Graph Based Geometric Data Analysis

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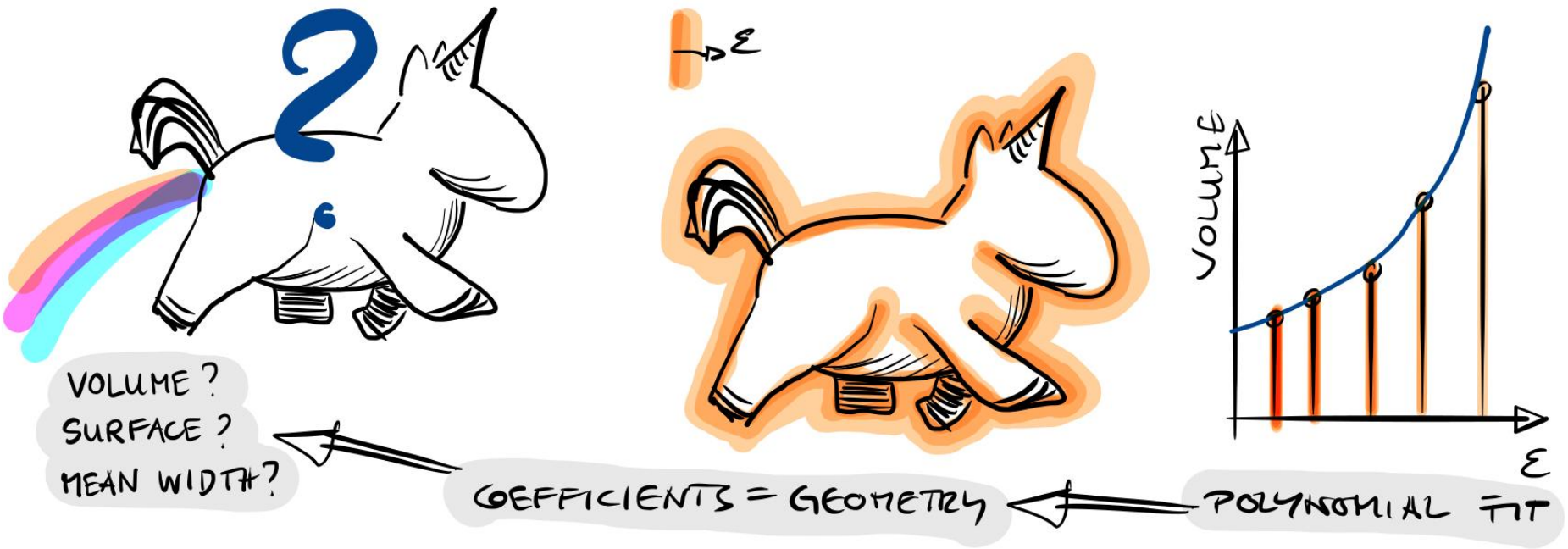
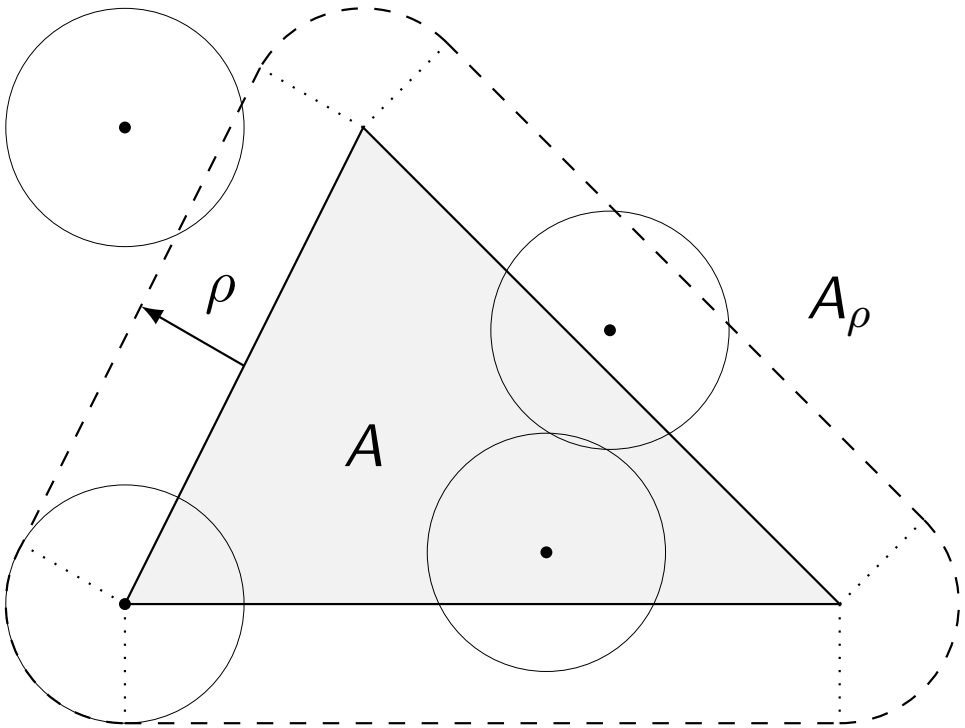
Introduction



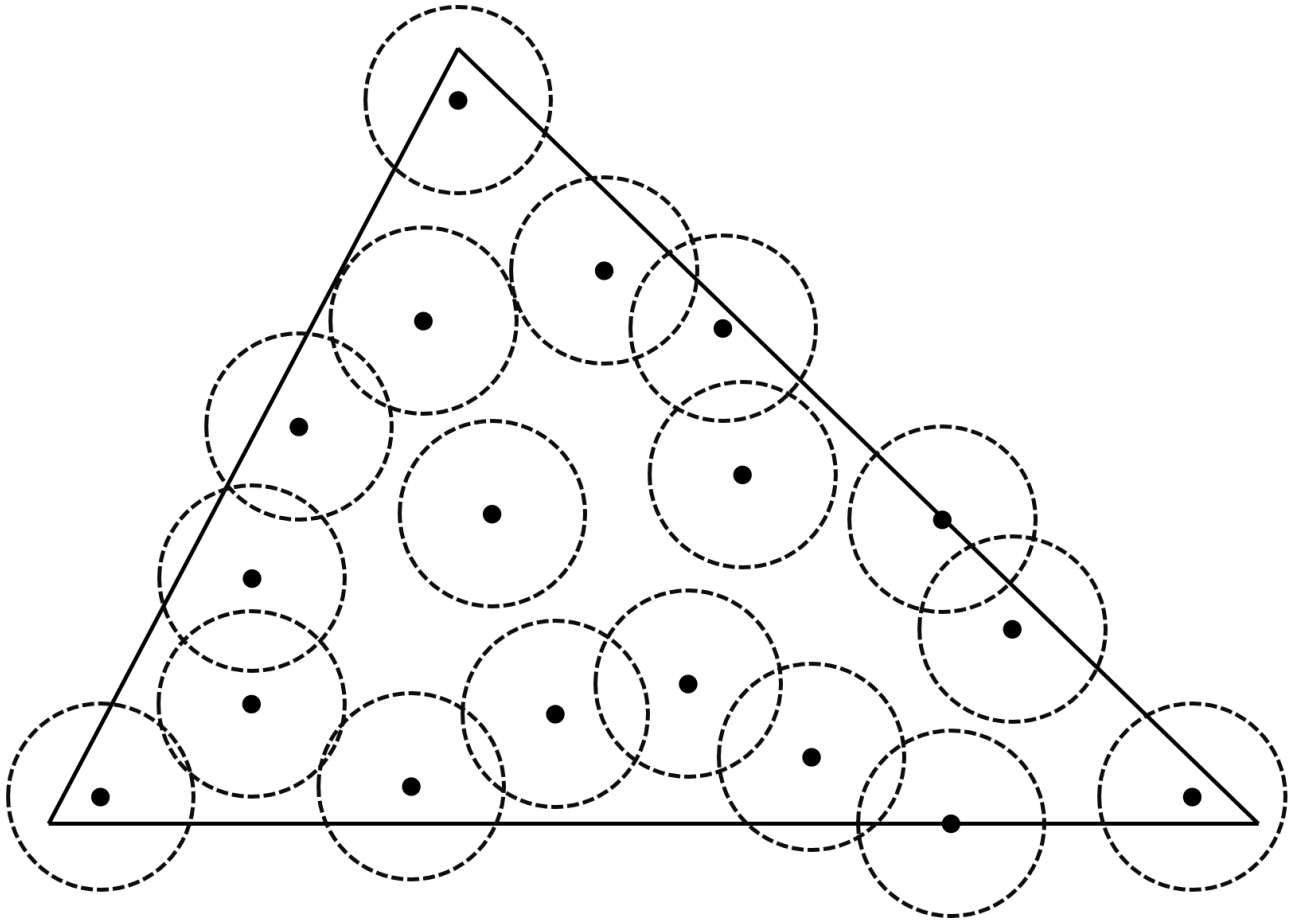
Background from Convex Geometry



$$\int \chi(A_r^N \cap B_\rho(x))dx$$

$$S = A + P * \rho + \chi * \pi * \rho^2$$

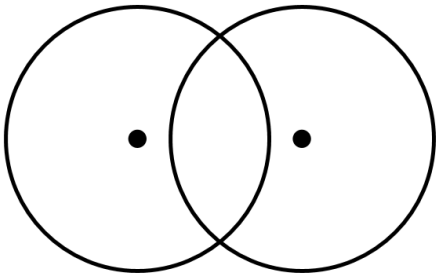
Samples from Triangle

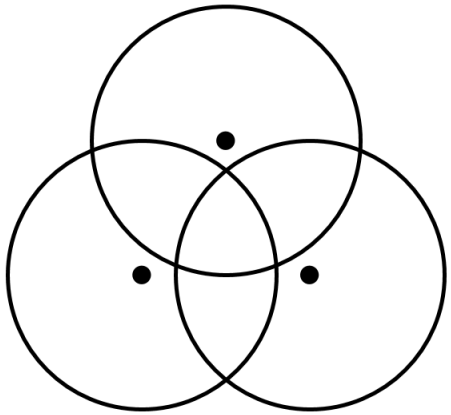


$$\int \chi(A_r^N \cap B_\rho(x))dx$$

$$S = A + P * \rho + \chi * \pi * \rho^2$$

Curse of Intersections





$$\int \chi(A_r^N \cap B_\rho(x))dx = \sum \int \chi(B_r(x_i) \cap B_\rho(x))dx -$$

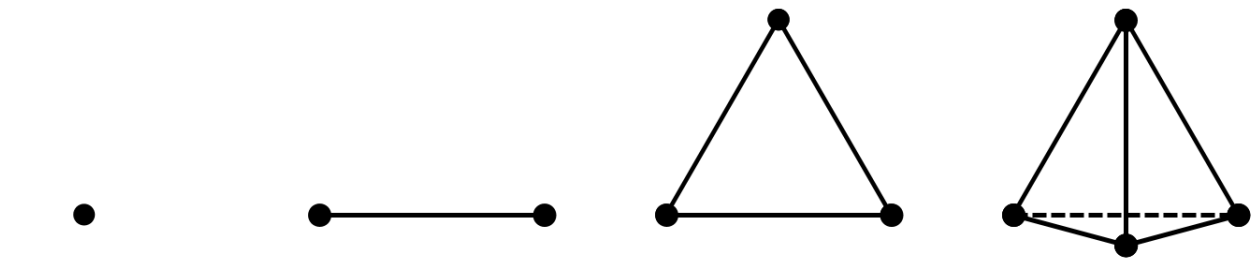
$$\sum\sum \int \chi(B_r(x_i) \cap B_r(x_j) \cap B_\rho(x))dx + \ldots - \ldots$$

Acceleration Process

- Classify intersections with inductive method
- Calculate each type of intersections with artificial neural network instead of pixel counting
- Add or Subtract to get total intersection

Background for Inductive Method

- Simplex
- Simplicial Complex

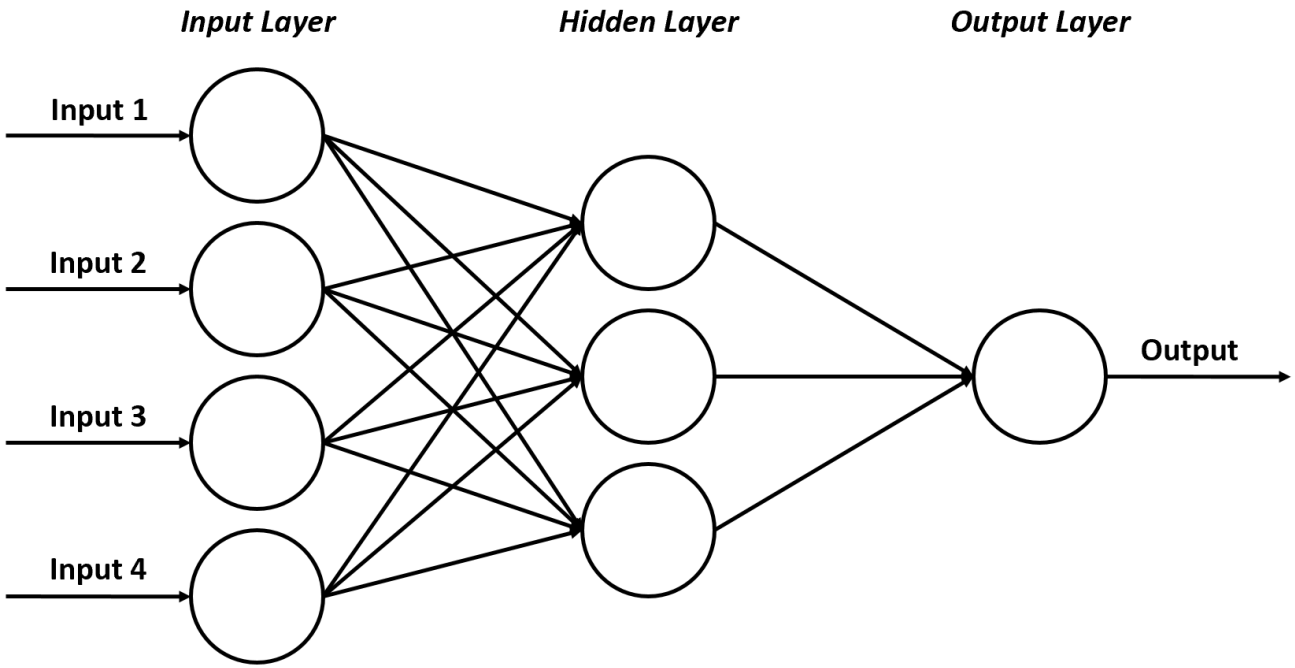


Inductive Method

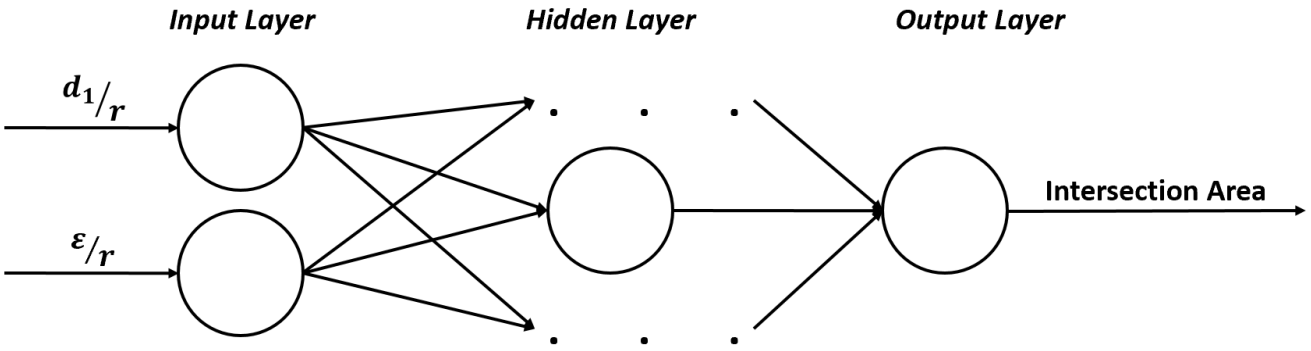
- 0-simplex: Point Cloud
- 1-simplex: Neighborhood
- 2-simplex: Add all common neighbors of 1-simplex
- k+1-simplex: Add all common neighbors of k-simplex

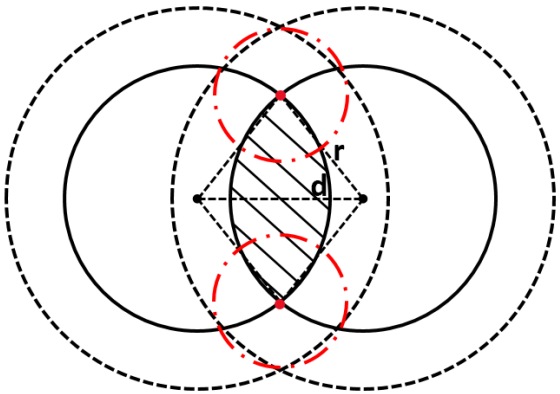
Artificial Neural Network

ANN: Build models with multiple coefficients efficiently



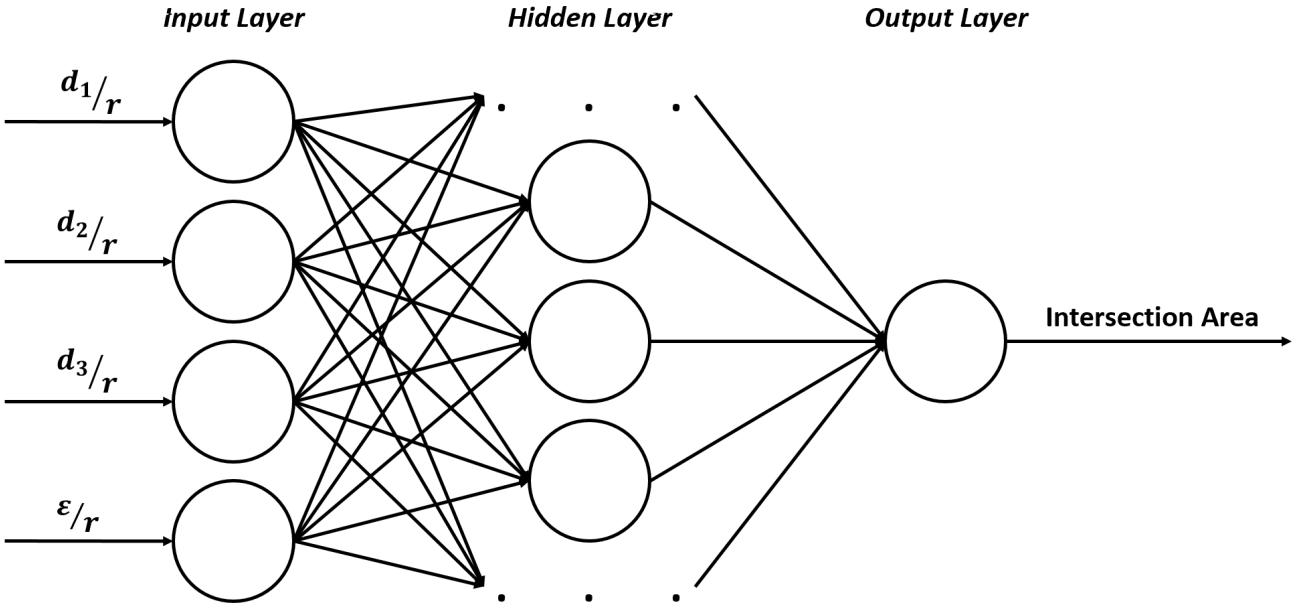
Sanity Check from 1-simplicial complex





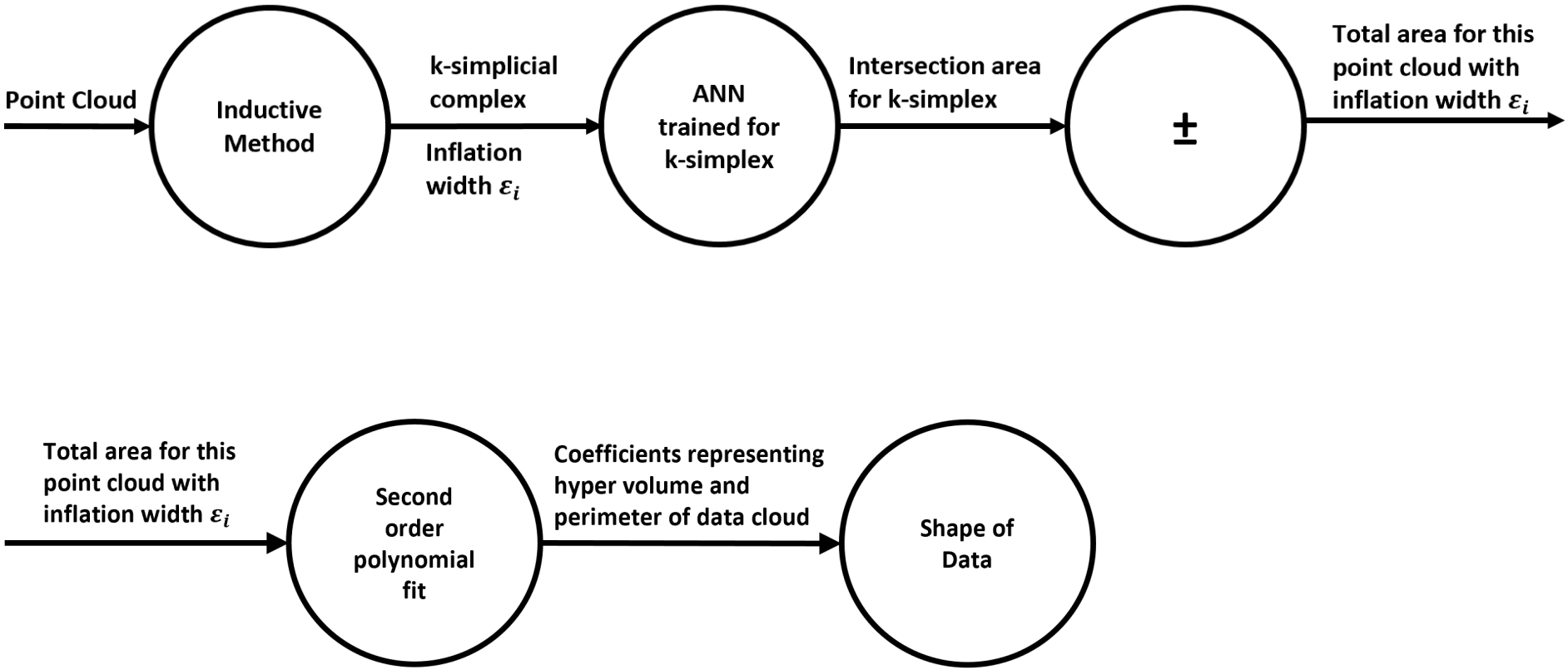
$$S = r^2 * sin^{-1}(\frac{\sqrt{r^2 - \frac{d^2}{4}}}{r}) - d * \sqrt{r^2 - \frac{d^2}{4}}$$

Example Training Process for 2-simplex



- ***Input:***
- d_1 between point 1 and point 2, scaled by radius of virtual ball
- d_2 between point 2 and point 3, scaled by radius of virtual ball
- d_3 between point 3 and point 1, scaled by radius of virtual ball
- Inflation width ϵ, scaled by radius of virtual ball
- ***Output:***
- Intersection area of 2-simplex

Complete Algorithm



Preliminary Results

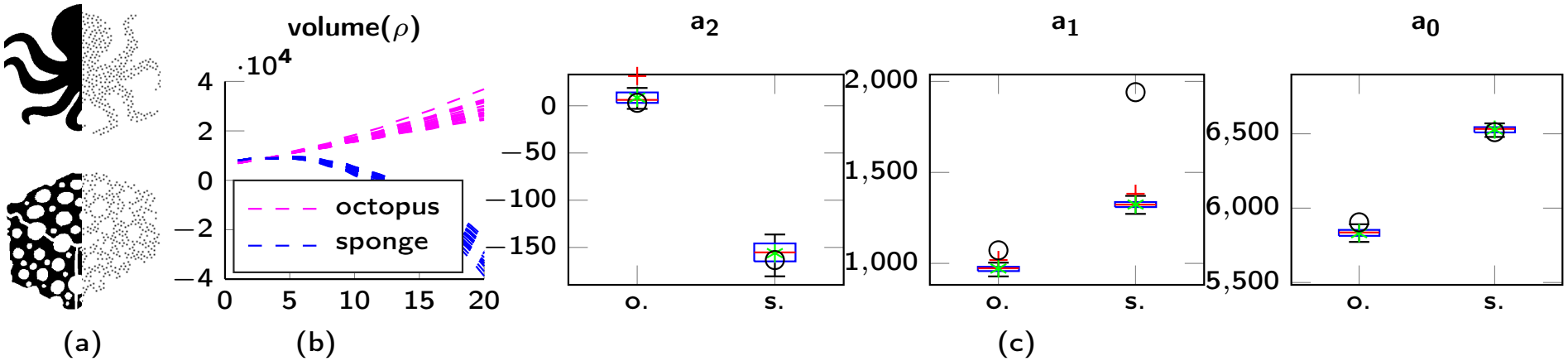


Figure: (a) Binary shapes (left) and sampling (right). (b) Volume with multiplicity, from 20 repeated shape-samplings, 625 points each, with fixed $r = 3$ and $\rho = 1 \ldots 20$. (c) Recovered coefficients of 20 replicates per shape (boxplots); composite estimate (green asterisk), and approximate ground truth (black circle). **Coefficients a_2, a_1, and a_0 capture shape-specific Euler–Poincaré characteristic, perimeter, and area, resp.**

Potential applications

- Algorithm has applications on pattern recognition.

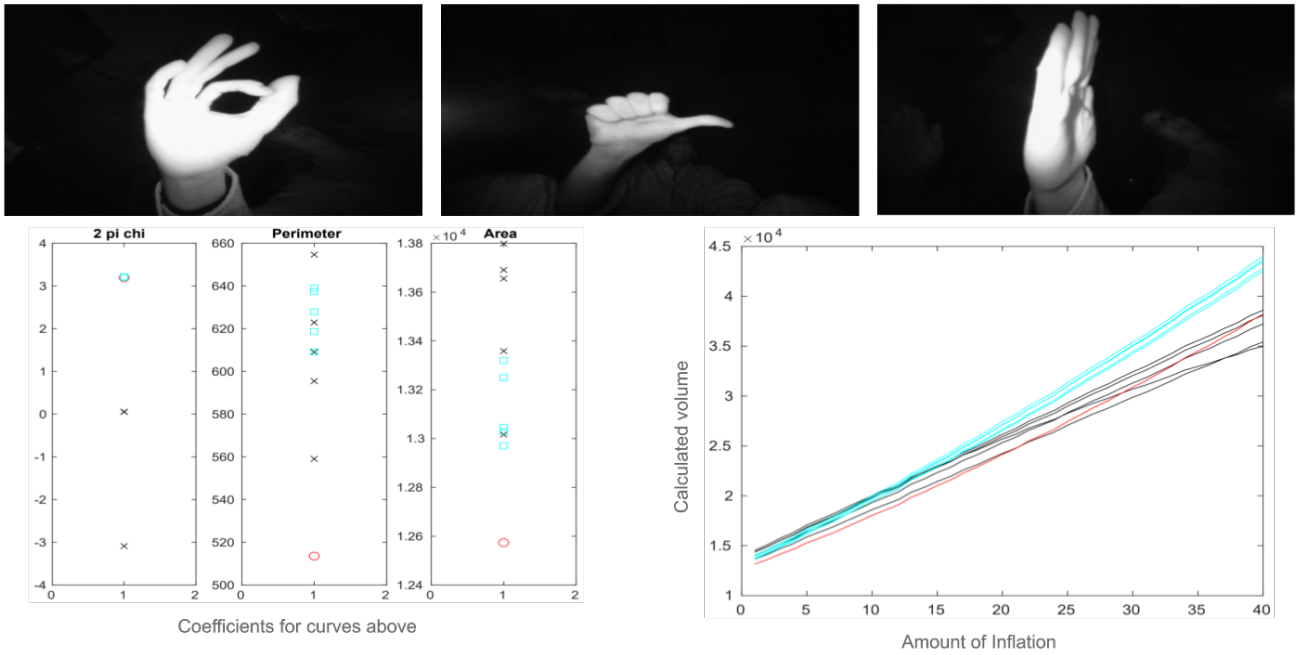


Figure: Geometric data analysis is performed on three types of gestures. Red corresponds to the ok gesture, black corresponds to the palm gesture, and cyan corresponds to the thumb gesture.

Acknowledgements

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