Untitled

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1 Machine Learning HW 3

1.0.1 Li Xingxuan 1002189

1.1 Problem 1: Matrix Factorization

In [78]:
```
import numpy as np
```

In [79]:
```
def matrix_factorization(R, P, Q, K, steps=5000, alpha=0.0002, beta=0.02):
    Q = Q.T
    for step in range(steps):
        for i in range(len(R)):
            for j in range(len(R[i])):
                if R[i][j] > 0:
                    eij = R[i][j] - np.dot(P[i,:],Q[:,j])
                    for k in range(K):
                        P[i][k] = P[i][k] + alpha * (2 * eij * Q[k][j] - beta * P[i][k
                        Q[k][j] = Q[k][j] + alpha * (2 * eij * P[i][k] - beta * Q[k][j
        eR = np.dot(P,Q)
        e = 0
        for i in range(len(R)):
            for j in range(len(R[i])):
                if R[i][j] > 0:
                    e = e + pow(R[i][j] - np.dot(P[i,:],Q[:,j]), 2)
                    for k in range(K):
                        e = e + (beta/2) * ( pow(P[i][k],2) + pow(Q[k][j],2) )
        if e < 0.001:
            break
    return P, Q.T
```

In [80]:
```
R = [
     [0,1,0],
     [1,0,1],
     [0,1,2],
    ]

R = np.array(R)
```

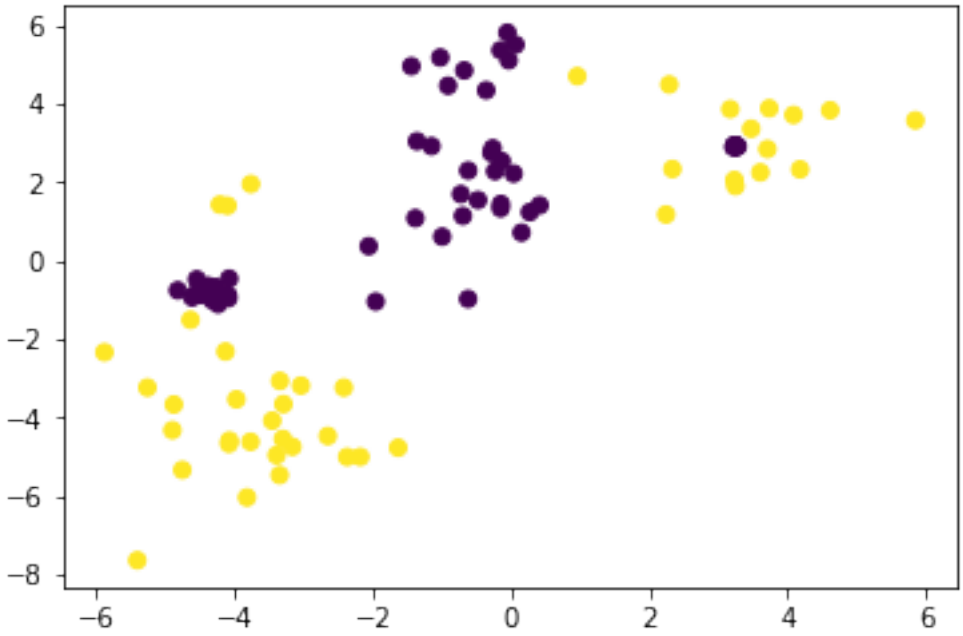
```
N = len(R)
M = len(R[0])
K = 2

P = np.random.rand(N,K)
Q = np.random.rand(M,K)

nP, nQ = matrix_factorization(R, P, Q, K)
print('A')
print(np.dot(nP, nQ.T))
print('U')
print(nP)
print('V')
print(nQ)
```

```
A
[[1.30635811 0.92144842 1.66127335]
 [0.85574216 0.6608885  1.12041975]
 [1.49200682 1.08095926 1.91340807]]
U
[[0.72308742 0.97381528]
 [0.80478295 0.30240525]
 [0.99094166 0.94492488]]
V
[[0.775665   0.76552969]
 [0.64584734 0.46666379]
 [1.04187208 0.93232133]]
```

```
In [82]: # Reference: http://www.quuxlabs.com/blog/2010/09/matrix-factorization-a-simple-tutor
```

1.2 Problem 2: Support Vector Machines

```
In [34]: %matplotlib inline
         import numpy as np
         import matplotlib.pyplot as plt
         csv = 'https://www.dropbox.com/s/wt45tvn9ig3o7vu/kernel.csv?dl=1'
         data = np.genfromtxt(csv, delimiter=',')
         X = data[:, 1:]
         Y = data[:,0]
         plt.scatter(X[:,0],X[:,1],c=Y)
         plt.show()
```



1.2.1 (a)

```
In [35]: from sklearn.svm import SVC
         clf = SVC(gamma=0.5, kernel='rbf')
         clf.fit(X, Y)
```

```
Out[35]: SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0,
           decision_function_shape='ovr', degree=3, gamma=0.5, kernel='rbf',
           max_iter=-1, probability=False, random_state=None, shrinking=True,
           tol=0.001, verbose=False)
```

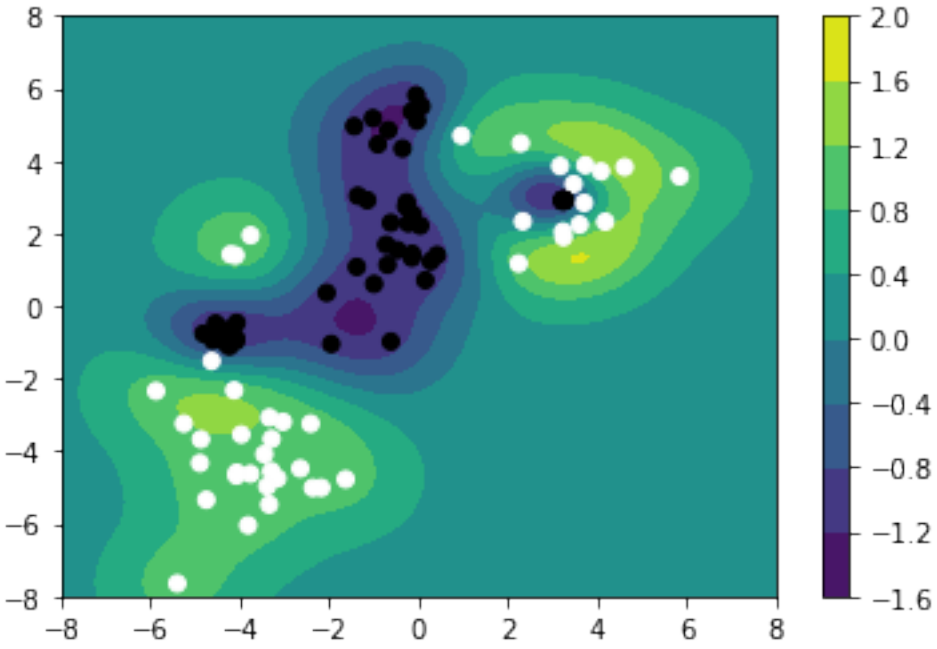
1.2.2 (b)

```
In [75]: def decision(x1, x2, clf):
             x = np.array([x1,x2])
             value = clf.decision_function([x])
             return value

         # decision(1,2,clf)
```

1.2.3 (c)

```
In [37]: vdecision = np.vectorize(decision, excluded=[2])
         x1list = np.linspace(-8.0,8.0,100)
         x2list = np.linspace(-8.0,8.0,100)
         X1, X2 = np.meshgrid(x1list,x2list)
```

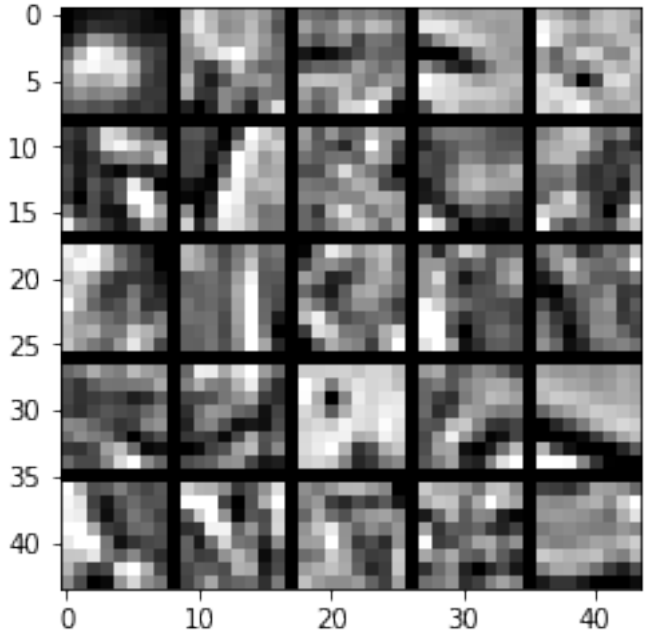
```
Z = vdecision(X1,X2,clf)
cp = plt.contourf(X1,X2,Z)
plt.colorbar(cp)
plt.scatter(X[:,0],X[:,1],c=Y,cmap='gray')
plt.show()
```



1.3 Problem 3: Deep Learning

In [57]:
```
%matplotlib inline
import numpy as np
from numpy.linalg import norm
import matplotlib.pyplot as plt
from scipy.optimize import fmin_l_bfgs_b as minimize
from utils import normalize, tile_raster_images, sigmoid
from utils import ravelParameters, unravelParameters
from utils import initializeParameters
from utils import computeNumericalGradient
nV = 8*8
nH = 25
dW = 0.0001
sW = 3
# number of visible units
# number of hidden units
# weight decay term
# sparsity penalty term
```

```
npy = 'images.npy'
X = normalize(np.load(npy))
plt.imshow(tile_raster_images(X=X,
    img_shape=(8,8),tile_shape=(5,5),
    tile_spacing=(1,1)),cmap='gray')
plt.show()
```



1.3.1 (a)

```
In [83]: def sparseAutoencoderCost(theta,nV,nH,dW,sW,X):
             W1,W2,b1,b2 = unravelParameters(theta,nH,nV)
             n = X.shape[0]
             one = np.ones((n,1))

             z2 = np.dot(X,W1) + np.dot(one,(b1.T))
             a2 = sigmoid(z2)
             z3 = np.dot(a2,W2) + np.dot(one,(b2.T))
             a3 = sigmoid(z3)

             eps = a3-X
             loss = (norm(eps)**2)/(2*n)
             decay = (norm(W1)**2 + norm(W2)**2)/2

             # Compute sparsity terms and total cost
             rho = 0.01
             a2mean = np.mean(a2,axis=0).reshape(nH,1)
```

```
    kl = np.sum(rho*np.log(rho/a2mean)+\
         (1-rho)*np.log((1-rho)/(1-a2mean)))
    dkl = -rho/a2mean+(1-rho)/(1-a2mean)
    cost = loss+dW*decay+sW*kl

    d3 = eps*a3*(1-a3)
    d2 = (sW*dkl.T+np.dot(d3,W2.T))*a2*(1-a2)
    W1grad = np.dot(X.T, d2)/n + dW*W1
    W2grad = np.dot(a2.T, d3)/n + dW*W2
    b1grad = np.dot(d2.T,one)/n
    b2grad = np.dot(d3.T,one)/n

    grad = ravelParameters(W1grad,W2grad,b1grad,b2grad)
    print(' .',end="")
    return cost,grad
```

```
In [85]: theta = initializeParameters(nH,nV)
         cost,grad = sparseAutoencoderCost(theta,nV,nH,dW,sW,X)
         print(cost, grad)
```

```
 .49.04830250632397 [ 0.77057418  0.81956205  0.78870559 ... -0.0053743   0.00737526
 -0.03224674]
```

1.3.2 (b)

```
In [66]: print('\nComparing numerical gradient with backdrop gradient')
         num_coords = 5
         indices = np.random.choice(theta.size,num_coords,replace=False)
         numgrad = computeNumericalGradient(lambda t:
                 sparseAutoencoderCost(t,nV,nH,dW,sW,X)[0],theta,indices)
         subnumgrad = numgrad[indices]
         subgrad = grad[indices]
         diff = norm(subnumgrad-subgrad)/norm(subnumgrad+subgrad)
         print('\n',np.array([subnumgrad,subgrad]).T)
         print('Relative difference: ', diff)

         if diff < 10**(-9):
             print("Pass")
         else:
             print("Too large")
```

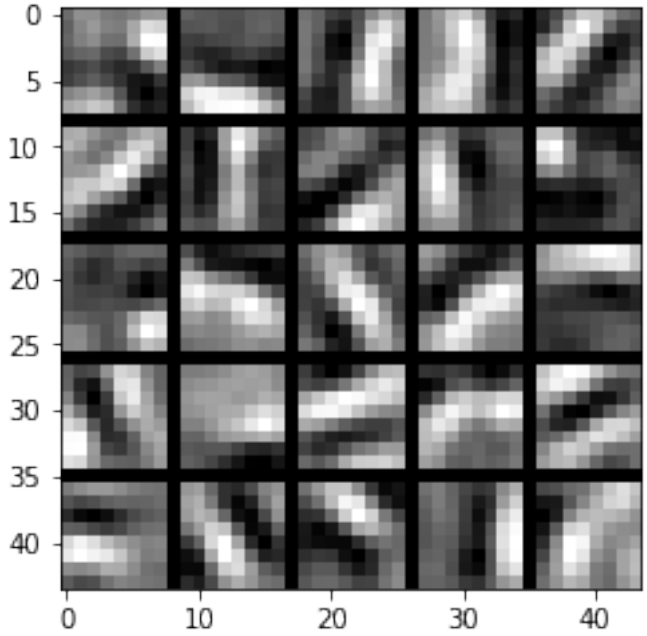
```
Comparing numerical gradient with backdrop gradient
 . . . . . .
 [[ 0.65180611  0.65180611]
 [ 0.51784787  0.51784787]
 [ 0.53142294  0.53142294]
 [ 0.92930343  0.92930343]
```

```
 [-0.01449782 -0.01449782]]
Relative difference:  1.3491014434412139e-10
Pass
```

1.3.3 (c)

```
In [69]: print('\nTraining neural network')
         theta = initializeParameters(nH,nV)
         opttheta,cost,messages = minimize(sparseAutoencoderCost,
                 theta,fprime=None,maxiter=400,args=(nV,nH,dW,sW,X))
         W1,W2,b1,b2 = unravelParameters(opttheta,nH,nV)
         plt.imshow(tile_raster_images(X=W1.T,
                 img_shape=(8,8),tile_shape=(5,5),
                 tile_spacing=(1,1)),cmap='gray')
         plt.show()
```

```
Training neural network
 . . . . . .
```



4. Proof of Two Equations

Date No.

(a) minimize $\frac{1}{n}\sum_{w\in W} -n(w)\log(\theta_w)$

subject to $\theta_w \geq 0$ for all $w \in W$, $\sum_{w\in W}\theta_w = 1$

~~$\nabla L_n(\theta) = 0 \Rightarrow$~~ $\hat{\theta}_w = \frac{n(w)}{\sum_{w'\in W} n(w')}$

Yes. If a dictionary is used instead of W, the denominator is different. So it matters.

(b) $L_n(\mu, \sigma^2) = \frac{d}{2}\log(2\pi\sigma^2) + \frac{1}{2n\sigma^2}\sum_{x\in S}\|x-\mu\|^2$

$$\frac{\partial L}{\partial \mu} = \frac{1}{2n\sigma^2}\sum_{x\in S} 2(x-\mu) = 0$$

$$\therefore \sum_{x\in S}(x-\mu) = 0$$

$$\therefore n\mu = \sum_{x\in S} x$$

$$\therefore \mu = \frac{1}{n}\sum_{x\in S} x$$

$$\frac{\partial L}{\partial \sigma} = \frac{d\,4\pi\sigma}{4\pi\sigma^2} + \frac{1}{n\sigma^3}\sum_{x\in S}\|x-\mu\|^2 = 0$$

$$\therefore \frac{d}{\sigma} = \frac{\sum_{x\in S}\|x-\mu\|^2}{n\sigma^3}$$

$$\therefore \sigma^2 = \frac{\sum_{x\in S}\|x-\mu\|^2}{nd}.$$