# **Redrafting the Introduction to a Research Paper on KL-based Ambiguity in Financial Markets**

## **1. Introduction**

This paper addresses the critical challenge of uncertainty in financial markets, specifically focusing on the advanced paradigm of distributionally robust optimization (DRO) and the pivotal role of Kullback-Leibler (KL) divergence in quantifying and managing ambiguity in financial decision-making. The introduction broadly establishes the context of uncertainty in finance, narrows the focus to distributional ambiguity, details the strengths of KL-based ambiguity sets for financial applications, outlines the paper's contributions to the literature, and concludes with a structural overview.

### **1.1. Background and Context: The Pervasive Challenge of Uncertainty in Financial Markets**

Financial markets are inherently complex systems operating under conditions of pervasive uncertainty. Achieving perfect information about future stock prices, asset returns, or macroeconomic variables is unattainable, necessitating robust methodologies capable of accounting for deviations from idealized models.1

Traditionally, uncertainty in finance has been addressed through established paradigms such as stochastic modeling and classical risk management. Stochastic models, often relying on assumptions of known probability distributions (e.g., Gaussian or log-normal for asset returns), optimize financial decisions based on these assumed statistical properties. Conversely, classical risk management typically focuses on bounded but unknown disturbances, aiming to guarantee performance within a worst-case scenario over a defined set of uncertainties, without necessarily relying on precise probabilistic models.

A crucial distinction in uncertainty modeling, particularly relevant in economics and finance, lies between "risk" and "ambiguity," also known as Knightian uncertainty. Risk refers to situations where the probabilities of various outcomes are objectively known, allowing for precise probabilistic analysis.2 For instance, the historical volatility of a well-established stock might be considered a known risk. In contrast,

**ambiguity is defined as a situation in which the first-order probabilities (the likelihoods of the states of nature or outcomes) are not uniquely assigned but are themselves treated as random variables**.2 Essentially, it's about uncertainty surrounding the probabilities themselves, rather than just the uncertainty of outcomes given known probabilities (which is risk).2 This fundamental distinction highlights that

**risk and ambiguity are independent concepts**: risk aversion occurs when the probabilities of outcomes are known, but a decision-maker still prefers a smaller payoff with a greater likelihood of success.3 Conversely, ambiguity aversion arises when the probabilities of outcomes are unknown, leading individuals to prefer options with known probabilities over those with unknown ones.2 This independence means that while both contribute to overall uncertainty, they represent distinct challenges in financial decision-making, as one deals with known odds and the other with unknown odds.2 This distinction is fundamental because economic agents and financial systems react differently to known probabilities versus unknown ones.2 Ambiguity aversion, a preference for known risks over unknown risks, is a well-documented behavioral phenomenon that can explain incomplete contracts, volatility in stock markets, and influence investor participation.2

The evolution of uncertainty modeling paradigms highlights a progression from simpler assumptions to more sophisticated considerations. Traditional financial models, while powerful, are inherently limited by their reliance on precisely known distributions for financial variables. In many practical financial scenarios, the exact statistical properties of market disturbances or asset returns are not perfectly characterized, leading to significant model misspecification.4 This inadequacy in accounting for situations where the

*form* of the uncertainty itself is unknown underscores a fundamental limitation of these earlier approaches. The recognition of "ambiguity" as a distinct form of uncertainty, where probabilities are unknown, represents a natural and necessary advancement in addressing increasingly complex real-world financial conditions. This progression in research reflects a growing need for sophisticated uncertainty quantification methods that move beyond simple parametric assumptions, directly leading to the development of frameworks like Distributionally Robust Optimization (DRO).4 This paradigm shift aims to actively robustify financial systems against the uncertainty in the uncertainty model itself, providing resilience to what can be termed "second-order uncertainty" and mitigating financial instability.

### **1.2. Quantifying Ambiguity: The Role of Information-Theoretic Measures**

To systematically address ambiguity in financial contexts, quantitative measures are essential. Kullback-Leibler (KL) divergence, a foundational concept in information theory, serves as a powerful tool for this purpose. It is defined as a "statistical distance" or "relative entropy" that quantifies how much a model probability distribution Q differs from a true probability distribution P.7 Mathematically, it measures the information lost when

Q is used to approximate P.12

In the context of financial modeling, the interpretation of KL divergence extends to quantifying uncertainty or information loss. It represents the "expected excess surprisal" when using an approximating distribution Q (our model's assumed distribution for, say, stock returns) instead of the true distribution P (the actual, unknown distribution of returns).7 This directly translates to a quantifiable measure of information loss or the additional uncertainty introduced by the approximation. For instance, if a financial model mistakenly predicts an event as impossible when it is merely rare, the KL divergence will be extremely large, reflecting significant information loss or "surprise".14 Furthermore, the rate of return expected by an investor can be directly related to the relative entropy between the investor's believed probabilities and the official odds, highlighting a general connection between financial returns and divergence measures.7

A notable property of KL divergence is its asymmetry; it is not a true metric distance because DKL​(P∣∣Q) is generally not equal to DKL​(Q∣∣P), nor does it satisfy the triangle inequality.7 This asymmetry, however, is not a limitation but a crucial feature. It reflects different "directions" of information gain or approximation error.16 For example,

DKL​(P∣∣Q) measures the cost of using Q when P is true, which is distinct from the cost of using P when Q is true. This distinction is particularly relevant in robust financial design, where the concern might be hedging against unforeseen true distributions that deviate from a nominal market model.

KL divergence serves as a robust proxy for model misspecification in financial applications. Since it quantifies the "information lost when Q is used to approximate P" 12, where P can be considered the true, unknown distribution of a financial variable's behavior or noise, and Q is our model's assumed distribution, a high KL divergence directly indicates that our model (Q) is a poor approximation of financial reality (P). This directly links an abstract information-theoretic concept to practical financial modeling challenges, providing a quantifiable measure of how "wrong" a current probabilistic financial model might be. This allows researchers to move beyond qualitative statements about "unknown probabilities" to a precise quantification of the severity of model misspecification, which is a primary source of ambiguity in financial markets.

Furthermore, the functional asymmetry of KL divergence is a significant advantage in robust optimization for finance. This asymmetry allows for nuanced modeling of different types of robustness concerns. When defining an ambiguity set for financial distributions, the choice between DKL​(P∣∣Q) and DKL​(Q∣∣P) implies different assumptions about the *direction* of deviation from the nominal distribution and the associated cost. This mathematical characteristic enables the design of robust financial strategies that specifically hedge against distributions that are "hard to distinguish" from the nominal, or those that represent a "maximal information gain" if they were true. This provides a rigorous mathematical foundation for designing financial systems that are robust to specific types of misspecification errors, rather than merely general bounded errors.

### **1.3. Research Motivation and Problem Statement: Addressing Distributional Ambiguity in Financial Markets**

Despite advancements in traditional financial modeling and risk management, these methods often struggle when the *probabilistic characteristics* of financial variables (e.g., asset returns, volatilities, economic shocks) are unknown or only partially known.4 This "distributional ambiguity" implies that even if the underlying economic dynamics are somewhat understood, the precise statistical properties of noise or external inputs are not precisely defined. Such scenarios are common in real-world financial applications where data is scarce (e.g., for new assets), models are simplified, or market conditions are highly variable and subject to structural breaks. This leads to suboptimal portfolio allocation, inaccurate risk management, and contributes to overall financial instability.2

To address this, Distributionally Robust Optimization (DRO) has emerged as a promising paradigm.4 In DRO, the objective is to synthesize a financial decision policy (e.g., portfolio weights, hedging strategies) that minimizes the expected cost (e.g., portfolio variance, downside risk) under the

*worst-case distribution* within a defined "ambiguity set" of possible probability distributions.4 This approach provides robustification in the space of probabilities, effectively hedging against model misspecifications and the inherent ambiguity in financial markets.4

The significance of addressing distributional ambiguity in finance is profound. Improving the reliability and safety of financial systems in real-world scenarios where precise probabilistic models are unavailable is critical for applications such as dynamic portfolio optimization, risk management, and the pricing of complex derivatives.6 For instance, in safety-critical financial tasks, the ability to quantify and manage uncertainty is paramount for bolstering model trustworthiness and ensuring safer real-world deployments.6 Beyond practical applications, addressing distributional ambiguity contributes to a more complete theoretical understanding of decision-making under deep uncertainty in economic contexts, bridging the gap between theoretical constructs and practical needs.7 The behavioral phenomenon of "ambiguity aversion" further underscores the importance of this research, as it directly impacts investor behavior and market dynamics, leading to outcomes like stock market volatility.2 Moreover, studies suggest that improved financial literacy can reduce perceived ambiguity among investors, contributing to a more stable economic system.3

Crucially, **taking ambiguity into account can bring additional returns to investors while maintaining a fixed level of risk**. By reasonably managing the ambiguity level through robust optimization, financial strategies can be designed to capture opportunities that might be overlooked by traditional models that assume known probabilities. This means that the **risk level can stay fixed while the return can be enhanced** by effectively navigating the uncertainty surrounding the probability distributions themselves. This is achieved by designing strategies that are robust to a set of possible distributions, rather than just a single nominal one, thereby exploiting potential upsides within the bounds of acceptable risk. This capability is particularly valuable in dynamic financial markets where precise probabilistic forecasts are often elusive, allowing for more adaptive and profitable investment decisions without necessarily increasing exposure to known risks.

DRO serves as a crucial bridging framework between theoretical ideals and practical necessities in finance. Traditional stochastic financial models often assume perfect knowledge of probability distributions, an assumption frequently violated in real-world financial systems. Concurrently, behavioral decision theory acknowledges that human decision-makers exhibit "ambiguity aversion," reacting differently to unknown probabilities compared to known risks.2 DRO provides a rigorous mathematical framework to quantify and manage this ambiguity in engineered financial systems. This implies that DRO is not merely a theoretical exercise but a practical imperative for deploying robust financial strategies in complex environments where "true" distributions remain elusive. This convergence of ideas from information theory, optimization theory, and decision theory allows for the design of financial systems that are not only mathematically optimal but also practically resilient to unforeseen statistical variations, thereby addressing a critical gap between idealized models and real-world operational challenges.

### **1.4. The Strength of KL-Based Ambiguity Sets in Financial Optimization**

The choice of discrepancy measure for defining ambiguity sets is critical in DRO. Kullback-Leibler divergence stands out as a particularly suitable and powerful choice for constructing these sets in financial applications due to its unique advantages compared to other measures.4

One significant advantage is its direct connection to system identification and Maximum Likelihood Estimation (MLE). MLE, a widely used method for estimating parameters of financial models from historical data, can be interpreted as searching for a model that minimizes the KL-divergence to the true system.4 This inherent link allows for a meaningful and data-driven estimation of the "radius" (

ρ) of the KL ambiguity set, as it can be directly related to the log-likelihood of observations.4 This provides an interpretable way to quantify the degree of trust in the nominal financial distribution, making the design of robust financial strategies more grounded in empirical data.

Furthermore, KL-based ambiguity sets exhibit a deep and clear connection with the theory of coherent risk measures, such as Conditional Value at Risk (CVaR).4 This relationship enables the design of financial policies that explicitly account for "tail-risk" or extreme, low-probability events, which is crucial in safety-critical financial applications like portfolio management during market crashes or stress testing.4 This integration with risk theory is less directly apparent when using other discrepancy measures, offering a unique capability for designing novel risk-aware financial formulations.4

KL divergence also possesses favorable mathematical properties that enhance its utility in financial optimization. Unlike Shannon entropy, which can become undefined or negative for continuous distributions (like asset returns), relative entropy (KL divergence) remains well-defined for both discrete and continuous probability distributions.7 It is also invariant under parameter transformations, ensuring dimensional consistency and robustness to changes in variable representation.7 Its convexity in the pair of probability measures is another valuable property for guaranteeing the well-posedness and solvability of optimization problems in finance.7 Moreover, in certain robust control problems, such as Distributionally Robust Linear Quadratic Gaussian (DR-LQG) problems, the use of KL ambiguity sets can preserve the linearity of the optimal control policy.4 This preservation of structure is a substantial practical advantage, as linear policies are generally easier to analyze and deploy in large-scale financial models.

While other measures exist for defining ambiguity sets, such as Wasserstein distance or moment-based constraints 4, KL divergence offers a unique blend of theoretical rigor, practical interpretability, and favorable mathematical properties for robust financial optimization. For example, while Wasserstein distance is a true metric and can be computationally tractable, it may not offer the same direct connections to financial system identification or risk measures as KL divergence.4 Moment-based approaches, on the other hand, rely on bounding specific moments of the distribution, which might not fully capture the complex shape of unknown financial distributions, especially during periods of market instability.4

KL divergence can be considered a "goldilocks" measure for robust financial optimization. Its properties, such as being well-defined for continuous distributions and invariant under parameter transformations 7, address limitations found in measures like Shannon entropy.7 Crucially, its explicit connections with system identification and risk measure theory 4 provide deeper theoretical links compared to simpler statistical distances. This positions KL divergence not merely as one option among many, but as a highly advantageous choice for specific classes of robust financial optimization problems, particularly those involving model misspecification and stringent risk management. Its mathematical properties simplify analysis, often leading to computationally tractable solutions, such as linear optimal policies 4, while providing meaningful interpretations for financial practitioners.

Furthermore, the interplay between epistemic uncertainty and KL ambiguity is significant in finance. Epistemic uncertainty, which stems from a lack of knowledge due to limited data or model inadequacies, is prevalent in financial markets, especially for new assets or during regime shifts, and is potentially reducible with more training data.6 KL divergence directly quantifies the "information lost when Q is used to approximate P" 12, effectively measuring how much a financial model Q deviates from a true P. This establishes a direct relationship: high epistemic uncertainty, reflecting a significant lack of knowledge about the true underlying distribution P of financial variables, will result in a larger KL divergence between our model Q and P.6 Consequently, this necessitates the definition of larger KL-based ambiguity sets to ensure the robustness of the financial system.6 The radius (

ρ) of the ambiguity set, therefore, becomes a quantifiable measure of the level of epistemic uncertainty that the financial system is designed to hedge against. This connection implies that methods for quantifying epistemic uncertainty can directly inform the design and calibration of KL-based ambiguity sets in robust financial optimization, providing a pathway for data-driven determination of the required robustness level.

**Table 1: Comparison of Key Ambiguity Measures for Robust Optimization in Finance**

| Measure Type | Definition/Concept | Key Properties | Strengths for Robust Optimization in Finance | Limitations/Challenges |
| --- | --- | --- | --- | --- |
| **Kullback-Leibler (KL) Divergence** | Measures information lost when Q approximates P; relative entropy.7 | Asymmetric, not a metric, well-defined for continuous distributions, invariant under parameter transformations, convex.7 | Direct connection to MLE/System ID for radius estimation from financial data 4; strong integration with coherent risk measures (e.g., CVaR) for tail-risk management 4; often preserves linear optimal policy structure for tractability.4 | Asymmetry can be counter-intuitive for general "distance," but is a feature for robustness 16; no upper bound in general case.7 |
| **Wasserstein Distance** | A true metric measuring the "cost" of transforming one distribution into another; "earth mover's distance".4 | Symmetric, satisfies triangle inequality, true metric. | Provides geometric intuition; often leads to convex optimization problems; useful for scenarios with outliers or heavy-tailed financial distributions. | Less direct connection to financial system identification or risk measures compared to KL divergence 4; computational complexity can be high for high-dimensional financial data. |
| **Moment-based Ambiguity Sets** | Defines ambiguity set by bounding moments (e.g., mean, variance) of the unknown distribution.4 | Focuses on statistical properties; can be computationally tractable. | Simple to define and interpret; useful when only partial information about moments of financial distributions is available. | May not capture the full shape or tail behavior of the unknown financial distribution, especially during instability; less robust to non-Gaussian or multimodal distributions.4 |
| **Behavioral Ambiguity Measures (e.g., Maxmin Expected Utility)** | Rationalizes ambiguity aversion by assuming multiple prior subjective probability distributions.2 | Axiomatic foundation for decision-making under ambiguity; separates belief from attitude.19 | Explains observed human behavior in financial markets (e.g., Ellsberg paradox) 2; provides a framework for decision-making when probabilities are unknown. | Less direct applicability to quantitative financial modeling for defining ambiguity sets; often leads to non-linear, complex optimization.2 |

### **1.5. Contributions of This Paper**

This paper makes several distinct contributions to the field of financial economics and distributionally robust optimization, specifically leveraging the unique advantages of Kullback-Leibler divergence for managing uncertainty in financial markets. The overall contribution lies in advancing the theoretical understanding and practical applicability of robust financial strategies in the face of significant distributional ambiguity.

Specifically, the novel contributions of this research are as follows:

* **Novel Framework for Financial Risk Management with KL-based Ambiguity Sets:** This paper proposes a novel framework for robust financial decision-making (e.g., optimal portfolio allocation, hedging strategies) by leveraging KL-based ambiguity sets. This extends the applicability of distributionally robust optimization to financial systems with time-varying and potentially non-Gaussian uncertainties in asset returns or economic shocks, thereby addressing a critical gap in current methodologies that often rely on restrictive distributional assumptions.4 This contribution directly addresses the limitations of traditional financial models under ambiguity, as identified in Section 1.3.
* **Rigorous Analysis of Optimal Financial Policies:** A rigorous proof is provided for the properties of optimal financial policies (e.g., the optimality of linear policies for certain robust portfolio problems) derived under KL-based distributional robustness. This offers foundational theoretical guarantees that were previously lacking or only conjectured for such complex uncertainty structures in finance, ensuring the analytical tractability and practical implementability of the derived financial strategies.4 This directly builds upon the strength of KL-based ambiguity sets in preserving desirable policy structures.
* **Development of an Efficient Computational Algorithm for Financial Optimization:** A novel computational scheme (e.g., an iterative best response algorithm) is developed that efficiently solves the resulting robust financial optimization problems, which can often be non-convex.4 This advancement offers significant computational advantages over existing methods, making the proposed robust financial framework practical for real-time implementation in complex financial systems. This contribution addresses the practical challenge of solving robust optimization problems arising from ambiguity in financial markets.
* **Empirical Demonstration of Enhanced Robustness and Performance in Financial Markets:** Through extensive numerical simulations and real-world case studies using financial data (e.g., historical stock prices, market indices), the practical efficacy and superior performance of the proposed approach are demonstrated. The results highlight its ability to significantly enhance the robustness of financial decisions against realistic distributional uncertainties, outperforming traditional financial models that fail to account for ambiguity and contribute to financial instability.9 This validates the theoretical underpinnings and showcases the practical benefits of the KL-based approach for managing financial risk, particularly in its capacity to  
  **improve investor returns while maintaining a consistent risk profile** by effectively navigating ambiguity.

These contributions are significant to the academic community as they fill identified gaps in the literature concerning robust financial decision-making under distributional ambiguity.4 By providing a method-based contribution through the novel framework and algorithm, a theory-based contribution through rigorous proofs, and findings-based contributions through empirical validation, this paper advances the state of the art in decision-making under deep uncertainty in financial contexts. The work offers important insights into designing more resilient and reliable financial systems, with potential implications for risk management, portfolio optimization, and regulatory frameworks.

The contributions presented in this paper are direct solutions to the identified gaps and limitations discussed earlier. The problem statement in Section 1.3 highlighted the inadequacies of traditional financial methods in handling distributional ambiguity and underscored the need for sophisticated DRO frameworks. The preceding discussion on the strengths of KL-based ambiguity sets in Section 1.4 then justified *why* this specific measure is particularly well-suited for such problems, citing its connections to MLE, risk measures, and favorable mathematical properties. Consequently, the contributions are precisely tailored to address these challenges and leverage these strengths. For instance, the development of an efficient computational algorithm directly tackles the potential complexity of solving robust financial optimization problems, while the rigorous proof of optimal policy structure reinforces the analytical benefits of using KL divergence. This structured approach ensures that the paper is perceived as a well-justified and impactful piece of research, where each component logically builds upon the preceding arguments to deliver a cohesive and necessary advancement in the field of financial economics.

### **1.6. Paper Structure**

The remainder of this paper is organized as follows.

Section 2 provides a comprehensive review of the theoretical foundations of Kullback-Leibler divergence, detailing its properties, interpretations as a measure of information loss and uncertainty, and its specific relevance to quantifying ambiguity in economic and financial contexts. This section also contextualizes KL divergence within the broader landscape of information-theoretic measures and statistical distances.

Section 3 formulates the robust financial optimization problem with KL-based ambiguity sets, explicitly defining the worst-case optimization problem for financial decision-making. This section then rigorously derives the optimal financial policy, demonstrating how the unique properties of KL divergence facilitate the preservation of desirable policy structures, such as linearity, for tractability.

Section 4 details the proposed computational algorithm (e.g., an iterative best response algorithm) for efficiently solving the formulated robust financial optimization problem. This section discusses the algorithmic steps, convergence properties, and computational complexity, highlighting its efficiency compared to alternative solution methods for financial applications.

Section 5 presents extensive numerical experiments and discusses the performance benefits of the proposed approach. This includes comparative studies against traditional financial models and other robust optimization techniques, demonstrating the enhanced robustness and resilience achieved by leveraging KL-based ambiguity sets in various financial scenarios, such as managing stock price uncertainty or optimizing portfolios under stress.

Finally, Section 6 concludes the paper with a summary of key findings and outlines promising directions for future research, including potential extensions to non-linear financial models, adaptive ambiguity set learning from market data, and broader applications in diverse economic and financial decision-making contexts.

This structured roadmap serves to reinforce the paper's contributions and its overall coherence. By briefly describing the purpose of each subsequent section, the roadmap subtly highlights how each part of the paper builds upon the foundational concepts and problem statement to achieve the stated objectives. This guides the reader through a logical progression of ideas, demonstrating that the paper is a cohesive unit delivering on its promises and providing a structured journey through complex material. This approach enhances readability and persuades the reader of the paper's rigor and completeness.

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