计算视觉与模式识别

几何摄像机模型

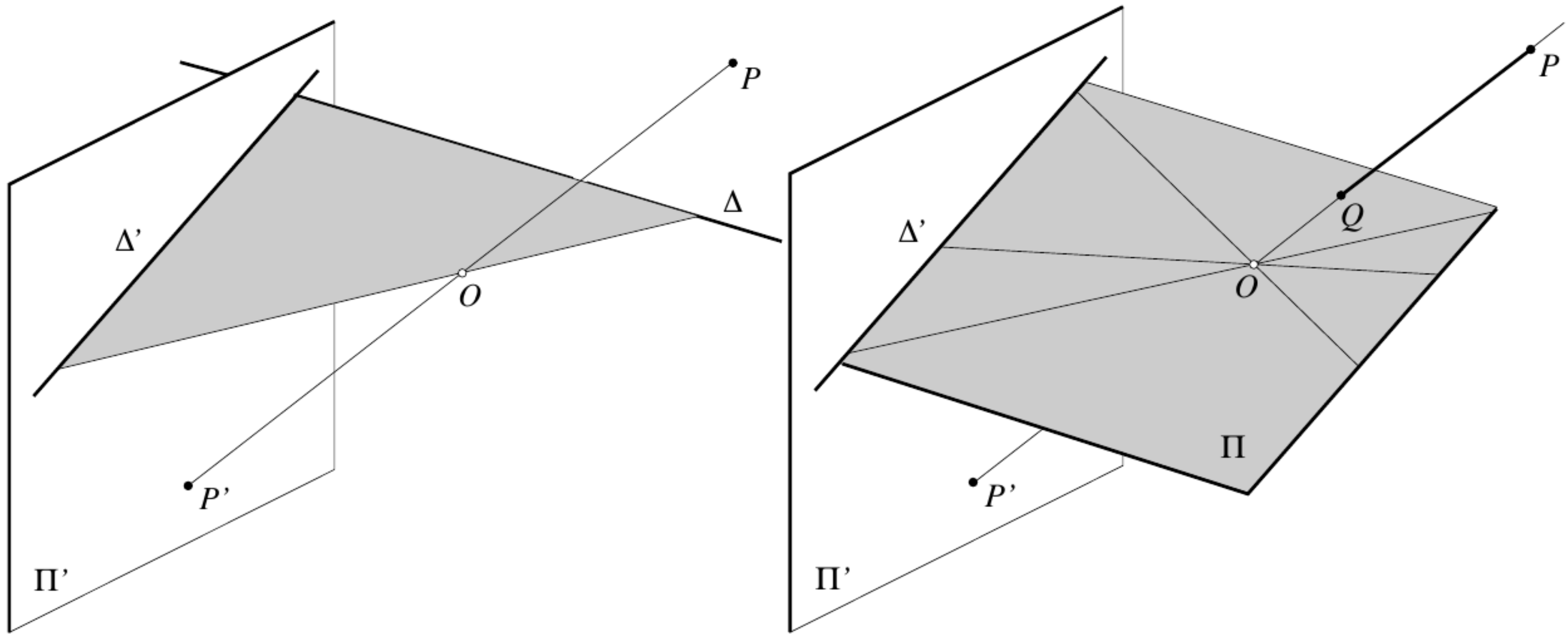

Figure 5.1. Point and line projections. Left: general case; the line Δ projects onto the line Δ', the grey triangle representing the plane defined by Δ and the pinhole O. Right: degenerate case; the line supporting the segment PQ and passing through O projects onto the point P', and the plane Π passing through O projects onto the line Δ'.

坐标系

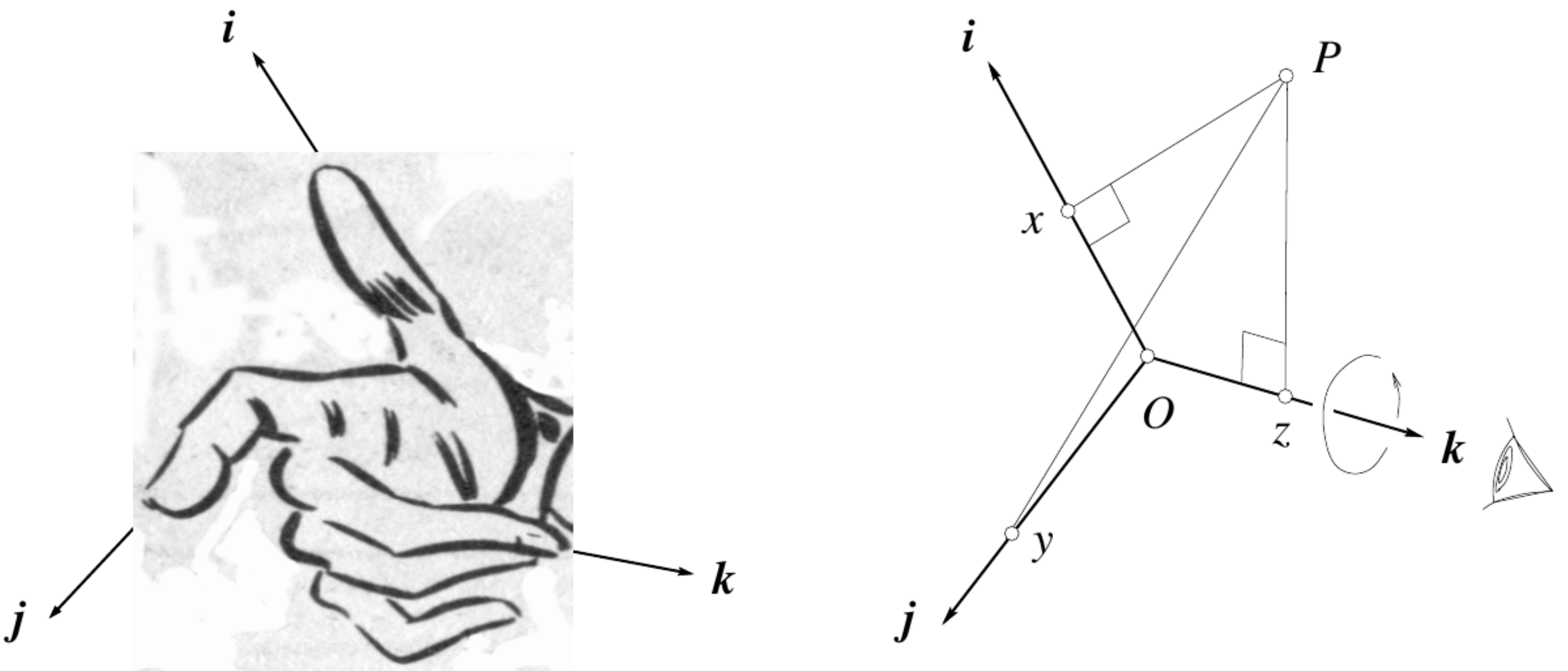

Figure 5.2. A right-handed coordinate system and the Cartesian coordinates x, y, z of a point P.

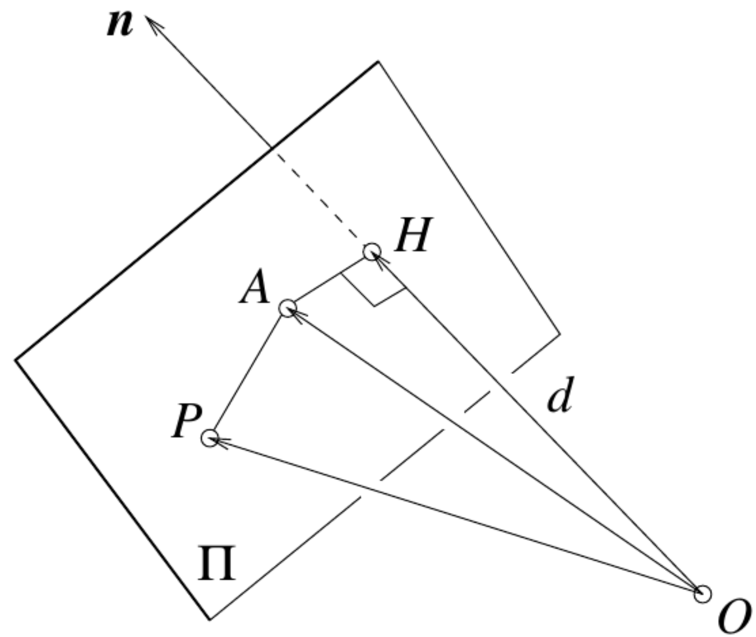

Figure 5.3. The geometric definition of the equation of a plane. The distance d between the origin and the plane is reached at the point H where the normal vector passing through the origin pierces the plane.

$$\overrightarrow{\mathrm{OP}} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$

where

$$\begin{aligned} x &= \overrightarrow{\mathrm{OP}} \cdot \boldsymbol{i} \\ y &= \overrightarrow{\mathrm{OP}} \cdot \boldsymbol{j} \\ z &= \overrightarrow{\mathrm{OP}} \cdot \boldsymbol{k} \end{aligned}$$

and

$$\boldsymbol{P} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$$

齐次坐标

$$
\begin{aligned}
\begin{pmatrix} a & b & c & -d \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} &= 0 \\
\mathbf{\Pi}\cdot\boldsymbol{P} &= 0 \\
\mathbf{\Pi} &\equiv \begin{pmatrix} a \\ b \\ c \\ -d \end{pmatrix} \\
\boldsymbol{P} &\equiv \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}
\end{aligned}
$$

坐标变换——平移

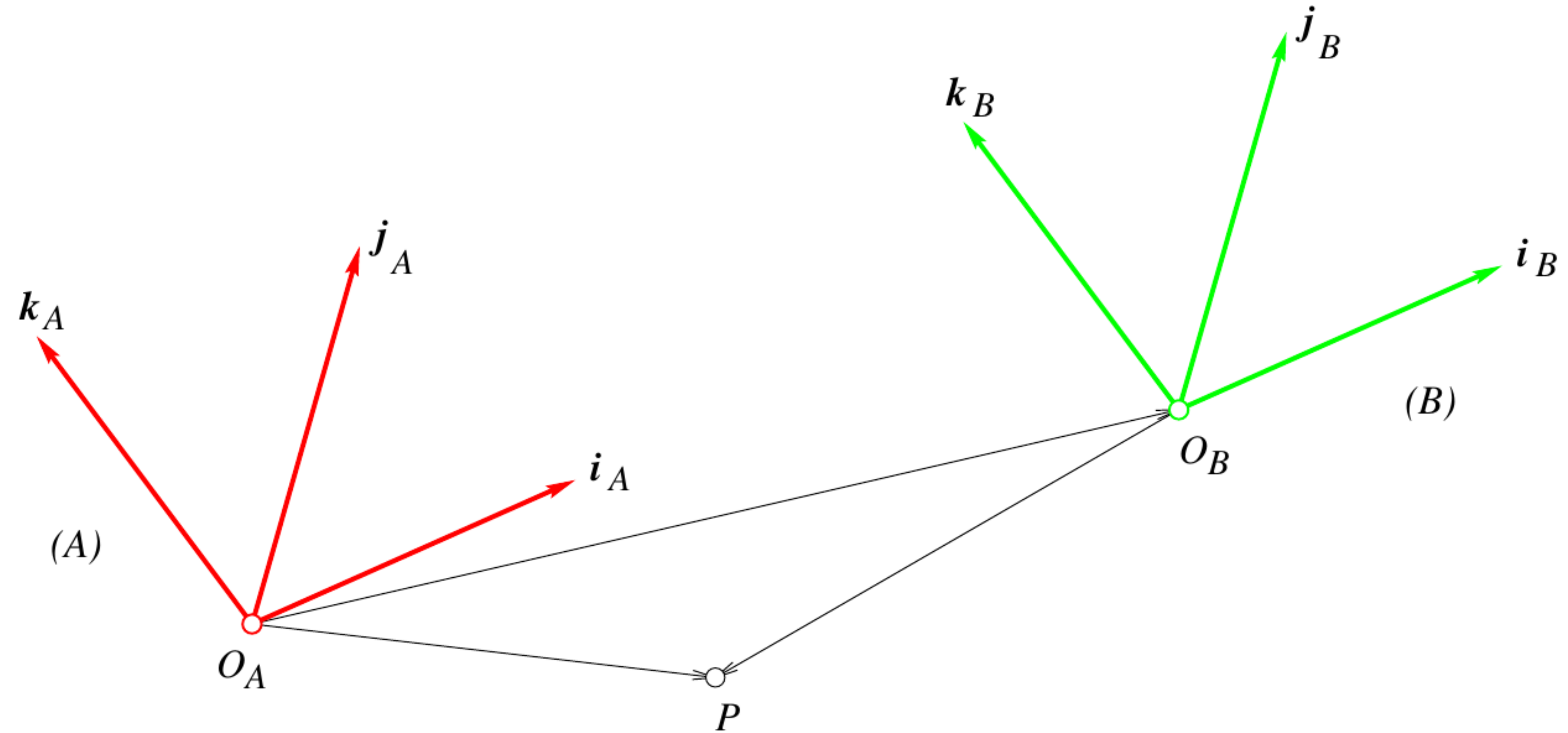

Figure 5.4. Change of coordinates between two frames: pure translation.

$$
\begin{aligned}
\overrightarrow{\mathrm{OP}} &= x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k} \\
{}^{F}P &= {}^{F}\overrightarrow{\mathrm{OP}} \\
&= \begin{pmatrix} x \\ y \\ z \end{pmatrix} \\
{}^{B}P &= {}^{A}P + {}^{B}O_{A}
\end{aligned}
$$

坐标变换——旋转

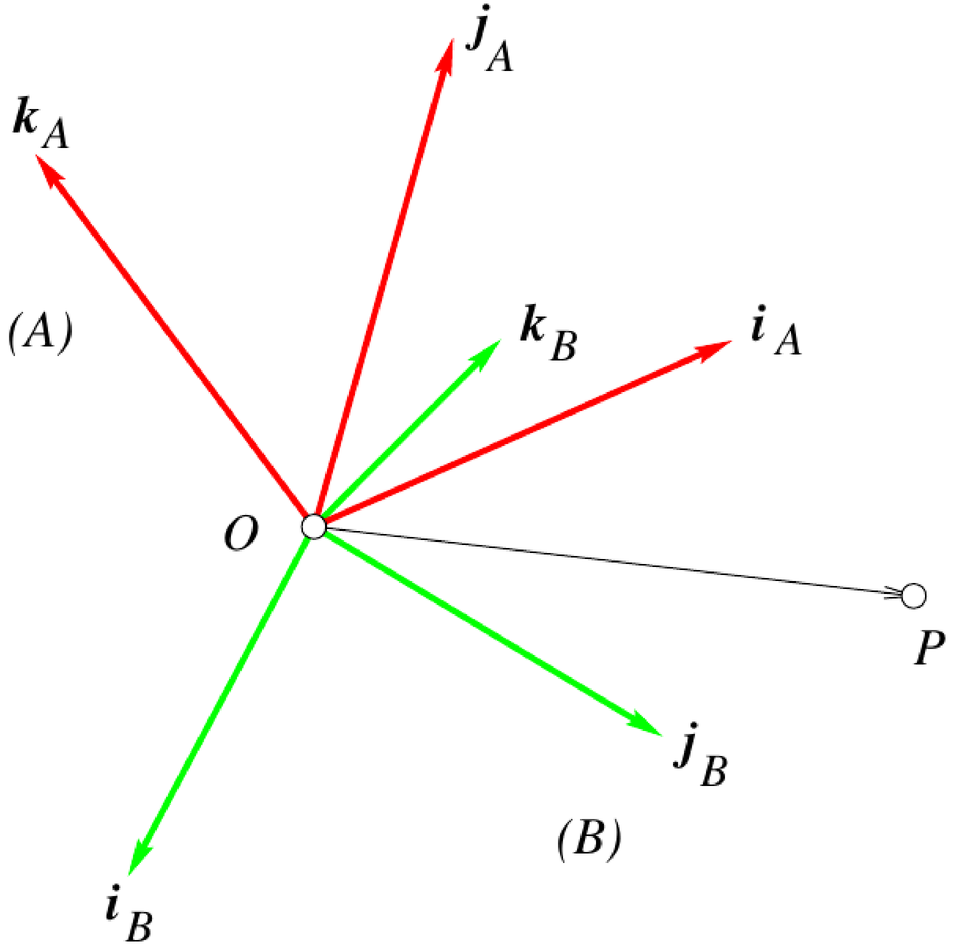

Figure 5.5. Change of coordinates between two frames: pure rotation.

$$\overrightarrow{\mathrm{OP}} = \begin{pmatrix} \boldsymbol{i}_A & \boldsymbol{j}_A & \boldsymbol{k}_A \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

$$= \begin{pmatrix} \boldsymbol{i}_B & \boldsymbol{j}_B & \boldsymbol{k}_B \end{pmatrix} \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}$$

$$^{B}P = {}^{B}_{A}\mathcal{R}\,{}^{A}P$$

$${}^{B}_{A}\mathcal{R} = \begin{pmatrix} \boldsymbol{i}_B & \boldsymbol{j}_B & \boldsymbol{k}_B \end{pmatrix}^T \begin{pmatrix} \boldsymbol{i}_A & \boldsymbol{j}_A & \boldsymbol{k}_A \end{pmatrix}$$

$$= \begin{pmatrix} \boldsymbol{i}_B\cdot\boldsymbol{i}_A & \boldsymbol{i}_B\cdot\boldsymbol{j}_A & \boldsymbol{i}_B\cdot\boldsymbol{k}_A \\ \boldsymbol{j}_B\cdot\boldsymbol{i}_A & \boldsymbol{j}_B\cdot\boldsymbol{j}_A & \boldsymbol{j}_B\cdot\boldsymbol{k}_A \\ \boldsymbol{k}_B\cdot\boldsymbol{i}_A & \boldsymbol{k}_B\cdot\boldsymbol{j}_A & \boldsymbol{k}_B\cdot\boldsymbol{k}_A \end{pmatrix}$$

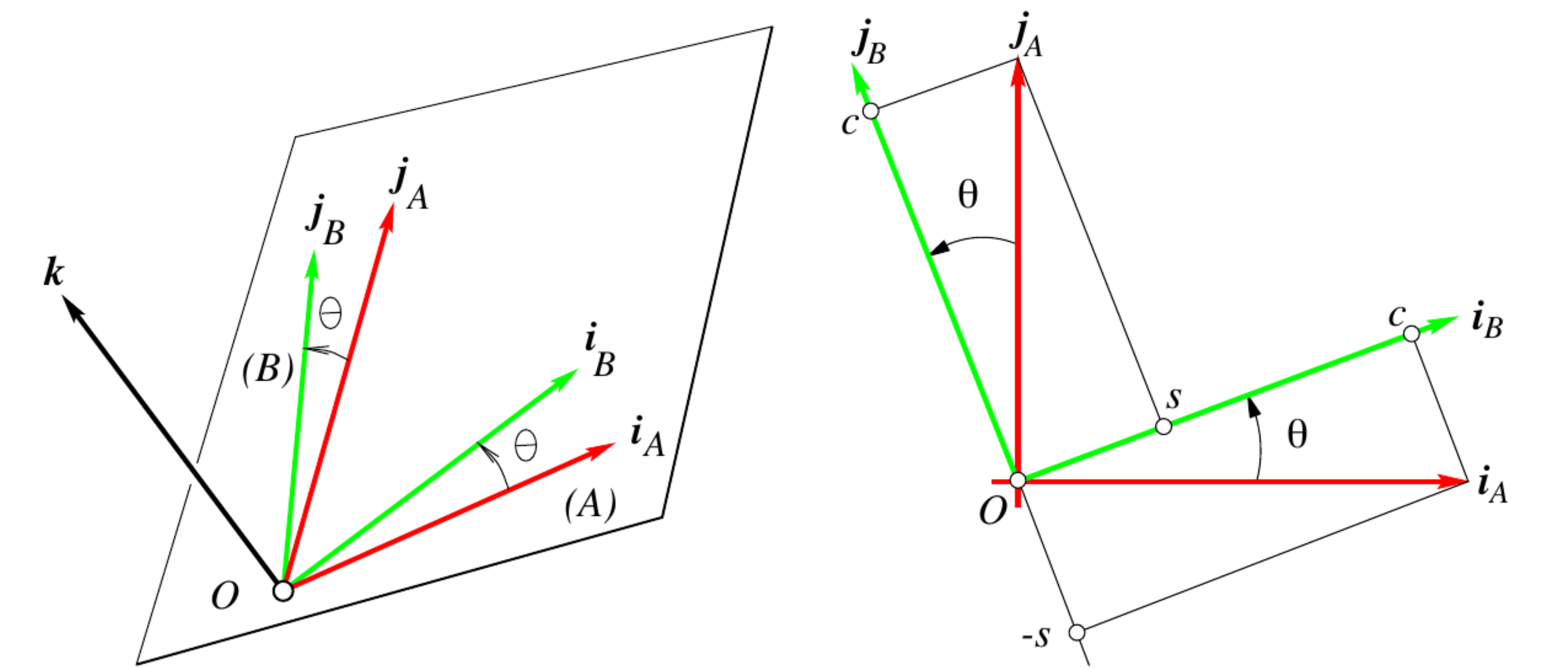

Figure 5.6. Two coordinate frames separated by a rotation of angle θ about their common $\boldsymbol{k}$ basis vector. As shown in the right of the figure, $\boldsymbol{i}_A = c\boldsymbol{i}_B - s\boldsymbol{j}_B$ and $\boldsymbol{i}_A = s\boldsymbol{i}_B + c\boldsymbol{j}_B$, where $c = \cos\theta$ and $s = \sin\theta$.

$$
{}^{B}_{A}\mathcal{R} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & o & 1 \end{pmatrix}
$$

坐标变换

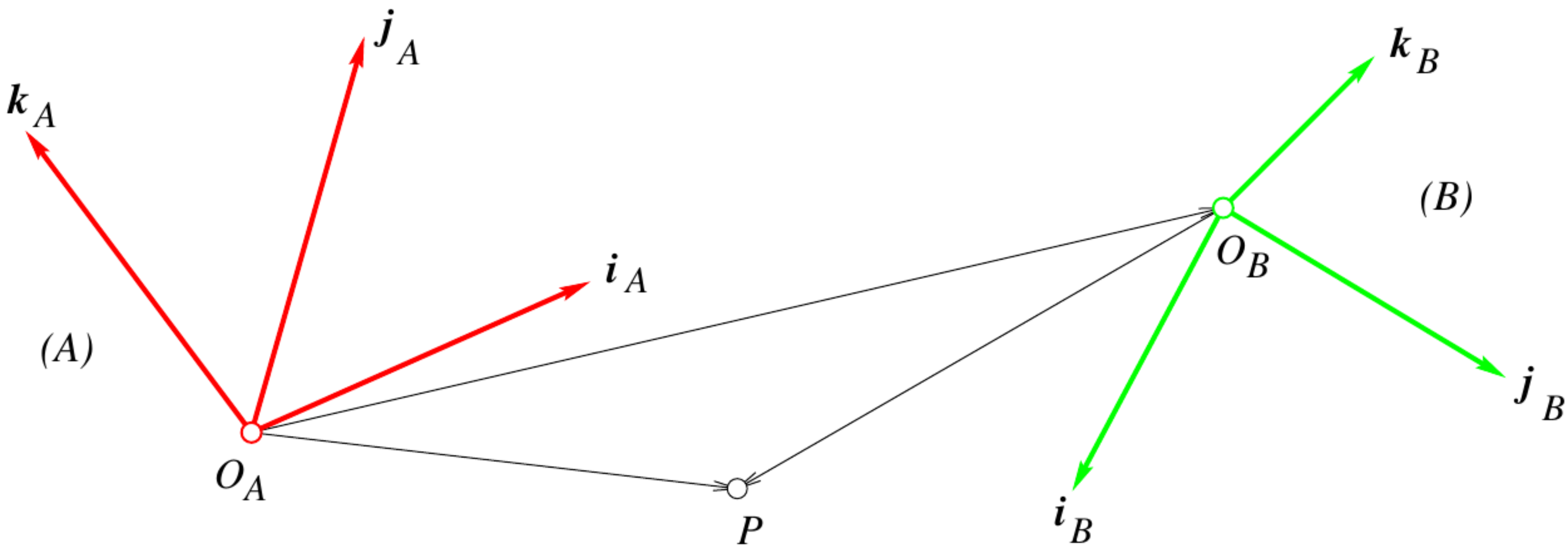

Figure 5.7. Change of coordinates between two frames: general rigid transformation.

$$\begin{pmatrix} {}^{B}P \\ 1 \end{pmatrix} = {}^{B}_{A}\mathcal{T} \begin{pmatrix} {}^{A}P \\ 1 \end{pmatrix}$$

$${}^{B}_{A}\mathcal{T} \equiv \begin{pmatrix} {}^{B}_{A}\mathcal{R} & {}^{B}O_{A} \\ \mathbf{0}^{T} & 1 \end{pmatrix}$$

坐标旋转示例

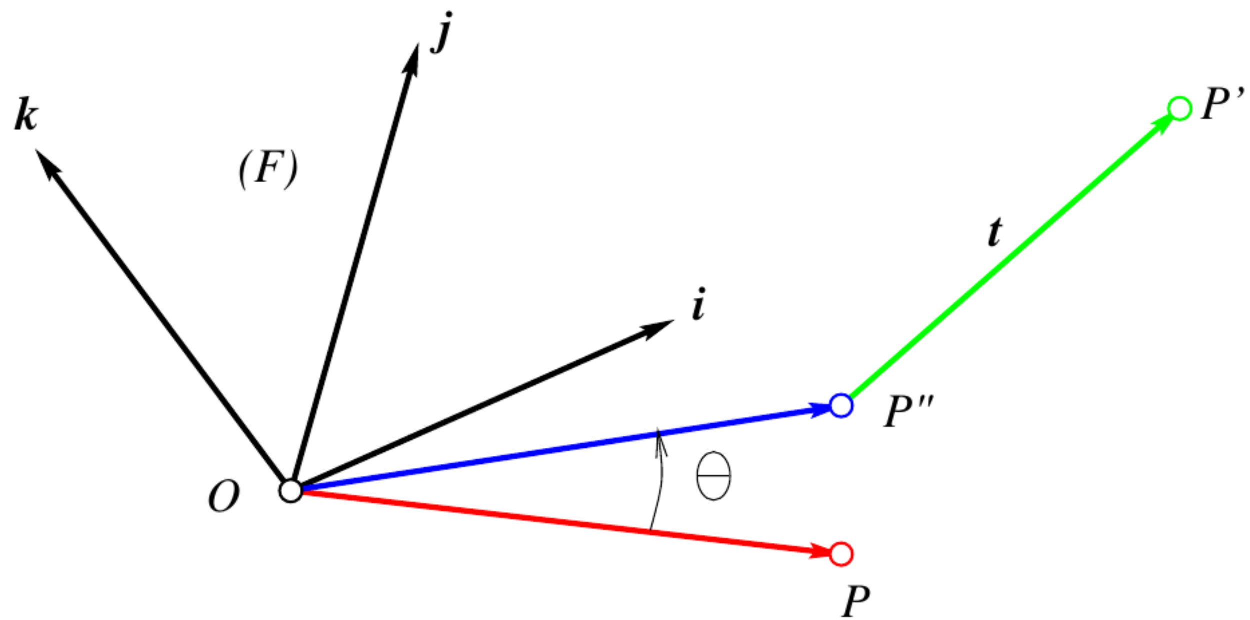

Figure 5.8. A rigid transformation maps the point P onto the point P'' through a rotation $\mathcal{R}$ before mapping P'' onto P' via a translation $\boldsymbol{t}$. In the example shown in this figure, $\mathcal{R}$ is a rotation of angle θ about the $\boldsymbol{k}$ axis of the coordinate system (F).

$$ {}^{F}P' = \mathcal{R}\,{}^{F}P $$

where

$$ \mathcal{R} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

几何相机参数——内参数

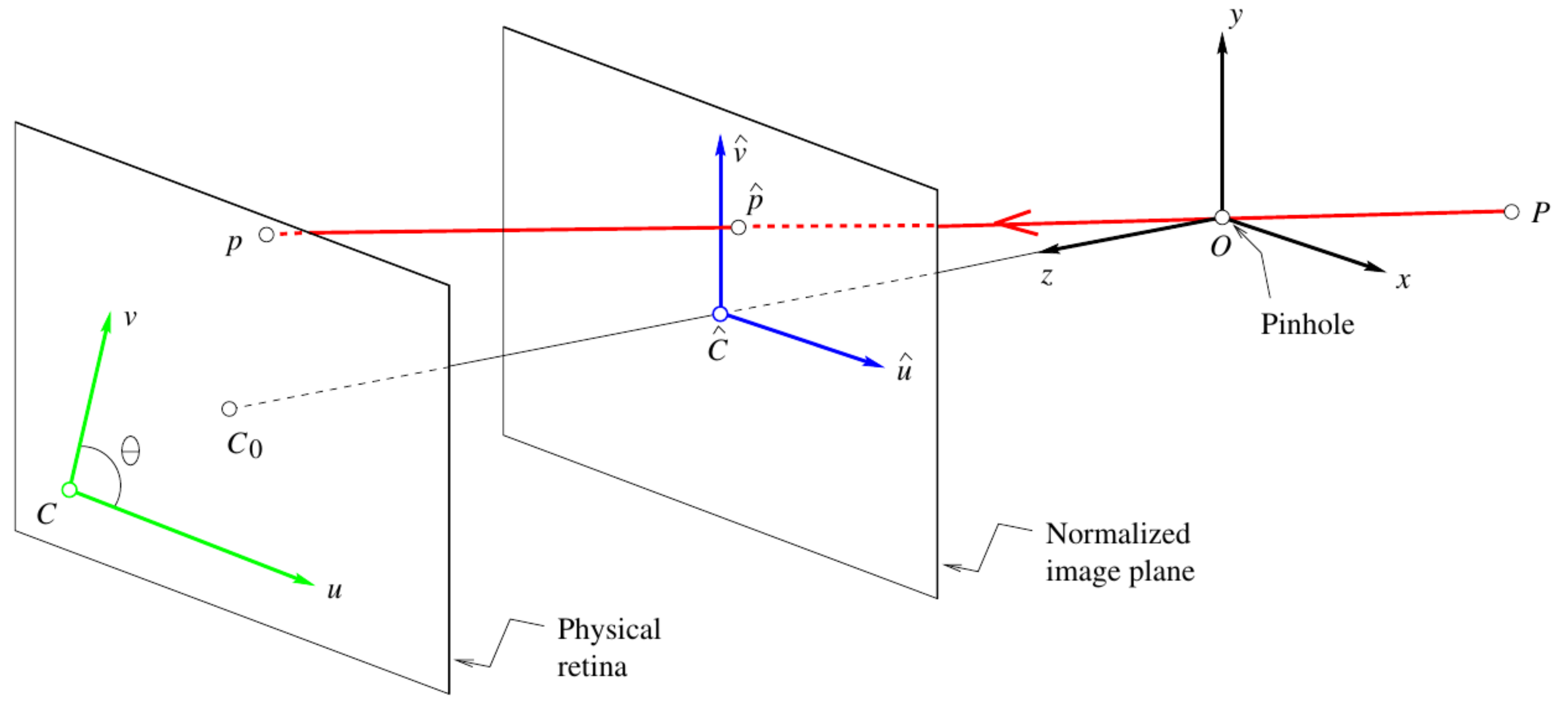

Figure 5.9. Physical and normalized image coordinate systems.

$$
\begin{aligned}
\hat{\boldsymbol{P}} &= \frac{1}{z}\begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} \end{pmatrix}\begin{pmatrix} \boldsymbol{P} \\ 1 \end{pmatrix} \\
\hat{u} &= \frac{x}{z} \\
\hat{v} &= \frac{y}{z} \\
u &= kf\frac{x}{z} \\
v &= lf\frac{y}{z}
\end{aligned}
$$

$$
\begin{aligned}
u &= \alpha\frac{x}{z} + u_0 \\
v &= \beta\frac{y}{z} + v_0
\end{aligned}
$$

$$
\begin{aligned}
u &= \alpha\frac{x}{z} - \alpha\cot\theta\frac{y}{z} + u_0 \\
v &= \frac{\beta}{\sin\theta}\frac{y}{z} + v_0
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{P} &= \frac{1}{z}\mathcal{M}\hat{\boldsymbol{P}} \\
\boldsymbol{P} &= \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \\
\mathcal{M} &= \begin{pmatrix} \mathcal{K} & \mathbf{0} \end{pmatrix} \\
\mathcal{K} &= \begin{pmatrix} \alpha & -\alpha\cot\theta & u_0 \\ 0 & \frac{\beta}{\sin\theta} & v_0 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned}
$$

几何相机参数——外参数

$$
{}^{C}P = \begin{pmatrix} {}^{C}_{W}\mathcal{R} & {}^{C}O_{W} \end{pmatrix} \begin{pmatrix} {}^{W}P \\ 1 \end{pmatrix}
$$

$$
\boldsymbol{p} = \frac{1}{z}\mathcal{M}\boldsymbol{P} \qquad \mathcal{M} = \mathcal{K}\begin{pmatrix} \mathcal{R} & \boldsymbol{t} \end{pmatrix} = \begin{pmatrix} \boldsymbol{m}_1^T \\ \boldsymbol{m}_2^T \\ \boldsymbol{m}_3^T \end{pmatrix}
$$

$$
u = \frac{\boldsymbol{m}_1^T\boldsymbol{P}}{\boldsymbol{m}_3^T\boldsymbol{P}}
$$

$$
v = \frac{\boldsymbol{m}_2^T\boldsymbol{P}}{\boldsymbol{m}_3^T\boldsymbol{P}}
$$

投影矩阵$\mathcal{M}$

$$
\begin{aligned}
\mathcal{M} &= \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \frac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \frac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix} \\
&= \begin{pmatrix} \mathcal{M} & \boldsymbol{b} \end{pmatrix} \\
\mathcal{R} &= \begin{pmatrix} \boldsymbol{r}_1^T \\ \boldsymbol{r}_2^T \\ \boldsymbol{r}_3^T \end{pmatrix} \\
\boldsymbol{t} &= \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}\begin{pmatrix} \boldsymbol{O} \\ 1 \end{pmatrix} &= 0 \\
\boldsymbol{O} &= -\mathcal{A}^{-1}\boldsymbol{b}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Det}(\mathcal{A}) &\neq 0 \\
(\boldsymbol{a}_1 \times \boldsymbol{a}_3)\cdot(\boldsymbol{a}_2 \times \boldsymbol{a}_3) &= 0 \\
|\boldsymbol{a}_1 \times \boldsymbol{a}_3| &= |\boldsymbol{a}_2 \times \boldsymbol{a}_3|
\end{aligned}
$$

where

$$
\mathcal{A} = \begin{pmatrix} \boldsymbol{a}_1^T \\ \boldsymbol{a}_2^T \\ \boldsymbol{a}_3^T \end{pmatrix}
$$

仿射投影

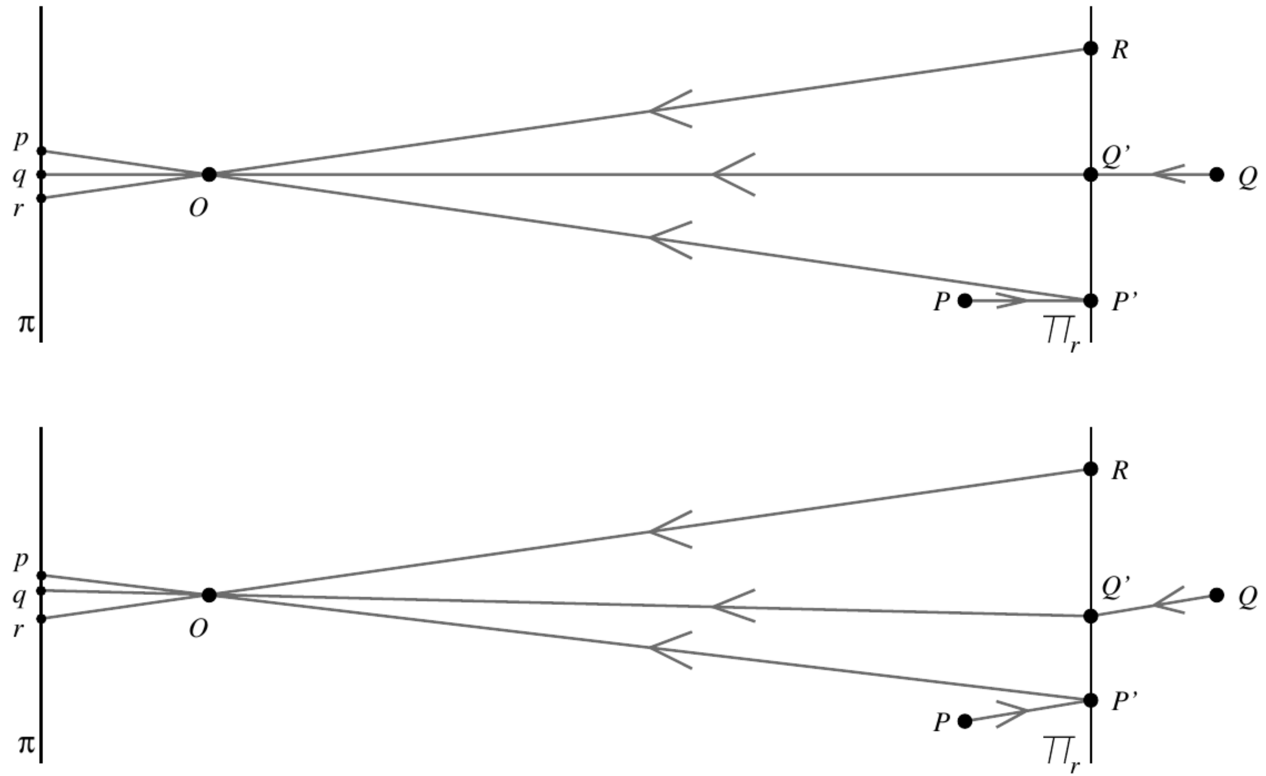

FIGURE 1.15: Affine projection models: (top) weak-perspective and (bottom) paraperspective projections.

弱透视投影模型

$$\boldsymbol{p} = \mathcal{M}\boldsymbol{P}$$

$$\begin{aligned}
\boldsymbol{p} &= \begin{pmatrix} x & y \end{pmatrix}^T \\
\mathcal{M} &= \frac{1}{Z_r}\mathcal{K}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & Z_r \end{pmatrix}\begin{pmatrix} \mathcal{R} & \boldsymbol{t} \\ \mathbf{0}^T & 1 \end{pmatrix} \\
\boldsymbol{P} &= \begin{pmatrix} X & Y & Z & 1 \end{pmatrix}^T \\
\mathcal{K} &= \begin{pmatrix} \mathcal{K}_2 & p_0 \\ \mathbf{0}^T & 1 \end{pmatrix} \\
\mathcal{K}_2 &= \begin{pmatrix} \alpha & -\alpha\cot\theta \\ 0 & \frac{\beta}{\sin\theta} \end{pmatrix} \\
p_0 &= \begin{pmatrix} x_0 & y_0 \end{pmatrix}^T
\end{aligned}$$

$$
\begin{aligned}
\mathcal{M} &= \left(\mathcal{A} \;\; \boldsymbol{b} \right) \\
&= \frac{1}{Z_r}\mathcal{K}\begin{pmatrix} \mathcal{R}_2 & \boldsymbol{t}_2 \\ \mathbf{0}^T & 1 \end{pmatrix} \\
&= \frac{1}{Z_r}\begin{pmatrix} \mathcal{K}_2 & p_0 \\ \mathbf{0}^T & 1 \end{pmatrix}\begin{pmatrix} \mathcal{R}_2 & \boldsymbol{t}_2 \\ \mathbf{0}^T & 1 \end{pmatrix} \\
&= \frac{1}{Z_r}\left(\mathcal{K}_2\mathcal{R}_2 \;\; \mathcal{K}_2\boldsymbol{t}_2 + p_0 \right) \\
&= \frac{1}{Z_r}\begin{pmatrix} k & s \\ 0 & 1 \end{pmatrix}\left(\mathcal{R}_2 \;\; \boldsymbol{t}_2 \right)
\end{aligned}
$$

类透视投影模型

$$\begin{pmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{pmatrix} = \begin{pmatrix} X - \frac{X_r}{Z_r}(Z - Z_r) \\ Y - \frac{Y_r}{Z_r}(Z - Z_r) \\ Z_r \end{pmatrix}$$

$$\mathcal{M} = \frac{1}{Z_r}\mathcal{K}\begin{pmatrix} 1 & 0 & -\frac{X_r}{Z_r} & Z_r \\ 0 & 1 & -\frac{Y_r}{Z_r} & Z_r \\ 0 & 0 & 0 & Z_r \end{pmatrix}\begin{pmatrix} \mathcal{R} & \boldsymbol{t} \\ \mathbf{0}^T & 1 \end{pmatrix}$$

$$\mathcal{M} = \frac{1}{Z_r}\begin{pmatrix} k & s \\ 0 & 1 \end{pmatrix}\left(\begin{pmatrix} 1 & 0 & -\frac{X_r}{Z_r} \\ 0 & 1 & -\frac{Y_r}{Z_r} \end{pmatrix}\mathcal{R} \quad \boldsymbol{t}_2\right)$$

相机标定

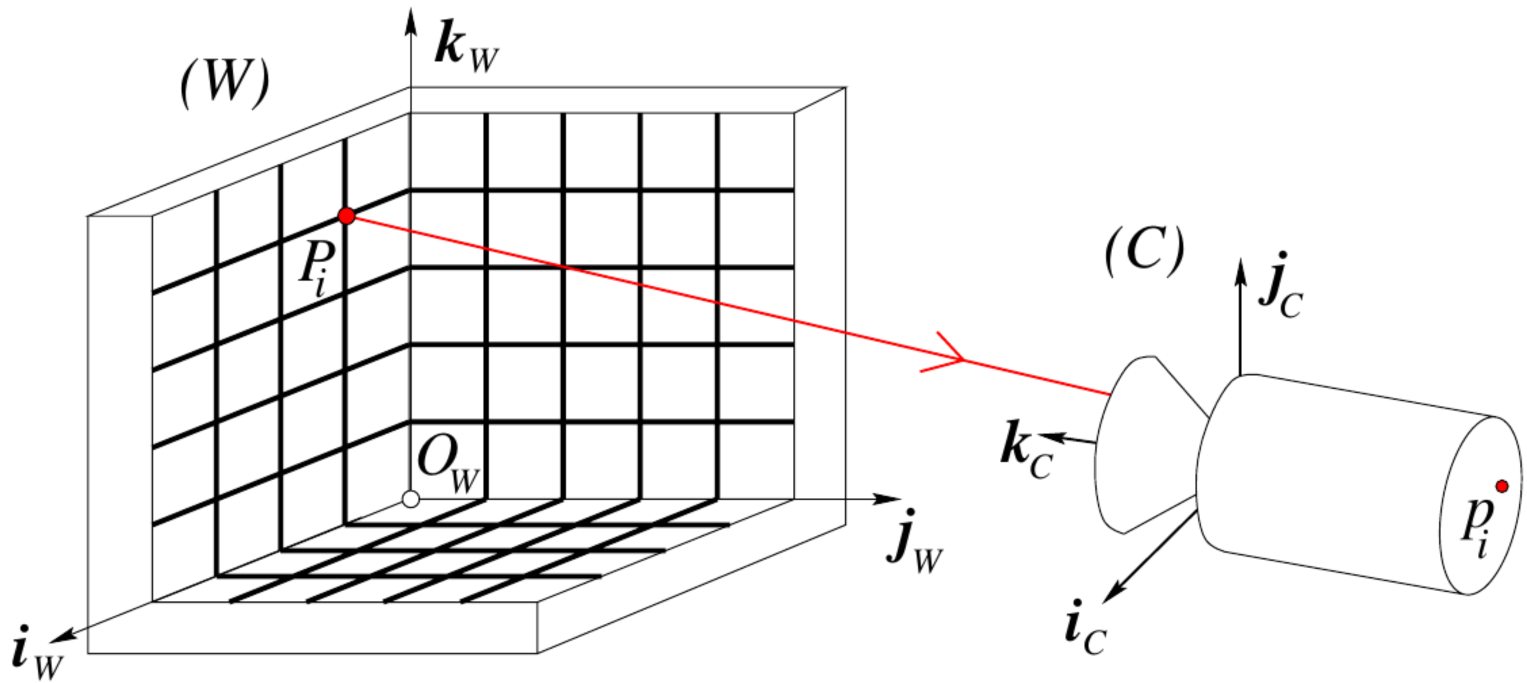

Figure 6.1. Camera calibration setup: in this example, the calibration rig is formed by three grids drawn in orthogonal planes. Other patterns could be used as well, and they may involve lines or other geometric figures.

估计$\mathcal{M}$

$$
\begin{aligned}
(\boldsymbol{m}_1 - u_i \boldsymbol{m}_3)\cdot \boldsymbol{P}_i &= 0 \\
(\boldsymbol{m}_{12} - u_v \boldsymbol{m}_3)\cdot \boldsymbol{P}_i &= 0 \\
\mathcal{P}\boldsymbol{m} &= 0
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{P} &= \begin{pmatrix}
\boldsymbol{P}_1^T & \boldsymbol{0}^T & -u_1 \boldsymbol{P}_1^T \\
\boldsymbol{0}^T & \boldsymbol{P}_1^T & -v_1 \boldsymbol{P}_1^T \\
\cdots & \cdots & \cdots \\
\boldsymbol{P}_n^T & \boldsymbol{0}^T & -u_n \boldsymbol{P}_n^T \\
\boldsymbol{0}^T & \boldsymbol{P}_n^T & -v_n \boldsymbol{P}_n^T
\end{pmatrix} \\
\boldsymbol{m} &= \begin{pmatrix}
\boldsymbol{m}_1 \\
\boldsymbol{m}_2 \\
\boldsymbol{m}_3
\end{pmatrix}
\end{aligned}
$$

估计内参数

$$\rho\left(\ \mathcal{A}\quad \boldsymbol{b}\ \right) = \mathcal{K}\left(\ \mathcal{R}\quad \boldsymbol{t}\ \right)$$

$$\rho\begin{pmatrix} \boldsymbol{a}_1^T \\ \boldsymbol{a}_2^T \\ \boldsymbol{a}_3^T \end{pmatrix} = \begin{pmatrix} \alpha\boldsymbol{r}_1^T - \alpha\cot\theta\boldsymbol{r}_2^T + u_0\boldsymbol{r}_3^T \\ \frac{\beta}{\sin\theta}\boldsymbol{r}_2^T + v_0\boldsymbol{r}_3^T \\ \boldsymbol{r}_3^T \end{pmatrix}$$

$\left(\ \boldsymbol{r}_1\quad \boldsymbol{r}_2\quad \boldsymbol{r}_3\ \right)$为单位正交阵：

$$\begin{aligned} \rho\boldsymbol{a}_3 &= \boldsymbol{r}_3 \\ \rho &= \frac{\varepsilon}{|\boldsymbol{a}_3|} \\ \rho^2(\boldsymbol{a}_1\cdot\boldsymbol{a}_3) &= u_0 \\ \rho^2(\boldsymbol{a}_2\cdot\boldsymbol{a}_3) &= v_0 \end{aligned}$$

利用叉乘：

$$
\begin{aligned}
\rho^2(\boldsymbol{a}_1\times\boldsymbol{a}_3) &= -\alpha\boldsymbol{r}_2-\alpha\cot\theta\boldsymbol{r}_1\\
\rho^2|\boldsymbol{a}_1\times\boldsymbol{a}_3| &= \frac{a}{\sin\theta}\\
\rho^2(\boldsymbol{a}_2\times\boldsymbol{a}_3) &= \frac{\beta}{\sin\theta}\boldsymbol{r}_1\\
\rho^2|\boldsymbol{a}_2\times\boldsymbol{a}_3| &= \frac{\beta}{\sin\theta}
\end{aligned}
$$

得：

$$
\begin{aligned}
\cos\theta &= -\frac{(\boldsymbol{a}_1\times\boldsymbol{a}_3)(\boldsymbol{a}_2\times\boldsymbol{a}_3)}{|\boldsymbol{a}_1\times\boldsymbol{a}_3||\boldsymbol{a}_2\times\boldsymbol{a}_3|}\\
\alpha &= \rho^2|\boldsymbol{a}_1\times\boldsymbol{a}_3|\sin\theta\\
\beta &= \rho^2|\boldsymbol{a}_2\times\boldsymbol{a}_3|\sin\theta
\end{aligned}
$$

估计外参数

$$
\begin{aligned}
\boldsymbol{r}_1 &= \frac{\rho^2 \sin\theta}{\beta}(\boldsymbol{a}_2 \times \boldsymbol{a}_3) \\
&= \frac{1}{|\boldsymbol{a}_2 \times \boldsymbol{a}_3|}(\boldsymbol{a}_2 \times \boldsymbol{a}_3) \\
\boldsymbol{r}_2 &= \boldsymbol{r}_3 \times \boldsymbol{r}_1
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{K}\boldsymbol{t} &= \rho \boldsymbol{b} \\
\boldsymbol{t} &= \rho \mathcal{K}^{-1} \boldsymbol{b}
\end{aligned}
$$

退化位形

$\mathcal{P}$的零空间

$$
\begin{aligned}
\mathbf{0} &= \mathcal{P}\boldsymbol{l} \\
&= \begin{pmatrix}
\boldsymbol{P}_1^T & \mathbf{0}^T & -u_1\boldsymbol{P}_1^T \\
\mathbf{0}^T & \boldsymbol{P}_1^T & -v_1\boldsymbol{P}_1^T \\
\cdots & \cdots & \cdots \\
\boldsymbol{P}_n^T & \mathbf{0}^T & -u_n\boldsymbol{P}_n^T \\
\mathbf{0}^T & \boldsymbol{P}_n^T & -v_n\boldsymbol{P}_n^T
\end{pmatrix}
\begin{pmatrix}
\boldsymbol{\lambda} \\ \boldsymbol{\mu} \\ \boldsymbol{\nu}
\end{pmatrix} \\
&= \begin{pmatrix}
\boldsymbol{P}_1^T\boldsymbol{\lambda} - u_1\boldsymbol{P}_1^T\boldsymbol{\nu} \\
\boldsymbol{P}_1^T\boldsymbol{\mu} - v_1\boldsymbol{P}_1^T\boldsymbol{\nu} \\
\cdots \\
\boldsymbol{P}_n^T\boldsymbol{\lambda} - u_n\boldsymbol{P}_n^T\boldsymbol{\nu} \\
\boldsymbol{P}_n^T\boldsymbol{\mu} - v_n\boldsymbol{P}_n^T\boldsymbol{\nu}
\end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{P}_i^T\boldsymbol{\lambda}-\frac{\boldsymbol{m}_1^T\boldsymbol{P}_i}{\boldsymbol{m}_3^T\boldsymbol{P}_i}\boldsymbol{P}_i^T\boldsymbol{\nu} &= 0\\
\boldsymbol{P}_i^T\boldsymbol{\mu}-\frac{\boldsymbol{m}_2^T\boldsymbol{P}_i}{\boldsymbol{m}_3^T\boldsymbol{P}_i}\boldsymbol{P}_i^T\boldsymbol{\nu} &= 0
\end{aligned}
$$

得

$$
\begin{aligned}
\boldsymbol{P}_i^T(\boldsymbol{m}_3\boldsymbol{\lambda}^T-\boldsymbol{m}_1\boldsymbol{\nu}^T)\boldsymbol{P}_i &= 0\\
\boldsymbol{P}_i^T(\boldsymbol{m}_3\boldsymbol{\mu}^T-\boldsymbol{m}_2\boldsymbol{\nu}^T)\boldsymbol{P}_i &= 0
\end{aligned}
$$

若存在平面Π使$\Pi\cdot\boldsymbol{P}_i=0$，则

$$
(\ \Pi\ \ 0\ \ 0\ ),(\ 0\ \ \Pi\ \ 0\ ),(\ 0\ \ 0\ \ \Pi\ ),\boldsymbol{m}
$$

张成$\mathcal{P}$的零空间。

考虑径向畸变

$$
\begin{aligned}
\boldsymbol{p} &= \frac{1}{z}\begin{pmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{pmatrix}\mathcal{M}\boldsymbol{P} \\
\lambda &= 1 + \sum_{p=1}^{q} \kappa_p d^{2p} \qquad\qquad q \leqslant 3
\end{aligned}
$$

$$
\begin{aligned}
\begin{pmatrix} \hat{u} \\ \hat{v} \end{pmatrix} &= \mathcal{K}^{-1} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} u\alpha^{-1} + v\beta^{-1}\cos\theta \\ v\beta^{-1}\sin\theta \\ 1 \end{pmatrix} \qquad (u_0 = v_0 = 0) \\
\mathcal{K}^{-1} &= \begin{pmatrix} \alpha^{-1} & \frac{\cos\theta}{\beta} & -\frac{u_0}{\alpha} - \frac{v_0\cos\theta}{\beta} \\ 0 & \frac{\sin\theta}{\beta} & -\frac{u_0\beta}{\sin\theta} \\ 0 & 0 & 1 \end{pmatrix} \\
d^2 &= \hat{u}^2 + \hat{v}^2 \\
&= u^2\alpha^{-2} + v^2\beta^{-2} + 2uv\alpha^{-1}\beta^{-1}\cos\theta
\end{aligned}
$$

估计投影矩阵

$$\begin{aligned}
\lambda\begin{pmatrix} u \\ v \end{pmatrix} &= \begin{pmatrix} \frac{\boldsymbol{m}_1 \cdot \boldsymbol{P}}{\boldsymbol{m}_3 \cdot \boldsymbol{P}} \\ \frac{\boldsymbol{m}_2 \cdot \boldsymbol{P}}{\boldsymbol{m}_3 \cdot \boldsymbol{P}} \end{pmatrix} \\
v(\boldsymbol{m}_1 \cdot \boldsymbol{P}) &= u(\boldsymbol{m}_2 \cdot \boldsymbol{P})
\end{aligned}$$

得

$$\begin{aligned}
\mathcal{Q}\boldsymbol{n} &= 0 \\
\mathcal{Q} &= \begin{pmatrix} v_1 \boldsymbol{P}_1^T & -u_1 \boldsymbol{P}_1^T \\ \cdots & \cdots \\ v_n \boldsymbol{P}_n^T & -u_n \boldsymbol{P}_n^T \end{pmatrix} \\
\boldsymbol{n} &= \begin{pmatrix} \boldsymbol{m}_1 \\ \boldsymbol{m}_2 \end{pmatrix}
\end{aligned}$$

内参数

$$
\begin{aligned}
\rho\begin{pmatrix} \boldsymbol{a}_1^T \\ \boldsymbol{a}_2^T \end{pmatrix} &= \begin{pmatrix} \alpha\boldsymbol{r}_1^T - \alpha\cot\theta\boldsymbol{r}_2^T \\ \frac{\beta}{\sin\theta}\boldsymbol{r}_2^T \end{pmatrix} \\
|\rho\boldsymbol{a}_1| &= \frac{\alpha}{\sin\theta} \\
|\rho\boldsymbol{a}_2| &= \frac{\beta}{\sin\theta} \\
\rho\boldsymbol{a}_1\cdot\rho\boldsymbol{a}_2 &= -\frac{\alpha\beta\cos\theta}{\sin^2\theta} \\
\cos\theta &= -\frac{\boldsymbol{a}_1\cdot\boldsymbol{a}_2}{|\boldsymbol{a}_1|\cdot|\boldsymbol{a}_2|} \\
\alpha &= |\rho\boldsymbol{a}_1|\sin\theta \\
\beta &= |\rho\boldsymbol{a}_2|\sin\theta
\end{aligned}
$$

外参数R

$$\boldsymbol{r}_1 = \frac{\varepsilon}{\sin\theta}\left(\frac{\boldsymbol{a}_1}{|\boldsymbol{a}_1|} + \frac{\boldsymbol{a}_2}{|\boldsymbol{a}_2|}\cos\theta\right)$$

$$\boldsymbol{r}_2 = \frac{\varepsilon \boldsymbol{a}_2}{|\boldsymbol{a}_2|}$$

外参数$\boldsymbol{t}$

$$
\begin{aligned}
\begin{pmatrix} \alpha t_x - \alpha t_y \cot\theta \\ \dfrac{\beta t_y}{\sin\theta} \end{pmatrix} &= \rho \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \\
\mathcal{M} &= \begin{pmatrix} \mathcal{A} & \boldsymbol{b} \end{pmatrix} \\
\boldsymbol{b} &= \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix}^T \\
t_x &= \frac{\varepsilon}{\sin\theta}\left(\frac{b_1}{|\boldsymbol{a}_1|} + \frac{b_2\cos\theta}{|\boldsymbol{a}_2|}\right) \\
t_y &= \frac{\varepsilon b_2}{|\boldsymbol{a}_2|}
\end{aligned}
$$

$$
\begin{aligned}
(\boldsymbol{m}_1 - \lambda u \boldsymbol{m}_3)\cdot\boldsymbol{P} &= 0 \\
(\boldsymbol{m}_2 - \lambda v \boldsymbol{m}_3)\cdot\boldsymbol{P} &= 0 \\
\boldsymbol{m}_3 &= \begin{pmatrix} \boldsymbol{r}_3 \\ t_z \end{pmatrix}
\end{aligned}
$$

径向畸变

$$
\begin{aligned}
d^2 &= \left|(u\alpha^{-1}+v\beta^{-1}\cos\theta)\boldsymbol{r}_2 - v\beta^{-1}\sin\theta\boldsymbol{r}_1\right|^2 \\
&= (\alpha^{-1}\beta^{-1}\sin\theta)^2\left|\left(\frac{u\beta}{\sin\theta}+v\alpha\cot\theta\right)\boldsymbol{r}_2 - v\alpha\boldsymbol{r}_1\right|^2 \\
&= \frac{1}{|\rho\boldsymbol{a}_1\times\rho\boldsymbol{a}_2|^2}\left|\frac{u\beta}{\sin\theta}\boldsymbol{r}_2 - v(\alpha\boldsymbol{r}_1-\alpha\cot\theta\boldsymbol{r}_2)\right|^2 \\
&= \frac{|u\rho\boldsymbol{a}_2 - v\rho\boldsymbol{a}_1|^2}{|\rho\boldsymbol{a}_1\times\rho\boldsymbol{a}_2|^2} \\
&= \frac{|u\boldsymbol{a}_2 - v\boldsymbol{a}_1|^2}{|\boldsymbol{a}_1\times\boldsymbol{a}_2|^2}
\end{aligned}
$$