MH4510 Project Report - Helvetios

IDA GOH YU XUAN, MA WENGANG, SU XINHUI, TEO ENXIN MELODY

Contents

Abstract

United States has a significant problem of police brutality due to its societal implications, with fatal shootings being a huge part of it. The aim of this project is to predict the number of shootings in each U.S state and which factor affects the number of shootings the most using machine learning. We used Decision Trees, Random Forest, Multiple Linear Regression (MLR), Artificial Neural Network (ANN) and XGBoost and compared their performances with one another. We concluded that Decision Trees is the best performing model to predict the number of shootings as it has the lowest mean squared error and mean absolute error.

Context

Police brutality in the United States have garnered significant attention due to their profound societal implications. In 2022, the U.S. saw a record-high of at least 1,176 fatalities at the hands of law enforcement, with 24% of the victims being Black despite Black people only making up 13% of the population. (Levin, 2023) Furthermore, U.S. officers often evade legal consequences for such actions, benefiting from qualified immunity that shields them from lawsuits. (Cheatham & Maizland, 2022) This issue has sparked concerns about racial inequality, social justice and the need for enhanced police accountability.

According to The Washington Post, the majority of police-involved deaths in the U.S., ranging from 65% to 95%, are the result of fatal shootings. (Shjarback, 2021) The ideal objective of the study would be to predict which police precinct has higher shooting rates and identify any patterns in the shooting cases. As most police departments in the US have limited manpower and resources (Berman, 2021), this can help them with effective resource allocation and management. While an accurate model can help policy makers look for characteristics of precincts with similar shooting rates, it can also help them identify precincts with fewer police shooting instances than predicted, potentially indicating that those precincts may have better training programs and policies that can be adapted to precincts with more shooting incidents. Furthermore, this can also contribute to the development of an early warning system to identify areas with higher risk of police brutality, so that agencies can intervene by providing preventive measures and additional training like de-escalation techniques, cultural sensitivity, and crisis intervention.

However, due to the lack of available information and the large scale of the objective, our study will instead focus on testing machine learning models on a smaller scale first. The ‘police_shooting.csv’ dataset will be analyzed, with the objective of predicting the annual number of shootings in each U.S. state. Subsequently, the results will be examined to assess the feasibility of the police applying the models on a broader scale with more information on their end.

Data Processing

In our final dataset, `police_shootings.csv`, we have joined 6 sets of data by state and year. The dataset contains 408 observations and 15 features, which their descriptions can be found in the appendix.

Feature	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	Type
state	-	-	-	-	-	-	categorical
year	2015	2017	2018	2018	2020	2022	categorical
high_sch_grads	0.5210	0.9167	0.9767	0.9727	1.0349	1.2236	numerical
unemployment_percent	2.000	3.500	4.200	4.395	5.200	9.500	numerical
total_labor_force	41.71	47.50	49.66	49.73	52.06	59.99	numerical
total_checked_firearm	0.1096	5.4608	8.0068	10.1702	10.6162	110.0390	numerical
white	18.47	46.14	56.64	54.82	63.06	76.70	numerical
black	0.3568	2.5457	5.6157	8.4082	11.3355	36.6857	numerical
american_indian_alaskan_native	0.1093	0.1928	0.3428	1.0868	0.7263	10.4842	numerical
asian	0.6547	1.3534	2.2890	3.5219	3.8363	31.4182	numerical
native_hawaiian_pacific_islander	0.01693	0.02976	0.04674	0.24727	0.09407	7.21659	numerical
two_or_more_race	0.5179	0.9124	1.0552	1.4538	1.2982	13.4021	numerical
hispanic_latino	1.021	3.250	6.287	8.214	9.155	36.749	numerical
population	577605	1788176	4468309	6423852	7376386	39501653	numerical
shooting_count_per_mil	0.000	1.945	3.193	3.599	4.495	17.200	numerical

Feature Engineering

When training our models, the variables `state` and `year` are not used as independent variables as neither the name of a state nor the numerical value of a year should directly affect the shooting count. Rather, the information of a certain state in a certain year are being represented by the other 12 variables, which can potentially have some correlations to the shooting count.

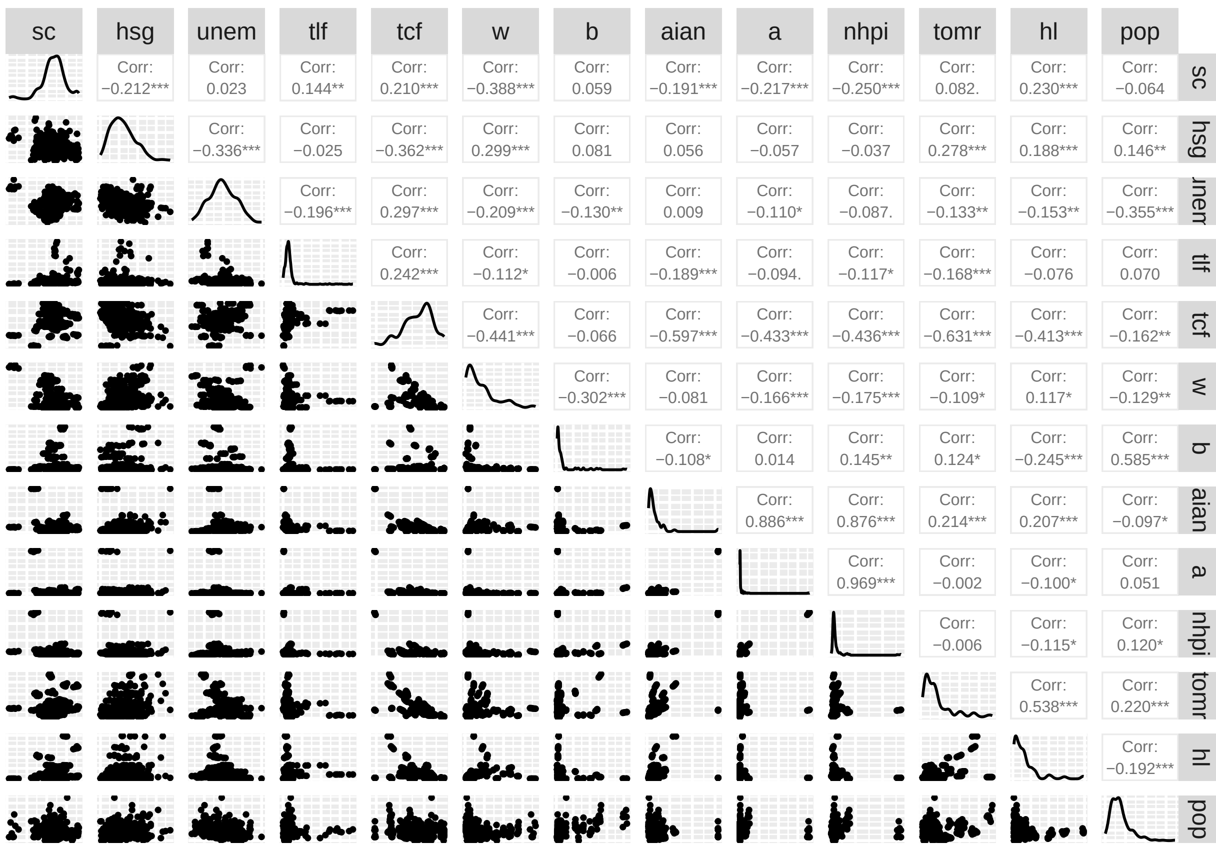
Further feature selection is also done through the use of L1 regularization in some of our models, such as Multiple Linear Regression and Artificial Neural Network.

Exploratory Data Analysis

Pairplots of numerical data

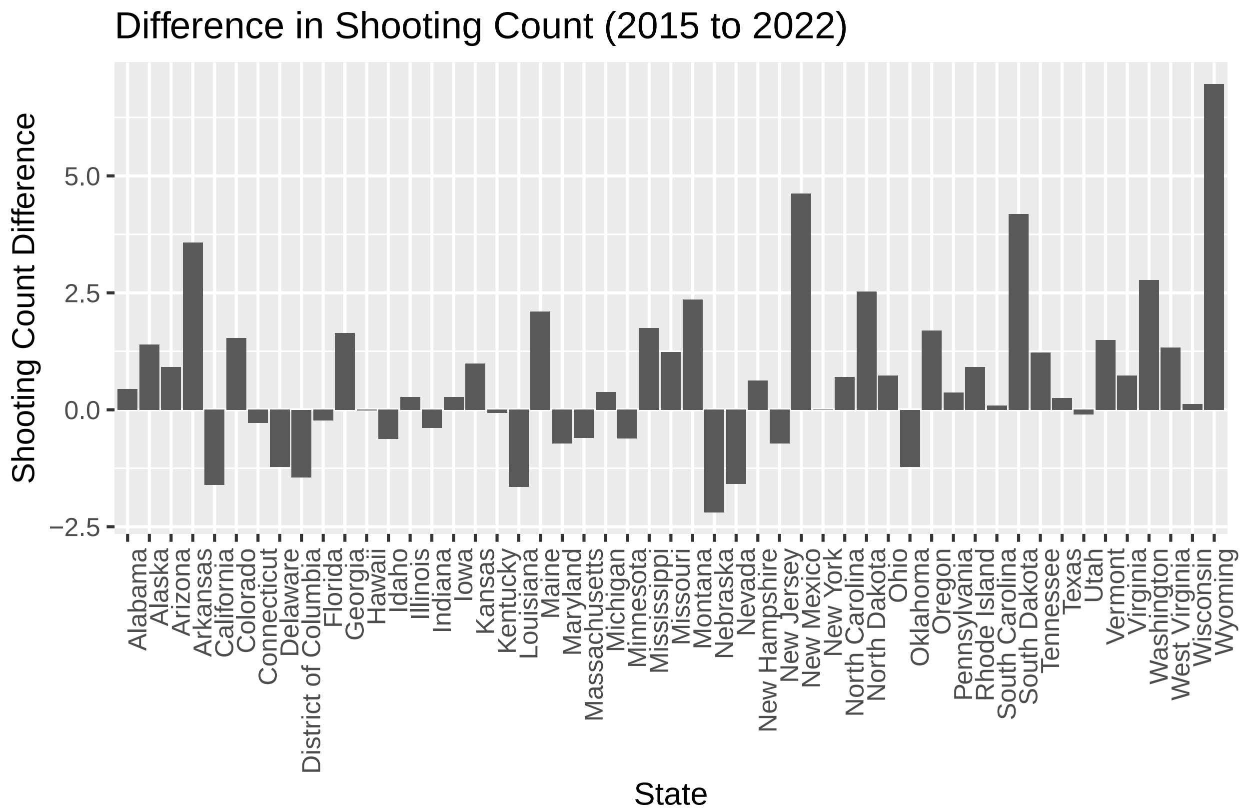
From the pairplots below, we can observe a few things:

1. Shooting Count can be seen to have a slightly negative linear relationship with Unemployment Percentage and White population, and a positive linear relationship with black population, Two or more race population, and Hispanic Latino population.
2. There are some features with high collinearity in the dataset, such as Asian Population and Native Hawaiian Pacific Islander Population.
3. Focusing on Shooting Count, we can observe that it has a higher correlation to Percentage of High School Graduates, Total Checked Firearm, White Population, American Indian Alaskan Native Population, Asian Population, Native Hawaiian Pacific Islander Population, and Hispanic Latino Population.



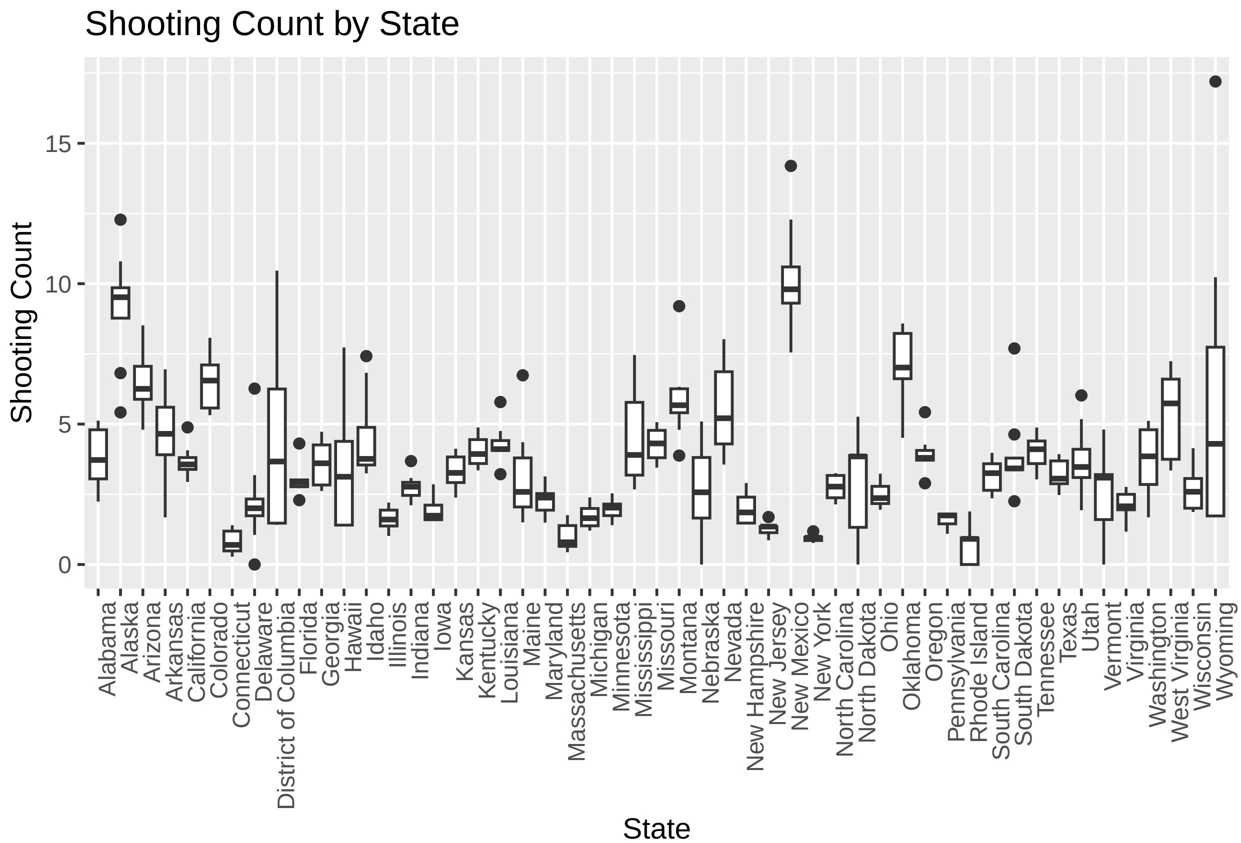
Bar graph of difference in shooting counts between 2015 and 2022

From the bar graph below, it can be observed that in general, there are more cases of police shootings in 2022 than 2015 in each state.



Shooting Count by State

From the boxplots below, it can be observed that New Mexico has the highest average number of police shooting cases while Connecticut has the lowest. We can also see that the number of police shooting cases varies the most in Wyoming, while the number of police shooing cases remains roughly the same in New York.



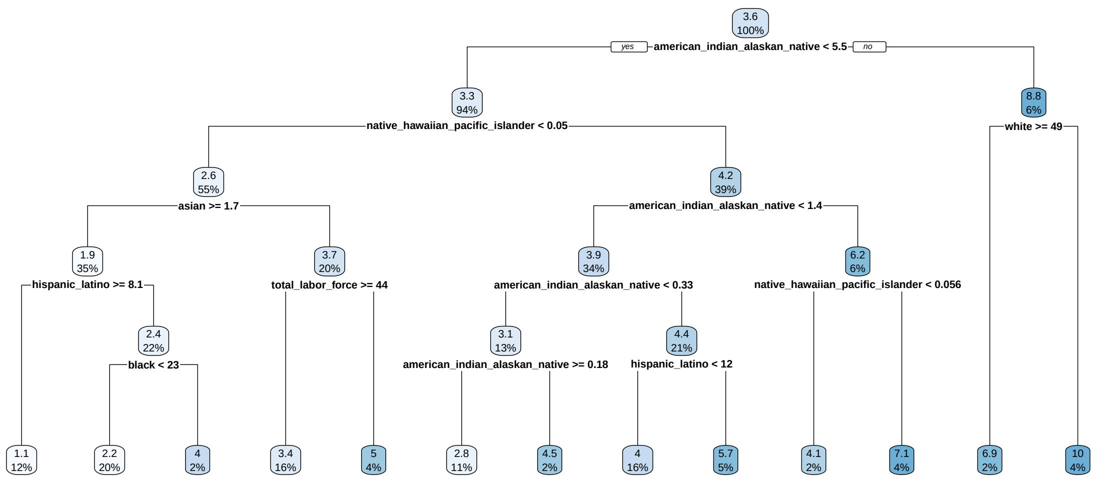
Methodology

Decision Trees

We will train two models, decision tree and random forest, to predict the number of police shootings.

First, the data is processed, 70% is used as the training set, and 30% is used as the test set.

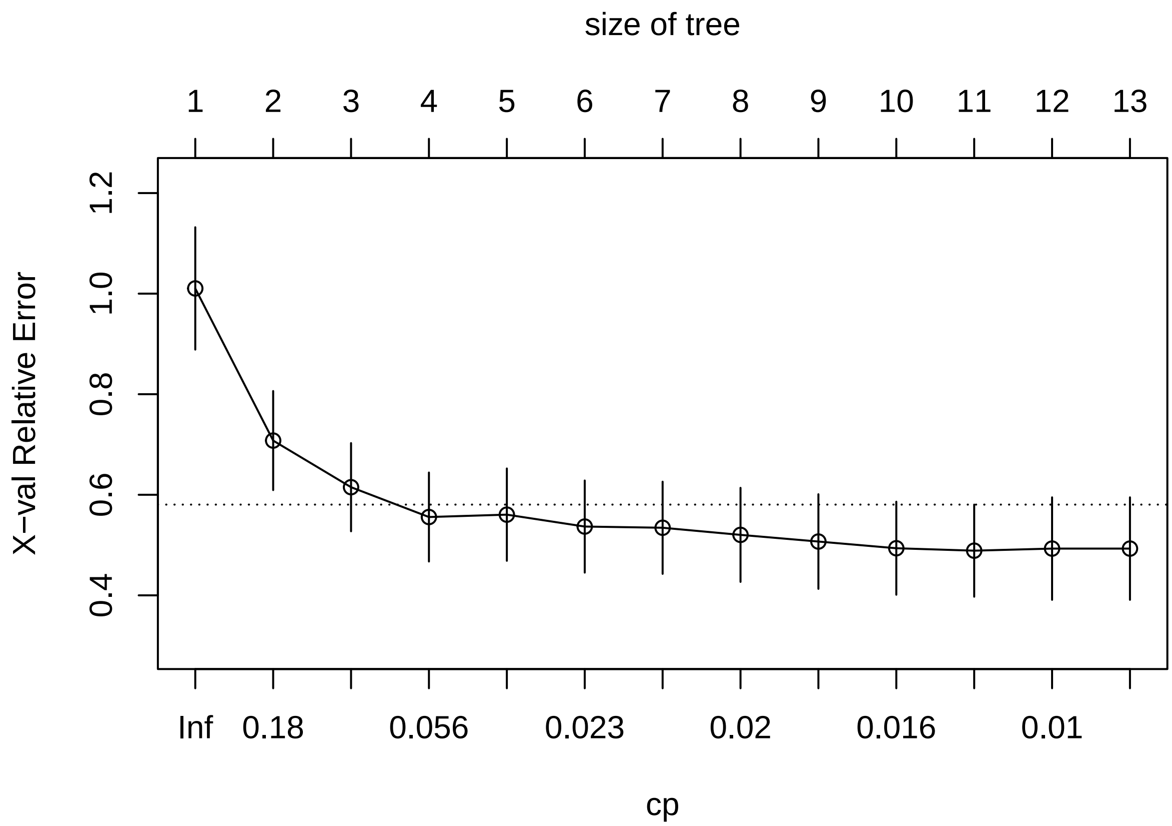
Then we train a decision tree model and plot it out:



To prune a decision tree, we first grow it and then we choose a sub-tree minimizing the regularized loss function

$$\sum_{m=1}^{|T|} \sum_{x^i \in R_m} (y^i - \hat{y}_{R_i})^2 + \alpha|T|$$

Below is the plot of cross-validation error vs α (parameter `cp`):



First, we will look at the table

CP	nsplit	rel error	xerror	xstd
0.3043822	0	1.0000000	1.0103506	0.1216897
0.1120597	1	0.6956178	0.7077939	0.0985250
0.0728857	2	0.5835581	0.6150736	0.0876907
0.0436065	3	0.5106724	0.5557583	0.0885225
0.0231614	4	0.4670659	0.5604920	0.0918373
0.0228732	5	0.4439044	0.5368004	0.0916156
0.0214133	6	0.4210313	0.5343914	0.0916712
0.0186478	7	0.3996180	0.5202588	0.0935197
0.0181823	8	0.3809702	0.5069789	0.0940588
0.0142784	9	0.3627879	0.4936753	0.0925140
0.0100557	10	0.3485094	0.4888518	0.0916169
0.0100266	11	0.3384537	0.4929632	0.1019396
0.0100000	12	0.3284271	0.4929632	0.1019396

We then found the optimal value of $\alpha = 0.01$ and pruned the tree.

Variable importance is measured as a total drop in residual sum of squares due to splits in each variable. Here is how we calculate it with the function `varImp` from `caret`:

	Overall
american_indian_alaskan_native	1.2498830

	Overall
hispanic_latino	1.0421042
total_labor_force	0.9934730
native_hawaiian_pacific_islander	0.8612360
population	0.8398427
black	0.8327607

And here is how we can extract it directly from the model:

	Var_importance
american_indian_alaskan_native	1008.9150
hispanic_latino	504.3055
two_or_more_race	345.7716
native_hawaiian_pacific_islander	332.3874
asian	261.0027
black	247.0228

Finally, let us construct predictions and report the test RMSE and MAE error calculated in ‘caret‘

RMSE	1.5260793
MAE	0.9802309

Now, we build and train a random forest model. Here we set train control to `oob`:

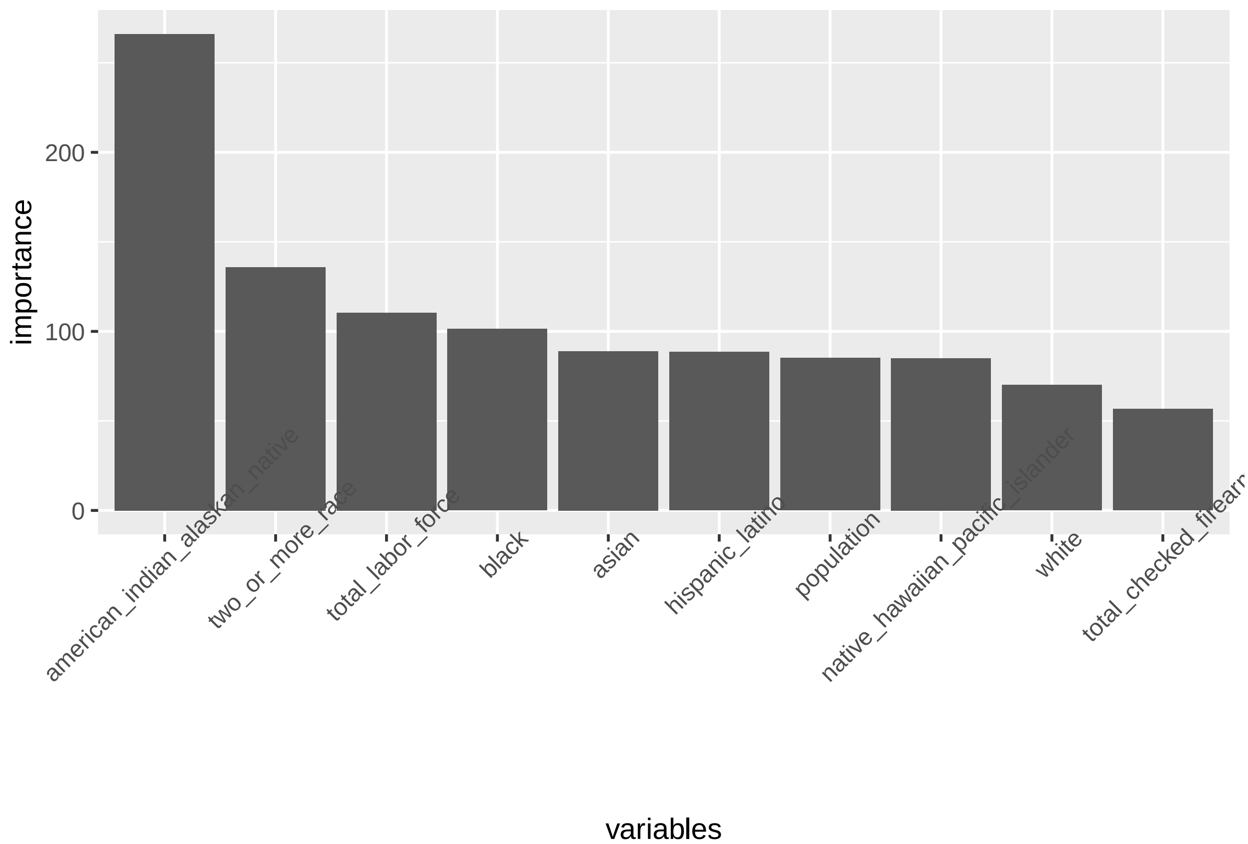
Let us construct predictions and report the test RMSE and MAE error calculated in ‘caret‘

RMSE	1.924458
MAE	1.140770

The top 10 most important variables using Random Forest are:

	Var_importance
american_indian_alaskan_native	266.11694
two_or_more_race	135.92768
total_labor_force	110.40747
black	101.41654
asian	88.85841
hispanic_latino	88.51447

Here is the a plot of variable importance:


importance
200
100
0
american_indian_alaskan_native
two_or_more_race
total_labor_force
black
asian
hispanic_latino
population
native_hawaiian_pacific_islander
white
total_checked_firear
variables

Multiple linear regression (MLR)

Multiple linear regression (MLR) is used to fit the data to a linear equation,

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_n X_n + \epsilon$$

, where Y is the dependent variable and X_n are the independent variables. This equation is then used to predict the value of the shootings_count_per_mil based on the values of the independent variables.

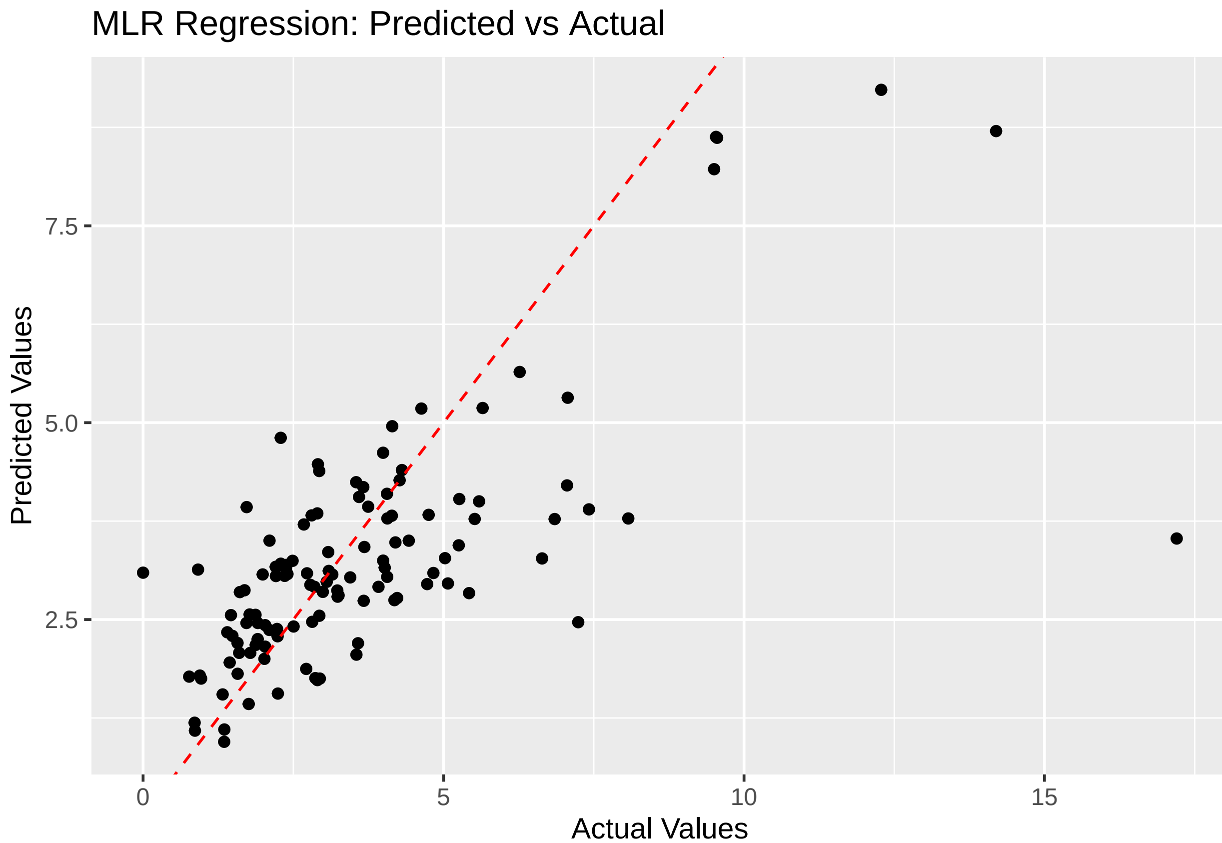
Before applying the MLR model, the data is split into a training set and a test set with a 70/30 ratio.

Firstly, the train_data will be fitted into the MLR model to find the coefficients to the linear equation that best fit the data. Then, the MLR model will be used to predict the shootings_count_per_mil in the test_data.

```
##
## ============================================================
##                                   Dependent variable:
##                               ---------------------------
##                                 shootings_count_per_mil
## ------------------------------------------------------------
## high_sch_grads                     -4.718*** (1.459)
## unemployment_percent                0.155* (0.090)
## total_checked_firearm               0.016** (0.008)
## white                              -0.233*** (0.079)
## black                              -0.231*** (0.088)
## american_indian_alaskan_native       0.088 (0.135)
## asian                              -0.612*** (0.092)
## native_hawaiian_pacific_islander    -0.535 (0.544)
## two_or_more_race                   0.887*** (0.289)
## hispanic_latino                     -0.133 (0.085)
## Constant                           23.937*** (7.250)
## ------------------------------------------------------------
## Observations                              289
## R2                                       0.493
## Adjusted R2                              0.474
## Residual Std. Error                 1.654 (df = 278)
## F Statistic                     26.984*** (df = 10; 278)
## ============================================================
## Note:                            *p<0.1; **p<0.05; ***p<0.01
```

From the summary table, we can see that there are a few insignificant variables. However, due to the lack of other information, we did not remove those variables.

So, to further evaluate the accuracy of the MLR model in predicting the shootings_count in the test_data, a "predicted values vs the actual values" scatter plot with a best fit line is created to visualize the results.



Based on the scatter plot, the points generally follow a linear trend, however a few outliers can be observed. The outliers could have been due to more extreme circumstances that is not taken into account by the MLR model. Thus, we identified the 10 outliers by calculating the residuals.

	28	44	47	48	100
state	Arkansas	Colorado	Colorado	Colorado	Idaho
year	2018	2018	2021	2022	2018
high_sch_grads	1.0294934	1.0062917	1.0255886	1.0279240	1.1038347
unemployment_percent	3.8	2.9	6.3	4.0	3.0
total_labor_force	44.53019	52.94992	53.66363	54.78679	48.62865
total_checked_firearm	8.247866	9.211089	10.820493	9.046947	11.832834
white	57.54194	55.53068	55.40046	55.33109	62.83838
black	11.2807715	3.0706379	3.1471632	3.1873863	0.5108802
american_indian_alaskan_native	0.5944237	0.5135897	0.5123985	0.5185168	0.8153765
asian	1.2178300	2.5925396	2.7307845	2.8105151	1.1507505
native_hawaiian_pacific_islander	0.21240564	0.10127862	0.11240176	0.11498433	0.13007441
two_or_more_race	1.0330125	1.3282770	1.4811324	1.5339578	1.1583415
hispanic_latino	4.760104	14.672516	15.413582	15.688641	7.990073
population	3012161	5697155	5811297	5839926	1752074
shootings_count_per_mil	6.639751	8.074205	7.055224	6.849402	7.419778
Predicted_Value	3.275855	3.783398	4.203500	3.775917	3.898927
Residual	3.363896	4.290808	2.851724	3.073485	3.520851

	255	256	318	389	408
state	New Mexico	New Mexico	Rhode Island	West Virginia	Wyoming
year	2021	2022	2020	2019	2022
high_sch_grads	0.9236175	0.9364306	0.9002641	1.0065378	1.1059873
unemployment_percent	8.2	5.4	3.6	5.0	3.5
total_labor_force	44.16583	45.05585	52.60415	44.22082	49.55064
total_checked_firearm	9.212034	8.223602	4.685478	11.485559	12.087770
white	31.01116	30.80346	60.21453	74.16897	66.13030
black	1.5315043	1.5620268	4.7495086	2.7819879	0.8180522
american_indian_alaskan_native	6.5908970	6.6346511	0.3242592	0.1835386	1.4511998
asian	1.3540564	1.4045040	2.7968386	0.6850250	0.8486689
native_hawaiian_pacific_islander	0.05499186	0.05583568	0.04761275	0.01782469	0.05762142
two_or_more_race	1.0190974	1.0467771	1.2657512	0.9034888	1.1641247
hispanic_latino	36.353917	36.749341	11.497111	1.181944	7.149872
population	2116677	2113344	1096345	1795263	581381
shootings_count_per_mil	12.283405	14.195512	0.000000	7.241279	17.200425
Predicted_Value	9.226647	8.702959	3.094993	2.466122	3.529035
Residual	3.056757	5.492553	-3.094993	4.775157	13.671389

A noteworthy observation is Rhode Island in 2020, where the shootings_per_mil is lower than the predicted value. This discrepancy may suggest the implementation of effective policies in the region. Otherwise, to understand why the shootings_per_mil are higher than predicted shootings for the other outliers, policymakers can conduct further research by employing additional information in hopes of identifying any trends within these outliers.

Furthermore, to improve the accuracy of the MLR predictions, we made use of LASSO regression to regularize the data.

Next, Mean Absolute Error (MAE), Root Mean Squared Error (RMSE) and R-Squared are computed to compare the accuracy of both models.

Model	MAE	MSE	RMSE	R_squared
MLR	1.125897	3.564029	1.887864	0.5086799
LASSO	1.114836	3.460447	1.860228	0.5134841

Based on the evaluation table, LASSO regression provides slightly more accurate predictions than MLR. Although both models may not be very accurate, MLR can still be useful for investigating the outliers.

Artificial Neural Network

We will be training a Feedforward Neural Network to predict the number of police shootings.

Before training the model, the dataset has to be preprocessed. The dataset is first split into train, test and validation set with a 60/20/20 ratio. The sets are then separated into feature sets ($train\_X$, $test\_X$, val_X) and response variable sets ($train\_y$, $test\_y$, val_y). We then normalized them with

$$x \mapsto \frac{x-m}{M-m},$$

where m and M are the minimum and the maximum of x in the respective datasets.

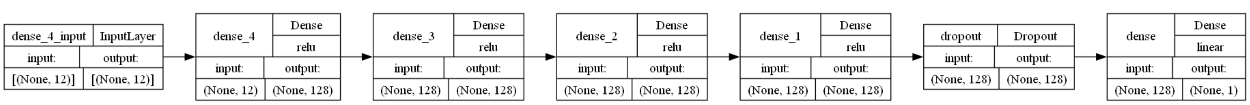
The dimensions of the datasets are as follows:

Dataset	Rows	Columns
train_X	253	12
train_y	253	1
test_X	75	12
test_y	75	1
val_X	80	12
val_y	80	1

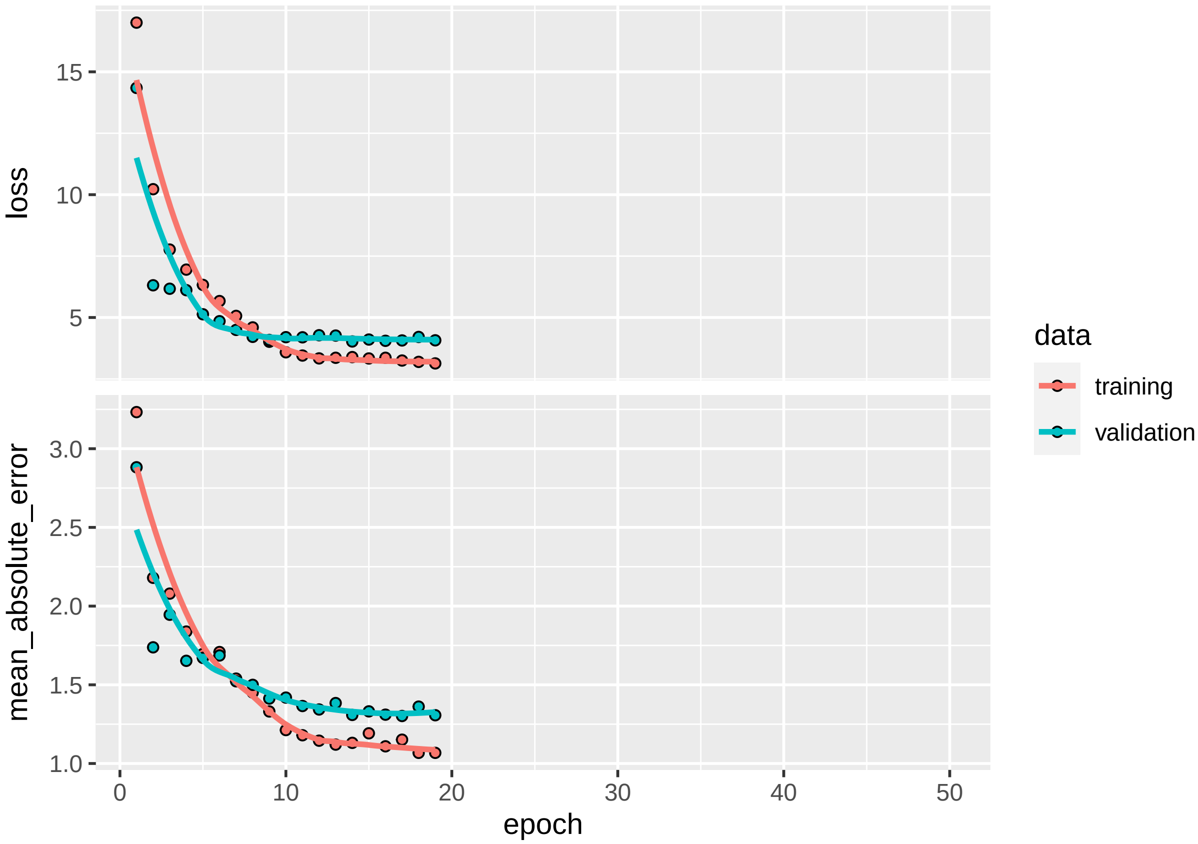
Our model uses `MSE` as the loss function, `adam` as the optimizer, and `MAE` as a metric. On experimenting with different widths (16, 32, 64, 128) and different depths (1, 2, 3, 4) several times, it is observed that different width size performs equally well with similar MSE and MAE values while deeper models give better predictions. Thus, our final model consists of 4 hidden layers with 128 units.

The architecture of our network is as follows:

- Input Layer: Input size of 12, regularized using L1 regularization with a regularization parameter of 0.005.
- Hidden Layer: A Dense layer with 128 units and ReLU activation function.
- Dropout Layer: A dropout layer with a dropout rate of 0.05.
- Output Layer: A Dense layer with 1 unit and Linear activation function.



The model will be fitted with the train and validation set and trained with 50 epochs. However, too much training will cause the model to overfit, resulting in poor performance on the test set. Hence, we implemented early stopping with $patience = 5$, where the training stops when the validation loss does not improve for 5 iterations in a row.



The metrics MSE and MAE are used to verify the performance of the model. On training, we found that

- MSE $\approx$ 2.8: The average squared difference between the predicted and actual values = 2.8.
- MAE $\approx$ 1.1: On average, the model's predictions are off by approximately 1.1 unit.

MSE	2.872759
MAE	1.129115

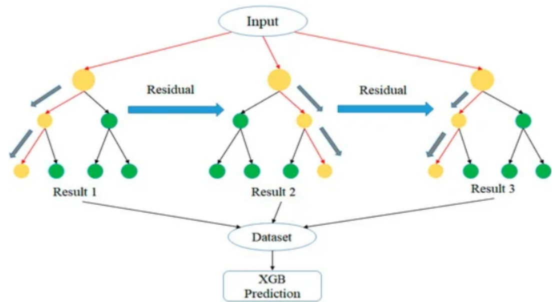
A table of the actual shooting count against the predicted shooting count is also shown below.

Actual	Predicted
2.241255	3.602570
12.279598	9.596032
6.815861	9.582149
5.013679	3.824813
6.639751	3.983049
6.951375	3.932378

The model is observed to perform well in the state level with only 408 observations, thus it can be expected to perform even better at the police precinct level, where there will be more data. However, as experiments are only done on the width and depth, more hyperparameters, such as optimizer, learning rate, dropout rate, can be tuned to achieve a more accurate model.

XGBoost

XGBoost (Extreme Gradient Boosting) is a type of ensemble learning method, which is a machine learning technique that combine the predictions from multiple models to improve the overall performance. In gradient boosting, decision trees are created in a sequential form and these subsequent trees will reduce the errors of the previous tree by learning from them. Weights are assigned to the independent predictors then they are fed into the decision tree. The weight of variables increase or decrease depending on the performance of the decision tree, which affects the residuals of the tree. The subsequent trees are then fitted according to the residuals of the previous trees and reduce their errors by learning from them. These trees are then ensembled to combine their predictive power, giving a stronger and more precise model. XGBoost is a more optimized version of gradient boosting.



The objective function at iteration t that we need to minimize is the following:

$$L^{(t)} = \sum_{x=1}^{n} l(y_i, \hat{y}^{(t-1)} + f_t(\mathbf{x}_i)) + \Omega(f_t)$$

where, $\Omega(f_t) = \gamma T + \frac{1}{2}\lambda \sum_{j=1}^{T} w_j^2$.

l is the loss function that measures the difference between prediction $\hat{y}$ and actual value y_i while $\Omega(f_t)$ is the regularization part of the function, penalizing the model's complexity. In the $\Omega(f_t)$ function, L1 regularization is controlled by γ while L2 regularization is controlled by λ. $\hat{y}^{(t-1)}$ denotes the prediction result for sample i after the t^{th} iteration. $f_t(\mathbf{x}_i)$ represents the t^{th} tree model.

Compared to normal gradient boosting, XGBoost uses L1 and L2 regularization techniques to prevent overfitting and has built-in functionality to handle missing values, whereas gradient boosting require explicit imputation of missing values. It is also faster than gradient boosting as it includes several optimization techniques.

Hyper parameters of XGBoost:

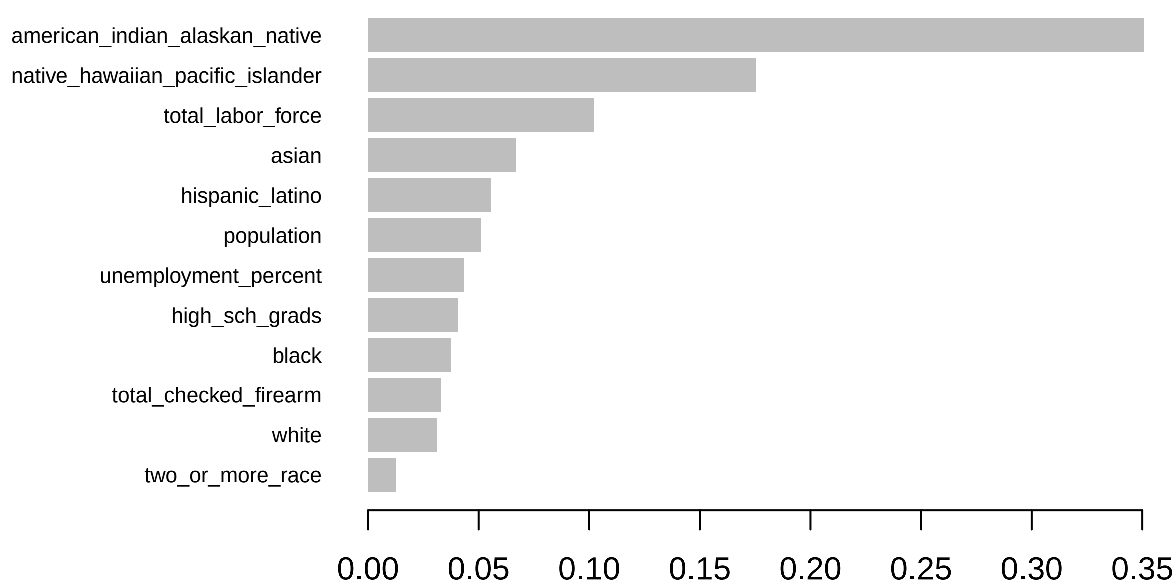
- objective: specify the learning task and the corresponding learning objective. In this case, we used regression with squared loss.
- learning_rate: controls the shrinkage of each tree's contribution to prevent overfitting
- subsample: Denotes the fraction of observations to be random samples for each tree. Used to control overfitting.
- colsample_bynode: the subsample ratio of columns for each node (split)
- lambda: L2 regularization on weights
- max_depth: maximum depth of a tree. Used to control overfitting.

Firstly, we split the data into train and test with a 70/30 ratio. We then fit a XGBoost model to the data:

Parameters.objective	reg:squarederror
Parameters.learning_rate	0.05

Parameters.subsample	0.9
Parameters.colsample_bynode	1
Parameters.lambda	2
Parameters.max_depth	5
Parameters.validate_parameters	TRUE
Number.of.Features	12
Number.of.Iterations	55
Best.Iteration	55
Best.Ntreelimit	55
Best.Score	2.041565
Best.Message	[55] valid-rmse:2.041565
Number.of.Features.Used	12

XGBoost has a built-in feature importance function, which allows us to carry out feature selection by finding out the feature importance values for all variables. Hence, we find the feature importance for each variable and plot them out:



From the variable importance model, the AMERICAN_INDIAN_ALASKAN_NATIVE variable is the most important, followed by the NATIVE_HAWAIIAN_PACIFIC_ISLAND and TOTAL_LABOR_FORCE variable. This suggests that the race of the population, specifically American Indian Alaskan Natives, significantly affect the number of shootings for the XGBoost model. We now find the error for the XGBoost model:

MSE	4.167986
MAE	1.151542

Conclusion

We have introduced 5 models (Decision tree, Random Forest, Multiple Linear Regression, Artificial Neural Network and XGBoost) to predict in police shooting rates across U.S. precinct and identifying patterns to help with resource deployment to control of shootings.

According to the Decision Tree and Random Forest models, we can know that the most important variable is AMERICAN_INDIAN_ALASKAN_NATIVE. Our Multiple Linear Regression model is regularized with LASSO, which gave a better performance as compared to vanilla linear regression. In the Artificial Neural Network model, it appears that every width size performs equally well with similar MSE and MAE values, while deeper models perform better with lower MSE and MAE values. Finally, for XGBoost, the most important variable is also AMERICAN_INDIAN_ALASKAN_NATIVE. While XGBoost has regularization techniques to prevent overfitting and is an efficient model, it is a 'black box' algorithm that is difficult to interpret.

Model	MSE	MAE
Artificial Neural Network	2.8272	1.133631
XGBoost	4.167986	1.151542
Decision Tree	2.328917	0.9802309
Random Forest	3.705386	1.14077
MLR	3.460447	1.114836

As shown in the table above, we can know that among all 5 models, Decision Tree has the highest accuracy and the smallest MSE and MAE.

In conclusion, our exploration into predicting police shooting rates in U.S. using various machine learning models has provided valuable insights. The predictive capabilities demonstrated by our models, particularly the Decision Tree model with its highest accuracy, lay the groundwork for future endeavors. As we navigate the complex landscape of law enforcement and social justice, the insights gained from this study can contribute to informed decision-making, resource allocation, and policy formulation.

In essence, our analysis serves as a testament to the potential of machine learning in shedding light on critical societal issues. While acknowledging the complexities involved, our findings encourage further research and collaboration between data scientists, policymakers, and law enforcement agencies to foster a more just and equitable society.

References

Cheatham, A., & Maizland, L. (2022, March 29). What Are Police Like in Other Countries? Council on Foreign Relations. Retrieved November 19, 2023, from https://www.cfr.org/backgrounder/how-police-compare-different-democracies

Levin, S. (2023, January 6). 'It never stops': killings by US police reach record high in 2022. The Guardian. Retrieved November 19, 2023, from https://www.theguardian.com/us-news/2023/jan/06/us-police-killings-record-number-2022

Shjarback, J. (2021, November 11). Lack of data on nonfatal police shootings hurts understanding of police use of force. The Washington Post. Retrieved November 19, 2023, from https://www.washingtonpost.com/crime-law/2021/11/11/non-fatal-police-shootings-disparities-data/

Berman, M. (2021, May 8). Most police departments in America are small. that's partly why changing policing is difficult, experts say. The Washington Post. Retrieved November 21, 2023 from https://www.washingtonpost.com/nation/2021/05/08/most-police-departments-america-are-small-thats-partly-why-changing-policing-is-difficult-experts-say/

Appendix

List of datasets combined

1. `fatal-police-shootings-data.csv`: Records death-related information about each incident and victim. This dataset is aggregated to obtain the total number of shootings in each US state from 2015 to 2022. The state abbreviations are renamed to their full form for ease of data joining in the later parts. To ensure that there are values for every state from 2015-2022, data of states with no data in a particular year are generated with `shootings_count` = 0.

2. `high_school_grads.csv`: Records the number of people whose highest qualification is high school in each US state from 1980 to 2026 (projected). The state abbreviations are renamed to their full form for ease of data joining in the later parts, and only data from 2015-2022 are selected. The dataset is also converted to a long data, where columns of 2015-2022 are converted to a single column ,`year`. The values are made into a new column, `high_sch_grads`.

3. `unemployment.csv`: Records the labour force, employment and unemployment rates in each US state from 1976 to 2022. Only data from 2015-2022, the unemployment percentage and total labor force are selected as features.

4. `nics-firearm-background-checks.csv`: Records the number of checked firearms in each US state from 1998 to 2023. This dataset is aggregated to obtain the total number of firearms in each US state from 2015 to 2022.

5. `race_population.csv`: Records the population of the races in each US state from 2000 to 2022. The races include: White, Black, American Indian and Alaskan Native, Asian, Native Hawaiian and Other Pacific Islander, Two or More Race Groups, and Hispanic or Latino. This dataset is converted to a wide data, where the value of the races are converted to columns.

6. `population_usa.csv`: Records the population of each US state from 1900 to 2022. This dataset is converted to a long data, where columns of 2015-2022 are converted to a single column ,`year`. The values are made into a new column, `population`.

Description of Features

Feature	Description
state	A state in US
year	Year

Feature	Description
high_sch_grads	% of people whose highest qualification is high school
unemployment_percent	% of unemployed people
total_labor_force	% Population of Labour Force
total_checked_firearm	Total # of checked firearms
white	% Population
black	% Population
american_indian_alaskan_native	% Population
asian	% Population
native_hawaiian_pacific_islander	% Population
two_or_more_race	% Population
hispanic_latino	% Population
population	Total population
shooting_count_per_mil	# of police shootings per 1 million people

Width and Depth Experiments

Depth	Width	MAE	MSE
1	16	0.9943658	2.658836
1	32	0.9886726	2.569897
1	64	1.0297630	2.707983
1	128	1.0083946	2.591784
2	16	1.0117185	2.616495
2	32	0.9991210	2.589253
2	64	1.0011917	2.642034
2	128	1.0233067	2.604966
3	16	1.0469836	2.691711
3	32	1.0198650	2.545069
3	64	1.0398048	2.564570
3	128	1.0550179	2.662320
4	16	1.0729930	2.742829
4	32	1.0339223	2.537532
4	64	1.0653225	2.742532
4	128	0.9874745	2.557349