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Defining futures

Formal Verification
Introduction and Applications

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Objectives

- Formal Verification
  - Why do we need it?
  - What is it?
  - How can we apply it for the analysis of real-world systems?
- Applications
  - Analyzing Thermal Management Algorithms for Multi-core Architectures
  - Reliability Analysis

System Analysis

- Does the system exhibit the desired behavior?
  - Mathematically using paper-and-pencil proof methods
  - Computer Simulations (Testing)

Paper-and-Pencil Proof Methods

- Construct a mathematical model of the system
- Mathematically verify that the system exhibits the desired characteristics
- Accurate
- Scalability
- Error-Prone

Simulation or Testing

- Construct a computer based model of the system
- Analyze the behavior of the system model under a number of test cases to deduce properties of interest

Simulation or Testing –
Example: Remainder and Quotient Program

```
var x, y, q, r: integer;
r := x; q := 0;
while y < r do {
        r := r - y;
        q := q + 1;}
return r, return q;
```

- Property: r and q represent the remainder and quotient for all values of variables x and y
- Test Cases

Input (x,y)	Output (r,q)	Property
(23,4)	(3,5)	True
(2,5)	(2,0)	True
(17,3)	(2,5)	True

- Infer: The property is true since it is found to be true for all the test vectors used

Simulation or Testing – Example: Remainder and Quotient Program

```
var x, y, q, r: integer;
r := x; q := 0;
while y < r do
    r := r - y;
    q := q + 1;
```

- Testing for (24, 4)
  - After the fifth iteration, r = 4, q = 5, y = 4, the loop ends and we get (5, 4), which is wrong
  - Fix: Replace the condition y < r with y ≤ r
- Testing for (23, -2)
  - r is always greater than x and thus we have an infinite loop
  - Fix: Constrain y to have positive values only

Simulation or Testing

- Easy to use
- May lead to wrong conclusions

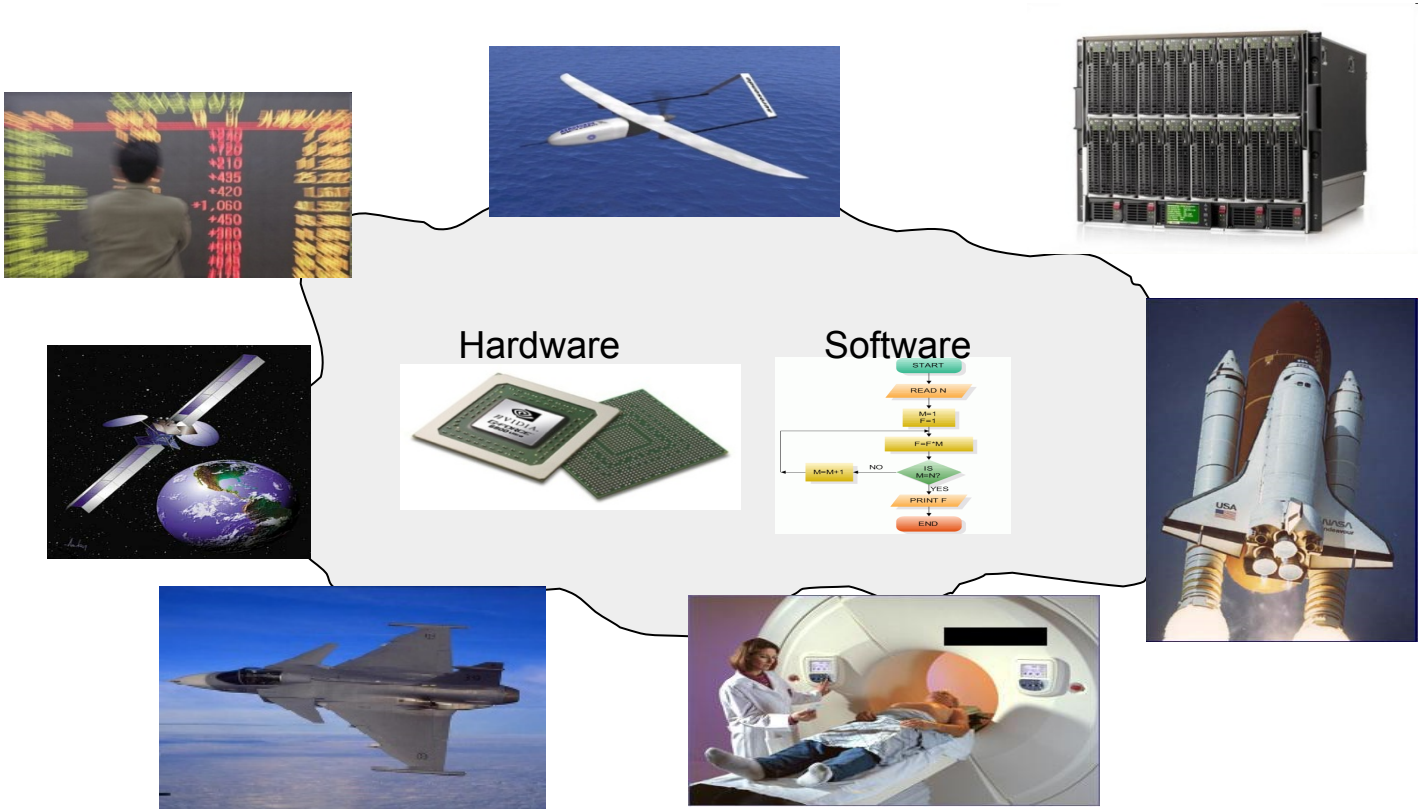
- Practically impossible to test for all possible cases when dealing with large systems
  - Suppose we want to test a 64-bit floating-point division routine. There are 2^{128} combinations.
  - At 1 test/micro second, it will take 10^{25} years

Program testing can be a very effective way to show the presence of bugs, but it is hopelessly inadequate for showing their absence.

Edsgar W. Dijkstra

System Analysis Accuracy

- Extremely Important



System Analysis Accuracy

- Faulty systems can be disastrous
- FDIV bug in Intel Pentium (60 Mhz, 90Mhz)
  - Hardware error in the floating point division unit
  - Expected precision up to 18 positions
  - in practice, only 4 positions
  - Example:
    - 5505001 / 294911
    - Answer from Intel Pentium: 18.666**00093**
    - Expected answer: 18.666**5197**

  - Resulted in net loss of approximately US $500M to the company in 1994

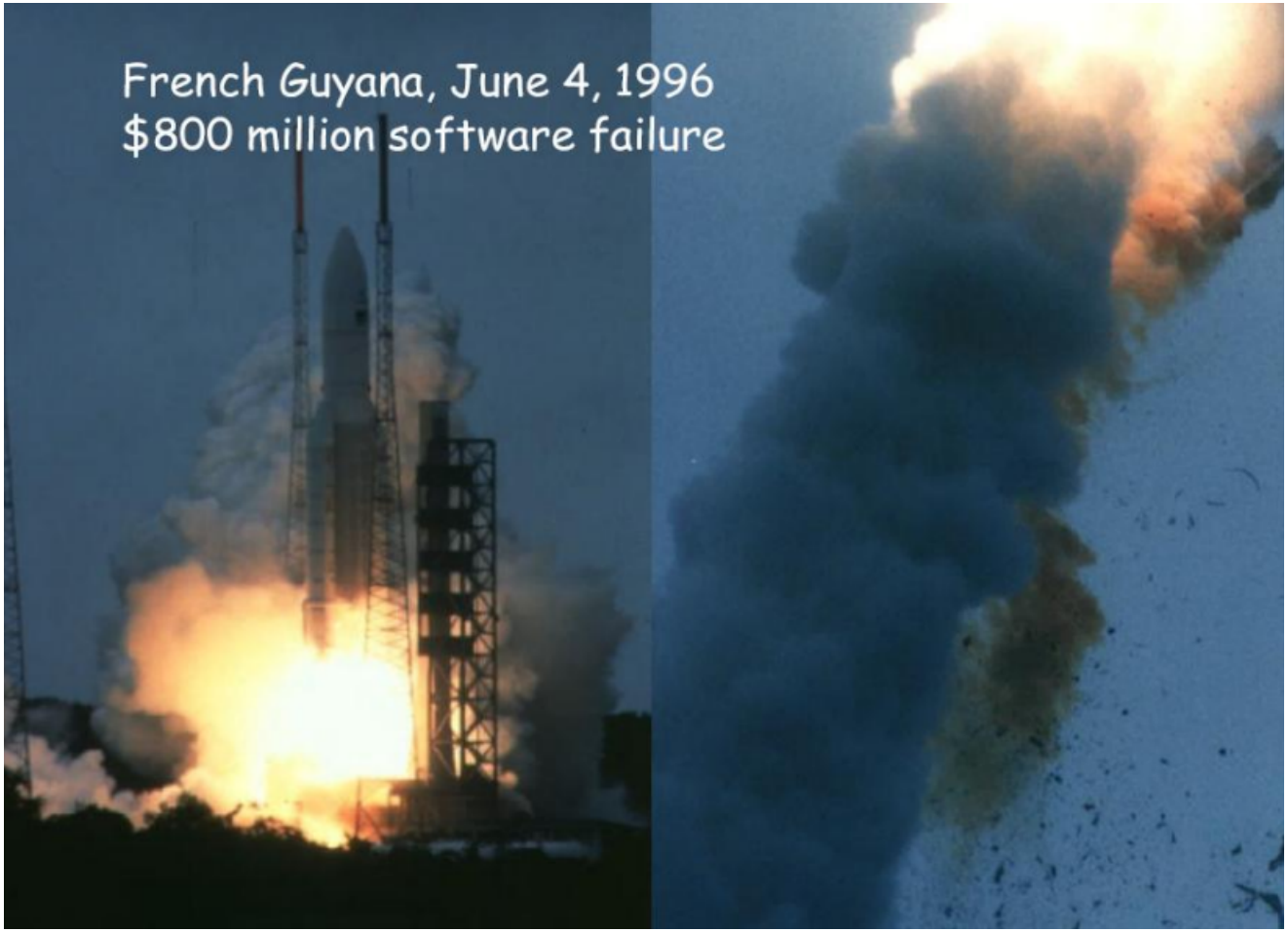
System Analysis Accuracy

- Faulty systems can be disastrous

- Therac-25
  - Software Bug in a Cancer Therapy Machine
  - 3 Deaths and 3 severe injuries between 1985-87

System Analysis Accuracy

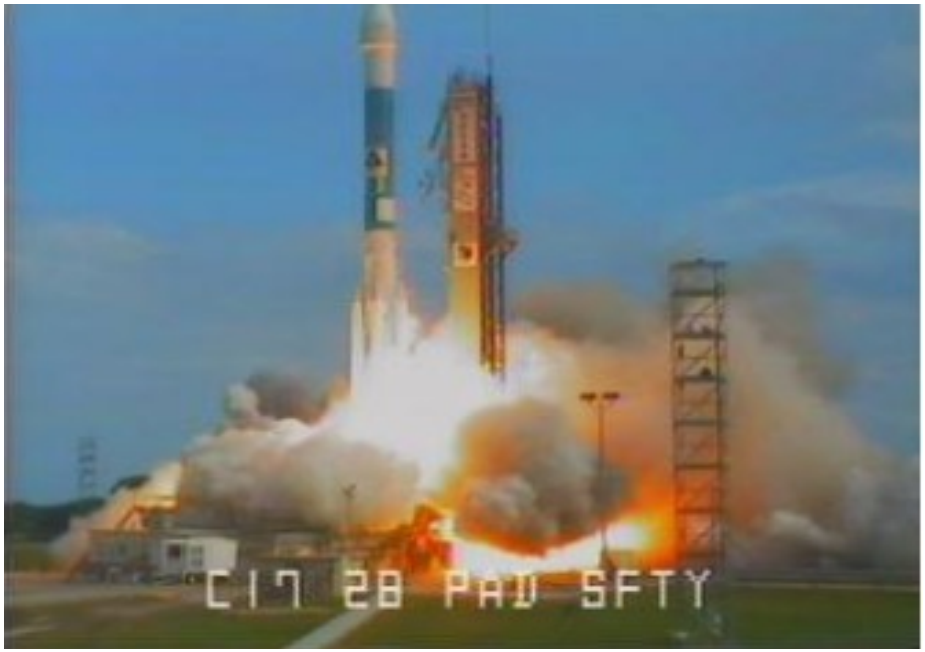
- Faulty systems can be disastrous



System Analysis Accuracy

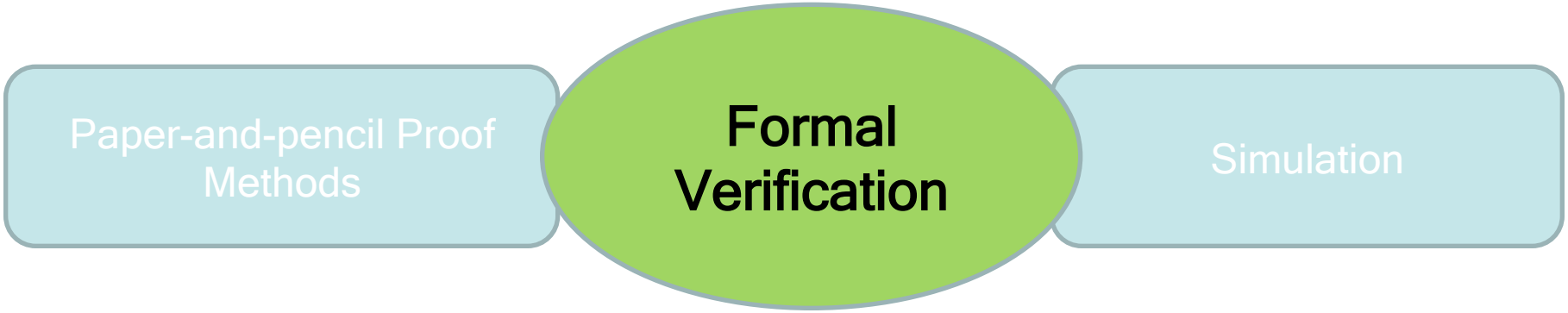
- Faulty systems can be disastrous

- Mars Climate Orbiter
  - Conversion error from English units to metric units
  - Resulted in a loss of US $125M in 1999



Formal Verification

- Bridges the gap between Paper-and-pencil proof methods and simulation



- Shares their advantages
  - As precise as a mathematical proof can be
  - Computers are used for book-keeping
- Not as straightforward to use as testing

Formal Verification

- Construct a computer-based mathematical model of the system (*implementation*)
- Use mathematical methods to check if the implementation satisfies the properties of interest (*specifications*) in a computerized environment

Formal Verification Methods

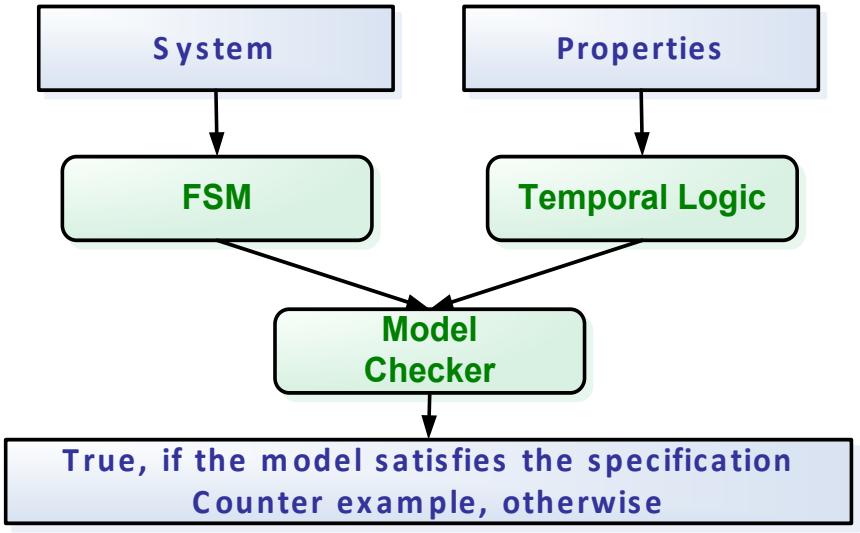
- Most widely used techniques
  - Model Checking
  - Theorem proving

Model Checking

- Typically used for verifying Concurrent Finite-State Systems
  - Rigorous testing is infeasible due to large number of possible scenarios

- Implementation
  - State-transition Graph

- Specifications
  - Temporal Logic



Temporal Logic

- Time is modeled as a sequence of states
- Atomic formulas associated with each state
- Temporal connectives allow us to refer to the future

- F: some Future state (Fp)

- X: neXt state (Xp)

- G: Globally (Gp)

- U: Until (pUq)

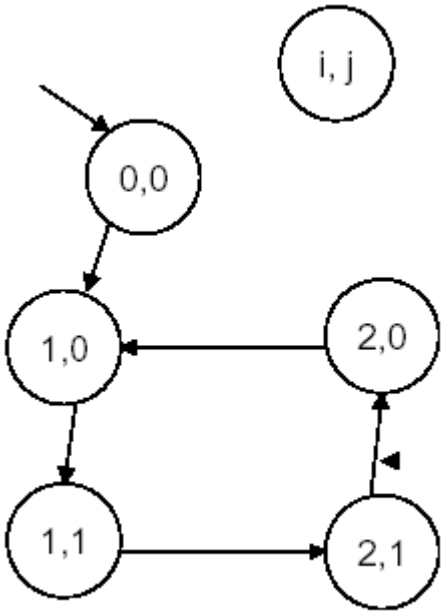
Model Checking

- The verification can be done automatically using rigorous state-exploration methods

- Counter Examples
  - In case of failing properties, counter examples are provided for debugging purposes

Model Checking -
Example – Simple Program

```
bar () {
  int i=0, j=0;
  while (i<2) {
        i++;
        j=i%2;
  }
  i=1;
}
```



- (i,j) = (0,0), (1,0), (1,1), (2,1), (2,0), (1,0)
- Property: j will eventually become 1: F(j = 1)
- Property: j is always less than or equal to 1: G(j ≤ 1)

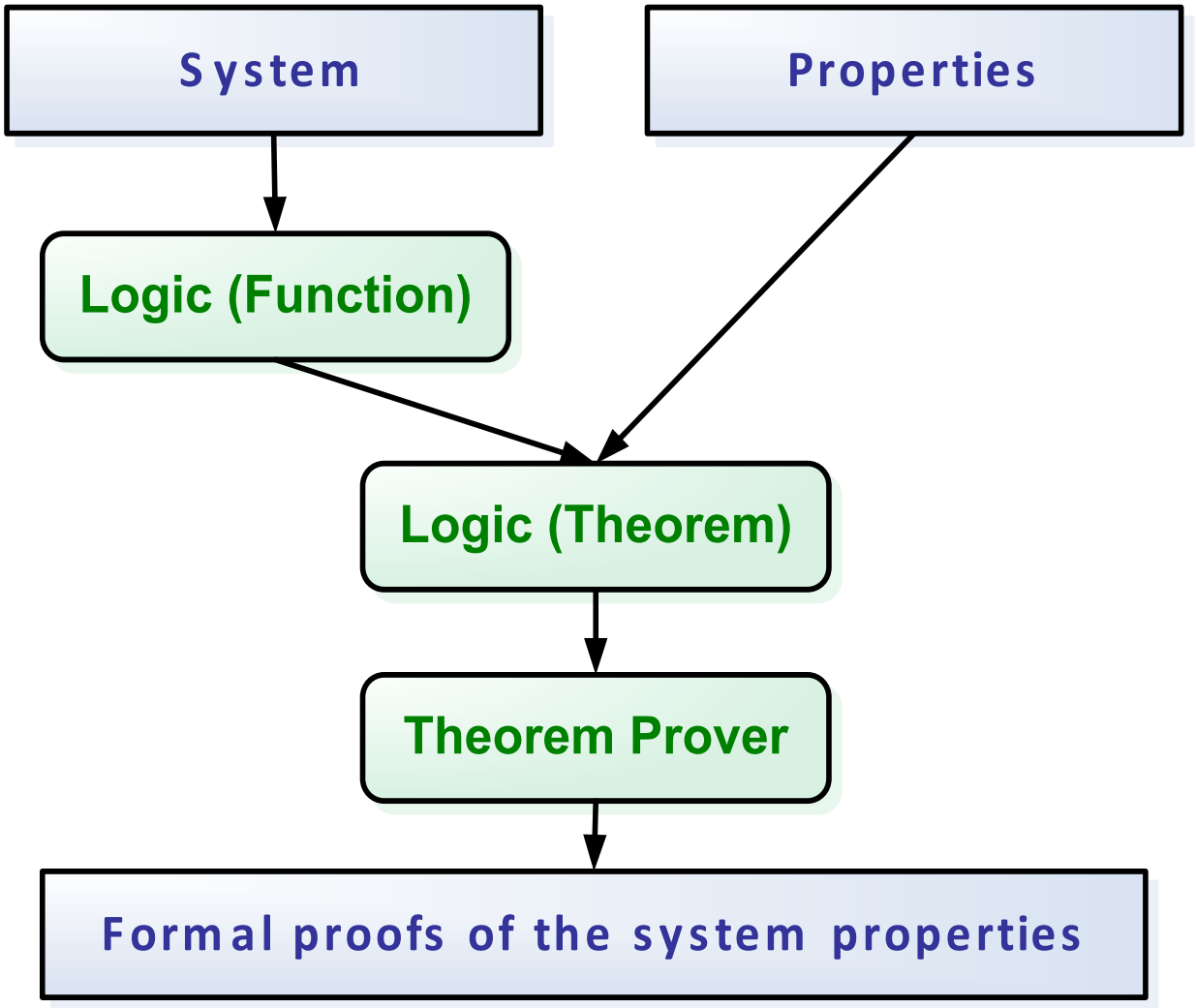
Model Checking

- Extensively used in verifying
  - Software
  - Security Protocols
  - Telecommunication Protocols
  - Digital hardware
  - Analog and mixed signal circuits
  - Microprocessors
  - Biological Systems

Model Checking

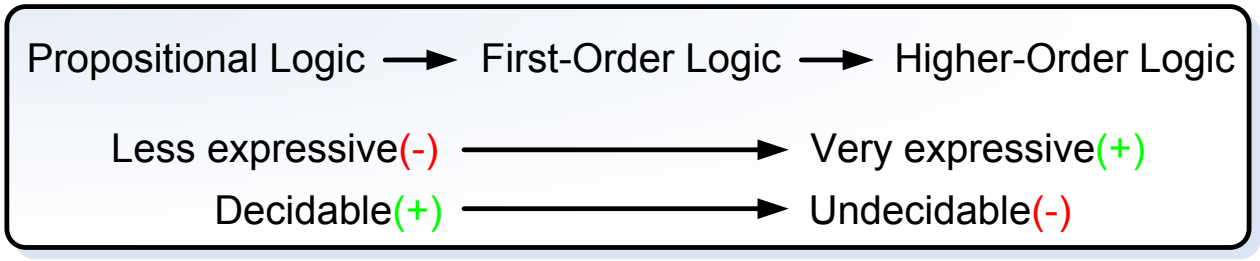
- Advantages
  - Automatic (Push button type analysis tools)
  - No proofs involved
  - Diagnostic counter examples
- Disadvantages
  - Limited expressiveness
  - State-space explosion problem
- Some commonly used Model Checking Tools
  - SPIN
  - NuSMV
  - PRISM (A probabilistic model checker)

Theorem Proving



Logic

- Study of drawing conclusions (reasoning)
  - Propositional logic
    - Supports statements that can be true or false
  - First-order logic (Predicate logic)
    - Quantification over variables (∀: For all, ∃: there exists)
  - Higher-order logic
    - Quantification over sets and functions



Theorem Prover

- A theorem prover consists of
  - A notation (Syntax)
  - A small set of fundamental axioms (facts)
    - Example: $(\neg\neg A) \Leftrightarrow A$
  - A small set of deduction rules
    - Example: Given $(A \rightarrow B)$ and A, we can deduce B
- Soundness is assured as every new theorem must be created from
  - The basic axioms and primitive inference rules
  - Any other already proved theorems (Theory Files)

Example 1: Equivalence Between Programs

```
If (!(x=y) && !(q<s)) then
    a = a + 5;
else {
    If (!(x=y)) then
        a = a - 5;
    else
        a = a +10;}
```

```
if (!a && !b) h();
else {
  if(!a) g();
  else f();
}
```

$(\texttt{if } x \texttt{ then } y \texttt{ else } z) \equiv (x \wedge y) \vee (\neg x \wedge z)$

$$(\neg a \wedge \neg b) \wedge h \vee \neg(\neg a \wedge \neg b) \wedge (\neg a \wedge g \vee a \wedge f)$$

$$\Leftrightarrow$$

$$a \wedge f \vee \neg a \wedge (b \wedge g \vee \neg b \wedge h)$$

```
If (x=y) then
    a = a + 10;
else {
    If (q<s) then
        a = a - 5;
    else
        a = a +5;}
```

```
if(a) f();
else {
  if(b) g();
  else h();
}
```

Theorem Proving

- Extensively used in verifying
  - Compilers
  - Floating-point algorithms
  - Digital Signal Processing systems
  - Optical systems
  - Wireless sensor networks
  - Kinematic Analysis of Robots

Theorem Proving

- Advantages
  - High expressiveness
  - No risk of mistakes (human errors)
  - Some parts of the proofs can be automated
- Disadvantages
  - Detailed and explicit human guidance required for verifying real-world systems
  - The state-of-the-art is limited
- Theorem Proving Tools
  - ACL2 (First-order Logic)
  - Coq (Higher-order Logic)
  - HOL (Higher-order-logic)

Applications

- Analyzing Thermal Management Algorithms for Multi-core Architectures
  - Model Checking
- Reliability Analysis
  - Theorem Proving

Analyzing Thermal Management Algorithms for Multi-core Architectures using nuXmv

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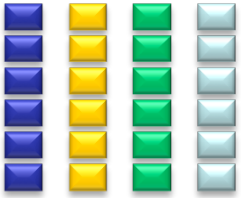
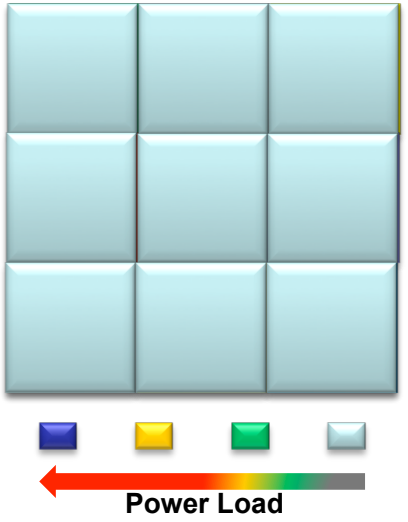
Thermal Hotspots

- Multiple-cores per chip
  - High Performance
  - Verification Re-usabilty
  - Imbalanced Task Mapping
    - Thermal Hot Spots

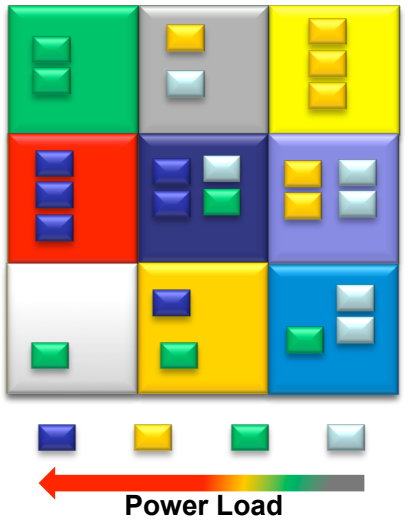




Dynamic Thermal Management

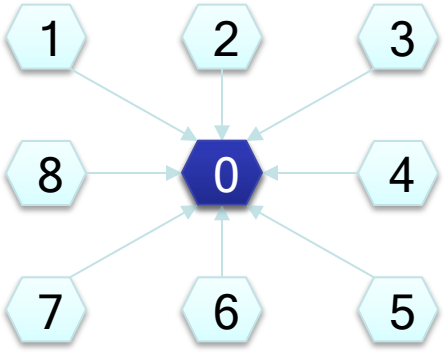
- Dynamic Thermal Management (DTM): An online scheme to manage the heating issues

- Distributes power and the resulting heat evenly by performing *task migration* from hotter regions to cooler ones



Centralized vs Decentralized DTM

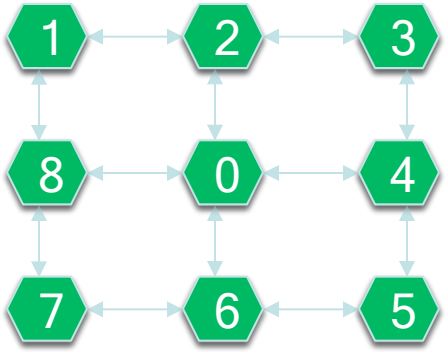
Central



Central Agent

- A central agent responsible for thermal management
- Leads to significant communication overhead

Decentralized



Distributed Agents

- Every node acts as an agent managing immediate neighbors only

Analysis of Decentralized DTM Techniques

- To ensure that the given DTM technique would reach the no hot-spot (stability) state
- To judge the effect of various parameters on time to stability of the DTM technique
- The decentralized nature makes the analysis quite complex

Decentralized DTM Analysis Techniques

Analytical

- Completeness

- Human-Error Prone
- Practically impossible to analyze large and complex systems

Simulation

- Quick insights about the working of the given dDTM scheme

- In-exhaustive
- Impossible to predict all corner cases

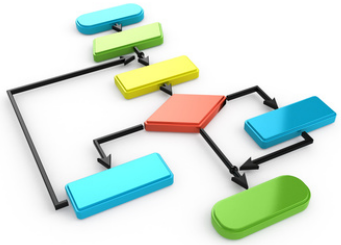
Emulation

- Real time Hardware/Software system interaction

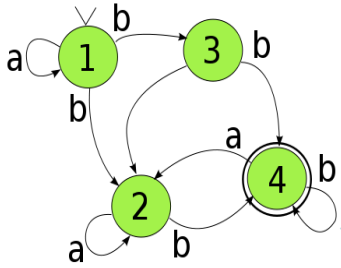
- High Cost
- In-exhaustive
- Impossible to predict all corner cases

- None of these techniques offers a complete and accurate analysis

Proposed Verification Methodology



DTM Algorithm



State Machine

Working Description

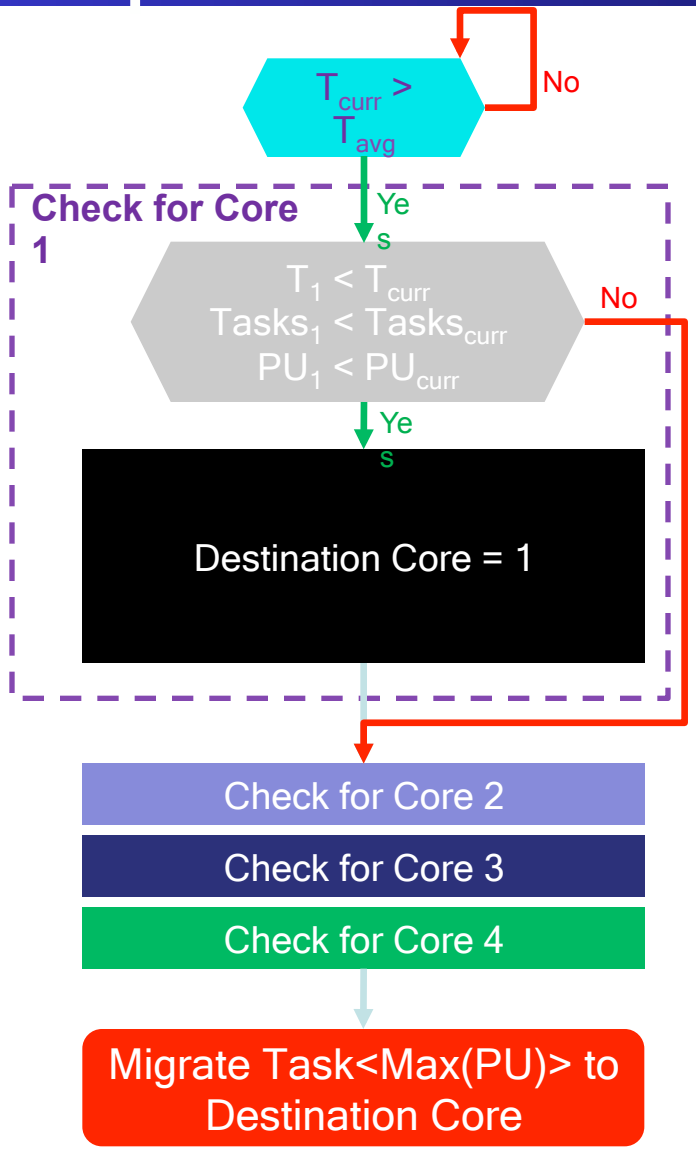
Properties

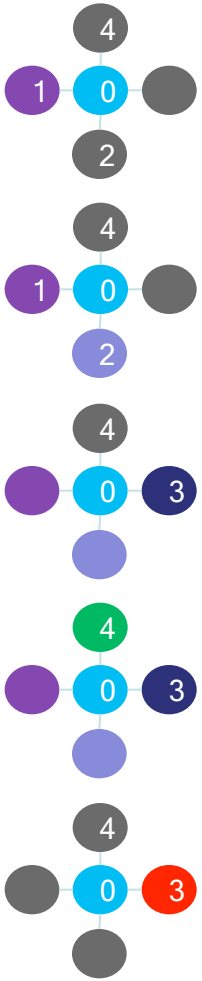
Property True / False
with Counter Example

nuXmv

- New set of types, namely 'Integer' and 'Reals'
- New model checking algorithms
  - SMT-based techniques
  - Abstraction based techniques

Task Migration for Thermal Hot Spot Reduction Algorithm



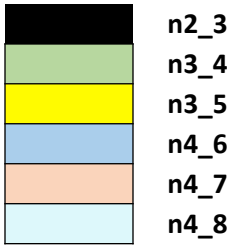


- Two Issues identified during the formal modeling phase
  - Ambiguity when multiple cores are trying to migrate the task on the same core
  - A core already with a hot spot can be assigned new tasks

Formal Modeling in nuXmv

- 6 Core Types were modeled

0	1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16	17
18	19	20	21	22	23	24	25	26
27	28	29	30	31	32	33	34	35
36	37	38	39	40	41	42	43	44
45	46	47	48	49	50	51	52	53
54	55	56	57	58	59	60	61	62
63	64	65	66	67	68	69	70	71
72	73	74	75	76	77	78	79	80



0	1	2	3	4
9	10	11	12	13
18	19	20	21	22
27	28	29	30	31
36	37	38	39	40

np_q: p: number of direct neighbors and q: number of second-degree neighbors

- Non-Deterministic Assignments
  - Number of Tasks in a core (0-14)
  - Number of Power units/Task (1-4)
- Temperature, modeled as a real number, is directly proportional to the number of power units

Stability

- Eventually Stability is achieved
  - We always eventually reach a condition when the temperature of all the cores will eventually be less than or equal to the estimated average temperature of the chip.

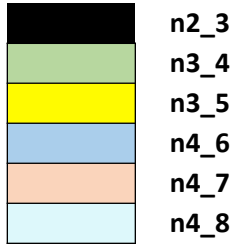
$$\mathbf{GF}(core_0.T_0 <= core_0.T_{avg}\ \&\ core_1.T_0 <= core_1.T_{avg}\ \&\ \ldots\ldots\ \&\ core_{80}.T_0 <= core_{80}.T_{avg})$$

Time to Stability

Experimental Setup					n2_3	n3_4	n3_5	n4_6	n4_7	n4_8
T0	T1	T2	T3	T4	0	1	2	10	11	20
56					49	52	51	63	67	62
56	56				53	53	54	61	63	61
56		56			61	57	55	71	73	69
56			56		-	55	57	72	79	80
56				56	-	-	-	89	87	91
56	56	56			63	59	53	94	92	85
56	56		56		-	61	54	91	98	96
56	56			56	-	57	51	107	114	113
56		56	56		-	62	59	117	112	105
56		56		56	-	58	57	103	109	112
56			56	56	-	60	65	100	105	102
56	56	56	56		-	-	-	115	120	117
56	56	56		56	-	-	-	119	124	118
56	56		56	56	-	-	-	121	125	131
56		56	56	56	-	-	-	132	129	141
56	56	56	56	56	-	-	-	129	131	137

Core number

0	1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16	17
18	19	20	21	22	23	24	25	26
27	28	29	30	31	32	33	34	35
36	37	38	39	40	41	42	43	44
45	46	47	48	49	50	51	52	53
54	55	56	57	58	59	60	61	62
63	64	65	66	67	68	69	70	71
72	73	74	75	76	77	78	79	80



Time To Stability

Experimental Setup					n2_3	n3_4	n3_5	n4_6	n4_7	n4_8
T0	T1	T2	T3	T4	0	1	2	10	11	20
56					49	52	51	63	67	62
56	56				53	53	54	61	63	61
56		56			61	57	55	71	73	69
56			56		-	55	57	72	79	80
56				56	-	-	-	89	87	91
56	56	56			63	59	53	94	92	85
56	56		56		-	61	54	91	98	96
56	56			56	-	57	51	107	114	113
56		56	56		-	62	59	117	112	105
56		56		56	-	58	57	103	109	112
56			56	56	-	60	65	100	105	102
56	56	56	56		-	-	-	115	120	117
56	56	56		56	-	-	-	119	124	118
56	56		56	56	-	-	-	121	125	131
56		56	56	56	-	-	-	132	129	141
56	56	56	56	56	-	-	-	129	131	137

0	1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16	17
18	19	20	21	22	23	24	25	26
27	28	29	30	31	32	33	34	35
36	37	38	39	40	41	42	43	44
45	46	47	48	49	50	51	52	53
54	55	56	57	58	59	60	61	62
63	64	65	66	67	68	69	70	71
72	73	74	75	76	77	78	79	80

Temperature of core is 56

Temperature of core < 56

0	1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16	17
18	19	20	21	22	23	24	25	26
27	28	29	30	31	32	33	34	35
36	37	38	39	40	41	42	43	44
45	46	47	48	49	50	51	52	53
54	55	56	57	58	59	60	61	62
63	64	65	66	67	68	69	70	71
72	73	74	75	76	77	78	79	80

T0, T1 T2 T3 and **T4** represents the temperature of core and its respective neighbors

Formal Reliability Analysis

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Defining Futures

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Concordia University
Hardware Verification Group

Reliability

- A measure of the continuity of service
- Probability that a system performs its intended function until some time t without failing

$$R(t) = Pr(X > t) = 1 - Pr(X \leq t) = 1 - F_X(t)$$

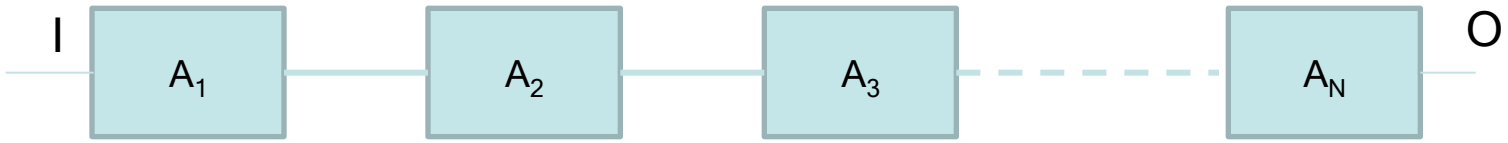
- X: random variable that models the time to failure of the system
  - Commonly used Distributions
    - Exponential
    - Weibul

Reliability Block Diagrams

- Used to asses the reliability of a complex system
  - Partition the system into sub-blocks and connectors (RBD)
  - Find the failure rates of sub-blocks
  - Judge the failure characteristics of the overall system
    - failure rates of individual components
    - RBD configuration

- The overall system failure happens if all the paths for successful execution fail
  - Add more parallelism to meet the reliability goals

Series Reliability Block Diagram

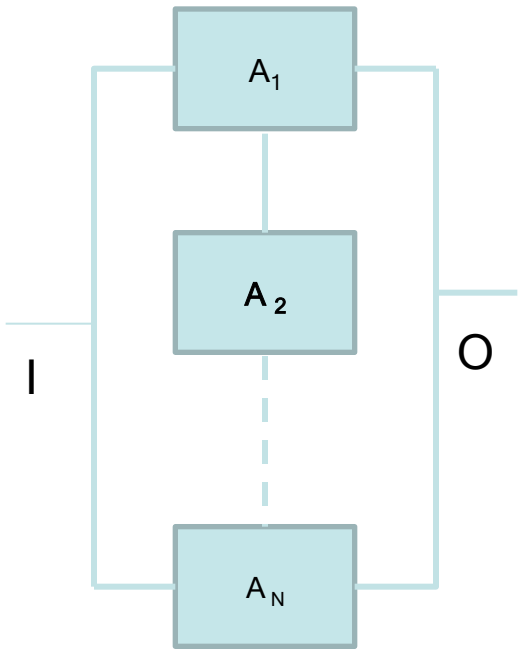


- The overall system is reliable only if all of its components are functioning reliably

$$R_{series}(t) = Pr(A_1(t) \cap A_2(t) \cap A_3(t) \cdots \cap A_N(t)) = \prod_{i=1}^{N} R_i(t)$$

Where $A_i(t)$ are the mutually independent events corresponding to *i serially-connected* components

Parallel Reliability Block Diagrams

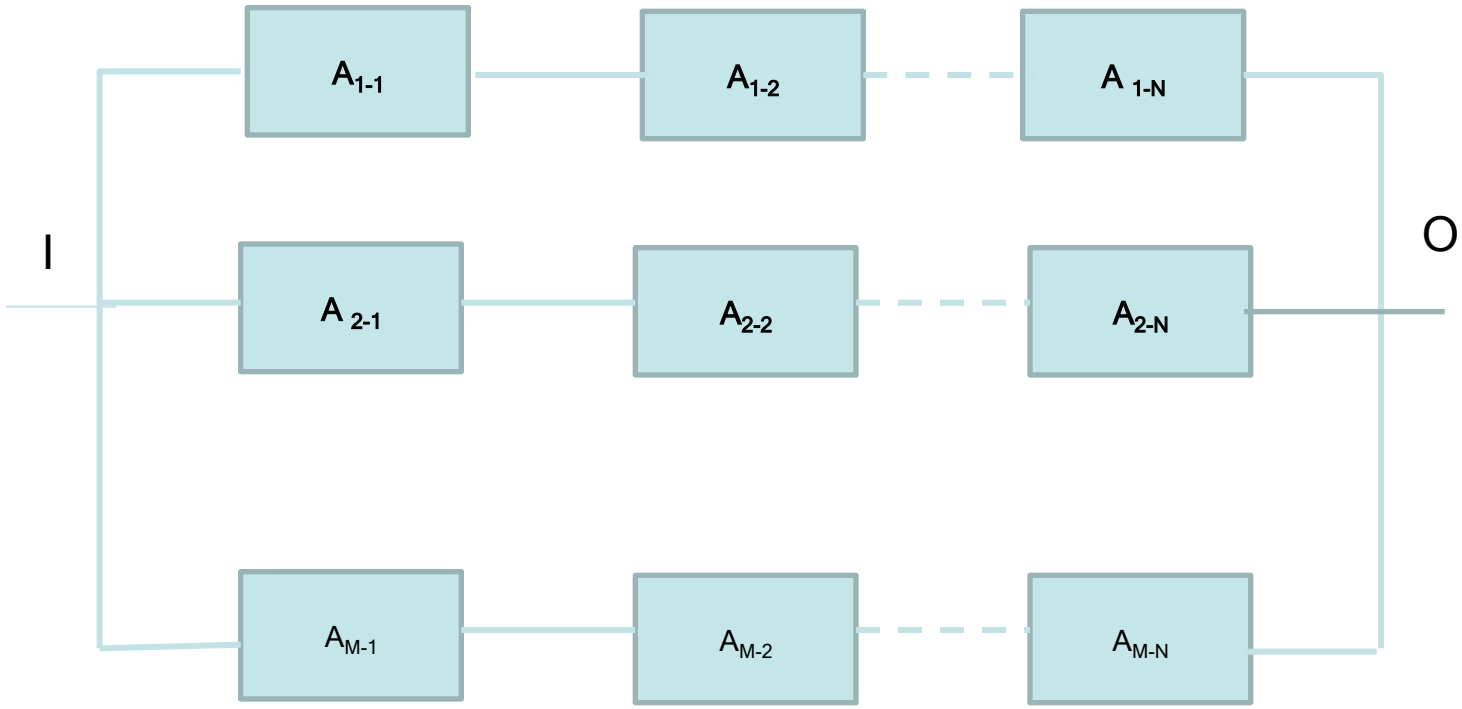


- The overall system reliability mainly depends on the component with the maximum reliability

$$R_{parallel}(t) = Pr(A_1 \cup A_2 \cup A_3 \cdots \cup A_N) = 1 - \prod_{i=1}^{N}(1 - R_i(t))$$

Where $A_i(t)$ are the mutually independent events corresponding to i *parallel-connected* components

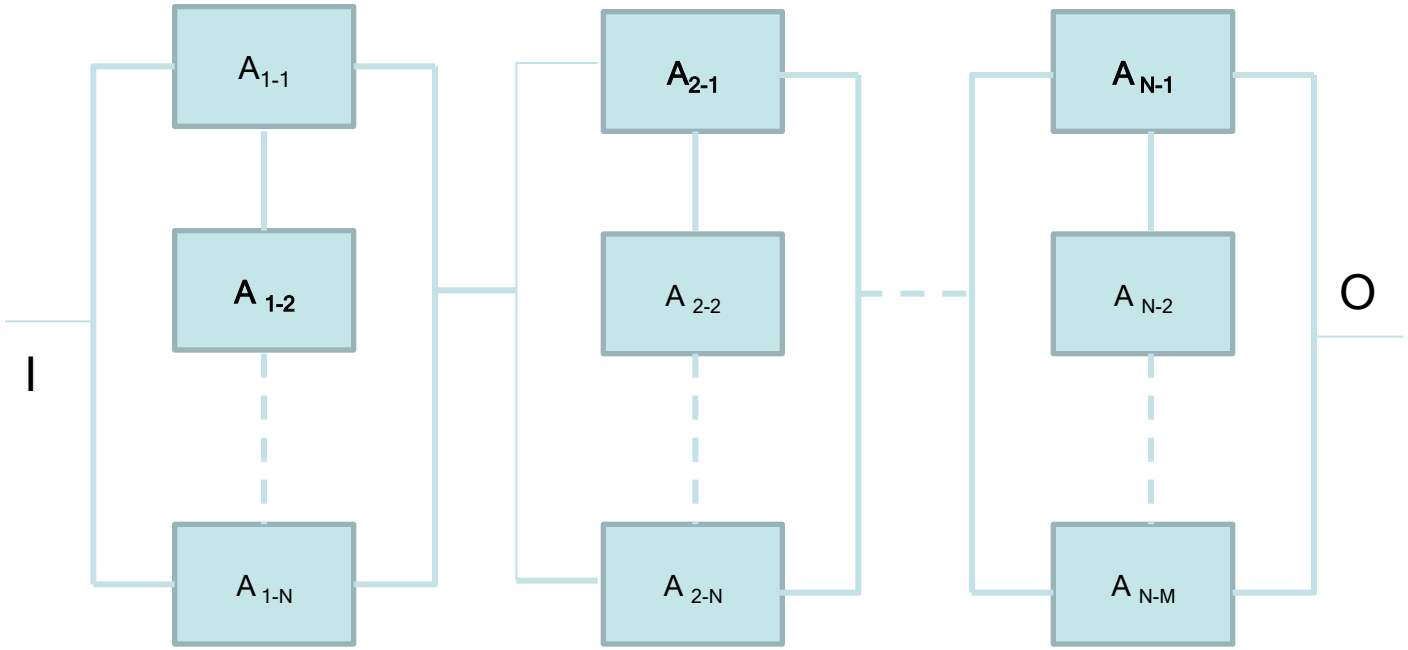
Series-Parallel Reliability Block Diagrams



- A combination of both series and parallel RBD

$$R_{Series-Parallel} = Pr(\bigcap_{i=1}^{N} \bigcup_{j=1}^{M} A_{ij}) = \prod_{i=1}^{N}(1 - \prod_{j=1}^{M}(1 - R_{ij}(t)))$$

Parallel- Series Reliability Block Diagrams



$$R_{Parallel-Series} = Pr(\bigcup_{i=1}^{M} \bigcap_{j=1}^{N} A_{ij}) = 1 - \prod_{i=1}^{M}(1 - \prod_{j=1}^{N}(R_{ij}(t)))$$

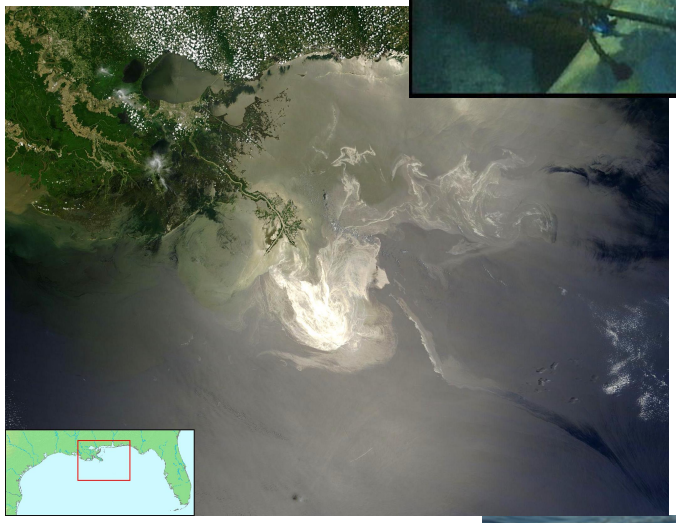
Example: Reliability Analysis of Oil and Gas Pipelines

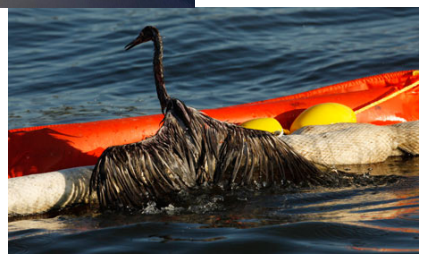
- There are tens of thousands of miles long oil and gas pipelines around the world
  - Some of them aging and are becoming more and more susceptible to failures

- Very important to rigorously analyze their reliability and thus plan timely replacements and maintenance

Methane Gas Leakage on the Deepwater Horizon oil rig – April 2010

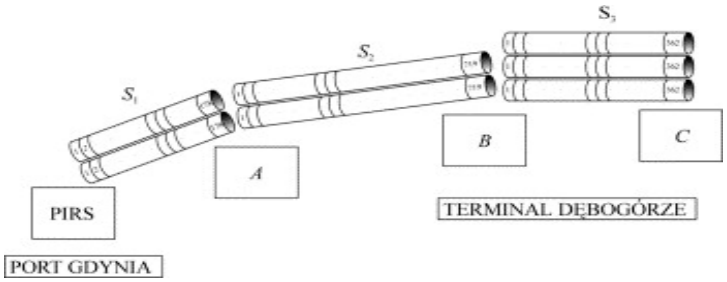
- Killed 11 workers
- Destroyed and sank the rig
- Caused millions of gallons of oil to pour into the Gulf of Mexico
- Took three months to bring the situation under control
- Damage to marine and wildlife habitats

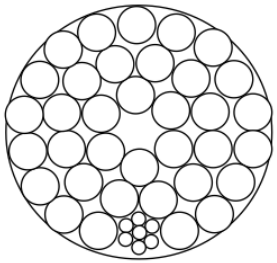


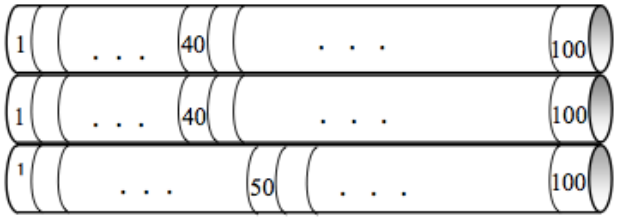


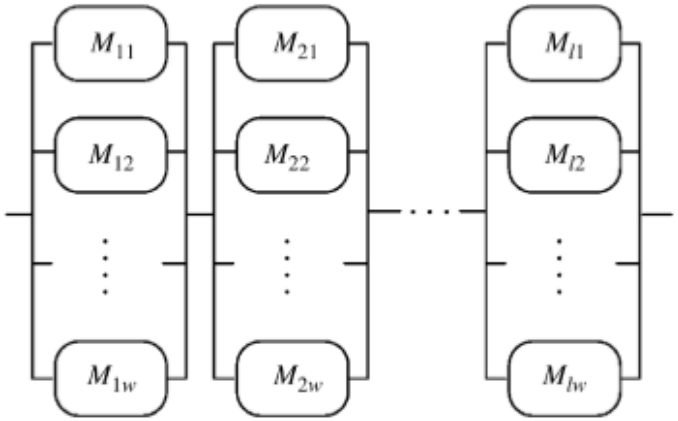
Reliability Analysis of Pipelines

- Partitioning the given pipeline into segments and constructing its equivalent reliability block diagram (RBD)





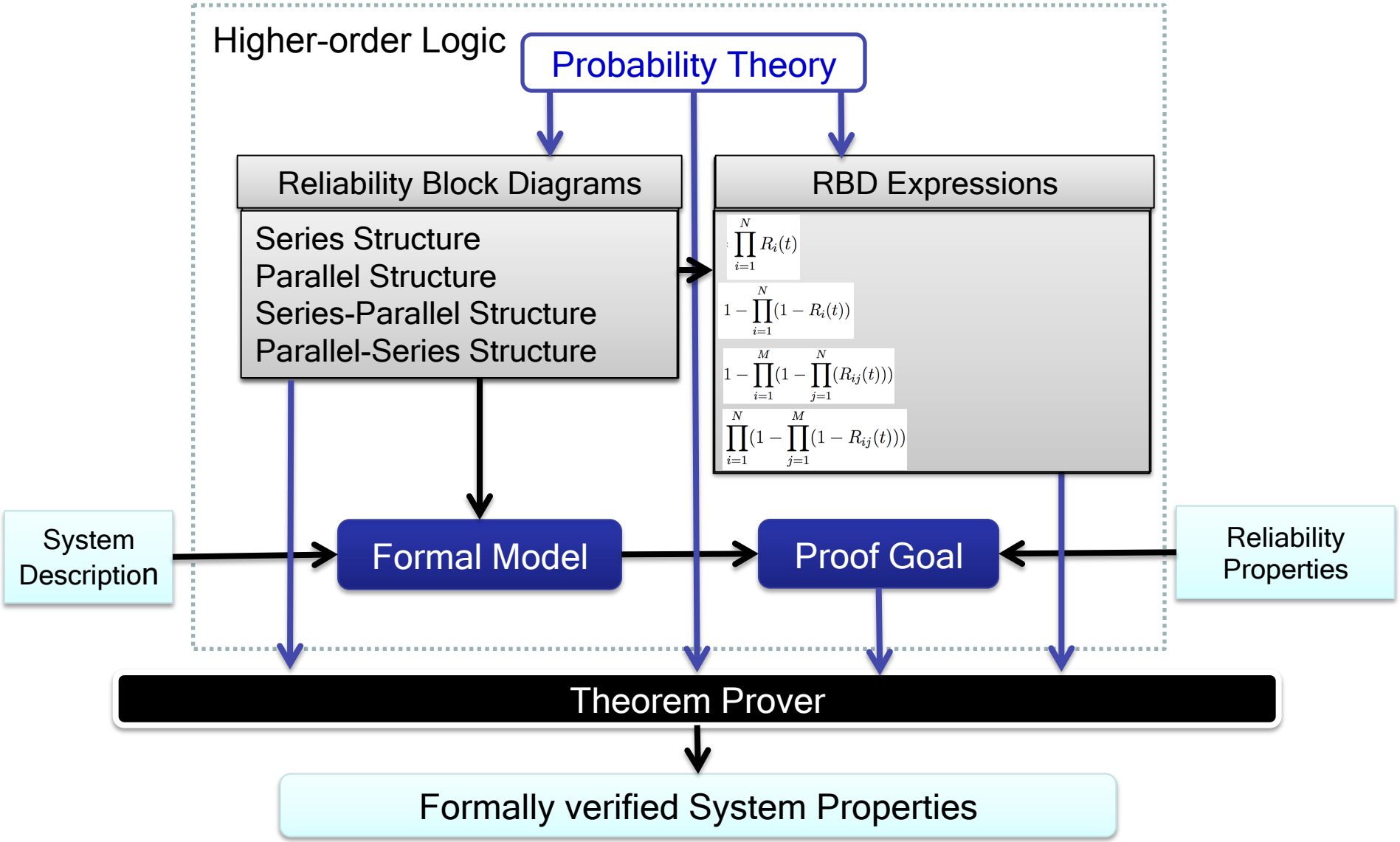




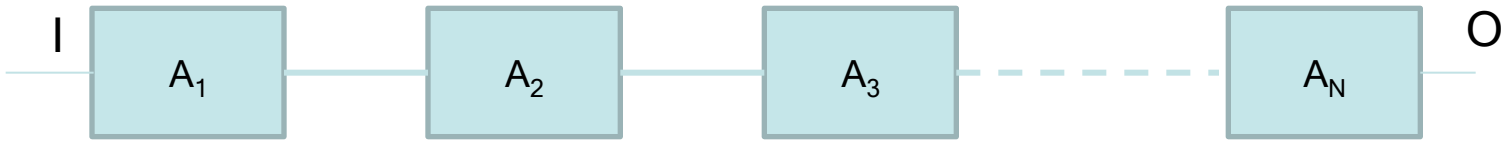
Analysis Techniques

Criteria	Paper-and-Pencil Proof	Simulation	Model Checking	Higher-order-logic Proof Assistants
Expressiveness	☑	☑	☒	☑
Accuracy	☑?	☒	☑	☑
Automation	☒	☑	☑	☑

Formal Reliability Analysis Methodology



Formalization of Series RBD



Definition 1: `⊢ ∀ p L. series_struct p L = inter_list p L`

$$R_{series}(t) = Pr(A_1(t) \cap A_2(t) \cap A_3(t) \cdots \cap A_N(t)) = \prod_{i=1}^{N} R_i(t)$$

Theorem 1:
```
⊢ ∀ p L. prob_space p ∧ (events p = POW (p_space p)) ∧
    1 ≤ LENGTH L ∧ mutual_indep p L ⇒
      (prob p (series_struct p L) = list_prod (list_prob p L))
```

Other RBDs

Definition 2: $\vdash \forall$ L . parallel_struct L = union_list L

Theorem 2: $\vdash \forall$ p L. (prob_space p) $\wedge$
(events p = POW (p_space p)) $\wedge$
(1 $\leq$ LENGTH L) $\wedge$ (mutual_indep p L) $\wedge$
($\forall$ x'. MEM x' L $\Rightarrow$ x' $\in$ events p) $\Rightarrow$
(prob p (parallel_struct L) =
1 - list_prod (one_minus_list (list_prob p L)))

Definition 3: $\vdash \forall$ p L. parallel_series_struct p L =
parallel_struct (list_inter_list p L)

Theorem 3: $\vdash \forall$p L. (prob_space p) $\wedge$
(events p = POW (p_space p)) $\wedge$
($\forall$z. MEM z L $\Rightarrow$ ~NULL z) $\wedge$ (mutual_indep p (FLAT L)) $\wedge$
($\forall$x'. MEM x'(FLAT L) $\Rightarrow$ x' $\in$ events p) $\Rightarrow$
(prob p (parallel_series_struct p L) =
1 − list_prod (one_minus_list(list_rel_list_prod p L)))

Other RBDs

Definition 4: $\vdash \forall$ p L. series_parallel_struct p L = series_struct p (list_union_list L)

Theorem 4: $\vdash \forall$ p L. (prob_space p) $\wedge$
(events p = POW (p_space p)) $\wedge$
($\forall$z. MEM z L $\Rightarrow$ $\sim$NULL z) $\wedge$ (mutual_indep p (FLAT L)) $\wedge$
($\forall$x'. MEM x'(FLAT L) $\Rightarrow$ x' $\in$ events p) $\Rightarrow$
(prob p (series_parallel_struct p L) =
list_prod (one_minus_list (list_compl_rel_list_prod p L)))

- Formalization took about 300 man-hours and more then 8000 lines of HOL4 proof script

Reliability Analysis Case Studies

- Virtual Data Centers
- Oil and Gas Pipelines
- Telecommunication Networks
- Smart Grids

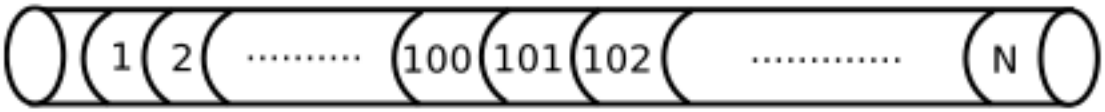
Case Study: A Simple Pipeline Structure

- A 60 segment pipeline with exponentially distributed failure rates, can be sub-divided into 3 categories according to their failure rates
  - 30 segments with $\lambda = 0.0025$
  - 20 segments with $\lambda = 0.0023$
  - 10 segments with $\lambda = 0.015$

Zhang, Z., Shao, B.: Reliability Evaluation of Different Pipe Section in Different Period. In: Service Operations and Logistics, and Informatics, IEEE (2008) 1779- 1782

Case Study: Formal Analysis

- The proposed approach for reliability analysis of pipelines allows us to formally verify generic expressions involving
  - any number of segments
  - arbitrary failure rates



Case Study: Formal Verification

Theorem 6: *Series Pipeline System*

```
⊢ ∀ p L x C. prob_space p ∧ (events p = POW (p_space p)) ∧
     0 ≤ x ∧ 2 ≤ LENGTH (rel_event_list p L x) ∧
     mutual_indep p (rel_event_list p L x) ∧
      list_exp p C L ∧ (LENGTH C = LENGTH L) ⇒
      (pipeline p (rel_event_list p L x) = exp (-list_sum C * x))
```

- The reasoning was very straightforward as we built upon our foundational results
  - About 100 lines of HOL code
- All the variables are universally quantified
  - Specialized to obtain the reliability of any given pipeline with series components
- Guaranteed correctness due to the involvement of a sound theorem prover
  - All the required assumptions for the validity of the result are accompanying the theorem

Objectives

- Formal Verification
  - Why do we need it?
    - Exact Answers (Useful for the analysis of Safety-critical systems)
  - What is it?
    - Mathematically reason about properties of a system using a computer-based tool
  - How can we apply it for the analysis of real-world systems?
    - Mathematically model the system (Implementation)
    - Mathematically model the desired properties (Specification)
    - Use tool support to mathematically prove that the specification holds for your implementation

Conclusions

- Formal Verification is not an alternative to simulation
  - Both techniques have to play together for a successful analysis framework
- Less critical sections of the system
  - Simulation/Testing
- Critical sections of the system that can be expressed as a FSM
  - Model Checking
- Critical sections of the system that cannot be handled by Model Checking
  - Theorem Proving

Thanks!

- For More Information
  - Visit our website
    - http://save.seecs.nust.edu.pk
  - Contact
    - osman.hasan@seecs.nust.edu.pk