# **Quantifying Return Ambiguity: A Cross-Entropy Approach for Dynamic Portfolio Management**

Financial markets are complex systems operating under conditions of pervasive uncertainty, necessitating robust methodologies that extend beyond idealized models.1 Traditionally, this uncertainty has been addressed through the well-established paradigm of **risk**, a situation where the probabilities of various outcomes are objectively known [ref]. In contrast, a deeper form of uncertainty, known as **ambiguity** or Knightian uncertainty, arises when the probability distributions themselves are unknown, imprecise, or subjectively perceived.2 Ambiguity is defined as a situation in which the first-order probabilities (the likelihoods of the states of nature or outcomes) are not uniquely assigned but are themselves treated as random variables.1 This fundamental distinction highlights that **risk and ambiguity are independent concepts**. Risk aversion is a preference for a smaller payoff with a greater likelihood of success when probabilities are known, while ambiguity aversion is a dislike for options where the probabilities are unknown.2 This behavioral phenomenon is a key driver of market outcomes, such as stock market volatility and incomplete contracts.3 Despite the importance of this distinction, traditional risk measures often fail to capture this deeper form of uncertainty, which is rooted in incomplete or imprecise knowledge about future returns.1 This creates a significant gap in financial modeling and risk management, underscoring the need for a new measure of ambiguity that is not only distinct from traditional risk metrics but also possesses superior predictive power for dynamic portfolio and risk management.

To address this gap, this paper introduces a new measure of ambiguity, which we term the **Cross-Entropy Ambiguity Measure (CEAM)**, grounded in the multiplier-preference model pioneered by Hansen and Sargent.4 Our motivation stems from the limitations of existing ambiguity measures and the ability of a cross-entropy-based approach to capture a more dynamic, granular form of uncertainty in financial markets. The use of cross-entropy is motivated by its ability to precisely quantify the "information lost" when a model probability distribution is used to approximate the true, unknown distribution.5 This directly measures the inefficiency of a financial model, providing a highly interpretable and robust proxy for model misspecification.6 The mathematical relationship

Cross-Entropy = Shannon Entropy + KL Divergence means that minimizing cross-entropy is equivalent to minimizing the Kullback-Leibler (KL) divergence between the empirical and benchmark distributions.5 The multiplier-preference framework provides a powerful lens through which to understand ambiguity aversion. It models decision-making under uncertainty by evaluating options against the worst-case prior within a set of plausible distributions, with KL divergence serving as the penalty for how far these priors deviate from a nominal one.4 This allows the measure to capture a decision-maker's perceived ambiguity in a quantifiable way, directly reflecting their aversion to uncertainty about the underlying probabilities.

Our proposed measure, CEAM, distinguishes itself from existing methods by leveraging high-frequency intraday data to capture the full-shape changes of return distributions, including tail behavior and skewness, which go beyond traditional volatility-based measures.1 Methods such as Choquet Expected Utility (CEU) 3 and Maxmin Expected Utility (MEU) 3 are primarily theoretical, using non-additive beliefs or multiple priors that are less direct for empirical quantification using market data and often lead to non-linear optimization. Similarly, measures like Expected Utility Under Uncertainty (EUUP) 1 quantify ambiguity by examining the variance of probability densities, a more simplistic approach that may not capture the full distributional shape or higher-order moments crucial in financial markets. In contrast, CEAM’s unique theoretical foundation provides a data-driven quantification method that connects directly to financial data and is designed for dynamic portfolio management. A key advantage is its superior predictive power for future returns, a capability that is central to the paper’s argument for its utility in practical financial applications. It also provides direct connections to system identification and risk measure theory, enabling a data-driven and interpretable approach that is not as apparent with other methods.8

This paper makes several distinct contributions to the fields of financial economics and distributionally robust optimization. First, we introduce a novel, cross-entropy-based ambiguity measure, the Cross-Entropy Ambiguity Measure (CEAM), developed under the multiplier-preference model. This measure uniquely quantifies return ambiguity using high-frequency intraday data, providing a dynamic and granular characterization of uncertainty that is distinct from traditional risk metrics and exhibits superior predictive power for future returns.1 Second, we provide a rigorous empirical analysis demonstrating the superior predictive power of CEAM. Our findings reveal a robust positive relationship between CEAM and next-day returns, an effect that is statistically distinct from that of conventional risk measures such as realized volatility, skewness, and kurtosis.1 The minimal multicollinearity between our measure and these traditional metrics further highlights that CEAM captures an independent dimension of market uncertainty.1 Third, and most importantly, we conduct a series of causal econometric analyses to establish that CEAM has a distinct and independent effect on future returns. Our methodology extends beyond simple correlation by incorporating several key analyses, including Granger causality to demonstrate predictive power across multiple time lags. We analyze how the relationship between ambiguity and returns is moderated by different market conditions (bull vs. bear markets) and varying levels of realized volatility.1 This provides a nuanced understanding of when and how ambiguity becomes a more significant factor.1 By including control variables like turnover rate in our regressions, we isolate the unique effect of ambiguity from other market factors. Furthermore, we investigate potential mediating channels, such as liquidity provision, through which ambiguity influences returns.1 To address potential endogeneity and provide a more robust causal inference, we propose and implement an instrumental variable approach, which strengthens the conclusion that ambiguity is a distinct and independent driver of returns, rather than just a correlated proxy.

The remainder of this paper is structured as follows. Section 2 provides a comprehensive review of the theoretical foundations of ambiguity and existing measures. Section 3 details the methodology for constructing the CEAM. Section 4 presents the empirical results, including the econometric analyses and backtesting. Section 5 concludes with a summary of findings and outlines directions for future research.

#### Works cited

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