



Two-party secure semiquantum summation against the collective-dephasing noise

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Abstract

In this paper, we propose a two-party semiquantum summation protocol, where two classical users can accomplish the summation of their private binary sequences with the assistance of a quantum semi-honest third party (TP). The term ‘semi-honest’ implies that TP cannot conspire with others but is able to implement all kinds of attacks. This protocol employs logical qubits as traveling particles to overcome the negative influence of collective–dephasing noise and need not make any two parties pre-share a random secret key. The security analysis turns out that this protocol can effectively prevent the outside attacks from Eve and the participant attacks from TP. Moreover, TP has no knowledge about the summation results.

Keywords Semiquantum summation · Logical qubit · Collective–dephasing noise · Participant attack

1 Introduction

As a fundamental branch of quantum secure computation, the problem of secure quantum summation can be described as follows: n users, P_1, P_2, \dots, P_n , who possess the private inputs X_1, X_2, \dots, X_n , respectively, want to securely compute the summation of X_1, X_2, \dots, X_n , i.e., $\text{sum}(X_1, X_2, \dots, X_n)$, on the basis that none of X_1, X_2, \dots, X_n is leaked out without being discovered. Recently, many researchers have shown their great interests on secure quantum summation so that it has been greatly developed. For instance, Heinrich not only introduced it into integration [1] but also considered quantum Boolean summation with repetitions under the worst-average setting [2]. Moreover, numerous secure quantum summation schemes [3–16]

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have appeared. However, each of these quantum summation schemes in Refs. [3–16] requires all parties to have complete quantum capabilities.

Recently, based on the well-known BB84 protocol [17], Boyer et al. [18, 19] suggested the completely novel concept named as semiquantum cryptography which does not have the demand that all parties must possess full quantum capabilities. In a semiquantum cryptography protocol, the classical party is always restricted within the four operations, i.e., sending the qubits, reordering the qubits, generating the qubits in the Z basis (i.e., $\{|0\rangle|1\rangle\}$) and measuring the qubits in the Z basis [18, 19]. It is apparent that in a semiquantum cryptography scheme, the classical party is free from the preparation and measurement of quantum superposition states and quantum entangled states. An important question naturally arises: Whether there exists semiquantum summation or not? In the year of 2021, Zhang et al. [20] gave a positive answer to this question by putting forward the first semiquantum summation scheme. Note that Zhang et al.'s scheme in Ref. [20] is the only semiquantum summation scheme at present. But it does not consider the negative influence of noise so that it is only adaptive for the ideal noiseless quantum channel. In practice, photons are inevitably affected by the fluctuation of the birefringence in optical fiber; hence, the negative influence of noise cannot be ignored. The channel noise can be considered as the collective noise, because photons travel inside a time window which is shorter than the variation of noise and are influenced by the same noise. [21] Hence, how to design a semiquantum summation scheme feasible for the collective noise quantum channel is urgent to solve.

Based on the above analysis, this paper concentrates on designing a semiquantum summation scheme immune to the collective-dephasing noise, which is one of the main kinds of collective noise. Compared with Zhang et al.'s semiquantum summation scheme [20], our scheme is more practical in reality, since it can resist the collective-dephasing noise; and our scheme has better privacy, since TP has no knowledge about the summation results.

2 Preliminary knowledge

$|0\rangle$ and $|1\rangle$ are the horizontal polarization and the vertical polarization of photon, respectively. When facing with the collective-dephasing noise, the former is kept unchanged, while the latter is turned into $e^{i\varphi}|1\rangle$, where φ is the parameter of collective-dephasing noise fluctuating with time [22]. $|0_{dp}\rangle = |01\rangle$ and $|1_{dp}\rangle = |10\rangle$ are two logical qubits immune to the collective-dephasing noise [22]. Naturally, $|\pm_{dp}\rangle = \frac{1}{\sqrt{2}}(|0_{dp}\rangle \pm |1_{dp}\rangle) = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ are also resistant against this kind of noise [23]. It is apparent that $Z_{dp} = \{|0_{dp}\rangle, |1_{dp}\rangle\}$ and $X_{dp} = \{+_{dp}\rangle, -_{dp}\rangle\}$ are two logical measuring bases under this kind of noise.

In addition, obviously, the four logical Bell entangled states defined as Eqs. (1–4) [24] are also resistant against this kind of noise, where $|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ are four Bell entangled states. After being imposed with twice Bell state measurements on the first and the third physical qubits and on the second and the fourth physical qubits, respectively, these four logical Bell states can be clearly discriminated among each other [24]. In this paper, for the sake of convenience,

this kind of quantum measurement is simply called as double Bell basis measurement.

$$\begin{aligned} |\Phi_{dp}^+\rangle_{1234} &= \frac{1}{\sqrt{2}}(|+dp\rangle|+dp\rangle + |-dp\rangle|-dp\rangle)_{1234} = \frac{1}{\sqrt{2}}(|0_{dp}\rangle|0_{dp}\rangle + |1_{dp}\rangle|1_{dp}\rangle)_{1234} \\ &= \frac{1}{\sqrt{2}}(|01\rangle|01\rangle + |10\rangle|10\rangle)_{1234} \\ &= \frac{1}{\sqrt{2}}(|00\rangle|11\rangle + |11\rangle|00\rangle)_{1324} = \frac{1}{\sqrt{2}}(|\phi^+\rangle|\phi^+\rangle - |\phi^-\rangle|\phi^-\rangle)_{1324}, \quad (1) \end{aligned}$$

$$\begin{aligned} |\Phi_{dp}^-\rangle_{1234} &= \frac{1}{\sqrt{2}}(|+dp\rangle|-dp\rangle + |-dp\rangle|+dp\rangle)_{1234} = \frac{1}{\sqrt{2}}(|0_{dp}\rangle|0_{dp}\rangle - |1_{dp}\rangle|1_{dp}\rangle)_{1234} \\ &= \frac{1}{\sqrt{2}}(|01\rangle|01\rangle - |10\rangle|10\rangle)_{1234} \\ &= \frac{1}{\sqrt{2}}(|00\rangle|11\rangle - |11\rangle|00\rangle)_{1324} = \frac{1}{\sqrt{2}}(|\phi^-\rangle|\phi^+\rangle - |\phi^+\rangle|\phi^-\rangle)_{1324}, \quad (2) \end{aligned}$$

$$\begin{aligned} |\Psi_{dp}^+\rangle_{1234} &= \frac{1}{\sqrt{2}}(|+dp\rangle|+dp\rangle - |-dp\rangle|-dp\rangle)_{1234} = \frac{1}{\sqrt{2}}(|0_{dp}\rangle|1_{dp}\rangle + |1_{dp}\rangle|0_{dp}\rangle)_{1234} \\ &= \frac{1}{\sqrt{2}}(|01\rangle|10\rangle + |10\rangle|01\rangle)_{1234} \\ &= \frac{1}{\sqrt{2}}(|01\rangle|10\rangle + |10\rangle|01\rangle)_{1324} = \frac{1}{\sqrt{2}}(|\psi^+\rangle|\psi^+\rangle - |\psi^-\rangle|\psi^-\rangle)_{1324}, \quad (3) \end{aligned}$$

$$\begin{aligned} |\Psi_{dp}^-\rangle_{1234} &= \frac{1}{\sqrt{2}}(|-dp\rangle|+dp\rangle - |+dp\rangle|-dp\rangle)_{1234} = \frac{1}{\sqrt{2}}(|0_{dp}\rangle|1_{dp}\rangle - |1_{dp}\rangle|0_{dp}\rangle)_{1234} \\ &= \frac{1}{\sqrt{2}}(|01\rangle|10\rangle - |10\rangle|01\rangle)_{1234} \\ &= \frac{1}{\sqrt{2}}(|01\rangle|10\rangle - |10\rangle|01\rangle)_{1324} = \frac{1}{\sqrt{2}}(|\psi^-\rangle|\psi^+\rangle - |\psi^+\rangle|\psi^-\rangle)_{1324}. \quad (4) \end{aligned}$$

According to Eqs. (1) and (3), it has

$$|+dp\rangle_{12}|+dp\rangle_{34} = \frac{1}{\sqrt{2}}(|\Phi_{dp}^+\rangle_{1234} + |\Psi_{dp}^+\rangle_{1234}), \quad (5)$$

which means that if $|+dp\rangle_{12}|+dp\rangle_{34}$ is performed with double Bell basis measurement, it will be collapsed into $|\phi^+\rangle_{13}|\phi^+\rangle_{24}$, $|\phi^-\rangle_{13}|\phi^-\rangle_{24}$, $|\psi^+\rangle_{13}|\psi^+\rangle_{24}$ or $|\psi^-\rangle_{13}|\psi^-\rangle_{24}$ with equal probability. Moreover, it can be obtained from Eqs. (1)-(4) that

$$|0_{dp}\rangle_{12}|0_{dp}\rangle_{34} = \frac{1}{\sqrt{2}}(|\Phi_{dp}^+\rangle_{1234} + |\Phi_{dp}^-\rangle_{1234}), \quad (6)$$

$$|0_{dp}\rangle_{12}|1_{dp}\rangle_{34} = \frac{1}{\sqrt{2}}(|\Psi_{dp}^+\rangle_{1234} + |\Psi_{dp}^-\rangle_{1234}), \quad (7)$$

$$|1_{dp}\rangle_{12}|0_{dp}\rangle_{34} = \frac{1}{\sqrt{2}}\left(|\Psi_{dp}^+\rangle_{1234} - |\Psi_{dp}^-\rangle_{1234}\right), \quad (8)$$

$$|1_{dp}\rangle_{12}|1_{dp}\rangle_{34} = \frac{1}{\sqrt{2}}\left(|\Phi_{dp}^+\rangle_{1234} - |\Phi_{dp}^-\rangle_{1234}\right). \quad (9)$$

3 The proposed two-party semiquantum summation protocol

Suppose that there are two classical users with limited quantum capabilities, Alice and Bob. Alice's private binary string is denoted as

$$X = (x_1, x_2, \dots, x_n), \quad (10)$$

while Bob's private binary string is represented as

$$Y = (y_1, y_2, \dots, y_n). \quad (11)$$

Here, $x_j, y_j \in \{0, 1\}, j = 1, 2, \dots, n$. Alice and Bob want to calculate the modulo 2 summation of their private binary strings over the collective-dephasing noise quantum channel with the aid of a semi-honest third party (TP). A semi-honest TP is supposed to have the ability to perform all kinds of attacks but is not allowed to collude with anyone else [25]. A genuine two-user secure semiquantum summation protocol with a semi-honest TP should satisfy the following requirements [5]:

- ①Correctness. The summation result of two users' private binary strings should be correct;
- ②Security. Two users' private binary strings cannot be leaked out to an outside eavesdropper without being detected.
- ③Privacy. Each user's private binary string should be kept secret from TP.

Recently, Zhang et al. [26] designed a semiquantum key distribution (SQKD) protocol robust against the collective-dephasing noise; Lin et al. [27] put forward a semiquantum private comparison (SQPC) protocol by using the quantum states $|+\rangle$, where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. Inspired by Refs. [26, 27], we design the following semiquantum summation protocol to compute the modulo 2 summation of Alice and Bob's private binary strings over the collective-dephasing noise quantum channel.

Step 1: TP generates $2n(4 + r + d + \delta) = 2nq$ particles all in the state of $|+_{dp}\rangle$. Here, r, d are integers greater than 0; δ is some fixed parameter greater than 0; and $q = (4 + r + d + \delta)$. Then, TP divides these particles into two sequences, $S_1 = \{s_1^1, s_1^2, \dots, s_1^{nq}\}$ and $S_2 = \{s_2^1, s_2^2, \dots, s_2^{nq}\}$, where s_i^j and s_j^i denote the i^{th} particle in S_1 and S_2 , respectively, and $i = 1, 2, \dots, nq$. Finally, TP sends the particles of S_1 and S_2 to Alice and Bob one by one, respectively. Except the first particle, TP sends out the next one only after obtaining the previous one.

Step 2: For each received particle in $S_1(S_2)$, Alice (Bob) immediately randomly performs one of the following two operations: directly reflecting it back to TP with

no disturbance (i.e., the CTRL operation) or measuring it with the Z_{dp} basis and resending the same state as found to TP (i.e., the SIFT operation). After Alice's (Bob's) operations, $S_1(S_2)$ is turned into $S'_1(S'_2)$. TP stores $S'_1(S'_2)$ in a quantum memory.

Step 3: TP picks out the i^{th} particle in S'_1 and the i^{th} particle in S'_2 to form the i^{th} particle group, where $i = 1, 2, \dots, nq$. For checking the transmission security toward an outside eavesdropper Eve, TP randomly chooses nr groups from these particle groups and tells Alice and Bob the positions of the chosen particle groups. Among these chosen particle groups, Alice (Bob) tells TP the positions of particles where she (he) chose the CTRL operations, the positions of particles where she (he) chose the SIFT operations as well as her (his) measurement results.

For the particles on which Alice (Bob) performed the CTRL operations, TP measures them with the X_{dp} basis. TP computes the error rate of CTRL particles by judging whether her measurement results are $|+_{dp}\rangle$ or not. The protocol is kept on only when the transmission of CTRL particles is secure.

For the particles on which Alice (Bob) performed the SIFT operations, TP measures them with the Z_{dp} basis. TP computes the error rate of SIFT particles by judging whether her measurement results are identical to Alice's (Bob's) corresponding measurement results or not. The protocol is kept on only when the transmission of SIFT particles is secure.

Step 4: Alice and Bob ask TP to perform the double Bell basis measurement on each of the remaining $n(4 + d + \delta)$ particle groups and announce them her corresponding measurement result. After confirming that TP have announced them all of the double Bell basis measurements on the remaining $n(4 + d + \delta)$ particle groups, Alice and Bob randomly choose nd groups from the remaining $n(4 + d + \delta)$ particle groups to check the honesty of TP. For the particle group on which both Alice and Bob performed the CTRL operations, if TP's measurement result is not $|\phi^+\rangle_{13}|\phi^+\rangle_{24}, |\phi^-\rangle_{13}|\phi^-\rangle_{24}, |\psi^+\rangle_{13}|\psi^+\rangle_{24}$ or $|\psi^-\rangle_{13}|\psi^-\rangle_{24}$, according to Eq. (5), Alice and Bob will think that TP is dishonest. For the particle group on which both Alice and Bob performed the SIFT operations, Alice and Bob check whether TP's measurement result satisfies Eqs. (6–9) or not; if the result is negative, Alice and Bob will think that TP is dishonest. For example, if Alice and Bob's measurement results are $|0_{dp}\rangle_{12}$ and $|0_{dp}\rangle_{34}$, respectively, according to Eq. (6), TP's measurement result should be $|\phi^+\rangle_{13}|\phi^+\rangle_{24}, |\phi^-\rangle_{13}|\phi^-\rangle_{24}, |\phi^+\rangle_{13}|\phi^-\rangle_{24}$ or $|\phi^-\rangle_{13}|\phi^+\rangle_{24}$; otherwise, Alice and Bob will conclude that TP is dishonest. The protocol is kept on only when TP is found to be honest in the end.

Step 5: For the remaining $n(4 + \delta)$ particle groups, Alice (Bob) tells Bob (Alice) the positions of particles where she (he) performed the SIFT operations. Note that there are $n(4 + \delta) \times \frac{1}{4} = n + \frac{n\delta}{4}$ particle groups where both Alice and Bob performed the SIFT operations. Alice and Bob choose the first n ones from these $n + \frac{n\delta}{4}$ particle groups to generate their private keys. The private key generating rule is that: if Alice's (Bob's) measurement result on the corresponding particle from the j^{th} particle group is $|0_{dp}\rangle$, her (his) j^{th} bit of private key, $k_j^a(k_j^b)$, will be 0; and if Alice's (Bob's) measurement result on it is $|1_{dp}\rangle, k_j^a(k_j^b)$ will be 1. Here, $j = 1, 2, \dots, n; K_A = [k_1^a, k_2^a, \dots, k_n^a]$ and $K_B = [k_1^b, k_2^b, \dots, k_n^b]$ are Alice and Bob's private keys, respectively. Then,

Alice (Bob) also derives a private bit string C_T from TP's measurement results on these n particle groups according to the following rule: If TP's measurement result on the j^{th} particle group is $|\phi^+\rangle_{13}|\phi^+\rangle_{24}, |\phi^-\rangle_{13}|\phi^-\rangle_{24}, |\phi^+\rangle_{13}|\phi^-\rangle_{24}$ or $|\phi^-\rangle_{13}|\phi^+\rangle_{24}, k_j^t$ will be 0; and if it is $|\psi^+\rangle_{13}|\psi^+\rangle_{24}, |\psi^-\rangle_{13}|\psi^-\rangle_{24}, |\psi^+\rangle_{13}|\psi^-\rangle_{24}$ or $|\psi^-\rangle_{13}|\psi^+\rangle_{24}, k_j^t$ will be 1. Here, k_j^t is the j^{th} bit of C_T , and $j = 1, 2, \dots, n$. Afterward, Alice (Bob) calculates $c_j^a = k_j^a \oplus x_j (c_j^b = k_j^b \oplus y_j)$, where \oplus is the modulo 2 summation, and $j = 1, 2, \dots, n$. Alice (Bob) sends $C_A(C_B)$ to Bob (Alice) via the classical channel, where $C_A = [c_1^a, c_2^a, \dots, c_n^a]$ ($C_B = [c_1^b, c_2^b, \dots, c_n^b]$). Finally, Alice (Bob) calculates $r_j = c_j^a \oplus c_j^b \oplus k_j^t$, where $j = 1, 2, \dots, n$, and obtains the summation result R , where $R = [r_1, r_2, \dots, r_n]$.

For the sake of clarity, the relations among different parameters for summation are listed in Table 1.

Obviously, TP needs to have full quantum capabilities. Both Alice and Bob only implement the following actions: ① measuring the particles with the Z_{dp} basis; ② preparing the particles in the Z_{dp} basis; ③ sending the particles. According to Refs. [18, 19], the Z basis is regarded to be classical, so the Z_{dp} basis is naturally classical. Consequently, both Alice and Bob only need limited quantum capabilities. In other words, this protocol is a semiquantum summation protocol.

4 Correctness analysis

In the proposed protocol, after receiving $C_A(C_B)$ from Alice (Bob), Bob (Alice) calculates $r_j = c_j^a \oplus c_j^b \oplus k_j^t$, where $j = 1, 2, \dots, n$. According to Eqs. (6)–(9), it is apparent that

$$k_j^a \oplus k_j^b \oplus k_j^t = 0. \quad (12)$$

Hence, it has

$$\begin{aligned} r_j &= c_j^a \oplus c_j^b \oplus k_j^t = (k_j^a \oplus x_j) \oplus (k_j^b \oplus y_j) \oplus k_j^t \\ &= (x_j \oplus y_j) \oplus (k_j^a \oplus k_j^b \oplus k_j^t) = x_j \oplus y_j. \end{aligned} \quad (13)$$

It can be concluded that the output correctness of the proposed protocol can be guaranteed.

5 Security analysis

5.1 Outside attack

An outside attacker, Eve, may try her best to extract $X(Y)$ from $C_A(C_B)$. Apparently, she should obtain $K_A(K_B)$ beforehand. Without loss of generality, here take Eve's trying to get K_A for example to analyze the outside attack.

Table 1 Relations among different parameters for summation

x_j	y_j	Alice's measurement result on the corresponding particle from the j^{th} particle group	Bob's measurement result on the corresponding particle from the j^{th} particle group	k_j^a	k_j^b	c_j^a	c_j^b	TP's double Bell basis measurement result on the j^{th} particle group	k_j^t	r_j
0	0	$ 0_{dp}\rangle$	$ 0_{dp}\rangle$	0	0	0	0	$ \phi^+\rangle_{13} \phi^+\rangle_{24} \phi^-\rangle_{13} \phi^-\rangle_{24} \phi^+\rangle_{13} \phi^-\rangle_{24} \phi^-\rangle_{13} \phi^+\rangle_{24}$	0	0
		$ 0_{dp}\rangle$	$ 1_{dp}\rangle$	0	1	0	1	$ \psi^+\rangle_{13} \psi^+\rangle_{24} \psi^-\rangle_{13} \psi^-\rangle_{24} \psi^+\rangle_{13} \psi^-\rangle_{24} \psi^-\rangle_{13} \psi^+\rangle_{24}$	1	0
		$ 1_{dp}\rangle$	$ 0_{dp}\rangle$	1	0	1	0	$ \psi^+\rangle_{13} \psi^+\rangle_{24} \psi^-\rangle_{13} \psi^-\rangle_{24} \psi^+\rangle_{13} \psi^-\rangle_{24} \psi^-\rangle_{13} \psi^+\rangle_{24}$	1	0
		$ 1_{dp}\rangle$	$ 1_{dp}\rangle$	1	1	1	1	$ \phi^+\rangle_{13} \phi^+\rangle_{24} \phi^-\rangle_{13} \phi^-\rangle_{24} \phi^+\rangle_{13} \phi^-\rangle_{24} \phi^-\rangle_{13} \phi^+\rangle_{24}$	0	0
0	1	$ 0_{dp}\rangle$	$ 0_{dp}\rangle$	0	0	0	1	$ \phi^+\rangle_{13} \phi^+\rangle_{24} \phi^-\rangle_{13} \phi^-\rangle_{24} \phi^+\rangle_{13} \phi^-\rangle_{24} \phi^-\rangle_{13} \phi^+\rangle_{24}$	0	1
		$ 0_{dp}\rangle$	$ 1_{dp}\rangle$	0	1	0	0	$ \psi^+\rangle_{13} \psi^+\rangle_{24} \psi^-\rangle_{13} \psi^-\rangle_{24} \psi^+\rangle_{13} \psi^-\rangle_{24} \psi^-\rangle_{13} \psi^+\rangle_{24}$	1	1
		$ 1_{dp}\rangle$	$ 0_{dp}\rangle$	1	0	1	1	$ \psi^+\rangle_{13} \psi^+\rangle_{24} \psi^-\rangle_{13} \psi^-\rangle_{24} \psi^+\rangle_{13} \psi^-\rangle_{24} \psi^-\rangle_{13} \psi^+\rangle_{24}$	1	1
		$ 1_{dp}\rangle$	$ 1_{dp}\rangle$	1	1	1	0	$ \phi^+\rangle_{13} \phi^+\rangle_{24} \phi^-\rangle_{13} \phi^-\rangle_{24} \phi^+\rangle_{13} \phi^-\rangle_{24} \phi^-\rangle_{13} \phi^+\rangle_{24}$	0	1
1	0	$ 0_{dp}\rangle$	$ 0_{dp}\rangle$	0	0	1	0	$ \phi^+\rangle_{13} \phi^+\rangle_{24} \phi^-\rangle_{13} \phi^-\rangle_{24} \phi^+\rangle_{13} \phi^-\rangle_{24} \phi^-\rangle_{13} \phi^+\rangle_{24}$	0	1
		$ 0_{dp}\rangle$	$ 1_{dp}\rangle$	0	1	1	1	$ \psi^+\rangle_{13} \psi^+\rangle_{24} \psi^-\rangle_{13} \psi^-\rangle_{24} \psi^+\rangle_{13} \psi^-\rangle_{24} \psi^-\rangle_{13} \psi^+\rangle_{24}$	1	1
		$ 1_{dp}\rangle$	$ 0_{dp}\rangle$	1	0	0	0	$ \psi^+\rangle_{13} \psi^+\rangle_{24} \psi^-\rangle_{13} \psi^-\rangle_{24} \psi^+\rangle_{13} \psi^-\rangle_{24} \psi^-\rangle_{13} \psi^+\rangle_{24}$	1	1
		$ 1_{dp}\rangle$	$ 1_{dp}\rangle$	1	1	1	1	$ \phi^+\rangle_{13} \phi^+\rangle_{24} \phi^-\rangle_{13} \phi^-\rangle_{24} \phi^+\rangle_{13} \phi^-\rangle_{24} \phi^-\rangle_{13} \phi^+\rangle_{24}$	0	1
1	1	$ 0_{dp}\rangle$	$ 0_{dp}\rangle$	0	0	1	1	$ \phi^+\rangle_{13} \phi^+\rangle_{24} \phi^-\rangle_{13} \phi^-\rangle_{24} \phi^+\rangle_{13} \phi^-\rangle_{24} \phi^-\rangle_{13} \phi^+\rangle_{24}$	0	1
		$ 0_{dp}\rangle$	$ 1_{dp}\rangle$	0	1	0	0	$ \psi^+\rangle_{13} \psi^+\rangle_{24} \psi^-\rangle_{13} \psi^-\rangle_{24} \psi^+\rangle_{13} \psi^-\rangle_{24} \psi^-\rangle_{13} \psi^+\rangle_{24}$	1	1
		$ 1_{dp}\rangle$	$ 0_{dp}\rangle$	1	0	0	0	$ \psi^+\rangle_{13} \psi^+\rangle_{24} \psi^-\rangle_{13} \psi^-\rangle_{24} \psi^+\rangle_{13} \psi^-\rangle_{24} \psi^-\rangle_{13} \psi^+\rangle_{24}$	1	1
		$ 1_{dp}\rangle$	$ 1_{dp}\rangle$	1	1	1	1	$ \phi^+\rangle_{13} \phi^+\rangle_{24} \phi^-\rangle_{13} \phi^-\rangle_{24} \phi^+\rangle_{13} \phi^-\rangle_{24} \phi^-\rangle_{13} \phi^+\rangle_{24}$	0	1
1	1	$ 0_{dp}\rangle$	$ 0_{dp}\rangle$	0	0	1	1	$ \phi^+\rangle_{13} \phi^+\rangle_{24} \phi^-\rangle_{13} \phi^-\rangle_{24} \phi^+\rangle_{13} \phi^-\rangle_{24} \phi^-\rangle_{13} \phi^+\rangle_{24}$	0	0
		$ 0_{dp}\rangle$	$ 1_{dp}\rangle$	0	1	0	0	$ \psi^+\rangle_{13} \psi^+\rangle_{24} \psi^-\rangle_{13} \psi^-\rangle_{24} \psi^+\rangle_{13} \psi^-\rangle_{24} \psi^-\rangle_{13} \psi^+\rangle_{24}$	1	0
		$ 1_{dp}\rangle$	$ 0_{dp}\rangle$	1	0	0	1	$ \psi^+\rangle_{13} \psi^+\rangle_{24} \psi^-\rangle_{13} \psi^-\rangle_{24} \psi^+\rangle_{13} \psi^-\rangle_{24} \psi^-\rangle_{13} \psi^+\rangle_{24}$	1	0
		$ 1_{dp}\rangle$	$ 1_{dp}\rangle$	1	1	1	0	$ \phi^+\rangle_{13} \phi^+\rangle_{24} \phi^-\rangle_{13} \phi^-\rangle_{24} \phi^+\rangle_{13} \phi^-\rangle_{24} \phi^-\rangle_{13} \phi^+\rangle_{24}$	0	0

Measure–resend attack. Eve measures the particles in S_1 from TP to Alice in Step 1 with the Z_{dp} basis and sends the new particles in the same states she found to Alice. However, she will be caught as she has no access to Alice's choices of operations in Step 2. Concretely speaking, for one particular particle chosen for detection, the probability that Alice chooses the CTRL operation is $\frac{1}{2}$; hence, the probability that Eve can be caught is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, since TP has a $\frac{1}{2}$ probability to get the wrong measurement result $| -_{dp} \rangle$ on the particle reflected by Alice. For nr particle groups chosen for the security check in Step 3, the probability of Eve's being discovered becomes $1 - \left(\frac{3}{4}\right)^{nr}$, which will converge to 1 if the value of nr is large enough.

Intercept–resend attack. Eve intercepts the particles in S_1 from TP to Alice in Step 1 and sends the fake particle sequence S_1^E she prepared in the Z_{dp} basis beforehand to Alice; Eve intercepts S_1^E after Alice's operations and sends S_1 to TP. For one particular particle chosen for detection, the probability that Alice chooses the SIFT operation is $\frac{1}{2}$; hence, the probability that Eve can be caught is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, as there is a $\frac{1}{2}$ probability that Alice's measurement result on Eve's fake particle is not same to TP's measurement result on the genuine one. For nr particle groups chosen for the security check in Step 3, the probability of Eve's being discovered becomes $1 - \left(\frac{3}{4}\right)^{nr}$.

DoubleCNOT attack. Eve may perform the first CNOT operations, defined as

$$CNOT = |00\rangle\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 10| + |10\rangle\langle 11|, \quad (14)$$

on the particles in S_1 and her own ancillary particles in the state of $|0_{dp}\rangle$ in Step 1. Here, the first physical qubit of each particle in S_1 acts as the control qubit, while two physical qubits of each ancillary particle play the role of target qubits. In order to escape the security check in Step 3, Eve has to perform the second CNOT operations on the particles in S_1' and her own ancillary particles in Step 2. Here, the first physical qubit of each particle in S_1' acts as the control qubit, while two physical qubits of each ancillary particle play the role of target qubits. If Eve can know Alice's choices of operations in Step 2 exactly through the double CNOT attacks, she will further perform the SIFT operations on the particles where Alice has implemented the same operations, in order to obtain K_A without being discovered. However, Eve cannot discriminate Alice's choices of operations at all. Concretely speaking, after the first CNOT operation, the composite system composed by one particle in S_1 and the ancillary particle $|0_{dp}\rangle$ is turned into

$$CNOT|+_{dp}\rangle_A|0_{dp}\rangle_E = \frac{1}{\sqrt{2}}(|0_{dp}\rangle_A|0_{dp}\rangle_E + |1_{dp}\rangle_A|1_{dp}\rangle_E) = |\Phi_{dp}^+\rangle_{AE}, \quad (15)$$

where the subscripts A and E represent Alice's particle and Eve's ancillary particle, respectively. If Alice chooses the CTRL operation, after Eve's second CNOT operation, the composite system will be changed into

$$CNOT|\Phi_{dp}^+\rangle_{AE} = |+_{dp}\rangle_A|0_{dp}\rangle_E. \quad (16)$$

If Alice chooses the SIFT operation and obtains the measurement result $|0_{dp}\rangle(|1_{dp}\rangle)$, according to Eq. (15), Eve's ancillary particle will be collapsed into $|0_{dp}\rangle(|1_{dp}\rangle)$. After Eve's second CNOT operation, the composite system is evolved into

$$CNOT|0_{dp}\rangle_A|0_{dp}\rangle_E = |0_{dp}\rangle_A|0_{dp}\rangle_E, \text{ if Alice's measurement result is } |0_{dp}\rangle, \quad (17)$$

$$CNOT|1_{dp}\rangle_A|1_{dp}\rangle_E = |1_{dp}\rangle_A|0_{dp}\rangle_E, \text{ if Alice's measurement result is } |1_{dp}\rangle. \quad (18)$$

According to Eqs. (16)–(18), no matter what operation Alice chooses in Step 2, after the second CNOT operation, Eve's ancillary particle is always in the state of $|0_{dp}\rangle$. In other words, Eve cannot discriminate Alice's choice of operation through her ancillary particle.

It is worthy of emphasizing that if Eve does not perform the second CNOT operation, Eve's CNOT attack will inevitably be detected in Step 3, as TP has a $\frac{1}{2}$ probability to get the wrong measurement result $| -_{dp} \rangle$ for the particle chosen for detection on which Alice has performed the CTRL operation, according to Eqs. (1) and (15).

Trojan horse attacks. As the traveling particles are transmitted in a circular way, the Trojan horse attacks from Eve should be paid special attention to, such as the invisible photon eavesdropping attack [28] and the delay-photon Trojan horse attack [29, 30]. It has been verified that the signal receiver can use a filter to resist the former attack and a photon number splitter (PNS) to defeat the latter attack [30, 31].

5.2 Participant attack

Gao et al. [32] firstly warned against the participant attack in the year of 2007 when designing a quantum cryptography protocol. It is natural that a m -user quantum summation protocol cannot resist the collusion attack from $m - 1$ users. The reason lies in that: when $m - 1$ users conspire together, they can easily derive the private input of the left user from the summation result. With regard to the proposed protocol, Alice (Bob) can easily deduce out $Y(X)$ from R . As a result, we only need to consider the participant attack from the semi-honest TP.

Apparently, if TP wants to derive $X(Y)$ from $C_A(C_B)$, she will need to get $K_A(K_B)$ beforehand. In order to achieve this goal, TP may launch the following attacks.

Attack I: TP may generate all particles in the Z_{dp} basis and transmits them to Alice and Bob in Step 1; moreover, TP always announces the genuine double Bell basis measurement results to Alice and Bob in Step 4. In this way, if TP's cheating behavior successfully passes the honesty check in Step 4, she will easily obtain K_A and K_B in Step 5. However, TP's cheating behavior is inevitably discovered by Alice and Bob during the honesty check in Step 4. Concretely speaking, for one particle group chosen for TP's honesty check, according to Eqs. (5)–(9), if both Alice and Bob choose the CTRL operations, TP will be detected with the probability of $\frac{1}{2}$; if both Alice and Bob choose the SIFT operations, TP will be detected with the probability of 0; and if only one chooses the CTRL operation, TP will be automatically not detected, since there is no check for this case. Thus, for one particle group chosen for TP's honesty check, the probability that TP will be detected is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$. For nd

particle groups chosen for TP's honesty check, the probability that TP will be detected is $1 - \left(\frac{7}{8}\right)^{nd}$, which will converge to 1 if the value of nd is large enough.

Attack II: TP measures all particles in the remaining $n(4 + d + \delta)$ particle groups with the Z_{dp} basis instead of the double Bell basis and announces the fake double Bell basis measurement results to Alice and Bob in Step 4, in hope of escaping the honesty check. However, TP's cheating behavior is undoubtedly detected as she cannot exactly know Alice and Bob's choices of operations in Step 2. Concretely speaking, for one particle group chosen for TP's honesty check, according to Eqs. (6)–(9), if TP's measurement result is $|0_{dp}\rangle|0_{dp}\rangle$ or $|1_{dp}\rangle|1_{dp}\rangle$, TP will randomly announce the fake measurement result $|\phi^+\rangle_{13}|\phi^+\rangle_{24}$, $|\phi^-\rangle_{13}|\phi^-\rangle_{24}$, $|\phi^+\rangle_{13}|\phi^-\rangle_{24}$ or $|\phi^-\rangle_{13}|\phi^+\rangle_{24}$; and if TP's measurement result is $|0_{dp}\rangle|1_{dp}\rangle$ or $|1_{dp}\rangle|0_{dp}\rangle$, TP will randomly announce the fake measurement result $|\psi^+\rangle_{13}|\psi^+\rangle_{24}$, $|\psi^-\rangle_{13}|\psi^-\rangle_{24}$, $|\psi^+\rangle_{13}|\psi^-\rangle_{24}$ or $|\psi^-\rangle_{13}|\psi^+\rangle_{24}$. As a result, if both Alice and Bob choose the SIFT operations, TP will be detected with the probability of 0; and if both Alice and Bob choose the CTRL operations, TP will be detected with the probability of $\frac{1}{2}$. Thus, for one particle group chosen for TP's honesty check, the probability that TP will be detected is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$. For nd particle groups chosen for TP's honesty check, the probability that TP will be detected is $1 - \left(\frac{7}{8}\right)^{nd}$.

It can be concluded that TP's attack is inevitably discovered by the honesty check in Step 4 when she tries to extract Alice and Bob's private keys.

6 Discussions and conclusions

We firstly discuss the qubit efficiency [33] defined as.

$$\eta = \frac{c}{p + v}, \quad (19)$$

where c is the total number of classical bits for summation; q is the total number of consumed qubits; and v is the total number of classical bits needed for summation. Here, the classical bits consumed for eavesdropping check are not taken into account. In this protocol, both the length of Alice's private binary string and the length of Bob's private binary string are n ; hence, we have $c = n$; TP needs to generate $2n(4 + r + d + \delta) = 2nq$ particles all in the state of $|+\rangle_{dp}$, while both Alice and Bob need to prepare $\frac{nq}{2}$ particles in the Z_{dp} basis when implementing the SIFT operations; hence, we have $p = 2nq \times 2 + \frac{nq}{2} \times 2 \times 2 = 6nq$; TP needs to publish Alice and Bob her double Bell basis measurement results on the n particle groups used for summation, while Alice and Bob need to send C_A and C_B to the other via the classical channel, respectively; hence, we have $v = 6n$. Consequently, it can be obtained that $\eta = \frac{n}{6nq + 6n} = \frac{1}{6(4+r+d+\delta)+6}$.

We compare this protocol with the only existing semiquantum summation protocol [20] in detail and list the comparison result in Table 2. In Table 2, with respect to this protocol, the two-qubit entangled state quantum resource

Table 2 Comparison results of this protocol and the previous semiquantum summation protocol

	The protocol of Ref [20]	This protocol
Characteristic	Measure-resend	Measure-resend
Number of communicants	Three	Two
Quantum resource	Single-qubit states	Two-qubit entangled states
Quantum measurement of TP	Single-qubit measurements and three-qubit entangled state measurements	Single-qubit measurements and two-qubit entangled state measurements
Quantum measurement of communicants	Single-qubit measurements	Single-qubit measurements
Type of TP	Semi-honest	Semi-honest
TP's knowledge about the summation result	Yes	No
Usage of quantum entanglement swapping	No	No
Usage of unitary operations	No	No
Usage of pre-shared key	No	No
Summation type	Modulo 2 addition	Modulo 2 addition
Computation way	bit-by-bit	bit-by-bit
Qubit efficiency	$\frac{2}{9(32+r+d+\delta)+6}$	$\frac{1}{6(4+r+d+\delta)+6}$
Quantum channel	Ideal noiseless quantum channel	Collective-dephasing noise quantum channel

refers to the state $|+_{dp}\rangle$; both TP's single-qubit measurements and communicants' single-qubit measurements refer to the Z_{dp} basis measurements, as the Z_{dp} basis measurement is composed by two composite Z basis measurements; and TP's two-qubit entangled state measurements refer to the X_{dp} basis measurements and the double Bell basis measurements. As for the protocol of Ref. [20], the single-qubit state quantum resource refers to the state $|+\rangle$; communicants' single-qubit measurements refer to the Z basis measurements; TP's single-qubit measurements refer to the X basis ($|+\rangle$, $|-\rangle$) measurements and the Z basis measurements; and TP's three-qubit entangled state measurements refer to the GHZ-type basis measurements. In addition, in the protocol of Ref. [20], the length of the private binary string from P_j is n , where $j = 1, 2, 3$; TP needs to generate $3n(32 + r + d + \delta) = 3nt$ particles all in the state of $|+\rangle$, while P_j needs to generate $\frac{nt}{2}$ particles in the Z basis when choosing the SIFT operations; and P_j needs to send C_j to TP through the classical channel; hence, we have $c = n, p = 3nt + \frac{nt}{2} \times 3 = \frac{9nt}{2}$ and $v = 3n$. Consequently, the qubit efficiency of the protocol of Ref. [20] is $\eta = \frac{n}{\frac{9nt}{2} + 3n} = \frac{2}{9(32+r+d+\delta)+6}$. From Table 1, it can be concluded that this protocol takes advantages over the protocol of Ref. [20] on the aspects of quantum measurement of TP, privacy of summation result towards TP and practical feasibility.

In conclusion, in this paper, a two-party secure semiquantum summation protocol is constructed, which need not require two communicants to possess full quantum capabilities. In this protocol, two classical communicants can successfully calculate the summation of their private binary sequences with the assistance from a quantum semi-honest TP, who is permitted to perform all kinds of attacks at her own will except colluding with anyone else. This protocol need not pre-share a random key between any two parties. This protocol resists the collective–dephasing noise by adopting logical qubits within decoherent free space as traveling particles. It has been validated that this protocol can overcome both the outside attacks from Eve and the participant attacks from TP. Moreover, this protocol can guarantee that TP has no knowledge about the summation results. In the future, we will concentrate on studying how to design the semiquantum summation protocol feasible over other noisy quantum channels, such as the collective-rotation noise channel and the amplitude damping channel, etc. As two classical communicants can know each other's private binary sequence, this protocol cannot be directly applied into SQPC [27, 34–38]. In the future, we will also study how to apply semiquantum summation into the other semiquantum secure computation, such as SQPC and semiquantum key agreement (SQKA) [39–41], etc.

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Data availability All data and models generated or used during the study appear in the submitted article.

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