Effective filed theory in the low energy regime

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1 Introduction

Modern renormalization theory shows that low energy behavior of a theory is independent of the details of the short-distance dynamics. For example, in the low energy scattering experiments, wavelength of the projectile can be much larger than the size of target, which make it impossible for the projectile to detect the inner details of the target. In other words, the scattering process will not be sensitive to the details of the potential. With long range force well studied, different theories for short distance can give similar results. Replacing the short distance potential by some simple form without changing the low-energy behavior is possible and can simplify the calculation a lot.

Nucleons interact with each other by exchanging pion particle. In the low energy region, where size of nucleon is negligible compared to the wavelength of scattered particle, interaction between nucleon can be described by contact potential (δ potential). As energy goes high, pion particle can be generated, interaction between nucleons can be described by one pion exchange model. In even higher energy, more pion can be exchanged between nucleons.

Usually, by solving Lippmann-Schwinger equation, we can obtain the phase shifts of a scattering system and relate them with those extracted from experimental cross section. If only the low-energy part is of interest, then Effective Field Theory model can be employed here to describe the scattering and simplify the interaction form. However, in LS equation the simpler interaction could usually result in UV divergence when the integral is taken to infinity in momentum space. We need to regularize the potential to cutoff the high-momentum modes in the loop integrals. By fitting the benchmark of phase shift, we can find the best interaction coefficients in our EFT model.

In this project, we consider a nucleon-nucleon interaction system. With pionless EFT potential to describe low energies part, the potential becomes a series of contact interactions. In order to tune the parameters in the contact interactions, we use our result in project 1 as a benchmark of the phase shift. In project 1 the toy model is sipn- and isospin-independent, therefore in this project the terms with Pauli spin/isospin matrices are not considered. Besides the contact interactions, we add the one-pion exchange term which can include the behavior at higher energies. Details are in section 3.

In this report, we showed the fitting results order by order of the contact interactions and it is clear that this EFT model can give a good description

up to some breakdown scale λ_b. In addition, we explore the cutoff dependence of our EFT models and it roughly revealed the theoretical errors in an EFT calculation scale.

2 Gradient descent method

We define the error function

$$f(\vec{c}) = \sqrt{\frac{\sum_{j=0}^{N}(\hat{\sigma}(E_j) - \sigma(E_j))^2}{N}}, \tag{1}$$

where N is the number of points along phase shift we chose to fit, $\hat{\sigma}(E_j)$ is the phase shift at energy E_j calculated using parameter set $\vec{c} = [c_1, c_2..c_n]$, and $\sigma(E_j)$ is the exact phase shift for three potential in Yukawa form from Project 1.

We would like to tune parameters from EFT to minimize the function $f(\vec{c})$. The method used in the project is gradient descent method, which is a first-order iterative optimization algorithm for finding the minimum of a function. The iteration is based on the following relation [2]

$$\vec{c}_{k+1} = \vec{c}_k - \gamma \nabla f(\vec{c}). \tag{2}$$

The $(k+1)^{\text{th}}$ point is obtained by moving the k^{th} point along the opposite direction of the gradient, γ is used to control the step size, which is tuned to make sure that minimum is not passed and move fast at relative flat region. The gradient is obtained by finite difference method:

$$\frac{\partial f}{\partial c_i} = \frac{f(c_i + h) - f(c_i - h)}{2h}, \tag{3}$$

with h being the small step size.

3 EFT model: pionless potential

In the low energy NN scattering, the interaction between two nucleons can be approximated as contact interaction. Without the non-central term, the EFT contact potential will have the following form [1, 3]:

$$\begin{aligned} V^{LO}(\vec{q},\vec{k}) =& C_S + C_T \vec{\sigma_1}\vec{\sigma_2}, \\ V^{NLO}(\vec{q},\vec{k}) =& C_1\vec{q}^2 + C_2\vec{k}^2 + \vec{\sigma_1}\vec{\sigma_2}\left(C_3\vec{q}^2 + C_4\vec{k}^2\right) \\ & iC_5\frac{\vec{\sigma_1}+\vec{\sigma_2}}{2}\cdot\vec{q}\times\vec{k} + C_6\vec{q}\cdot\vec{\sigma_1}\vec{q}\cdot\vec{\sigma_2} + C_7\vec{k}\cdot\vec{\sigma_1}\vec{k}\cdot\vec{\sigma_2} \end{aligned} \tag{4}$$

As the potential used in project 1 does not include any spin/isospin-dependent terms or non-central terms, the effective potential we choose to optimized only

include central potential with spin/isospin-independence. Projecting into the ${}^1S_0$ channel, the leading order (LO) pionless EFT potential is:

$$V_{{}^1S_0}^{LO}(p',p) = C_0. \tag{5}$$

The next leading order term (NLO) and next to next leading order terms are:

$$\begin{aligned} V_{{}^1S_0}^{NLO}(p',p) &= C_0 + C_2(p^2 + p'^2). \\ V_{{}^1S_0}^{NNLO}(p',p) &= C_0 + C_2(p^2 + p'^2). + C_4(p^4 + p'^4) + C_4' P^2 p'^2, \end{aligned} \tag{6}$$

where $C_4 = C_4'$ is assumed in this project.

To avoid the UV divergence in solving the LS equation, instead using a sharp cutoff by multiplying $\theta(\Lambda - p)\theta(\Lambda - p')$, we instead use a smooth UV regulator

$$V(p',p) \Rightarrow \exp[-p'^4/\Lambda^4]V(p',p)\exp[-p^4/\Lambda^4] \tag{7}$$

3.1 Orders of contact interactions

By including different order potential to the fitting process, we obtained the parameters for LO, NLO and NNLO. The cutoff parameter used for optimization in this section is $\Lambda = 0.7\, fm^{-1}$

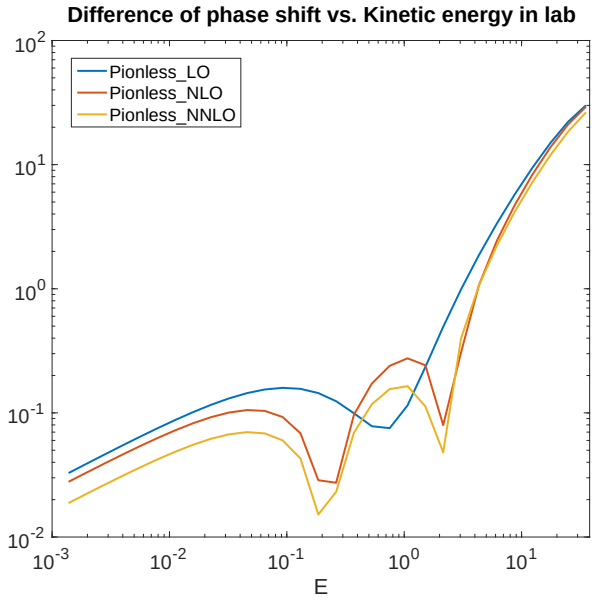

Figure 1: Error dependence of order of potential included in pionless EFT. Horizontal axis is the scattering energy in unit of MeV in lab frame. Vertical axis shows the difference between the analytical result and EFT result in unit of deg.

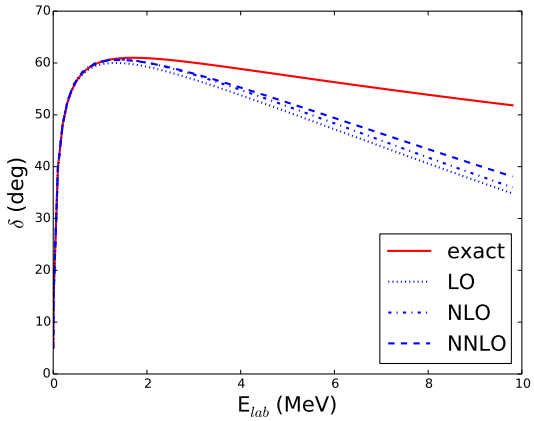

Figure 2: The comparison between exact phase shift and the phase shift for different order of EFT pionless potential

Results shown in Fig. (1) are the difference between analytical result and EFT calculation with different order. In the low energy region, the difference are very small, which shows the EFT can represent the low energy scattering behavior very well. Also, with higher order EFT potential, the error get smaller as expected. The curves comes from the shape of plateau form of the phase shift at its maximum, which has no physical interests and is not discussed here.

In the higher energy region, error of LO, NLO, NNLO all goes up and converges at the same point. This is because higher energy can detect the details of nucleon inner structure, pionless model can not describe the interaction in this region. The break up energy E_b for EFT-pionless method is around 1 MeV.

In Fig. (2), we show the phase shift obtained using the optimized parameter for different order of EFT potential. The red line shows the exact phase shift. In the low energy region, results for three different potential do not show much difference and all match the exact value very well. Difference between EFT results and exact results gets bigger as energy goes higher and eventually EFT pionless model fail to describe the NN scattering experiments.

3.2 Cutoff Λ dependence

When solving the LS equation to obtain the phase shift, the divergence issue is dealt by setting a cutoff Λ. In the section, we optimized the NLO potential using different cutoff Λ. Results are shown in Fig. (3). The vertical axis shows the difference between the calculated phase shift and exact value.

As the cutoff goes down from above 137.9 MeV, the errors also decrease. However, when it goes below 137.9 MeV, the errors become larger again. Since we are fitting the low energy region, larger lambda includes higher-momentum mode which is not explained by pionless EFT in low energies. If the cutoff

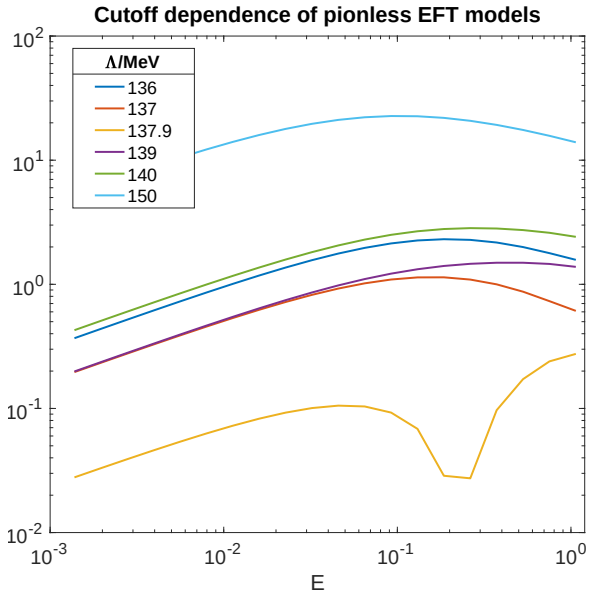

Figure 3: Cutoff dependence of the optimization for NLO potential.

is smaller than lambda b, too few momentum modes are included in the loop integral to describe the physics. Therefore, the cutoffs much further away from lambda b would give larger errors. We denote a generic momentum of some low-energy process by Q, then the theoretical errors in an EFT calculation scale roughly as

$$\Delta(Q) \sim \mathrm{Max}((Q/\Lambda)^n, (Q/\Lambda_b)^n). \tag{8}$$

4 One Pion exchange potential

To account for scattering behavior in higher energy region, we would like to include one pion exchange potential besides contact term. One pion potential used in this part has the following form in the coordinate space

$$V(r) = V_\pi \frac{e^{-m_\pi r}}{m_\pi r}, \tag{9}$$

where m_π is pion mass Projecting the potential into the $^1S_0$ channel in the momentum space, the potential is:

$$V(p,p') = \frac{V_\pi}{4m_\pi pp'} log\left[\frac{m_\pi^2 + (p+p')^2}{m_\pi^2 + (p-p')^2}\right] \tag{10}$$

The LO term including one pion exchange potential is

$$V_{^1S_0}^{LO}(p',p) = V_\pi(p,p') + C_0. \tag{11}$$

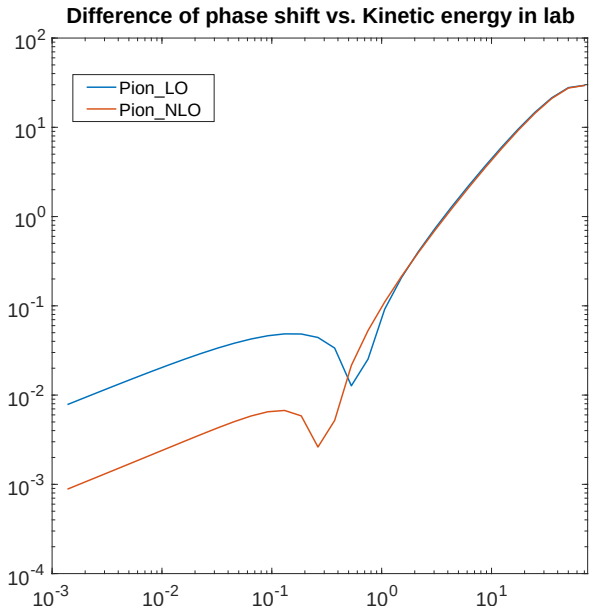

Figure 4: Error of phase shift for LO and NLO EFT potential including one pion exchange interaction

And the NLO term with one pion exchange potential is

$$V_{^1S_0}^{NLO}(p', p) = V_\pi(p, p') + C_0 + C_2(p^2 + p'^2). \tag{12}$$

V_π is another parameter that needed to be adjusted in the calculation and the cutoff $\Lambda = 0.8\, fm^{-1}$ (157.6 MeV) is used in this part.

Results for LO and NLO EFT potential with one pion exchange interaction are shown in Fig. (4). NLO shows better results than LO in the low energy region and both approximations fail in the high energy region as expected. Again, in higher energy region, they converge at the same energy scale.

In Fig. (5), we put the results for pionless and one pion exchange potential together. Red lines shows the difference of phase shift for pionless EFT approximation relative to the exact result and blue line shows the results for potential with one pion exchange interaction. The figure on the left side shows the calculation for LO potential and the right side one is for NLO. In both LO and NLO case, including one pion exchange potential in the calculation improves the results overall. Although the higher energy region is still not precise, including one pion exchange potential here did improve the results a little.

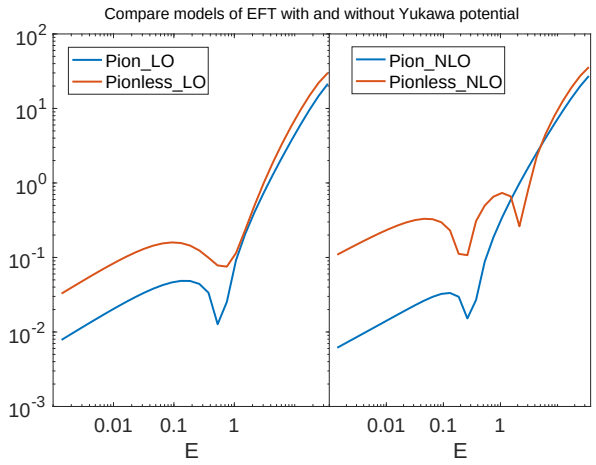

Figure 5: Comparison between pionless EFT and one pion exchange potential

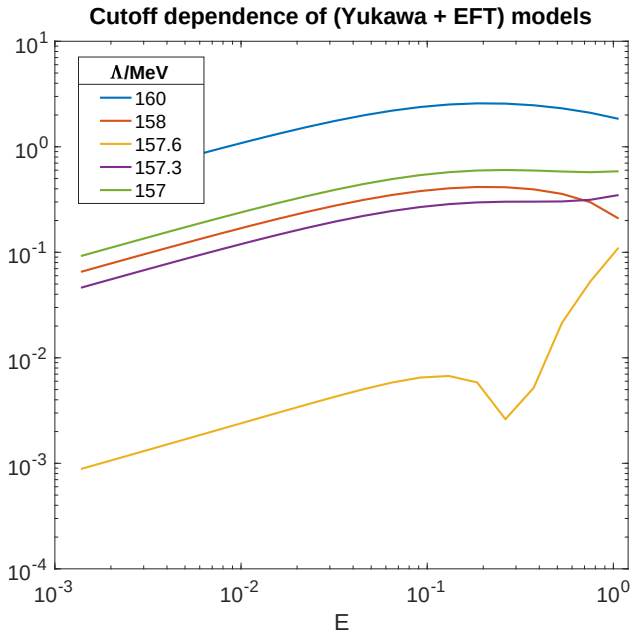

Figure 6: Cutoff dependence for one pion exchange potentiall

Fig. (6) is similar to Fig. (3) and shows the cutoff dependence of the one pion exchange potential. The best cutoff resides at $\Lambda = 157.6$ MeV. Cutoffs with larger and smaller values all shows less precise results.

5 Summary

In summary, we optimized the parameters for LO, NLO and NNLO pionless EFT and one-pion-exchange EFT potential in the low energy region. Higher order approximation gives better description for low energy scattering. The errors of the phase shift become large in the higher energy region and converge to the same point at Λ_b . Since EFT is effective where $\Lambda_b \sim m_\pi$, the cutoffs larger or smaller than Λ_b both give imprecise fitting and larger errors. In general, including one-pion-exchange potential gives better results than pionless EFT potential.

References

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[3] Evgeny Epelbaum Nuclear forces from chiral effective filed theory arXiv:1001.3229v1 [nucl-th] 19 Jan 2010