$$

得

$$\boldsymbol{p}^T\mathcal{E}\boldsymbol{p}' = 0$$

其中

$$\mathcal{E} = [\boldsymbol{t}]_\times\mathcal{R}$$

$$\begin{aligned} f(s) &= \boldsymbol{p}^T[\boldsymbol{t}(s)]_\times \mathcal{R}(s)\boldsymbol{p}'(s) \\ &= 0 \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{t}(s) &= s\boldsymbol{v} \\ \mathcal{R}(s) &= \boldsymbol{I} + s[\boldsymbol{\omega}]_\times \\ \boldsymbol{p}'(s) &= \boldsymbol{p} + s\dot{\boldsymbol{p}} \end{aligned}$$

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}s} f(s) &= \boldsymbol{p}^T[\boldsymbol{v}]_\times(\boldsymbol{I} + s[\boldsymbol{\omega}]_\times)(\boldsymbol{p} + s\dot{\boldsymbol{p}}) + \boldsymbol{p}^T[s\boldsymbol{v}]_\times[\boldsymbol{\omega}]_\times(\boldsymbol{p} + s\dot{\boldsymbol{p}}) \\
&\quad + \boldsymbol{p}^T[s\boldsymbol{v}]_\times(\boldsymbol{I} + s[\boldsymbol{\omega}]_\times)\dot{\boldsymbol{p}} \\
\left.\frac{\mathrm{d}}{\mathrm{d}s} f(s)\right|_{s=0} &= \boldsymbol{p}^T[\boldsymbol{v}]_\times \boldsymbol{p} \\
&= 0 \\
\left.\frac{\mathrm{d}^2}{\mathrm{d}s^2} f(s)\right|_{s=0} &= \boldsymbol{p}^T[\boldsymbol{v}]_\times[\boldsymbol{\omega}]_\times \boldsymbol{p} + \boldsymbol{p}^T[\boldsymbol{v}]_\times \dot{\boldsymbol{p}} + \boldsymbol{p}^T[\boldsymbol{v}]_\times[\boldsymbol{\omega}]_\times \boldsymbol{p} + \boldsymbol{p}^T[\boldsymbol{v}]_\times \dot{\boldsymbol{p}} \\
&= 2(\boldsymbol{p}^T[\boldsymbol{v}]_\times[\boldsymbol{\omega}]_\times \boldsymbol{p} + \boldsymbol{p}^T[\boldsymbol{v}]_\times \dot{\boldsymbol{p}})
\end{aligned}
$$

得

$$
\boldsymbol{p}^T[\boldsymbol{v}]_\times[\boldsymbol{\omega}]_\times \boldsymbol{p} + \boldsymbol{p}^T[\boldsymbol{v}]_\times \dot{\boldsymbol{p}} = 0
$$

平动

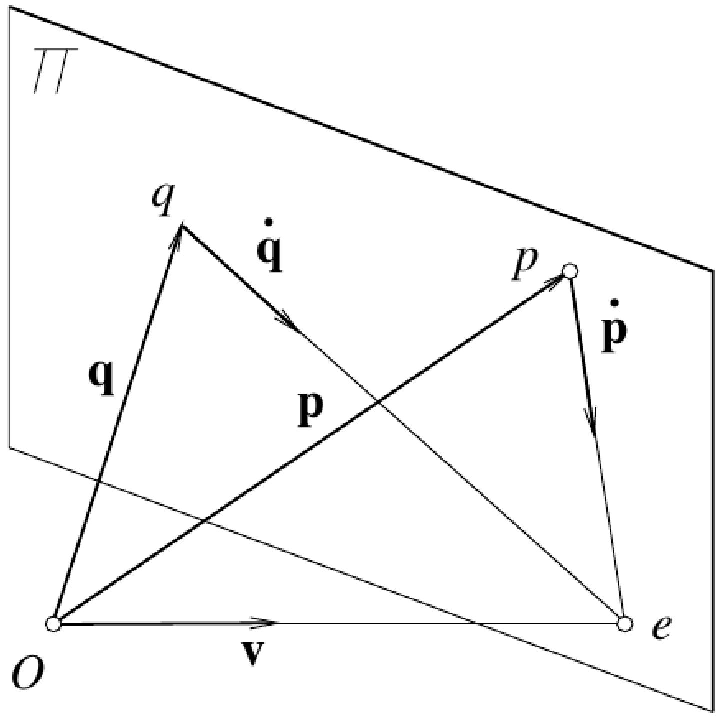

Figure 12.3. Focus of expansion: under pure translation, the motion field at every point in the image points toward the focus of expansion.

$$\boldsymbol{p}\mathcal{F}\boldsymbol{p}' = 0$$

where

$$\begin{aligned}\boldsymbol{p} &= \mathcal{K}\hat{\mathbf{p}}\\ \boldsymbol{p}' &= \mathcal{K}'\hat{\mathbf{p}}'\\ \mathcal{F} &= \mathcal{K}^{-T}\mathcal{E}\mathcal{K}'^{-1}\end{aligned}$$

$$\mathcal{F} = \begin{pmatrix} b & a & -a\beta - b\alpha \\ -d & -c & c\beta + d\alpha \\ d\beta' - b\alpha' & c\beta' - a\alpha' & -c\beta\beta' - d\beta'\alpha + a\beta\alpha' + b\alpha\alpha' \end{pmatrix}$$

弱标定

$$\begin{pmatrix} u & v & 1 \end{pmatrix} \mathcal{F} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0$$

$$\mathrm{tr}\left[\mathcal{F} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} \begin{pmatrix} u & v & 1 \end{pmatrix} \right] = 0$$

$$\mathrm{tr}\left[\mathcal{F} \begin{pmatrix} uu' & vu' & u' \\ uv' & vv' & v' \\ u & v & 1 \end{pmatrix} \right] = 0$$

$$\mathrm{tr}\left[\begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix}\begin{pmatrix} uu' & vu' & u' \\ uv' & vv' & v' \\ u & v & 1 \end{pmatrix}\right] = 0$$

$$\begin{pmatrix} u\,u' & u\,v' & u & v\,u' & v\,v' & v & u' & v' & 1 \end{pmatrix}\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$

eight-point algorithm

$$\begin{pmatrix} u_1 u_1' & u_1 v_1' & u_1 & v_1 u_1' & v_1 v_1' & v_1 & u_1' & v_1' \\ u_2 u_2' & u_2 v_2' & u_2 & v_2 u_2' & v_2 v_2' & v_2 & u_2' & v_2' \\ u_3 u_3' & u_3 v_3' & u_3 & v_3 u_3' & v_3 v_3' & v_3 & u_3' & v_3' \\ u_4 u_4' & u_4 v_4' & u_4 & v_4 u_4' & v_4 v_4' & v_4 & u_4' & v_4' \\ u_5 u_5' & u_5 v_5' & u_5 & v_5 u_5' & v_5 v_5' & v_5 & u_5' & v_5' \\ u_6 u_6' & u_6 v_6' & u_6 & v_6 u_6' & v_6 v_6' & v_6 & u_6' & v_6' \\ u_7 u_7' & u_7 v_7' & u_7 & v_7 u_7' & v_7 v_7' & v_7 & u_7' & v_7' \\ u_8 u_8' & u_8 v_8' & u_8 & v_8 u_8' & v_8 v_8' & v_8 & u_8' & v_8' \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = -\mathbf{1}$$

三视图

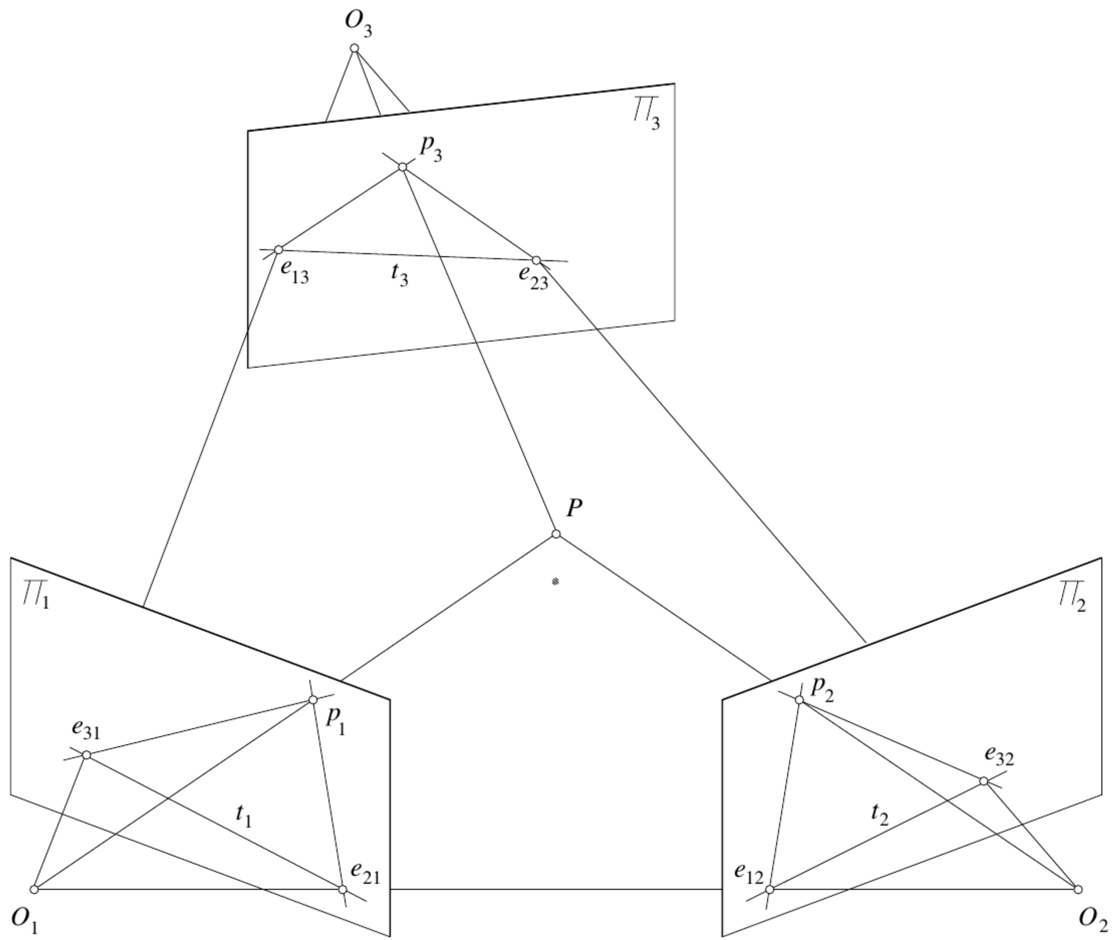

Figure 12.5. Trinocular epipolar geometry.

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$$
\begin{aligned}
\boldsymbol{p}_1^T \mathcal{E}_{12} \boldsymbol{p}_2 &= 0 \\
\boldsymbol{p}_2^T \mathcal{E}_{23} \boldsymbol{p}_3 &= 0 \\
\boldsymbol{p}_3^T \mathcal{E}_{31} \boldsymbol{p}_1 &= 0
\end{aligned}
$$

直线

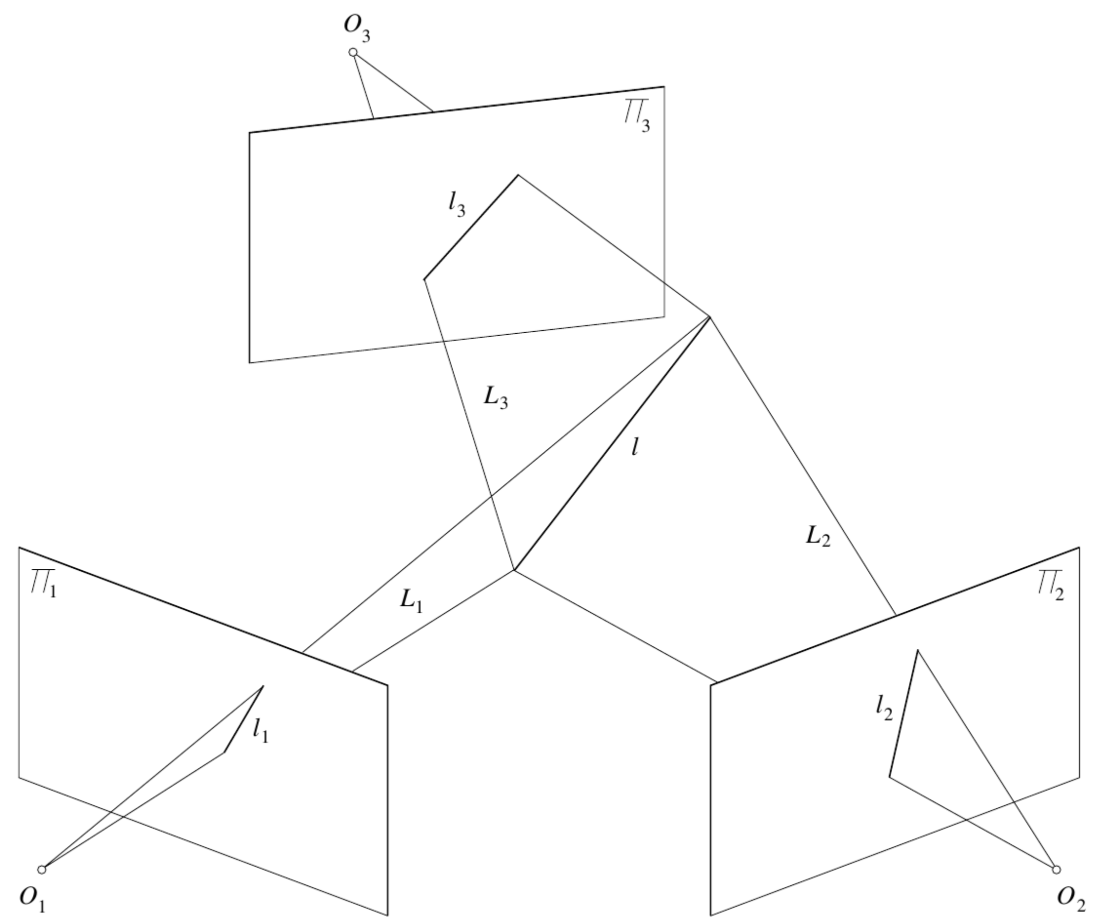

Figure 12.6. Three images of a line define it as the intersection of three planes in the same pencil.

$$\boldsymbol{l}^T \mathcal{M} \boldsymbol{P} = \boldsymbol{0}$$

得

$$\begin{aligned}
\mathcal{L}\boldsymbol{P} &= \boldsymbol{0} \\
\mathcal{L} &\equiv \begin{pmatrix} \boldsymbol{l}_1^T \mathcal{M}_1 \\ \boldsymbol{l}_2^T \mathcal{M}_2 \\ \boldsymbol{l}_3^T \mathcal{M}_3 \end{pmatrix} \\
&= \begin{pmatrix} \boldsymbol{L}_1^T \\ \boldsymbol{L}_2^T \\ \boldsymbol{L}_3^T \end{pmatrix} \\
\boldsymbol{L}_i &= \mathcal{M}^T \boldsymbol{l}_i
\end{aligned}$$

已标定情况

$$
\begin{aligned}
\mathcal{M}_1 &= \begin{pmatrix} \boldsymbol{I} & \mathbf{0} \end{pmatrix} \\
\mathcal{M}_2 &= \begin{pmatrix} \mathcal{R}_2 & \boldsymbol{t}_2 \end{pmatrix} \\
\mathcal{M}_1 &= \begin{pmatrix} \mathcal{R}_3 & \boldsymbol{t}_3 \end{pmatrix}
\end{aligned}
$$

得

$$
\begin{aligned}
\mathcal{L} &= \begin{pmatrix} \boldsymbol{l}_1^T & 0 \\ \boldsymbol{l}_2^T \mathcal{R}_2 & \boldsymbol{l}_2^T \boldsymbol{t}_2 \\ \boldsymbol{l}_3^T \mathcal{R}_3 & \boldsymbol{l}_3^T \boldsymbol{t}_3 \end{pmatrix} \\
&= \begin{pmatrix} \boldsymbol{l}_1^1 & \boldsymbol{l}_1^2 & \boldsymbol{l}_1^3 & 0 \\ \boldsymbol{l}_2^T \mathcal{R}_2^1 & \boldsymbol{l}_2^T \mathcal{R}_2^2 & \boldsymbol{l}_2^T \mathcal{R}_2^3 & \boldsymbol{l}_2^T \boldsymbol{t}_2 \\ \boldsymbol{l}_3^T \mathcal{R}_3^1 & \boldsymbol{l}_3^T \mathcal{R}_3^2 & \boldsymbol{l}_3^T \mathcal{R}_3^3 & \boldsymbol{l}_3^T \boldsymbol{t}_3 \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\begin{vmatrix} \boldsymbol{l}_1^2 & \boldsymbol{l}_1^3 & 0 \\ \boldsymbol{l}_2^T\mathcal{R}_2^2 & \boldsymbol{l}_2^T\mathcal{R}_2^3 & \boldsymbol{l}_2^T\mathbf{t}_2 \\ \boldsymbol{l}_3^T\mathcal{R}_3^2 & \boldsymbol{l}_3^T\mathcal{R}_3^3 & \boldsymbol{l}_3^T\mathbf{t}_3 \end{vmatrix} &= 0 \\
\boldsymbol{l}_1^2(\boldsymbol{l}_2^T\mathcal{R}_2^3\boldsymbol{l}_3^T\mathbf{t}_3 - \boldsymbol{l}_3^T\mathcal{R}_3^3\boldsymbol{l}_2^T\mathbf{t}_2) - \boldsymbol{l}_1^3(\boldsymbol{l}_2^T\mathcal{R}_2^2\boldsymbol{l}_3^T\mathbf{t}_3 - \boldsymbol{l}_3^T\mathcal{R}_3^2\boldsymbol{l}_2^T\mathbf{t}_2) &= 0 \\
\boldsymbol{l}_1^2(\boldsymbol{l}_2^T\mathcal{R}_2^3\mathbf{t}_3^T\boldsymbol{l}_3 - \boldsymbol{l}_2^T\mathbf{t}_2\mathcal{R}_3^{3T}\boldsymbol{l}_3) - \boldsymbol{l}_1^3(\boldsymbol{l}_2^T\mathcal{R}_2^2\mathbf{t}_3^T\boldsymbol{l}_3 - \boldsymbol{l}_2^T\mathbf{t}_2\mathcal{R}_3^{2T}\boldsymbol{l}_3) &= 0 \\
\boldsymbol{l}_1^2[\boldsymbol{l}_2^T(\mathcal{R}_2^3\mathbf{t}_3^T - \mathbf{t}_2\mathcal{R}_3^{3T})\boldsymbol{l}_3] - \boldsymbol{l}_1^3[\boldsymbol{l}_2^T(\mathcal{R}_2^2\mathbf{t}_3^T - \mathbf{t}_2\mathcal{R}_3^{2T})\boldsymbol{l}_3] &= 0 \\
\begin{vmatrix} \boldsymbol{l}_1^1 & \boldsymbol{l}_1^3 & 0 \\ \boldsymbol{l}_2^T\mathcal{R}_2^1 & \boldsymbol{l}_2^T\mathcal{R}_2^3 & \boldsymbol{l}_2^T\mathbf{t}_2 \\ \boldsymbol{l}_3^T\mathcal{R}_3^1 & \boldsymbol{l}_3^T\mathcal{R}_3^3 & \boldsymbol{l}_3^T\mathbf{t}_3 \end{vmatrix} &= 0 \\
\begin{vmatrix} \boldsymbol{l}_1^1 & \boldsymbol{l}_1^2 & 0 \\ \boldsymbol{l}_2^T\mathcal{R}_2^1 & \boldsymbol{l}_2^T\mathcal{R}_2^2 & \boldsymbol{l}_2^T\mathbf{t}_2 \\ \boldsymbol{l}_3^T\mathcal{R}_3^1 & \boldsymbol{l}_3^T\mathcal{R}_3^2 & \boldsymbol{l}_3^T\mathbf{t}_3 \end{vmatrix} &= 0
\end{aligned}
$$

三焦张量

$$
\begin{aligned}
\boldsymbol{l}_1 \times \begin{pmatrix} \boldsymbol{l}_2^T \mathcal{G}_1^1 \boldsymbol{l}_3 \\ \boldsymbol{l}_2^T \mathcal{G}_1^2 \boldsymbol{l}_3 \\ \boldsymbol{l}_2^T \mathcal{G}_1^3 \boldsymbol{l}_3 \end{pmatrix} &= \mathbf{0} \\
\mathcal{G}_1^i &= \boldsymbol{t}_2 \boldsymbol{R}_3^{iT} - \boldsymbol{R}_2^i \boldsymbol{t}_3^T \qquad i = 1, 2, 3
\end{aligned}
$$

$$\boldsymbol{l}_1 \propto \begin{pmatrix} \boldsymbol{l}_2^T \mathcal{G}_1^1 \boldsymbol{l}_3 \\ \boldsymbol{l}_2^T \mathcal{G}_1^2 \boldsymbol{l}_3 \\ \boldsymbol{l}_2^T \mathcal{G}_1^3 \boldsymbol{l}_3 \end{pmatrix}$$

$$\boldsymbol{p}_1^T \begin{pmatrix} \boldsymbol{l}_2^T \mathcal{G}_1^1 \boldsymbol{l}_3 \\ \boldsymbol{l}_2^T \mathcal{G}_1^2 \boldsymbol{l}_3 \\ \boldsymbol{l}_2^T \mathcal{G}_1^3 \boldsymbol{l}_3 \end{pmatrix} = 0$$

where

$$\begin{aligned} \boldsymbol{P} &\in \boldsymbol{l} \\ \boldsymbol{p}_1 &= \mathcal{M}_1 \boldsymbol{P} \\ \boldsymbol{p}_1 &\in \boldsymbol{l}_1 \\ \boldsymbol{p}_1 \boldsymbol{l}_1 &= 0 \end{aligned}$$

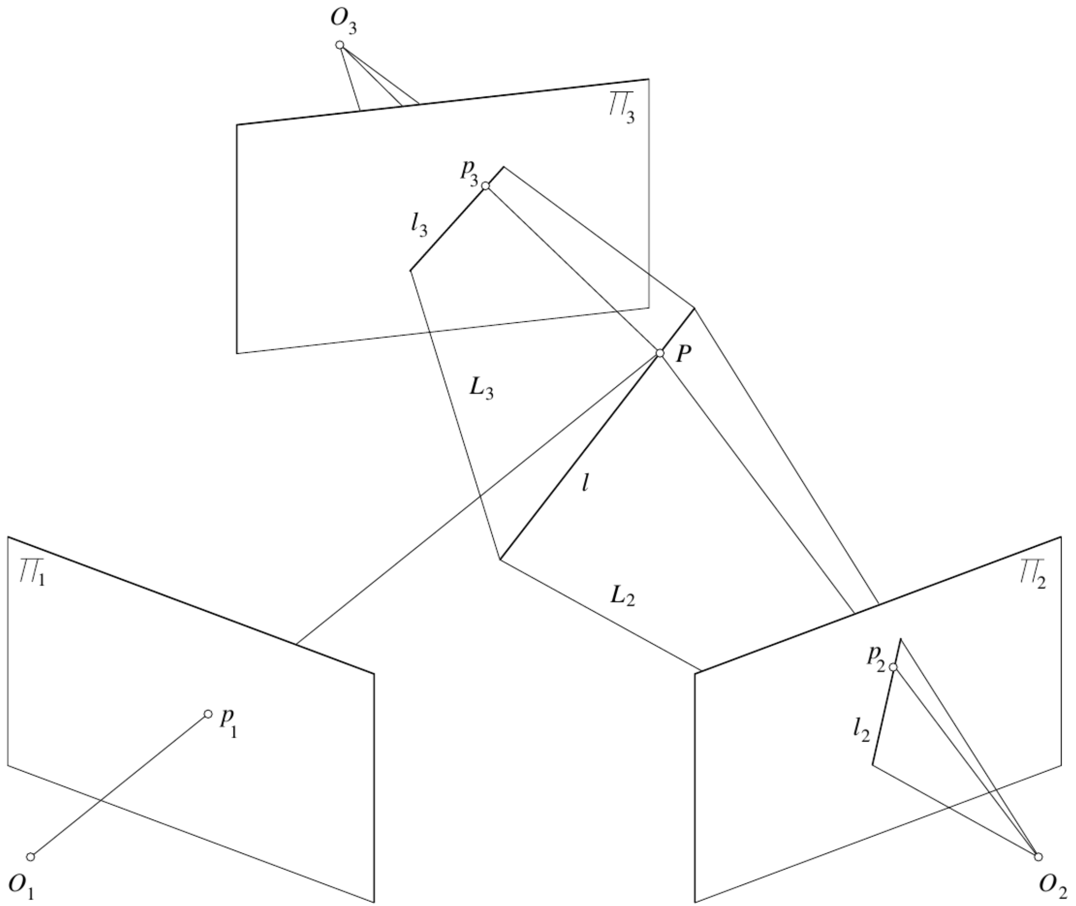

Figure 12.7. Given three images p_1, p_2 and p_3 of the same point P, and two arbitrary images l_2 and l_3 passing through the points p_2 and p_3, the ray passing through O_1 and p_1 must intersect the line where the planes L_2 and L_3 projecting onto l_2 and l_3 meet in space.

未标定情况

$$
\begin{aligned}
\boldsymbol{p} &= \mathcal{K}\hat{\boldsymbol{p}} \\
\boldsymbol{l}^T\boldsymbol{p} &= 0 \\
\boldsymbol{l}^T\mathcal{K}\hat{\boldsymbol{p}} &= 0
\end{aligned}
$$

得

$$
\mathcal{L} = \begin{pmatrix} \boldsymbol{l}_1^T\mathcal{K}_1 & 0 \\ \boldsymbol{l}_2^T\mathcal{K}_2\mathcal{R}_2 & \boldsymbol{l}_2^T\mathcal{K}_2\boldsymbol{t}_2 \\ \boldsymbol{l}_3^T\mathcal{K}_3\mathcal{R}_3 & \boldsymbol{l}_3^T\mathcal{K}_3\boldsymbol{t}_3 \end{pmatrix}
$$

$$\mathcal{L}\begin{pmatrix} \mathcal{K}_1^{-1} & \mathbf{0} \\ \mathbf{0} & 1 \end{pmatrix} = \begin{pmatrix} \boldsymbol{l}_1^T & 0 \\ \boldsymbol{l}_2^T\mathcal{A}_2 & \boldsymbol{l}_2^T\boldsymbol{b}_2 \\ \boldsymbol{l}_3^T\mathcal{A}_3 & \boldsymbol{l}_3^T\boldsymbol{b}_3 \end{pmatrix}$$

$$\begin{aligned} \mathcal{A}_i &\equiv \mathcal{K}_i\mathcal{R}_i\mathcal{K}_1^{-1} \\ \boldsymbol{b}_i &= \mathcal{K}_i\boldsymbol{t}_i \qquad i=2,3 \end{aligned}$$

对应

$$\begin{aligned} \mathcal{M}_1 &= \begin{pmatrix} \mathcal{K}_1 & \mathbf{0} \end{pmatrix} \\ \mathcal{M}_2 &= \begin{pmatrix} \mathcal{A}_2\mathcal{K}_1 & \boldsymbol{b}_2 \end{pmatrix} \\ \mathcal{M}_3 &= \begin{pmatrix} \mathcal{A}_3\mathcal{K}_1 & \boldsymbol{b}_3 \end{pmatrix} \end{aligned}$$

且有

$$\begin{aligned} \mathcal{G}_1^i &= \boldsymbol{b}_2\boldsymbol{A}_3^{iT} - \boldsymbol{A}_2^i\boldsymbol{b}_3^T \\ \mathcal{A}_i &= \begin{pmatrix} \boldsymbol{A}_i^1 & \boldsymbol{A}_i^2 & \boldsymbol{A}_i^3 \end{pmatrix} \end{aligned}$$