Age of Information in Random Access Channels

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***Abstract*—In applications of remote sensing, estimation, and control, timely communication is not always ensured by high-rate communication. Oftentimes, it is observed that as the capacity of a system is approached, delay increases significantly and so does age of information – a metric recently proposed to capture freshness and timeliness of information. This work proposes *distributed* age-efficient transmission policies for random access channels with M transmitters and provides asymptotic results for the age of information as $M \to \infty$. Slotted ALOHA-type algorithms are shown to be asymptotically age-optimal for arrival rates below $\frac{1}{eM}$ and far from optimal for larger arrival rates. For larger arrival rates, novel distributed age-based policies are proposed that benefit from the availability of fresh packets to reduce age of information. For arrival rates θ, $\theta = \frac{1}{o(M)}$[1], the proposed algorithms provide a multiplicative gain factor of at least two compared to the state-of-the-art schemes. We conclude that, as opposed to the common practice, it is beneficial to increase the sampling rate (and hence the arrival rate) and transmit packets selectively based on their "age-gains", a notion defined in the paper.**

I. Introduction

Communication networks have witnessed rapid growth in the past few decades. Today, state-of-the-art network communication strategies are considered reliable and high speed; nevertheless, they often do not perform satisfactorily for time-sensitive applications. Age of information (AoI), introduced in [1], [2], measures the freshness of information. AoI is a function of both how often packets are transmitted and how much delay they experience in the system. The metric of AoI is of great importance in the Internet of Things applications where timeliness of information is crucial, e.g. in monitoring the status of a system or estimating a Markov process.

Assuming a first come first serve (FCFS) policy, the works in [3], [4] show in queue theoretic setups that AoI is minimized at an optimal update rate. Relaxing the restriction of FCFS policies, [4], [5] propose packet management policies that discard old packets and improve AoI in wide regimes of operation. As a matter of fact, under the metric of AoI, rate and reliability have little relevance in the design of communication schemes. This is because AoI implicitly assumes that the information content of the packets form a Markov process and hence fresh packets render older packets obsolete. In the past few years, various aspects of AoI have been studied in communication networks: source and channel coding were studied in [6]–[9], multi-hop networks were studied in [10]–[12], and scheduling algorithms were studied in [13]–[20].

We consider the problem of minimizing age of information over a random access (RA) channel. In this setup, centralized scheduling policies are not practical especially in modern IoT applications where millions of devices are involved. This is due to the very large scale of coordination and communication that they entail. Furthermore, designing distributed transmission policies is challenging for two reasons: (i) Transmitters cannot coordinate to avoid collision, and (ii) Transmitters do not share information about their arrivals and actions; it is hence not clear how they can concur on which transmitter should be prioritized for transmission. Considering that transmitters can only act based on their local information, one might wonder if the transmitters can do any better than stationary randomized policies. We design distributed policies in which transmitters exploit the available collision feedback to track the local age of information at the source and destination and make a transmission decision accordingly.

Towards designing distributed algorithms for minimizing age of information, [21], [22] analyze stationary randomized policies under the assumption that sources actively generate packets in every time slot. For the more realistic scenario where packets are generated at random times, [23] analyzes round-robin scheduling with and without packet management and also presents partial results for stationary randomized policies (without explicit characterizations). In [24], we provide a closed form expression for AoI under stationary randomized policies, also accounting for the stochastic nature of arrivals which was disregarded in [21], [22]. In [25], it is additionally assumed that nodes are provided with carrier sensing capabilities and a distributed scheme is proposed and shown to have good performance in simulations. Nevertheless, [25] does not address how the parameters of the scheme are to be designed; nor does it provide theoretical guarantees on the attained AoI.

In this work, we design distributed age-based transmission policies and provide asymptotic analytical results on the achievable AoI in interesting regimes of operation. The major part of this paper deals with random access technologies such as slotted ALOHA but extensions to general random access techniques are also addressed at the end. The contributions of this paper are as follows. In presenting our results below, we assume large symmetric networks in which we have M transmitters and each transmitter has the arrival rate θ.

- We derive lower bounds on the normalized AoI in terms of the capacity of the underlying channel and show the asymptotic tightness of them in interesting regimes of operation.
- We prove that when the sum arrival rate $M\theta$ is below the infamous critical point $\frac{1}{e}$, the normalized age performance of the slotted ALOHA algorithm is approximately $\frac{1}{M\theta}$ in the limit of large M and it is optimal.

[1] $o(\cdot)$ represents the Bachmann-Landau's little o notation.

- The maximum sum-rate of slotted ALOHA is provably $\frac{1}{e}$. Hence, as the sum arrival rate increases beyond this critical point, the age of slotted ALOHA and its unit-buffer-size variants such as [21], [23] increase. We first introduce the age-gain of a packet to quantify how much it would reduce the instantaneous AoI upon successful delivery. We then propose two age-based thinning methods (based on adaptive and stationary thresholding) in which transmitters disregard packets in order to mimic an effective (sum) arrival rate equal to $\frac{1}{e}$. In particular, we develop a threshold policy that can be implemented in a distributed manner at the transmitters and in which only packets that offer large age-gains are transmitted. In the limit of $M \to \infty$, using the stationary thinning method, we prove that the normalized age is approximately $\frac{e}{2}$ for all $\theta = \frac{1}{o(M)}$.
- Finally, we demonstrate applications of our proposed thinning mechanism, more generally, for other random access technologies. In particular, we prove that given a technology that can achieve the sum-rate C, age-based thinning methods attain the normalized age of $\frac{1}{2C}$. In particular, using CSMA, we approach optimality in the limit of large M.

II. System Model and Notation

We consider a wireless symmetric architecture where a controller monitors the status of M identical source nodes over a shared wireless medium. Let time be slotted. At the beginning of every slot k, $k = 1, 2, \ldots$, the source node i, $i = 1, \ldots, M$, generates a new packet encoding information about its current status with probability θ and this packet becomes available at the transmitter immediately. We denote this generation/arrival process at the transmitter by $A_i(k)$, where $A_i(k) = 1$ indicates that a new packet is generated at time slot k and $A_i(k) = 0$ corresponds to the complementary event. New packets are assumed to replace undelivered older packets at the transmitters, relying on the fact that the underlying processes are oftentimes Markovian. It is straightforward to show that this assumption holds at no loss of generality when AoI is to be minimized and the buffer size can be assumed to be one (see [24, Appendix A]).

The communication medium is modeled by a collision channel: If two or more source nodes transmit at the beginning of the same slot, then the packets collide and do not get delivered at the receiver. We use the binary variable $d_i(k)$ to indicate whether a packet is transmitted from source i and received at the destination in time slot k. Specifically, $d_i(k) = 0$ if source i does not transmit at the beginning of time slot k or if collision occurs; $d_i(k) = 1$ otherwise.

We assume a delay of one time unit in the delivery of packets, i.e., packets are transmitted at the beginning of a time slot and, if there is no collision, they are delivered at the end of the same slot. We assume that all transmitters are provided with channel collision feedback at the end of each time slot. Specifically, at the end of time slot k, $c(k) = 1$ if collision happened and $c(k) = 0$ otherwise. In the event of collision, the involved transmitters can keep the undelivered packets and retransmit them according to their transmission policy.

Our objective is to design *distributed* transmission policy to minimize the age of information (defined below). A distributed policy is one in which the action of transmitter i at time k is dependent only on its history of actions, the arrival process $\{A_i(j)\}_{j=1}^{k}$, and the collision feedback $\{c(j)\}_{j=1}^{k-1}$.

We extend the definition of AoI a bit further to also account for the aging of information at the transmitter (due to the *stochastic nature of arrivals*). Consider a source-destination pair. Let $\{k'_\ell\}_{\ell\geq 1}$ denote the delivery times of the received packets and $\{k_\ell\}_{\ell\geq 1}$ denote the corresponding generation times. At time τ, denote the index of the last generated (resp. delivered) packet by $n_s(\tau) = \max\{\ell | k_\ell \leq \tau\}$ (resp. $n_d(\tau) = \max\{\ell | k'_\ell \leq \tau\}$). The *source's AoI* is $w(k) = k - k_{n_s(k)}$ and the *destination's AoI* is $h(k) = k - k_{n_d(k)}$.

Following this definition, let $h_i(k)$ (resp. $w_i(k)$) denote the destination's (resp. source's) AoI at time slot k with respect to source i. The age $h_i(k)$ increases linearly with k when there is no packet delivery from source i and it drops with every successful delivery to a value that represents how old the received packet is; in our framework, this would be the corresponding source's AoI (in previous time slot) plus one.

For any transmission policy π, the performance metric that we consider is the *Normalized Expected Weighted Sum AoI (NEWSAoI)* defined below[2]:

$$NEWSAoI = \lim_{K\to\infty} \mathbb{E}[J_K^\pi],\ \ J_K^\pi = \frac{1}{M^2}\sum_{i=1}^{M}\frac{1}{K}\sum_{k=1}^{K} h_i^\pi(k). \quad (1)$$

III. Lower Bound

We start by finding two lower bounds on the achievable age performance. The first lower bound is derived by assuming that there is always a fresh packet to be transmitted. The second lower bound is derived by assuming that all packets are delivered instantaneously upon their arrivals (with unit-time delays, but without collisions). We will see that the former is asymptotically tight as θ approaches 1 and the latter is asymptotically tight when θ is small (i.e., when the inter-arrival time is the dominant term of the inter-delivery time).

Fix a large time horizon K and look at the packets of source i. Let $N_i(K)$ denote the number of delivered packets (from source i) up to and including time slot K. Now consider the m^{th} and $(m+1)^{th}$ deliveries at the receiver and denote the delivery time of them by $T_i(m)$ and $T_i(m+1)$, respectively. The inter-delivery time $I_i(m) = T_i(m+1) - T_i(m)$ is the time between these two consecutive deliveries. Upon arrival of the m^{th} delivered packet at the receiver, the age of information at the receiver drops to the value $D_i(m)$ which represents how much delay the packet has experienced in the system. Now define $\Gamma_i(m)$ as the sum of $h_i(k)$, $k \in [T_i(m), T_i(m+1))$:

$$\Gamma_i(m) = \sum_{k=T_i(m)}^{T_i(m)+I_i(m)-1} h_i(k) \quad (2)$$

$$= \frac{1}{2} I_i^2(m) - \frac{1}{2} I_i(m) + D_i(m) I_i(m). \quad (3)$$

[2]Since age increases linearly with the number of source nodes M when θ is fixed, we further normalize by M to look at the slope of the increase.

It follows that in the limit of large K, we have

$$\lim_{K\to\infty}\mathbb{E}[J_K^\pi]=\lim_{K\to\infty}\mathbb{E}\left[\frac{1}{M^2}\sum_{i=1}^{M}\frac{1}{K}\sum_{m=1}^{N_i(K)}\Gamma_i(m)\right]. \tag{4}$$

Using this formulation, we next lower bound NEWSAoI. Let C_{RA} denote the sum-capacity of the underlying random access channel. Note that in the limit of large K, $\frac{N_i(K)}{K}$ is the throughput of transmitter i and

$$\lim_{K\to\infty}\sum_{i=1}^{M}\frac{N_i(K)}{K}\le C_{RA}. \tag{5}$$

Using (5) and the lower bound derivations in [13], [26], we prove the following propositions (see [24] for the proofs).

Proposition 1. *For any transmission policy π, the normalized age in* (1) *is lower bounded as follows:*

$$\lim_{K\to\infty}\mathbb{E}[J_K^\pi]\ge\frac{1}{2C_{RA}}+\frac{1}{2M}. \tag{6}$$

Proposition 2. *For any transmission policy π, the normalized age in* (1) *is lower bounded as follows:*

$$\lim_{K\to\infty}\mathbb{E}[J_K^\pi]\ge\frac{1}{M\theta}. \tag{7}$$

Let us give an example of how Proposition 1 can be utilized. Note that C_{RA} is not known in general. Nevertheless, any upper bound on C_{RA} gives a lower bound on the normalized age. Based on [27], the capacity of the random access channel with collision feedback, in the limit of large M, is upper bounded by $\lim_{M\to\infty}C_{RA}\le 0.568$ and hence

$$\lim_{M\to\infty}\lim_{K\to\infty}\mathbb{E}[J_K^\pi]\ge .88. \tag{8}$$

Remark 1. *The lower bound in* (8) *does not assume CSMA capabilities. For CSMA, we have $C_{CSMA}\le 1$ and hence*

$$\lim_{K\to\infty}\mathbb{E}[J_K^\pi]\ge\frac{1}{2}+\frac{1}{2M}. \tag{9}$$

We show the asymptotic optimality of this bound in Section IV-E as $M\to\infty$.

IV. Distributed Age-Based Policies

Consider two regimes of operation:

- The regime of infrequent arrivals, where $\theta\le\frac{1}{eM}$,
- The regime of frequent arrivals, where $\theta>\frac{1}{eM}$.

For simplicty and clarity of ideas, we will develop our framework for the slotted-ALOHA random access technology, but will expand on how it generalizes to other random access technologies in Section IV-E.

The basic idea of slotted ALOHA is as follows: At every time slot k, transmitters send their packets immediately upon arrival unless they are "backlogged" after a collision in which case they transmit with a backoff probability. Here, we focus on Rivest's stabilized slotted ALOHA. In this algorithm, all arrivals are regarded as backlogged nodes that transmit with the backoff probability $p_b(k)$. Note that since buffer sizes are assumed to be one, the number of backlogged nodes is at most M. Let $c(k)=1$ denote the event that collision occurred at time k and $c(k)=0$ denote the complementary event. The backoff probability is calculated through a pseudo-Bayesian algorithm based on an estimate of the number of backlogged nodes $n(k)$:

$$\begin{aligned}&p_b(k)=\min\{1,\frac{1}{n(k)}\}\\&n(k)=\begin{cases}\min\{n(k-1)+M\theta+(e-2)^{-1},M\} & \text{if } c(k)=1\\ \min\{\max\{M\theta,n(k-1)+M\theta-1\},M\} & \text{if } c(k)=0.\end{cases}\end{aligned} \tag{10}$$

Performing slotted ALOHA, transmitters can reliably send packets with a sum-rate up to $\frac{1}{e}$ in a distributed manner. Assuming $M\to\infty$, the probability of delivering a packet in each time slot is $1/e$, the probability of collisions is $1-2/e$, and the probability of having an idle channel is $1/e$. Note that since $M\theta\le\frac{1}{e}$, the expected total number of delivered packets in every time slot is $M\theta$. We find the asymptotic NEWSAoI of any stabilized slotted ALOHA algorithm in Theorem 1 below (and refer to [24] for the proof).

Theorem 1. *Consider the limit of large M and suppose that $\lim_{M\to\infty}M\theta$ exists and is smaller than $\frac{1}{e}$. Any stabilized slotted ALOHA scheme achieves*

$$\lim_{M\to\infty}NEWSAoI=\frac{1}{M\theta}.$$

Hence, for $M\to\infty$, the normalized age of slotted ALOHA matches the lower bound in Proposition 2 and is optimal .

A. Age-Based Thinning

When the arrival rate θ increases beyond $\frac{1}{eM}$, slotted ALOHA is not stablized anymore, leading to a large delay and age. Noting that the maximum rate that ALOHA can support is $\frac{1}{eM}$ per source, a natural question rises: How could we attain a smaller age of information at the destination when $\theta>\frac{1}{eM}$? A naive solution to the above question is to have the transmitters randomly drop packets and perform at the effective rate $\frac{1}{eM}$. But as we argued in Theorem 1, this only leads to $NEWSAoI\approx e$ which means that we would not benefit from the frequency of fresh packets to reduce AoI.

To benefit from the availability of fresh packets, we devise an age-based transmission policy in which transmitters prioritize packets that have larger age-gains. In particular, in each time slot k, transmitters find a proper threshold $\mathtt{T}(k)$ in order to distinguish and prioritize packets that offer high age-gains. The core idea is to still use the channel at high rate but to carefully select, in a distributed manner, which packets to send in order to minimize age.

Note that no matter how the transmission policy is designed, since it is distributed, it may happen that multiple transmitters try to access the channel at the same time, leading to collision. For simplicity and clarity of ideas, we will restrict attention to slotted ALOHA techniques to resolve such collisions. More general techniques are discussed in Section IV-E.

Consider transmitter i at time k and recall the definition of $h_i(k)$ and $w_i(k)$ from Section II. We introduce the age-gain of (the packet of) transmitter i to quantify how much the instantaneous receiver's AoI reduces upon successful delivery from transmitter i. Denote this quantity by $\delta_i(k)$:

$$\delta_i(k) := h_i(k) - w_i(k). \tag{11}$$

At time k, we propose to discard a fresh packet at transmitter i if $0 \le \delta_i(k) < \mathtt{T}(k)$ and to keep it otherwise. We refer to this process as thinning and note that this is done based on the local AoI at the source/destination. The main underlying challenge is in the design of $\mathtt{T}(k)$. We propose two algorithms: an adaptive method of calculating $\mathtt{T}(k)$ for each time slot based on the local collision feedback and a fixed threshold value $\mathtt{T}^*$ (designed in advance). In the remainder of this section, we assume that $\theta > \frac{1}{eM}$, M is large, and we use the following definition.

Definition 1. *Transmitter i is said to be an m-order node if $\delta_i(k) = m$. We use $\ell_m(k)$ to denote the fraction of m-order nodes in time slot k and the node distribution (of the age-gain) at time k is given by $\{\ell_m(k)\}_{m=0}^{\infty}$.*

B. Adaptive Thresholding

Let $\mathtt{T}(k)$ be the threshold for decision making in slot k. Suppose the (estimated) node distribution $\{\ell_m(k-1)\}_{m=0}^{\infty}$ is known at the end of time slot $k-1$. For clarity of ideas, let us view time slot k in three stages: The first stage corresponds to the beginning of the time slot when new packets may arrive and replace the old packets. We denote the time just before the arrival of new packets by k^- and the time just after the arrival of packets by k^+. In the second stage, transmitters determine the threshold $\mathtt{T}(k)$ based on $\{\ell_m(k^+)\}_{m=0}^{\infty}$. Transmissions happen according to the designed threshold $\mathtt{T}(k)$. In the third stage, at the end of time slot k when collision feedback is also available, the node distribution is updated. We slightly abuse notation and denote the final estimate of the node distribution at the end of time slot k by $\{\ell_m(k)\}_{m=0}^{\infty}$.

Stage 1: Suppose the node distribution $\{\ell_m(k-1)\}_m$ is known at the beginning of slot k before the arrival of new packets. The expected fraction of $m-$order nodes, $m \ge 0$, that receive new packets is $\theta\ell_m(k-1)$. The order of these nodes increase and the expected node distribution changes to $\{\ell_m(k^+)\}_m$. Let $a_m(k)$ denote the expected fraction of nodes that have just become $m-$order at time k^+. It is straightforward to show

$$a_m(k) = \theta^2 \sum_{j=0}^{m-1} \ell_j(k-1)(1-\theta)^{m-j-1} \tag{12}$$

$$\ell_m(k^+) = \begin{cases} (1-\theta)\ell_0(k-1) & m = 0 \\ (1-\theta)\ell_m(k-1) + a_m(k) & m \ge 1. \end{cases} \tag{13}$$

Stage 2: The threshold $\mathtt{T}(k)$ is determined based on $\{\ell_m(k^+)\}_m$. We design $\mathtt{T}(k)$ such that the *effective arrival rate* of packets that have an age-gain above $\mathtt{T}(k)$ is close to $\frac{1}{e}$. In other words, we *thin* the arrival process. The critical point $\frac{1}{e}$ is the maximum sum arrival rate that ALOHA can support. So if the effective sum arrival rate falls below $\frac{1}{e}$, we do not use the available throughput and if we operate above $\frac{1}{e}$, we may incur additional collisions and delay.

Recall that $a_m(k)$ is the expected fraction of nodes that have just become m-order at time k^+ (coming from lower order nodes). So the total fraction of nodes whose age-gain would, for the first time, pass the threshold $\mathtt{T}(k)$ is $\sum_{m\ge \mathtt{T}(k)} a_m(k)$. We propose to choose $\mathtt{T}(k)$ as follows:

$$\mathtt{T}(k) = \max\left\{ t | \sum_{m\ge t} a_m(k) \ge \frac{1}{eM} \right\}. \tag{14}$$

Remark 2. *The threshold $\mathtt{T}(k)$ is computed based on $\{\ell_m(k-1)\}$. Note that $\{\ell_m(k-1)\}$ is an estimate of the node distribution and as we describe in stage 3, this estimate is found recursively by computing the expected node distribution condition on the estimated distribution in the previous time slot and the available feedback.*

Stage 3: Once the threshold $\mathtt{T}(k)$ is determined, each transmitter verifies locally if its age-gain is above the specified threshold. If so, it mimics slotted ALOHA and transmits its packet with probability $p_b(k)$ defined in (10) where $M\theta$ is replace by the effective arrival rate $\frac{1}{e}$. If collision happens or if all nodes abstain from transmitting, then AoI at the destination increases by 1 for all sources. If only one node transmits, then its packet will be delivered successfully and its age at the destination drops to the source's AoI.

C. Estimating the node distribution

It remains to estimate $\ell_m(k)$ at the end of time slot k, which will serve in computing $\mathtt{T}(k+1)$ in the next time slot. We assume that at the end of time slot k, all transmitters are provided with collision feedback from the channel and we hence consider two cases separately: $c(k) = 0$ and $c(k) = 1$. If $c(k) = 1$, the order of nodes will not change:

$$\ell_m(k) = \ell_m(k^+), \quad m \ge 0. \tag{15}$$

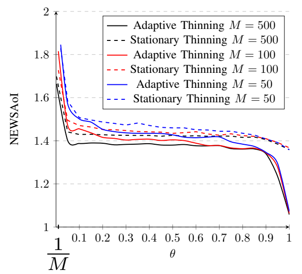
If $c(k) = 0$, we compute the conditional expectation of $\{\ell_m(k)\}_m$, condition on $\ell_m(k^+)$ and $c(k) = 0$, to find the following update rule (see [24] for details):

$$\begin{aligned} \ell_0(k) =& \ell_0(k^+) + \sum_{m=\mathtt{T}(k)}^{\infty} \min\left(\frac{r_m(k)}{2M}, \ell_m(k^+)\right) \\ \ell_m(k) =& \ell_m(k^+), \qquad\qquad 1 \le m \le \mathtt{T}(k) - 1 \\ \ell_m(k) =& \left(\ell_m(k^+) - \frac{r_m(k)}{2M}\right)^+ \qquad m \ge \mathtt{T}(k). \end{aligned} \tag{16}$$

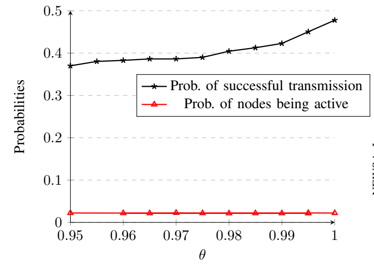
Remark 3. *Note that in Stage 3, we updated $\ell_m(k)$ as a function of $\ell_m(k^+)$ and the collision feedback $c(k)$, hence the name adaptive. If we update $\ell_m(k)$ by the conditional expectation of $\ell_m(k)$, condition on $\ell_m(k^+)$ but not on $c(k)$, we will find a fixed limiting threshold $\mathtt{T}^*$ discussed next.*

D. Stationary Thresholding

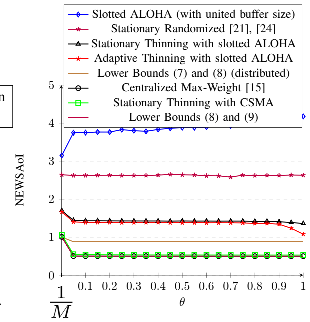
A simple variant of the age-based thinning method is found when the threshold $\mathtt{T}(k) = \mathtt{T}^*$ is fixed throughout the transmission phase. In particular, we design $\mathtt{T}^*$ ahead of time based on (14) as a function of the node distribution in



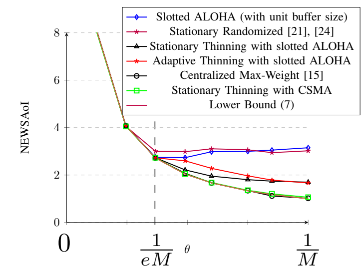
(a) NEWSAoI for stationary and adaptive age-based policies.



(b) Probabilities of successful transmission and being active in adaptive thinning policies.



(c) $M = 500$, $\theta \in (\frac{1}{M}, 1]$



(d) $M = 500$, $\theta \in (0, \frac{1}{M}]$

Fig. 1: Normalized age $NEWSAoI$ and success transmission probabilities

the *stationary regime*. By doing so, we cannot benefit from the collision feedback to adaptively choose $\mathtt{T}(k)$. However, this framework is preferable for deriving analytical results. In particular, we show in [24] that $\mathtt{T}^*$ takes the following form:

$$\mathtt{T}^* = \max\big(1, \lfloor eM - \frac{1}{\theta} + 1 \rfloor\big). \tag{17}$$

Our stationary age-based thinning method (followed by slotted ALOHA) is summarized in Algorithm 1 and its age-performance is analyzed in Theorem 2 next.

Algorithm 1 Stationary Age-based Thinning

Set the time horizon K.
Set initial points: $h_i(0) = 1, w_i(0) = 0$ for $i = 1, 2, \cdots, M$; $c(0) = 0$; $T(0) = 1$; $p_b(0) = 1$; $n(0) = 0$; $k = 1$.
Calculate $\mathtt{T}^* = \max\big(1, \lfloor eM - \frac{1}{\theta} + 1 \rfloor\big)$.
repeat
  Step 1: For transmitter i: compute $\delta_i(k) = h_i(k) - w_i(k)$; If $\delta_i(k) \geq \mathtt{T}^*$, transmit packet with probability $p_b(k)$. Otherwise, remain silent.
  Step 2: Calculate $p_b(k)$ using (10) in which $M\theta$ is replaced by $\min(M\theta, e^{-1})$.
until $k = K$
Calculate $NEWSAoI = \frac{1}{M^2} \sum_{i=1}^{M} \frac{1}{K} \sum_{k=0}^{K} h_i(k)$.

Theorem 2. *For any $\theta = \frac{1}{o(M)}$, the asymptotic NEWSAoI of Algorithm 1 is given by*

$$\lim_{M \to \infty} NEWSAoI = \frac{e}{2}.$$

E. Extensions to Other Random Access Technologies

So far, we restricted attention to the slotted ALOHA scheme. Theorem 3 formalizes extensions of our thinning framework to other RA techniques. Consider a random access scheme that attains the sum throughput C with a transmission policy π_C that does not employ coding across packets. All existing collision avoidance and resolution techniques such as [29]–[32] fall into this class. Now develop a variant of the transmission policy π_C in which only the most recent packet of each transmitter is preserved and all older packets are discarded. Denote this policy by $\pi_C^{(1)}$. We have the following general result (see [24] for the proof).

Theorem 3. *Suppose $\theta = \frac{1}{o(M)}$ and choose $\mathtt{T}^* = \max\big(1, \lfloor \frac{M}{C} - \frac{1}{\theta} + 1 \rfloor\big)$. The stationary age-based thinning method using $\mathtt{T}^*$, followed by the transmission policy $\pi_C^{(1)}$, attains the following normalized age in the limit of large M:*

$$\lim_{M \to \infty} NEWSAoI = \frac{1}{2C}.$$

V. Numerical Results

In this section, we verify our findings through simulations. Figure 1a shows the normalized age under adaptive and stationary age-based transmission policies for $M = 50, 100, 500$. For stationary age-based policies, the normalized age converges to $\frac{e}{2}$ when M is large, validating our findings in Theorem 2. The performance of the adaptive policy is better than that of the stationary age-based policy for $\theta > \frac{1}{M}$ and the efficacy (the gap between the two curves) increases with θ. Since the maximum sum throughput of slotted ALOHA is $\frac{1}{e}$, one may ask if this contradicts the lower bound of Proposition 5. To answer this question, we remark that the adaptive age-based transmission policy is *not* a slotted ALOHA scheme and therefore the maximum throughput of slotted ALOHA would not apply. As a matter of fact, Fig. 1b shows that the throughput of the scheme increases beyond $\frac{1}{e}$ with θ, supporting Proposition 5. One can also observe that adaptive policy performs worse than the stationary policy for $\theta \leq \frac{1}{M}$. We believe this is because the estimation of the node distribution $\{\ell_m\}_{m \geq 0}$ is imprecise for small θ.

Finally the age-performance of our proposed distributed age-based policies are compared with the lower bounds of Section III, state-of-the-art distributed schemes such as [21], as well as centralized Max-Weight policies such as [15]. For clarity, we consider two regimes of θ: $\theta \in (\frac{1}{M}, 1]$ (see Fig. 1c), and $\theta \in (0, \frac{1}{M}]$ (see Fig. 1d). Fig. 1d, in particular, shows that when $\theta \leq \frac{1}{eM}$, the normalized age of slotted ALOHA coincides with centralized Max-Weight policies and the lower bound of Proposition 7. When θ increases beyond $\frac{1}{eM}$, our proposed age-based thinning methods provide significant gains compared to randomized stationary and slotted ALOHA schemes. Finally, we numerically observe that the normalized age of the centralized Max-Weight policy is approximately attained by stationary age-based thinning in CSMA.

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